Optique quantique électronique
-
Electron quantum optics

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Classical vs. quantum description

Classical

Evolution Newton's (Einstein's) laws

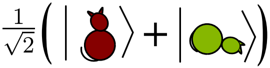
- Defined position and velocity
- One state at a time : No superposition

Electromagnetic field Maxwell :
Light is a wave

Quantum world

Evolution Schrödinger's equation

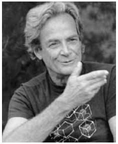
- Heisenberg inequalities
- Linear superpositions : Schrödinger's cat



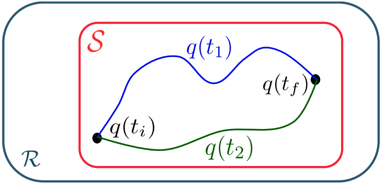
Electromagnetic field QED :

Light ≡ wave **and** particle

Quantum coherence



Linear superpositions $\equiv$ **Interferences**



Probability $i \to f$ $\mathcal{P}_{if} = |\mathcal{A}[q_1] + \mathcal{A}[q_2]|^2$

Classical terms $|\mathcal{A}[q_1]|^2 + |\mathcal{A}[q_2]|^2$

Quantum terms $2\Re(\mathcal{A}[q_1]\mathcal{A}^*[q_2])$

Coupling to an external environment $\mathcal{R} \Rightarrow$ Reduction of visibility !

$$2\Re(\mathcal{A}[q_1]\mathcal{A}^*[q_2])\underbrace{\mathcal{F}[q_1, q_2]}_{\textit{visibility}}$$

- R. P. Feynman Rev. Mod. Phys. **20**, 367 (1948)
- R. P. Feynman, F. L. Vernon, Ann. Phys. (N. Y.) 24, 118 (1963)

Quantum coherence in solid state systems

Feynman - Vernon picture OK if one can separate the system $\mathcal{S}$ and the environment $\mathcal{R}$

Approach well suited for :

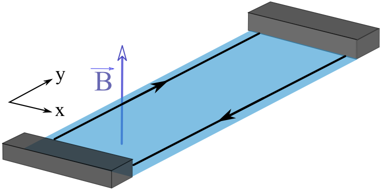
- Coupling of a two level system to a quantum bath
- Quantum impurity problems
- Atom in a cavity
- Josephson junctions

In a metal : Not so evident to single out an electron ...

Loss of quantum coherence in a metallic environment ?

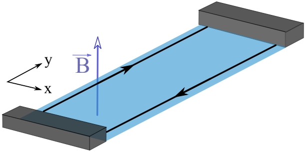
2D electron gas in a magnetic field

- Interface of two semiconductors : 2D electron gas
- In high magnetic field ($\simeq 5\ T$), at low temperature ($100\ mK$)
  $\Rightarrow$ Quantum Hall effect
- Electrical current at the edge



- Klitzing, K. von ; Dorda, G. ; Pepper, M., Phys. Rev. Lett. **45** 494-497 (1980)

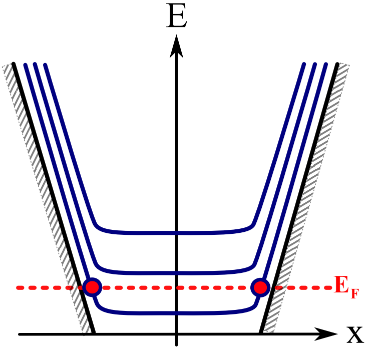
Current transport in the IQHE



- Electrons in a magnetic field : Landau levels

- Sample edges ↔ Confinment potential
  ⇒ Deformed Landau levels

- Intersection Landau level/Fermi energy :
  Conduction channels !



- M. Buttiker, Phys. Rev B, **38** 9375 (1988)
- B.I. Halperin, Phys. Rev. B **25** 2185-2190 (1982)

Quantum optics with electrons ?

Chirality of edge channels :

$\Rightarrow$ Robustness against disorder and interactions

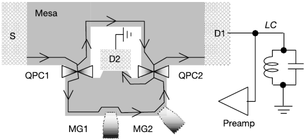
$\rightarrow$ Edge channels $\equiv$ coherent electronic waveguides

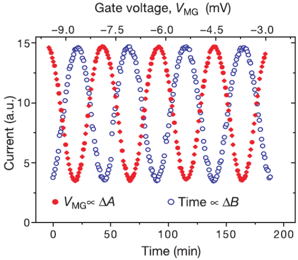
Engineering the electron gas $\Rightarrow$ electronic mirrors, beamsplitters ...

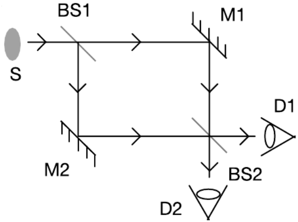
Is it possible to perform interference experiments with chiral electrons ?

- M. Buttiker, Phys. Rev B, **38** 9375 (1988)

A Mach-Zehnder interferometer for electrons





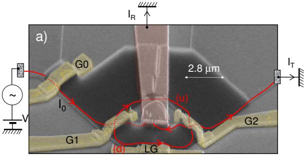


- Playing with the 2DEG : analogues of optics components
- Current oscillations : **Quantum coherence !**

- Yang Ji, *et al*, Nature **422**, 415 (2003)

Electron - photon analogy

Photons	Electrons
Photons beams	QHE chiral edge channels
Beamsplitter	QPC
Mirrors	Sample edges
Light source	Driven reservoir
Single photon source	Single electron source[ii]

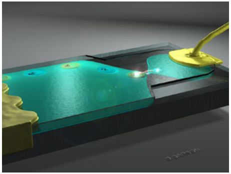


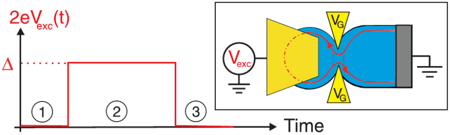
Electronic MZI (CEA)
$l_\phi = 20\mu m$ at $20mK$ [i]

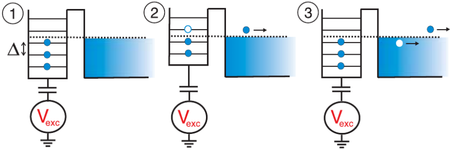
- P. Roulleau & al. Phys. Rev. Lett. **100**, 126802 (2008)
- G. Fève *et al.*, Science **316**, 1169 (2007)

The single electron source

...The missing piece to perform single quanta experiments

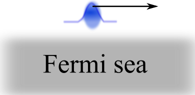






- Electrostatic gate : constriction
- Discrete level structure
- Control via V_{exc}
- **Single electron emission !**

Possible realization of the **quantum optics paradigm** :



Main questions

i) A formalism for electron quantum optics ?
$\Rightarrow$ Render the analogies and differences with photons

ii) Inclusion of interactions ?

iii) Relation to experimentally relevant quantities ?

