

# Semi-active SOBEN suspension modeling and control

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**PhD. defense, October, 25th, 2010**

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# Summary

- 1 Introduction
  - Context
  - Objectives and contribution
- 2 System analysis and modeling
  - Description
  - Modeling
- 3 Control strategy
  - Introduction
  - Observer design and experimental results
  - Local damper control and experimental results
  - Global vehicle control and simulation results
- 4 Conclusion and perspectives

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# The industrial context

## Manufacturing

- Damper prototypes
- Mass-production
- Helicopter undercarriages
- Testing benches

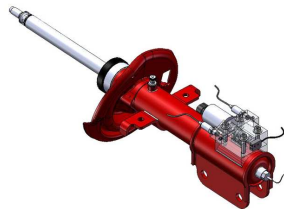


## Tests & Validation



- Testing benches
- Testing cars
- Sensors & acquisition boards

## Research & Development



- Dampers for private and racing car
- Adjustable motorcycle dampers
- Helicopter undercarriages
- Semi-active dampers development

# The collaboration context

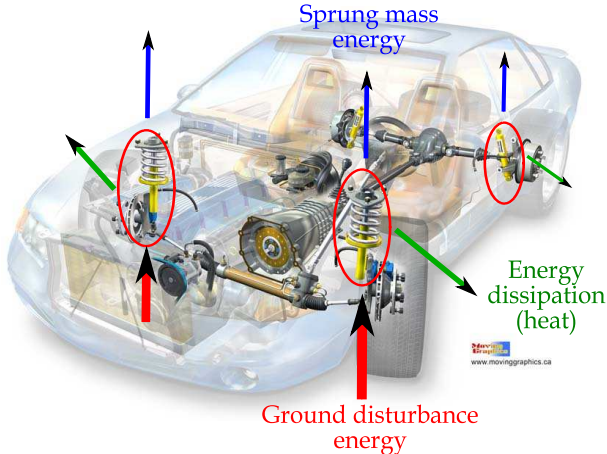
- **SLR team - GIPSA-lab**

- Semi-active suspension modeling and control
  - ▶ [Sammier, 2001, PhD. thesis]
- Active suspension control toward Global Chassis Control
  - ▶ [Zin, 2005, PhD. thesis]
- Robust multivariable LPV automotive Global Chassis Control
  - ▶ [C. Pousot-Vassal, 2008, PhD. thesis]
- Robust vehicle dynamics LPV control : comfort and safety improvement
  - ▶ [A. Lam Do, PhD. thesis] (in progress)

- **TEC Monterrey (PCP program - 6 months visit)**

- Magneto-rheological (MR) dampers modeling
- Semi-active damper control

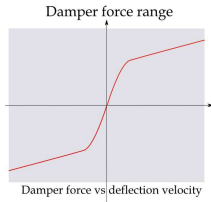
# The role of suspensions



- Spring
  - Chassis holding
- Damper
  - Minimize :
    - sprung mass vibrations
      - ▶ Comfort
    - wheel displacements
      - ▶ Roadholding

# Various suspension types

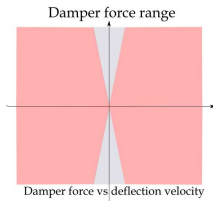
## Passive



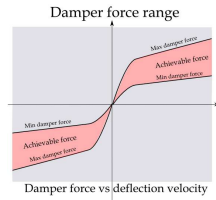
- Most widely used suspension type
- Simple and economical
- Can be optimized ► CRONE approach [Moreau, 1995], [Oustaloup et Mathieu, 1999]

## Active

- Widely studied, rarely used [Hrovat, 1997], [Karnopp, 1983], [Sammier, 2003]
- Expensive, high power consumption, high performance



## Semi-active



- Under development, already used
- Moderate cost and power consumption, improved performances

## Previous studies on semi-active suspension control

- Clipped semi-active suspension control
  - LQ : complex, state measurement
    - ▶ [Tseng et al., 1994]
  - MPC based : involve optimization, state measurement
    - ▶ [Canale et al., 2006], [Giorgetti et al., 2006], [Sammier et al., 2003]
  - $\mathcal{H}_\infty$ 
    - ▶ [Sammier et al., 2003]
- Dedicated semi-active suspension control
  - $\mathcal{H}_\infty$ /LPV control
    - ▶ [Poussot-Vassal, 2008], [Do et al., 2010]
  - ADD, Mixed SH-ADD : easily implementable, deflection velocity measurement, comfort oriented
    - ▶ [Savaresi et al., 2005, 2007]



# PhD contributions

- Control oriented damper models
- $\mathcal{H}_\infty$  observer
  - Unknown input observer
  - Few sensors required
- LPV/ $\mathcal{H}_\infty$  suspension control
  - Local(damper) and global(vehicle) control
  - Adjustable performance
  - Damper non linearities/abilities (LPV)
- System development
  - Sensors and actuator choice
  - Vehicle preparation (cables, sensors, dampers, control boards)
  - Control boards programming

# Publications

## • Conference papers

### • Vehicle modeling and analysis

- ▶ 17th IFAC World Congress 2008
- ▶ 7th EUROSIM Congress on modeling and simulation (co-author)
- ▶ "Journées Automatique et Automobile" (JAA GRDMACS) 2007

### • Observer design methodology

- ▶ IEEE Multi-Conference on Systems and Control 2009
- ▶ IFAC Symposium Advances in Automotive Control 2010

### • $\mathcal{H}_\infty$ /LPV control strategy

- ▶ European Control Conference 2009
- ▶ 11th Mini Conference on Vehicle System Dynamics, Identification and Anomalies 2008

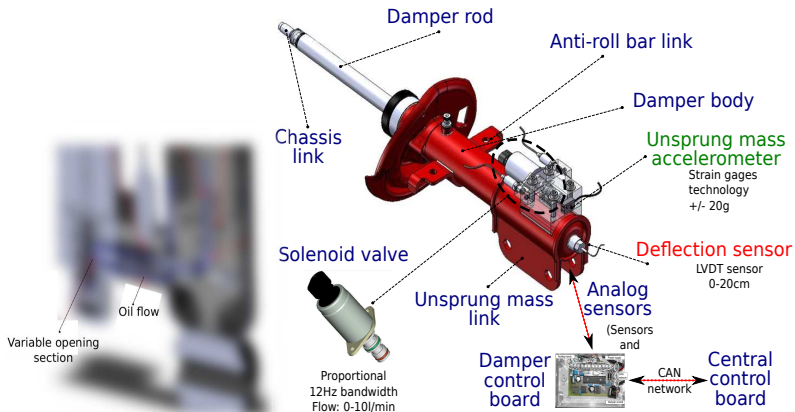
## • Journal papers in preparation

- "Full vertical car observer design methodology for on-board suspension control applications"
- "Semi-active  $\mathcal{H}_\infty$ /LPV control for a full vertical car equipped with industrial hydraulic dampers"

