Dynamique hors d'équilibre classique et quantique. Formalisme et applications.

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20 Septembre 2010

Membres du jury

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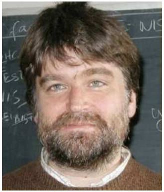

Out-of-equilibrium dynamics

Situations

- Changing a parameter: in the system or the environment (*e.g.* quenching a coupling constant, the temperature, ...)
- Applying a drive: external force or non-equilibrium environment (*e.g.* shear, voltage biais, ...)

Systems of interest

Macroscopic systems exhibiting slow dynamics

- domain growth (*e.g.* ferromagnets, binary liquids, ...)
- disordered interactions
  - weak disorder (*e.g.* random fields)
  - strong disorder (*e.g.* glasses)

General questions

- How does the system relax ?
- What is similar to equilibrium ?
- What are the effects of
  - disorder ?
  - quantum fluctuations ?
- What is universal ?

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What can we do ?

- Equations for the dynamics
  - Classical: stochastic processes (Langevin, Fokker-Planck, ...)
  - Quantum: Schwinger-Keldysh
- Solving the dynamics
  - analytically
    - 1d systems
    - mean-field models
  - numerical simulations for small d
- exact statements
  - fluctuation theorems
  - bounds on entropy creation

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Roadmap

- In and out-of-equilibrium dynamics, symmetry approach
- Out-of-equilibrium classical dynamics after a quench
- Driven out-of-equilibrium quantum dynamics

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Part I

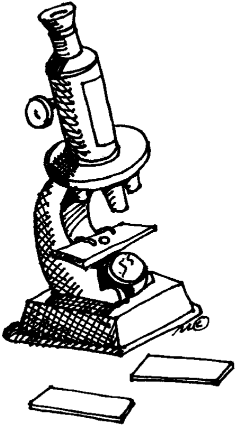
Symmetries of Langevin and Quantum Generating Functionals

C. A., L. F. Cugliandolo, G. Biroli

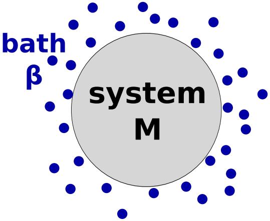
arXiv:1007.5059 (2010)

Robert Brown's experiment (1828)

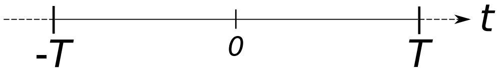
[Video]



Albert Einstein's understanding:



Paul Langevin's equation (1908)



Initial conditions $P_{\mathrm{i}}(\psi, \dot{\psi})$

Langevin equation

$$m\ddot{\psi} = F + F_{\mathrm{bath}\to\mathrm{system}}$$

with the heuristic force

$$F_{\mathrm{bath}\to\mathrm{system}} = -\eta_0\dot{\psi} + \xi$$

Gaussian white noise

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle \propto \delta(t - t')$$

Bath equilibrium condition

$$\langle \xi(t)\xi(t') \rangle = 2\beta^{-1}\eta_0\delta(t - t')$$

Generalized Langevin equation

- Multiplicative noise

$$F_{\text{bath}\to\text{system}} = -\eta_0 M'(\psi)^2\,\dot{\psi} + M'(\psi)\,\xi$$

Generalized Langevin equation

- Multiplicative noise

$$F_{\text{bath}\to\text{system}} = -\eta_0 M'(\psi)^2 \, \dot{\psi} + M'(\psi) \, \xi$$

- Colored noise

$$\langle \xi(t)\xi(t') \rangle = \beta^{-1} \aleph(t-t')$$

ex: Ornstein-Uhlenbeck process: $\aleph(t-t') = \eta_0 \tau^{-1} \mathrm{e}^{-|t-t'|/\tau}$
Bath equilibrium condition

$$F_{\text{bath}\to\text{system}} = -\int_{-T}^{t} \mathrm{d}t' \; \aleph(t-t') \dot{\psi}(t') + \xi(t)$$

Generalized Langevin equation

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- Multiplicative & Colored noise

Classical field theory

Martin-Siggia-Rose-Janssen-deDominicis path-integral formalism

$$\psi_{\rm sol}(t) \Longrightarrow P[\psi(t)] \propto \mathcal{J} \int \mathcal{D}[\hat\psi] \;\; \mathrm{e}^{S[\psi,\hat\psi]}$$