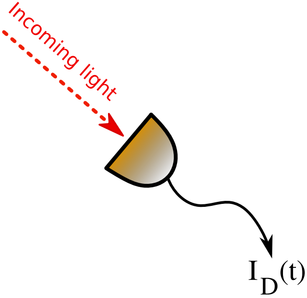
iv) Validity of the electron quantum optics paradigm ?

Outline

Outline

Photodetection and coherence function

- Photons



For a photodetector :

$$I_D(t) = \int_0^t \underbrace{\mathcal{G}^{(1)}(x_D, \tau | x_D, \tau')}_{\text{Single photon coherence}} \underbrace{K_D(\tau - \tau')}_{\text{Detector properties}} \, d\tau \, d\tau'$$

$$\mathcal{G}^{(1)}(x_D, t | x_D, t') = \langle \underbrace{\mathrm{E}^{(-)}(x_D, t)}_{\text{creation operators}} \overbrace{\mathrm{E}^{(+)}(x_D, t')}^{\text{annihilation operators}} \rangle$$

Coherent wavepacket :

$$\mathcal{G}^{(1)}(x, y) = E(x) E^*(y)$$

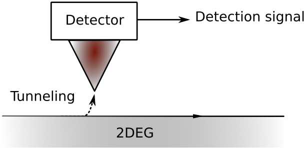
Electron coherence function

- Electrons

Simple model for tunnelling detection :

$$\mathcal{H} = \hbar(\psi^\dagger(x_D)O + O^\dagger\psi(x_D))$$

Current flow from edge channel to the detector [i] :



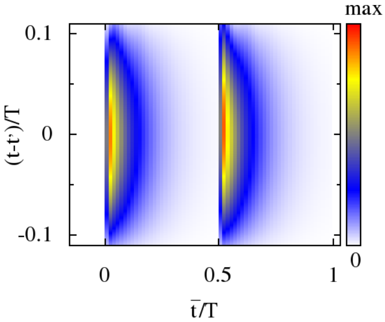
$$I_D(t) = \int_0^t \underbrace{\mathcal{G}^{(e)}(x_D,\tau|x_D,\tau')}_{\text{Single electron coherence}} \underbrace{K_D(\tau-\tau')}_{\text{Detector properties}} \, d\tau \, d\tau'$$

$$\boxed{\mathcal{G}^{(e)}(x,t|x',t') = \langle\psi^\dagger(x',t')\psi(x,t)\rangle_\rho}$$

- C. Altimiras *et al.*, Nature Physics **6**, 34-39 (2009)

Electron coherence functions : different contributions

Single electron coherence $\mathcal{G}_\rho^{(e)}(x,t;x',t') = \mathrm{Tr}\left(\psi(x,t)\rho\psi^\dagger(x',t')\right)$



For a wavepacket ϕ_e :

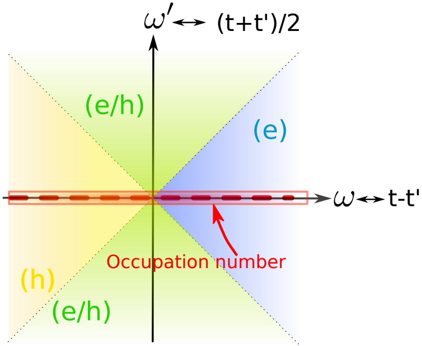
$$\mathcal{G}^{(e)}(t,t') = \mathcal{G}_F^{(e)}(t-t') + \phi_e(v_F t)\phi_e^*(v_F t')$$

- Vacuum contribution
- Excess contribution

Quantity of interest : $\Delta\mathcal{G}^{(e)}(t,t') = \mathcal{G}^{(e)}(t,t') - \mathcal{G}_F^{(e)}(t-t')$

Properties of the single electron coherence function

At fixed position : $\Delta\mathcal{G}^{(e)}(t-t',(t+t')/2) \to \Delta\mathcal{G}^{(e)}(\omega,\omega')$ in Fourier space



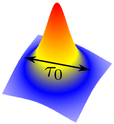
- Electronic excitations
- Hole excitations
- Electron-hole coherences

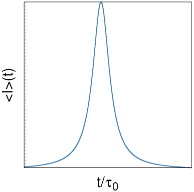
Access to **occupation number** through **spectroscopy** experiments

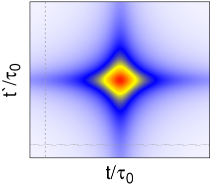
- C. Altimiras *et al.*, Nature Physics **6**, 34-39 (2009)

An electronic wavepacket

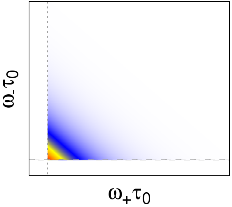
One lorentzian wavepacket of characteristic time τ_0



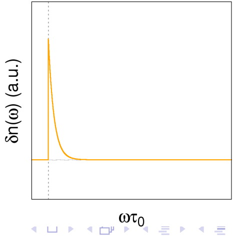




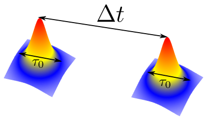
Coherence function



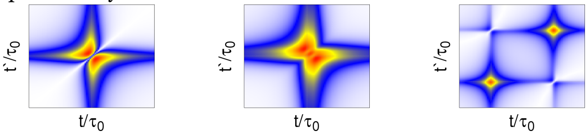
Diagonal part : Occupation number



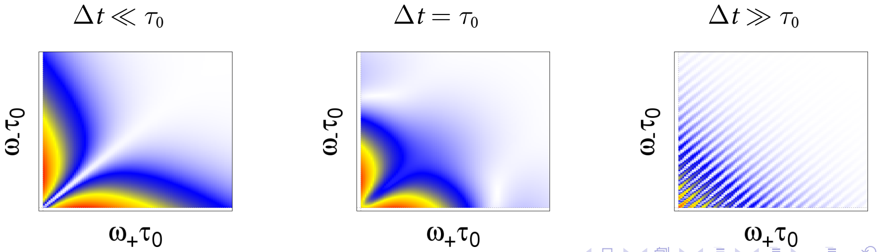
Two electrons



Two identical lorentzian wavepackets of characteristic time τ_0 separated by Δt



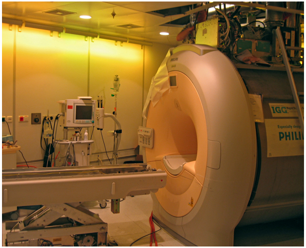
In Fourier space :

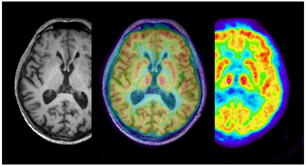


Outline

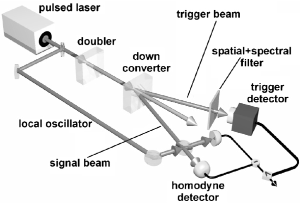
What is tomography ?