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# System description



# Identified damper model

- Quasi-static damper behavior

[Guo et al., 2006]

$$F(t) = (A_1 u_d + A_2) \tanh(A_3 \dot{z}_{def} + A_4 z_{def}) + A_5 \dot{z}_{def} + A_6 z_{def} + A_7$$

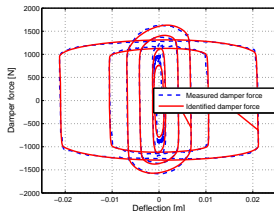
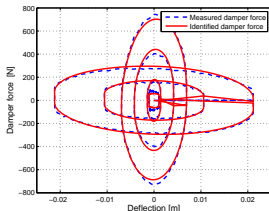
- ▶ Constant parameters  $A_i$  identified from experimental data

- ▶ Damper non-linearity & hysteresis

- Dynamical damper behavior

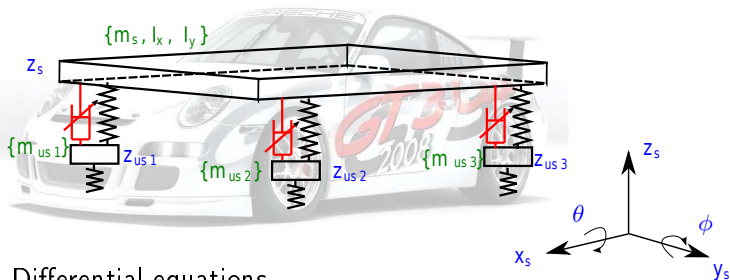
$$\frac{l(s)}{u_d(s)} = \frac{G}{\left(\frac{s}{\omega_d}\right)^2 + 2\zeta_d \frac{s}{\omega_d} + 1}$$

- ▶ Actuator bandwidth



# The vehicle model

- Vertical full-car model (7 DOF)



- Differential equations

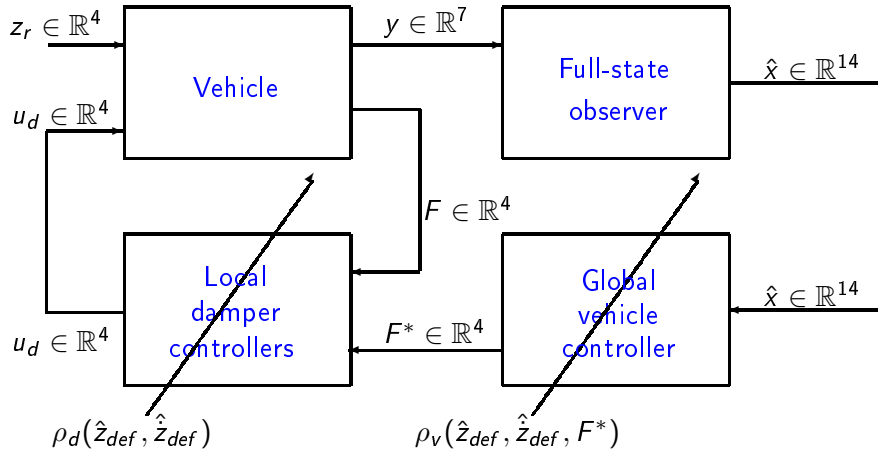
$$\begin{cases} m_s \ddot{z}_s = -(F_{s1} + F_{s2} + F_{s3} + F_{s4}) \\ m_{us_i} \ddot{z}_{us_i} = F_{s_i} - F_{t_i}, i = 1, \dots, 4 \\ I_x \ddot{\theta} = (F_{s1} - F_{s2}) t_f + (F_{s3} - F_{s4}) t_r \\ I_y \ddot{\phi} = (F_{s4} + F_{s3}) l_r - (F_{s2} + F_{s1}) l_f \end{cases} \quad \begin{cases} F_{s_i} = k_i z_{def_i} + F(z_{def}, \dot{z}_{def}, u_i) \\ F_{t_i} = k_{t_i} (z_{us_i} - z_{r_i}) \\ z_{def_i} = z_{s_i} - z_{us_i} \end{cases}$$

$F(z_{def}, \dot{z}_{def}, u_i)$  linear for synthesis, nonlinear for simulations

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# Hierarchical control architecture

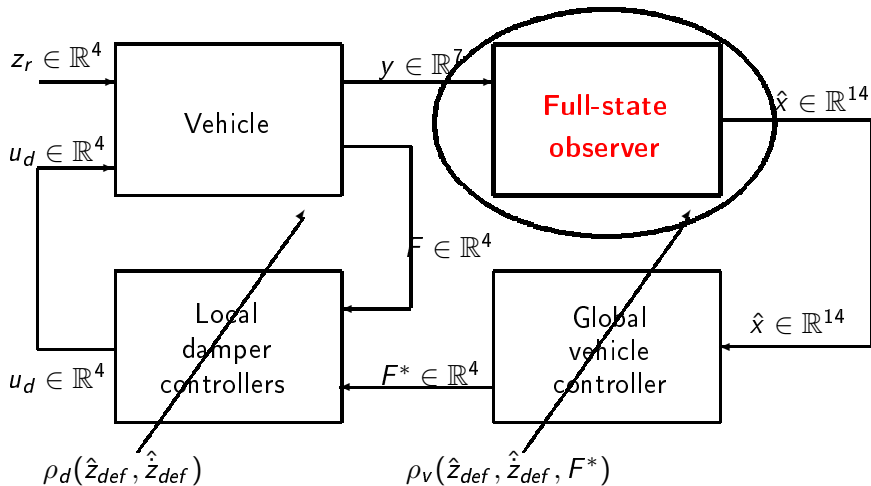




# Architecture characteristics

- Full-state observer
  - To reduce the number of sensors
- Global vehicle controller
  - $\mathcal{H}_\infty$  LPV state-feedback controller
  - LPV to ensure dissipativity constraint
  - Damper force reference generation for each damper
  - Comfort/roadholding trade-off optimization
- Local damper controllers
  - Mixed  $\mathcal{H}_\infty/\mathcal{H}_2$  LPV dynamic output feedback controller
  - LPV to account for damper nonlinearities

# Control architecture



## Full vertical car observer [Koenig, 2008]

Measured variables :

4 unsprung mass accelerations, 3 sprung mass accelerations

Full-car linear model :

$$\begin{cases} \dot{x} = Ax + D_x v \\ y = Cx + D_y v \end{cases}$$

Full-state observer :

$$\begin{cases} \dot{z} = Nz + Ly \\ \hat{x} = z - Ey \end{cases}$$

$$\dot{e} = \dot{x} - \dot{\hat{x}} = Ne + (PA - KC - N)x + (PD_x - LD_y)v + ED_y \dot{v}$$

Where  $P = I + EC$  and  $K = L + NE$ .