Action

$$S = S^{\rm det} + S^{\rm diss}$$

$$S^{\rm det}[\psi,\hat\psi] \equiv \ln P_{\rm i}\left(\psi(-T),\dot\psi(-T)\right) - \int \mathrm{d}u \; \mathrm{i}\hat\psi(u) \left[m\ddot\psi(u) - F([\psi],u)\right]$$

$$S^{\rm diss}[\psi,\hat\psi] \equiv \eta_0 \int \mathrm{d}u \; \mathrm{i}\hat\psi(u) \left[\beta^{-1}\mathrm{i}\hat\psi(u) - \dot\psi(u)\right]$$

Additive white noise

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$$S^{\rm diss}[\psi,\hat\psi] \equiv \int {\rm d}u \int^u {\rm d}v \; {\rm i}\hat\psi(u) M'(\psi(u)) \aleph(u-v) M'(\psi(v)) \left[\beta^{-1}{\rm i}\hat\psi(v) - \dot\psi(v)\right]$$

Multiplicative & colored noise

Conditions for equilibrium dynamics

- preparation: Gibbs-Boltzmann initial distribution
  $P_{\rm i}(\psi(-T),\dot\psi(-T)) \propto {\rm e}^{-\beta_{\rm i}\mathcal{H}_{\rm i}[\psi(-T)]}$, $\mathcal{H}_{\rm i}[\psi] \equiv \frac{1}{2}m\dot\psi^2 + V_{\rm i}(\psi)$
- evolution: same potential and time-independent forces
  $F = -V_{\rm i}'(\psi)$
- equilibrium bath at temperature $\beta = \beta_{\rm i}$

Action

$$S = S^{\rm det} + S^{\rm diss}$$

$$S^{\rm det} = -\beta\mathcal{H}[\psi(-T)] + \iint {\rm d}u\,{\rm d}v\; {\rm i}\hat\psi(u)\frac{\delta\mathcal{L}[\psi(v)]}{\delta\psi(u)}$$

$$S^{\rm diss} = \int {\rm d}u \int^{u} {\rm d}v\; {\rm i}\hat\psi(u) M'(\psi(u))\aleph(u-v)M'(\psi(v))\left[\beta^{-1}{\rm i}\hat\psi(v) - \dot\psi(v)\right]$$

Supersymmetric representation

$$\mathcal{J} = \det \frac{\delta\xi}{\delta\psi} = \int \mathcal{D}[c,\bar{c}] \;\; \mathrm{e}^{S^{\mathcal{J}}[\psi,c,\bar{c}]}$$

Superfield:

$$\left\{\psi, \hat{\psi}, c, \bar{c}\right\} \mapsto \Psi(t,\theta,\bar{\theta}) \equiv \psi + \bar{\theta}c + \bar{c}\theta + \bar{\theta}\theta \left(\mathrm{i}\hat{\psi} + \bar{c}c\frac{M''(\psi)}{M'(\psi)}\right)$$

Action

$$S = S^{\mathrm{det}} + S^{\mathrm{diss}}$$

$$S^{\mathrm{det}}[\Psi] \equiv -\beta\mathcal{H}[\Psi(-T,0,0)] + \int \mathrm{d}\Upsilon \; \mathcal{L}[\Psi(\Upsilon)]$$

$$S^{\mathrm{diss}}[\Psi] \equiv \frac{1}{2}\iint \mathrm{d}\Upsilon' \, \mathrm{d}\Upsilon \; M(\Psi(\Upsilon'))\, \mathbf{D}^{(2)}(\Upsilon',\Upsilon)\, M(\Psi(\Upsilon))$$

$$\Upsilon \equiv \{t,\theta,\bar{\theta}\}$$

Supersymmetry of the action

S is invariant under

$$\begin{aligned} \Psi &\longmapsto \Psi + \bar{\epsilon}\,\mathbf{Q}\Psi, \quad \mathbf{Q} \equiv \frac{\partial}{\partial\bar{\theta}} \\ \Psi &\longmapsto \Psi + \epsilon\,\bar{\mathbf{Q}}\Psi, \quad \bar{\mathbf{Q}} \equiv \beta^{-1}\frac{\partial}{\partial\theta} + \bar{\theta}\frac{\partial}{\partial t} \end{aligned}$$

Ward identities $\Rightarrow$ equilibrium relations

- stationarity, time-translational invariance (TTI)
- fluctuation-dissipation theorem

