String theory flux vacua on twisted tori and Generalized Complex Geometry

David ANDRIOT

PhD defence, in presence of

Iosif BENA, Michela PETRINI, Henning SAMTLEBEN, Dimitrios TSIMPIS, Daniel WALDRAM, Jean-Bernard ZUBER

arXiv:0804.1769 by D. A.
arXiv:0903.0633 by D. A., R. Minasian, M. Petrini
arXiv:1003.3774 by D. A., E. Goi, R. Minasian, M. Petrini

01/07/2010, LPTHE, UPMC Univ Paris 6, France

Introduction

- String theory considers extended objects: strings, of typical length l_s.

Introduction

- String theory considers extended objects:
  strings, of typical length l_s.
  String theory is a perturbative theory of quantum gravity.

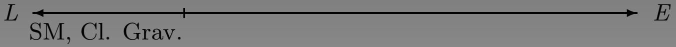
Introduction

- String theory considers extended objects:
  strings, of typical length l_s.
  String theory is a perturbative theory of quantum gravity.
  Only few candidates...

Introduction

- String theory considers extended objects:
  strings, of typical length l_s.
  String theory is a perturbative theory of quantum gravity.
  Only few candidates...

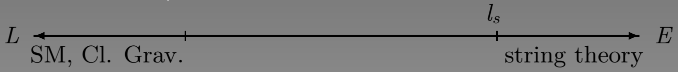
  Quantum gravity effects are expected to play a role at very high energy/tiny length scale



Introduction

- String theory considers extended objects:
  strings, of typical length l_s.
  String theory is a perturbative theory of quantum gravity.
  Only few candidates...

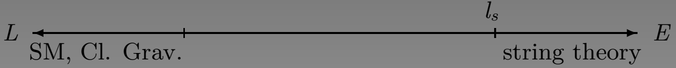
  Quantum gravity effects are expected to play a role at very high energy/tiny length scale



Introduction

- String theory considers extended objects:
  strings, of typical length l_s.
  String theory is a perturbative theory of quantum gravity.
  Only few candidates...

  Quantum gravity effects are expected to play a role at very high energy/tiny length scale



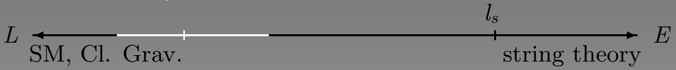
- LHC ≈ giant microscope

Zoom on particles we know, and even tinier things.

Introduction

- String theory considers extended objects:
  strings, of typical length l_s.
  String theory is a perturbative theory of quantum gravity.
  Only few candidates...

  Quantum gravity effects are expected to play a role at very high energy/tiny length scale



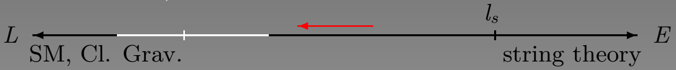
- LHC $\approx$ giant microscope

Zoom on particles we know, and even tinier things.
But quantum gravity effects are still at smaller length.

Introduction

- String theory considers extended objects: strings, of typical length l_s.
  String theory is a perturbative theory of quantum gravity.
  Only few candidates...

  Quantum gravity effects are expected to play a role at very high energy/tiny length scale



- LHC ≈ giant microscope

Zoom on particles we know, and even tinier things.
But quantum gravity effects are still at smaller length.

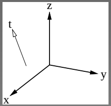
By “zooming-out” from string theory, can we recover particle physics, or predict things to be discovered at the LHC?

- Problem

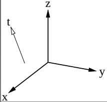
- Problem : superstring theory has several characteristic features not observed

- Problem : superstring theory has several characteristic features not observed
    - Dimensions of space-time: $1 + 9$

- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1 + 9$



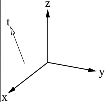
- Problem : superstring theory has several characteristic features not observed
    - Dimensions of space-time: $1 + 9$

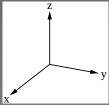


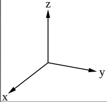
$+$??

$\hookrightarrow$ 6 space dimensions more !

- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1 + 9$

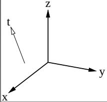


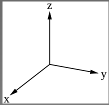


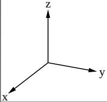


$\hookrightarrow$ 6 space dimensions more !

- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1 + 9$

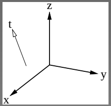


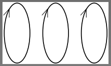


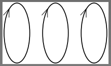


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed.

- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1+9$

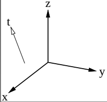


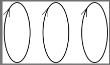


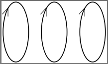


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed.