Stability conditions :

$$\begin{cases} N \text{ is Hurwitz} \\ N = PA - KC \end{cases}$$

Decoupling conditions :

$$\begin{cases} ED_y = 0 \\ LD_y - PD_x = 0 \end{cases}$$

$LD_y - PD_x = 0$  may not be solvable or lead to inconvenient pole placement  $\Rightarrow \mathcal{H}_\infty$ -observer  $\Rightarrow N, L$  and  $E$

# $\mathcal{H}_\infty$ observer synthesis

- Method

- 1  $E$  parameterized s.t.  $ED_y = 0$
- 2  $N = PA - KC$

$$\dot{e} = Ne + (PD_x - LD_y)v \Leftrightarrow \dot{e} = \mathcal{A}e + \mathcal{B}v$$

- $\mathcal{H}_\infty$  disturbance attenuation

Application of the Bounded Real Lemma [Scherer and Weiland, 1999]

$$\left( \begin{array}{c|c|c} \mathcal{A}^T P + P \mathcal{A} & P \mathcal{B} & C^T \\ \hline * & -\gamma \mathcal{I} & \mathcal{D}^T \\ \hline * & * & -\gamma \mathcal{I} \end{array} \right) \prec 0$$

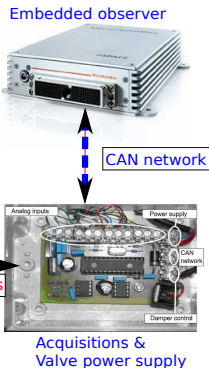
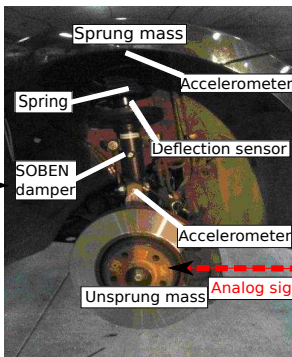
- + Observer pole placement [Chilali et al., 1999]

$$\left( \begin{array}{c|c|c} \mathcal{D} = \{z \in \mathbb{C} : L + zM + z^*M^T \prec 0\} \\ L \otimes X + M \otimes (XA) & M_1^T \otimes (XB) & M_2^T \otimes C^T \\ + M^T \otimes (\mathcal{A}^T X) & & \\ \hline * & -\gamma \mathcal{I} & \mathcal{D}^T \\ \hline * & * & -\gamma \mathcal{I} \end{array} \right) \prec 0$$

# Experimental set-up and measured variables



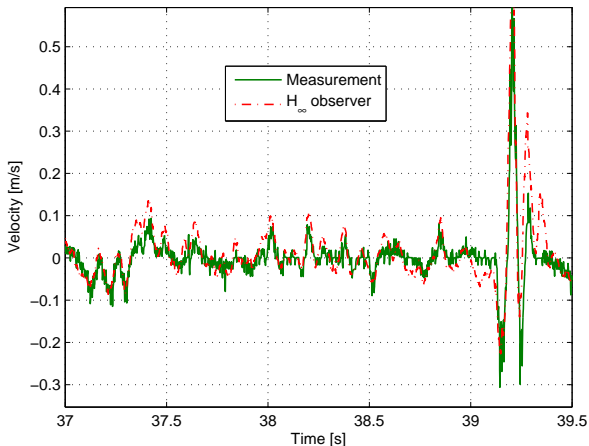
SOBEN testing car  
 (Renault Laguna GT)



Notation	Description	Full scale	Used in...
$\ddot{z}_{us_{1,2,3,4}}$	Wheel vertical accelerations	+/- 50g	► Observer input
$\ddot{z}_{s_{1,2,3}}$	Chassis vertical acceleration	+/- 5g	► Observer input
$z_{def_{1,2,3,4}}$	Front left suspension deflection	0-20cm	► Validation
$\dot{\theta}, \dot{\phi}$	Roll and pitch velocities	0-150deg/s	► Validation
$F_{1,2,3,4}$	Damper forces	Confidential	► Local damper control

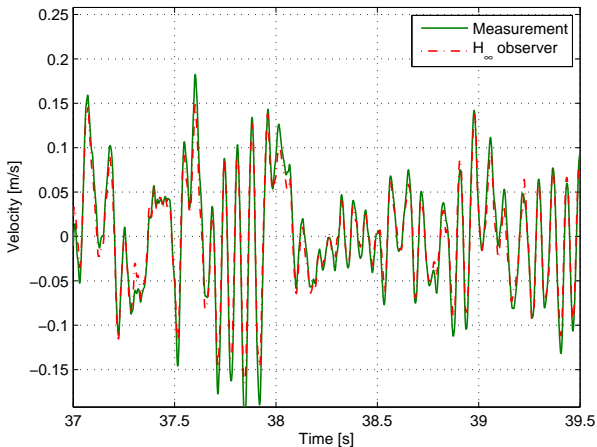
# Real time estimation results from experiments

## Deflection velocity (front left)



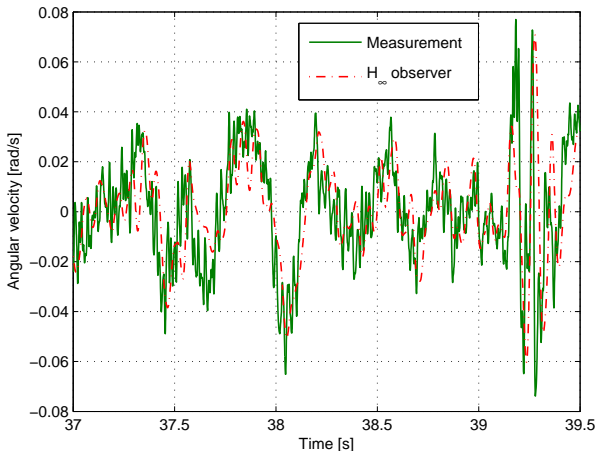
# Real time estimation results from experiments

## Unsprung mass velocity (front left)



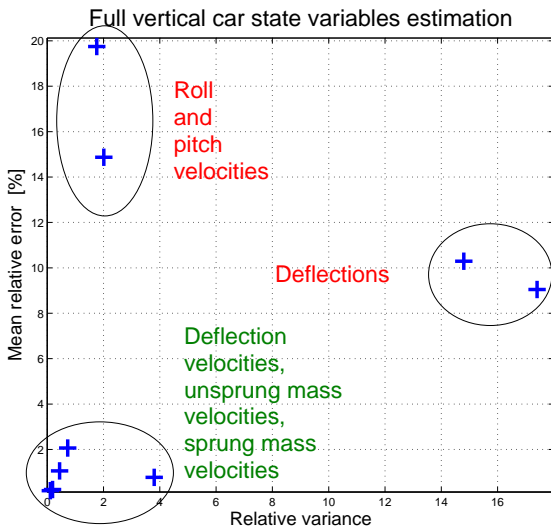
# Real time estimation results from experiments