Symmetry of the action

S is invariant under

$$\mathcal{T}_{\text{eq}} \equiv \begin{cases} \psi(t) & \longmapsto & \psi(-t) \\ \mathrm{i}\hat{\psi}(t) & \longmapsto & \mathrm{i}\hat{\psi}(-t) + \beta \partial_t \psi(-t) \end{cases}$$

Equilibrium relations

Ward identities

$$\langle A[\psi, \hat{\psi}] \rangle_S = \langle A[\mathcal{T}_{\text{eq}}\psi, \mathcal{T}_{\text{eq}}\hat{\psi}] \rangle_S$$

- stationarity, TTI
- equipartition theorem
- fluctuation-dissipation th.
- Onsager relations
- many more...

Broken symmetry

S is no longer invariant

$$S[\psi,\hat{\psi}] \xrightarrow{\mathcal{T}_{\rm eq}} S_{\rm r}[\psi,\hat{\psi}] + \mathcal{S}$$

Out-of-equilibrium relations

$$\langle A[\psi,\hat{\psi}]\rangle_S = \langle A[\mathcal{T}_{\rm eq}\psi,\mathcal{T}_{\rm eq}\hat{\psi}]\,{\rm e}^{\mathcal{S}}\rangle_{S_{\rm r}}$$

- Kawasaki identity (1967): $\langle {\rm e}^{-\mathcal{S}}\rangle_S = 1$
  Jarzynski equality (1997): ${\rm e}^{\beta\Delta\mathcal{F}}\langle {\rm e}^{-\beta\mathcal{W}}\rangle_S = 1$
- Fluctuation theorem (FT, 1993): $P(\mathcal{S}) = P_{\rm r}(-\mathcal{S})\,{\rm e}^{\mathcal{S}}$
  Crooks FT (1998): $P(\mathcal{W}) = P_{\rm r}(-\mathcal{W}){\rm e}^{\beta(\mathcal{W}-\Delta\mathcal{F})}$
- many more...

Out-of-equilibrium symmetry

Generalized Langevin equation:

$$\underbrace{m\ddot{\psi}(t) - F([\psi],t) + M'(\psi(t))\int^t \mathrm{d}u\, \aleph(t-u) M'(\psi(u))\dot{\psi}(u)}_{\equiv \mathrm{LHS}([\psi],t)} = M'(\psi(t))\xi(t)$$

S is invariant under

$$\mathcal{T}_{\mathrm{eom}} \equiv \begin{cases} \psi(u) & \mapsto \quad \psi(u) \\ \mathrm{i}\hat{\psi}(u) & \mapsto \quad -\mathrm{i}\hat{\psi}(u) + \dfrac{2\beta}{M'(\psi(u))}\displaystyle\int \mathrm{d}v\, \aleph^{-1}(u-v)\, \dfrac{\mathrm{LHS}([\psi],v)}{M'(\psi(v))} \end{cases}$$

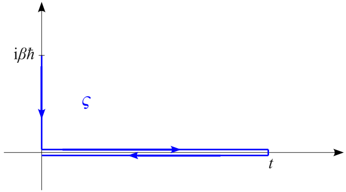
Ward identities ⇒ ‘Schwinger-Dyson’ out-of-equilibrium relations

For additive white noise:

$$m\partial_{t'}^2 C(t,t') + \eta_0 \partial_{t'} C(t,t') - \langle \psi(t) F([\psi],t')\rangle = 2\beta^{-1}\eta_0 R(t,t')$$
$$m\partial_t^2 R(t,t') + \eta_0 \partial_t R(t,t') - \langle \mathrm{i}\hat{\psi}(t') F([\psi],t)\rangle_S = \delta(t-t')$$

Quantum generalization

Schwinger-Keldysh approach



$$\begin{aligned} \langle A(t)\rangle &= \mathcal{Z}^{-1}\mathrm{Tr}\left[\mathrm{e}^{\frac{\mathrm{i}}{\hbar}Ht}A(t)\,\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}Ht}\mathrm{e}^{-\beta H}\right] \\ &\propto \int \mathcal{D}[\phi]\,\mathrm{e}^{\frac{\mathrm{i}}{\hbar}\int_{\varsigma}\mathrm{d}u\,\mathcal{L}[\phi(u)]}\,A[\phi(t)] \end{aligned}$$

