Extended H2 - H∞ controller synthesis for linear time invariant descriptor systems

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PÔLE DE RECHERCHE ET D'ENSEIGNEMENT SUPÉRIEUR

ECOLE DES MINES DE NANTES

THESE

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ECOLE DOCTORALE : Sciences et Technologies de l'Information et Mathématiques
THESE N° 2011 EMNA 0006

Thèse présentée en vue de l'obtention du grade de
Docteur de l'Ecole des Mines
Sous le label de l'Université Nantes Angers Le Mans
Discipline Automatique et Informatique Appliquée

Soutenue le 13 décembre 2011

Commande H_2-H_∞ non standard des systèmes implicites

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Notations

$\mathbb{C}$	filed of complex numbers
$\mathbb{R}$	filed of real numbers
$\mathbb{R}^n$	space of n-dimensional real vectors
$\mathbb{R}^{n \times m}$	space of $n \times m$ real matrices
$\in$	'belongs to'
$\times$	inner product
$\oplus$	sum of vector spaces
$\otimes$	Kronecker product
$\text{A} \iff \text{B}$	statements A and B are equivalent
$\text{A} \implies \text{B}$	statement A implies statement B
$\mathcal{L}[\cdot]$	Laplace transform of an argument
$\mathcal{F}_l(\cdot, \cdot)$	lower linear fractional transformation
$\text{rank}(\cdot)$	rank of a matrix
$\det(\cdot)$	determinant of a matrix
$\deg(\cdot)$	degree of a polynomial
$\text{Re}(\cdot)$	real part of a complex number
$\lambda_{min}(\cdot)$	minimum eigenvalue of a real matrix
$\lambda_{max}(\cdot)$	maximum eigenvalue of a real matrix
$\alpha(\cdot, \cdot)$	generalized spectral abscissa of a matrix
$\alpha(\cdot)$	spectral abscissa of a matrix
$\rho(\cdot, \cdot)$	generalized spectral radius of a matrix
$\rho(\cdot)$	spectral radius of a matrix
$\sigma_{\max}(\cdot)$	maximum singular value of a matrix
$\text{vec}(\cdot)$	ordered stack of the columns of a matrix from left to right starting with the first column

I_n	identity matrix of the size $n \times n$
$0_{n\times m}$	zero matrix of the size $n \times m$
$X^\top$	transpose of matrix X
X^{-1}	inverse of matrix X
X^*	conjugate transpose of matrix X
$X^\dagger$	Moore-Penrose inverse of matrix X
$\mathrm{Im}(X)$	range space of matrix X
$\mathrm{Ker}(X)$	Kernel (null) space of matrix X
$\mathrm{diag}(X_1, \ldots, X_m)$	block diagonal matrix with blocks X_1, ..., X_m
$X \geq (>) 0$	X is real symmetric positive semi-definite (positive definite)
$\mathbf{He}\{X\}$	$X^\top + X$
$\Gamma(X, Y)$	$X^\top Y + Y^\top X$
$\bullet$	offdiagonal blocks of a symmetric matrix represented blockwise, e.g. $\begin{bmatrix} X_{11} & X_{12} \\ X_{12}^\top & X_{22} \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} \\ \bullet & X_{22} \end{bmatrix} = \begin{bmatrix} X_{11} & \bullet \\ X_{12}^\top & X_{22} \end{bmatrix}$
L_2	space of square integrable functions on $[0, \infty)$
RH_∞	set of all rational proper stable transfer matrices
RH_2	set of strictly proper and real rational stable transfer matrices
$\left\{E, \left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]\right\}$	descriptor system associated with system data (E, A, B, C, D)
$\\|\cdot\\|_2$	L_2 norm
$\\|G\\|_\infty$	H_∞ norm of transfer function G

ARE	algebraic Riccati equation
BMI	bilinear matrix inequality
DAE	differential algebraic equation
ESPR	extended strictly positive real
GARE	generalized algebraic Riccati equation
GEP	generalized eigenvalue problem
LME	linear matrix equality
LMI	linear matrix inequality
LPV	linear parameter varying
LTI	linear time invariant
MIMO	multi-input multi-output
PID	proportional-integral-derivative
SISO	single-input single-output
SVD	singular value decomposition

Acknowledgements

The present dissertation is the outcome of three years of research carried out at the École des Mines de Nantes. I would like to thank, above all, my supervisor, Prof. Philippe Chevrel, for his valuable ideas and suggestions throughout this academic experience. He was also the advisor for both my M.E. and M.S. research. I shall never forget his support and help during the whole 6 years of my study in France.

I would also like to thank my co-advisor, Dr. Mohamed Yagoubi, for guiding me in my research in such an excellent way and always taking time to answer my questions. It has been a great pleasure to work with him and I am truly grateful for his outstanding support and encouragement during the work of this dissertation.

Moreover, my thanks go to Prof. Germain Garcia, from INSA of Toulouse, Dr. Olivier Bachelier, from University of Poitiers and Dr. Yoshio Ebihara, from Kyoto University, for reviewing this dissertation and providing valuable and pertinent suggestions. Especially, Dr. Olivier Bachelier, together with Dr. Guy Lebret, from IRCCyN, has been in the annual evaluation committee, since the very beginning of my Ph.D. study. My thanks also go to Prof. Michel Malabre, from IRCCyN, and Prof. Gilles Duc, from Supélec of Paris, for accepting being members of the Ph.D. viva committee.

I would like to express my sincere gratitude to everyone at the Département d'Automatique-Productique of the École des Mines de Nantes for providing a pleasant working atmosphere and also to everyone met in the Institut de Recherche en Communications et Cybernétique de Nantes for enjoyable conversations. I would especially like to mention Anita Niebroj-Kaelbel, Isabelle Laine and Emily Thureau, our friendly secretaries, for their kind help in handling administration procedures.

Furthermore, this work has been supported by the "Allocations de Recherche MESR" which is gratefully acknowledged.

My special thanks are for my parents, Meiying Xu and Jinlin Feng, who have been giving me unconditional supports and standing by me at every stage in my life. I would not be here without them.

Although a long-term graduate study in abroad causes to lose many friendships in my home country, I would like to thank Lele Yang, Jun Xu, Zhenyu Xu, Zhe Wang, Yue Ying, Sha Yang, Huan Ni and Ying Zeng (it is difficult to mention all the names) for everything that they did in spite of the distance.

Finally, my “non-academic” friends made during my overseas life also deserve my gratitude for spending their time with me.

Yu Feng
Nantes, December 2011

Abstract

Descriptor systems constitute an important class of systems of both theoretical and practical interests and have been attracting the attention of many researchers over recent decades. This dissertation is concerned with non-standard H_2 and H_∞ control for linear time-invariant descriptor systems. Within the descriptor framework, the contributions of this dissertation are threefold: i) review of existing results for state-space systems by the use of the descriptor representation, ii) generalizations of some classical results to descriptor systems, iii) exact and analytical solutions to nonstandard control problems with unstable and nonproper weighting functions or subject to regulation constraints.

The first part of this dissertation is concerned with a development of useful tools for analysis and control synthesis for descriptor systems. A realization independent Kalman-Yakubovich-Popov lemma is deduced in terms of strict linear matrix inequalities (LMIs) for descriptor systems. This new condition removes the equality constraints found in the reported results and outperforms the existing methods in the viewpoint of numerical computation. The issue of dilated LMI characterizations, which have been extensively studied for conventional state-space systems, is also investigated in this dissertation. Based on reciprocal application of the projection lemma, dilated LMI conditions with regard to admissibility, H_2 and dissipative properties are derived for both continuous-time and discrete-time settings. The proposed formulations review the existing results reported in the literature, and also complete some missing conditions. The known dilated LMIs for state-space systems can be regarded as special cases of the proposed results.

As known, the Sylvester equations and Riccati equations play an important role in control theory, and some control issues are directly concerned with these equations. This dissertation deals with these two topics for descriptor systems as well. The solvability of a generalized Sylvester equation is transformed into a linear programming problem which can be solved efficiently using available techniques. Moreover, a generalized algebraic Riccati equation (GARE) is considered and a numerical algorithm relying on a generalized eigenvalue problem (GEP) is given for solving it.

Moreover, the strong H_∞ stabilization and simultaneous H_∞ control problems for continuous-time descriptor systems are considered. As a generalization of the existing

results to descriptor systems, it states that the simultaneous H_∞ control problem for a set of descriptor systems is achieved if and only if the strong H_∞ stabilization problem of a corresponding augmented system is solvable. Then, a sufficient condition for the existence of an observer-based controller solving the strong H_∞ stabilization problem is proposed. The proposed result is based on a combination of a GARE and a set of LMIs and outperforms some reported methods in the literature.

Another subject discussed in this dissertation is the extended control problem where unstable and nonproper weighting functions are used to present design specifications. Due to the weights involved, the overall weighted plant is neither wholly stabilizable nor detectable and the two crossed systems induced from the overall weighted plant have zeros on the imaginary axis including infinity which leads to a singular problem. The standard procedures fail to give a solution in such a case. In order to solve such a nonstandard problem, the concept of extended (comprehensive) admissibility is used, within which only the internal stability of a part of the underlying closed-loop system is sought. It is proved that this requirement is achieved if and only if some generalized Sylvester equations associated with the weighted plant admit solutions. The class of all controllers achieving extended admissibility is then parameterized by the Youla-Kučera parametrization.

Based on this result, H_2 and H_∞ performance control problems are further integrated. Similarly, the so-called quasi-admissible solution is adapted instead of the normal admissible solution for GAREs. It shows that a quasi-admissible solution to the underlying GARE consists of an admissible solution to a reduced GARE and the solution to the underlying generalized Sylvester equation. The resulting controllers are constructed in terms of quasi-admissible solutions and a parametrization of all controllers solving the extended control problem is also provided.

The last topic addressed in this dissertation is the problem of H_2 and H_∞ control with output regulation constraints. In this problem, an output is to be regulated asymptotically in the presence of an infinite-energy exo-system, while a desired performance by the H_2 or H_∞ norm from a finite external disturbance to a tracking error must also be satisfied. As pointed out, this problem can be viewed as a modified extended control problem.

Like the extended control problem, a generalized Sylvester equation is deduced for the objective of asymptotical regulation. A specific structure of the resulting controllers is derived and this coincides with the internal model principle for the conventional state-space systems. Relying on this structure, the state feedback H_2 optimal control and H_∞ output feedback control problems are considered under regulation constraints. The solution of state feedback H_2 optimal control is given by solving a GARE, and as mentioned in the literature, the optimum is not unique and a parametrization of all optimal state feedback controllers is established. Concerning the H_∞ output feedback control issue, an LMI-based approach is given, which is well tractable.

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Chapter 1

Synthèse des travaux de thèse

1.1 Introduction

Ce mémoire présente une synthèse des travaux de thèse que j'ai effectués à l'Institut de Recherche en Communications et en Cybernétique de Nantes (IRCCyN).

Ces travaux ont porté sur la commande optimale des systèmes implicites. Rosenbrock fut parmi les premiers à introduire les modèles implicites, qui permettent de décrire une très large gamme de comportements externes. Leur étude a connu un regain d'intérêt ces dernières années, suite à la prise de conscience de l'importance qu'ils peuvent revêtir, même en dehors du cas strictement singulier (nécessairement implicite).

Des réalisations implicites peuvent ainsi être utilisées pour modéliser macroscopiquement l'implémentation d'une loi de commande, de manière plus rigoureuse que par le passé, permettant de généraliser nombre de travaux antérieurs de Gevers et al. sur ce thème, les variables de description pouvant contenir, en sus des variables d'état, des variables intermédiaires de calcul, permettant un choix de paramétrisation élargi [HCW07, HCW10].

L'étude des systèmes rationnels en les paramètres peut également être ramenée à l'étude de systèmes implicites affines en introduisant des variables de description additionnelles. Cette propriété peut être utilisée pour résoudre le problème d'analyse ou de commande (H_∞) des systèmes LPV [BYC08].

L'utilisation de réalisations implicites enfin, est bien adaptée à la description de modèles physiques : elle permet de préserver une paramétrisation explicite en les paramètres physiques et en tant que telle, doit s'affirmer comme le support privilégié de la simplification de modèles physiques, ou de l'identification.

1.1.1 Equations algébro-différentielles

Un système dynamique est souvent modélisé par un ensemble d'équations différentielles ordinaires (ODEs en anglais), se ramenant à une description sous forme d'état (système d'équations différentielles matricielles du 1er ordre) [BCP96] définie par :

$$\boldsymbol{F}\left(\dot{x}(t), x(t)\right) = 0, \tag{1.1}$$

où $\boldsymbol{F}$ et x sont des fonctions vectorielles. La forme ci-dessus comporte non seulement des équations différentielles, mais aussi potentiellement une classe de contraintes algébriques, on parle alors d'équations algébro-différentielles.

Dans le domaine automatique, on suppose le plus souvent les ODE considérées sous la forme explicite (d'état) ci-dessous

$$\dot{x}(t) = \boldsymbol{f}\left(x(t)\right), \tag{1.2}$$

où $\boldsymbol{f}$ est une fonction vectorielle. C'est sur cette classe d'ODE qu'ont eu lieu la plupart des travaux portant sur l'analyse et la conception de lois de commande.

Une telle représentation d'état peut être obtenue sous la condition que le processus considéré soit régi par le principe de causalité. Il est possible cependant, dans certain cas, que l'état passé dépende de l'état futur, auquel cas l'hypothèse de causalité n'est plus respectée. Il existe par ailleurs des situations pratiques où

i) Certaines variables physiques ne peuvent pas être choisies comme les variables d'état de la représentation (1.2),

ii) les sens physique de certaines variables ou coefficients sont perdus après une transformation sous la forme (1.2).

Illustrons la limite de l'utilisation de la représentation d'état par un exemple concret. Soit un processus économique dans lequel n secteurs de production interdépendants sont parties prenantes. Les relations entre les niveaux de production des différents secteurs peuvent être décrites par le modèle de Leontieff [Lue77b] :

$$x(k) = Ax(k) + Ex(k+1) - Ex(k) + u(k), \tag{1.3}$$

où $x(k) \in \mathbb{R}^n$ est le vecteur du niveau de production des différents secteurs à l'instant k. $Ax(k)$ représente le capital requis comme entrée directe pour la production de x. La matrice A est appelée matrice de coefficients de flux dont chaque coefficient a_{ij} indique le volume de produit i demandé en vue de fabriquer un volume unitaire du produit j. $Ex(k+1) - Ex(k)$ désigne le capital stocké en vue de la production de x à la prochaine période. La matrice E est appelée matrice de stockage dont e_{ij} indique le volume du produit i qui doit être stocké afin de fabriquer un volume unitaire du produit j à la prochaine période. Par ailleurs, le vecteur $u(k)$ est une consigne de demande de production. Ce modèle (1.3) a été étudié par Leontieff en cas continu et discret [Leo53].

La matrice E est normalement très creuse, et ses éléments sont majoritairement nuls, conduisant à une matrice souvent singulière. On observe que l'équation (2.3) peut être transformée comme suit :

$$Ex(k+1) = (I - A + E)x(k) - u(k), \tag{1.4}$$

Si la matrice E est inversible, la multiplication à gauche de l'égalité (1.4) par la matrice E^{-1} permet de se ramener à une représentation d'état. Lorsque E est singulière cependant, cette transformation ne peut être réalisée. On montre que la transformation de l'équation (1.4) sous forme d'état dépend des propriétés de la paire $(E, I - A + E)$. Ce sujet n'est pas détaillé dans cette synthèse, mais discuté dans le mémoire de thèse (c.f. chapitre 3). D'autres exemples de systèmes implicites y sont également introduits.

1.1.2 Systèmes implicites

On peut décomposer la DAE (1.1) en deux parties :

$$\dot{x}(t) = \phi\left(x(t)\right), \tag{1.5a}$$

$$0 = \varphi\left(x(t)\right), \tag{1.5b}$$

où ϕ et φ sont deux fonctions vectorielles. Comparée avec la forme (1.2), la DAE ci-dessus associe aux équations différentielles des contraintes algébriques qui ne peuvent trouver place en (1.2).

Pour un système linéaire et invariant dans le temps, la deuxième condition de (1.5) est susceptible de conférer au système des propriétés atypiques, telles que l'impulsivité. On désigne dans la littérature le système (1.5) de différentes façons par : système implicite, système d'état généralisé (ou étendu), système descripteur, système algébro-différentiel ou système semi-état. Dans ce mémoire, la terminologie "système implicite" est adoptée, et l'étude limitée au cas linéaire et invariant dans le temps.

Considérons donc désormais le système implicite :

$$E\dot{x} = Ax + Bu, \tag{1.6a}$$

$$y = Cx, \tag{1.6b}$$

(1.6) où $x \in \mathbb{R}^n$ est le vecteur des variables "descripteurs", $u \in \mathbb{R}^m$ est le vecteur des entrées de commandes et $y \in \mathbb{R}^p$ le vecteur de sorties mesurées. Les matrices E, A, B et C sont à coefficients réels et dimensions compatibles avec celles de x, u et y. On peut, de plus, considérer sans perte de généralité, que E et A sont des matrices carrées, quitte à compléter par des lignes nulles jusqu'à obtention de matrices de dimensions $n \times n$. Par ailleurs, la matrice E n'est pas nécessairement de plein rang, auquel cas $rang(E) = r \leq n$. Le fait de ne pas considérer de terme de transfert direct entre y et u dans la deuxième équation n'est pas restrictif. Il suffit en effet, partant de (1.7), d'augmenter le vecteur d'état comme ci-dessous, en incluant la commande u dans le nouveau vecteur de description afin d'annuler la matrice D.

$$E\dot{x} = Ax + Bu, \tag{1.7a}$$
$$y = Cx + Du, \tag{1.7b}$$

peut se mettre sous la forme (1.6) en écrivant

$$\underbrace{\begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}}_{\mathcal{E}} \begin{bmatrix} \dot{x} \\ \dot{\zeta} \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ 0 & -I \end{bmatrix}}_{\mathcal{A}} \begin{bmatrix} x \\ \zeta \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ I \end{bmatrix}}_{\mathcal{B}} u, \tag{1.8a}$$
$$y = \underbrace{\begin{bmatrix} C & D \end{bmatrix}}_{\mathcal{C}} \begin{bmatrix} x \\ \zeta \end{bmatrix}. \tag{1.8b}$$

Les systèmes implicites constituent un puissant outil de modélisation dans la mesure où ils peuvent décrire des processus régis à la fois par des équations dynamiques et des équations statiques. Ce type de systèmes apparaît dans différentes situations. Citons par exemple certains systèmes interconnectés de grandes dimensions [Lue77a, SL73], circuits électriques [ND89], systèmes de puissance [Sto79], systèmes économiques[Lue77a, Lue77b], processus chimiques [KD95], systèmes mécaniques [HW79], robots [MG89], ou encore modèle d'avions [SL91]. L'étude des systèmes implicites a motivé de nombreuses recherches depuis le début des années 1970. En effet, le livre [Dai89] et l'état de l'art présenté dans [Lew86], ainsi que les articles inclus, sont des références de choix dans ce domaine.

Un système implicite, nous l'avons dit, possède des spécificités importantes vis-à-vis d'un système d'état [YS81, VLK81, BL87] :

- la fonction de transfert d'un système implicite, lorsqu'elle existe, peut être impropre (strictement);
- pour une condition initiale arbitraire, la réponse temporelle d'un système implicite peut être impulsive (cas continu) ou acausale (cas discret);
- un système implicite comporte trois types de modes : les modes dynamiques finis, les modes infinis (sortie à caractère impulsionnel) et les modes statiques;
- même si un système implicite est non impulsif, sa sortie peut présenter des discontinuités finies à cause de conditions initiales incohérentes.

Notons que même lorsque E est inversible, permettant *a priori* de se ramener au formalisme d'état classique, on peut craindre des erreurs numériques importantes en cas de mauvais conditionnement de la matrice E, et préférer ainsi le formalisme (1.6).

Les travaux exposés dans le présent mémoire ont pour objet la commande optimale non standard de systèmes implicites. Nous étudierons pour commencer certains sujets classiques et les étendrons au cadre implicite. Les caractérisations de la dissipativité, les caractérisations de la stabilité et des performances à base d'inégalités linéaires matricielles (LMI) étendues, les équations de Sylvester et de Riccati seront revisitées, et leurs solutions étendues au cas implicite.

Nous aborderons dans un deuxième temps, le problème de stabilisation simultanée, avec ou sans objectif de performance H_∞. La solution proposée s'appuie sur la combinaison d'une solution d'une équation algébrique de Riccati généralisée (GARE) et la faisabilité d'une LMI stricte.

Nous traiterons enfin les problèmes H_2 et H_∞ non standard, en présence de pondérations instables voire impropres. Le problème multiobjectif de minimisation de performance H_2 ou H_∞ sous contraintes de régulation sera également généralisé au cas implicite.

1.1.3 Organisation du mémoire

Le mémoire de thèse est organisé comme indiqué ci-dessous; les résultats clés y sont soulignés.

Le deuxième chapitre motive l'étude des systèmes implicites et balaie les développements et notation de base relatifs à ce type de systèmes.

Le troisième chapitre donne une introduction basique à l'étude des systèmes implicites linéaires, rappelant quelques définitions et résultats fondamentaux tels que la régularité, l'admissibilité, les relations d'équivalence, la décomposition de systèmes, l'expression et le calcul de la réponse temporelle, les propriétés de commandabilité, d'observabilité et de dualité.

Le quatrième chapitre énonce certains résultats utiles concernant la caractérisation de la dissipativité, les LMI étendues, l'équation de Sylvester généralisée et l'équation de Riccati étendue (GARE), ceci dans le cadre des systèmes implicites. Une nouvelle condition caractérisant la propriété de dissipativité est donnée au travers d'une LMI stricte, s'affranchissant des contraintes d'égalités habituellement présentes dans la littérature. Des conditions LMI étendues appliquées aux systèmes implicites sont également obtenues, l'utilisation inverse du Lemme de projection permettant de retrouver les résultats existants en complétant certaines conditions LMI manquantes. Par ailleurs, la résolution d'une équation de Sylvester généralisée et de la GARE associée à la représentation implicite est formulée de manière à permettre la mise en œuvre d'algorithmes numériques stables et efficaces. Les résultats de ce chapitre jouent un rôle important dans la thèse. Ils supporteront par la suite la caractérisation des performances en terme de dissipativité et la commande H_∞ sous contrainte de régulation. Les algorithmes de résolution de l'équation de Sylvester généralisée et la GARE seront utilisés dans le cadre des problèmes de commande H_2 ou H_∞ étendue, dans le chapitre 6.

Le problème de la conception de lois de commande H_∞ simultanée est au cœur du cinquième chapitre. Nous y étendons la résolution du problème de stabilisation simultanée sous contrainte de performance H_∞ au cas implicite. Nous montrons, dans le cas de la commande H_∞ simultanée de deux systèmes en utilisant l'approche par factorisation co-première des systèmes considérés, que ce problème peut être résolu si et seulement si un problème de commande H_∞ sur un système augmenté relié aux systèmes originaux admet une solution sous contrainte d'admissibilité forte. Une condition suffisante est ensuite établie, exprimée au travers d'une GARE et d'un ensemble de LMI ; le régulateur résultant admet une forme retour d'état /observateur. La généralisation au cas de n systèmes est ainsi présentée.

Dans le sixième chapitre, nous étudions le problème de la commande étendue des systèmes implicites à temps continu. L'adjectif "étendu" indique ici que le régulateur doit rendre admissible de manière interne une partie seulement de la boucle fermée, laissant la possibilité d'occultation de pôles et de zéros instables ou à l'infini des pondérations, elles-mêmes formulées sous forme implicite. L'utilisation de pondérations instables voire impropres est autorisée en ce cas. Le problème d'admissibilisation étendu est résolu en premier lieu. Une condition nécessaire et suffisante d'existence d'une solution est donnée au travers du caractère résoluble ou non de deux équations de Sylvester généralisées. Ces deux équations se ramènent aux équations d'occultation déjà présentes dans la littérature (pour le problème de régulation dans [FW75] et dans [Che02] pour le dual). Une paramétrisation de l'ensemble des régulateurs garantissant l'admissibilité étendue est également donnée. S'appuyant sur ce résultat, les commandes H_2 et H_∞ sous contrainte d'admissibilité étendue sont considérées. En ce cas, pour les GARE en question, une nouvelle définition nommée "solution quasi-admissible" est adoptée. Grâce à cette relaxation, une solution exacte est analytiquement établie pour le problème de commande étendue. De plus, l'ensemble des régulateurs H_2 ou H_∞ étendus seront également paramétrés.

Le chapitre 7 aborde les problèmes de commandes H_2 et H_∞ sous contrainte de régulation. Ces problèmes formalisent la recherche de régulateurs assurant en boucle fermée : i) la régulation asymptotique d'une sortie donnée en dépit de signaux exogènes à énergie non bornée modélisés par un exo-système *ad hoc* et ii) une performance H_2 ou H_∞ donnée entre une perturbation externe et l'erreur de sortie. Nous prouvons que l'objectif de régulation asymptotique peut être atteint sous réserve de résolubilité d'une équation de Sylvester généralisée associée au systéme augmenté de l'exo-système. Nous explicitons également la structure de régulateurs satisfaisant la condition de régulation asymptotique. En s'appuyant sur cette structure, nous réduisons ce problème non standard en un problème standard sur un systéme auxiliaire dont la solution est caractérisée par une GARE ou un ensemble de LMI.

La conclusion générale et les perspectives se trouvent dans le dernier chapitre, où les contributions de cette thèse sont résumées et les sujets de recherche pour la suite discutés.

1.1.4 Publications

Les résultats principaux de cette thèse ont été développés en coopération avec le Professeur Philippe CHEVREL et le Docteur Mohamed YAGOUBI. On trouvera ci-dessous la liste des publications relatives aux travaux exposés

- Articles de revues

  1. **Y. Feng**, M. Yagoubi and P. Chevrel. H_∞ control under regulation constraints for descriptor systems. In preparation.
  2. **Y. Feng**, M. Yagoubi and P. Chevrel. H_∞ control with unstable and non-proper weights for descriptor systems. *Automatica*. Submitted.
  3. **Y. Feng**, M. Yagoubi and P. Chevrel. Extended H_2 controller synthesis for continuous descriptor systems. *IEEE Transactions on Automatic Control*. Accepted.
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3. **Y. Feng**, M. Yagoubi and P. Chevrel. On dissipativity of continuous-time singular systems. In: *Proceedings of the 18th Mediterranean Conference on Control & Automation*, Marrakesh, Morocco, June 2010, pp. 839-844.

Un rappel non exhaustif des propriétés et des définitions associées aux systèmes implicites est donné dans le mémoire de thèse (chapitre 3). Nous estimons qu'il n'est pas nécessaire ici d'en faire un récapitulatif dans cette synthèse.

1.2 Outils précieux pour les systèmes implicites

Cette partie de thèse est consacrée à certains résultats développés pour des systèmes implicites. Quatre thèmes différents sont explorés ici, à savoir la caractérisation de performance dissipative, les formes LMI étendues, l'équation de Sylvester généralisée et la GARE. Le présent chapitre développe des outils qui serviront à l'analyse et à la synthèse des différents problèmes traités dans cette thèse. Nous fournirons dans ce qui suit les résultats principaux, et les détails se trouvent dans le mémoire de thèse.

1.2.1 Performance Dissipative

La notion de "dissipativité" est un aspect important dans le domaine des systèmes et de la commande, à la fois pour des raisons théoriques et des considérations pratiques. Généralement parlant, un système dissipatif est caractérisé par la propriété qu'en tout moment la quantité d'énergie que le système peut fournir à son environnement ne peut pas dépasser la quantité d'énergie qui lui a été fournie. Autrement dit, un système dissipatif peut absorber une partie des énergies de son environnement, et il transforme ces énergies sous différentes formes, par exemple, la chaleur, la radiation électromagnétique, etc.

Soit un système continu à temps invariant, dynamique donné ci-dessous :

$$\dot{x} = f(x, w), \tag{1.9a}$$

$$z = g(x, w), \tag{1.9b}$$

avec $x(0) = x_0$. x, w et z sont respectivement l'état prenant sa valeur dans un espace d'état $\mathcal{X}$, l'entrée prenant sa valeur dans un espace d'entrée $\mathcal{W}$, et la sortie prenant sa valeur dans un espace de sortie $\mathcal{Z}$. Et soient $f : \mathcal{X} \times \mathcal{W} \to \mathcal{X}$ et $g : \mathcal{X} \times \mathcal{W} \to \mathcal{Z}$. Introduisons la fonction $s(w(t), z(t))$ qui caractérise le flux d'énergie à travers le système, définie ci-dessous :

$$s(w, z) = \begin{bmatrix} w \\ z \end{bmatrix}^\top S \begin{bmatrix} w \\ z \end{bmatrix}, \tag{1.10}$$

où $S = \begin{bmatrix} S_1 & S_2 \\ \bullet & S_3 \end{bmatrix}$ est une matrice symétrique de dimension compatible avec les dimensions de w et z.

Définition 1.2.1 (Dissipativité) *Le système dynamique* (1.9) *est dit dissipatif vis-à-vis de la fonction* $s(\cdot,\cdot)$*, s'il existe une fonction non-négative, dite fonction de stockage,* $\mathcal{V} : \mathcal{X} \to \mathbb{R}$*, pour tout temps* $t_0 \leq t_1$ *et* $w \in \mathcal{L}_2[t_0\ t_1]$ *telle que l'inégalité ci-dessous est satisfaite :*

$$\mathcal{V}(x(t_1)) - \mathcal{V}(x(t_0)) \leq \int_{t_0}^{t_1} s(w(t), z(t))\, dt. \tag{1.11}$$

La notion de "dissipativité stricte" peut être définie par modification simple de la définition ci-dessus.

Définition 1.2.2 (Dissipativité stricte) *Le système dynamique* (1.9) *est dit strictement dissipatif vis-à-vis de la fonction* $s(\cdot,\cdot)$*, s'il existe une fonction non-négative, dite fonction de stockage,* $\mathcal{V} : \mathcal{X} \to \mathbb{R}$ *et un scalaire* $\epsilon > 0$*, pour tout temps* $t_0 \leq t_1$ *et* $w \in \mathcal{L}_2[t_0\ t_1]$ *telle que l'inégalité ci-dessous est satisfaite :*

$$\mathcal{V}(x(t_1)) - \mathcal{V}(x(t_0)) \leq \int_{t_0}^{t_1} s(w(t), z(t))\, dt - \epsilon^2 \int_{t_0}^{t_1} \|w(t)\|^2 dt. \tag{1.12}$$

Il est connu que de nombreux problèmes d'analyse et de commande peuvent être formulés via la propriété de dissipativité associée à une fonction $s(w,z)$ quadratique, par exemple, la réelle positivité, le lemme réel borné et le critère du cercle.

Une des formulations importantes caractérisant la propriété d'un système dissipatif est le lemme de Kalman-Yakubovich-Popov (KYP), qui souligne la relation entre la performance dissipative et la propriété fréquentielle. Ce lemme était proposé dans [Kal63, Yak63, Pop64], et en suite généralisé au cas multivariable par [And67, AV73] pour des systèmes explicites continus.

La propriété de dissipativité ou ses réalisations concrètes pour les systèmes d'état usuels ont été largement étudiées dans la littérature [AV73, GG97, HB91, HIS99, Ran96, SKS94]. Ces problèmes ont aussi été étendus au cas implicite [FJ04, WC96, ZLX02, MKOS97, TMK94, WYC98]. Cependant, la majorité des résultats développés demande certaines conditions sur la réalisation des systèmes implicites, en plus des hypothèses de régularité et de commandabilité. Par exemple, en considérant le système donné par (1.7), les critères donnés dans [WC96, ZLX02] requièrent $D^\top + D > 0$, alors que la condition $D = 0$ est supposée pour le lemme réel borné dans [WYC98]. Afin de retirer ces restrictions, les auteurs ont depuis peu de temps proposé une caractérisation LMI indépendante de la réalisation du système sous contraintes LMI pour la performance dissipative dans le cas des systèmes implicites [Mas06, Mas07, CT08].

Motivé par les résultats de [Mas06, Mas07] qui sont formalisés sous forme LMI non-strictes, nous introduisons un nouveau lemme KYP pour évaluer la propriété dissipative des systèmes implicites au cas continu. Ce formalisme est caractérisé par des LMI strictes, numériquement fiables et faciles à résoudre par les solveurs classiques.

Nous présentons ci-dessous ce nouveau lemme KYP. La preuve de ce résultat et une application concernant la commande du type retour d'état peuvent être trouvées dans le mémoire de thèse.

Lemme 1.2.1 (Lemme de KYP) *[FYC10b] Soit un système implicite donné par* (1.7) *et la matrice M définie par*

$$M = \begin{bmatrix} 0 & I \\ C & D \end{bmatrix}^\top S \begin{bmatrix} 0 & I \\ C & D \end{bmatrix}, \tag{1.13}$$

avec $S_3 \geq 0$. Alors, les deux conditions ci-dessous sont équivalentes.

i. *Le système* (1.7) *est admissible et strictement dissipatif;*

ii. *Il existe les matrices $P = P^\top \in \mathbb{R}^{n\times n} > 0$, $Q \in \mathbb{R}^{(n-r)\times n}$ et $R \in \mathbb{R}^{(n-r)\times m}$ telles que*

$$M + \begin{bmatrix} (PE+UQ)^\top \\ R^\top U^\top \end{bmatrix} \begin{bmatrix} A & B \end{bmatrix} + \begin{bmatrix} A^\top \\ B^\top \end{bmatrix} \begin{bmatrix} PE+UQ & UR \end{bmatrix} < 0, \tag{1.14}$$

où $U \in \mathbb{R}^{n\times(n-r)}$ est une matrice arbitraire de rang plein par colonne et satisfaisant $E^\top U = 0$.