## Pitch angular velocity

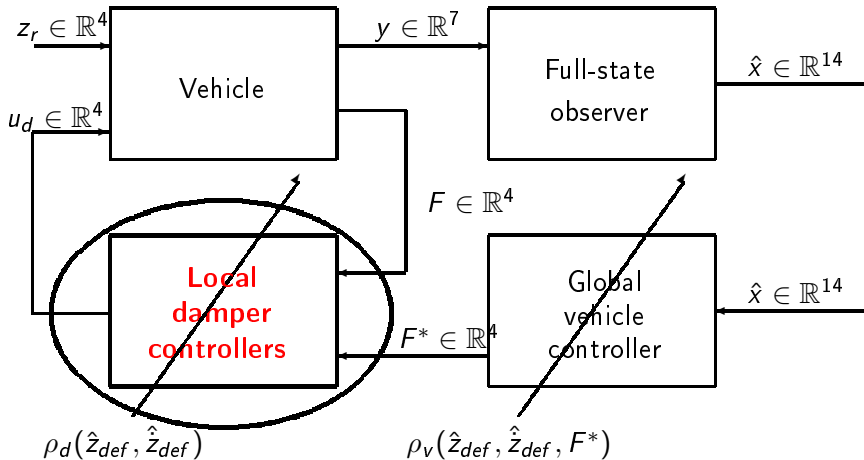




# Real time estimation results from experiments



# Control architecture



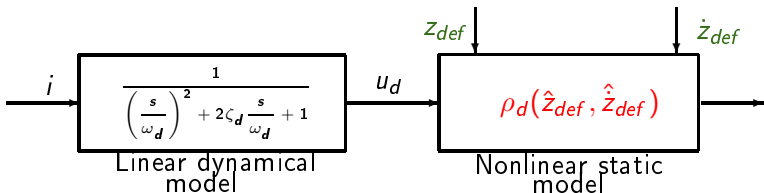
# LPV damper model

- **Static damper model** ▶ Nonlinear static gain

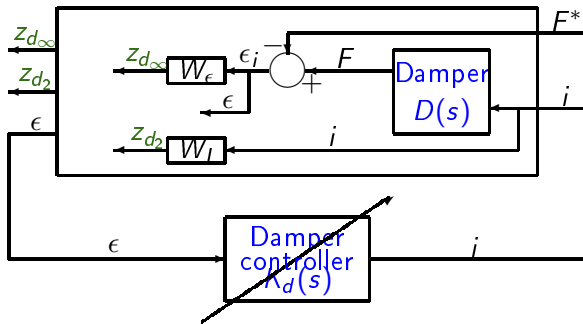
$$\begin{aligned}
 F &= (A_1 u_d + A_2) \tanh(A_3 \dot{z}_{def} + A_4 z_{def}) + A_5 \dot{z}_{def} + A_6 z_{def} + A_7 \\
 &= \rho_d(z_{def}, \dot{z}_{def}) u_d + F_0(z_{def}, \dot{z}_{def})
 \end{aligned}$$

- **Damper dynamical behavior** ▶ Bandwidth characterization

$$D(s) = \frac{\rho_d(z_{def}, \dot{z}_{def})}{\left(\frac{s}{\omega_d}\right)^2 + 2\zeta_d \frac{s}{\omega_d} + 1}$$



# Controller synthesis



$$\rho_d(\hat{z}_{def}, \hat{\dot{z}}_{def})$$

Mixed  $\mathcal{H}_\infty/\mathcal{H}_2$  LPV synthesis

- Tracking performances  $\blacktriangleright$   $\mathcal{H}_\infty$  constraint
- Control signal energy bound  $\blacktriangleright$   $\mathcal{H}_2$  constraint
- Stability of the closed-loop LPV system

# Damper controller synthesis

- Damper model

$$D(s) = \frac{\rho_d(z_{def}, \dot{z}_{def})}{\left(\frac{s}{\omega_d}\right)^2 + 2\zeta_d \frac{s}{\omega_d} + 1}$$

- Generalized system

$$\begin{pmatrix} \dot{x}_{da} \\ z_{d\infty} \\ z_{d2} \\ y \end{pmatrix} = \left( \begin{array}{c|c|c} A_{da} & B_1 & B_2(\rho_d) \\ \hline C_\infty & D_{\infty 1} & D_{\infty 2} \\ \hline C_2 & D_{21} & D_{22} \\ \hline C_y & D_{y1} & \mathcal{O} \end{array} \right) \begin{pmatrix} x_{da} \\ F^* \\ i \end{pmatrix}$$

- LPV controller

$$K_d(\rho_d) : \begin{pmatrix} \dot{\zeta} \\ i \end{pmatrix} = \left( \begin{array}{c|c} A_{K_d}(\rho_d) & B_{K_d}(\rho_d) \\ \hline C_{K_d}(\rho_d) & \mathcal{O} \end{array} \right) \begin{pmatrix} \zeta \\ \epsilon \end{pmatrix}$$

- Closed-loop system

$$\begin{pmatrix} \dot{x}_{cl} \\ z_{d\infty} \\ z_{d2} \end{pmatrix} = \left( \begin{array}{c|c} A(\rho_d) & B \\ \hline C_1 & \mathcal{D}_1 \\ \hline C_2 & \mathcal{D}_2 \end{array} \right) \begin{pmatrix} x_{cl} \\ F^* \end{pmatrix}$$

# Synthesis inequalities

- $\mathcal{H}_\infty$  performance for LPV systems (Bounded Real Lemma)  
 [Apkarian et al., 1995], [Scherer and Weiland, 2005]

$$\left( \begin{array}{c|c|c} \mathcal{A}^T(\rho_d)P + PA(\rho_d) & PB & C_\infty^T \\ \hline * & -\gamma_0 I & \mathcal{D}_\infty^T \\ \hline * & * & -\gamma_0 I \end{array} \right) \prec 0$$

- $\mathcal{H}_2$  performance for LPV systems

$$\left( \begin{array}{c|c} \mathcal{A}^T(\rho_d)P + PA(\rho_d) & PB \\ \hline * & -I \end{array} \right) \prec 0, \left( \begin{array}{c|c} K & C_2^T \\ \hline * & Z \end{array} \right) \succ 0, \text{Tr}(Z) > \sigma_0$$