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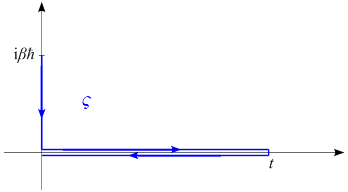

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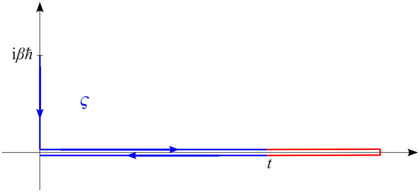

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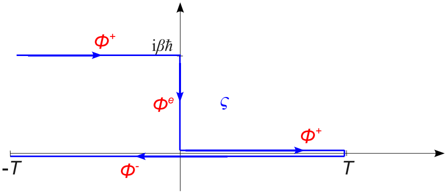
Symmetry



Action

$$S = \int_{-T+\mathrm{i}\beta\hbar}^{\mathrm{i}\beta\hbar} \mathrm{d}u \, \mathcal{L}[\phi^+(u)] + \int_{\mathrm{i}\beta\hbar}^{0} \mathrm{d}u \, \mathcal{L}[\phi^e(u)] + \int_{0}^{T} \mathrm{d}u \, \mathcal{L}[\phi^+(u)] + \int_{T}^{-T} \mathrm{d}u \, \mathcal{L}[\phi^-(u)]$$

S is invariant under

$$\mathcal{T}_{\mathrm{eq}}^{Q} \equiv \begin{cases} \phi^+(u) & \longmapsto & \phi^+(\mathrm{i}\beta\hbar - u) \\ \phi^-(u) & \longmapsto & \phi^-(-u) \\ \phi^e(u) & \longmapsto & \phi^e(\mathrm{i}\beta\hbar - u) \end{cases}$$

Part II

Scalings and Super-Universality in Coarsening versus Glassy Dynamics

C. A., C. Chamon, L. F. Cugliandolo, M. Picco

J. Stat. Mech. (2008) P05016

8 Random Field Ising Model

9 3d Edwards-Anderson Model

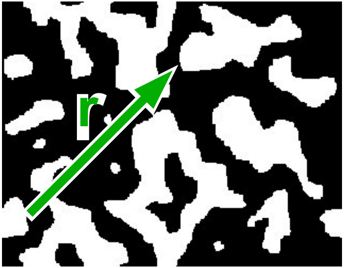
Overview of the 3d Random Field Ising Model

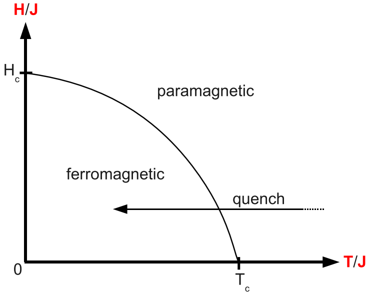
Hamiltonian

$$\mathcal{H} = -\mathbf{J} \sum_{<i,j>} s_i s_j - \sum_i h_i s_i$$

$$h_i = \pm \mathbf{H}$$

Coarsening





Growing length

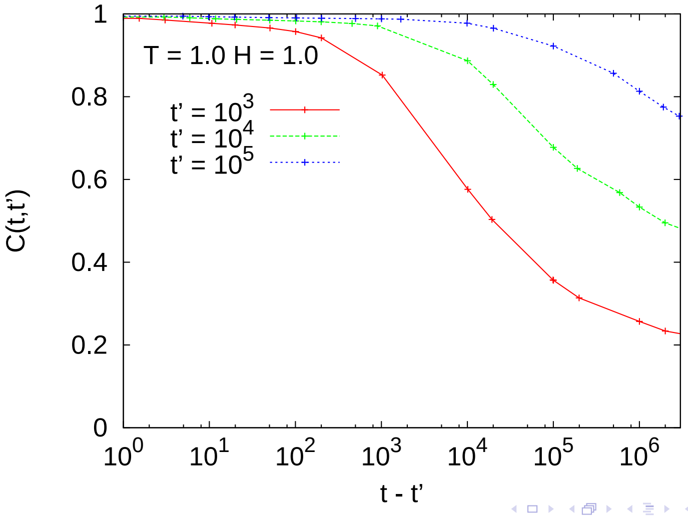
$$C_2(r,t) \equiv \langle s_i(t) s_{i+r}(t) \rangle_i$$