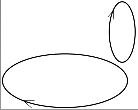
- Problem : superstring theory has several characteristic features not observed
    - Dimensions of space-time: $1+9$



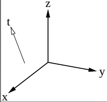


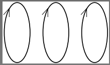


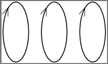
$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed. Several possibilities, a simple example:



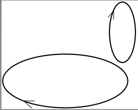
- Problem : superstring theory has several characteristic features not observed
    - Dimensions of space-time: $1+9$





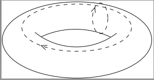


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed. Several possibilities, a simple example:

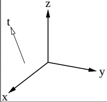


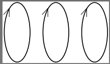
$\Rightarrow$

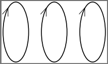
Torus: T^2



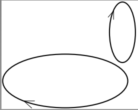
- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1 + 9$





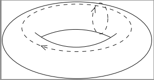


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed. Several possibilities, a simple example:



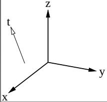
$\Rightarrow$

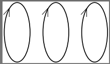
Torus: T^2

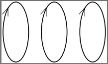


In general: compact six-dimensional space $\mathcal{M}$ (manifold).

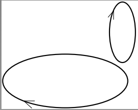
- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1+9$





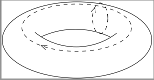


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed. Several possibilities, a simple example:



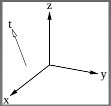
$\Rightarrow$

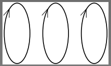
Torus: T^2

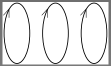


In general: compact six-dimensional space $\mathcal{M}$ (manifold). We don't observe $\mathcal{M}$

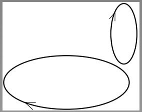
- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1+9$





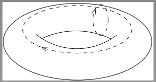


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed.
Several possibilities, a simple example:

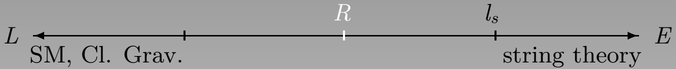


$\Rightarrow$

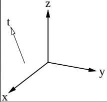
Torus: T^2

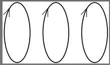


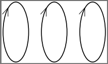
In general: compact six-dimensional space $\mathcal{M}$ (manifold).
We don't observe $\mathcal{M}$ $\Rightarrow$ size too small to be detected:



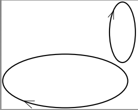
- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1+9$





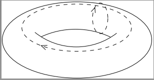


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed. Several possibilities, a simple example:

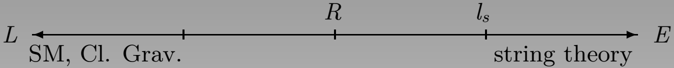


$\Rightarrow$

Torus: T^2

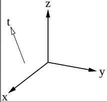


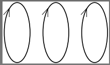
In general: compact six-dimensional space $\mathcal{M}$ (manifold). We don't observe $\mathcal{M}$ $\Rightarrow$ size too small to be detected:

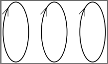


  - Supersymmetry (SUSY)

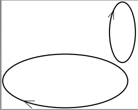
- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1 + 9$





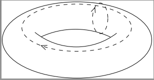


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed. Several possibilities, a simple example:

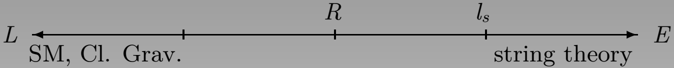


$\Rightarrow$

Torus: T^2

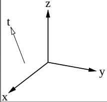


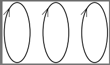
In general: compact six-dimensional space $\mathcal{M}$ (manifold). We don't observe $\mathcal{M} \Rightarrow$ size too small to be detected:

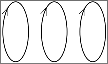


  - Supersymmetry (SUSY): exchanges fermions/bosons.

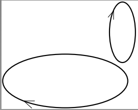
- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1 + 9$





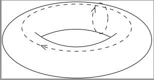


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed. Several possibilities, a simple example:

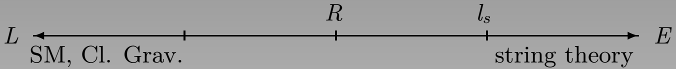


$\Rightarrow$

Torus: T^2

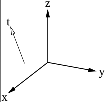


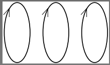
In general: compact six-dimensional space $\mathcal{M}$ (manifold). We don't observe $\mathcal{M}$ $\Rightarrow$ size too small to be detected:

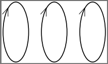


  - Supersymmetry (SUSY): exchanges fermions/bosons. Not observed

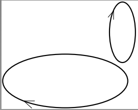
- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1+9$





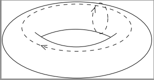


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed. Several possibilities, a simple example:

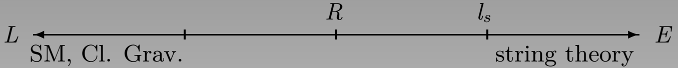


$\Rightarrow$

Torus: T^2



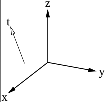
In general: compact six-dimensional space $\mathcal{M}$ (manifold). We don't observe $\mathcal{M}$ $\Rightarrow$ size too small to be detected:

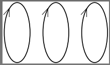


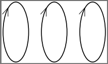
  - Supersymmetry (SUSY): exchanges fermions/bosons. Not observed $\Rightarrow$ SUSY broken at a length l_{SUSY}.