- Pole placement constraints

$$\mathcal{D} = \{z \in \mathbb{C} : L + zM + z^*M^T \prec 0\}$$

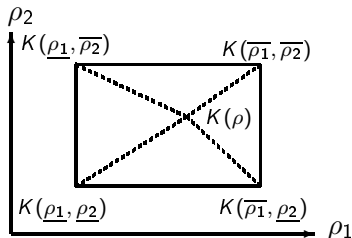
$$\left( \begin{array}{c|c|c} L \otimes X + M \otimes (XA(\rho_d)) & M_1^T \otimes (XB) & M_2^T \otimes C_1^T \\ + M^T \otimes (\mathcal{A}^T(\rho_d)X) & \hline * & -\gamma I & \mathcal{D}_1^T \\ \hline * & * & -\gamma I \end{array} \right) \prec 0$$

# LPV control - Polytopic approach

- Polytopic approach [Apkarian et al., 1995], [Scherer and Weiland, 2005]
  - Extremal parameters ► polytope
  - Single Lyapunov function ► conservatism
  - Synthesis ► one controller for each vertex
- LPV controller

$$K_d(\rho) = \sum_{k=1}^{2^P} \alpha_k(\rho) K_{d_k}$$

$$\alpha_k(\rho) = \frac{\prod_{j=1}^P |\rho(j) - \Theta_k|}{\prod_{j=1}^P |\bar{\rho} - \underline{\rho}|}$$



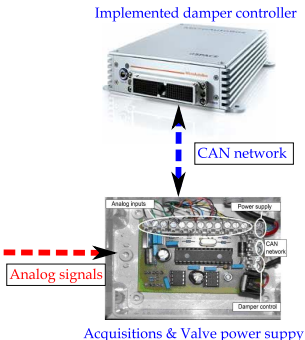
## Experimental set-up



Testing bench



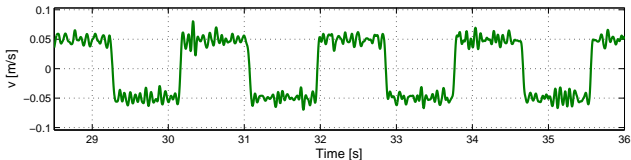
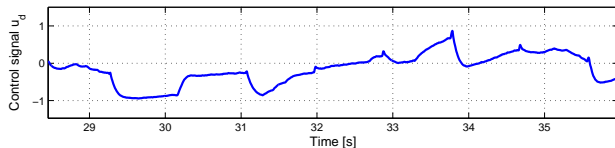
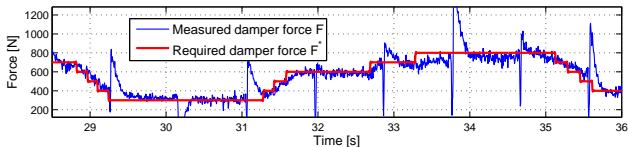
Controlled damper



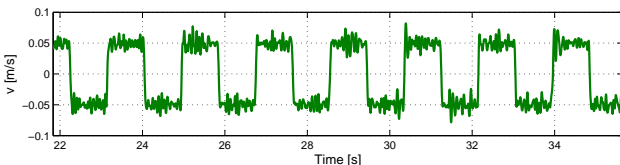
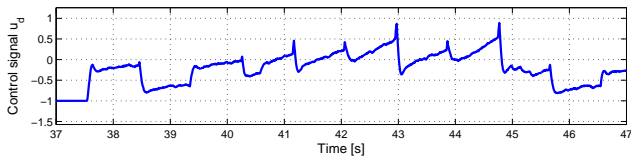
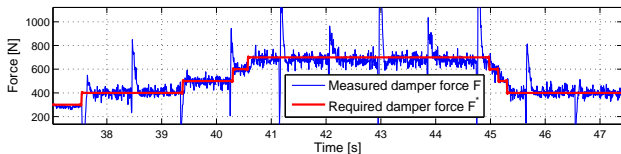
- Testing bench
- Ramp deflections ( $\dot{z}_{def} = 0.05m/s$ )
- Dspace acquisition & damper control board
- Closed-loop damper (measured force)



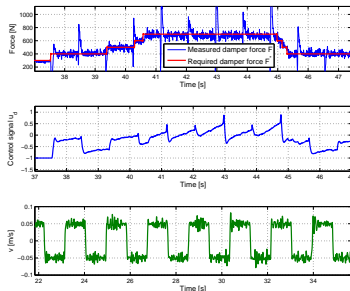
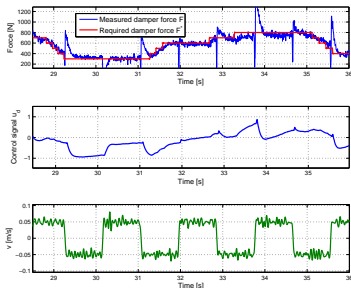
# Experimental results : LTI control



# Experimental results : LPV control



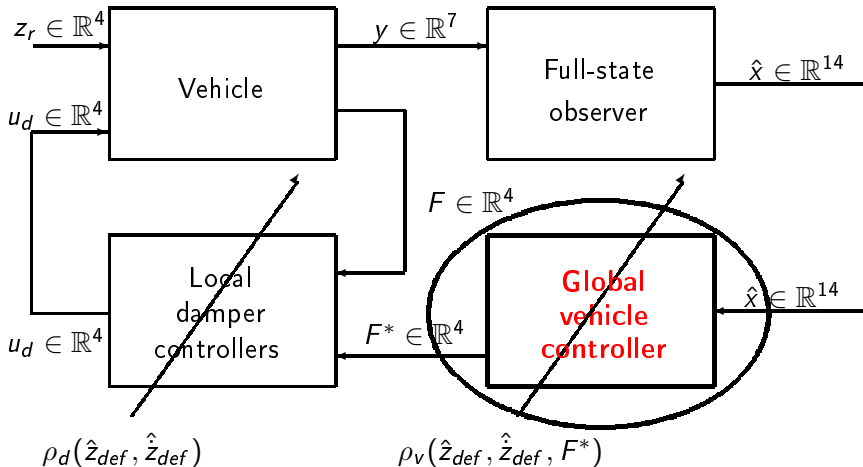
# Experimental results : synthesis



Mean relative errors [%] :

	Experiments	Simulations
LTI control	14	11
LPV control	6	5

# Control architecture



# Objectives

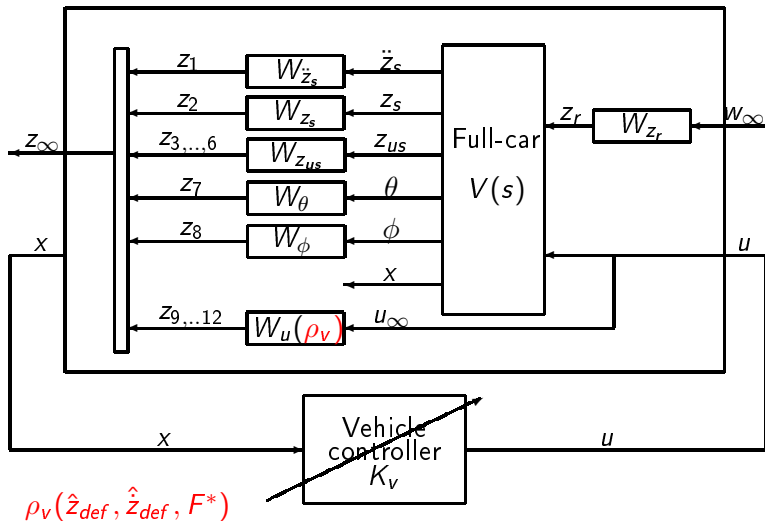
- Minimize the  $\mathcal{H}_\infty$ -norm of the input-output relations