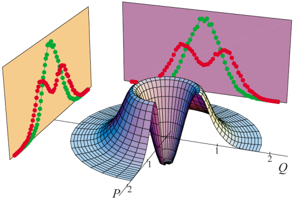
Classical tomography





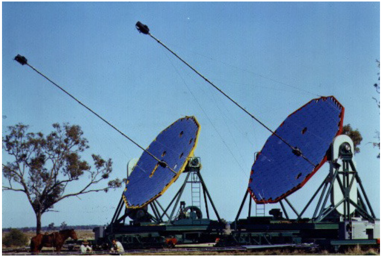
Quantum tomography

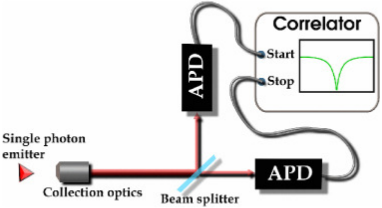




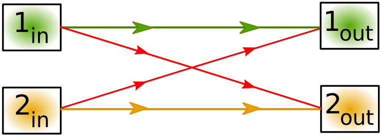
- A. I. Lvovsky and M. G. Raymer Rev. Mod. Phys. **81**, 299 (2009)

HBT effect





- Nature **178**, 1046 (1956)
- Am. J. Phys. **29**, 539 (1961)



- 2 particle interferences
- Correlations without interactions :

Classical	(0,2) (2,0) (1,1)
Bosons	(0,2) (2,0)
Fermions	(1,1)

HBT effect with electrons :

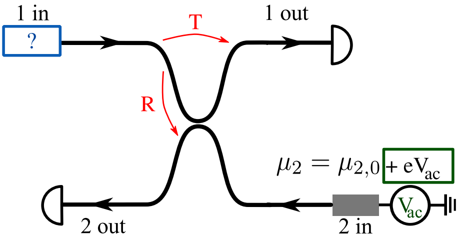
Liu et al, Nature **391**, 263 (1998)

Henny et al, Science **284**, 396 (1999)

Oliver et al, Science **284**, 299 (1999)

HBT interferometry for electrons

In our case : reconstruction of the single particle coherence



- Controlled source : driven ohmic contact
- Outcoming current correlations :

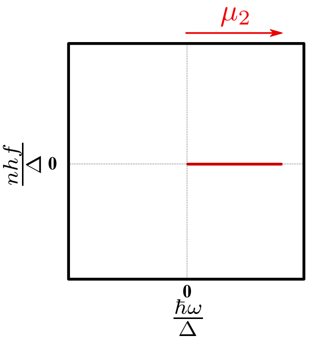
$$S^{(out)}_{\alpha\beta}(t, t') = \langle i_\alpha(t) i_\beta(t') \rangle_c$$

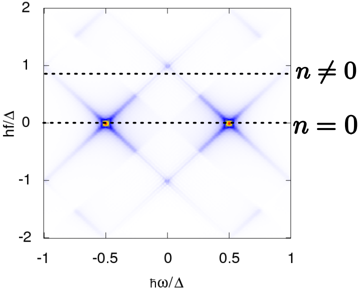
Experimental signal

Low frequency current correlations $\equiv$ **Overlap of single particle coherences**

- Samuelsson and Büttiker, Phys. Rev. B **73**, 041305R (2006)

Extraction of the single particle coherence



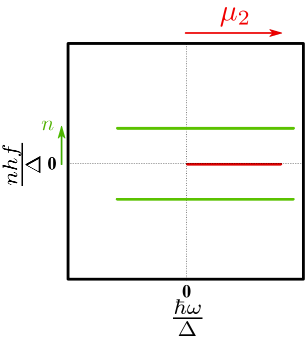


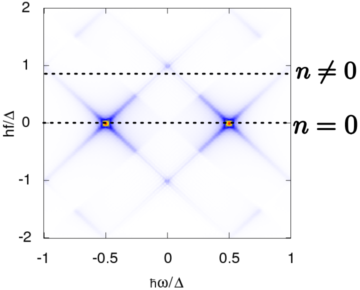
Controlled source : tunable $\mu_{2,0}$ and $V_{ac} = V_0 \cos\left(n\Omega_T t + \phi\right)$

$\Delta\mathcal{G}^{(e)}_{n=0}$ $\rightarrow$ Current correlations in terms of the DC bias

$\Delta\mathcal{G}^{(e)}_{n\neq 0}$ $\rightarrow$ Response of current correlations to AC voltage

Extraction of the single particle coherence





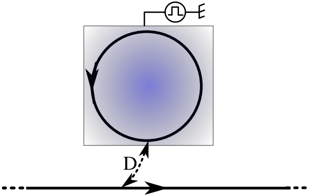
Controlled source : tunable $\mu_{2,0}$ and $V_{ac} = V_0 \cos\left(n\Omega_T t + \phi\right)$

$\Delta\mathcal{G}^{(e)}_{n=0}$ → Current correlations in terms of the DC bias

$\Delta\mathcal{G}^{(e)}_{n\neq 0}$ → Response of current correlations to AC voltage

The mesoscopic capacitor - Modelling

Square voltage Amplitude A pulsation Ω



- Free electrons
- Quantum dot & T - periodic driving $\Rightarrow$ Scattering
- $S(t,t') = S_0(t-t')\exp\left(\frac{ie}{\hbar}\int_{t'}^{t} V(\tau)d\tau\right)$