$$\Downarrow$$

We extract $R_{T,H}(t)$

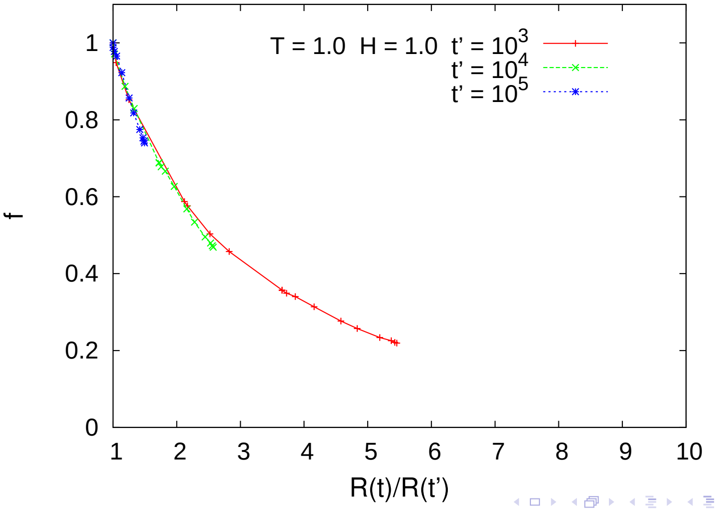
Dynamical scaling

$C_{\mathbf{T},\mathbf{H}}(t,t') \equiv \langle s_i(t) s_i(t') \rangle_i \qquad C^{ag}_{\mathbf{T},\mathbf{H}}(t,t')$



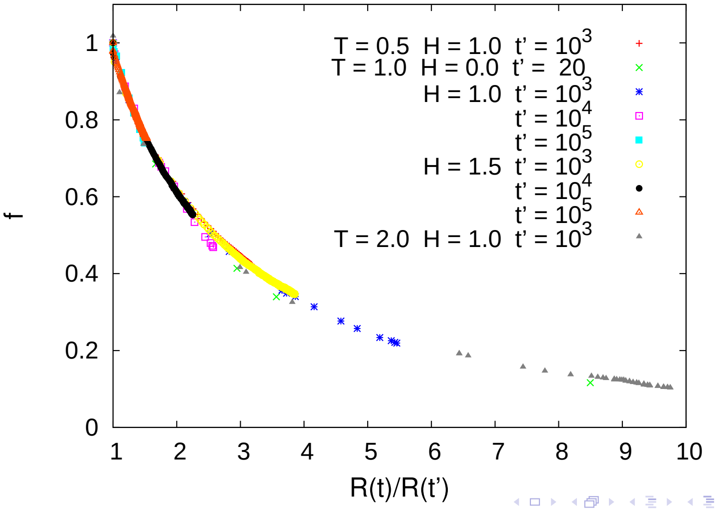
Dynamical scaling

$$C_{\mathbf{T},\mathbf{H}}(t,t') \equiv \langle s_i(t)s_i(t')\rangle_i \qquad C^{ag}_{\mathbf{T},\mathbf{H}}(t,t') = f_{\mathbf{T},\mathbf{H}}\left(\frac{R_{\mathbf{T},\mathbf{H}}(t)}{R_{\mathbf{T},\mathbf{H}}(t')}\right)$$



Dynamical scaling: super-universality

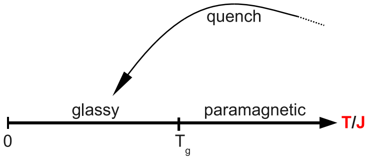
$$C_{\mathbf{T},\mathbf{H}}(t,t') \equiv \langle s_i(t)s_i(t')\rangle_i \qquad C^{ag}_{\mathbf{T},\mathbf{H}}(t,t') = \mathbf{f}\left(\frac{R_{\mathbf{T},\mathbf{H}}(t)}{R_{\mathbf{T},\mathbf{H}}(t')}\right)$$



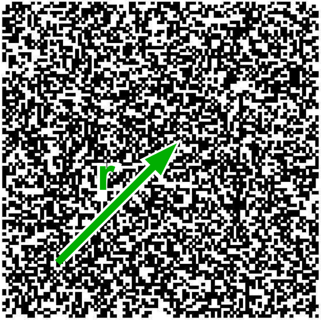
Overview of the 3d Edwards-Anderson model

Hamiltonian

$$\mathcal{H} = -\sum_{<i,j>} \mathbf{J_{ij}} s_i s_j$$
$$\mathbf{J_{ij}} = \pm \mathbf{J}$$