Relation	Frequency range
$z_{r_i} \mapsto \ddot{z}_s, i \in [1, 4]$	[4-20] Hz
$z_{r_i} \mapsto z_s, i \in [1, 4]$	[0-5] Hz
$z_{r_i} \mapsto z_{us_i}, i \in [1, 4]$	[0-20] Hz
$M_x \mapsto \theta$	[0-5] Hz
$M_y \mapsto \phi$	[0-5] Hz

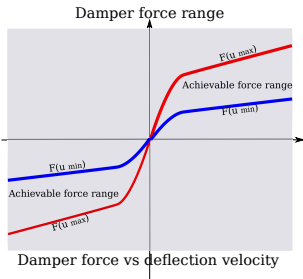
[Sammier, 2001]

- Compute achievable semi-active force references
- Ensure the dissipativity constraint

# Generalized plant



# Scheduling strategy to ensure dissipativity constraint



- Achievable damper force range

$$F(u_{min}) = (A_1 u_{d_{min}} + A_2) \tanh(A_3 \dot{z}_{def} + A_4 z_{def}) + A_5 \dot{z}_{def} + A_6 z_{def} + A_7$$

$$F(u_{max}) = (A_1 u_{d_{max}} + A_2) \tanh(A_3 \dot{z}_{def} + A_4 z_{def}) + A_5 \dot{z}_{def} + A_6 z_{def} + A_7$$

- Dynamical force reference saturation

$$\underline{F} = \min(F(u_{min}), F(u_{max}))$$

$$\overline{F} = \max(F(u_{min}), F(u_{max}))$$

$$F_{rea} = \min(\max(F^*, \underline{F}), \overline{F})$$

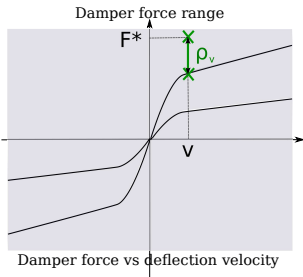
- Parameter calculation

$$W_u(\rho_v) = \rho_v = (\overline{\rho}_v - \underline{\rho}_v) \min(\overline{\epsilon}, |F_{rea} - F^*|) + \underline{\rho}_v$$

- Extension of previous results [Poussot-Vassal, 2008]

- Full vertical car
- Static state-feedback controller
- Identified damper model based scheduling

# Scheduling strategy to ensure dissipativity constraint



- Achievable damper force range

$$F(u_{min}) = (A_1 u_{d_{min}} + A_2) \tanh(A_3 \dot{z}_{def} + A_4 z_{def}) + A_5 \dot{z}_{def} + A_6 z_{def} + A_7$$

$$F(u_{max}) = (A_1 u_{d_{max}} + A_2) \tanh(A_3 \dot{z}_{def} + A_4 z_{def}) + A_5 \dot{z}_{def} + A_6 z_{def} + A_7$$

- Dynamical force reference saturation

$$\underline{F} = \min(F(u_{min}), F(u_{max}))$$

$$\overline{F} = \max(F(u_{min}), F(u_{max}))$$

$$F_{rea} = \min(\max(F^*, \underline{F}), \overline{F})$$

- Parameter calculation

$$W_u(\rho_v) = \rho_v = (\overline{\rho}_v - \underline{\rho}_v) \min(\overline{\epsilon}, |F_{rea} - F^*|) + \underline{\rho}_v$$

- Extension of previous results [Poussot-Vassal, 2008]

- Full vertical car
- Static state-feedback controller
- Identified damper model based scheduling



## Complete control strategy simulations

### Simulation of the whole system :

- Nonlinear vehicle model
- Observer  $\Rightarrow$  State estimation
- Local controller  $\Rightarrow$  Force reference tracking
- Global controller  $\Rightarrow$  Force reference generation

### Vehicles under study :

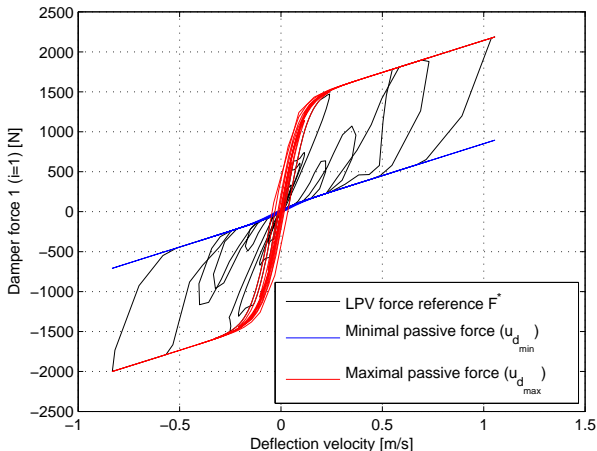
- 1 Passive linear damper with minimal damping rate,
- 2 Passive linear damper with maximal damping rate,
- 3 Active damper,
- 4 Semi-active LPV control,
- 5 Semi-active ADD control (Acceleration Driven Damper)

[Savaresi et al., 2005]

$$\text{damping rate} = \begin{cases} c_{max} & \text{if } \ddot{z}_{s_i} (\dot{z}_{s_i} - \dot{z}_{u s_i}) > 0 \\ c_{min} & \text{if } \ddot{z}_{s_i} (\dot{z}_{s_i} - \dot{z}_{u s_i}) < 0 \end{cases}$$

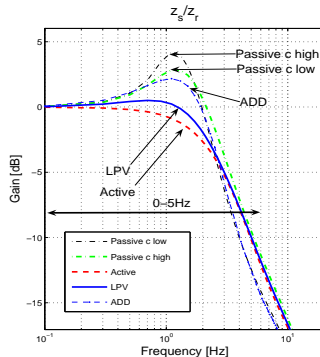
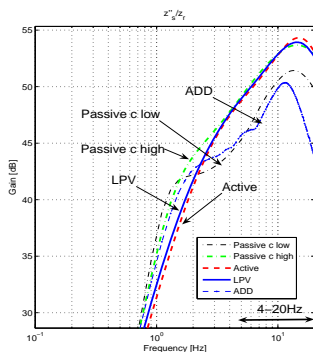
# Time-domain simulations with random road disturbance