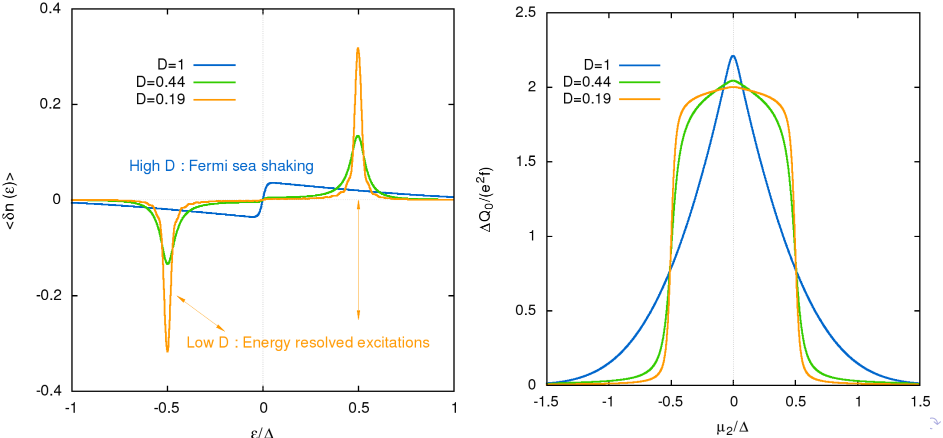
- $S \equiv$ Floquet scattering : relates *in* and *out* electronic modes
- Periodic driving $\Rightarrow \mathcal{G}^{(e)}$ periodic in $\frac{t+t'}{2}$
- Floquet scattering $\Rightarrow$ Harmonics of $\mathcal{G}^{(e)}$

- M. Moskalets & M. Büttiker, Phys.Rev.B **66**, 205320 (2002)

$n = 0$ harmonic : occupation number

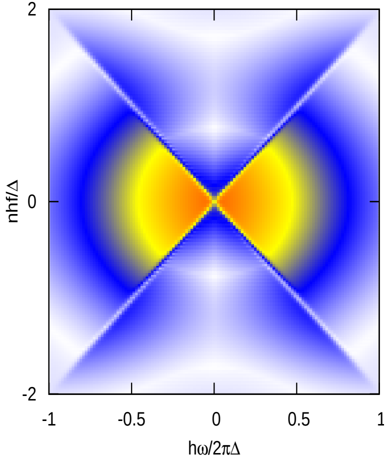
→ Spectroscopy of the source

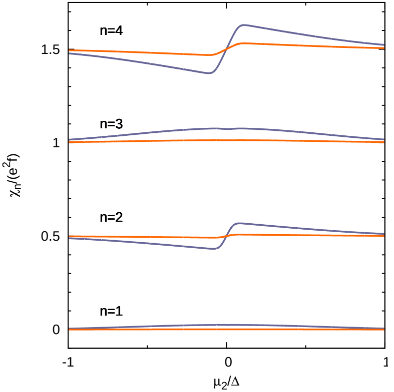
$$f = \frac{\Omega}{2\pi} = 3GHz \quad T_{el} = 50mK \quad A = \frac{\Delta}{e}$$



Higher order harmonics

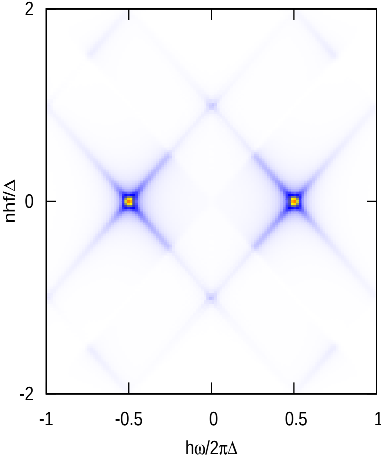
At high transparency

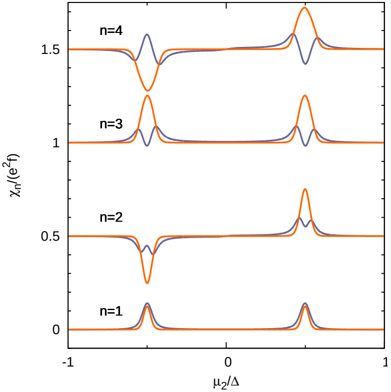




Higher order harmonics

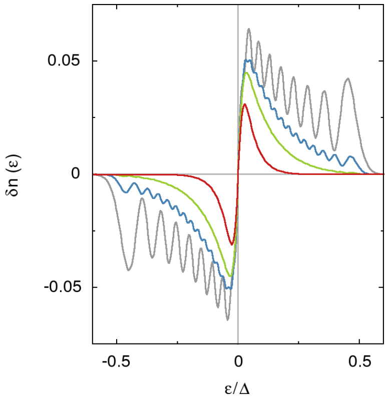
At low transparency



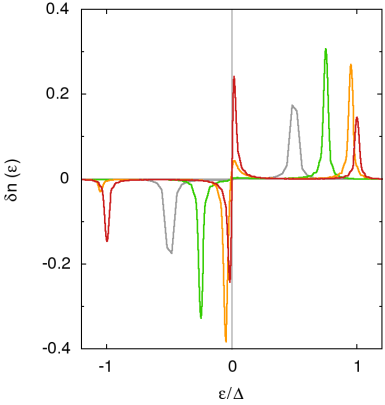


Changing parameters

Transition to the adiabatic regime



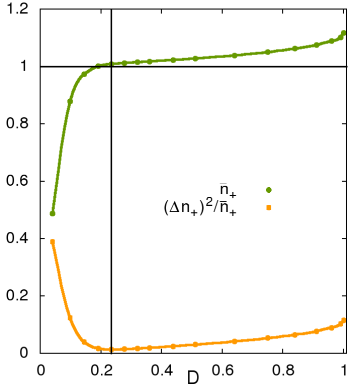
Level shift



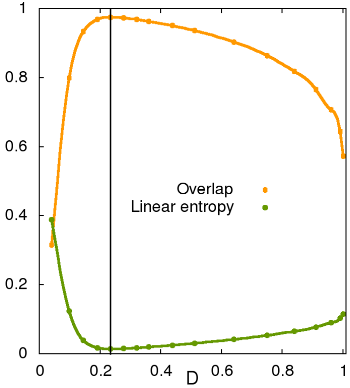
Quantum quality of the SES

Can be extracted from $\Delta\mathcal{G}^{(e)}$:

Charge average and fluctuations



Purity and Fidelity to RL wavefunction



- M. Albert *at al.*, Phys. Rev. B **82**, 041407 (2010)

Summary - Up to now ...

i. Formalism for electron quantum optics
  - Suitable for the description of quantum optics experiments with electrons
  - Underlines the photon analogy

ii. Relation to experimental quantities
  - Electronic quantum tomography protocol
  - Access to single particle coherence through current noise measurements
  - Experimental signal predictions

Question : What happens in the presence of interactions ?