1.2.2 Inégalités linéaires matricielles étendues

Les techniques d'analyse et de synthèse de lois de commande basées sur la formulation LMI [IS94, GA94, Sch92, CG96] ont connu un essor important grâce à leur efficacité inspirée de l'utilisation d'algorithmes d'optimisation convexes et au soutien numérique très puissant des boîtes à outils disponibles [GNLC95]. Ces techniques ont aussi permis la simplification d'hypothèses nécessaires dans le cadre de l'utilisation d'équations de Riccati. Elles ont aussi permis l'accès à des solutions numériques d'une grande classe de problèmes d'analyse et de commande. La stabilité, le placement de pôles, la commande H_2 ou H_∞, la synthèse multicritère et la commande LPV peuvent ainsi être interprétés et reformulés sous forme de problèmes de faisabilité ou d'optimisation sous contraintes LMI (voir les références suivantes [BGFB94, SGC97, MOS98], à titre d'exemple).

Néanmoins, un certain conservatisme des techniques LMI classiques apparait lors du traitement de certains problèmes d'analyse ou de commande "complexes". Par exemple, en utilisant les LMI standards pour résoudre un problème de commande multicritères, une matrice de Lyapunov commune peut être envisagée en vue de rendre le problème de synthèse convexe. Il est évident que cette démarche induit un conservatisme dans la méthode de conception de la loi de commande. Pour réduire ce conservatisme, une nouvelle caractérisation dite LMI étendue (ou dilatée, généralisée) a été introduite par [GdOH98] pour des systèmes d'état continus. Désormais, de nombreuses études ont été lancées afin d'explorer les apports de ces nouvelles caractérisations LMI, et des résultats constructifs concernant l'analyse et la synthèse de lois de commande ont été traités dans une littérature abondante sur le sujet [ATB01, BBdOG99, EH04, EH05, dOBG99, dOGB99, dOGH99, dOGB02, PABB00, Xie08, PDSV09]. De manière synthétique, les avantages de ces LMI étendues par rapport aux LMI standards peuvent être résumés comme suit :

- Les LMIs étendues ne comportent pas de produits entre la matrice de Lyapunov et la matrice système A. Cette séparation permet l'utilisation de fonctions de Lyapunov dépendantes des paramètres, dans le cas de l'analyse et de la synthèse robuste;
- Il n'existe pas de termes quadratiques indéfinis fonction de la matrice A;
- Les variables auxiliaires introduites induisent l'utilisation de variables de décision supplémentaires. Cela peut éventuellement réduire le conservatisme.

Par ailleurs, la formulation LMI étendue a été généralisée au cas implicite. Certains résultats relatifs aux LMI étendues pour les systèmes implicites ont été introduits par [XL06, Yag10, Seb07, Seb08].

Motivé par les travaux de [PDSV09], nous nous appuyons sur l'approche inverse du lemme de projection pour revisiter les LMI étendues associées à la caractérisation de la stabilité, la performance H_2 et la performance dissipative pour des systèmes implicites. Pour ce faire nous rappelons ci-dessous le lemme de projection.

Lemme 1.2.2 (Lemme de projection) *[BGFB94, IS94] Soit une matrice symétrique $\Xi \in \mathbb{R}^{n\times n}$ et deux matrices $\Psi \in \mathbb{R}^{n\times m}$ et $\Upsilon \in \mathbb{R}^{k\times n}$ avec $rang(\Psi) < n$ et $rang(\Upsilon) < n$. Il existe un matrice non structurée Θ telle que*

$$\Xi + \Upsilon^\top \Theta^\top \Psi + \Psi^\top \Theta \Upsilon < 0 \tag{1.15}$$

si et seulement si, les inégalités de projection vis-à-vis de Θ suivantes sont satisfaites

$$N_\Psi^\top \Xi N_\Psi < 0, \quad N_\Upsilon^\top \Xi N_\Upsilon < 0, \tag{1.16}$$

où N_Ψ et N_Υ sont des matrices arbitraires dont les colonnes forment une base du noyau respectivement de Ψ et de Υ.

La méthodologie adoptée consiste à transformer les LMI standards en formes quadratiques qui seront interprétées comme la première inégalité de (1.16), où N_Ψ est traduit en fonction des données du système. Ensuite, les LMI étendues peuvent être déduites en appliquant le lemme de projection. Quatre types différents de LMI étendues seront explorés selon la construction de la matrice N_Υ.

I $N_\Upsilon = [\ \ ]$. Dans ce cas, la deuxième inégalité de (1.16) disparaît et $\Upsilon = I$.

II Choix de N_Υ telle que la deuxième inégalité de (1.16) est équivalente au fait qu'une partie de la matrice P est définie positive;

III Choix de N_Υ telle que la deuxième inégalité de (1.16) soit triviale;

IV Combinaison des deux stratégies II et III.

Nous présentons, à titre d'exemple, dans l'ordre de leur introduction ci-dessus, les différentes formulations LMI étendues de la stabilité. D'autres caractérisations, notamment celles associées à la performance H_2 et la dissipativité se trouvent dans le mémoire de thèse.

Soit un système dynamique $\Sigma(\lambda)$ donné par :

$$\Sigma(\lambda) : \begin{cases} E\sigma x = Ax + Bw, \\ z = Cx + Dw, \end{cases} \tag{1.17}$$

où $x \in \mathbb{R}^n$, $z \in \mathbb{R}^p$ et $w \in \mathbb{R}^m$ sont respectivement le vecteur de variables descripteurs, le vecteur de sortie à contrôler et le vecteur de perturbation appartenant à $\mathcal{L}_2[0\ +\infty)$. La matrice E peut être singulière, i.e. $rank(E) = r \leq n$. Pour le cas continu $\sigma x = \frac{dx}{dt}$ et $\lambda = s$, et pour le cas discret, σ représente l'opérateur q et $\lambda = z$.

Caractérisation I 1.2.1 (Admissibilité, cas continu) *Le système implicite continu (1.17) est admissible, si et seulement si, il existe $P = P^\top \in \mathbb{R}^{n\times n} > 0$, $Q \in \mathbb{R}^{(n-r)\times n}$ et $\Theta_1, \Theta_2 \in \mathbb{R}^{n\times n}$ telles que*

$$\begin{bmatrix} 0 & (PE+UQ)^\top \\ \bullet & 0 \end{bmatrix} + \begin{bmatrix} \Theta_1^\top \\ \Theta_2^\top \end{bmatrix} \begin{bmatrix} A & -I \end{bmatrix} + \begin{bmatrix} A^\top \\ -I \end{bmatrix} \begin{bmatrix} \Theta_1 & \Theta_2 \end{bmatrix} < 0, \tag{1.18}$$

où $U \in \mathbb{R}^{n\times(n-r)}$ est une matrice arbitraire de rang plein par colonne satisfaisant $E^\top U = 0$.

Caractérisation I 1.2.2 (Admissibilité, cas discret) *Le système implicite discret (1.17) est admissible, si et seulement si, il existe $P = P^\top \in \mathbb{R}^{n\times n} > 0$, $Q \in \mathbb{R}^{(n-r)\times n}$ et $\Theta_1, \Theta_2 \in \mathbb{R}^{n\times n}$ telles que*

$$\begin{bmatrix} -E^\top PE & Q^\top U^\top \\ \bullet & P \end{bmatrix} + \begin{bmatrix} \Theta_1^\top \\ \Theta_2^\top \end{bmatrix} \begin{bmatrix} A & -I \end{bmatrix} + \begin{bmatrix} A^\top \\ -I \end{bmatrix} \begin{bmatrix} \Theta_1 & \Theta_2 \end{bmatrix} < 0, \tag{1.19}$$

où $U \in \mathbb{R}^{n\times(n-r)}$ est une matrice arbitraire de rang plein par colonne satisfaisant $E^\top U = 0$.

Caractérisation II 1.2.1 (Admissibilité, cas continu) *Pour un système implicite continu, la condition LMI (1.18) est équivalente à*

$$\begin{bmatrix} 0 & (PE+UQ)^\top \\ \bullet & 0 \end{bmatrix} + \begin{bmatrix} E^\top\Theta_1^\top + V\Theta_2^\top \\ \epsilon\Theta_1^\top \end{bmatrix} \begin{bmatrix} A & -I \end{bmatrix} + \begin{bmatrix} A^\top \\ -I \end{bmatrix} \begin{bmatrix} \Theta_1 E + \Theta_2 V^\top & \epsilon\Theta_1 \end{bmatrix} < 0, \tag{1.20}$$

où $U \in \mathbb{R}^{n\times(n-r)}$ et $V \in \mathbb{R}^{n\times(n-r)}$ sont des matrices arbitraires respectivement de rang plein par colonne et de rang plein par ligne satisfaisants $E^\top U = 0$ et $EV = 0$. ϵ est un scalaire positif, $\Theta_1 \in \mathbb{R}^{n\times n}$ et $\Theta_2 \in \mathbb{R}^{n\times(n-r)}$ sont des matrices auxiliaires.

Caractérisation II 1.2.2 (Admissibilité, cas discret) *Pour un système implicite discret, la condition LMI (1.19) est équivalente à*

$$\begin{bmatrix} -E^\top PE & Q^\top U^\top \\ \bullet & P \end{bmatrix} + \begin{bmatrix} V\Theta_1^\top \\ \Theta_2^\top \end{bmatrix} \begin{bmatrix} A & -I \end{bmatrix} + \begin{bmatrix} A^\top \\ -I \end{bmatrix} + \begin{bmatrix} \Theta_1 V^\top & \Theta_2 \end{bmatrix} < 0, \qquad (1.21)$$

où $U \in \mathbb{R}^{n\times(n-r)}$ *et* $V \in \mathbb{R}^{n\times(n-r)}$ *sont des matrices arbitraires respectivement de rang plein par colonne et de rang plein par ligne satisfaisants* $E^\top U = 0$ *et* $EV = 0$. $\Theta_1 \in \mathbb{R}^{n\times(n-r)}$ *et* $\Theta_2 \in \mathbb{R}^{n\times n}$ *sont des matrices auxiliaires.*

1.2.3 Equation de Sylvester généralisée

Plusieurs problèmes de commande peuvent être liés à la résolution des équations de Sylvester. En effet, ce type d'équation a des applications importantes en analyse de stabilité, en synthèse d'observateurs et dans le cadre de certains problèmes de régulation et de placement de pôles [Tsu88, Doo84, FKKN85, Dua93].

Une forme d'équation matricielle ayant un intérêt particulier dans la théorie de la commande peut être décrite comme suit :

$$\sum_{i=1}^{k} A_i X S_i = R, \qquad (1.22)$$

où A_i, S_i et R sont des matrices données de dimensions appropriées et X est la matrice inconnue.

Un exemple souvent utilisé de l'équation (1.22) est celui communément appelé équation de Sylvester

$$AX - XS = R, \qquad (1.23)$$

où A et S sont de matrices carrées. Sylvester a prouvé dans [Syl84] que l'équation (1.23) peut être résolue, si et seulement si, les matrices A et S ne comportent pas de valeurs propres identiques.

Un résultat concernant l'équation (1.22), dans le même esprit que celui de l'équation de Sylvester, n'est, cependant, pas encore obtenu. Les chercheurs focalisent souvent leur attention sur certains cas particuliers. Par exemple, dans [Chu87, HG89, GLAM92], les auteurs ont présenté des conditions sous lesquelles l'équation matricielle suivante

$$AXB - CXD = E. \qquad (1.24)$$

admet une solution.

En outre, une équation de Sylvester généralisée décrite comme suit

$$\begin{aligned} AX - YB &= C, && (1.25a) \\ DX - YE &= F, && (1.25b) \end{aligned}$$

a été introduite et étudiée dans la littérature, [Ste73, KW89, Wim94]. Dans ces références, on montre que pour le cas où les paramètres de (1.25) sont réels, et A, B, D et E sont toutes des matrices carrées, l'équation (1.25) admet une solution unique, si et seulement si, les polynômes $\det(A - sB)$ et $\det(D - sE)$ sont copremiers entre eux [Ste73]. Sous ces hypothèses, un algorithme de résolution de (1.25) est proposé en s'appuyant sur une méthode de Schur généralisée [KW89]. Par ailleurs, sous aucune hypothèse, Wimmer a étendu le théorème d'équivalence de Roth [Rot52] à une paire d'équations de Sylvester, et a donné une condition nécessaire et suffisante sous laquelle (1.25) admet une solution.

Dans le cadre des systèmes implicites, les équations de Sylvester généralisées ont attiré l'attention des chercheurs, et différents types de formes relatifs aux équations de Sylvester généralisées ont été explorés dans les références suivantes [Chu87, HG89, GLAM92, Dua96, CdS05, Dar06, Ben94]. Nous nous intéressons ici à une formulation plus générale et particulière de l'équation de Sylvester qui s'écrit comme suit

$$AXB - CYD = E, \tag{1.26a}$$
$$FXG - HYJ = K, \tag{1.26b}$$

où A, B, C, D, E, F, G, H, J et K sont de matrices connues, et de dimensions appropriées, et X et Y sont des variables matricielles à déterminer.

En utilisant la propriété du produit de Kronecker, nous pouvons réécrire (1.26) sous la forme suivante :

$$\begin{bmatrix} B^\top \otimes A & D^\top \otimes C \\ G^\top \otimes F & J^\top \otimes H \end{bmatrix} \begin{bmatrix} vec(X) \\ vec(Y) \end{bmatrix} = \begin{bmatrix} vec(E) \\ vec(K) \end{bmatrix}. \tag{1.27}$$

Sans contrainte de structure, la solution de cette équation peut être trivialement obtenue par résolution d'un système d'équations linéaires.

Dans les chapitres 6 et 7, un cas particulier de l'équation (1.26) sera utilisé dans le cadre des problèmes d'admissibilité étendue et de minimisation de critère H_2 ou H_∞ sous contrainte de régulation.

1.2.4 Equation algébrique de Riccati généralisée

En théorie des systèmes et de la commande, le terme "équation de Riccati" est utilisé pour indiquer des équations matricielles ayant un terme quadratique, qui apparait dans le cadre des problèmes de commande linéaire quadratique et linéaire quadratique gaussienne (LQ, LQG) en cas continu et discret. L'équation algébrique de Riccati (ARE), ou la version non dynamique de l'équation de Riccati, permet de résoudre deux problèmes des plus fondamentaux en automatique Une littérature abondante existe autour des ARE dans le cas continu et dans le cas discret (à titre d'exemple, voir [WAL84, LR95]).

Nous présentons dans cette section le résultat concernant l'équation algébrique de Riccati généralisée (GARE) pour des systèmes implicites continus.

Soit un système implicite décrit par (1.7). Nous considérons une GARE ayant la forme suivante

$$E^\top P = P^\top E, \tag{1.28a}$$

$$A^\top P + P^\top A - (P^\top B + S)R^{-1}(P^\top B + S)^\top + Q = 0, \tag{1.28b}$$

où $Q \in \mathbb{R}^{n\times n}$, $S \in \mathbb{R}^{n\times m}$ et $R \in \mathbb{R}^{m\times m}$ sont des matrices réelles et constantes.

Nous définissons par la suite la solution admissible de la GARE (1.28).

Définition 1.2.3 *Une solution P de la GARE* (1.28) *est dite solution admissible, si à la fois, la paire $\big(E, A - BR^{-1}(B^\top P + S^\top)\big)$ est régulière, non-impulsive, stable, et $E^\top P \geq 0$.*

Nous remarquons aussi que la solution admissible peut être éventuellement non unique, pourtant $E^\top P$ est unique.

Avant de présenter et rappeler un algorithme numérique de résolution de la GARE (1.28), nous considérons quelques hypothèses sous lesquelles la GARE, donnée ci-dessus, admet une solution admissible.

Hypothèses 1.2.1

H1 (E, A) *est régulière;*

H2 $D^\top D > 0$*;*

H3 (E, A, B) *est à dynamique finie stabilisable et état impulsif commandable (les définitions associées à ces notions sont introduites dans le chapitre 3 de la thèse);*

H4 $\begin{bmatrix} A - sE & B \\ C & D \end{bmatrix}$ *ne contient pas de zéros invariants sur l'axe imaginaire y compris à l'infini.*

En se basant sur le problème des valeurs propres généralisées, des algorithmes numériques pour résoudre la GARE (1.28) ont été proposés dans [TMK94, TK98, KK97, KM92, WYC98]. Nous rappelons ici l'essentiel de l'approche adoptée. Tout d'abord, nous construisons le faisceau hamiltonien ci-dessous:

$$H - \lambda J = \begin{bmatrix} A & 0 & B \\ -Q & -A^\top & -S \\ S^\top & B^\top & R \end{bmatrix} - \lambda \begin{bmatrix} E & 0 & 0 \\ 0 & E^\top & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{1.29}$$

avec $\lambda \in \mathbb{C}$. Sous les hypothèses faites plus haut H1-H4, il est facile de vérifier que (J, H) est régulière, non-impulsive et n'a pas de zéros invariants sur l'axe imaginaire y compris à l'infini. En outre, ce faisceau matriciel comporte r valeurs propres stables finies, r valeurs propres instables et $2n + m - 2r$ valeurs propres impulsives. Soit $\Lambda = [\Lambda_1^\top \ \Lambda_2^\top \ \Lambda_3^\top]^\top \in \mathcal{C}^{(2n+m)\times n}$ la matrice des vecteurs propres généralisés et vecteurs principaux généralisés relatifs aux valeurs propres finies et stables. Nous avons :

$$\begin{bmatrix} E & 0 & 0 \\ 0 & E^\top & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{bmatrix} \Delta = \begin{bmatrix} A & 0 & B \\ -Q & -A^\top & -S \\ S^\top & B^\top & R \end{bmatrix} \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{bmatrix}, \tag{1.30}$$

où $\Delta \in \mathcal{C}^{r\times r}$ est la forme de Jordan avec toutes les valeurs propres dans le demi-plan complexe gauche.

Selon [TMK94], une solution admissible P de la GARE (1.28) est donnée par

$$P = \begin{bmatrix} \Lambda_2 & W_2 P_r \end{bmatrix} \begin{bmatrix} \Lambda_1 & W_1 \end{bmatrix}^{-1}, \tag{1.31}$$

où P_r satisfait l'équation de Riccati suivante :

$$A_r^\top P_r + P_r^\top A_r - (P_r^\top B_r + S_r) R^{-1} (P_r^\top B_r + S_r)^\top + Q_r = 0, \tag{1.32a}$$

$$A_r = W_2^\top A W_1, \quad B_r = W_2^\top B, \quad Q_r = W_1^\top Q W_1, \quad S_r = W_1^\top S, \tag{1.32b}$$

$W_1 \in \mathbb{R}^{n\times(n-r)}$ et $W_2 \in \mathbb{R}^{n\times(n-r)}$ sont des matrices arbitraires de rang plein satisfaisant respectivement $EW_1 = 0$ et $E^\top W_2 = 0$.

1.3 Commande H_∞ simultanée et Commande H_∞ forte

La stabilisation forte consiste à trouver un régulateur stable qui stabilise un système donné. Cette stabilisation a plusieurs intérêts pratiques, et ce sujet a été largement étudié depuis les années 1970. Vidyasagar a montré que les régulateurs instables sont très sensibles, et leurs réponses aux défaillances de capteurs et aux incertitudes/nonlinéarités du système sont imprédictibles [Vid85]. A contrario, les régulateurs stables permettent de réaliser des tests hors ligne pour vérifier l'existence d'éventuelles fautes dans la démarche d'implémentation, et aussi de comparer les résultats obtenus avec les spécifications du cahier des charges.

Notons qu'une condition nécessaire et suffisante pour l'existence d'un régulateur stable est introduite dès 1974 [YBL74] et appelée PIP (pour l'anglais Parity Interlacing Property). Cette condition consiste en la propriété suivante : le système comporte des pôles en nombre pair entre une paire quelconque de ses zéros sur $\mathbb{R}^+$. Par ailleurs, différentes approches ont été proposées pour résoudre le problème de stabilisation forte [YBL74, Vid85, SGP97, SGP98].

D'autre part, le problème de stabilisation simultanée consistant à trouver un seul régulateur tel qu'un ensemble de systèmes soit stabilisé a été introduit par Sake et al. [SM82] et Vidyasagar et al. [VV82] au début des années 1980. Ces auteurs ont prouvé que le problème de stabiliser k systèmes simultanément peut toujours être réduit en un problème de stabilisation de $k-1$ systèmes simultanément avec cette fois-ci un régulateur stable. Ainsi, dans le cas où $k = 2$, ce problème peut être considéré comme un problème de stabilisation forte.

Au-delà du problème de stabilisation forte, Zeren et Özbay ont introduit dans [ZO99] le problème de commande H_∞ sous contrainte de stabilité forte. Ce problème consiste à trouver un régulateur stable tel qu'un système donné est stabilisé et la norme H_∞ de la boucle fermée est aussi bornée. Une condition suffisante a été proposée en se basant sur l'existence d'une solution définie positive d'une ARE. Récemment, ce problème a été développé, et une condition suffisante en termes de contraintes LMI a été proposée dans [GO05]. En outre, les auteurs dans [CDZ03] ont présenté une approche pour synthétiser un régulateur H_2 ou H_∞ stable en optimisant de manière directe les matrices de transfert libres dans une paramétrisation particulière des régulateurs H_2 ou H_∞ sous-optimaux. D'autres méthodes relatives à la synthèse de régulateurs H_∞ stables peuvent être trouvées dans [ZO00, CDZ01, CC01, LS02] et les références incluses.

Un autre problème de commande H_∞ simultanée a été introduit par Cao et Lam dans [CL00]. Dans ce problème on s'intéresse à rechercher un seul régulateur qui stabilise un ensemble de systèmes tout en garantissant un niveau de performance H_∞ γ donné sur chaque boucle fermée. Pour résoudre ce problème les auteurs proposent un problème équivalent, appelé problème de “commande γ-H_∞ forte”, qui consiste à trouver un régulateur solution du problème de commande H_∞ forte et garantissant en plus que la norme H_∞ du régulateur lui-même soit inférieure à γ.

En se basant sur la factorisation co-première des systèmes considérés, ces auteurs ont montré que le problème de commande H_∞ simultanée vis-à-vis d'un ensemble de systèmes admet une solution si et seulement si le problème de commande γ-H_∞ forte associé à un certain système augmenté admet une solution. Par ailleurs, Cheng et al. dans [CCS07, CCS08, CCS09] ont étendu ces travaux au cas du problème de commande γ_k-γ_{cl} H_∞ forte i.e. en imposant de manière séparée la norme H_∞ de la boucle fermée γ_{cl} et celle du régulateur γ_k.

Nous nous intéressons dans cette partie à la commande H_∞ simultanée et à la commande H_∞ forte pour des systèmes implicites continus. Il nous semble que les récents résultats cités ci-dessus n'ont pas été encore étendus au cas des systèmes implicites.

Nous proposons de tirer au clair la relation entre ces deux problèmes dans le cadre des systèmes implicites. Nous montrons donc dans le cas des systèmes implicites, que le problème de commande H_∞ simultanée admet une solution si et seulement si le problème de commande H_∞ forte associé à un système implicite augmenté admet une solution. Nous proposons aussi une nouvelle condition suffisante pour résoudre le problème de commande H_∞ forte en termes d'une GARE et d'un ensemble de contraintes LMI.

Nous présentons brièvement dans ce qui suit les résultats principaux de cette partie des travaux de thèse, les preuves et les exemples numériques illustrant les résultats obtenus se trouvant dans le mémoire de thèse.

Soient k systèmes implicites continus G_i donnés par :

$$G_i = \left\{ E_i, \left[\begin{array}{c|cc} A_i & B_{iw} & B_i \\ \hline C_{iz} & 0 & D_{izu} \\ C_i & D_{iyw} & 0 \end{array} \right] \right\}. \tag{1.33}$$

Nous supposons que ces systèmes satisfont les hypothèses suivantes :

Hypothèses 1.3.1

(H1) (E_i, A_i) *est régulière;*

(H2) (E_i, A_i, B_i) *est à dynamique finie stabilisable et état impulsif commandable;*

(H3) (E_i, A_i, C_i) *est à dynamique finie détectable et état impulsif observable;*

(H4) $\begin{bmatrix} -sE_i + A_i & B_i \\ C_{iz} & D_{izu} \end{bmatrix}$ *ne contient pas de zéros invariants sur l'axe imaginaire y compris à l'infini;*

(H5) $\begin{bmatrix} -sE_i + A_i & B_{iw} \\ C_i & D_{iyw} \end{bmatrix}$ *ne contient pas de zéros invariants sur l'axe imaginaire y compris à l'infini.*

Définissons M_i, N_i, X_i, Y_i, $\tilde{M}_i$, $\tilde{N}_i$, $\tilde{X}_i$ et $\tilde{Y}_i$ les factorisations co-premières associées aux systèmes G_i.

Théorème 1.3.1 *Soient les systèmes implicites G_i et un scalaire $\gamma > 0$ donné. Sous les hypothèses (H1)-(H5), il existe un régulateur H_∞ stabilisant simultanément G_i tel que $\|T^i_{zw}\|_\infty < \gamma$, où T^i_{zw} est la fonction de transfert en boucle fermée de chaque sous système, si et seulement si, il existe $Q_1 \in RH_\infty$ avec $\|Q_1\|_\infty < \gamma$ telle que :*

$$Q_i = (\Pi_{i1} + Q_1\Pi_{i3})^{-1} (\Pi_{i2} + Q_1\Pi_{i4}) \in RH_\infty \tag{1.34}$$

et

$$\|Q_i\|_\infty < \gamma, \tag{1.35}$$

où

$$\begin{bmatrix} \Pi_{i1} & \Pi_{i2} \\ \Pi_{i3} & \Pi_{i4} \end{bmatrix} := \begin{bmatrix} -\tilde{Y}_1 & \tilde{X}_1 \\ -\tilde{N}_1 & \tilde{M}_1 \end{bmatrix} \begin{bmatrix} M_i & -X_i \\ N_i & -Y_i \end{bmatrix}. \tag{1.36}$$

Théorème 1.3.2 *Soient les systèmes implicites G_i et un scalaire $\gamma > 0$ donné. Sous les hypothèses (H1)-(H5), il existe un régulateur H_∞ stabilisant simultanément G_i tel* *que $\|T^i_{zw}\|_\infty < \gamma$, si et seulement si, le problème de commande H_∞ forte pour les $k-1$ systèmes augmentés donnés par*

$$S_i := \begin{bmatrix} {\Pi_{i1}}^{-1}\Pi_{i2} & {\Pi_{i1}}^{-1} \\ \Pi_{i4} - \Pi_{i3}{\Pi_{i1}}^{-1}\Pi_{i2} & -\Pi_{i3}{\Pi_{i1}}^{-1} \end{bmatrix}, i = 2, \ldots, k, \tag{1.37}$$

admet une solution.

Nous attirons l'attention du lecteur sur le fait suivant : au-delà de la nécessité d'étendre un résultat connu dans le cadre explicite au cas des systèmes implicites continus, ce dernier résultat présenté nous permettra de mettre en œuvre la nouvelle condition suffisante que nous proposons par la suite dans le cadre de la commande H_∞ simultanée.

Nous proposons dans ce qui suit une nouvelle condition suffisante pour la conception de lois de commande H_∞ forte dans le cas des systèmes implicites.

Considérons un système implicite G défini par :

$$G = \left\{ E, \ \left[\begin{array}{c|cc} A & B_w & B \\ \hline C_z & 0 & D_{zu} \\ C & D_{yw} & 0 \end{array}\right] \right\}, \tag{1.38}$$

Où les matrices E, A, B_w, B, C_z, D_{zu}, C, et D_{yw} sont constantes et de dimensions appropriées.

Théorème 1.3.3 *Soit le système implicite (1.38) et un scalaire $\gamma > 0$ donné. Supposons que les conditions (H1)-(H5) sont satisfaites et que la GARE*

$$\begin{cases} E^\top X = X^\top E \\ A^\top X + X^\top A + X^\top(\mu B_w B_w^\top - BB^\top)X + C_z^\top C_z = 0 \end{cases} \tag{1.39}$$

admet une solution admissible X. Alors, il existe un régulateur K_G, tel que le problème de commande H_∞ forte associé au système G est résolu, s'il existe des matrices $P = P^\top \in \mathbb{R}^{n\times n} > 0$, $R = R^\top \in \mathbb{R}^{n\times n} > 0$, $Q \in \mathbb{R}^{(n-r)\times n}$, $S \in \mathbb{R}^{(n-r)\times n}$, $W \in \mathbb{R}^{(n-r)\times n}$ et $Y \in \mathbb{R}^{n\times p}$ telles que

$$\begin{bmatrix} \Gamma(\Phi(P,Q), A_X) + \Gamma(Y,C) & \bullet & \bullet \\ -Y^\top & -\gamma^2 I & 0 \\ -B^\top X & 0 & -I \end{bmatrix} < 0, \tag{1.40}$$

$$\begin{bmatrix} \Xi_{11} & \bullet & \bullet \\ \Xi_{21} & \Xi_{22} & \bullet \\ \Xi_{31} & \Xi_{32} & -\gamma^2 I \end{bmatrix} < 0, \tag{1.41}$$

où

$$\begin{aligned}
&\Phi(P,Q) = (PE+UQ)^\top, \quad \Omega(R,S) = (RE+US)^\top,\\
&A_X = A - BB^\top X, \quad \tilde{C}_1 = C_z - D_{zu}B^\top X, \quad \tilde{C}_2 = -D_{zu}B^\top X,\\
&\Xi_{11} = \Gamma(\Omega(R,S), A_X) + \tilde{C}_1^\top \tilde{C}_1, \quad \Xi_{31} = B_w^\top \Omega(R,S)^\top,\\
&\Xi_{21} = W^\top U^\top A_X - X^\top BB^\top \Omega(R,S)^\top + \tilde{C}_1^\top \tilde{C}_2,\\
&\Xi_{22} = \Gamma(\Phi(P,Q), A) + \Gamma(Y,C) - \Gamma(W^\top U^\top, BB^\top X) + \tilde{C}_2^\top \tilde{C}_2,\\
&\Xi_{32} = B_w^\top W^\top U^\top - B_w^\top \Phi(P,Q)^\top - D_{yw}^\top Y^\top,
\end{aligned} \tag{1.42}$$

U est une matrice de rang plein par colonne vérifiant $E^\top U = 0$. En plus, sous ces conditions, le régulateur K_G est donné par

$$K_G = \left\{ E, \; \left[\begin{array}{c|c} A - BB^\top X + \Phi(P,Q)^{-1} YC & -\Phi(P,Q)^{-1} Y \\ \hline -B^\top X & 0 \end{array} \right] \right\}. \tag{1.43}$$

1.4 Stabilisation et commande H_2-H_∞ étendues

De nombreux problèmes de commande nécessitent la définition d'un modèle standard qui est souvent construit à partir du modèle physique du système, des modèles des perturbations et des signaux de référence, ainsi que les objectifs de commande. Dans ce cas, l'utilisation de pondérations décrites par la représentation d'état devient restrictive car les modèles cités ci-dessus sont généralement instables voire non-propres [HZK92, Mei95, SSS00a, Che02].

Utiliser, par exemple, un intégrateur ou un dérivateur comme pondération induit éventuellement dans le modèle global des dynamiques non stabilisables ou non détectables voire des éléments impulsifs non commandables

Dans cette configuration, la notion de stabilité est bien différente de celle dans le cas standard, car nous savons d'emblée que la stabilité interne n'est plus réalisable à cause des pondérations instables ou non-propres qui sont soit non-stabilisables soit non détectables. Pour surmonter cette difficulté, la notion de "stabilité étendue" ou "stabilité compréhensive" est adoptée. Elle peut être vue comme une généralisation de la notion de stabilité interne et elle peut aussi être reliée de manière évidente aux exigences de la pratique [LM94].

1.4.1 Intérêt des pondérations instables et non-propres

Comme présenté plus haut, les cahiers des charges de synthèse d'une loi de commande peuvent souvent être interprétés par des pondérations fréquentielles. Par exemple, une pondération ayant un pôle à l'origine est généralement utilisée pour imposer un rejet parfait d'une perturbation constante (ou constante par morceaux) voire un suivi d'une référence constante (ou constante par morceaux).

Nous prenons ici le cas de la commande H_∞ comme exemple afin d'illustrer l'importance de l'utilisation de pondérations instables et/ou non-propres.

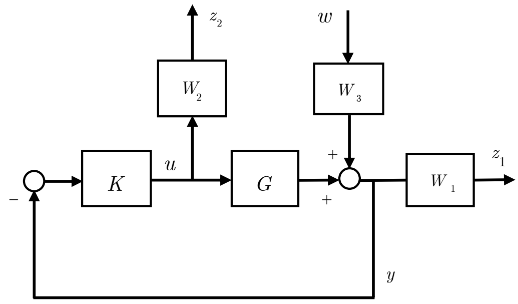

Figure 1.1: Problème de sensibilité mixte

Examinons alors le problème de sensibilité mixte présenté par la Fig. 1.1, où G représente le système physique, K est un régulateur à synthétiser et W_1, W_2 et W_3 sont les pondérations fréquentielles en entrée et en sortie. Ce schéma est emprunté à [Mei95] et admet les matrices de transfert suivantes :

$$T_{zw} = \begin{bmatrix} W_1(I+GK)^{-1}W_3 \\ W_2K(I+GK)^{-1}W_3 \end{bmatrix}. \tag{1.44}$$

Selon le problème de sensibilité mixte introduit par Kwakernaak [Kwa93], le choix souvent fait est celui d'imposer que :

- W_1 ait un pôle à l'origine;
- W_2 soit non-propre.