## Achievable force range and force reference



# Frequency-domain simulations of the nonlinear vehicle model

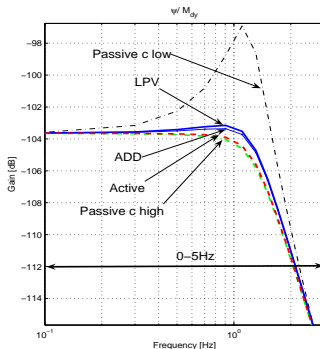
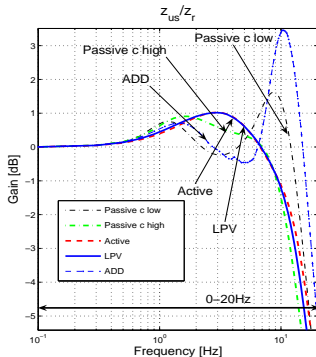
Classical calculation method [Savaresi et al., 2010]



- ADD ► Good high-frequency comfort level
- LPV ► Good low frequency comfort level

# Frequency-domain simulations of the nonlinear vehicle model

Classical calculation method [Savaresi et al., 2010]



- LPV ► Good roadholding performance
- ADD ► Poor roadholding performance

## Performance criteria

Performance criteria :

$$PSD_{f_1 \rightarrow f_2}(X(f)) = \sqrt{\int_{f_1}^{f_2} X^2(f) \cdot df}$$

Comparison to nominal passive :

	$\ddot{z}_s/z_r$	$z_s/z_r$	$z_{us}/z_r$	$\psi/z_r$
Active $\mathcal{H}_\infty$	11%	17 %	24 %	2%
LPV $\mathcal{H}_\infty$	8%	15 %	22 %	-2%
ADD	18%	7 %	-7 %	-4%

# Summary

- 1 Introduction
  - Context
  - Objectives and contribution
- 2 System analysis and modeling
  - Description
  - Modeling
- 3 Control strategy
  - Introduction
  - Observer design and experimental results
  - Local damper control and experimental results
  - Global vehicle control and simulation results
- 4 Conclusion and perspectives

# Conclusion

## • Contribution

- Observer design methodology
- Complete and flexible suspension control methodology
- Improved comfort and roadholding
- Adjustable performance specifications
- Achievable force reference
- Local damper control  $\Rightarrow$  Nonlinearities
- Implementable control strategy

## • Perspectives

- Vehicle controller : implementation and tests in practice
- Global chassis control
- Feedforward control

# Conclusion

## Questions

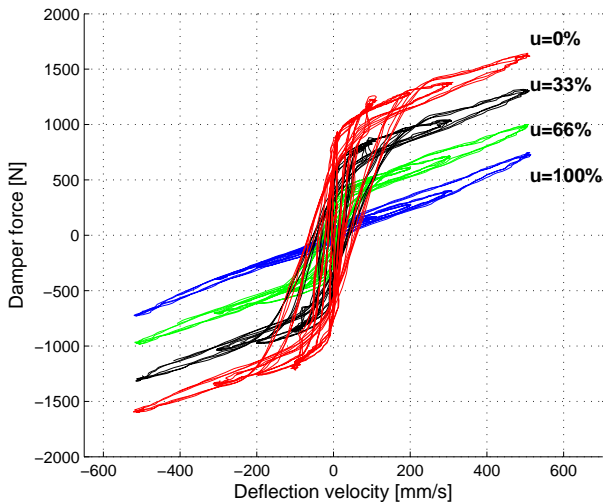


## Experiment 1 : protocol

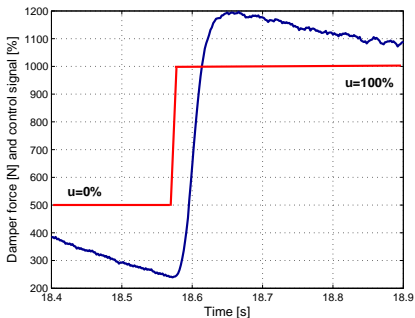
- **Sine deflections** : various amplitudes, frequencies and control signals (constant)

Sine wave	Freq. [Hz]	Amp. [mm]	Control [%]
Sine 1	1.5	1	0, 33, 66 and 100
Sine 2	1.5	10.5	0, 33, 66 and 100
Sine 3	1.5	21	0, 33, 66 and 100
Sine 4	12	1.4	0, 33, 66 and 100
Sine 5	12	4.1	0, 33, 66 and 100
Sine 6	12	6.9	0, 33, 66 and 100

## Experiment 1 : results



## Experiment 2

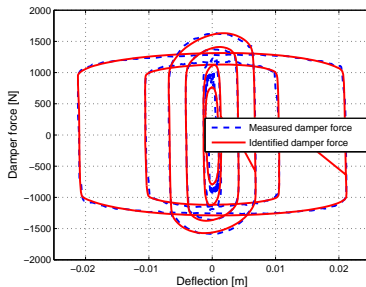
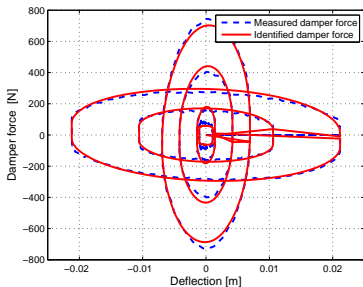


### Step response :

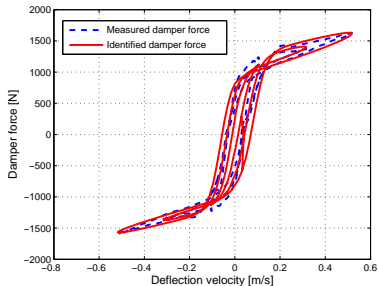
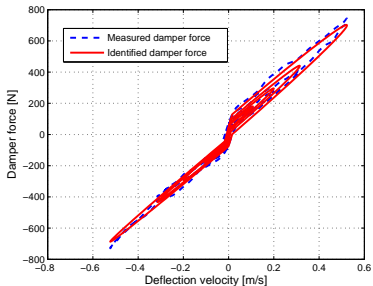
- Testing bench
- Ramp deflections (0.1m/s and -0.1m/s)
- Control signal step : 0% to 100%

## Identified model validation : force deflection

- Minimal control signal
- Other sine deflections (different amplitudes and frequencies)  
⇒ Max relative error : 5.3%