And now ?

i. How to include interactions in the electron quantum optics formalism ?

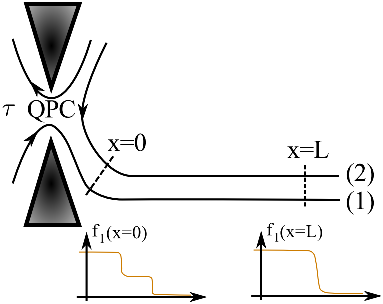
ii. Is the quantum optics paradigm valid in the presence of interactions ?

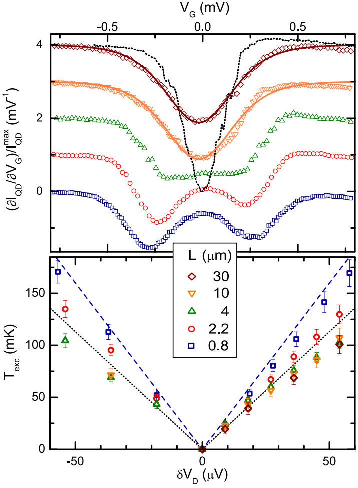
iii. Predictions for quasiparticle relaxation ?

iv. Is it possible to get information on relaxation mechanism through electron quantum optics experiments ?

Outline

Setup and results



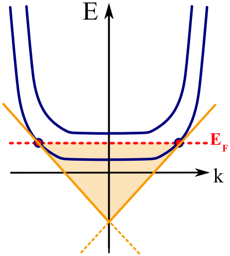


- Relaxation of a nonequilibrium distribution
- Interchannel interactions
- Thermalization over $L = \hbar v / |\Delta\mu|$

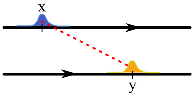
- H. Le Sueur *at al.*, Phys. Rev. Lett., **105** 056803 (2010)

Interaction modelling

$\Rightarrow$ Bosonization formalism



- Coulombic interactions $\equiv$ density / density coupling :



$$\mathcal{H}_{int} = \int dx\, dy\, \rho(x) V(x,y) \rho(y)$$

- **Coupling over a finite length** $\equiv$ Scattering $\mathcal{S}(\omega)$
- Scattering $\leftrightarrow$ Admittances : $Y(\omega) = \frac{e^2}{h}(1 - \mathcal{S}(\omega))$

Dealing with interactions

- Plasmon scattering
- Plasmon scattering related to finite frequency admittances

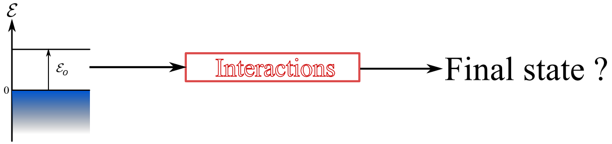
- I. Safi and H. Schulz, Phys. Rev. B **52**, 1740 (1995)
- Eur. Phys. J. D, **12** 451 (1999)

Energy resolved excitation

Motivations :

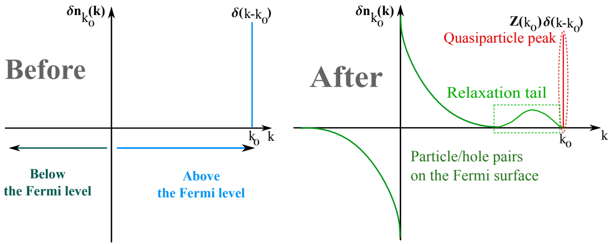
a- Idealization of the SES

b- Solution to Landau's problem



- Interaction region $\equiv$ linear environment : external circuit, other edge channel ...
- What happens at low $\mathcal{E}_0$? At high $\mathcal{E}_0$?
- Single particle coherence in the outcoming region ? Energy relaxation ?
- Under what conditions does the excess quasiparticle survives ?

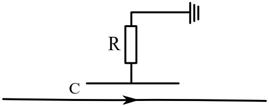
Quasiparticle relaxation



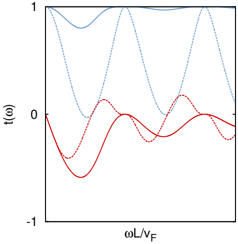
- Before interaction : QP peak
- After interaction : two contributions
  - Regular part : $\delta n^r_{k_0}(k)$
  - Singular part : $Z(k_0)\delta(k-k_0)$
- $Z(k_0)$: Elastic scattering probability

Two different illustrations

RC- circuit

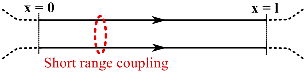


- Finite bandwidth
- 2 parameters : $\frac{l}{v_F R_K C}$ & $\frac{R}{R_K}$

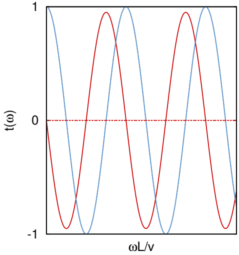


- A. Prêtre, *et al.*, Phys. Rev. B **54**,8130 (1996)

Coupled channels

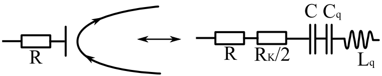


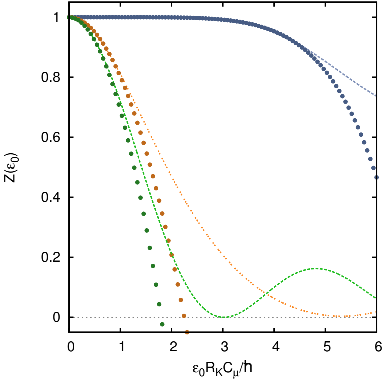
- Infinite bandwidth
- Coupling strength $\leftrightarrow \theta$



Low energy regime - Elastic scattering probability

At low energy → Circuit equivalent :





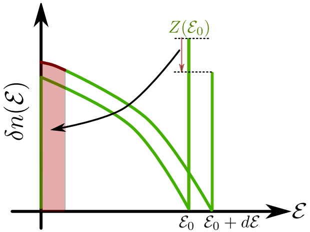
- $\mathcal{Z}(\epsilon_0) \to 1$ at low energy
  $t(\omega) \to 1 \equiv$ Capacitive coupling
- $1 - \epsilon^2 : R \neq 0$
- $1 - \epsilon^6 : R = 0 \;\to$ passive gate
- Always resistive for coupled channels