Il est normalement souhaitable d'imposer le fait que la pondération W_1 ait un pôle à l'origine, car $\|T_{zw}\|$ est finie, si et seulement si, la fonction de sensibilité $(I+GK)^{-1}$ a un zéro à l'origine. Ceci permettra au régulateur solution de garantir un rejet parfait de perturbations constantes ou un suivi parfait de références constantes.

Un autre fait pour argumenter ce choix serait le cas où le système G ne comporte pas de pôle à l'origine. Dans ce cas, le régulateur désiré devra contenir une action intégrale.

De plus, pour éviter une grande sensibilité aux bruits de mesures en hautes fréquences et par là même éviter une performance robuste limitée, il est également d'usage de choisir une pondération W_2 non-propre. En particulier, $\|W_2\|_\infty$ doit être grande hors de la bande-passante désirée de la boucle fermée en raison du fait que ce choix assure que le régulateur soit faible (i.e. a peu d'effet) hors de la bande-passante de la boucle fermée.

1.4.2 Approches existantes

L'objectif des problèmes non-standards ainsi définis est bien évidemment différent de celui associé aux problèmes "classiques". Le système pondéré dans ces cas ne peut

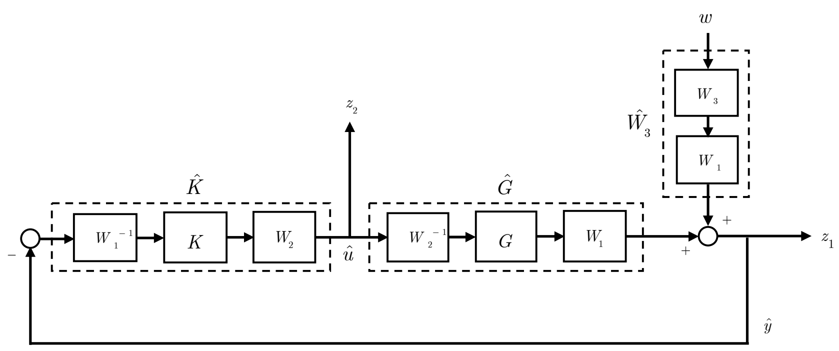

Figure 1.2: Problème de sensibilité mixte modifiée

être stabilisé de manière interne à cause des pondérations non-stabilisables ou non détectables.

Il existe plusieurs techniques et approches pour traiter ces problèmes dans la littérature. Afin d'en faire un rappel [Mei95], nous considérons ici le problème de sensibilité mixte (voir Fig. 1.1) à nouveau.

Méthode 1 Cette méthode consiste à traiter ces éléments "indésirables" par des petites perturbations pour rendre le problème global standard [CS92a]. Par exemple, nous pourrons considérer $W_1(s) = 1/(s + 0.0001)$ au lieu de $W_1(s) = 1/s$. Egalement, nous remplaçerons $W_2(s) = s$ par $W_2(s) = s/(1 + 0.0001s)$. Ceci est évidement une approximation du problème originel. Un inconvénient majeur de cette démarche est qu'elle aboutit un problème standard très sensible aux pôles peu amortis, et peut donc générer des régulateurs non strictement propres et/ou de dimensions élevées.

Méthode 2 Cette méthode consiste en une augmentation du système, semblable à la démarche décrite dans [Kra92, Mei95]. La Fig. 1.2 montre comment les pondérations peuvent être absorbées dans la boucle. Considérant le problème modifié de la Fig. 1.2, un régulateur $\hat{K}$ peut être trouvé, et le régulateur final K peut être déduit comme suit : $K = W_1 \hat{K} W_2^{-1}$. Cette approche est facile à expliquer et à implémenter. Nous observons tout de même que si une simplification pôle-zéro instable s'opère dans le système modifié, i.e. $\hat{G} = W_2^{-1} G W_1$, alors les propriétés de stabilité de la boucle originale et celles de la boucle modifiée ne sont plus les mêmes. Autrement dit, les pondérations W_1 et W_2 doivent être choisies en conséquence. En plus, cette méthode demande sans doute une procédure de prétraitement afin de pouvoir établir un problème modifié équivalent.

Méthode 3 Décrite dans le cadre des systèmes explicites, cette méthode se base sur le théorème de simplification de modèles ou de stabilisation étendue (dite aussi stabilisation compréhensive) [LM94, LM95, LZM97, MXA00]. L'idée principale de cette approche est de faire en sorte que les éléments non-stabilisables (non détectables) soient inobservables (non-commandables) dans la boucle fermée. Cependant, cette

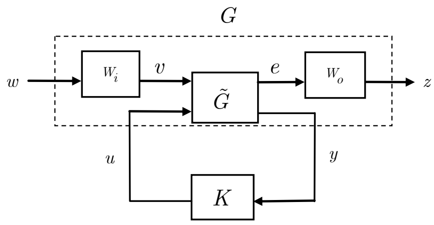

Figure 1.3: Problème de commande étendue

méthode ne peut pas prendre en compte des pondérations non-propres.

Dans une partie de nos travaux de thèse nous nous sommes intéressés à la résolution des problèmes de commande non standard dans le cas implicite en optant pour une généralisation de la dernière méthode. L'approche développée ainsi permet de traiter ces problèmes sans aucune approximation, ni transformation de boucle. Elle permet aussi d'aborder le cas des pondérations non-propres.

1.4.3 Commande étendue

Cette partie traite le problème de commande étendue pour des systèmes implicites continus. Ici, le terme "étendue" sous entend que le régulateur désiré doit stabiliser de manière interne uniquement une partie de la boucle fermée. Les systèmes physiques et les pondérations sont tous décrits par des représentations implicites.

Dans ce problème il est possible de considérer des pondérations instables et/ou des pondérations non-propres. Nous définissons alors un problème non-standard où les techniques existantes déjà évoquées ne sont plus applicables.

Soit le système implicite $\widetilde{G}(s)$ (voir Fig.1.3) :

$$\begin{bmatrix} e(s) \\ y(s) \end{bmatrix} = \widetilde{G} \begin{bmatrix} v(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} \widetilde{G}_{ev} & \widetilde{G}_{eu} \\ \widetilde{G}_{yv} & \widetilde{G}_{yu} \end{bmatrix} \begin{bmatrix} v(s) \\ u(s) \end{bmatrix} \tag{1.45}$$

où $e \in \mathbb{R}^q$, $y \in \mathbb{R}^p$, $v \in \mathbb{R}^l$ et $u \in \mathbb{R}^m$ représentent respectivement le vecteur de sorties à contrôler, le vecteur de sorties mesurées, le vecteur de perturbations et le vecteur de commandes. Ce système peut s'écrire comme suit :

$$\widetilde{G} = \left\{ E_g, \left[\begin{array}{c|cc} A_g & B_{g1} & B_{g2} \\ \hline C_{g1} & D_{g11} & D_{g12} \\ C_{g2} & D_{g21} & D_{g22} \end{array} \right] \right\} \tag{1.46}$$

où $E_g \in \mathbb{R}^{n_g \times n_g}$, A_g, B_{g1}, B_{g2}, C_{g1}, C_{g2}, D_{g11}, D_{g12}, D_{g21} et D_{g22} sont des matrices réelles, constantes et de dimensions appropriées. La matrice E_g peut être singulière, i.e. $rang(E_g) = r_g \leq n_g$.

Supposons que la pondération en entrée W_i et la pondération en sortie W_o sont aussi décrites par des réalisations implicites

$$W_i = \left\{ E_i, \left[\begin{array}{c|c} A_i & B_i \\ \hline C_i & D_i \end{array} \right] \right\}, \quad W_o = \left\{ E_o, \left[\begin{array}{c|c} A_o & B_o \\ \hline C_o & D_o \end{array} \right] \right\}, \tag{1.47}$$

où $E_i \in \mathbb{R}^{n_i \times n_i}$, $E_o \in \mathbb{R}^{n_o \times n_o}$, $A_i \in \mathbb{R}^{n_i \times n_i}$, $A_o \in \mathbb{R}^{n_o \times n_o}$, $B_i \in \mathbb{R}^{n_i \times m_i}$, $B_o \in \mathbb{R}^{n_o \times q}$, $C_i \in \mathbb{R}^{l \times n_i}$, $C_o \in \mathbb{R}^{p_o \times n_o}$, $D_i \in \mathbb{R}^{l \times m_i}$ et $D_o \in \mathbb{R}^{p_o \times q}$ sont des matrices réelles, constantes et de dimensions appropriées. Les matrices E_i et E_o peuvent être singulières, i.e. $rang(E_i) = r_i \leq n_i$ and $rang(E_o) = r_o \leq n_o$.

Alors, le système pondéré G est donné par :

$$G = \left\{ \begin{bmatrix} E_o & 0 & 0 \\ 0 & E_g & 0 \\ 0 & 0 & E_i \end{bmatrix}, \left[\begin{array}{ccc|cc} A_o & B_oC_{g1} & B_oD_{g11}C_i & B_oD_{g11}D_i & B_oD_{g12} \\ 0 & A_g & B_{g1}C_i & B_{g1}D_i & B_{g2} \\ 0 & 0 & A_i & B_i & 0 \\ \hline C_o & D_oC_{g1} & D_oD_{g11}C_i & D_oD_{g11}D_i & D_oD_{g12} \\ 0 & C_{g2} & D_{g21}C_i & D_{g21}D_i & D_{g22} \end{array} \right] \right\}. \tag{1.48}$$

Par ailleurs, nous utiliserons la notation suivante pour le système G :

$$G = \left\{ E, \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array} \right] \right\} \triangleq \begin{bmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{bmatrix}. \tag{1.49}$$

Définition 1.4.1 (Admissibilité étendue) *Le système $\mathcal{F}_l(G, K)$ est dit admissible de manière étendue si $\mathcal{F}_l(\widetilde{G}, K)$ est stable de manière interne, et le système en boucle fermée défini par*

$$T_{zw} = \mathcal{F}_l(G, K) = G_{zw} + G_{zu}K(I - G_{yu}K)^{-1}G_{yw} \tag{1.50}$$

est admissible.

Problème 1.4.1 (Commande étendue) *Le problème de commande étendue associé à G donné par (1.49) consiste à trouver un régulateur K tel que*

(1) *(Admissibilité étendue) Le système bouclé formé par G et K est admissible de manière étendue.*

(2) *(Performance H_2 ou H_∞) la performance (H_2, H_∞) de la fonction de transfert T_{zw} est inférieure à $\gamma > 0$ donné.*

Nous montrons que la première condition est satisfaite, si et seulement si, deux équations de Sylvester généralisées admettent une solution. Dans le cas où $E = I$, ces équations se réduisent aux résultats existants [SSS00a, SSS00b, LZM97, MXA00]. L'objectif de la performance H_2 ou H_∞ est réalisé en résolvant cette fois-ci deux GARE. Le système pondéré G n'est pas totalement stabilisable, ni détectable, les deux GARE associées ne peuvent admettre des solutions admissibles. Pour ces raisons, la notion de “solution quasi-admissible” est introduite au lieu de la notion de “solution admissible”. Nous prouvons qu'une solution quasi-admissible d'une GARE est construite par une solution admissible d'une GARE réduite et la solution de l'équation de Sylvester généralisée correspondante. De plus, nous proposons une paramétrisation de l'ensemble des régulateurs solution du problème de commande étendue défini ci-dessus.

Nous présentons succinctement dans la suite les résultats principaux concernant la commande étendue. Les démonstrations et les exemples illustratifs se trouvent dans le mémoire de thèse.

Avant d'exposer les résultats, nous donnons deux partitions différentes de G. Soit la partition de G par rapport à la pondération W_i suivante :

$$G = \left\{ \begin{bmatrix} \bar{E} & 0 \\ 0 & E_i \end{bmatrix}, \left[\begin{array}{cc|cc} \bar{A}_{11} & \bar{A}_{12} & \bar{B}_{11} & \bar{B}_{12} \\ 0 & A_i & \bar{B}_{21} & 0 \\ \hline \bar{C}_{11} & \bar{C}_{12} & D_{11} & D_{12} \\ \bar{C}_{21} & \bar{C}_{22} & D_{21} & D_{22} \end{array}\right] \right\}, \tag{1.51}$$

avec

$$\bar{E} = \begin{bmatrix} E_o & 0 \\ 0 & E_g \end{bmatrix}, \ \bar{A}_{11} = \begin{bmatrix} A_o & B_oC_{g1} \\ 0 & A_g \end{bmatrix}, \ \bar{A}_{12} = \begin{bmatrix} B_oD_{g11}C_i \\ B_{g1}C_i \end{bmatrix}, \tag{1.52a}$$

$$\bar{B}_{11} = \begin{bmatrix} B_oD_{g11}D_i \\ B_{g1}D_i \end{bmatrix}, \ \bar{B}_{12} = \begin{bmatrix} B_oD_{g12} \\ B_{g2} \end{bmatrix}, \ \bar{B}_{21} = B_i, \tag{1.52b}$$

$$\bar{C}_{11} = \begin{bmatrix} C_o & D_oC_{g1} \end{bmatrix}, \bar{C}_{12} = D_oD_{g11}C_i, \tag{1.52c}$$

$$\bar{C}_{21} = \begin{bmatrix} 0 & C_{g2} \end{bmatrix}, \ \bar{C}_{22} = D_{g21}C_i. \tag{1.52d}$$

Soit la partition de G par rapport à la pondération W_o suivante :

$$G = \left\{ \begin{bmatrix} E_o & 0 \\ 0 & \hat{E} \end{bmatrix}, \left[\begin{array}{cc|cc} A_o & \hat{A}_{12} & \hat{B}_{11} & \hat{B}_{12} \\ 0 & \hat{A}_{22} & \hat{B}_{21} & \hat{B}_{22} \\ \hline \hat{C}_{11} & \hat{C}_{12} & D_{11} & D_{12} \\ 0 & \hat{C}_{22} & D_{21} & D_{22} \end{array}\right] \right\}, \tag{1.53}$$

avec

$$\hat{E} = \begin{bmatrix} E_g & 0 \\ 0 & E_i \end{bmatrix}, \hat{A}_{12} = \begin{bmatrix} B_oC_{g1} & B_oD_{g11}C_i \end{bmatrix}, \hat{A}_{22} = \begin{bmatrix} A_g & B_{g1}C_i \\ 0 & A_i \end{bmatrix}, \tag{1.54a}$$

$$\hat{B}_{11} = B_oD_{g11}D_i, \hat{B}_{12} = B_oD_{g12}, \hat{B}_{21} = \begin{bmatrix} B_{g1}D_i \\ B_i \end{bmatrix}, \hat{B}_{22} = \begin{bmatrix} B_{g2} \\ 0 \end{bmatrix}, \tag{1.54b}$$

$$\hat{C}_{11} = C_o; \hat{C}_{12} = \begin{bmatrix} D_oC_{g1} & D_oD_{g11}C_i \end{bmatrix}, \hat{C}_{22} = \begin{bmatrix} C_{g2} & D_{g21}C_i \end{bmatrix}. \tag{1.54c}$$

1.4.4 Cas de l'admissibilité étendue

Nous considérons ici les hypothèses suivantes :

Hypothèses 1.4.1

H1 *Les pondérations ne contiennent que les modes instables ou impulsifs.*

H2 $(\bar{E}, \bar{A}_{11}, \bar{B}_{12})$ *est à dynamique finie stabilisable et état impulsif commandable.*

H3 $(\hat{E}, \hat{A}_{22}, \hat{C}_{22})$ *est à dynamique finie détectable et état impulsif observable.*

Théorème 1.4.1 (Admissibilité étendue) *Soit le système pondéré G donné par (1.49). Sous les hypothèses (H1)-(H3), l'ensemble des régulateurs stabilisants de manière étendue peut être paramétré comme suit :*

$$\mathbf{K} = \mathcal{F}_l(J, Q), \quad \forall Q \in RH_\infty, \tag{1.55}$$

avec

$$J = \left\{ E, \left[\begin{array}{c|cc} A + B_2F + LC_2 & -L & B_2 \\ \hline F & 0 & I \\ -C_2 & I & 0 \end{array} \right] \right\}, \tag{1.56a}$$

$$F = \begin{bmatrix} F_1 & F_a + F_1V_i \end{bmatrix}, \tag{1.56b}$$

$$L = \begin{bmatrix} L_a - U_oL_2 \\ L_2 \end{bmatrix}, \tag{1.56c}$$

où F_a, V_i, L_a, U_o, V_o and U_i sont les solutions des équations de Sylvester généralisées ci-dessous

$$\bar{B}_{12}F_a = \bar{A}_{11}V_i - \bar{A}_{12} - U_iA_i, \tag{1.57a}$$

$$D_{12}F_a = \bar{C}_{11}V_i - \bar{C}_{12}, \tag{1.57b}$$

$$\bar{E}V_i = U_iE_i. \tag{1.57c}$$

$$L_a\hat{C}_{22} = A_oV_o - \hat{A}_{12} - U_o\hat{A}_{22}, \tag{1.58a}$$

$$L_aD_{21} = -U_o\hat{B}_{21} - \hat{B}_{11}, \tag{1.58b}$$

$$U_o\hat{E} = E_oV_o. \tag{1.58c}$$

F_1 est tel que la paire $(\bar{E}, \bar{A}_{11} + \bar{B}_{12}F_1)$ soit admissible et L_2 est tel que la paire $(\hat{E}, \hat{A}_{22} + L_2\hat{C}_{22})$ soit admissible.

1.4.5 Cas de la commande H_2 étendue

Nous donnons aussi ci-dessous les hypothèses adoptées dans le cas de la commande H_2 étendue.

Hypothèses 1.4.2

(H4) (E, A) *est régulière;*

(H5) $R_1 := D_{12}^\top D_{12} > 0$, $R_2 := D_{21} D_{21}^\top > 0$ *et* $D_{22} = 0$;

(H6) $\begin{bmatrix} \bar{A}_{11} - s\bar{E} & \bar{B}_{12} \\ \bar{C}_{11} & D_{12} \end{bmatrix}$ *ne contient pas de zéros invariants sur l'axe imaginaire y compris à l'infini;*

(H7) $\begin{bmatrix} \hat{A}_{22} - s\hat{E} & \hat{B}_{21} \\ \hat{C}_{22} & D_{21} \end{bmatrix}$ *ne contient pas de zéros invariants sur l'axe imaginaire y compris à l'infini.*

Nous rappelons dans ce qui suit les deux GARE associées au problème H_2 standard.

$$\begin{cases} E^\top X = X^\top E, \\ A^\top X + X^\top A + C_1^\top C_1 \\ -(C_1^\top D_{12} + X^\top B_2) R_1^{-1} (D_{12}^\top C_1 + B_2^\top X) = 0; \end{cases} \tag{1.59}$$

$$\begin{cases} Y^\top E^\top = EY, \\ AY + Y^\top A^\top + B_1 B_1^\top \\ -(B_1 D_{21}^\top + Y^\top C_2^\top) R_2^{-1} (D_{21} B_1^\top + C_2 Y) = 0. \end{cases} \tag{1.60}$$

Nous définissons la nouvelle notion de "solution quasi-admissible" ci-dessous.

Soient X et Y les solutions des GARE (1.59) et (1.60), respectivement. Définissons

$$F = -R_1^{-1}(D_{12}^\top C_1 + B_2^\top X), \tag{1.61}$$

$$\textit{resp.} \quad L = -(B_1 D_{21}^\top + Y^\top C_2^\top) R_2^{-1}. \tag{1.62}$$

Alors, une solution X (resp. Y) de la GARE (1.59) (resp. (1.60)) est dite une solution quasi-admissible, si $E^\top X \geq 0$ (resp. $Y^\top E^\top \geq 0$), ainsi que la boucle fermée $\left[\begin{array}{c|c} A + B_2 F - sE & B_2 \\ \hline C_1 + D_{12} F & D_{12} \end{array}\right]$ $\left(\textit{resp.} \left[\begin{array}{c|c} A + LC_2 - sE & B_1 + LD_{21} \\ \hline C_2 & D_{21} \end{array}\right]\right)$ est admissible.

Lemme 1.4.1 *Supposons que les hypothèses (H1), (H2), (H4)-(H6) sont satisfaites et il existe,* $U_i \in \mathbb{R}^{(n_g+n_o)\times n_i}$, $V_i \in \mathbb{R}^{(n_g+n_o)\times n_i}$ *et* $F_a \in \mathbb{R}^{m \times n_i}$ *telles que l'équation de Sylvester généralisée* (1.57) *est satisfaite. Alors, la GARE* (1.59) *admet une solution quasi-admissible. En plus, cette solution quasi-admissible peut être construite à partir de la solution admissible* X_c *de la GARE suivante :*

$$\begin{cases} \bar{E}^\top X_c = X_c^\top \bar{E}, \\ \bar{A}_{11}^\top X_c + X_c^\top \bar{A}_{11} + \bar{C}_{11}^\top \bar{C}_{11} \\ -(\bar{C}_{11}^\top D_{12} + X_c^\top \bar{B}_{12}) R_1^{-1} (D_{12}^\top \bar{C}_{11} + \bar{B}_{12}^\top X_c) = 0, \end{cases} \tag{1.63}$$

grâce à la relation suivante :

$$X = \begin{bmatrix} I & U_i \end{bmatrix}^\top X_c \begin{bmatrix} I & V_i \end{bmatrix}. \tag{1.64}$$

Lemme 1.4.2 *Supposons que les hypothèses (H1), (H3)-(H5), (H7) sont satisfaites et il existe, $U_o \in \mathbb{R}^{n_o \times (n_g+n_i)}$, $V_o \in \mathbb{R}^{n_o \times (n_g+n_i)}$ et $L_a \in \mathbb{R}^{n_o \times p}$ telles que l'équation de Sylvester généralisée (1.58) est satisfaite. Alors, la GARE (1.60) admet une solution quasi-admissible. En plus, cette solution quasi-admissible peut être construite à partir de la solution admissible Y_o de la GARE suivante :*

$$\begin{cases} Y_o^\top \hat{E}^\top = \hat{E} Y_o, \\ \hat{A}_{22} Y_o + Y_o^\top \hat{A}_{22}^\top + \hat{B}_{21} \hat{B}_{21}^\top \\ -(\hat{B}_{21} D_{21}^\top + Y_o^\top \hat{C}_{22}^\top) R_2^{-1} (D_{21} \hat{B}_{21}^\top + \hat{C}_{22} Y_o) = 0. \end{cases} \tag{1.65}$$

grâce à la relation suivante :

$$Y = \begin{bmatrix} V_o^\top & I \end{bmatrix}^\top Y_o \begin{bmatrix} U_o^\top & I \end{bmatrix}. \tag{1.66}$$

Lemme 1.4.3 *Supposons que les GARE (1.59) et (1.60) admettent des solutions quasi-admissibles X et Y, et définissons*

$$T_1 := \left[\begin{array}{c|c} \begin{bmatrix} A_F & -B_2 F \\ 0 & A_L \end{bmatrix} - s \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix} & \begin{matrix} B_1 \\ B_L \end{matrix} \\ \hline \begin{matrix} C_F & -D_{12} F \end{matrix} & D_{11} \end{array}\right], \tag{1.67}$$

$$T_2 := \left[\begin{array}{c|c} A_F - sE & B_2 \\ \hline C_F & D_{12} \end{array}\right], \tag{1.68}$$

$$T_3 := \left[\begin{array}{c|c} A_L - sE & B_L \\ \hline C_2 & D_{21} \end{array}\right], \tag{1.69}$$

où

$$A_F = A + B_2 F, \quad A_L = A + L C_2, \quad C_F = C_1 + D_{12} F, \quad B_L = B_1 + L D_{21},$$

avec F et L définis respectivement par (1.61) et (1.62). Alors, les systèmes T_1, T_2 et T_3 sont admissibles.

Théorème 1.4.2 (Commande H_2 étendue) *Supposons que les hypothèses (H1)-(H7) sont satisfaites et les deux équations de Sylvester généralisées admettent des solutions. Alors, le problème de commande H_2 étendue est résolu, si et seulement si, les deux conditions suivantes sont satisfaites.*

(I) *Il existe une matrice constante de dimension compatible Θ telle que $T_1(\infty) - T_2(\infty)\Theta T_3(\infty) = 0$;*

(II) *$(E, A + B_2 F + L C_2 + B_2 \Theta C_2)$ est régulière.*

De plus, le régulateur H_2 optimal est donné par

$$K := \left[\begin{array}{c|c} A + B_2 F + L C_2 + B_2 \Theta C_2 - sE & -B_2 \Theta - L \\ \hline F + \Theta C_2 & -\Theta \end{array}\right], \tag{1.70}$$

avec F et L définis respectivement par (1.61) et (1.62).

1.4.6 Cas de la commande H_∞ étendue

Nous adoptons les mêmes notations que dans les les deux sections précédentes.

Théorème 1.4.3 (Commande H_∞ étendue) *Soit $\gamma > 0$. Supposons que les hypothèses (H1)-(H7) sont satisfaites. Alors, le problème de commande H_∞ étendue est résolu, si et seulement si, les quatre conditions suivantes sont satisfaites.*

(i) *Il existe $U_i \in \mathbb{R}^{(n_g+n_o)\times n_i}$, $V_i \in \mathbb{R}^{(n_g+n_o)\times n_i}$, $F_a \in \mathbb{R}^{m\times n_i}$, $U_o \in \mathbb{R}^{n_o\times(n_g+n_i)}$, $V_o \in \mathbb{R}^{n_o\times(n_g+n_i)}$ et $L_a \in \mathbb{R}^{n_o\times p}$ telles que les deux équations de Sylvester généralisées (1.57) et (1.58) admettent une solution.*

(ii) *La GARE ci-dessous admet une solution admissible X_c*

$$\begin{cases} \bar{E}^\top X_c = X_c^\top \bar{E} \geq 0, \\ \boldsymbol{He}\{(\bar{A}_{11} - \bar{B}_{12}R_1^{-1}D_{12}^\top \bar{C}_{11})^\top X_c\} + \bar{C}_{11}^\top (I - D_{12}R_1^{-1}D_{12}^\top)\bar{C}_{11} \\ +X_c^\top \left(\frac{1}{\gamma^2}(\bar{B}_{11} + U_i\bar{B}_{21})(\bar{B}_{11} + U_i\bar{B}_{21})^\top - \bar{B}_{12}R_1^{-1}\bar{B}_{12}^\top\right) X_c = 0. \end{cases} \tag{1.71}$$

(iii) *La GARE ci-dessous admet une solution admissible Y_o*

$$\begin{cases} Y_o^\top \hat{E}^\top = \hat{E}Y_o \geq 0, \\ \boldsymbol{He}\{(\hat{A}_{22} - \hat{B}_{21}D_{21}^\top R_2^{-1}\hat{C}_{22})Y_o\} + \hat{B}_{21}(I - D_{21}^\top R_2^{-1}D_{21})\hat{B}_{21}^\top \\ +Y_o^\top \left(\frac{1}{\gamma^2}(\hat{C}_{11}V_o - \hat{C}_{12})^\top(\hat{C}_{11}V_o - \hat{C}_{12}) - \hat{C}_{22}^\top R_2^{-1}\hat{C}_{22}\right) Y_o = 0. \end{cases} \tag{1.72}$$

(iv) *$\rho(YX) < \gamma^2$, avec*

$$X = \begin{bmatrix} I & U_i \end{bmatrix}^\top X_c \begin{bmatrix} I & V_i \end{bmatrix}, \quad Y = \begin{bmatrix} -V_o^\top & I \end{bmatrix}^\top Y_o \begin{bmatrix} -U_o^\top & I \end{bmatrix}. \tag{1.73}$$

De plus, l'ensemble des régulateurs solutions de ce problème peut être paramétré par

$$K_\infty = F_l(J_\infty, Q_\infty), \tag{1.74}$$

où

$$J_\infty := \left[\begin{array}{c|cc} A_\infty - sE & B_{1\infty} & B_{2\infty} \\ \hline C_{1\infty} & 0 & R_1^{-1}D_{12}^\top \\ C_{2\infty} & D_{21}^\top R_2^{-1} & 0 \end{array}\right], \tag{1.75}$$

avec

$$A_\infty = A + B_2C_{1\infty} - B_{1\infty}C_2 + \frac{1}{\gamma^2}(B_1 - B_{1\infty}D_{21})B_1^\top X, \tag{1.76a}$$

$$Z = (I - \frac{1}{\gamma^2}YX)^{-1}Y, \tag{1.76b}$$

$$B_{1\infty} = Z^\top(C_2 + \frac{1}{\gamma^2}D_{21}B_1^\top X)^\top R_2^{-1} + B_1D_{21}^\top R_2^{-1}, \tag{1.76c}$$

$$B_{2\infty} = (B_2 - Z^\top C_{1\infty}^\top)R_1^{-1}D_{12}^\top, \tag{1.76d}$$

$$C_{1\infty} = -R_1^{-1}(B_2^\top X + D_{12}^\top C_1), \tag{1.76e}$$

$$C_{2\infty} = -D_{21}^\top R_2^{-1}(C_2 + \frac{1}{\gamma^2}D_{21}B_1^\top X). \tag{1.76f}$$

et $Q_\infty \in RH_\infty$ est un paramètre libre qui vérifie $\|Q_\infty\|_\infty < \gamma$.

1.5 Commande sous contrainte de régulation

Le sujet abordé ici revêt une grande importance dans la théorie de la commande linéaire. L'objectif principal du problème de commande sous contrainte de régulation concerne la détermination d'un régulateur stabilisant de manière interne un système donné tout en garantissant que la sortie de la boucle fermée correspondante converge vers ou suit un signal de référence prédéfini, en présence de perturbations externes. Ces signaux de références et de perturbations externes sont généralement représentés par des exo-systèmes (ou systèmes exogènes).

Pour résoudre le probléme de régulation, un résultat séminal, dit “Principe du Modèle Interne”, a été développé dans les années 1970 [FSW74, FW75]. Basé sur ce principe, le rejet ou suivi asymptotiques sont réalisés par un régulateur structuré qui contient une copie des dynamiques de l'exo-système en question. D'autres facettes associées à ce problème ne se limitent pas seulement au principe du modèle interne, au caractère de bien posé, et à la stabilité structurée, qui ont fait l'objet de nombreuses recherches pendant les années 1960, 1970 et les décennies suivantes. Des extensions du principe du modèle interne ont été considérées en intégrant d'autres objectifs ou critères de performance, H_2 ou H_∞ à titre d'exemple De tels problèmes multi-objectifs ont été largement étudiés dans la littérature, voir [ANP94, ANKP95, HHF97, SSS00a, SSS00b, KS08, KS09] et les références incluses.

Par ailleurs, le problème de commande sous contrainte de régulation a été aussi étendu au cas des systèmes implicites. Par exemple, Dai a proposé une solution en termes d'un ensemble d'équations matricielles non-linéaires dépendantes des coefficients du système et d'autres paramètres dans [Dai89]. Une solution plus concise a été obtenue par résolution d'une équation de Sylvester généralisée dans [LD96]. En outre, dans [IK05], les auteurs ont aussi abordé le problème de commande sous contrainte de régulation pour le cas des systèmes implicites à coefficients périodiques et quasi-périodiques.

Nous présentons ici un problème de commande multicritères non-standard pour des systèmes implicites continus. Pour ce type de problème, une sortie doit être régulée asymptotiquement en présence d'un exo-système à énergie infinie, en même temps une performance H_2 ou H_∞ entre une perturbation externe finie et l'écart de sortie doit être réalisée.

Nous montrons par la suite que l'objectif de régulation asymptotique est atteint, si et seulement si, une équation de Sylvester généralisée associée au système implicite en considération et l'exo-système correspondant admet une solution. Ensuite, nous prouvons aussi que chaque régulateur solution du problème de régulation proposé contient une structure spécifique. En utilisant cette structure, nous arrivons à réduire le problème de commande multicritères à un problème standard dont la solution peut être obtenue et caractérisée par une GARE ou un problème d'optimisation sous contraintes

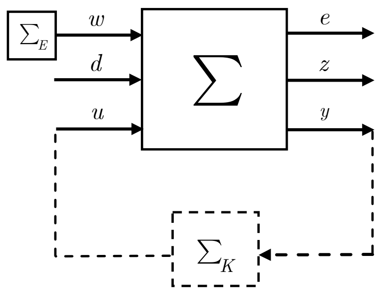

Figure 1.4: Performance de commande sous contrainte de régulation

LMI.