Frustration



Results

Dynamical scalings: yes
Super-universality: no

Part III

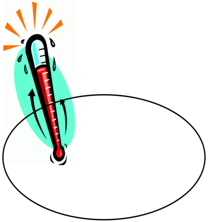
Driven Quantum Coarsening

C. A., G. Biroli, L. F. Cugliandolo

Phys. Rev. Lett. **102**, 050404 (2009)
arXiv:1005.2414 (2010)

Quench

System

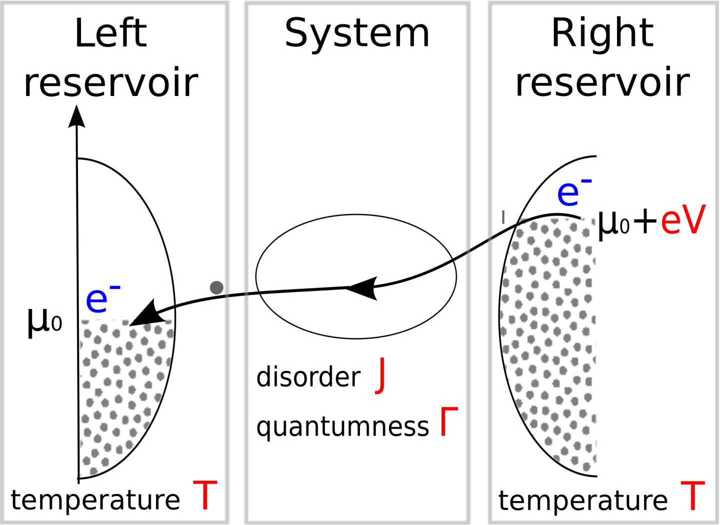


disorder J

quantumness Γ

temperature T_0

Quench


Left reservoir
System
Right reservoir
e-
μ0
e-
μ0+eV
disorder J
quantumness Γ
temperature T
temperature T

System

- N n-component quantum rotors: $\mathbf{n}_i \in \mathbb{R}^n, i = 1 \ldots N$
- unit length: $\mathbf{n}_i^2 = 1$
- mass $\propto 1/\boldsymbol{\Gamma}$
- fully connected via random couplings: $\mathbf{J_{ij}} \leftarrow \text{Gauss}(0, \mathbf{J})$

Hamiltonian

$$\mathcal{H}_S = \frac{\boldsymbol{\Gamma}}{2n} \sum_{i=1}^{N} \mathbf{L}_i^2 - \frac{n}{\sqrt{N}} \sum_{i<j} \mathbf{J_{ij}}\, \mathbf{n}_i \cdot \mathbf{n}_j$$

$\mathbf{L}_i^2 = \sum_{\mu<\nu} (L_i^{\mu\nu})^2$ with $L_i^{\mu\nu} = -i\hbar \left(n_i^\mu \frac{\partial}{\partial n_i^\nu} - n_i^\nu \frac{\partial}{\partial n_i^\mu} \right)$

Reservoirs

- Free fermions 'ψ_L' and 'ψ_R' in equilibrium at temperature **T**
- applied voltage **eV** between 'L' and 'R' reservoirs.

Coupling System/Reservoirs

$$\mathcal{H}_{SB} = -\mathbf{g}\frac{\sqrt{n}}{N_s}\sum_{i=1}^{N}\sum_{k,k'=1}^{N_s}\sum_{l,l'=1}^{M}\mathbf{n}_i \cdot [\psi^\dagger_{Likl}\ \boldsymbol{\sigma}_{ll'}\ \psi_{Rik'l'} + L \leftrightarrow R]$$

$$M^2 - 1 = n$$

Treatment

- Integration over the reservoirs: 2^{nd} order in **g**
- Average over disorder

$$[\ldots]_J \equiv \int \prod_{i<j} \mathrm{d}J_{ij} \; P(J_{ij}) \ldots$$

$$\Downarrow$$

Quartic terms in $\mathbf{n_i}$

- large n limit

Effect of the environment

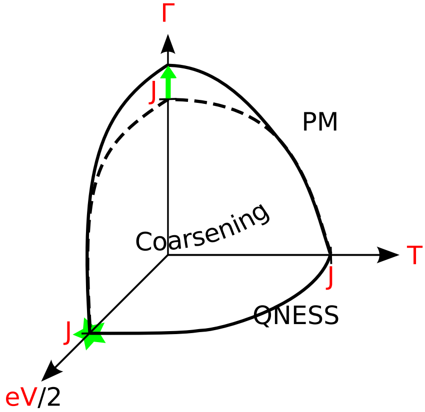
Non-equilibrium environment: $eV \neq 0$