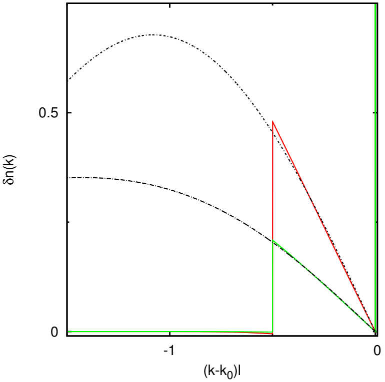
- A. Prêtre, *et al.*, Phys. Rev. B **54**,8130 (1996)
- Y. M. Blanter *et al.*, Phys. Rev. Lett. **81**, 1925 (1998)

Quasiparticle relaxation - At low energy

Phenomenological model :

- Spectator Fermi sea
  $\hookrightarrow$ Fermi golden rule
- $\delta n^r_{k_0}(k) = -Z'(k_0 - k)$





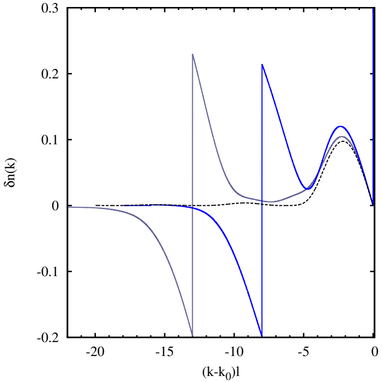
Universal conclusion

At low energy, Pauli principle ensures quasiparticle survival

Quasiparticle relaxation - At high energy

Low coupling - Finite bandwidth

- Fermi sea = effective environment
- Supplementary electron singled out from the Fermi sea
- For a wavepacket,
  $\Delta\mathcal{G}^{(e)}(x,y) = \mathcal{D}(x-y)\varphi(x)\varphi^*(y)$
  $\mathcal{D}$: decoherence coefficient
- **Quantum optics paradigm valid**

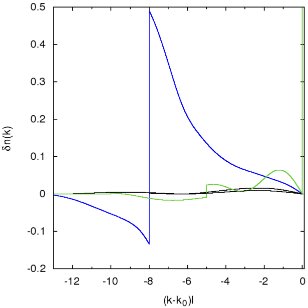


- G.-L. Ingold and Yu.V. Nazarov. NATO ASI Series B **294** 21-107. Plenum Press, New York (1992).

Quasiparticle relaxation - At high energy

Strong coupling - Infinite bandwidth

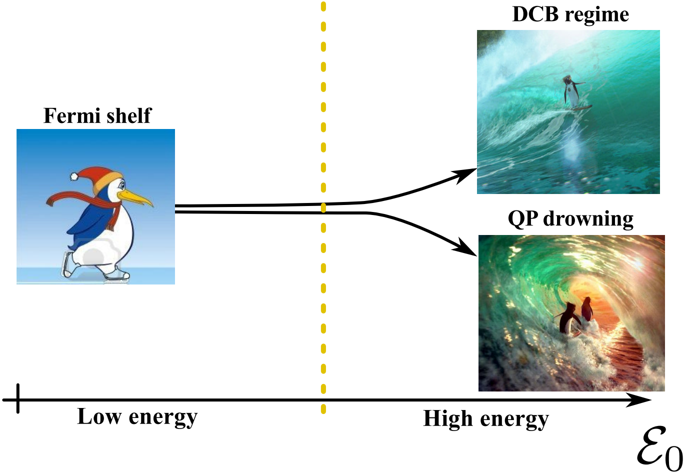
- Fermi sea $\neq$ effective environment
- Drowning of the quasiparticle
- **Quantum optics paradigm wrong**



Two limiting regimes

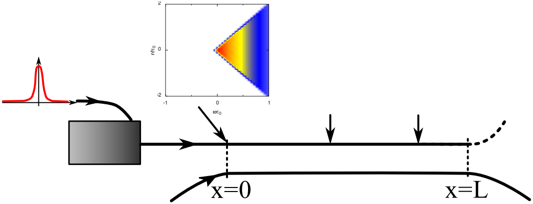
- "Dynamical Coulomb blockade" for low coupling and finite bandwidth
- QP decay for strong coupling and large bandwidth

Single electron relaxation - Summary



What for a time resolved excitation ?

Time resolved excitation



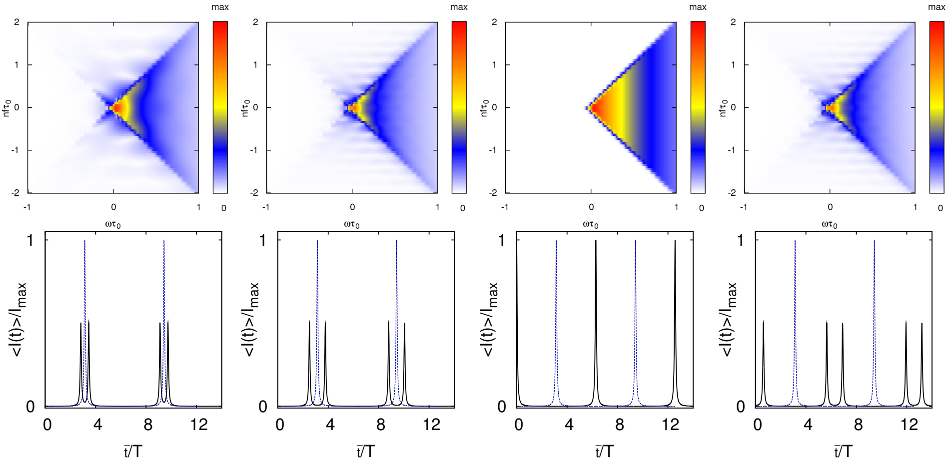
- Quantized lorentzian pulse
- Purely electronic state
- No particle/hole pair

In the presence of interactions :

- Particle/hole pair generation ?
- Possibility to gain information on relaxation mechanisms ?