1.5.1 Formalisation du problème

Soit un système implicite décrit par la réalisation suivante :

$$(\Sigma):\begin{bmatrix} E\dot{x} \\ \hline e \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|ccc} A & B_w & B_d & B \\ \hline C_e & D_{ew} & D_{ed} & D_{eu} \\ C_z & D_{zw} & D_{zd} & D_{zu} \\ C & D_{yw} & D_{yd} & 0 \end{array}\right]\begin{bmatrix} x \\ \hline w \\ d \\ u \end{bmatrix} \tag{1.77}$$

où $e \in \mathbb{R}^{q_e}$, $z \in \mathbb{R}^{q_z}$, $y \in \mathbb{R}^{p}$, $w \in \mathbb{R}^{n_w}$, $d \in \mathbb{R}^{m_d}$ et $u \in \mathbb{R}^{m}$ sont respectivement les vecteurs d'écarts de sortie, de sorties à contrôler, de sorties mesurées, de perturbations exogènes (associé à l'exo-système), de perturbations externes et de commandes. La perturbation exogène w est générée par un exo-système Σ_E qui est sopposé être implicite :

$$(\Sigma_E): E_w\dot{w} = A_w w, \tag{1.78}$$

où la matrice E_w peut être singulière, i.e. $rank(E_w) = r_w \leq n_w$. La configuration du système et l'exo-système sont donnés par le schéma de la Fig. 1.4.

En introduisant le nouveau vecteur de variables descripteurs $\zeta^\top = [x^\top \; w^\top]$, nous obtenons le système augmenté G ci-dessous :

$$(G):\left\{\begin{bmatrix} E & 0 \\ 0 & E_w \end{bmatrix}, \left[\begin{array}{cc|cc} A & B_w & B_d & B \\ 0 & A_w & 0 & 0 \\ \hline C_e & D_{ew} & D_{ed} & D_{eu} \\ C_z & D_{zw} & D_{zd} & D_{zu} \\ C & D_{yw} & D_{yd} & 0 \end{array}\right]\right\} := \begin{bmatrix} G_{ed}(s) & G_{eu}(s) \\ G_{zd}(s) & G_{zu}(s) \\ G_{yd}(s) & G_{yu}(s) \end{bmatrix}. \tag{1.79}$$

Nous cherchons un régulateur qui admet aussi une réalisation implicite :

$$(\Sigma_K):\begin{cases} E_K\dot{\xi} = A_K\xi + B_K y, \\ u = C_K\xi + D_K y, \end{cases} \tag{1.80}$$

où $E_K \in \mathbb{R}^{n_k \times n_k}$ peut être singulière, i.e. $rang(E_K) = r_k \leq n_k$.

Problème 1.5.1 (commande H_2 ou H_∞ sous contrainte de régulation) *Le problème de commande H_2 ou H_∞ sous contrainte de régulation consiste à trouver un régulateur Σ_K tel que la boucle fermée formée par G et Σ_K vérifie les conditions suivantes :*

C.1 *(Stabilité interne) En absence des perturbations w et d, la boucle fermée est stable de manière interne;*

C.2 *(Régulation asymptotique) $\lim_{t\to\infty} e(t) = 0$ pour tout $d \in L_2$, et tout $x(0) \in \mathbb{R}^n$ et $w(0) \in \mathbb{R}^{n_w}$;*

C.3 *(Performance) Soit $\gamma > 0$. La norme H_2 ou H_∞ de la boucle fermée définie par*

$$T_{zd} = G_{zd} + G_{zu}\Sigma_K(I - G_{yu}\Sigma_K)^{-1}G_{yd}, \tag{1.81}$$

satisfait $\|T_{zd}\|_p < \gamma$, $p = 2, \infty$.

1.5.2 Commande H_2 sous contrainte de régulation : retour d'état

Un cas particulier retient notre attention. Il s'agit de la commande par retour d'état H_2 optimal sous contrainte de régulation. Nous nous sommes intéressés à ce cas particulier partant du constat que le problème de commande par retour d'état H_2 optimal dans le cas implicite n'admet pas forcément une solution unique voire statique. Une paramétrisation de l'ensemble des régulateurs solutions dans ce cas a été proposée par [IT02]. Motivé par ces travaux nous avons souhaité donner une paramétrisation de l'ensemble des retours d'état (statiques et dynamiques) solutions du problème de commande H_2 sous contrainte de régulation.

Avant de présenter succinctement les résultats obtenus, nous définissons les notions suivantes : U, V, E_L et E_R sont respectivement des matrices de rang plein par colonne vérifiant $U^\top E = 0$, $VE = 0$, $E_L^\top E = 0$ et $E_R^\top E = 0$. Sous ces définitions, la matrice E est décomposée sous forme SVD telle que $E = E_L \Omega E_R^\top$, avec $\Omega \in \mathbb{R}^{r\times r}$ non-singulière. Nous définissons aussi la matrice $M = \begin{bmatrix} \Omega^{-1}(E_L^\top E_L)^{-1}E_L^\top \\ U^\top \end{bmatrix}$.

Nous considérons les hypothèses ci-dessous :

Hypothèses 1.5.1

H1 (E, A, B) *est à dynamique finie stabilisable et état impulsif commandable;*

H2 *L'exo-système Σ_E ne contient que des modes instables et impulsifs;*

H3 (E, A) *est régulière ;*

H4 D_{zu} *est de rang plein par colonne;*

H5 $\begin{bmatrix} A - j\omega E & B \\ C_z & D_{zu} \end{bmatrix}$ *ne contient pas de zéros invariants sur l'axe imaginaire y compris à l'infini;*

H6 $Ker \begin{bmatrix} U^\top AV & U^\top B \\ C_e V & D_{zu} \end{bmatrix}^\top \subseteq Ker \begin{bmatrix} U^\top B_d \\ D_{zd} \end{bmatrix}^\top$;

H7 $Ker\ U^\top B_d \subseteq Ker D_{zd}$.

Théorème 1.5.1 *Soit le système implicite G (1.79). Supposons que les hypothèses (H1)-(H7) sont satisfaites. Alors, le problème H_2 optimal par retour d'état sous contrainte de régulation admet une solution, si et seulement si, il existe des matrices $R \in \mathbb{R}^{n \times n_w}$, $T \in \mathbb{R}^{n \times n_w}$ et $\Pi \in \mathbb{R}^{m \times n_w}$ telles que*

$$B\Pi = AT - B_w - RA_w, \tag{1.82a}$$

$$D_{eu}\Pi = C_e T - D_{ew}, \tag{1.82b}$$

$$RE_w = ET. \tag{1.82c}$$

De plus, l'ensemble des régulateurs solutions de ce problème est paramétré comme suit

$$F(s) = \begin{bmatrix} \mathcal{F}(s) & \Pi + \mathcal{F}(s)T \end{bmatrix}, \tag{1.83}$$

avec

$$\mathcal{F}(s) := F_c + (I + (\Psi + W(s)\Upsilon)B)^{-1} (\Psi + W(s)\Upsilon)(sE - A - BF_c), \tag{1.84}$$

où

i) $F_c := -(D_{zu}^\top D_{zu})^{-1}(D_{zu}^\top C_z + B^\top X)$, *et X est une solution admissible de la GARE suivante*

$$\begin{cases} E^\top X = X^\top E, \\ A^\top X + X^\top A + C_z^\top C_z \\ -(C_z^\top D_{zu} + X^\top B)(D_{zu}^\top D_{zu})^{-1}(D_{zu}^\top C_z + B^\top X) = 0; \end{cases} \tag{1.85}$$

ii) $\Upsilon := I - B_d B_d^\dagger$, *où $B_d^\dagger$ est la pseudo-inverse de B_d;*

iii) $\Psi := -\begin{bmatrix} 0 & \Theta \left(U^\top (A + BF_c)V\right)^{-1} \end{bmatrix} M$, *où Θ est la solution de l'équation :*

$$\begin{aligned} &\left(D_{zu} - (C_z + D_{zu}F_c)V \left(U^\top (A + BF_c)V\right)^{-1} U^\top B\right) \\ &\qquad \Theta \left(U^\top (A + BF_c)V\right)^{-1} U^\top B_d \\ &= D_{zd} - (C_z + D_{zu}F_c)V \left(U^\top (A + BF_c)V\right)^{-1} U^\top B_d; \end{aligned} \tag{1.86}$$

iv) $W(s) \in RH_\infty$ *tel que* $det\,(I + (\Psi + W(s)\Pi)B) \neq 0$.

En plus, la valeur minimale de la norme H_2 de la boucle fermée est donnée par $\|T_{zd}\|_2 = \|G_{F_c\Psi}\|_2$, où

$$(G_{F_c\Psi}) : \left\{ E, \left[\begin{array}{c|c} A + BF_c & B_d + B\Psi B_d \\ \hline C_z + D_{zu}F_c & D_{zd} + D_{zu}\Psi B_d \end{array} \right] \right\}. \tag{1.87}$$

1.5.3 Une solution LMI au problème de commande sous contrainte de régulation

Nous adoptons ici les mêmes définitions que les sections précédentes. Nous considérons en outre une hypothèse supplémentaire :

Hypothèse 1.5.1

H8 $\left(\begin{bmatrix} E & 0 \\ 0 & E_w \end{bmatrix}, \begin{bmatrix} A & B_w \\ 0 & A_w \end{bmatrix}, \begin{bmatrix} C & D_{yw} \end{bmatrix} \right)$ *est à dynamique finie détectable et état impulsif observable.*

Lemme 1.5.1 (Structure du régulateur) *Soit le système G (1.79). Supposons que les hypothèses (H1), (H2), (H8) sont satisfaites. Les conditions C.1 et C.2 du problème de commande sous contrainte de régulation sont satisfaites par un régulateur dynamique Σ_K (1.80), si et seulement si, il existe des matrices $R \in \mathbb{R}^{n \times n_w}$, $T \in \mathbb{R}^{n \times n_w}$ et $\Pi \in \mathbb{R}^{m \times n_w}$ telles que l'équation de Sylvester généralisée (1.82) admet une solution. Sous cette condition, une réalisation du régulateur est donnée par*

$$\left\{ \begin{array}{l} \begin{bmatrix} E_w & 0 \\ 0 & \tilde{E}_k \end{bmatrix} \dot{\xi} = \begin{bmatrix} A_w + \tilde{D}_{k2}(CT - D_{yw}) & \tilde{C}_{k2} \\ \tilde{B}_k(CT - D_{yw}) & \tilde{A}_k \end{bmatrix} \xi + \begin{bmatrix} \tilde{D}_{k2} \\ \tilde{B}_k \end{bmatrix} y, \\ u = \begin{bmatrix} \Pi + \tilde{D}_{k1}(CT - D_{yw}) & \tilde{C}_{k1} \end{bmatrix} \xi + \tilde{D}_{k1} y, \end{array} \right. \tag{1.88}$$

où $\tilde{E}_k$, $\tilde{A}_k$, $\tilde{B}_k$, $\tilde{C}_{k1}$, $\tilde{C}_{k2}$, $\tilde{D}_{k1}$ et $\tilde{D}_{k2}$ sont matrices du régulateur $\tilde{\Sigma}_c$

$$(\tilde{\Sigma}_c) : \left\{ \begin{array}{l} \tilde{E}_k \dot{x}_c = \tilde{A}_k x_c + \tilde{B}_k y_c, \\ u_c = \begin{bmatrix} \tilde{C}_{k1} \\ \tilde{C}_{k2} \end{bmatrix} x_c + \begin{bmatrix} \tilde{D}_{k1} \\ \tilde{D}_{k2} \end{bmatrix} y_c, \end{array} \right. \tag{1.89}$$

stabilisant de manière interne le système $\tilde{G}$ ci-dessous :

$$(\tilde{G}) : \left\{ \begin{bmatrix} E & 0 \\ 0 & E_w \end{bmatrix}, \left[\begin{array}{cc|ccc} A & B\Pi & B_d & B & 0 \\ 0 & A_w & 0 & 0 & I \\ \hline C_e & D_{eu}\Pi & D_{ed} & D_{eu} & 0 \\ C & CT - D_{yw} & D_{yd} & 0 & 0 \end{array} \right] \right\}. \tag{1.90}$$

Nous avons donc proposé une structure du régulateur qui assure les deux premières conditions du problème de commande considéré. Considérons dans la suite cette structure (1.88) et le système augmenté décrit ci-dessous

$$(\tilde{\mathcal{G}}) : \left\{ \begin{array}{rcl} \mathcal{E}\dot{\bar{\zeta}} & = & \mathcal{A}\bar{\zeta} + \mathcal{B}_d \bar{d} + \mathcal{B}(R) u_c, \\ \bar{z} & = & \mathcal{C}_z(T, \Pi)\bar{\zeta} + D_{zd}\bar{d} + \mathcal{D}_{zu} u_c, \\ y_c & = & \mathcal{C}\bar{\zeta} + D_{yd}\bar{d}, \end{array} \right. \tag{1.91}$$

où

$$\mathcal{E} = \begin{bmatrix} E & 0 \\ 0 & E_w \end{bmatrix}, \mathcal{A} = \begin{bmatrix} A & -B_w \\ 0 & A_w \end{bmatrix}, \mathcal{B}_d = \begin{bmatrix} B_d \\ 0 \end{bmatrix}, \mathcal{B}(R) = \begin{bmatrix} B & R \\ 0 & I \end{bmatrix}, \tag{1.92a}$$

$$\mathcal{C}_z(T,\Pi) = \begin{bmatrix} C_z & D_{zu}\Pi - C_zT \end{bmatrix}, \mathcal{C} = \begin{bmatrix} C & -D_{yw} \end{bmatrix}, \mathcal{D}_{zu} = \begin{bmatrix} D_{zu} & 0 \end{bmatrix}. \tag{1.92b}$$

Théorème 1.5.2 *Soit $\gamma > 0$. Il existe un régulateur dynamique $\tilde{\Sigma}_c$ tel que la norme H_∞ de la boucle fermée formée par $\tilde{\Sigma}_c$ (1.89) and $\tilde{\mathcal{G}}$ (1.91) est inférieure à γ, si et seulement si, il existe des matrices $R \in \mathbb{R}^{n\times n_w}$, $T \in \mathbb{R}^{n\times n_w}$, $\Pi \in \mathbb{R}^{m\times n_w}$, $X \in \mathbb{R}^{(n+n_w)\times(n+n_w)}$, $Y \in \mathbb{R}^{(n+n_w)\times(n+n_w)}$, $W \in \mathbb{R}^{(n+n_w)\times m_d}$ et $Z \in \mathbb{R}^{m_d\times(n+n_w)}$ telles que l'équation de Sylvester généralisée (1.82) est satisfaite, ainsi que les LMI/LME suivantes :*

$$\begin{bmatrix} \mathcal{E}^\top & 0 \\ 0 & \mathcal{E} \end{bmatrix}\begin{bmatrix} X & M^{-\top} \\ N & Y \end{bmatrix} = \begin{bmatrix} X^\top & N^\top \\ M^{-1} & Y^\top \end{bmatrix}\begin{bmatrix} \mathcal{E} & 0 \\ 0 & \mathcal{E}^\top \end{bmatrix} \geq 0, \tag{1.93}$$

$$\mathcal{E}^\top W = 0, \quad \mathcal{E}Z^\top = 0, \tag{1.94}$$

$$\begin{bmatrix} \mathcal{N}_o & 0 \\ 0 & I \end{bmatrix}^\top \begin{bmatrix} \mathcal{A}^\top X + X^\top\mathcal{A} & X^\top\mathcal{B}_d + \mathcal{A}^\top W & \mathcal{C}_z(T,\Pi)^\top \\ \bullet & W^\top\mathcal{B}_d + \mathcal{B}_d^\top W - \gamma^2 I & D_{zd}^\top \\ \bullet & \bullet & -I \end{bmatrix}\begin{bmatrix} \mathcal{N}_o & 0 \\ 0 & I \end{bmatrix} < 0, \tag{1.95}$$

$$\begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix}^\top \begin{bmatrix} A^\top\mathcal{Y}^\top + \mathcal{Y}A & \mathcal{Y}^\top C_z^\top & B_d + A\mathcal{Z}^\top \\ \bullet & -\gamma^2 I & C_z\mathcal{Z}^\top + D_{zd} \\ \bullet & \bullet & -I \end{bmatrix}\begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix} < 0, \tag{1.96}$$

où $M = \begin{bmatrix} I & R \\ 0 & I \end{bmatrix}$, $N = \begin{bmatrix} I & -T \\ 0 & I \end{bmatrix}$, $\mathcal{N}_o = Ker\left(\begin{bmatrix} \mathcal{C} & D_{yd} \end{bmatrix}\right)$, $N_c = Ker\left(\begin{bmatrix} B^\top & D_{zu}^\top \end{bmatrix}\right)$, $\mathcal{Y} = \begin{bmatrix} I & 0 \end{bmatrix} Y \begin{bmatrix} I \\ 0 \end{bmatrix}$ and $\mathcal{Z} = Z\begin{bmatrix} I \\ 0 \end{bmatrix}$, soient vérifiées.

1.6 Conclusion et perspectives

Les travaux présentés dans la présente synthèse s'inscrivent dans le cadre de la commande H_2-H_∞ non standards des systèmes implicites, linéaires continus à temps invariant.

L'étude qui nous avons menée a été effectuée à l'Institut de Recherche en Communications et Cybernétique de Nantes (IRCCyN).

Nous résumons dans la suite les principales contributions de ce travail de thèse. Nous nous intéressons aussi à en présenter les perspectives qui nous semblent possibles et envisageables.

1.6.1 Conclusion

Dans ce mémoire de thèse, nous avons, dans un premier temps, montré l'intérêt particulier que revêt l'étude des systèmes implicites, et aussi donné une sorte de revue globale sur les développements théoriques et pratiques concernant le champ d'application des systèmes implicites. Nous avons ensuite rappelé les notions et les propriétés caractéristiques de ces systèmes. Un ensemble d'outils importants et nécessaires au traitement des différents problèmes de commande non standards abordés dans cette thèse est introduit dans le chapitre 4. Ces résultats concernent la caractérisation de la performance dissipative, les caractérisations de l'admissibilité et des performances H_2 et H_∞ sous forme de problèmes de faisabilité ou d'optimisation sous contraintes LMI étendues, l'équation de Sylvester généralisée et l'équation algébrique de Riccati généralisée.

Ces développements généralisent certains concepts correspondants aux systèmes d'état "classiques" et servent comme outils pour les développements objet des chapitres 5-7, de cette thèse.

Notons que la nouvelle condition KYP proposée est caractérisée par une LMI stricte. Ceci permet d'enlever la contrainte d'égalité existant dans la caractérisation classique. Cette différence permet d'éviter des problèmes numériques potentiels à cause des erreurs de troncature. Par ailleurs, nous avons revisité différentes caractérisations de stabilité et de performances par des LMI étendues en utilisant le lemme de projection. Ces nouvelles formulations couvrent ainsi les LMI étendues connues dans la littérature et fournissent des caractérisations intéressantes nouvelles à notre connaissance, en dépit d'une littérature abondante sur le sujet. Ces nouvelles caractérisations peuvent réduire le conservatisme intrinsèque aux problèmes d'analyse robuste et de synthèse multicritères. Dans ce même chapitre nous proposons des procédures numériques permettant de résoudre les équations de Sylvester généralisées et nous rappelons aussi la méthode numérique de résolution des GARE.

Dans le chapitre 5 nous abordons le problème de commande H_∞ simultanée dans le cadre des systèmes implicites. Nous généralisons ensuite les résultats récents sur ce sujet dans le cadre des modèles d'états classiques. Considérant la factorisation copremière des systèmes implicites en question, nous avons prouvé, nous appuyant sur les résultats existants, que le problème de commande H_∞ simultanée est équivalent à un problème de commande H_∞ forte associé à un système implicite augmenté. Nous avons par ailleurs, développé une nouvelle condition suffisante permettant la résolution du problème de commande H_∞ forte sous-optimale. La condition proposée fait un usage combiné d'une GARE et d'un ensemble de contraintes LMI strictes. Ce résultat, moins conservatif que les résultats existants, même dans le cas de systèmes explicites, permet de résoudre avec un conservatisme limité le problème de commande H_∞ simultanée des systèmes implicites.

Les chapitres 6 et 7 traitent respectivement du problème de commande étendue et du problème de commande sous contrainte de régulation. Pour ces deux problèmes, la stabilité interne ne peut plus être réalisée à cause d'éléments non-stabilisables ou non détectables introduits soit par des pondérations fréquentielles soit par des exosystèmes non stables ou impulsifs. Par conséquent, la notion d'admissibilité étendue (une extension de la notion de stabilité compréhensive) est adoptée. Cette nouvelle définition induit la stabilité interne uniquement d'une partie de la boucle fermée.

Concernant le problème de commande étendue, nous avons tout d'abord exhibé les conditions sous lesquelles la stabilité étendue est satisfaite. Ces conditions sont données sous formes de deux équations de Sylvester généralisées. Une paramétrisation de l'ensemble des régulateurs garantissant l'admissibilité étendue a été introduite. Partant de ces résultats nous avons traité les problèmes de commande H_2 et H_∞ sous contrainte de stabilité étendue. La relaxation des hypothèses standard, et l'introduction de la notion de "solution quasi-admissible" ont permis de résoudre de manière analytique et exacte, sans approximation ni tranformation de boucle, les problèmes H_2 et H_∞ sous contrainte de stabilité étendue. La solution est donnée ici en fonction des solutions quasi-admissibles de deux GARE et des solutions de deux équations de Sylvester généralisées. Là encore la paramétrisation de l'ensemble des régulateurs solution a été présentée.

Dans le cadre du problème de commande sous contrainte de régulation dans le cas implicite, nous nous sommes intéressés à l'extension du principe du modèle interne. Nous avons ainsi proposé la structure du régulateur solution et nous avons montré que la contrainte de régulation ne peut être satisfaite que sous la condition nécessaire et suffisante décrite sous forme d'une équation de Sylvester généralisée associée au système physique et à l'exo-système considéré. Par ailleurs, le cas de performances H_2 ou H_∞ sous contrainte de régulation a été traité sous les formalismes LMI et Riccati.

1.6.2 Perspectives

Nous proposons comme perspectives à ces travaux de thèse de traiter trois points ouverts qui nous semblent être des verrous théoriques importants.

La solution proposée dans le cadre du problème de commande H_∞ forte représente une condition suffisante qui induit forcement un certain conservatisme. Ce dernier vient principalement du fait qu'une matrice de Lyapunov unique est utilisée pour prouver la stabilité de la boucle fermée et du régulateur. Ce choix particulier permet de relaxer le problème BMI sous-jacent mais induit une perte de certains degrés de liberté quant à la synthèse du régulateur. Partant du fait que ce problème est équivalent à un problème de commande multi-objectif, nous estimons qu'une piste éventuelle de réduction de ce conservatisme serait de développer une approche par LMI étendues permettant l'utilisation de matrices de Lyapunov différentes pour la boucle fermée et pour le régulateur lors de la synthèse.

Le deuxième point que nous souhaitons aborder à la suite de ces travaux de thèse concerne le problème de commande étendue introduit dans le chapitre 6. Ce problème tel que défini est limité au cas régulier. Nous supposons que les deux systèmes $\widetilde{G}_{eu}$ et $\widetilde{G}_{yv}$ inclus dans la définition du modèle standard $\widetilde{G}$ (1.45) ne contiennent pas de zéros invariants sur l'axe imaginaire y compris à l'infini. Cette hypothèse limite naturellement l'étendue des résultats développés. En effet, les problèmes de commande singuliers dans le cadre des systèmes implicites n'ont pas encore été complètement résolus. Les approches LMI [RA99, ILU00, Mas07, XL06] existant dans la littérature ne possèdent pas cette restriction mais aboutissent à des régulateurs propres qui sont des solutions sous-optimales. Dans le cas explicite, il est prouvé par [Sto92, CS92b] que le régulateur H_2 ou H_∞ optimal solution des problèmes singuliers peut être impropre (strictement). Une perspective naturelle à ce travail de thèse consiste à chercher à caractériser l'ensemble des régulateurs H_2 et H_∞ optimaux dans le cas des problèmes de commande étendue singuliers. Ceci est envisageable par la résolution de la GARE associée à un faisceau hamiltonien singulier.

Le troisième point concerne les problèmes de commande H_2 ou H_∞ étendue et les problèmes de commande H_2 ou H_∞ sous contrainte de régulation (asymptotique ou non) pour un système LPV (linéaire à paramètres variant) avec des pondérations (respectivement un exo-système) ne dépendant pas des paramètres voire le cas où le système est à temps invariant mais les pondérations (respectivement l'exo-système) sont dépendants des paramètres. Dans ce cadre, la résolution de LMI/LME dépendant des paramètres ainsi que leurs relaxations éventuelles peuvent être investiguées.

Chapter 2

Introduction

2.1 Why Differential Algebraic Equations

A classical dynamic system is, in systems and control theory, often considered as a set of *ordinary differential equations* (ODEs), which describe relations between the system variables (usually known as state variables). For the most general purpose of system analysis, the first order system described as follows is widely used [BCP96]:

$$\boldsymbol{F}\left(\dot{x}(t), x(t)\right) = 0, \tag{2.1}$$

where $\boldsymbol{F}$ and x are vector-value functions. The form (2.1) contains not only differential equations, but also a set of algebraic constraints. It is referred to as *differential algebraic equations* (DAEs).

For control engineering, it is usually assumed that the considered ODEs can be expressed in an explicit form

$$\dot{x}(t) = \boldsymbol{f}\left(x(t)\right), \tag{2.2}$$

where $\boldsymbol{f}$ is a vector-value function. A set of ODEs of the form (2.2) is generally referred to a state-space description. This representation has been the predominant tool in systems and control theory, on which many theorems and techniques have been based.

One notes that a state-space system model is obtained on the assumption that the plant is governed by the causality principle. However, in certain cases, the state in the past may depend on its state in the future, which violates the causality assumption. There are practical situations in which:

i) physical variables can not be chosen as state variables in a natural way to meet the form (2.2),

ii) physical senses of variables or coefficients are lost after transformation to (2.2).

Even in the area of signal processing where significant results have also expressed the filter in the state-space form, the limitations of the use of the state space system model have been recognized by some scholars. As pointed out in [HCW07], the analysis of the rounding effect of a specific coefficient in a particular realization form can become very difficult after transformation to the state-space form. Moreover, many realization forms require the computation of intermediate variables that cannot be expressed in the state-space form.

Here, an example is given to show the limitations of the use of state-space systems. Let us consider an economic process where n interrelated production sectors are involved [Lue77b]. The relationships between the production levels of the sectors can be described by the *Leontieff Model* of the form:

$$x(k) = Ax(k) + Ex(k+1) - Ex(k) + u(k), \tag{2.3}$$

where $x(k) \in \mathbb{R}^n$ is the vector of production level of the sectors at time k. $Ax(k)$ is interpreted as the capital required as direct input for production of x, and a coefficient a_{ij} of A called the *flow coefficient matrix* indicates the amount of product i needed to produce one unit of product j. $Ex(k+1) - Ex(k)$ stands for the stocked capital for producing x in the next time period, and a coefficient e_{ij} of E called the *stock coefficient matrix* indicates the amount of product i that has to be in stock in order to produce one unit of product j in the next time period. Moreover, $u(k)$ is the demanded production level. The type of econometric model shown in (2.3) was firstly studied by Leontieff and both continuous-time and discrete-time cases were considered in [Leo53].

The stock coefficient matrix E is, in general, quite sparse, and most of its entries are zero which means that E is often singular. The singularity of E can be explained by the fact that the productions of one sector do not generally require the capital in stock from all the other sectors. In addition, in many cases, there are few sectors offering capital in stock to other sectors. The equation (2.3) can be rewritten as:

$$Ex(k+1) = (I - A + E)x(k) - u(k), \tag{2.4}$$

which is similar to, but not exactly the representation given in (2.2). If the matrix E is invertible, we can left-multiply the above equation by E^{-1}, and then a state-space model can be obtained. For the case where E is singular, it is clear that this economic process can not be represented by a state-space model via simply inverting the matrix E. In fact, the feasibility of casting this process onto a state-space model depends on the properties of the matrix pencil $(E, I - A + E)$, which will be discussed in quite some details in Section 3.5 of Chapter 3.

This is one of the concrete examples for which the conventional state-space form fails to give a representation. Another example concerning an electrical circuit which cannot be described by a state-space form either will be given in the next section.

2.2 Descriptor Systems

Let us decompose the DAEs (2.1) into two parts:

$$\dot{x}(t) = \phi(x(t)), \tag{2.5a}$$
$$0 = \varphi(x(t)), \tag{2.5b}$$

where ϕ and φ are both vector-value functions. Compared with the form (2.2), the DAEs in (2.5) contain not only differential equations, which are also included in (2.2), but also the algebraic constrains that do not exist in (2.2).

For a linear time-invariant system, the second equation related to the algebraic constraints in (2.5) concerns the static properties and impulsive behaviors of the system. These two concepts will be detailed in Chapter 3. Thanks to this add-on, the systems for which the writing of (2.2) is impossible or undesirable can be represented by the DAEs (2.5).

Dynamic systems of the form (2.5) have different nomenclature depending on the research fields. For example, control theorists and mathematicians call them *singular systems* [Dai89, Lew86, Ail89, Cob84, XL06] due to the fact that the matrix on the derivative of the state ("generalized state" is more appropriate in this case), that is E in (2.4), is generally singular. They sometimes use the name *generalized (extended) state-space systems* [Ail87, VLK81, Cam84, HFA86] since the form (2.5) can be viewed as a generalization (extension) of state-space systems. In the engineering economic systems community, the terminology *descriptor system* [Lue77a, BL87, HM99b, WYC06] is most frequently adopted for the reason that the form (2.5) offers a fairly natural description of systems' properties; while numerical analysts like to call this representation *differential algebraic equations* [BCP96, GSG+07, KM06]. Besides, in circuit theory, the form (2.5) is named a *semistate system* [ND89, RS86] because it describes "almost state" of the underlying system. Sometimes the term *implicit system* [SGGG03, IS01] is also used by some researchers to mention systems of form (2.5). Throughout this dissertation, the name descriptor system will be used and the studies will focus on LTI dynamic descriptor systems.

Descriptor systems defined by DAEs do not evidently belong to the class of ODEs since an ODE does not include any algebraic constraints. Hence, descriptor systems contain conventional state-space systems as a special case and behave much more powerful in the terms of system modeling than their state-space counterpart. Compared with state-space systems, they can not only preserve the structure of physical systems, but also describe static constraints and impulsive behaviors. Such systems arise in real systems, for instance, large-scaled systems networks [Lue77a, SL73], electrical circuits [ND89], boundary control systems [Pan90], power systems [Sto79], economic systems [Lue77a, Lue77b], chemical processes [KD95], mechanical engineering systems [HW79], robotics [MG89] and aircraft modeling [SL91].

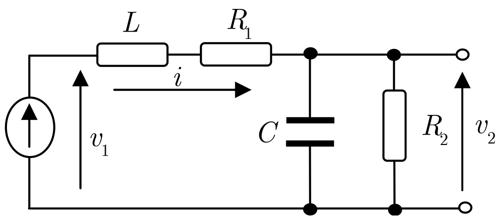

Figure 2.1: Electrical circuit

Generally speaking, the following features of descriptor systems are not found in state-space systems [YS81, VLK81, BL87]:

- The transfer function of a descriptor system may be not proper;
- For an arbitrary initial condition, the time response of a descriptor system may be impulsive or non-causal;
- A descriptor system generally contains three types of modes: finite dynamic modes, infinite dynamic modes (related to impulsive behaviors) and static modes.
- Even if a descriptor system is impulse-free, it can still possess finite discontinuities due to inconsistent initial conditions.

Example 2.1 (Electrical Circuit) *[ZHL03]*

Consider the electrical circuit depicted by Figure 2.1. In this circuit, L and C are the inductance and capacitance, respectively, while $v_k(k = 1, 2)$ and i denote the voltages and current flow, respectively. According to Ohm's law and Kirchhoff's circuit laws, we can deduce the following differential equations:

$$v_1 = v_2 + R_1 i + L\frac{\partial i}{\partial t}, \tag{2.6a}$$

$$C\frac{\partial v_2}{\partial t} = -\frac{v_2}{R_2} + i. \tag{2.6b}$$

Here, we assume that $i = u + w$, where u is the control input and w is the white noise disturbance with zero mean and unit intensity. We also define the controlled output as $z = v_2$ and the measured output as $y = v_1 + v_2$. Hence, this dynamic system has the

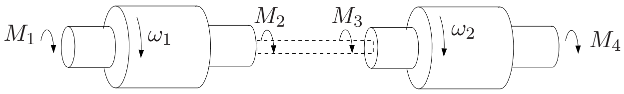

Figure 2.2: Two interconnected rotating masses

form of

$$\begin{bmatrix} L & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{i} \\ \dot{v}_2 \\ \dot{v}_1 \end{bmatrix} = \begin{bmatrix} -R_1 & -1 & 1 \\ 0 & -1/R_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i \\ v_2 \\ v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} w + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} u, \tag{2.7a}$$

$$z = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i \\ v_2 \\ v_1 \end{bmatrix}, \tag{2.7b}$$

$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} i \\ v_2 \\ v_1 \end{bmatrix}. \tag{2.7c}$$

It is observed that the matrix on the derivative of the state is singular, hence the electrical circuit cannot be represented by a conventional state-space system. Moreover, if we calculate the generalized eigenvalues associated with the matrix pencil $\left(\begin{bmatrix} L & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -R_1 & -1 & 1 \\ 0 & -1/R_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right)$*, we have*

$$\det \left(s \begin{bmatrix} L & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} -R_1 & -1 & 1 \\ 0 & -1/R_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right) = sC + \frac{1}{R_2}. \tag{2.8}$$

Hence, this matrix pencil has a finite eigenvalue $-\frac{1}{CR_2}$ *and two infinite eigenvalues at* ∞*. Note that the number of states is smaller than the degree of the polynomial of the matrix pencil determinant. This fact indicates that, for this electrical circuit, one of the infinite eigenvalues is related to the impulsive mode, that is,* i*, while the other one is the static mode, that is,* v_1*.*

Example 2.2 (Two Interconnected Rotating Masses) *[SGGG03]*

Let us treat a system comprised by two rotating masses, as shown in Figure 2.2. The two rotating parts are described by the torques denoted M_1*,* M_2*,* M_3 *and* M_4 *and* *the angular velocities denoted ω_1 and ω_2. To describe this dynamic system, we have the following equations:*

$$\begin{aligned} J_1\dot{\omega}_1 &= M_1 + M_2, && (2.9a)\\ J_2\dot{\omega}_2 &= M_3 + M_4, && (2.9b)\\ M_2 &= -M_3, && (2.9c)\\ \omega_1 &= \omega_2. && (2.9d) \end{aligned}$$

The first two equations of (2.9) describe the relationship between angular accelerations and torques, while the last two describe how the two masses are connected. This system can be described by the descriptor representation:

$$\begin{bmatrix} J_1 & 0 & 0 & 0\\ 0 & J_2 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\omega}_1\\ \dot{\omega}_2\\ \dot{M}_2\\ \dot{M}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -1 & -1\\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_1\\ \omega_2\\ M_2\\ M_3 \end{bmatrix} + \begin{bmatrix} 1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0 \end{bmatrix} \begin{bmatrix} M_1\\ M_4 \end{bmatrix}. \qquad (2.10)$$

Similarly, the matrix on the the state derivative is singular, hence the state-space representation is not able to describe this system. By computing the matrix pencil determinant, we have

$$\det\left(s\begin{bmatrix} J_1 & 0 & 0 & 0\\ 0 & J_2 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -1 & -1\\ -1 & 1 & 0 & 0 \end{bmatrix} \right) = -s(J_1 + J_2). \qquad (2.11)$$

Hence, this system has a finite eigenvalue at 0 and three infinite eigenvalues at ∞. It can also be observed that the number of states is two and the degree of the polynomial of the matrix pencil determinant is one. This fact indicates that, for this system, one of the infinite eigenvalues is related to the impulsive mode, while the other two are static modes.