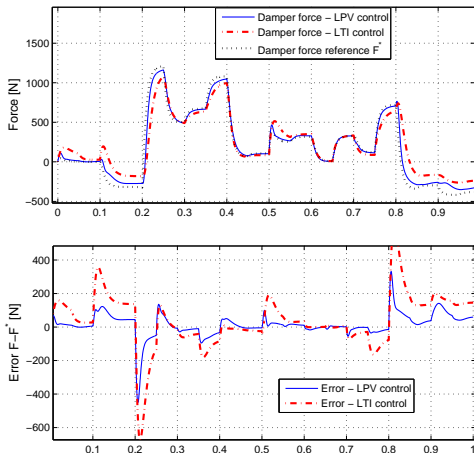


# Identified model validation : force velocity



## Simulation results : LTI vs LPV controller

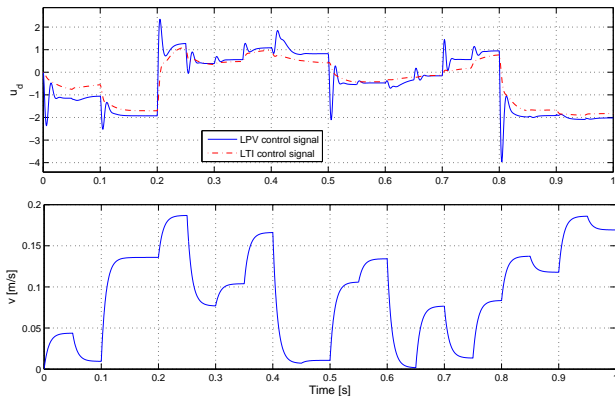
Damper force and tracking error :



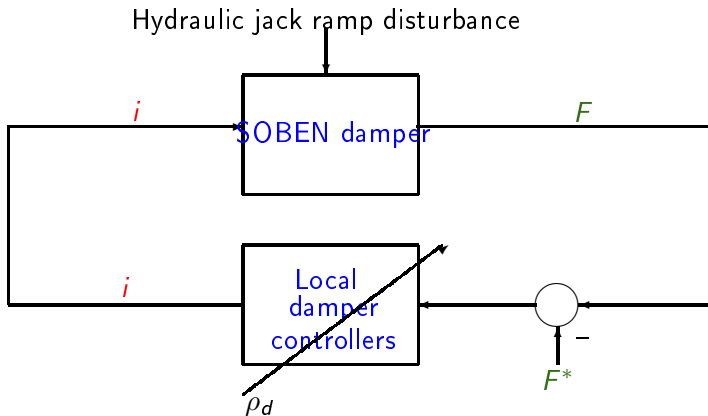
- **LTI controller** :  
 Synthesized with nominal constant  $\rho_{d_i} (\dot{z}_{def_i} = 0.1, z_{def_i} = 0)$
- **LPV controller** :  
 Synthesized with polytopic approach,  $\rho_{d_i}$  scheduled controller

## Simulation results : LTI vs LPV controller

Damper control signal and deflection velocity :



# Simulations





## Simulation results

- Mean relative error :

$$\epsilon_r(F - F^*) = \frac{\sum_{k=1}^n |F(k) - F^*(k)|}{F(k)}$$

- ▶  $F^*$  : Force reference
- ▶  $F$  : Damper force
- ▶  $n$  : Number of samples

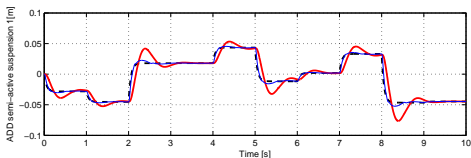
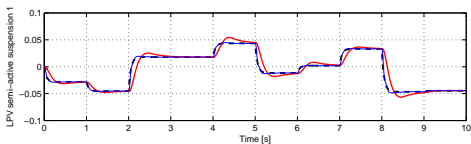
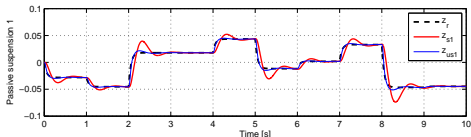
- Results :

Case	MRE*
LTI	11%
LPV	5%
* Mean Relative Error	

# Weighting filters

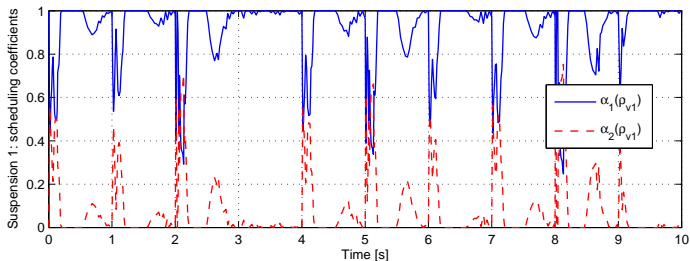
System	Filter (Frequency unit : Hz)
$\ddot{z}_s \mapsto z_1$	$W_{\ddot{z}_s} = G_{\ddot{z}_s} \frac{s}{s+2\pi f_{\ddot{z}_s}}$ $f_{\ddot{z}_s} = 4$ $G_{\ddot{z}_s} = 0.01$
$z_s \mapsto z_2$	$W_{z_s} = G_{z_s} \frac{2\pi f_{z_s}}{s+2\pi f_{z_s}}$ $f_{z_s} = 5$ $G_{z_s} = 2$
$\theta \mapsto z_7$	$W_{\theta} = G_{\theta} \frac{2\pi f_{\theta}}{s+2\pi f_{\theta}}$ $f_{\theta} = 5$ $G_{\theta} = 2$
$\phi \mapsto z_8$	$W_{\phi} = G_{\phi} \frac{2\pi f_{\phi}}{s+2\pi f_{\phi}}$ $f_{\phi} = 5$ $G_{\theta} = 2$
$z_{us_j} \mapsto z_j$ $i \in [1, 4], j \in [3, 6]$	$W_{z_{us}} = G_{z_{us}} \frac{2\pi f_{z_{us}}}{s+2\pi f_{z_{us}}}$ $f_{z_{us}} = 20$ $G_{z_{us}} = 1$
$w_{\infty} \mapsto z_{r_i}$ $i \in [1, 4]$	$W_{z_r}^{-1} = G_{z_r} \frac{2\pi f_{z_r}}{s+2\pi f_{z_r}}$ $f_{z_{r_i}} = 20$ $G_{z_{r_i}} = 1$
$u \mapsto z_j$ $i \in [1, 4], j \in [9, 12]$	$W_{u_i}(\rho v_i) = \rho v_i$

# Simulation results : Sprung and unsprung mass positions



- Sprung mass position :
  - Passive : many oscillations
  - Semi-active : smooth
  - ADD : smooth, less oscillations
- Tire deflection :
  - Passive : large values
  - Semi-active : small values
  - ADD : large values

## Simulation results : scheduling coefficients



- $(\alpha_1, \alpha_2) = (1, 0)$  if  $F^*$  achievable
- $(\alpha_1, \alpha_2) = (0, 1)$  if  $F^*$  not achievable
- $u$  automatically decreased if  $F^*$  outside the damper force range

# Simulation results