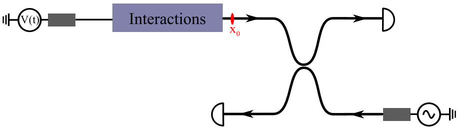
• L. Levitov *et al.*, J. Math. Phys. **37** 4845 (1996)

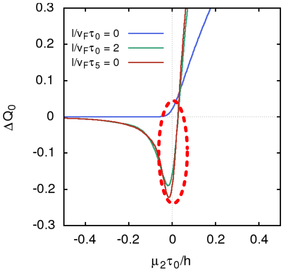
Coherence function and interactions

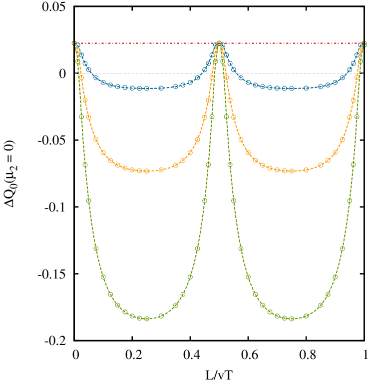


Is it possible to probe the interactions with these excitations ?

HBT signals and particle hole/pair formation







$\mu_2 = 0$ value of $n = 0$ HBT correlation

$\Rightarrow$ Interaction induced hole production

Conclusions

i. A formalism for electron quantum optics

ii. Measurement of single particle coherence

iii. Predictions for decoherence and relaxation

iv. Test for interaction pictures in $\nu = 2$ systems :
$\rightarrow$ **LPN experiments**

v. Proposition of noise measurement protocols with experimental signal estimation :
$\rightarrow$ **HBT interferometry at the LPA**

To do list ...

i. Quantum optics for plasmons - Relation to radiation statistics (Beenakker-Schömerus)

ii. Coherent spin transport

iii. Fractional quantum Hall regime
- Coherence of Laughlin's quasiparticles
- Electron quantum optics in a Luttinger liquid

iv. Far from equilibrium distributions $\rightarrow$ Adaptation of nonequilibrium bosonization formalism

Merci !

Tomography - Formulae1

Experimental signal : $S_{\alpha\beta} = 2\int d\tau \overline{S_{\alpha\beta}(\bar{t}+\frac{\tau}{2}, \bar{t}-\frac{\tau}{2})}^{\bar{t}}$

$$S_{11}^{out}(t,t') = \mathcal{R}^2 S_{11}(t,t') + \mathcal{T}^2 S_{22}(t,t') + \mathcal{R}\mathcal{T}\ \boxed{\mathcal{Q}(t,t')}$$

$$S_{22}^{out}(t,t') = \mathcal{T}^2 S_{11}(t,t') + \mathcal{R}^2 S_{22}(t,t') + \mathcal{R}\mathcal{T}\ \boxed{\mathcal{Q}(t,t')}$$

$$S_{12}^{out}(t,t') = S_{21}^{out}(t,t') = \mathcal{R}\mathcal{T}\big(S_{11}(t,t') + S_{22}(t,t') - \boxed{\mathcal{Q}(t,t')}\big)$$

Quantum contribution to HBT correlations :

$$\mathcal{Q}(t,t') = (ev_F)^2\left[\mathcal{G}_1^{(e)}(t',t)\mathcal{G}_2^{(h)}(t',t) + \mathcal{G}_2^{(e)}(t',t)\mathcal{G}_1^{(h)}(t',t)\right]$$

$$\mathcal{Q} = \underbrace{\mathcal{G}_{\mu_1}\mathcal{G}_{\mu_2}}_{\text{stationnary excess noise} \propto \frac{e^2\mu_2}{h}} + \underbrace{\mathcal{G}_{\mu_1}\Delta\mathcal{G}_2}_{\text{photoassisted noise} \propto \frac{e^2\mu_2}{h}\frac{eV_0}{\hbar\omega_d}} + \overbrace{\boxed{\underbrace{\mathcal{G}_{\mu_2}\Delta\mathcal{G}_1}_{V_0=0 \text{ contribution}} + \underbrace{\Delta\mathcal{G}_1\Delta\mathcal{G}_2}_{V_0\neq 0 \text{ contribution}}}}^{\Delta\mathcal{Q}}$$

Tomography - Formulae 2

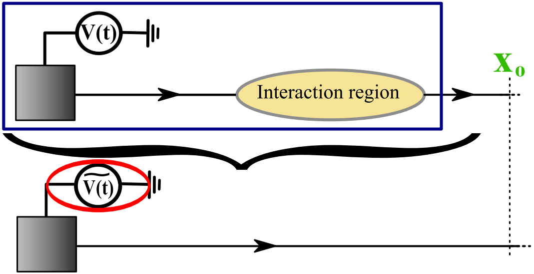
Coherence function harmonics :

order 0 $\delta\bar{n}_1(\omega) \stackrel{!}{=} \bar{n}_1(\omega) - \bar{n}_{\mu_1}(\omega) = -\frac{R_K}{2}\left(\frac{\partial\Delta\mathcal{Q}_0}{\partial\mu_2}\right)_{\mu_2=\hbar\omega}$

order n $\overline{\chi_n}(t-t') = \overline{\frac{\partial\Delta\mathcal{Q}}{\partial(eV_0/n\hbar\Omega)}(t,t')}^{\bar{t}}$

$$\frac{\partial\overline{\chi_n}}{\partial\mu_2}(\mu_2,\phi) = \frac{v_F e^2}{h}\Re\left[e^{i\phi}\left(\mathcal{G}^{(e)}_{1,n}(\mu_2/\hbar + n\frac{\Omega}{2}) - \mathcal{G}^{(e)}_{1,n}(\mu_2/\hbar - n\frac{\Omega}{2})\right)\right]$$

Single electron pulse and interactions



- Source with driving voltage V
- Interactions $\equiv$ scattering
- Coherence measurement at x_0

Source $\oplus$ interaction region $\equiv$ Renormalized driving voltage $\widetilde{V}$:

$$\widehat{\widetilde{V}}(\omega) = \widehat{V}(\omega) \times \left(1 - \frac{h}{e^2}\widehat{Y}(\omega)\right)$$

$\widehat{Y}(\omega)$: finite frequency admittance

Coherence function from Floquet theory

Floquet matrix relates in and out modes :

$$\mathfrak{c}^{(out)}_{\omega} = \sum_{n\in\mathbb{Z}} S_n(\omega)\,\mathfrak{c}^{(in)}_{\omega+n\Omega}$$

$$S_n(\omega) = \sum_{k\in\mathbb{Z}} S_0(\omega - k\Omega)\,c_k[V]\,c^*_{k+n}[V],$$

and allows to compute the harmonics of the coherence function :