2.3 Literature Review for Descriptor Systems

As descriptor systems describe an important class of systems of both theoretical and practical significance, they have been a subject of research for many years. The history of studying descriptor systems dates back to the 1860s. The foundation for the study of linear descriptor systems was laid by Weierstrass. In [Wei67], he developed the theory of elementary divisor for systems of the form:

$$E\dot{x} = Ax + Bu. \qquad (2.12)$$

His results were restricted in the regular case, that is, the determinant $sE - A$ is not identically zero. Then, by using the notion of minimal indices, Kronecker extended

Figure 2.3: Karl Theodor Wilhelm Weierstrass

this theory to general cases where $|sE - A| = 0$ or E and A are rectangular [Kro90, LOMK91, BM95].

Foundational research of descriptor systems in the system theoretic context began from the 1970s. Frequency domain approaches and their counterparts, time domain approaches, were developed for the theory of descriptor systems. The 1970s and 1980s were characterized as the development of basic yet essential notions for descriptor systems, such as, the structure of matrix pencils, impulsive behavior, solvability, controllability, reachability, observability and system equivalence [Ros70, Ros74, VLK81, Lue77a, YS81, Cob81, Cob84].

From the beginning of the 1990s, scholars began to generalize the classic control issues to descriptor systems, for both continuous-time and discrete-time settings. By further studying the classic Riccati equations, so-called generalized algebraic Riccati equations (GAREs) were introduced for the H_2 and H_∞ performance control within the descriptor framework. Satisfactory results similar yet more complicated than those for the state-space systems were reported in the literature [TMK94, KK97, TK98]. Moreover, linear matrix inequality (LMI) approaches were also used for solving underlying control problems for descriptor systems [MKOS97, UI99, ZXS08, ZHL03].

Let us recall the main research outcomes of descriptor systems. Avoiding giving an exhaustive list, we give a briefly reminder as follows:

- controllability and observability [Cob84, Lew85, YS81, Ail87, Hou04]
- system equivalence [BS00, VLK81, ZST87]
- regularity and regularization [BKM97, CH99, KLX03, WS99]
- stability and stabilization [Var95, XY99, XY00, XL04, SC04, XL06, SBMar, Bar11]

- linear quadratic optimal control [Cob83, BL87, Wan04]
- pole assignment [Ail89, FKN86, DP98, MKG03, VK03, YW03, RBCM08]
- generalized Lyapunov and Riccati equations [Ben87, ZLX02, TMK95, IT02, SMA95]
- positive real lemma [FJ04, WC96, ZLX02, Mas06, Mas07, CT08]
- H_2 analysis and control synthesis [ILU00, IT02, TK98]
- H_∞ analysis and control synthesis [MKOS97, Mas06, Mas07, RA99, TMK94, UI99, ZXS08]
- observer design [DZH96, HM99a, Gao05, WD06, WDF07]
- generalized Sylvester equation [KW89, GLAM92, Dua96, CdS05, Dar06]
- output regulation problem [LD96, IK05]
- model reduction problem [XLLZ03, WLZX04, WVS06, LZD07]

On the other hand, descriptor systems bring extra complexities to system analysis and controller design synthesis. Roughly speaking, given a descriptor system (2.12), the major difficulties of studying these systems are rooted in the analysis of the matrix pair (E, A) instead of a single matrix A for the state-space case. Two new concepts called regularity and impulsiveness (resp. causality for the discrete setting) which are not necessarily considered for the state-space case need to be taken into account. For instance, in order to stabilize a descriptor system, the closed-loop system must be stable, as well as regular and impulsive-free. The latter two are intrinsic properties of conventional state-space systems. Another example is the indefiniteness of the Lyapunov matrix for descriptor systems. As known, in the state-space setting, a desirable Lyapunov matrix should be symmetric and positive definite. However, this is not the case within the descriptor framework. A desirable Lyapunov matrix associated with a descriptor system is indefinite, only the part related to the image of the matrix E is supposed to be positive definite. Furthermore, contrary to state-space systems, the Lyapunov matrix for a given descriptor system is not unique, and it is possible to find some Lyapunov matrices for this system. The requirement of uniqueness is no longer imposed on the Lyapunov matrix, but on the product of $E^\top$ and the Lyapunov matrix. Hence, some control problems which have been successfully solved within the state-space framework are still open and deserve further investigation for descriptor systems.

This dissertation deals with the nonstandard optimal control problem for linear time-invariant descriptor systems. Although descriptor systems have been attracting the attention of many researchers since the seventies and many control issues have been extended to descriptor systems, as mentioned before, some control problems which have been successfully solved for state-space systems are still open within the descriptor framework. For example, it is well known that, to solve the standard H_2 or H_∞ control problem for descriptor systems, a set of assumptions [TK98, TMK94, WYC06] should be satisfied for the existence of admissible solutions to the underlying GAREs. Even the use of the LMI approach [MKOS97, Mas07] requires that the given system is stabilizable and detectable. However, for some real cases, these theoretical conditions do not always hold. For instance, the mixed sensitivity problem with the presence of unstable and nonproper weighting functions yields an unstabilizable/unobservable weighted plant for which standard solution procedures fail to provide a solution. The solution to these nonstandard problems within the descriptor framework has not yet been reported so far.

This dissertation generalizes some control issues to descriptor systems circumstances. A realization independent Kalman-Yakubovich-Popov (KYP) lemma and dilated LMI characterizations are deduced for descriptor systems. The solvability and corresponding numerical algorithms of generalized Sylvester equations and GAREs associated with descriptor systems are provided. In addition, the simultaneous H_∞ control problem is considered by using coprime factorization. Moreover, an extended control problem where unstable and nonproper weights are considered in the overall feedback model is addressed. The aforementioned mixed sensitivity problem can be viewed as a special case of this. Finally, the subject of performance measure with regulation constraints is also extended to descriptor systems.

2.4 Highlights of the Dissertation

The remainder of this dissertation is organized in the following and the key results are highlighted accordingly.

Chapter 3 recalls the basic concepts concerning linear time-invariant descriptor systems, for both continuous-time and discrete-time settings, which will be used subsequently. In this chapter, fundamental definitions and results of descriptor systems, such as regularity, admissibility, equivalent realizations, system decomposition, temporal response, controllability, observability and duality are reviewed.

Chapter 4 provides some preliminary and useful results for descriptor systems. The issues of dissipativity, dilated LMI characterization, generalized Sylvester equations and GARE are addressed. A new KYP-type lemma for dissipativity is characterized in terms of a strict LMI condition. Dilated LMI characterizations within the descriptor framework are also studied and the deduced formulations cover not only the existing results, but also complete some missing LMI conditions. Moreover, generalized Sylvester equations and GARE associated with descriptor systems are investigated in this chapter. Numerical algorithms for solving these equations are presented. The reported results play an important role in the following chapters. The LMI condition for dissipativity will be used to solve the performance control problem under regulation constraints for descriptor systems discussed in Chapter 7. The issue of generalized Sylvester equations and GAREs will be adopted to solve the extended control problem and regulation problem in Chapters 6 and 7, respectively.

Chapter 5 is devoted to the strong H_∞ stabilization problem and simultaneous H_∞ control problem for continuous-time descriptor systems. We first attempt to explore the relation between these two problems within the descriptor framework. Based on coprime factorization, it shows that the simultaneous H_∞ control problem for a set of descriptor systems is achieved if and only if the strong H_∞ stabilization problem associated with a corresponding augmented system is solvable. Furthermore, a sufficient condition of solvability of strong H_∞ stabilization is proposed in terms of a GARE and a set of LMIs and an observer-based controller solving this problem is deduced.

Chapter 6 considers the extended control problem (the "extended" term indicates here that the desirable controller can and must stabilize a part of the overall closed-loop system) for continuous-time descriptor systems. Systems and their weights are all described within the descriptor framework. Hence, it is possible to take into account not only unstable weights, but nonproper weights as well. We first study the stabilization issue in these circumstances and the necessary and sufficient conditions for the existence of a solution are derived based on two generalized Sylvester equations. A parametrization of all controllers achieving extended stabilization is also given. Moreover, the H_2 and H_∞ performance control subjects to extended stabilization constraints are investigated. The so-called quasi-admissible solution is defined instead of the conventional admissible solution for these nonstandard cases. Relying on this relaxation, a complete, exact and analytical solution to the extended control problem is given in terms of underlying GAREs, and the set of desirable controllers is also parameterized.

Chapter 7 addresses the state feedback H_2 optimal control and H_∞ output feedback control problems under regulation constraints for continuous-time descriptor systems. In this problem, an output is to be regulated asymptotically in the presence of an infinite-energy exosystem, while a desired performance by the H_2 or H_∞ norm from a finite external disturbance to a tracking error has also to be satisfied. It shows that the asymptotical regulation objective is satisfied if and only if a generalized Sylvester equation associated with the given descriptor system and exosystem is solvable. Moreover, any controller achieving asymptotical regulation constraints possesses a specific structure. Thus, using this structure, the defined multi-objective control problems reduce to the standard control problems for an associated descriptor plant, whose solution is characterized based on the solvability of a GARE and a set of LMIs.

Chapter 8 contains concluding remarks which summarize the achievements in this dissertation and discuss future topics.

Remark 2.4.1 *Appendix I-Appendix VIII consist of relevant publications which con-*

tain the main results of each chapter.

2.5 Related Publications

The main results of this dissertation have been developed in cooperation with Dr. Mohamed Yagoubi and Prof. Philippe Chevrel and have previously been published or submitted in

- Journal publications

  1. **Y. Feng**, M. Yagoubi and P. Chevrel. H_∞ Control Under Regulation Constraints For Descriptor Systems. In preparation.
  2. **Y. Feng**, M. Yagoubi and P. Chevrel. H_∞ control with unstable and non-proper weights for descriptor systems. *Automatica.* Submitted.
  3. **Y. Feng**, M. Yagoubi and P. Chevrel. Extended H_2 controller synthesis for continuous descriptor systems. *IEEE Transactions on Automatic Control.* Accepted.
  4. **Y. Feng**, M. Yagoubi and P. Chevrel. Parametrization of extended stabilizing controllers for continuous-time descriptor Systems. *Journal of The Franklin Institute.* vol 348, (9), pp. 2633-2646, 2011.
  5. **Y. Feng**, M. Yagoubi and P. Chevrel. State feedback H_2 optimal controllers under regulation constraints for descriptor systems. *International Journal of Innovative Computing, Information and Control.* vol 7, (10), pp. 5761-5770, 2011.
  6. **Y. Feng**, M. Yagoubi and P. Chevrel. Simultaneous H_∞ control for continuous-time descriptor systems. *IET Control Theory & Applications.* vol. 5, (1), pp. 9-18, 2011.
  7. **Y. Feng**, M. Yagoubi and P. Chevrel. Dilated LMI characterizations for linear time-invariant singular systems. *International Journal of Control.* vol. 83, (11), pp. 2276-2284, 2010.

- Conference publications

  1. **Y. Feng**, M. Yagoubi and P. Chevrel. Extended H_2 output feedback control for continuous descriptor systems. In: *Proceedings of the 49th IEEE Conference on Decision & Control*, Atlanta, GA, USA, December 2010, pp. 6016-6021.
  2. **Y. Feng**, M. Yagoubi and P. Chevrel. Extended stabilizing controllers for continuous-time descriptor systems. In: *Proceedings of the 49th IEEE Conference on Decision & Control*, Atlanta, GA, USA, December 2010, pp. 726-731.

3. **Y. Feng**, M. Yagoubi and P. Chevrel. On dissipativity of continuous-time singular systems. In: *Proceedings of the 18th Mediterranean Conference on Control & Automation*, Marrakesh, Morocco, June 2010, pp. 839-844.

2.6 Conclusion

This chapter presents the importance of the use of descriptor systems and provides a brief literature review on analysis and control problems with respect to descriptor systems. The outline of the dissertation is also given and the main published or submitted results of the Ph.D. work are listed.

Chapter 3

Linear Time-Invariant Descriptor Systems

In this chapter, we recall some basic concepts concerning linear time-invariant descriptor systems both for continuous-time and discrete-time settings, which will be used subsequently. We give here a quick reminder of the fundamental definitions and results of descriptor systems, such as regularity, admissibility, equivalent realizations, system decomposition, temporal response, controllability, observability and duality.

3.1 Introduction

Let us recall the first order DAE discussed in the previous chapter:

$$\boldsymbol{F}\left(\dot{x}(t), x(t)\right) = 0, \tag{3.1}$$

where $\boldsymbol{F}$ and x are vector-value functions. Representing the Jacobians as

$$E \triangleq \frac{\partial \boldsymbol{F}}{\partial \dot{x}(t)}, \quad A \triangleq -\frac{\partial \boldsymbol{F}}{\partial x(t)}, \tag{3.2}$$

we can write

$$E d\dot{x}(t) = A dx(t) + \left(d\boldsymbol{F} - \frac{\partial \boldsymbol{F}}{\partial t} dt \right). \tag{3.3}$$

As mentioned before, if the matrix E is not singular, i.e. $|E| \neq 0$, we can convert this system into a conventional state-space system by left-multiplying the two sides by E^{-1}. On the other hand, if E is singular, this conversion is not possible. In the parts to follow, we omit the time index t for continuous-time descriptor systems to simplify the writing.

A linear time-invariant version of (3.1) including a control input u and a measurement output y can be written as:

$$E\dot{x} = Ax + Bu, \tag{3.4a}$$

$$y = Cx, \tag{3.4b}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$ and $u \in \mathbb{R}^m$ are the descriptor variable, measurement and control input vector, respectively. The matrix $E \in \mathbb{R}^{n \times n}$ may be singular, *i.e.* $rank(E) = r \leq n$.

Note that the form (3.4) can be used without loss of generality. In the case where the feedthrough matrix from u to y is not null, we can introduce an extra descriptor variable to render the D matrix zero. For example, consider the following system:

$$E\dot{x} = Ax + Bu, \tag{3.5a}$$

$$y = Cx + Du, \tag{3.5b}$$

which can be equivalently represented by

$$\underbrace{\begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}}_{\mathcal{E}} \begin{bmatrix} \dot{x} \\ \dot{\zeta} \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ 0 & -I \end{bmatrix}}_{\mathcal{A}} \begin{bmatrix} x \\ \zeta \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ I \end{bmatrix}}_{\mathcal{B}} u, \tag{3.6a}$$

$$y = \underbrace{\begin{bmatrix} C & D \end{bmatrix}}_{\mathcal{C}} \begin{bmatrix} x \\ \zeta \end{bmatrix}. \tag{3.6b}$$

By introducing the auxiliary variable ζ, this system is rewritten as the form of (3.4).

A descriptor system G associated with the system data $(E, A, B, C, (D))$ can also be represented by the form of

$$G(s) := \left\{ E, \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] \right\} \tag{3.7}$$

Example 3.1 *Let us take the example of the electrical circuit (Figure 3.1), used in the previous chapter, to illustrate the use of descriptor systems. We also assume that* $i = u$ *where* u *is the control input, and define the measured output as* $y = v_1 + v_2$*. Hence, this dynamic system can be written within the descriptor framework as*

$$\begin{bmatrix} L & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{i} \\ \dot{v}_2 \\ \dot{v}_1 \end{bmatrix} = \begin{bmatrix} -R_1 & -1 & 1 \\ 0 & -1/R_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i \\ v_2 \\ v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} u, \tag{3.8a}$$

$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} i \\ v_2 \\ v_1 \end{bmatrix}. \tag{3.8b}$$

3.2 Regularity

One of the basic notions of descriptor systems is *regularity* or *solvability*. If a descriptor system is regular, then it has a unique solution for any initial condition and any continuous input function [VLK81, Cob83].

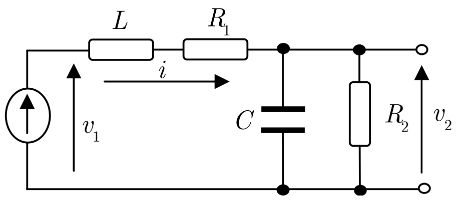

Figure 3.1: Electrical circuit

Definition 3.2.1 (Regularity) *The linear descriptor system in* (3.4) *is said to be regular if* $\det(sE - A)$ *is not identically null.*

This definition is the same as the one called solvability used by Yip and Sincovec in [YS81]. To illustrate the physical mean of regularity, let us examine the Laplace transformed version of $E\dot{x} = Ax + Bu$:

$$sE\mathcal{L}[x] - Ex(0) = A\mathcal{L}[x] + B\mathcal{L}[u], \tag{3.9}$$

which can be arranged as:

$$(sE - A)\mathcal{L}[x] = B\mathcal{L}[u] + Ex(0). \tag{3.10}$$

It observes that, if the system is regular, then

$$\mathcal{L}[x] = (sE - A)^{-1}(B\mathcal{L}[u] + Ex(0)), \tag{3.11}$$

which guarantees the existence and uniqueness of $\mathcal{L}[x]$ for any initial condition and any continuous input function.

On the other hand, if the system is not regular, or equivalently, the matrix $sE - A$ is rank deficient, there exists a non-zero vector $\theta(s)$ such that

$$(sE - A)\theta(s) \equiv 0. \tag{3.12}$$

Consequently, one can state that, if the system has a solution denoted $\mathcal{L}[x]$, then $\mathcal{L}[x] + \alpha\theta(s)$ is also its solution for any α. It is clear that a solution to this system is not unique, and it is also obvious that there may be no solution for this system.

The following characterizations of regularity are given in [YS81].

Lemma 3.2.1 (Regularity) *The following statements are equivalent.*

a) (E, A) *is regular.*

b) *If* X_0 *is the null space of* A *and* $X_i = \{x : Ax \in EX_{i-1}\}$ *then* $KerE \bigcap X_i = 0$ *for* $i = 0, 1, 2, \cdots$.

c) *If* Y_0 *is the null space of* A^T *and* $Y_i = \{x : A^T x \in E^T Y_{i-1}\}$ *then* $KerE^T \bigcap Y_i = 0$ *for* $i = 0, 1, 2, \cdots$.

d) *The matrix*

$$G(n) = \left.\begin{bmatrix} E & 0 & \cdots & 0 \\ A & E & \cdots & 0 \\ 0 & A & \cdots & 0 \\ & & & E \\ 0 & & \cdots & A \end{bmatrix}\right\} n+1 \tag{3.13}$$

has full column rank for $n = 1, 2, \cdots$.

e) *The matrix*

$$F(n) = \underbrace{\begin{bmatrix} E & A & 0 & \cdots & 0 \\ 0 & E & A & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & & & E & A \end{bmatrix}}_{n+1} \tag{3.14}$$

has full row rank for $n = 1, 2, \cdots$.

f) *There exist nonsingular matrices* M *and* N *such that* $E\dot{x} = Ax + Bu$ *is decomposed into possibly two subsystems: a subsystem with only a state variable, and an algebraic-like subsystem, i.e.* $MENN^{-1}\dot{x} = MANN^{-1}x + MBu$ *has one of the following forms.*

i)

$$\dot{x}_1 = E_1 x_1 + B_1 u, \tag{3.15a}$$

$$E_2 \dot{x}_2 = x_2 + B_2 u, \quad E_2^k = 0, \quad E_2^{k-1} \neq 0. \tag{3.15b}$$

In this case, both E *and* A *are singular, or* A *is nonsingular and* E *is singular but not nilpotent, i.e.* $E^k \neq 0$ *for all positive integers* k.

ii)

$$\dot{x}_1 = E_1 x_1 + B_1 u. \tag{3.16}$$

In this case, E *is nonsingular.*

iii)

$$E_2 \dot{x}_2 = x_2 + B_2 u, \quad E_2^k = 0, \quad E_2^{k-1} \neq 0. \tag{3.17}$$

In this case, A *is nonsingular and* E *is nilpotent.*

In all cases,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = N^{-1}x, \quad \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = MB, \tag{3.18}$$

and the exact solution is

$$x_1(t) = e^{E_1 t}x_{10} + \int_0^t e^{(t-\tau)E_1} B_1 u(\tau)d\tau, \tag{3.19a}$$

$$x_2(t) = -\sum_{i=0}^{k-1} E_2^i B_2 u^{(i)}(t), \tag{3.19b}$$

where x_{10} *is the transformed initial condition, i.e.* $\begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = N^{-1}x_0$.

Among these equivalent statements, the easiest one to characterize regularity for a given descriptor system is the condition d) or its dual version e). In order to avoid computing a matrix with a huge dimension, Luenberger, in [Lue78], proposed the so-called shuffle algorithm which requires manipulations only on the rows and columns of the matrix $[E \quad A]$. For convenience, we usually check regularity directly from its definition, that is, $sE - A \not\equiv 0$ for all $s \in \mathbb{C}$. In addition, if the descriptor system (3.4) is regular, $(sE - A)^{-1}$ is a rational matrix and we can further define its transfer function as:

$$G(s) = C(sE - A)^{-1}B. \tag{3.20}$$

Example 3.2 *Let us study the regularity of Example 3.1. Computing the determinant of the matrix* $sE - A$ *gives*

$$|sE - A| = sC + \frac{1}{R_2}, \tag{3.21}$$

which is not identically zero. Hence this electrical circuit is regular. Moreover, we obtain its transfer function

$$\begin{aligned} G(s) &= \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} sL & 0 & 0 \\ 0 & sC & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} -R_1 & -1 & 1 \\ 0 & -1/R_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \\ &= -sL - R_1 - \tfrac{2R_2}{sCR_2+1}. \end{aligned} \tag{3.22}$$

It is observed that this transfer function is nonproper. This fact can be further explained through the impulsive property of this system, which will be discussed in Section 3.5.

3.3 Equivalent Realizations & System Decomposition

To model a physical system, one has to choose a set of states which are related to the same performance, such as acceleration, velocity, position, temperature and mass. The choice of these states is in general not unique, and this fact leads to a set of different models (realizations) which yield the same input-output relationship for a given system. Consequently, it is of great interest to determine the relation of equivalence for these different representations.

Definition 3.3.1 (Restricted System Equivalence) *[Ros74] Consider two descriptor systems G and $\bar{G}$ given by*

$$E\dot{x} = Ax + Bu, \tag{3.23a}$$
$$y = Cx, \tag{3.23b}$$

and

$$\bar{E}\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u}, \tag{3.24a}$$
$$\bar{y} = \bar{C}\bar{x},. \tag{3.24b}$$

The two systems G and $\bar{G}$ are termed restricted system equivalent if there exist nonsingular matrices M and N such that

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \left[\begin{array}{c|c} sE - A & B \\ \hline C & 0 \end{array}\right] \begin{bmatrix} N & 0 \\ 0 & I \end{bmatrix} = \left[\begin{array}{c|c} s\bar{E} - \bar{A} & \bar{B} \\ \hline \bar{C} & 0 \end{array}\right]. \tag{3.25}$$

Compared with state-space systems, the transformation matrices for descriptor systems have no explicit relation between each other. As known, in the state-space setting, we have $M = N^{-1}$. In [VLK81], the authors proposed a new term called *strong equivalence*.

Definition 3.3.2 (Strong Equivalence) *[VLK81] Consider two descriptor systems G and $\bar{G}$ given in (3.23) and (3.24), respectively. The two systems G and $\bar{G}$ are termed strongly equivalent if there exist nonsingular matrices M, N and two matrices Q, R such that*

$$\begin{bmatrix} M & 0 \\ Q & I \end{bmatrix} \left[\begin{array}{c|c} sE - A & B \\ \hline C & 0 \end{array}\right] \begin{bmatrix} N & R \\ 0 & I \end{bmatrix} = \left[\begin{array}{c|c} s\bar{E} - \bar{A} & \bar{B} \\ \hline \bar{C} & 0 \end{array}\right], \tag{3.26a}$$
$$QE = ER = 0. \tag{3.26b}$$

Note that, in this dissertation, the term "equivalence" means "restricted system equivalence".

Among the many equivalent representations, there are two particular realizations of great importance for system analysis and control. They are referred to as the Kronecker-Weierstrass form [Wei67, Kro90] and the singular value decomposition (SVD) [BL87] form.

Lemma 3.3.1 (Kronecker-Weierstrass Decomposition) *[Dai89] The descriptor system (3.4) is regular if and only if there exists nonsingular matrices M_1 and N_1 such that*

$$M_1 E N_1 = \begin{bmatrix} I_{n_1} & 0 \\ 0 & \mathcal{N} \end{bmatrix}, \quad M_1 A N_1 = \begin{bmatrix} \mathcal{A} & 0 \\ 0 & I_{n_2} \end{bmatrix}, \tag{3.27}$$

where $n_1 + n_2 = n$ and $\mathcal{N}$ is a nilpotent matrix.

The Kronecker-Weierstrass decomposition can be viewed as an equivalent condition for regularity, and this form is also referred to by some scholars as *slow-fast decomposition* [Cob84]. The subsystem related to $\mathcal{A}$ is called the slow subsystem; while what is related to $\mathcal{N}$ is called the fast subsystem. The two matrices M_1 and N_1 are not unique.

Although the Kronecker-Weierstrass decomposition divides the systems into two parts which may bring simplicity for analysis, the use of this decomposition requires that the underlying system is regular. If the regularity of the system is not known, then this form cannot be applied. Moreover, the Kronecker-Weierstrass decomposition is sometimes numerically unreliable, especially in the case where the order of the system is relatively large.

Another decomposition which does not depend on the regularity of systems is called the singular value decomposition form. This form can be obtained via a singular value decomposition on E and followed by scaling of the bases. Under the SVD form, the pair (E, A) is decomposed by two nonsingular matrices M_2 and N_2 as:

$$M_2 E N_2 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad M_2 A N_2 = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}. \tag{3.28}$$

Similar to the Kronecker-Weierstrass decomposition, M_2 and N_2 for SVD form are not unique. Now we give one of possible constructions of M_2 and N_2. Let us define constant matrices $U, V \in \mathbb{R}^{n\times(n-r)}$ with full column rank satisfying

$$E^\top U = 0, \quad EV = 0, \tag{3.29}$$

and also $E_L, E_R \in \mathbb{R}^{n\times(n-r)}$ with full column rank satisfying

$$E_L^\top U = 0, \quad E_R^\top V = 0. \tag{3.30}$$

With this matrix definition, E can be decomposed as $E = E_L \Sigma E_R^\top$, with $\Sigma \in \mathbb{R}^{r\times r}$ nonsingular. Then we define the following matrices $\tilde{M}, \tilde{N} \in \mathbb{R}^{n\times n}$

$$\tilde{M} = \begin{bmatrix} (E_L^\top E_L)^{-1} E_L^\top \\ U^\top \end{bmatrix}, \quad \tilde{N} = \begin{bmatrix} E_R (E_R^\top E_R)^{-1} & V \end{bmatrix}. \tag{3.31}$$

It is easy to see that these two matrices are nonsingular. In fact, we have

$$\tilde{M}^{-1} = \begin{bmatrix} E_L & U(U^\top U)^{-1} \end{bmatrix}, \ \tilde{N}^{-1} = \begin{bmatrix} E_R^\top \\ (V^\top V)^{-1} V^\top \end{bmatrix}. \tag{3.32}$$

Moreover, the transform matrices M_2 and N_2 can be formed as

$$M_2 = \begin{bmatrix} \Sigma^{-1} & 0 \\ 0 & I \end{bmatrix} \tilde{M}, \quad N_2 = \tilde{N}. \tag{3.33}$$

Example 3.3 *Let us transform the realization (3.8) into the SVD form for Example 3.1. To this end, we choose*

$$M_2 = \begin{bmatrix} 1/L & 0 & 0 \\ 0 & 1/C & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad N_2 = I_3. \tag{3.34}$$

Under this transformation, we have

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{i} \\ \dot{v}_2 \\ \dot{v}_1 \end{bmatrix} &= \begin{bmatrix} -\frac{R_1}{L} & -\frac{1}{L} & \frac{1}{L} \\ 0 & -\frac{1}{CR_2} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i \\ v_2 \\ v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \\ -1 \end{bmatrix} u, & (3.35a) \\ y &= \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} i \\ v_2 \\ v_1 \end{bmatrix}. & (3.35b) \end{aligned}$$

3.4 Temporal Response

Suppose that the descriptor system (3.4) is regular. According to the Kronecker-Weierstrass decomposition, there exists M_1 and N_1 such that

$$\begin{aligned} \dot{x}_1 &= \mathcal{A}x_1 + B_1 u, & (3.36a) \\ y_1 &= C_1 x_1, & (3.36b) \end{aligned}$$

$$\begin{aligned} \mathcal{N}\dot{x}_2 &= x_2 + B_2 u, & (3.37a) \\ y_2 &= C_2 x_2, & (3.37b) \end{aligned}$$

where

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = N_1^{-1} x, \quad \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = M_1 B, \quad \begin{bmatrix} C_1 & C_2 \end{bmatrix} = C N_1.$$

Suppose that h is the nilpotent degree of the matrix $\mathcal{N}$, that is, $\mathcal{N}^{h-1} \neq 0$ and $\mathcal{N}^h = 0$. The subsystem (3.36) is a normal state-space system, whose temporal response for a given input $u(t)$ and initial condition x_{10} can be written as:

$$y_1(t) = C_1 e^{\mathcal{A}t} x_{10} + C_1 \int_0^t e^{\mathcal{A}(t-\tau)} B_1 u(\tau) d\tau. \tag{3.38}$$

Then we consider the subsystem (3.37). Suppose that $u(t) \in \mathcal{C}^{h-1}$, where $\mathcal{C}^{h-1}$ stands for the set of $h-1$ times continuously differentiable functions. Then we have the following relations:

$$\begin{array}{rcl} \mathcal{N}\dot{x}_2(t) & = & x_2(t) + B_2u(t), \\ \mathcal{N}^2x_2^{(2)}(t) & = & \mathcal{N}\dot{x}_2(t) + \mathcal{N}B_2\dot{u}(t), \\ \cdots & , & \\ \mathcal{N}^k x_2^{(k)}(t) & = & \mathcal{N}^{k-1}x_2^{(k-1)}(t) + \mathcal{N}^{k-1}B_2u^{(k-1)}(t), \\ \cdots & , & \\ \mathcal{N}^{h-1}x_2^{(h-1)}(t) & = & \mathcal{N}^{h-2}x_2^{(h-2)}(t) + \mathcal{N}^{h-2}B_2u^{(h-2)}(t), \\ 0 & = & \mathcal{N}^{h-1}x_2^{(h-1)}(t) + \mathcal{N}^{h-1}B_2u^{(h-1)}(t). \end{array} \tag{3.39}$$

Hence, the expression of $x_2(t)$ can be obtained as:

$$\begin{array}{rcl} x_2(t) & = & \mathcal{N}\dot{x}_2 - B_2u(t), \\ x_2(t) & = & \mathcal{N}^2x_2^{(2)}(t) - B_2u(t) - \mathcal{N}B_2\dot{u}(t), \\ \cdots & , & \\ x_2(t) & = & -\sum_{k=0}^{h-1}\mathcal{N}^kB_2u^{(k)}(t), \end{array} \tag{3.40}$$

which gives

$$y_2(t) = -\sum_{k=0}^{h-1} C_2\mathcal{N}^kB_2u^{(k)}(t). \tag{3.41}$$

Hence the temporal response $y(t)$ of the descriptor system (3.4) is

$$y(t) = C_1\left(e^{\mathcal{A}t}x_{10} + \int_0^t e^{\mathcal{A}(t-\tau)}B_1u(\tau)d\tau\right) - \sum_{k=0}^{h-1} C_2\mathcal{N}^kB_2u^{(k)}(t). \tag{3.42}$$

It is observed that the response of the subsystem (3.36) depends on the matrix $\mathcal{A}$, initial condition x_{10}, as well as the input $u(t)$, while the response of the subsystem (3.37) depends only on the derivative of the input $u(t)$ on time t. That is why we sometimes call these two subsystems slow subsystem and fast subsystem, respectively. If $t \to 0^+$, then we can deduce the following constraint on the initial condition:

$$x(0^+) = N_1\begin{bmatrix} I \\ 0 \end{bmatrix} x_{10} - N_1\begin{bmatrix} 0 \\ I \end{bmatrix}\sum_{k=0}^{h-1}\mathcal{N}^kB_2u^{(k)}(0^+). \tag{3.43}$$

Any initial condition satisfying (3.43) is called an *admissible condition*. From this point of view, only one initial condition is allowed and hence only one solution is allowed for each choice of $u(t)$. In [VLK81, Cob83], the authors used the theory of distributions and generalized this viewpoint to allow arbitrary initial conditions. Under this theory, for the fast subsystem, we have:

$$x_2(t) = -\sum_{k=1}^{h-1}\delta^{(k-1)}\mathcal{N}^kx_{20} - \sum_{k=0}^{h-1}\mathcal{N}^kB_2u^{(k)}(t), \tag{3.44}$$

where δ is the Dirac delta. As pointed out in [Cob81], the form of (3.44) suggests that in any conventional sense the dynamics of the overall system are concentrated in the slow subsystem in (3.36). With the theory of distributions, we can represent the systems whose initial conditions are not admissible or those who contain "jump" behaviors (for example, when we switch an electrical circuit on, there will be a jump in the current or voltage at this moment.). For these cases, the first term of (3.44) can transform the system into an admissible state.