The effect of the reservoirs on the low frequency dynamics is expected to be the one of an equilibrium bath at

$$T^* \equiv \frac{eV}{2} \coth\left(eV/2T\right)$$

- Equilibrium ($eV = 0$): $T^* = T$
- Zero temperature ($T = 0$): $T^* = eV/2$

Critical manifold: with drive ($eV \neq 0$)



New 'drive induced' critical point

$$eV_c/2 \propto J$$

Long-time dynamics

Long-time dynamics described by a classical Langevin equation

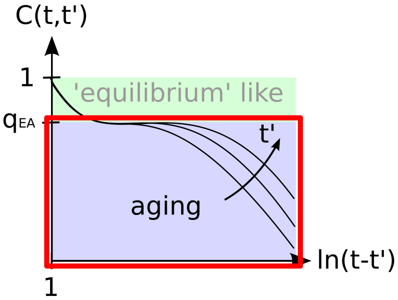
$$\eta_0 \dot{\mathbf{n}} = \ldots + \xi(\mathbf{t})$$

white noise statistics: $\langle \xi(t)\xi(t') \rangle = 2\eta_0 \mathbf{T}^* \delta(t - t')$

temperature $\mathbf{T}^* = \frac{\mathbf{eV}}{2} \coth\left(\mathbf{eV}/2\mathbf{T}\right)$

Long-time dynamics

Classical scenario



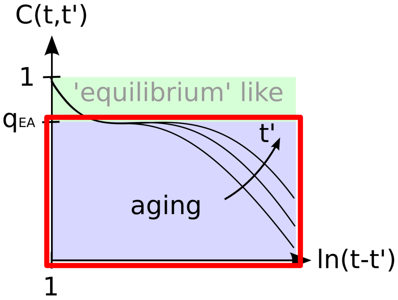
$$C_{\text{aging}}(t,t') \propto \mathbf{f}(t/t') \quad \mathbf{f}(x) = 2\sqrt{2}\frac{x^{3/4}}{(1+x)^{3/2}}$$

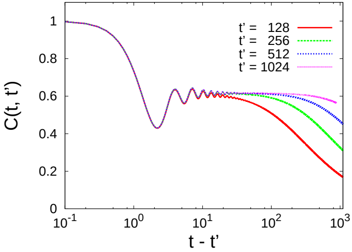
Quantum driven scenario: we expect universal dynamics !

Same t/t' scaling, same $\mathbf{f}(x)$

Long-time dynamics

Classical scenario | Quantum scenario





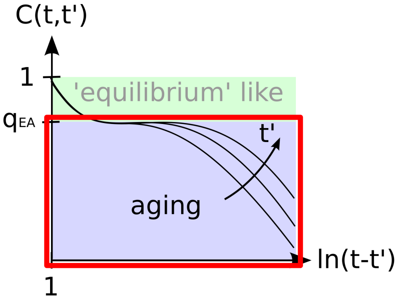
$$C_{\text{aging}}(t,t') \propto \mathbf{f}(t/t') \qquad \mathbf{f}(x) \equiv 2\sqrt{2}\frac{x^{3/4}}{(1+x)^{3/2}}$$

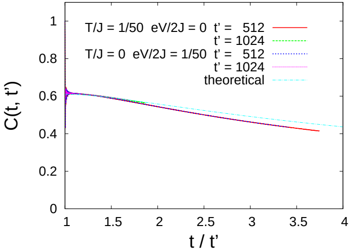
Quantum driven scenario: we expect universal dynamics !

Same t/t' scaling, same $\mathbf{f}(x)$

Long-time dynamics

Classical scenario | Quantum scenario





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Conclusions & Outlook

- - Symmetry associated to equilibrium in the MSRJD formalism
  - another symmetry is also valid out of equilibrium
  - handy formalism to obtain relations for non-Markovian systems and multiplicative noise
  - quantum generalization ?
- check of the super-universality hypothesis in the ordering dynamics of the 3d RFIM Vs 3dEA
- Driven quantum coarsening:
  - dynamical scalings and universality in **J**, **T**, **Γ**, **eV**
  - does this picture survive for non-quadratic models, other couplings ?
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