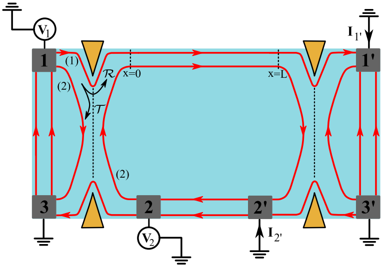
$$\mathcal{G}(t,t') = \langle \Psi^{\dagger}_{out}(t')\Psi_{out}(t)\rangle$$

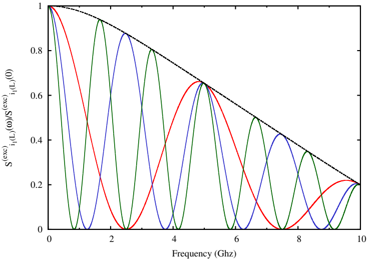
$$= \sum_{n\in\mathbb{Z}} \mathrm{g}_n(t-t')\,e^{-ni\Omega\bar{t}} \quad ,\bar{t} = \frac{t+t'}{2}.$$

$$\widetilde{\mathrm{g}}_n(\omega) = \frac{1}{v_F}\sum_{k\in\mathbb{Z}} S^*_{n+k}(\omega - \frac{n\Omega}{2})\; S_k(\omega + \frac{n\Omega}{2})\; \bar{n}_F(\omega + (\frac{n}{2}+k)\Omega)$$

i. M. Moskalets & M. Büttiker, Phys.Rev.B **66**, 205320 (2002)

Noise measurements in $\nu = 2$





i. Unitarity : $\mathcal{S} \in SU(2)$

ii. OB relations : symmetry

iii. Capacitive coupling : $\mathcal{S} \to 1$

iv. Simplest dependence in ω

$\Rightarrow \mathcal{S}(\omega) = e^{i\omega L/v_0}\, e^{i\omega L/v \cos\theta\sigma^z \sin\theta\sigma^x}$

$$\left(\frac{k_b T_{exc}}{\Delta\mu}\right)^2 = \frac{3}{\pi^2}\tau(1-\tau)\left(T_\infty + (1-T_\infty)\frac{\mathrm{sinc}^2(L/L_{\Delta\mu})}{\mathrm{sinhc}^2(L/L_{th})}\right),$$

$$\mathcal{J}_{\alpha,qp} = \frac{\pi^2}{6h}(k_BT)^2 + \frac{R_K}{2}\left(\frac{e^2}{2\pi}\right)^2\tau(1-\tau)\int_{-\infty}^{+\infty} T(\omega,L)F(\omega,\mu_2-\mu_1,\beta)\,\mathrm{d}\omega\,,$$

Scattering matrices - Derivation

$$\psi(t+\tau, x+v_F\tau) = \psi(x,t)\exp\left(\frac{ie}{\hbar}\int_0^\tau V(x+v_Ft', t+t')\mathrm{d}t'\right). \quad (1)$$

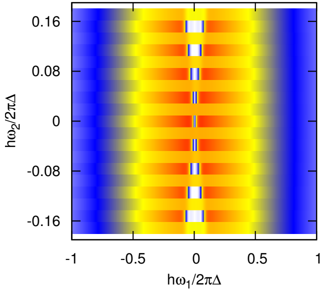
$$(\partial_t + v_F\partial_x)\phi(x,t) = \frac{e\sqrt{\pi}}{h}V(x,t). \quad (2)$$

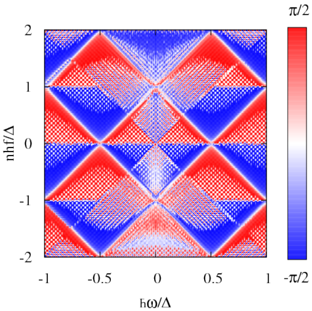
$$V(x,t) = V_{cond}(t)K(x) - \frac{e}{\epsilon\sqrt{\pi}}\int \mathrm{d}y(-\Delta+\chi)^{-1}(x,y)(\partial_y\phi)(y,t). \quad (3)$$

$$t_b(\omega) = \frac{\mu^*(\omega) - iRC\omega|\beta(\omega)|^2\Lambda^*(\omega)}{\mu^*(\omega) + iRC\omega|\beta(\omega)|^2\Lambda^*(\omega)} \quad (4)$$

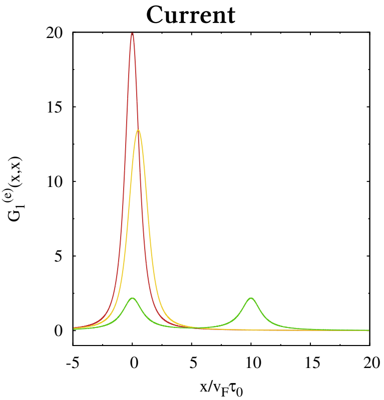
$$\beta(\omega) = \frac{i}{2v_FRC}\sqrt{\frac{2R}{R_K}}\tilde{f}(\omega/v_F) \quad (5)$$

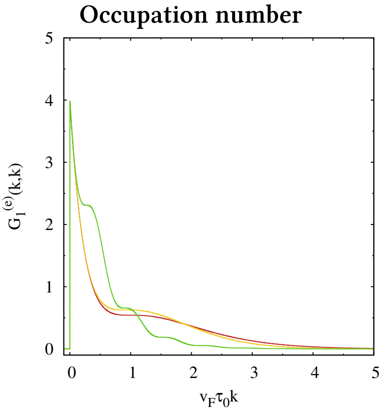
Tomography - Zoom and phase





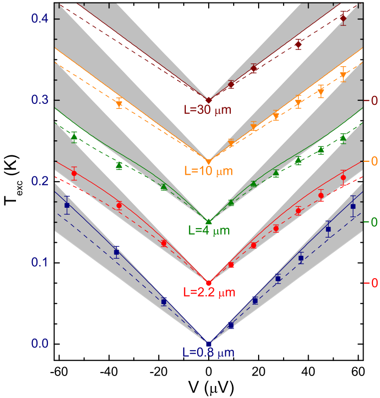
Superposition of 2 lorentzian wavepackets



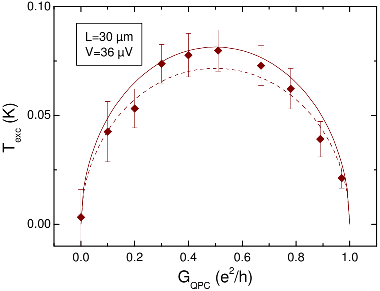


Relaxation results vs bosonization

T_{exc} vs. V



T_{exc} vs. G



Comparison with FCS