3.5 Admissibility

Stability is a fundamental concept for state-space systems, which can be characterized, by one of the definitions, that the underlying system has no poles located in the right-hand plane including the imaginary axis. Under the descriptor framework, a similar yet more general concept called *admissibility* plays the same role as stability in state-space systems.

Definition 3.5.1 (Admissibility) *[Dai89, Lew86]*

a) *The descriptor system (3.4) is said to be regular if* $det(sE - A)$ *is not identically null;*

b) *The descriptor system (3.4) is said to be impulse-free if* $deg(det(sE - A)) = rank(E)$*;*

c) *The descriptor system (3.4) is said to be stable if all the roots of* $det(sE - A) = 0$ *have negative real parts;*

d) *The descriptor system (3.4) is said to be admissible if it is impulse-free and stable.*

From the definition, the admissibility of a descriptor system concerns stability, as well as regularity and impulsiveness. The latter two are intrinsic properties of conventional state-space systems and are not necessarily considered in the state-space case. Furthermore, it can be deduced that if a descriptor system is impulse-free, then it is regular.

Now we give some equivalent conditions for admissibility.

Lemma 3.5.1 *[Dai89] Suppose that the descriptor system (3.4) is regular and there exist nonsingular matrices* M_1 *and* N_1 *such that the Kronecker-Weierstrass form (3.27) holds. Then*

i) *this system is impulse-free if and only if* $\mathcal{N} = 0$*;*

ii) *this system is stable if and only if* $\alpha(\mathcal{A}) < 0$*;*

iii) *this system is admissible if and only if* $\mathcal{N} = 0$ *and* $\alpha(\mathcal{A}) < 0$*.*

Lemma 3.5.2 *[Dai89] Consider the descriptor system (3.4) and suppose that there exist nonsingular matrices M_2 and N_2 such that the SVD form (3.28) holds. Then*

i) *this system is impulse-free if and only if $|A_4| \neq 0$;*

ii) *this system is admissible if and only if $|A_4| \neq 0$ and $\alpha(A_1 - A_2A_4^{-1}A_3) < 0$.*

If a descriptor system is impulse-free, then it can be transformed into a conventional state-space system. For instance, if the descriptor system (3.4) is impulse-free, then this system is equivalent to

$$\dot{\bar{x}}_1 = (A_1 - A_2A_4^{-1}A_3)\bar{x}_1 + (B_1 - A_4^{-1}B_2)u, \tag{3.45a}$$

$$\bar{y} = (C_1 - C_2A_4^{-1}A_3)\bar{x}_1 - C_2A_4^{-1}B_2u, \tag{3.45b}$$

where A_i, $i = 1, 2, 3, 4$, B_i and C_i, $i = 1, 2$ are partitions of the matrices A, B and C in the SVD form (3.28).

Furthermore, provided that the descriptor system is regular and the matrices M_1 and N_1 exist to render it Kronecker-Weierstrass form. The transfer function of this system can be written as:

$$G(s) = C_1(sI - \mathcal{A})^{-1}B_1 + C_2(s\mathcal{N} - I)^{-1}B_2. \tag{3.46}$$

For an impulse-free system, that is, $\mathcal{N} = 0$, we have

$$G(s) = C_1(sI - \mathcal{A})^{-1}B_1 - C_2B_2. \tag{3.47}$$

It is noted that the term $C_2(s\mathcal{N} - I)^{-1}B_2$ creates polynomial terms of s if both B_2 and C_2 are nonzero. Hence the impulse-free assumption guarantees the properness of the transfer function. The converse statement is, however, not true. Clearly, if either B_2 or C_2 vanishes, the transfer function is still proper, even if the system is impulsive. Hence, given a stable transfer function $G(s)$ and its corresponding system data $(E, A, B, C, (D))$, the admissibility of this system cannot be concluded.

Now we discuss briefly the issue of generalized eigenvalues of a matrix pencil. The theory mentioned here has been reported in the literature, for instance see [GvL96, BDD+00].

Consider a matrix pencil $\lambda E - A$, where E and A are both real $n \times n$ matrices, and λ is a scalar. First, we assume that this pencil is regular, that is, $|\lambda E - A| \not\equiv 0$ for all λ. The generalized eigenvalues are defined as those λ for which

$$|\lambda E - A| = 0. \tag{3.48}$$

Definition 3.5.2 (Infinite Generalized Eigenvectors) *[BL87]*

1. *Grade 1 infinite generalized eigenvectors of the pencil $(sE - A)$ satisfy*

$$Ev_i^1 = 0. \tag{3.49}$$

2. Grade k ($k \geq 2$) infinite generalized eigenvectors of the pencil $(sE - A)$ corresponding to the i^{th} grade 1 infinite generalized eigenvectors satisfy

$$Ev_i^{k+1} = Av_i^k. \tag{3.50}$$

Moreover, the finite generalized eigenvalues of $sE - A$ are called the finite dynamic modes. The infinite generalized eigenvalues of $sE - A$ with the grade one infinite generalized eigenvectors determine the static modes, while the infinite generalized eigenvalues with the grade k ($k \geq 2$) infinite generalized eigenvectors are the impulsive modes.

Let q be the degree of the polynomial $|\lambda E - A|$. One can state that the matrix pencil $\lambda E - A$ has q finite generalized eigenvalues and $n - q$ infinite generalized eigenvalues where the number of static modes is $n - r$ and the number of impulsive modes is $r - q$.

Example 3.4 *Consider Example 3.1. We have*

$$E = \begin{bmatrix} L & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -R_1 & -1 & 1 \\ 0 & -1/R_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}. \tag{3.51}$$

Note $r = rank(E) = 2$ and

$$\det(\lambda E - A) = \lambda C + \frac{1}{R_2}, \tag{3.52}$$

which implies that the degree of $\det(\lambda E - A)$ is $q = 1$. The system is impulsive, since $q < r$. This fact can also be verified from Example 3.2 because its transfer function is nonproper. Moreover, this system has one finite eigenvalue $-\frac{1}{CR_2}$ and two infinite eigenvalues at ∞. For this electrical circuit, one of the infinite eigenvalues is related to the impulsive mode, that is, i, while the other one is the static mode, that is, v_1.

3.6 Controllability

In this section, we introduce controllability for descriptor systems in such a way that it reduces to the state-space definition when $E = I$. We suppose that the descriptor system (3.4) is regular and it is transformed into the Kronecker-Weierstrass form of the form:

$$\theta_s: \quad \dot{x}_1 = \mathcal{A}x_1 + B_1 u, \quad y_1 = C_1 x_1, \tag{3.53a}$$

$$\theta_f: \quad \mathcal{N}\dot{x}_2 = x_2 + B_2 u, \quad y_2 = C_2 x_2, \tag{3.53b}$$

$$y = y_1 + y_2, \tag{3.53c}$$

where $x_1 \in \mathbb{R}^{n_1}$, $x_2 \in \mathbb{R}^{n_2}$, $n_1 + n_2 = n$ and $\mathcal{N}$ is a nilpotent matrix with degree h.

Let us define

- $\mathcal{C}_p^i$ be the i times piecewise continuously differentiable maps on $\mathbb{R}$ with range depending on context;

- $\mathcal{I}$ be the set of admissible initial conditions, that is,

$$\mathcal{I} = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1 \in \mathbb{R}^{n_1}, x_2 = -\sum_{k=0}^{h-1} \mathcal{N}^k B_2 u^{(k)}(0), u \in \mathcal{C}_m^{h-1} \right\}; \tag{3.54}$$

- $\langle X, Y \rangle = \beta + X\beta + X^2\beta + \cdots + X^{n-1}\beta$, where X is a square matrix, n is the order of X, the product XY is well defined and $\beta = Im(Y)$.

Definition 3.6.1 (Reachable State) *[YS81] A state x_r is reachable from a state x_0 if there exists $u(t) \in \mathcal{C}_m^{h-1}$ such that $x(t_r) = x_r$ for some $t_r > 0$.*

Lemma 3.6.1 *[YS81] Let $\mathcal{R}(0)$ be the set of reachable states from $x_0 = 0$. Then we have*

$$\mathcal{R}(0) = \langle \mathcal{A}, B_1 \rangle \oplus \langle \mathcal{N}, B_2 \rangle. \tag{3.55}$$

Lemma 3.6.2 *[YS81] Let $\mathcal{R}(x)$ be the set of reachable states from $x \in \mathcal{I}$. Then the complete set of reachable states $\mathcal{R}$ is*

$$\mathcal{R} = \bigcup_{x \in \mathcal{I}} \mathcal{R}(x) = \mathbb{R}^{n_1} \oplus \langle \mathcal{N}, B_2 \rangle. \tag{3.56}$$

We can adapt the conventional definition of controllability for descriptor systems.

Definition 3.6.2 ($\mathcal{C}$-controllability) *The descriptor system (3.53) is said to be completely controllable ($\mathcal{C}$-controllable) if one can reach any state from any initial state in finite time.*

Within the descriptor framework, we also define two different types of controllability as follows.

Definition 3.6.3 ($\mathcal{R}$-controllability) *The descriptor system (3.53) is said to be controllable within the set of reachable states ($\mathcal{R}$-controllable) if, from any initial state $x_0 \in \mathcal{I}$, there exists $u(t) \in \mathcal{C}_m^{h-1}$ such that $x(t_f) \in \mathcal{R}$ for any $t_f > 0$.*

Note that, in the case of the state variable systems, $\mathcal{C}$-controllability and $\mathcal{R}$-controllability are equivalent. This is, however, not so with descriptor systems.

Definition 3.6.4 (Imp-controllability) *[Cob84] The descriptor system (3.53) is said to be impulse controllable (Imp-controllable) if for every $w \in \mathbb{R}^{n_2}$ there exists $u(t) \in \mathcal{C}_m^{h-1}$ such that the fast subsystem θ_f satisfies*

$$x_2(t_f) = \sum_{k=1}^{h-1} \delta_{t_f}^{(k-1)} \mathcal{N}^k w, \quad \forall t_f > 0. \tag{3.57}$$

Theorem 3.6.1 (Regarding $\mathcal{C}$-controllability) *[YS81, Cob84, Dai89, Lew86]*

(1) *The following statements are equivalent.*

(1i) *The descriptor system (3.53) is $\mathcal{C}$-controllable.*

(1ii) *θ_s and θ_f are both controllable.*

(1iii) $\langle \mathcal{A}, B_1 \rangle \oplus \langle \mathcal{N}, B_2 \rangle = \mathbb{R}^{n_1+n_2}$.

(1iv) *$rank\,([sE - A\ B]) = n$, for a finite $s \in \mathbb{C}$ and $rank\,([E\ B]) = n$.*

(1v) *$Im(\lambda E - A) \oplus Im(B) = \mathbb{R}^n$ and $Im(E) \oplus Im(B) = \mathbb{R}^n$.*

(1vi) *The matrix $\mathfrak{C}$ is full row rank,*

$$\mathfrak{C} = \begin{bmatrix} -A & & & & B & & & \\ E & -A & & & & B & & \\ & E & \ddots & & & & B & \\ & & \ddots & -A & & & & \ddots \\ & & & E & & & & & B \end{bmatrix}$$

(2) *The following statements are equivalent.*

(2i) *θ_s is controllable.*

(2ii) *The descriptor system (3.53) is $\mathcal{R}$-controllable.*

(2iii) $\langle \mathcal{A}, B_1 \rangle = \mathbb{R}^{n_1}$.

(2iv) *$rank\,([sE - A\ B]) = n$, for a finite $s \in \mathbb{C}$.*

(2v) $Im(\lambda E - A) \oplus Im(B) = \mathbb{R}^n$.

(3) *The following statements are equivalent.*

(3i) *θ_f is controllable.*

(3ii) $\langle \mathcal{N}, B_2 \rangle = \mathbb{R}^{n_2}$.

(3iii) $rank\,([E\ B]) = n$.

(3iv) $Im(E) \oplus Im(B) = \mathbb{R}^n$.

(3v) $Im(\mathcal{N}) \oplus Im(B_2) = \mathbb{R}^{n_2}$.

(3vi) *The rows of B_2 corresponding to the bottom rows of all Jordan blocks of $\mathcal{N}$ are linearly independent.*

(3vii) *$v^\top (s\mathcal{N} - I)^{-1} B_2 = 0$ for constant vector v implies that $v = 0$.*

Theorem 3.6.2 (Regarding $\mathcal{R}$-controllability) *[YS81, Cob84, Dai89] The following statements are equivalent.*

1. *The descriptor system (3.53) is $\mathcal{R}$-controllable.*

2. θ_s *is controllable.*

3. $\langle \mathcal{A}, B_1 \rangle = \mathbb{R}^{n_1}$.

4. $rank\,([sE - A \;\; B]) = n$, *for a finite* $s \in \mathbb{C}$.

5. $Im(\lambda E - A) \oplus Im(B) = \mathbb{R}^n$.

Theorem 3.6.3 (Regarding Imp-controllability) *[Cob84, Dai89, Lew86] The following statements are equivalent.*

1. *The descriptor system* (3.53) *is Imp-controllable.*

2. θ_f *is Imp-controllable.*

3. $Ker(\mathcal{N}) \oplus \langle \mathcal{N}, B_2 \rangle = \mathbb{R}^{n_2}$.

4. $Im(\mathcal{N}) = \langle \mathcal{N}, B_2 \rangle$.

5. $Im(\mathcal{N}) \oplus Ker(\mathcal{N}) \oplus Im(B_2) = \mathbb{R}^{n_2}$.

6. $rank\left(\begin{bmatrix} A & E & B \\ E & 0 & 0 \end{bmatrix}\right) = n + r$.

7. *The rows of* B_2 *corresponding to the bottom rows of the nontrivial Jordan blocks of* $\mathcal{N}$ *are linearly independent.*

8. $v^\top \mathcal{N}(s\mathcal{N} - I)^{-1} B_2 = 0$ *for constant vector* v *implies that* $v = 0$.

It is observed that the conditions characterizing $\mathcal{R}$-controllability are only concerned with the slow subsystem θ_s. The response of the fast subsystem depends only on $u(t)$ and its derivatives. Any reachable state of θ_f $w \in \langle \mathcal{A}, B_1 \rangle$ can be written as $w = \sum_{k=0}^{h-1} \eta_k \mathcal{N}^k B_2$. Then it is easy to find an input $u(t)$ satisfying $u^{(k)}(t_f) = \eta_k$, for $k = 0, 1, \ldots, h-1$ (for example $u(t) = \sum_{k=0}^{h-1} \eta_k (t - t_f)^k / k!$) which leads to $x_2(t_f) = w$. Hence, the fast subsystem has no influence on $\mathcal{R}$-controllability.

Imp-controllability guarantees our ability to generate a maximal set of impulses, at each instant, in the following sense: Suppose E and A are given but B and u are allowed to vary over all values.

The following scheme is borrowed from [Mar03] to illustrate the relations between $\mathcal{C}$-controllability, $\mathcal{R}$-controllability and Imp-controllability.

$$\begin{array}{c} (E, A, B) \\ \mathcal{C} - controllable \end{array} \iff \left\{ \begin{array}{ccc} \begin{array}{c} (I_{n_1}, \mathcal{A}, B_1) \\ controllable \end{array} & \iff & \begin{array}{c} (E, A, B) \\ \mathcal{R} - controllable \end{array} \\ \\ \begin{array}{c} (\mathcal{N}, I_{n_2}, B_2) \\ controllable \end{array} & \Longrightarrow & \begin{array}{c} (E, A, B) \\ Imp - controllable \end{array} \end{array} \right.$$

Example 3.5 *For Example 3.1, we have*

$$E = \begin{bmatrix} L & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -R_1 & -1 & 1 \\ 0 & -1/R_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}. \tag{3.58}$$

Then

$$rank\,([sE - A \;\, B]) = 3 = n, \tag{3.59}$$

$$rank\,([E \;\, B]) = 3 = n, \tag{3.60}$$

$$rank\left(\begin{bmatrix} A & E & B \\ E & 0 & 0 \end{bmatrix}\right) = 5 = n + r. \tag{3.61}$$

Hence, this circuit is $\mathcal{C}$-controllable, $\mathcal{R}$-controllable and Imp-controllable.

Remark 3.6.1 *Note that the terms "finite dynamics controllable" and "impulse controllable" are also widely used to refer to as $\mathcal{R}$-controllable and Imp-controllable, respectively.*

3.7 Observability

In this section, we introduce observability for descriptor systems in a way that allows for a set of results analogous to the last section. Similarly, the concepts, that is, $\mathcal{C}$-observability, $\mathcal{R}$-observability and Imp-observability are defined.

Definition 3.7.1 ($\mathcal{C}$-observability) *The descriptor system (3.53) is said to be completely observable ($\mathcal{C}$-observability) if knowledge of $u(t)$ and $y(t)$ for $t \in [0, \infty]$ is sufficient to determine the initial condition x_0.*

Definition 3.7.2 ($\mathcal{R}$-observability) *The descriptor system (3.53) is said to be observable within the set of reachable states ($\mathcal{R}$-observable) if , for $t \geq 0$, $x(t) \in \mathcal{I}$ can be computed from $u(\tau)$ and $y(\tau)$ for any $\tau \in [0, t]$.*

Definition 3.7.3 (Imp-observability) *[Cob84] The descriptor system (3.53) is said to be impulse observable (Imp-observable) if, for every $w \in \mathbb{R}^{n_2}$, knowledge of $y(t)$ for $t \in [0, \infty]$ is sufficient to determine $x_2(t)$.*

$$x_2(t) = \sum_{k=1}^{h-1} \delta_{t_f}^{(k-1)} \mathcal{N}^k w. \tag{3.62}$$

Theorem 3.7.1 (Regarding $\mathcal{C}$-observability) *[YS81, Cob84, Dai89, Lew86]*

(1) *The following statements are equivalent.*

(1i) *The descriptor system (3.53) is $\mathcal{C}$-observable.*

(1ii) *θ_s and θ_f are both observable.*

(1iii) $\langle \mathcal{A}^\top, C_1^\top \rangle \oplus \langle \mathcal{N}^\top, C_2^\top \rangle = \mathbb{R}^{n_1+n_2}$.

(1iv) *rank* $\left([sE^\top - A^\top \; C^\top]\right) = n$, *for a finite* $s \in \mathbb{C}$ *and rank* $\left([E^\top \; C^\top]\right) = n$.

(1v) $Ker(\lambda E - A) \bigcap Ker(C) = \{0\}$ *and* $Ker(E) \bigcap Ker(C) = \{0\}$.

(1vi) *The matrix $\mathfrak{O}$ is full row rank,*

$$\mathfrak{O} = \begin{bmatrix} -A^\top & & & & C^\top & & & \\ E^\top & -A^\top & & & & C^\top & & \\ & E^\top & \ddots & & & & C^\top & \\ & & \ddots & -A^\top & & & & \ddots \\ & & & E^\top & & & & & C^\top \end{bmatrix}$$

(2) *The following statements are equivalent.*

(2i) *θ_s is observable.*

(2ii) *The descriptor system (3.53) is $\mathcal{R}$-observable.*

(2iii) $\langle \mathcal{A}^\top, C_1^\top \rangle = \mathbb{R}^{n_1}$.

(2iv) *rank* $\left([sE^\top - A^\top \; C^\top]\right) = n$, *for a finite* $s \in \mathbb{C}$.

(2v) $Ker(\lambda E - A) \bigcap Ker(B) = \{0\}$.

(3) *The following statements are equivalent.*

(3i) *θ_f is observable.*

(3ii) $\langle \mathcal{N}^\top, C_2^\top \rangle = \mathbb{R}^{n_2}$.

(3iii) *rank* $\left([E^\top \; C^\top]\right) = n$.

(3iv) $Ker(E) \bigcap Ker(C) = 0$.

(3v) $Ker(\mathcal{N}) \bigcap Ker(C_2) = \{0\}$.

(3vi) *The rows of $C_2^\top$ corresponding to the bottom rows of all Jordan blocks of $\mathcal{N}^\top$ are linearly independent.*

(3vii) $C_2(s\mathcal{N} - I)^{-1}v = 0$ *for constant vector v implies that $v = 0$.*

Theorem 3.7.2 (Regarding $\mathcal{R}$-observability) *[YS81, Cob84, Dai89] The following statements are equivalent.*

1. *The descriptor system (3.53) is $\mathcal{R}$-observable.*
2. *θ_s is observable.*
3. $\langle \mathcal{A}^\top, C_1^\top \rangle = \mathbb{R}^{n_1}$.
4. *rank* $\left([sE^\top - A^\top \; C^\top]\right) = n$, *for a finite* $s \in \mathbb{C}$.

5. $Ker(\lambda E - A) \bigcap Ker(C) = \{0\}$.

Theorem 3.7.3 (Regarding Imp-observability) *[Cob84, Dai89, Lew86] The following statements are equivalent.*

1. *The descriptor system (3.53) is Imp-observable.*
2. *θ_f is Imp-observable.*
3. $Im(\mathcal{N}^\top) \bigcap Ker\left(\langle \mathcal{N}^\top, C_2^\top \rangle\right) = \{0\}$.
4. $Ker(\mathcal{N}^\top) = \mathcal{N} Ker\left(\langle \mathcal{N}^\top, C_2^\top \rangle\right)$.
5. $Ker(\mathcal{N}) \bigcap Im(\mathcal{N}) \bigcap Ker(C_2) = \{0\}$.
6. $rank\left(\begin{bmatrix} A^\top & E^\top & C^\top \\ E^\top & 0 & 0 \end{bmatrix}\right) = n + r$.
7. *The rows of $C_2^\top$ corresponding to the bottom rows of the nontrivial Jordan blocks of $\mathcal{N}^\top$ are linearly independent.*
8. *$C_2(s\mathcal{N} - I)^{-1}\mathcal{N}v = 0$ for constant vector v implies that $v = 0$.*

Similar to $\mathcal{R}$-controllability, the characterizations for evaluating $\mathcal{R}$-observability are only concerned with the slow subsystem θ_s.

We use the following scheme borrowed from [Mar03] to illustrate the relations between $\mathcal{C}$-observability, $\mathcal{R}$-observability and Imp-observability.

$$\begin{array}{c} (E, A, C) \\ \mathcal{C}-observable \end{array} \Longleftrightarrow \left\{ \begin{array}{ccc} \begin{array}{c}(I_{n_1}, \mathcal{A}, C_1) \\ observable\end{array} & \Longleftrightarrow & \begin{array}{c}(E, A, C) \\ \mathcal{R}-observable\end{array} \\ \\ \begin{array}{c}(\mathcal{N}, I_{n_2}, C_2) \\ observable\end{array} & \Longrightarrow & \begin{array}{c}(E, A, C) \\ Imp-observable\end{array} \end{array} \right.$$

Example 3.6 *For Example 3.1, we have*

$$E = \begin{bmatrix} L & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -R_1 & -1 & 1 \\ 0 & -1/R_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}. \tag{3.63}$$

Then

$$rank\left([sE^\top - A^\top \; C^\top]\right) = 3 = n, \tag{3.64}$$

$$rank\left([E^\top \; C^\top]\right) = 3 = n, \tag{3.65}$$

$$rank\left(\begin{bmatrix} A^\top & E^\top & C^\top \\ E^\top & 0 & 0 \end{bmatrix}\right) = 5 = n + r. \tag{3.66}$$

Hence, this circuit is $\mathcal{C}$-observable, $\mathcal{R}$-observable and Imp-observable.

Remark 3.7.1 *Note that the terms "finite dynamics observable" and "impulse observable" are also widely used to refer to as $\mathcal{R}$-observable and Imp-observable, respectively.*

3.8 Duality

As known, there is a strong sense of symmetry between controllability and observability for the state-space setting. We now extend this idea to descriptor systems. Corresponding to (3.4), we define the dual system $\bar{\theta}$

$$E^\top \dot{x} = A^\top x + C^\top u, \tag{3.67a}$$
$$y = B^\top x. \tag{3.67b}$$

Then we have the following statements.

Theorem 3.8.1 (Duality)

1. *The descriptor system (3.4) is $\mathcal{C}$-controllable ($\mathcal{C}$-observable) if and only if the system (3.67) is $\mathcal{C}$-observable ($\mathcal{C}$-controllable).*

2. *The descriptor system (3.4) is $\mathcal{R}$-controllable ($\mathcal{R}$-observable) if and only if the system (3.67) is $\mathcal{R}$-observable ($\mathcal{R}$-controllable).*

3. *The descriptor system (3.4) is Imp-controllable (Imp-observable) if and only if the system (3.67) is Imp-observable (Imp-controllable).*

3.9 Discrete-time Descriptor Systems

Consider the following linear time-invariant discrete-time descriptor system:

$$Ex(k+1) = Ax(k) + Bu(k), \tag{3.68a}$$
$$y(k) = Cx(k) \tag{3.68b}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the descriptor variable and control input vector, respectively. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular, *i.e.*, $rank(E) = r \leq n$.

The aforementioned notations for continuous-time descriptor systems can be adapted directly for the discrete-time setting. The only two differences between continuous-time and discrete-time settings are impulsiveness and stability. For discrete-time descriptor systems, we use the term *non causality* instead of impulsiveness; while the discrete-time descriptor system (3.68) is said to be stable if $\rho(E, A) < 1$. Moreover, the definition of the transfer function for a regular discrete-time descriptor system is the same as that defined in the continuous-time setting, except for the use of the shift operator z instead of the Laplace operator s.

Interested readers are referred to [Dai89, XL06] for a comprehensive discussion of discrete-time descriptor systems.

3.10 Conclusion

This chapter recalls some basic concepts for linear time-invariant descriptor systems. Some fundamental and important results, such as regularity, admissibility, equivalent realizations, system decomposition and temporal response, are reviewed. The definitions of controllability and observability are also presented. Compared with state-space systems, for a descriptor system, three types of controllability are involved, that is, $\mathcal{C}$-controllability, $\mathcal{R}$-controllability and Imp-controllability. This is also the case for observability. In addition, the duality notion for descriptor systems is stated. The notations and definitions presented in this chapter will be frequently used throughout this dissertation.

Chapter 4

General Useful Results

In this chapter, some preliminary and useful results concerning continuous-time linear descriptor systems are presented. Four issues, which will be used subsequently, are addressed, namely, dissipativity, dilated linear matrix inequality (LMI) characterization, generalized Sylvester equations and generalized algebraic Riccati equations (GAREs). The main results of this chapter have been reported in [FYC10b, FYC10a].

A new Kalman-Yakubovich-Popov (KYP)-type lemma for dissipativity is first characterized in terms of a strict LMI condition. Compared with the existing results, this condition does not involve equality constraints which may result in numerical problems in checking inequality conditions owing to roundoff errors in digital computation. Dilated LMI characterizations within the descriptor framework are also studied in the current chapter. The deduced formulations cover not only the existing results reported in the literature, but also complete some missing conditions. This work also highlights the mutual relations of these characterizations and clarifies the relation between the dilated LMIs for conventional state-space systems and those for descriptor systems. The well-known LMI conditions for the state-space setting reported in the literature can be viewed as special cases. Moreover, generalized Sylvester equations are investigated in this part. Solvability of a generalized Sylvester equation is deduced which will be used in Chapters 6 and 7 to achieve comprehensive admissibility and to meet regulation constraints, respectively. Finally, GAREs associated with descriptor systems are discussed and the numerical algorithm for solving GAREs is provided.

4.1 Dissipativity

The notion of dissipativity plays a crucial role in systems and control theory both for theoretical considerations as well as from a practical point of view. Roughly speaking, a dissipative system is characterized by the property that at any time the amount of energy which the system can conceivably supply to its environment cannot exceed the amount of energy that has been supplied to it. In other words, a dissipative system can absorb part of the energy supplied from its environment with which it interacts, and transforms this energy into other forms, for instance, heat, electro-magnetic radiation, rotation, and so on [SW09].

The concept of dissipativity firstly emerged in the field of circuit theory, stemming from the phenomenon of energy dissipation across resistors [Zam66, Vid77]. Wu and Desoer investigated, from a more general operator theoretical viewpoint, the dissipative systems to give a different research direction cast in terms of the system input-output properties [WD70]. In addition, Willems [Wil72a, Wil72b] further studied this issue inspired by circuit theory, thermodynamics and mechanics, and connected this topic with control theory.

For a continuous, time-invariant dynamic system described as follows:

$$\dot{x} = f(x, w), \tag{4.1a}$$

$$z = g(x, w), \tag{4.1b}$$

with $x(0) = x_0$. x, w and z are the state taking its values in a state space $\mathcal{X}$, the input taking its values in an input space $\mathcal{W}$ and the output taking its values in the output space $\mathcal{Z}$, respectively. Moreover, $f : \mathcal{X} \times \mathcal{W} \to \mathcal{X}$ is a smooth mapping of its arguments, and $g : \mathcal{X} \times \mathcal{W} \to \mathcal{Z}$. Let $s(w(t), z(t))$ be a mapping with the following form:

$$s : \mathcal{W} \times \mathcal{Z} \to \mathbb{R}. \tag{4.2}$$

Assume that for all t_0, $t_1 \in \mathbb{R}$ and for all input-output pairs $(w(t), z(t))$ satisfying (4.1), $s(w(t), z(t))$ is locally absolutely integrable, that is, $\int_{t_0}^{t_1} |s(w(t), z(t))| dt < \infty$. The mapping s is referred to as the *supply function* (*supply rate*).

Definition 4.1.1 (Dissipativity) *The dynamic system* (4.1) *is said to be dissipative with respect to the supply function* $s(\cdot, \cdot)$ *if there exists a non-negative function, addressed as the storage function,* $\mathcal{V} : \mathcal{X} \to \mathbb{R}$*, such that for any time* $t_0 \leq t_1$ *and for any* $w \in \mathcal{L}_2[t_0\ t_1]$ *the following inequality holds*

$$\mathcal{V}(x(t_1)) - \mathcal{V}(x(t_0)) \leq \int_{t_0}^{t_1} s(w(t), z(t))\, dt. \tag{4.3}$$

The notion of strict dissipativity, which is the primary topic in the sequel, can also be defined through a simple modification of Definition 4.1.1.

Definition 4.1.2 (Strict dissipativity) *The dynamic system* (4.1) *is said to be strictly dissipative with respect to the supply function* $s(\cdot, \cdot)$ *if there exists a non-negative function,* $\mathcal{V} : \mathcal{X} \to \mathbb{R}$ *and an* $\epsilon > 0$ *such that any time* $t_0 \leq t_1$ *and for any* $w \in \mathcal{L}_2[t_0\ t_1]$ *the following inequality holds*

$$\mathcal{V}(x(t_1)) - \mathcal{V}(x(t_0)) \leq \int_{t_0}^{t_1} s(w(t), z(t))\, dt - \epsilon^2 \int_{t_0}^{t_1} \|w(t)\|^2 dt. \tag{4.4}$$

As known, many important control issues can be formulated as dissipativity with quadratic supply functions, for instance, positive realness, bounded realness and circle criterion. For example, the time domain property of positive realness can be viewed as follows.

Definition 4.1.3 (Positive realness) *[AV73] The system (4.1) is said to be positive real if for all $w \in \mathcal{W}$, $t \geq 0$*

$$\int_0^t z^\top(\tau)w(\tau)d\tau \geq 0, \tag{4.5}$$

whenever the system is relaxed at time $t = 0$ (i.e. $x(0) = 0$), where the integral is considered along the system trajectories.

Moreover, the definition of bounded realness is given as:

Definition 4.1.4 (Bounded realness) *[BGFB94] The system (4.1) is said to be bounded real if it is* nonexpansive*, that is,*

$$\int_0^t z^\top(\tau)z(\tau)d\tau \leq \int_0^t w^\top(\tau)w(\tau)d\tau, \tag{4.6}$$

whenever the system is relaxed at time $t = 0$ (i.e. $x(0) = 0$), where the integral is considered along the system trajectories.

It can be deduced, from the above definitions, that the system (4.1) is positive real and bounded real if it is dissipative with respect to the supply functions $s(w(t), z(t)) = z^\top(t)w(t)$, and $s(w(t), z(t)) = -z^\top(t)z(t) + w^\top(t)w(t)$, respectively. One notes that the latter is also referred to as *finite gain stability* [GS84].

One of the most important formulations characterizing the property of a dissipative system is called the Kalman-Yakubovich-Popov (KYP) lemma, which highlights the relation between dissipativity and frequency domain properties. This lemma was firstly proposed in [Kal63, Yak63, Pop64], and was then generalized to multivariable systems by [And67, AV73] for linear continuous-time systems.

Dissipativity (or its specializations) for conventional state-space systems has been well studied in the literature [AV73, GG97, HB91, HIS99, Ran96, SKS94, IH05]. Parallel to the conventional linear system theory, this problem has also been extended to descriptor systems by a number of scholars. For example, positive realness is studied in [FJ04, WC96, ZLX02] and bounded realness is investigated in [MKOS97, TMK94, WYC98]. Moreover, the dissipativity theory is also studied for nonlinear descriptor systems [Kab05]. However, most of the reported results require some prior conditions on the realization of descriptor systems, besides the assumptions of regularity and controllability. For example, the criteria in [WC96, ZLX02] require $D^\top + D > 0$, while $D = 0$ is supposed for the bounded real lemma in [WYC98]. To remove this restriction, the authors recently proposed LMI-based realization-independent characterizations for the dissipativity of descriptor systems [Mas06, Mas07, CT08].

Motivated by the results in [Mas06, Mas07] which are characterized by non-strict LMIs, we introduce a new KYP-type lemma for dissipativity of continuous descriptor systems. The solutions given in this part are all characterized in terms of a strict LMI, therefore they are very tractable and reliable in numerical computations.

Remark 4.1.1 *The main results of this section have been reported in [FYC10b]. See Appendix I.*

4.2 Dilated Linear Matrix Inequalities

The history of the use of LMIs in the context of dynamic system and control goes back more than 120 years. This story probably begins in about 1890, when Aleksandr Mikhailovich Lyapunov published his fundamental work on the stability of motion. Lyapunov showed that the differential equations of the form

$$\dot{x}(t) = Ax(t) \tag{4.7}$$

are stable if and only if there exists a positive definite matrix P such that

$$A^\top P + PA < 0. \tag{4.8}$$

This statement is now called Lyapunov theory and the requirement $P > 0$ together with (4.8) is what we now call a Lyapunov inequality on P (referred to as a Lyapunov matrix), having a special form of an LMI. This inequality can be solved analytically by solving a set of linear equations. In the early 1980s, it was observed that many LMIs arising in systems and control theory can be formulated as convex optimization problems and these problems can be reliably solved by computer, even if for many of them no analytical solution has been found.

Over the past two decades, LMI-based techniques [IS94, GA94, Sch92, CG96] have been employed as an important tool in systems analysis and controller design synthesis because of its efficient and reliable solvability through convex optimization algorithms and powerful numerical supports of LMI toolboxes available in popular application software [GNLC95]. This method benefits not only from simplifying in a wide sense the necessity of certain cumbersome material of Riccati (Riccati-like) equations when the classical approaches are used, but also from its capability of gaining access to a vast panorama of control problems. Stability and many performance specifications, such as, eigenvalue assignment, H_2 and H_∞ control, multiobjective design problems and linear parameter-varying (LPV) synthesis, can be interpreted into LMIs [BGFB94, SGC97, MOS98].

However, the conservatism of the LMI formulations emerges when handling some complicated control problems. For instance, while using standard LMIs for solving a

Figure 4.1: Aleksandr Mikhailovich Lyapunov

multiobjective control design problem, a common Lyapunov matrix is imposed on all equations involved to render the synthesis problem convex, which is referred to as the *Lyapunov Shaping Paradigm* in [SGC97]. This restriction inherently causes conservatism into design procedure. In order to reduce this conservatism, a new characterization named the dilated (extended/enhanced) LMI was first introduced in [GdOH98] for continuous-time state-space systems. From then on, tremendous investigations have been launched to explore new dilated LMI characterizations, and constructive results have been reported in the literature for analysis and controller design synthesis in both discrete-time and continuous-time settings [ATB01, BBdOG99, EH04, EH05, dOBG99, dOGB99, dOGH99, dOGB02, PABB00, Xie08, PDSV09]. Generally speaking, the advantages of these dilated LMIs over the standard ones can be resumed as follows:

- The dilated LMIs do not involve product terms of the Lyapunov matrix and the system matrix A. This separation enables the use of parameter-dependent Lyapunov functions for robust system analysis and controller synthesis;
- No indefinite quadratic terms of the system matrix A;
- Auxiliary (slack) variables are introduced which means that more decision variables are involved. This fact might reduce the conservatism in robust analysis and controller synthesis.

Besides, the same enthusiasm has been witnessed for descriptor systems and the resulting dilated LMIs have also been studied in [XL06, Yag10, Seb07, Seb08].

Motivated by [PDSV09], we explore dilated LMIs with regard to admissibility and performance specifications (H_2 and dissipativity) for linear descriptor systems through reciprocal application of the projection lemma. The purpose is not only to revisit the existing LMI catechizations, but to complete some missing conditions as well.

Lemma 4.2.1 (Projection Lemma) *[BGFB94, IS94] Given a symmetric matrix $\Xi \in \mathbb{R}^{n\times n}$ and two matrices $\Psi \in \mathbb{R}^{n\times m}$ and $\Upsilon \in \mathbb{R}^{k\times n}$ with $rank(\Psi) < n$ and $rank(\Upsilon) < n$. There exists an unstructured matrix Θ such that*

$$\Xi + \Upsilon^\top \Theta^\top \Psi + \Psi^\top \Theta \Upsilon < 0 \tag{4.9}$$

if and only if the following projection inequalities with respect to Θ are satisfied

$$N_\Psi^\top \Xi N_\Psi < 0, \quad N_\Upsilon^\top \Xi N_\Upsilon < 0, \tag{4.10}$$

where N_Ψ and N_Υ are any matrices whose columns form a basis of the nullspaces of Ψ and Υ, respectively.

The main idea used here is that the standard LMI characterizations are transformed into quadratic forms, as the first inequality in (4.10), in which N_Ψ relates to the system data. Then, the dilated LMI conditions can be derived by applying Projection Lemma. Four different types of dilated LMIs are explored, according to the constructions of N_Υ:

I $N_\Upsilon = [\ ]$. In this case, the second inequality of (4.10) vanishes and $\Upsilon = I$;

II Choose N_Υ such that the second inequality of (4.10) is equivalent to the positive definiteness of partial entries of P;

III Choose N_Υ such that a trivial matrix inequality is introduced;

IV Combine the strategies II and III.

Remark 4.2.1 *The main results of this section have been reported in [FYC10a]. See Appendix II*

4.3 Generalized Sylvester Equation

4.3.1 Sylvester Matrix Equation

Many problems in systems and control theory are related to solvability of Sylvester equations. As known, these equations have important applications in stability analysis, observer design, output regulation problems and eigenvalue assignment [Tsu88, Doo84, FKKN85, Dua93]. In this section, we deal with the problem of generalized Sylvester equations associated with descriptor systems.

A matrix equation of interest in control theory is of the form,

$$\sum_{i=1}^{k} A_i X S_i = R, \tag{4.11}$$

where A_i, S_i and R are given matrices and X is an unknown. In [Hau83, Hau94], Hautus provided a detailed discussion on such equations while recalling historical origins of them.

A well-known example of the linear matrix equation (4.11) is what is referred to as the Sylvester equation,

$$AX - XS = R, \tag{4.12}$$

where A and S are square matrices. As proved by Sylvester [Syl84], the equation (4.12) is universally solvable[1] if and only if the matrices A and S have no eigenvalues in common. A result for the general equation (4.11), in the same spirit as that of the Sylvester equation, is still not known. Thus researchers restrict themselves to some special cases. For example, the authors in [Chu87, HG89, GLAM92] considered the solvability for the matrix equation of the form,

$$AXB - CXD = E. \tag{4.13}$$

It has been proved that the equation (4.13) has a unique solution if and only if the matrix pencils $A - \lambda C$ and $D - \lambda B$ are regular and the spectrum of one is disjoint from the negative of the spectrum of the other.

The generalized Sylvester equation of the form

$$AX - YB = C, \tag{4.14a}$$
$$DX - YE = F, \tag{4.14b}$$

has also been studied in the literature, e.g. see [Ste73, KW89, Wim94]. It shows that in the case where the matrices of (4.14) are real and A, B, D and E are square, the generalized Sylvester equation has a unique solution if and only if the polynomials $\det(A - sB)$ and $\det(D - sE)$ are coprime [Ste73]. With these assumptions, the authors in [KW89] deduced a solution of (4.14) by applying generalized Schur methods. Moreover, Wimmer extended Roth's equivalence theorem [Rot52] to a pair of Sylvester equations and concluded the following statement for the consistency of (4.14) without assumptions.

Theorem 4.3.1 *[Wim94] The equation (4.14) has a solution X and Y if and only if there exist nonsingular matrices R and S with appropriate dimensions such that*

$$S\left[\begin{bmatrix} A & C \\ 0 & B \end{bmatrix} - \lambda \begin{bmatrix} D & F \\ 0 & E \end{bmatrix}\right] = \left[\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} - \lambda \begin{bmatrix} D & 0 \\ 0 & E \end{bmatrix}\right] R. \tag{4.15}$$

The above theorem can also be interpreted as the polynomial matrices $\begin{bmatrix} A - \lambda D & C - \lambda F \\ 0 & B - \lambda E \end{bmatrix}$ and $\begin{bmatrix} A - \lambda D & 0 \\ 0 & B - \lambda E \end{bmatrix}$ are unimodularly equivalent.

[1]Equation (4.11) is said to be universally solvable if it has a solution for every R.

Within the descriptor framework, the so-called generalized Sylvester equations have also been received wide attention from scholars [Chu87, HG89, Dua96, Ben94, CdS05, Dar06]. In [Dua96], Duan considered the generalized Sylvester matrix equation of the form

$$AV + BW = EVC, \tag{4.16}$$

where $A \in \mathbb{C}^{m\times n}$, $B \in \mathbb{C}^{m\times r}$, $C \in \mathbb{C}^{p\times p}$, $E \in \mathbb{C}^{m\times n}(p \leq n)$ are known, and $V \in \mathbb{C}^{n\times p}$ and $W \in \mathbb{C}^{r\times p}$ are to be determined. This equation is directly related to the eigenstructure assignment and observer design for linear descriptor systems. Based on the Smith canonical form of the matrix $[A - \lambda E \;\; B]$, the author provided a simple, direct, complete and explicit parametric solution of (4.16) for the matrix C in the Jordan form with arbitrary eigenvalues.

Moreover, combined with some rank and regional pole placement constraints, the authors investigated the following problem in [CdS05, Dar06].

Problem 4.3.1 *Consider a linear descriptor system represented by*

$$\begin{aligned} E\dot{x} &= Ax + Bu, & (4.17a)\\ y &= Cx, & (4.17b) \end{aligned}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$ and $u \in \mathbb{R}^m$, respectively, are the descriptor variable, the measured output and the control input vectors. The matrix $E \in \mathbb{R}^{n\times n}$ is such that $rank(E) = r < n$ and $p < r$. Let $\mathcal{D}$ be a region in the open left-half complex plane, $\mathcal{D} \subseteq \mathbb{C}^-$, symmetric with respect to the real axis. Find matrices $T \in \mathbb{R}^{(r-p)\times n}$, $Z \in \mathbb{R}^{(r-p)\times p}$ and $H \in \mathbb{R}^{(r-p)\times(r-p)}$ such that

$$TA - HTE = -ZC, \quad \sigma(H) \subset \mathcal{D}, \tag{4.18}$$

under the rank constraint

$$rank\left(\begin{bmatrix} TE \\ LA \\ C \end{bmatrix}\right) = n, \tag{4.19}$$

where $L \in \mathbb{R}^{(r-p)\times n}$ is any full row rank matrix satisfying $LE = 0$.

The main motivation of solving this problem is directly concerned with the design of a reduced-order observer of minimal order $r - p$ under the form

$$\begin{aligned} \dot{z}(t) &= Hz(t) + TBu(t) - Zy(t), & (4.20a)\\ \hat{x}(t) &= Sz(t) + \bar{N}\bar{y}(t) + Ny(t), & (4.20b) \end{aligned}$$

where $z \in \mathbb{R}^{(r-p)}$ is the state of the observer and $\bar{y} \in \mathbb{R}^{(n-r)}$ is a fictitious output. As shown in [CdS05], if Problem 4.3.1 is solved for some matrices T, Z and H and if we compute the matrices S, $\bar{N}$ and N satisfying

$$\begin{bmatrix} S & \bar{N} & N \end{bmatrix} \begin{bmatrix} TE \\ LA \\ C \end{bmatrix} = I, \tag{4.21}$$

then, the corresponding minimal order observer given by (4.20) is such that

(i) the observer state verifies

$$\lim_{t\to\infty} [z(t) - TEx(t)] = 0, \quad \forall z(0), \quad Ex(0); \tag{4.22}$$

(ii) for $\bar{y}(t) = -LBu(t)$, the estimated state $\hat{x}(t)$ satisfies

$$\lim_{t\to\infty} [x(t) - \hat{x}(t)] = 0, \quad \forall x(0), \quad \hat{x}(0). \tag{4.23}$$

Note that for $L = 0$, Problem 4.3.1 reduces to finding matrices $T \in \mathbb{R}^{(n-p)\times n}$, $Z \in \mathbb{R}^{(n-p)\times p}$ and $H \in \mathbb{R}^{(n-p)\times(n-p)}$ such that

$$TA - HTE = -ZC, \quad \sigma(H) \subset \mathcal{D}, \tag{4.24}$$

under the rank constraint

$$rank\left(\begin{bmatrix} TE \\ C \end{bmatrix}\right) = n. \tag{4.25}$$

They are required conditions for the reduced-order observer design with order $n - p$, see for example [DB95, DZH96, Var95].

The solvability of Problem 4.3.1 was deduced in terms of the concept of $\mathcal{D}$-strong detectability.

Definition 4.3.1 ($\mathcal{D}$-strong detectability) *The descriptor system (4.17) is $\mathcal{D}$-strongly detectable if and only if the following conditions are satisfied:*

(1) $rank\left(\begin{bmatrix} A - \lambda E \\ C \end{bmatrix}\right) = n,\ \forall \lambda \in \mathbb{C},\ \lambda \notin \mathcal{D},$

(2) $rank\left(\begin{bmatrix} E \\ LA \\ C \end{bmatrix}\right) = n.$

Theorem 4.3.2 *[CdS05, Dar06] There exists $T \in \mathbb{R}^{(r-p)\times n}$, $Z \in \mathbb{R}^{(r-p)\times p}$ and $H \in \mathbb{R}^{(r-p)\times(r-p)}$ with $\sigma(H) \subset \mathcal{D} \subseteq \mathcal{C}^-$ solving Problem (4.3.1), if and only if the descriptor system (4.17) is $\mathcal{D}$-strongly detectable and*

$$rank\left(\begin{bmatrix} LA \\ C \end{bmatrix}\right) = n - r + p. \tag{4.26}$$

4.3.2 Considered Generalized Sylvester Equation

For a general case, we define the following matrix equation:

$$\sum_{1\leq i\leq f, 1\leq j\leq k} \Phi_{ij}\Theta_j\Psi_{ij} = P_i, \tag{4.27}$$

where Φ_{ij}, Ψ_{ij} and P_i are constant matrices with appropriate dimensions, while Θ_j is the matrix variable. It is worth pointing that (4.27) can be regarded as a generalized Sylvester equation, which covers the aforementioned generalized Sylvester equations reported in [KW89, Chu87, HG89, GLAM92, Dua96].

For example, the Sylvester equation (4.13) can be obtained by setting $f = 1$, $k = 2$, $\Phi_{11} = A$, $\Phi_{12} = C$, $\Psi_{11} = -B$, $\Psi_{12} = D$, $P_1 = E$, and $\Theta_1 = \Theta_2 = X$ in (4.27); the equation (4.14) can be viewed as (4.27) with $f = k = 2$, $\Phi_{11} = A$, $\Psi_{11} = I$, $\Phi_{12} = -I$, $\Psi_{12} = B$, $\Phi_{21} = D$, $\Psi_{21} = I$, $\Phi_{22} = -I$, $\Psi_{22} = E$, $P_1 = C$, $P_2 = F$, $\Theta_1 = X$ and $\Theta_2 = Y$; while the generalized Sylvester equation (4.16) can be regarded as (4.27) with $f = 1$, $k = 2$, $\Phi_{11} = [A\ B]$, $\Phi_{12} = [E\ 0]$, $\Psi_{11} = I$, $\Psi_{12} = C$, $P_1 = 0$, and $\Theta_1 = \Theta_2 = [V^\top\ W^\top]^\top$.

Now, we discuss briefly the solvability of a special case of (4.27). According to the properties of the Kronecker product, we have the following relationship

$$AXB = (B^\top \otimes A)vec(X). \tag{4.28}$$

Then, the matrix equation (4.27) can be written as

$$(\mathbf{N}^\top \otimes \mathbf{M})vec\left(\mathbf{diag}(I_f \otimes \Theta_1, \cdots, I_f \otimes \Theta_k)\right) = vec(\mathbf{L}) \tag{4.29}$$

where

$$\mathbf{M} = \left[\ diag(\Phi_{11}, \cdots, \Phi_{f1}) \,\middle|\, \cdots \,\middle|\, diag(\Phi_{1k}, \cdots, \Phi_{fk})\ \right], \tag{4.30a}$$

$$\mathbf{N} = \left[\ diag(\Psi_{11}^\top, \cdots, \Psi_{f1}^\top) \,\middle|\, \cdots \,\middle|\, diag(\Psi_{1k}^\top, \cdots, \Psi_{fk}^\top)\ \right]^\top, \tag{4.30b}$$

$$\mathbf{N} = \left[\ P_1^\top \quad \cdots \quad P_k^\top\ \right]^\top. \tag{4.30c}$$

In the parts to follow, we focus on the special case of the generalized Sylvester equation (4.27). Let us consider the following matrix equation

$$AXB - CYD = E, \tag{4.31a}$$

$$FXG - HYJ = K, \tag{4.31b}$$

where A, B, C, D, E, F, G, H, J and K are known matrices with appropriate dimensions, and X and Y are the matrix variables to determine.

Using the Kronecker product, (4.31) can be written by

$$\begin{bmatrix} B^\top \otimes A & D^\top \otimes C \\ G^\top \otimes F & J^\top \otimes H \end{bmatrix} \begin{bmatrix} vec(X) \\ vec(Y) \end{bmatrix} = \begin{bmatrix} vec(E) \\ vec(K) \end{bmatrix}. \tag{4.32}$$

Figure 4.2: Jacopo Francesco Riccati

Clearly, the solution to this equation can easily be obtained through a linear program.

In Chapters 6 and 7, a simplified version of the generalized Sylvester equation (4.27) is used to achieve comprehensive admissibility and to meet regulation constraints, respectively.

4.4 Generalized Algebraic Riccati Equation

In mathematics, a Riccati equation is named after Count Jacopo Francesco Riccati and is referred to any ordinary differential equation which is quadratic in the unknown function. In other words, it is an equation of the form

$$\dot{y}(x) = q_0(x) + q_1(x)y(x) + q_2(x)y^2(x), \tag{4.33}$$

where $q_0(x) \neq 0$ and $q_2(x) \neq 0$. Clearly, if $q_0(x) = 0$, (4.33) is a Bernoulli equation, and $q_2(x) = 0$ is a first order linear ordinary differential equation.

In systems and control theory, the term "Riccati equation" is used to refer to matrix equations with an analogous quadratic term, which occur in both continuous-time and discrete-time linear-quadratic-Gaussian (LQG) control problems. The steady-state (non-dynamic) version of these is referred to as the algebraic Riccati equation (ARE). The ARE is either of the following matrix equations: the continuous-time algebraic Riccati equation (CARE)

$$A^\top P + PA - PBR^{-1}B^\top P + Q = 0, \tag{4.34}$$

or the discrete-time algebraic Riccati equation (DARE)

$$A^\top PA - (A^\top PB)(R + B^\top PB)^{-1}(B^\top PA) + Q = P, \tag{4.35}$$

where $P \in \mathbb{R}^{n\times n}$ is the unknown symmetric matrix and A, B, Q, R are known real coefficient matrices with appropriate dimensions. The ARE determines the solution of two of the most fundamental problems in control theory, namely, the infinite horizon time-invariant Linear-Quadratic Regulator (LQR) problem as well as that of the infinite horizon time-invariant LQG control problem. Comprehensive studies on the AREs in both continuous-time and discrete-time settings have been reported in the literature, for example, see [WAL84, LR95].

In this section, we study the continuous-time generalized algebraic Riccati equation (GARE) of the form

$$E^\top P = P^\top E, \tag{4.36a}$$

$$A^\top P + P^\top A - (P^\top B + S)R^{-1}(P^\top B + S)^\top + Q = 0, \tag{4.36b}$$

where $E \in \mathbb{R}^{n\times n}$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $Q \in \mathbb{R}^{n\times n}$, $S \in \mathbb{R}^{n\times m}$, $R \in \mathbb{R}^{m\times m}$ and $rank(E) = r \leq n$, associated with the descriptor system

$$E\dot{x} = Ax + Bu, \tag{4.37a}$$

$$y = Cx + Du. \tag{4.37b}$$

Note that Katayama and Minamino, in [KM92], proposed an asymmetric GARE to study the linear quadratic optimal regulator problem for descriptor systems. The GARE (4.36) can be viewed as the symmetric version of the one proposed by Katayama and Minamino. The authors in [WFC93] also used a symmetric GARE for the robustness properties of the LQR problem within the descriptor framework.

Definition 4.4.1 (Admissible solution) *A solution P to the GARE (4.36) is called an admissible solution if $\big(E, A - BR^{-1}(B^\top P + S^\top)\big)$ is regular, impulsive-free and stable as well as $E^\top P \geq 0$.*

It is noted that the admissible solution P might not be unique, but $E^\top P$ is unique.

The H_2 and H_∞ control problems for descriptor systems reported in [TK98, TMK94, KK97] are directly related to admissible solutions to the underlying GAREs. For example, for the H_2 control, we set

$$\begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix} = \begin{bmatrix} C^\top \\ D^\top \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix} \tag{4.38}$$

Furthermore, the H_∞ control problem needs to solve the following H_∞-like Riccati equation

$$E^\top P = P^\top E, \tag{4.39a}$$

$$A^\top P + P^\top A + P^\top(\gamma^{-2}B_1B_1^\top - B_2B_2^\top)P + C^\top C = 0, \tag{4.39b}$$

where $\gamma \in \mathbb{R}^+$. The second equation can be rewritten in the GARE format as

$$A^\top P + P^\top A + P^\top \underbrace{\begin{bmatrix} B_1 & B_2 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} -\gamma^2 & 0 \\ 0 & I \end{bmatrix}}_{R} P \begin{bmatrix} B_1^\top \\ B_2^\top \end{bmatrix} + \underbrace{C^\top C}_{Q} = 0. \tag{4.40}$$

Let us make some assumptions which guarantee the solvability of the GARE.

Assumption 4.4.1

(A1). *(E, A) is regular;*

(A2). *$D^\top D > 0$;*

(A3). *(E, A, B) is finite dynamics stabilizable and impulse controllable;*

(A4). $\begin{bmatrix} A - sE & B \\ C & D \end{bmatrix}$ *has no invariant zeros on the imaginary axis including infinity;*

These assumptions are quite standard and coincide with the classical assumptions for conventional state-space systems [ZDG96]. Note that (A2) is widely made for state-space systems to guarantee a regular problem, i.e. one without zeros at infinity. However, descriptor systems can still have zeroes at infinity, even if D is full column rank. This condition is made here in order to deduce the controller in terms of the GARE. Moreover, Assumption (A2) can be made without loss of generality within the descriptor framework. If it does not hold, an equivalent realization satisfying this assumption can always be obtained [MKOS97, TK98]. Assumption (A3) is obviously essential to the existence of an admissible solution. Assumption (A4) is made to guarantee a regular problem. Moreover, for the state-space case where $E = I$, (A1) always holds.

Based on the generalized eigenvalue problem (GEP), numerical methods for solving the GARE (4.36) have been reported in [TMK94, TK98, KK97, KM92, WYC98]. Now we recall these processes. To this end, let us construct the Hamiltonian pencil of the form

$$H - \lambda J = \begin{bmatrix} A & 0 & B \\ -Q & -A^\top & -S \\ S^\top & B^\top & R \end{bmatrix} - \lambda \begin{bmatrix} E & 0 & 0 \\ 0 & E^\top & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{4.41}$$

with $\lambda \in \mathbb{C}$. Under Assumptions (A1)-(A4), (J, H) is regular, impulse-free and has no finite dynamic modes on the imaginary axis including infinity. In addition, this matrix pencil contains r stable finite eigenvalues, r unstable eigenvalues, and $2n + m - 2r$ infinite eigenvalues. Let $\Lambda = [\Lambda_1^\top \; \Lambda_2^\top \; \Lambda_3^\top]^\top \in \mathcal{C}^{(2n+m)\times n}$ be the matrix consisting of the generalized eigenvectors and the generalized principal vectors related to the stable finite eigenvalues. We have

$$\begin{bmatrix} E & 0 & 0 \\ 0 & E^\top & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{bmatrix} \Delta = \begin{bmatrix} A & 0 & B \\ -Q & -A^\top & -S \\ S^\top & B^\top & R \end{bmatrix} \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{bmatrix}, \tag{4.42}$$

where $\Delta \in \mathcal{C}^{r\times r}$ is a Jordan form with all eigenvalues in the open left-half complex plane.

According to [TMK94], we can state that any admissible solution P to the GARE (4.36) is given by

$$P = \begin{bmatrix} \Lambda_2 & W_2 P_r \end{bmatrix} \begin{bmatrix} \Lambda_1 & W_1 \end{bmatrix}^{-1}, \tag{4.43}$$

where P_r satisfies

$$A_r^\top P_r + P_r^\top A_r - (P_r^\top B_r + S_r) R^{-1} (P_r^\top B_r + S_r)^\top + Q_r = 0, \tag{4.44a}$$

$$A_r = W_2^\top A W_1, \quad B_r = W_2^\top B, \quad Q_r = W_1^\top Q W_1, \quad S_r = W_1^\top S, \tag{4.44b}$$

and $W_1 \in \mathbb{R}^{n\times(n-r)}$, $W_2 \in \mathbb{R}^{n\times(n-r)}$ are any full column rank matrices satisfying $EW_1 = 0$ and $E^\top W_2 = 0$, respectively.

4.5 Conclusion

This chapter presents some useful results for descriptor systems. Dissipativity is characterized via a strict LMI which removes the non-strict constraints in the existing results and is very tractable and reliable in numerical computations. Based on projection lemma, dilated LMI conditions for linear descriptor systems is also explored and the well-known results for the state-space setting can be viewed as special cases of these formulations. Furthermore, the issues of generalized Sylvester equations and GAREs associated with descriptor systems are also addressed in this chapter. Numerical algorithms for solving these equations are provided.

The present results play an important role in the following chapters of this dissertation. The LMI condition for dissipativity will be used to solve the dissipativity control problem under regulation constraints for descriptor systems in Chapter 7. The issue of a generalized Sylvester equation will be adapted in Chapters 6 and 7 to achieve comprehensive admissibility in the presence of unstable or nonproper weighting functions and to satisfy regulation constraints in the presence of an infinite-energy exo-system, respectively. In addition, the desired H_2 or H_∞ controllers in Chapters 6 and 7 are based on the solvability of underlying GAREs. Hence, the result concerning the GARE is indispensable for these chapters.

Chapter 5

Simultaneous H_∞ Control

5.1 Strong and Simultaneous Stabilization

Strong stabilization which consists in finding a stable controller to stabilize a given plant has several practical reasons [Vid85] and was well studied in the 1970s. As pointed out by Vidyasagar, unstable controllers are highly sensitive and their responses to sensor-fault and plant uncertainties/nonlinearities are unpredictable [Vid85]. It is also known that if a controller contains unstable poles, the tracking or disturbance rejection performance of the underlying closed-loop system will be degraded [DFT92, FL85]. Unlike unstable controllers, stable controllers allow to realize off-line test for checking some potential faults in the process of implementation and for comparing with the required theoretical design specifications.

A necessary and sufficient condition for the existence of a stable controller is called the *Parity Interlacing Property* (PIP) [YBL74], that is, the plant has even number of poles between any pair of its zeros on the extended positive real axis. Different methods have been proposed in the literature to solve the strong stabilization problem [YBL74, Vid85, SGP97, SGP98].

On the other hand, the simultaneous stabilization problem which is concerned with the design of a single linear time-invariant controller such that a set of plants is stabilized was introduced by Sake *et al.* [SM82] and Vidyasagar *et al.* [VV82] in the early 1980s. The issue of simultaneous stabilization of a set of plants arises in the linearization of nonlinear process plants at various operating points. In [Vid85], it is shown that the problem of simultaneously stabilizing k plants can always be reduced to simultaneously stabilizing $k-1$ associated plants by a stable controller. Moreover, for the case where $k = 2$, the problem can be equivalently considered as a strong stabilization problem. In this single system, the strong stabilization problem can be checked by the PIP. For three or more plants, there are, however, no necessary and sufficient conditions available to check the feasibility of strong stabilization [BCG93, BGMR94] in the general case.

5.2 Strong and Simultaneous $\mathbf{H_\infty}$ Control

H_∞ control plays an essential role in systems and control theory. It is always chosen as an appropriate criterion for disturbance minimization problems owing to the fact that the H_∞ norm can be interpreted as the worst-case gain of the system. H_∞ control theory for state-space systems was extensively studied and reached a fairly mature state in the late 1980s. The solution was given by solving two algebraic Riccati equations (AREs) together with a spectral radius condition, and the set of all H_∞ controllers was parameterized via the stabilizing solutions to the AREs with a free parameter Q [GD88, DGKF89, ZDG96]. However, the Riccati-based method giving an analytical solution to this problem imposes some restrictions on the realization of systems. To offer a large framework of resolution, LMI-based approaches were introduced to solve the H_∞ control problem [Sch92, IS94, GA94].

Based on the context of a strong stabilization problem, Zeren and Özbay, in [ZO99], introduced the *stable H_∞ stabilization problem* by integrating the performance index into consideration, which requires the design of a stable controller such that a given plant is stabilized and the H_∞ norm of the underlying closed-loop system is bounded by some specified performance level γ as well. A sufficient condition of this problem was also given by the existence of a positive definite solution to an ARE. Recently, this problem was further developed and a sufficient condition in terms of LMIs was derived by Gümuüssoy and Özbay in [GO05]. Besides, Campos-Delgado and Zhou presented an approach for designing stable H_2 and H_∞ controllers using the direct optimization of the free transfer matrices in the suboptimal H_2 and H_∞ controller parameterization [CDZ03]. Other related results for stable H_∞ controller design can be found in [ZO00, CDZ01, CC01, LS02] and the references therein.

The problem of *simultaneous H_∞ control* was introduced by Cao and Lam in [CL00]. This issue can be regarded as the aforementioned simultaneous stabilization problem with guaranteed H_∞ performance. It means that a single controller is sought to stabilize a collection of systems such that each closed-loop transfer function has an H_∞ norm less than a given level γ. In order to solve this problem, the authors discussed a new problem named *strong γ-H_∞ stabilization* requiring the construction of a controller such that the stable H_∞ stabilization problem is solved and the H_∞ norm of the controller is also bounded by γ. It proved, via coprime factorization, that the simultaneous H_∞ control for a set of plants is achieved if and only if the strong γ-H_∞ stabilization of a corresponding augmented system is solvable. Cheng *et al.* extended this issue to a *strong γ_k-γ_{cl} H_∞ stabilization problem*, by considering the H_∞ norms of the controller and closed-loop separately [CCS07, CCS08, CCS09].

In this chapter, we investigate strong H_∞ stabilization and simultaneous H_∞ control problems for continuous-time descriptor systems, which have not been studied in the literature, though these issues also have significant means for descriptor cases. For instance, the controllers obtained by some existing methods [MKOS97, Mas07, XL06], possess potentially unstable modes, hence they are hard to be realized. Moreover, stable (admissible) descriptor controllers can be equivalently represented by the conventional state-space form, which is a nice property in terms of implementation. Similar to what has been done for the conventional state-space systems, we first attempt to explore the relation between the strong H_∞ stabilization problem and simultaneous H_∞ control problem within the descriptor framework. It shows that the simultaneous H_∞ control problem for a set of descriptor systems is achieved if and only if the strong H_∞ stabilization problem of the corresponding augmented system is solvable. Furthermore, a sufficient condition of solvability of strong H_∞ stabilization is proposed in terms of a GARE and a set of LMIs.

Remark 5.2.1 *The main results of this chapter have been reported in [FYC11b]. See Appendix III. We would also like to mention here that the deduced dilated LMI conditions could be applied to this subject and less conservative results may be obtained.*

Chapter 6

Extended Control

In systems and control theory, many problems need the definition of a standard model that is necessarily based on the physical model of the system, the models of disturbances and reference signals (exo-system) together with the control objectives (controlled signals and weights). For example, for the synthesis of optimal servo systems, one of the important approaches is to optimize some weighted closed-loop transfer matrix with weighting filters which are normally interpreted as the models of reference input and disturbances. In this context, it is well known that the use of exclusively state-space stable weighting filters is restrictive since the exo-system models and weights are generally unstable, or even nonproper [HZK92, Mei95, SSS00a, Che02]. Using for instance integral or derivative weighting filters may introduce some unstabilizable (undetectable) finite dynamics, even uncontrollable (unobservable) impulsive elements, in the generalized model.

In this circumstance, the issue of stability is quite different from that of the general setting. In this case, the requirement of the well-known internal stability is to ask for stabilization of weighting filters together with internal stabilization of the feedback system formed by the actual physical system and the controller. Since some weights are unstable, or even nonproper, it is impossible to achieve the internally stabilization for the overall feedback system. Instead, the notion of *extended stability* or *comprehensive stability* is introduced. This concept can be regarded as a generalization of internal stability and is highly related to practical concerns, for instance, the regulator and servomechanism problems [LM94].

The main results of this chapter have been reported in [FYC11a, FYC, FYCed].

6.1 Why Unstable and Nonproper Weights

As mentioned before, for many problems, control specifications are usually interpreted by weighting filters. For example, the use of weights having a pole at the origin is in general adapted to achieve perfect rejection or tracking of constant disturbances or

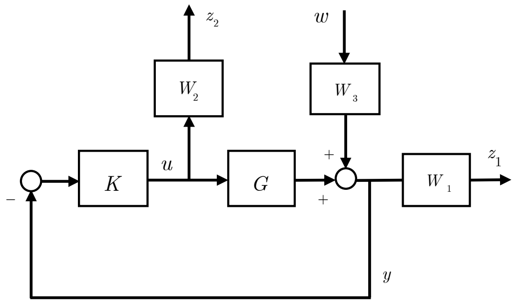

Figure 6.1: A mixed sensitivity configuration

references.

Now let us take the H_∞ control problem to show the importance of the use of unstable and nonproper weights. Examine the mixed sensitivity problem represented in Fig. 6.1, where G stands for the given plant, K the controller to be determined, and W_1, W_2 and W_3 input and output weighting filters. This diagram is borrowed from [Mei95] and yields the following transfer matrix

$$T_{zw} = \begin{bmatrix} W_1(I+GK)^{-1}W_3 \\ W_2K(I+GK)^{-1}W_3 \end{bmatrix}. \tag{6.1}$$

According to Kwakernaak's mixed sensitivity problem [Kwa93], the weights should be appropriately chosen such that stabilizing controllers with which the infinity norm of T_{zw} is less than a prescribed bound γ make the closed-loop system behave well. For this problem, "standard procedure" is available under Matlab routines through transforming it into a standard H_∞ control problem together with the assumptions for the standard problem [BDG+91, CS92a]. The desirable choices for weighting filters are such that

- W_1 has a pole at the origin;
- W_2 is nonproper.

As known, it is in general desirable to choose the weight W_1 having a pole at the origin, since the $\|T_{zw}\|_\infty$ is finite only if the sensitivity transfer function, that is, $(I+GK)^{-1}$ has a zero at the origin. This fact indicates that the underlying controller which stabilizes and makes $\|T_{zw}\|_\infty < \gamma$ achieves perfect rejection or tracking of constant disturbances or references. Another well-known fact which can explain this choice is that if the plant G does not have a pole at the origin, then any desirable controller K has integration action.

To avoid undesirable high frequency noise sensitivity and limited robustness, it is also often desirable to select a nonproper weight W_2. In particular, $\|W_2\|_\infty$ should be large outside the desirable closed-loop bandwidth due to the fact that this choice ensures that the controller is small outside the closed-loop bandwidth.

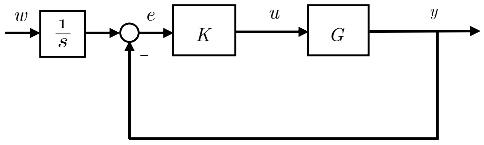

Figure 6.2: Asymptotic tracking problem

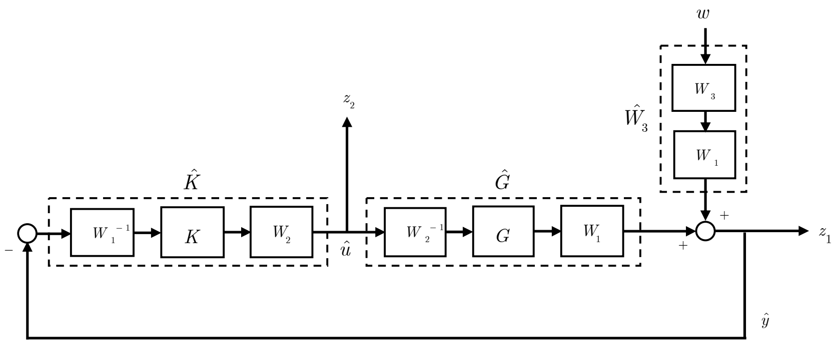

Figure 6.3: Modified mixed sensitivity configuration

6.2 Existing Approaches

The importance of the use of unstable and nonpropoer weights has been discussed in the preceding section. The control objective for such nonstandard problems is quite different from the conventional ones, since the weighted system cannot be internally stabilized owing to the presence of these weights which are neither stabilizable nor detectable.

To illustrate this situation, let us consider an asymptotic tracking problem depicted in Fig. 6.2, where w is a step reference. The input-output relation is given by:

$$T_{ew} = \frac{1}{s}(I + GK)^{-1}. \tag{6.2}$$

In this case, the dynamic of the integrator is not stabilizable by the controller K. Hence, the internal stability of the weighted closed-loop system cannot be achieved. However, we know that the weight stands for our specifications which will not be realized in real devices, and we are only interested in the internal stability of the feedback system formed by G and K which is independent on the weight. Hence, this asymptotic tracking problem can still be solved by finding a controller internally stabilizing G which is called the actual physical plant and making T_{ew} stable.

To handle such nonstandard problems, there are several techniques existing in the literature. Let us take the mixed sensitivity problem presented in Fig. 6.1 as an example to give a brief reminder of these known approaches [Mei95].

Method 1 One method is to treat these undesirable elements by slight perturbation to render the problem standard [CS92a]. For example, one takes $W_1(s) = 1/(s + 0.0001)$ instead of $W_1(s) = 1/s$. Similarly, one can also replace $W_2(s) = s$ with $W_2(s) = s/(1 + 0.0001s)$. This treatment is obviously an approximation and is widely used. The disadvantage of this approach is that it is vulnerable to the troubles related with lightly-damped poles and may lead to higher order and non strictly proper controllers.

Method 2 The second method includes plant augmentations as well as philosophically similar "plant state tapping" techniques [Kra92, Mei95]. Let us call it here the filter absorption method. Fig. 6.3 shows how to absorb the weights into the loop. With this modified problem, the controller $\hat{K}$ can be constructed, and the corresponding controller K is $K = W_1\hat{K}W_2^{-1}$. This method is easy to explain and not difficult to implement. Note that if there exists an unstable pole-zero cancelation in the modified plant, that is, $\hat{G} = W_2^{-1}GW_1$, then the stability properties of the original loop and the modified loop are not the same. In other words, the weights W_1 and W_2 must be appropriately chosen. Moreover, this method requires a pretreatment to absorb the weights into the loop.

Method 3 The theory of mode cancelation or comprehensive stabilization [LM94, LM95, LZM97, MXA00] has been proposed for solving these nonstandard problems. Roughly speaking, the main idea is to make, respectively, the unstabilizable and undetectable elements unobservable and uncontrollable by feedback in the underlying closed-loop. This theory was developed for both H_2 and H_∞ control problems, and does not allow nonproper weights.

6.3 Extended Control Problem

In this chapter, the extended control problem (the "extended" term indicates here that the desirable controller can and must stabilize a part of the generalized closed-loop system) for linear continuous-time descriptor systems is investigated. Systems and their weights are all described within the descriptor framework. Hence, it is possible to take into account not only unstable weights, but nonproper weights as well. This case results in nonstandard control problems for which the standard solution procedures fail. It shows here that the existence of a solution to this extended problem is directly concerned with the solvability of two generalized Sylvester equations.

Consider a descriptor system $\widetilde{G}(s)$ (see Fig.6.4):

$$\begin{bmatrix} e(s) \\ y(s) \end{bmatrix} = \widetilde{G} \begin{bmatrix} v(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} \widetilde{G}_{ev} & \widetilde{G}_{eu} \\ \widetilde{G}_{yv} & \widetilde{G}_{yu} \end{bmatrix} \begin{bmatrix} v(s) \\ u(s) \end{bmatrix} \tag{6.3}$$

where $e \in \mathbb{R}^q$, $y \in \mathbb{R}^p$, $v \in \mathbb{R}^l$ and $u \in \mathbb{R}^m$ are the controlled output, measurement, disturbance input and control input vector, respectively. The system (6.3) can be

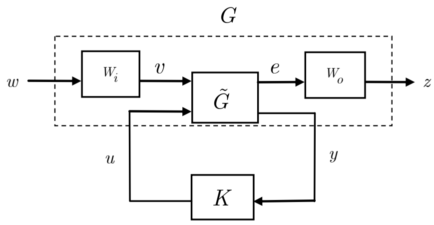

Figure 6.4: Extended control problem

rewritten as:

$$\widetilde{G} = \left\{ E_g, \left[\begin{array}{c|cc} A_g & B_{g1} & B_{g2} \\ \hline C_{g1} & D_{g11} & D_{g12} \\ C_{g2} & D_{g21} & D_{g22} \end{array} \right] \right\} \tag{6.4}$$

where $E_g \in \mathbb{R}^{n_g \times n_g}$, A_g, B_{g1}, B_{g2}, C_{g1}, C_{g2}, D_{g11}, D_{g12}, D_{g21} and D_{g22} are all known real constant matrices. The matrix E_g may be singular, i.e. $rank(E_g) = r_g \leq n_g$.

Suppose that the input weight W_i and the output weight W_o are both descriptor systems described as:

$$W_i = \left\{ E_i, \left[\begin{array}{c|c} A_i & B_i \\ \hline C_i & D_i \end{array} \right] \right\}, \quad W_o = \left\{ E_o, \left[\begin{array}{c|c} A_o & B_o \\ \hline C_o & D_o \end{array} \right] \right\}, \tag{6.5}$$

where $E_i \in \mathbb{R}^{n_i \times n_i}$, $E_o \in \mathbb{R}^{n_o \times n_o}$, $A_i \in \mathbb{R}^{n_i \times n_i}$, $A_o \in \mathbb{R}^{n_o \times n_o}$, $B_i \in \mathbb{R}^{n_i \times m_i}$, $B_o \in \mathbb{R}^{n_o \times q}$, $C_i \in \mathbb{R}^{l \times n_i}$, $C_o \in \mathbb{R}^{p_o \times n_o}$, $D_i \in \mathbb{R}^{l \times m_i}$ and $D_o \in \mathbb{R}^{p_o \times q}$ are all known real constant matrices. The matrix E_i and E_o may be singular, i.e. $rank(E_i) = r_i \leq n_i$ and $rank(E_o) = r_o \leq n_o$.

For simplicity of the presentation, W_i and W_o are supposed to have only unstable and/or impulsive modes. Note that this assumption causes no loss of generality, since the stable and static modes of the weights decay to zero eventually and do not affect the admissibility of the closed-loop system.

Then the resulting generalized weighted plant G is written as:

$$G = \left\{ \begin{bmatrix} E_o & 0 & 0 \\ 0 & E_g & 0 \\ 0 & 0 & E_i \end{bmatrix}, \left[\begin{array}{ccc|cc} A_o & B_oC_{g1} & B_oD_{g11}C_i & B_oD_{g11}D_i & B_oD_{g12} \\ 0 & A_g & B_{g1}C_i & B_{g1}D_i & B_{g2} \\ 0 & 0 & A_i & B_i & 0 \\ \hline C_o & D_oC_{g1} & D_oD_{g11}C_i & D_oD_{g11}D_i & D_oD_{g12} \\ 0 & C_{g2} & D_{g21}C_i & D_{g21}D_i & D_{g22} \end{array} \right] \right\}. \tag{6.6}$$

Moreover, we denote in the sequel G as:

$$G = \left\{ E, \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array} \right] \right\} \triangleq \begin{bmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{bmatrix}. \tag{6.7}$$

Definition 6.3.1 (Extended admissibility) *The feedback system $\mathcal{F}_l(G, K)$ is said to be extended admissible if $\mathcal{F}_l(\widetilde{G}, K)$ is internally stable and the closed-loop system defined as:*

$$T_{zw} = \mathcal{F}_l(G, K) = G_{zw} + G_{zu}K(I - G_{yu}K)^{-1}G_{yw} \tag{6.8}$$

is admissible.

Problem 6.3.1 (Extended Control Problem) *The extended control problem associated with G shown in (6.7) consists in finding, if possible, a controller K such that the following conditions hold.*

(1) *(Extended admissibility) The overall feedback system formed by G and K is extended admissible.*

(2) *(Performance measure) A desirable performance measure (H_2, H_∞) based on the transfer matrix T_{zw} is achieved.*

It shows that the first condition is satisfied if and only if two generalized Sylvester equations admit solutions. The Sylvester equations make the modes which are neither finite dynamics stabilizable nor impulse controllable, unobservable and the modes which are neither finite dynamics detectable nor impulse observable, non-controllable by feedback in the underlying closed-loop system. When $E = I$, these equations reduce to well-known results [SSS00a, SSS00b, LZM97, MXA00] for conventional state-space systems.

The additional performance objectives, such as H_2, H_∞, can be achieved by the solvability of two underlying GAREs. As the weighted plant G is neither wholly stabilizable nor detectable, the GAREs have no admissible solutions (see definition in Section 4.4 of Chapter 4). While similar to the definition of extended admissibility, the concept of so-called quasi-admissible solution is adopted. It observes that the quasi-admissible solutions to the GAREs are formed by admissible solutions to two reduced GAREs and solutions to the two generalized Sylvester equations. Then, the controller solving Problem 6.3.1 is constructed in terms of the quasi-admissible solutions, and the set of desirable controllers is also parameterized.

Note that the viability of the proposed methods depends on having numerically sound algorithms which are able to solve the two generalized Sylvester equations in addition to the two GAREs. Related numerical procedures for the solvability of these equations can be found in Section 4.3.2 and Section 4.4 of Chapter 4.

Remark 6.3.1 *The related results of this chapter can be found in [FYC11a, FYC, FYCed]. See Appendix IV-Appendix VI, respectively*

Chapter 7

Output Regulation Problem

The subject of output regulation occupies an important theme in all endeavors of both theoreticians and practitioners alike. Generally speaking, the main objective arising in output regulation consists in finding a feedback controller such that the given plant is internally stabilized and the output of the resulting closed-loop system converges to, or tracks, a prescribed reference signal in the presence of external disturbances. The reference signals and external disturbances are usually described by the so-called exo-system or exogenous system.

In order to deal with the output regulation problem, the seminal result, known as the *Internal Model Principle*, was developed in the 1970s [FSW74, FW75]. Based on this principle, exact asymptotic rejection/tracking is achieved by a structured controller containing a copy of the dynamics of the exo-system. The facets associated with this subject are of course not limited to internal model principle, well posedness and structured stability which have been the subject of many studies during the late sixties, seventies and thereafter. Extensions of internal model principle have been considered by integrating other performance objectives, for instance, H_2 and H_∞ performance. Such multi-objective problems have been extensively investigated in the literature, e.g. see [ANP94, ANKP95, HHF97, SSS00a, SSS00b, KS08, KS09] and the references therein. An alternative method for solving these problems consists in reformulating the problems through the use of unstable weighting filters [LZM97, MXA00]. Moreover, the regulation problem has also been studied for descriptor systems. For example, in [Dai89], the author has provided a solution to this problem in terms of a set of non-linear matrix equations depending on system parameters and some other parameters. In [LD96], a more clear-cut solution of this problem has been obtained via solving a generalized Sylvester equation. The authors have also investigated the regulation problem for descriptor systems with periodic and almost periodic coefficients [IK05].

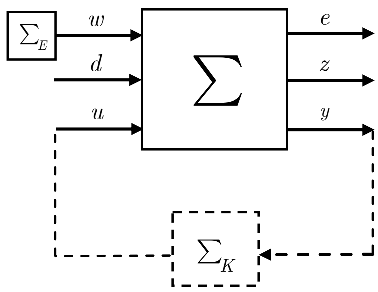

Figure 7.1: Performance requirements subject to regulation constraints

7.1 Synthesis with Regulation Constraints

The current chapter is concerned with a nonstandard multi-objective output control problem for continuous-time descriptor systems. In this problem an output is to be regulated asymptotically in the presence of an infinite-energy exo-system, while a desired performance by the H_2 or H_∞ norm from a finite external disturbance to a tracking error has also to be satisfied.

It shows here that the asymptotical regulation objective is satisfied if and only if a generalized Sylvester equation associated with the given descriptor system and exo-system is solvable. In addition, every controller achieving asymptotical regulation constraints possesses a specific structure. Furthermore, using this structure, the defined multi-objective control problem reduces to the standard control problem for an associated descriptor plant, whose solution is characterized based on the solvability of a GARE or a set of LMIs.

This chapter explores state feedback H_2 optimal control and H_∞ output feedback control problems under regulation constraints for continuous-time descriptor systems. Thanks to the descriptor framework, not only unstable but also nonproper behaviors can be treated. For the issue of H_2 control, the class of optimal state feedback controllers is explicitly characterized based on the results in [ITS03], while for the H_∞ output feedback control, an LMI-based approach is proposed.

7.2 Problem Formulation

Consider the following descriptor plant:

$$(\Sigma): \left[\begin{array}{c} E\dot{x} \\ \hline e \\ z \\ y \end{array}\right] = \left[\begin{array}{c|ccc} A & B_w & B_d & B \\ \hline C_e & D_{ew} & D_{ed} & D_{eu} \\ C_z & D_{zw} & D_{zd} & D_{zu} \\ C & D_{yw} & D_{yd} & 0 \end{array}\right] \left[\begin{array}{c} x \\ \hline w \\ d \\ u \end{array}\right] \tag{7.1}$$

where $e \in \mathbb{R}^{q_e}$, $z \in \mathbb{R}^{q_z}$, $y \in \mathbb{R}^{p}$, $w \in \mathbb{R}^{n_w}$, $d \in \mathbb{R}^{m_d}$ and $u \in \mathbb{R}^{m}$ are the tracking error, controlled output, measurement, exogenous disturbance, external disturbance and control input vector, respectively. The exogenous disturbance w is generated by an exo-system Σ_E within the descriptor framework:

$$(\Sigma_E) : E_w \dot{w} = A_w w, \tag{7.2}$$

where the matrix E_w may be singular, *i.e.* $rank(E_w) = r_w \leq n_w$. The given plant and the exo-system are graphically depicted in Fig. 7.1.

Denote the new descriptor variable as $\zeta^\top = [x^\top \ w^\top]$. Then the descriptor plant G can be rewritten as:

$$(G) : \left\{ \begin{bmatrix} E & 0 \\ 0 & E_w \end{bmatrix}, \left[\begin{array}{cc|cc} A & B_w & B_d & B \\ 0 & A_w & 0 & 0 \\ \hline C_e & D_{ew} & D_{ed} & D_{eu} \\ C_z & D_{zw} & D_{zd} & D_{zu} \\ C & D_{yw} & D_{yd} & 0 \end{array} \right] \right\} := \begin{bmatrix} G_{ed}(s) & G_{eu}(s) \\ G_{zd}(s) & G_{zu}(s) \\ G_{yd}(s) & G_{yu}(s) \end{bmatrix}. \tag{7.3}$$

We make the following assumptions subsequently:

(A.1) (E, A, B) is finite dynamics stabilizable and impulse controllable;

(A.2) $\left(\begin{bmatrix} E & 0 \\ 0 & E_w \end{bmatrix}, \begin{bmatrix} A & B_w \\ 0 & A_w \end{bmatrix}, \begin{bmatrix} C & D_{yw} \end{bmatrix} \right)$ is finite dynamics detectable and impulse observable;

(A.3) The exo-system Σ_w has only unstable and impulsive modes.

Note that in the plant Σ, the zero feedthrough matrix from u to y is assumed, without loss of generality, to simplify the computations. If it does not hold, an equivalent realization satisfying this assumption can always be obtained. Assumptions (A.1)-(A.3) coincide with the standard assumptions in the regulator theory for the conventional state-space systems [SSS00b, SSS00a]. Note that for the state-space systems, the assumptions related to the impulse controllability and observability vanish. Assumption (A.1) together with another assumption that (E, A, C) is finite dynamics detectable and impulsive observable is obviously essential to the existence of a measurement feedback controller internally stabilizing the given system. The condition (A.3) is assumed without loss of generality due to the fact that the stable and static modes of G_w decay to zero and do not affect the regulation objective.

We seek a measurement feedback controller which is also represented within the descriptor framework as:

$$(\Sigma_K) : \left\{ \begin{array}{l} E_K \dot{\xi} = A_K \xi + B_K y, \\ u = C_K \xi + D_K y, \end{array} \right. \tag{7.4}$$

where $E_K \in \mathbb{R}^{n_k \times n_k}$ may be singular, *i.e.* $rank(E_K) = r_k \leq n_k$.

Now we are in a position to state the multi-objective control problem of *performance control with asymptotic regulation constraints.*

Problem 7.2.1 (Performance Control with Regulation Constraints) *The performance control problem with asymptotic regulation constraints consists in finding, if possible, a controller Σ_K such that the closed-loop system formed by G and Σ_K satisfies the following conditions.*

C.1 *(Internal stability) In the absence of the disturbances w and d, the closed-loop system is internally stable (admissible);*

C.2 *(Asymptotic regulation) $\lim_{t\to\infty} e(t) = 0$ for any $d \in L_2$, and for all $x(0) \in \mathbb{R}^n$ and $w(0) \in \mathbb{R}^{n_w}$;*

C.3 *(Performance measure) Given $\gamma > 0$. The closed-loop system defined by*

$$T_{zd} = G_{zd} + G_{zu}\Sigma_K(I - G_{yu}\Sigma_K)^{-1}G_{yd}, \tag{7.5}$$

satisfies $\|T_{zd}\|_p < \gamma$, $p = 2, \infty$.

The present multi-objective problem can be viewed as a generalization of the control problem subject to regulation constraints for state-space systems defined in [SSS00b] to descriptor systems. In addition, if we relax the asymptotic regulation requirement, we can equally define the control problem subject to almost asymptotic regulation constraints discussed in [KS08, KS09] for descriptor systems. More precisely, we can redefine the condition C.2 as follows:

C'.2 (Almost asymptotic regulation of level $\kappa \geq 0$) In the absence of d, there exist positive scalars α, η such that $\|e(t)\| \triangleq \sqrt{e(t)^\top e(t)} \leq \alpha e^{-\eta t} + \kappa\|w(t)\|$, $\forall t \geq 0$, for any $x(0) \in \mathbb{R}^n$ and $w(0) \in \mathbb{R}^{n_w}$.

Remark 7.2.1 *The main results of this chapter are found in [FYC11c, FYCon]. See Appendix VII and Appendix VIII, respectively.*

Chapter 8

Concluding Remarks

This dissertation is concerned with non-standard H_2 and H_∞ control for descriptor systems. The contributions of this dissertation can be resumed as follows. By using the descriptor representation, some existing results for state-space systems are reviewed. Some classical control issued are also extended to descriptor systems. Moreover, without approximation and transformation, an exact and analytical solution to the nonstandard control problems is given. This allows dealing with many practical problems interpreted by H_2 or H_∞ control, where the control signals are penalized at high frequency or unstable internal models specified by external signals are involved.

The results reported in this dissertation can be viewed as extensions of the underlying existing results to descriptor systems. Moreover, by using the descriptor framework, solutions to non standard control problems with unstable and non-proper weights and the output regulation problem with the presence of an infinite-energy exo-system are also proposed. The main achievements are summarized and future research topics are discussed in this concluding chapter.

8.1 Summary

Chapter 1 constants a French summary of this thesis work.

Chapter 2 discusses the theoretical and practical interest of the use of descriptor systems and provides a brief literature review on analysis and control problems within the descriptor framework. The outline of the dissertation is also given and the key results are highlighted accordingly.

Basic concepts for linear time-invariant descriptor systems are recalled in Chapter 3 as preliminaries. Fundamental and important results, such as regularity, admissibility, equivalent realizations, system decomposition and temporal response are reviewed. The definitions of controllability and observability are also presented. In addition, the duality notion is stated.

Chapter 4 serves to present some useful results concerning dissipative properties, dilated LMI conditions, generalized Sylvester equations and GAREs for descriptor systems. By removing equality constraints, a new KYP-type lemma is characterized in terms of a strict LMI which overcomes numerical problems in checking inequality conditions owing to roundoff errors in digital computation. Dilated LMI characterizations with regard to stability, H_2 performance and dissipativity are also deduced through reciprocal application of the projection lemma. The deduced formulations cover the existing results reported in the literature, and complete some missing conditions as well. Moreover, generalized Sylvester equations and GAREs associated with descriptor systems are also investigated. Numerical algorithms for solving these matrix equations are provided.

Chapter 5 considers the strong H_∞ stabilization and simultaneous H_∞ control problems for continuous-time descriptor systems. As a generalization of the existing results to descriptor systems, it stats that the simultaneous H_∞ control problem for a set of descriptor systems is achieved if and only if the strong H_∞ stabilization problem of a corresponding augmented system is solvable. Then, a sufficient condition for the existence of an observer-based controller solving the strong H_∞ stabilization problem is proposed. The proposed result is based on a combination of a GARE and a set of LMIs and outperforms some reported methods in the literature.

Chapter 6 is devoted to the control problem subject to extended (comprehensive) stabilization. In such a problem, the conventional internal stability of the overall feedback system cannot be achieved due to the use of unstable and nonproper weighting functions. Hence, in this case, a desired controller has to satisfy that the underlying closed-loop system is admissible and only the internal stability of a part of the closed-loop system is sought.

Extended stabilization is first investigated and the existence of a solution is directly concerned with the solvability of two generalized Sylvester equations. With these Sylvester equations, the modes which are neither finite dynamics stabilizable nor impulse controllable are rendered unobservable, while the modes which are neither finite dynamics detectable nor impulse observable are made non-controllable in the overall closed-loop system. This fact cancels these undesirable elements in the closed-loop system and guarantees extended stability. A set of controllers achieving extended stability is also parameterized by the Youla-Kučera parametrization.

Relying on the result of extended stabilization, H_2 and H_∞ performance requirements are further integrated for this nonstandard problem. As classic assumptions by which the standard H_2 and H_∞ control problems are solvable do not hold due to the weights involved, relaxed assumptions are made and the so-called quasi-admissible solution is adapted, instead of the admissible solution. The quasi-admissible solutions for underlying GAREs are formed by admissible solutions to two reduced GAREs and the solutions to the two generalized Sylvester equations. Then, the resulting controller is obtained through these quasi-admissible solutions, and the class of desirable controllers is also parameterized. The reported results for the extended control problem obviously encompasses the state-space case.

Chapter 7 deals with the H_2 and H_∞ control with output regulation constraints. In this problem an output is to be regulated asymptotically in the presence of an infinite-energy exo-system, while a desired performance by the H_2 or H_∞ norm from a finite external disturbance to a tracking error must also be satisfied. This problem can be viewed as a special case of the extended control problem.

Based on a generalized Sylvester equation, the asymptotical regulation objective is achieved and a specific structure of the resulting controller is deduced. The obtained structure coincides with the well known internal model principle developed for state-space systems. Using this structure, the defined multi-objective control problem reduces to the standard performance control problem for an augmented descriptor plant.

8.2 Perspectives

In closing, we describe some future topics and possible extensions of the results obtained in this dissertation.

As for the strong H_∞ stabilization problem, although a promising method is deduced, the result is still somewhat conservative. This conservatism is mainly rooted in the choice of the common Lyapunov matrix. The Lyapunov matrices related, respectively, to the H_∞ norms of the closed-loop system and the resulting controller are chosen to be the same in order to render the optimization problem convex. Like other multi-objective control problems, the strong H_∞ stabilization problem is still open and needs further investigation. One of our future research aims is to develop a less conservative LMI-based approach for this problem. Dilated LMIs may allow the using of independent Lyapunov matrices with respect to the closed-loop system and the controller.

On the other hand, the extended control problem addressed in this dissertation is restricted in the regular case, that is, the across coupling transfer functions $\widetilde{G}_{eu}(s)$ and $\widetilde{G}_{yv}(s)$ induced from the physical plant $\widetilde{G}$ (6.3) have no zeros on the imaginary axis including infinity. This assumption obviously limits the application of the deduced results. Indeed, the singular control problems [Sto92, CS92b] for descriptor systems have not yet been completely solved. The LMI-based solutions [RA99, ILU00, Mas07, XL06] reported in the literature remove this restriction on the realization of systems, but they yield proper controllers, which may in some circumstances, be only suboptimal solutions. As pointed out in [CS92b], for the state-space setting, the real H_2 or H_∞ optimal controller solving singular problems may be nonproper. Hence, there is no reason that the singular problem for descriptor systems is an exception. Another future topic is to deduce the solution for a singular Hamiltonian pencil, with which the GARE associated with the singular case is solved, and to generalize the results in [Sto92, CS92b] to descriptor systems. Then, we will also attempt to handle the extended control problem in singular cases and to provide a wholly complete solution to this problem.

The last point is concerned with the extended control problem and the regulation problem for LPV systems with parameter independent weights (resp. the parameter independent exo-system) or the cases where the system is time invariant, but the weights (resp. the exo-system) are parameter dependent. Under this circumstance, the parametric LMI/LME-based solutions as well as their potential relaxations with introducing supplementary degrees of freedom need further investigation.

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Appendix I

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Appendix IV

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Appendix V

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Appendix VI

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Appendix VII

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Appendix VIII

Y. Feng, M. Yagoubi and P. Chevrel. H_∞ Control Under Regulation Constraints For Descriptor Systems. In preparation.

Yu Feng

Commande H_2-H_∞ non standard des systèmes implicites

Résumé

Les systèmes implicites (dits aussi « descripteurs ») peuvent décrire des processus régis à la fois par des équations dynamiques et statiques et permettent de préserver la structure des systèmes physiques. Ils comportent trois types de modes : dynamiques finis, infinis (réponse temporelle impulsive (en cas continu) ou acausale (en cas discret)) et statiques.
Dans le cadre du formalisme descripteur, les contributions de cette thèse sont triples : i) revisiter des résultats existants pour les systèmes d'état, ii) étendre certains résultats classiques au cas des systèmes implicites, iii) résoudre rigoureusement des problèmes de commande non standard.
Ainsi, le présent mémoire commence par revisiter les résultats concernant la caractérisation LMI stricte de la dissipativité, les caractérisations de l'admissibilité et des performances H_2 ou H_∞ par LMI étendues et les équations de Sylvester et de Riccati généralisées.
Il aborde dans un deuxième temps, le problème de stabilisation simultanée, avec ou sans critère H_∞, à travers l'extension de certains résultats récents au cas des systèmes implicites. La solution proposée s'appuie sur la résolution combinée d'une équation algébrique de Riccati généralisée (GARE) et d'un problème de faisabilité sous contrainte LMI stricte.
Il traite enfin des problèmes H_2 et H_∞ non standards : i) en présence de pondérations instables voire impropres, ii) sous contraintes de régulation; dans le cas des systèmes implicites. Ces dernières contributions permettent désormais de traiter rigoureusement, sans approximations ou transformations, de nombreux problèmes H_2 ou H_∞ formalisant des problèmes pratiques de commande, dont ceux faisant intervenir une pénalisation haute fréquence de la commande ou un modèle interne instable des signaux exogènes.

Mots clés
Systèmes implicites, GARE, commande H_2-H_∞, LMI, pondérations instables et impropres, stabilisation simultanée

Abstract

The descriptor systems have been attracting the attention of many researchers over recent decades due to their capacity to preserve the structure of physical systems and to describe static constraints and impulsive behaviors.
Within the descriptor framework, the contributions of this dissertation are threefold: i) review of existing results for state-space systems, ii) generalization of classical results to descriptor systems, iii) exact and analytical solutions to non standard control problems.
A realization independent Kalman-Yakubovich-Popov (KYP) lemma and dilated LMI characterizations are deduced for descriptor systems. The solvability and corresponding numerical algorithms of generalized Sylvester equations and generalized algebraic Riccati equations (GARE) associated with descriptor systems are provided.
In addition, the simultaneous H_∞ control problem is considered through extending recently reported results. A sufficient condition is proposed through a combination of a generalized algebraic Riccati equation and a set of LMIs.
Moreover, the nonstandard H_2 and H_∞ control problems with unstable and/or nonproper weighting functions or subject to regulation constraints are addressed. These contributions allow, without approximation or transformation, dealing with many practical problems defined within H_2 or H_∞ control methodologies, where the control signals are penalized at high frequency or unstable internal models specified by external signals is involved.

Key Words
Descriptor system, H_2-H_∞ control problems, GARE, LMI, simultaneous stabilization, unstable and nonproper weights

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