- Problem : superstring theory has several characteristic features not observed

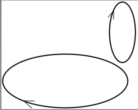
  - Dimensions of space-time: $1 + 9$





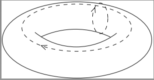


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed.
Several possibilities, a simple example:



$\Rightarrow$

Torus: T^2



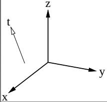
In general: compact six-dimensional space $\mathcal{M}$ (manifold).
We don't observe $\mathcal{M}$ $\Rightarrow$ size too small to be detected:

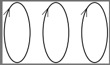
$L \longleftarrow l_{SUSY} \;—\; R \;—\; l_s \longrightarrow E$

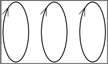
SM, Cl. Grav. ... string theory

  - Supersymmetry (SUSY): exchanges fermions/bosons.
Not observed $\Rightarrow$ SUSY broken at a length l_{SUSY}.
Hypothesis: $l_{SUSY} > R$

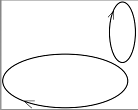
- Problem : superstring theory has several characteristic features not observed
  - Dimensions of space-time: $1 + 9$





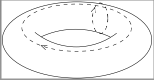


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed. Several possibilities, a simple example:

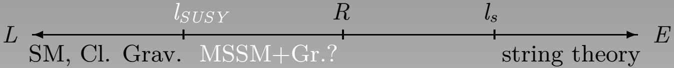


$\Rightarrow$

Torus: T^2



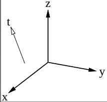
In general: compact six-dimensional space $\mathcal{M}$ (manifold). We don't observe $\mathcal{M} \Rightarrow$ size too small to be detected:

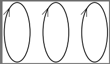


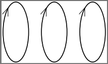
  - Supersymmetry (SUSY): exchanges fermions/bosons. Not observed $\Rightarrow$ SUSY broken at a length l_{SUSY}. Hypothesis: $l_{SUSY} > R$ ($4d$ extensions of SM, Higgs hierarchy problem, GUT...). Discovered at the LHC?

- Problem : superstring theory has several characteristic features not observed

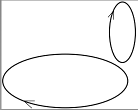
  - Dimensions of space-time: $1+9$





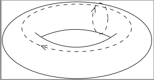


$\hookrightarrow$ 6 space dimensions more ! Not extended, but closed.
Several possibilities, a simple example:



$\Rightarrow$

Torus: T^2



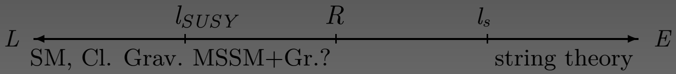
In general: compact six-dimensional space $\mathcal{M}$ (manifold).
We don't observe $\mathcal{M} \Rightarrow$ size too small to be detected:

L ← l_{SUSY} — R — l_s → E

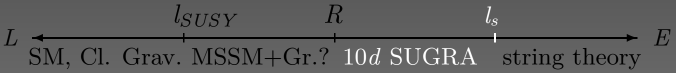
SM, Cl. Grav. MSSM+Gr.? ... string theory

  - Supersymmetry (SUSY): exchanges fermions/bosons.
    Not observed $\Rightarrow$ SUSY broken at a length l_{SUSY}.
    Hypothesis: $l_{SUSY} > R$ ($4d$ extensions of SM, Higgs hierarchy problem, GUT...). Discovered at the LHC?
    Technical simplification.

"Zooming-out"

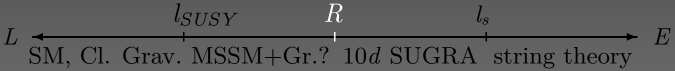


“Zooming-out”



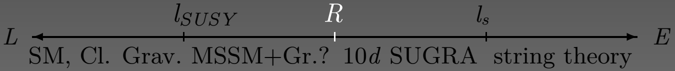
- “Zoom-out” from superstring theory: below l_s, consider its low energy effective theory: supergravity (SUGRA). SUGRA: $10d$ supersymmetric theory, including gravity.

"Zooming-out"



- "Zoom-out" from superstring theory: below l_s, consider its low energy effective theory: supergravity (SUGRA). SUGRA: 10d supersymmetric theory, including gravity.
- To lower further the energy: $10d \rightarrow 4d$. Go through R...

“Zooming-out”



- “Zoom-out” from superstring theory: below l_s, consider its low energy effective theory: supergravity (SUGRA). SUGRA: $10d$ supersymmetric theory, including gravity.
- To lower further the energy: $10d \to 4d$. Go through R...

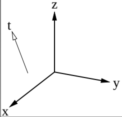
Technical procedure: Kaluza-Klein (KK) reduction:

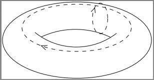
"Zooming-out"

L ← l_{SUSY} — R — l_s → E

SM, Cl. Grav. MSSM+Gr.? 10d SUGRA string theory

- "Zoom-out" from superstring theory: below l_s, consider its low energy effective theory: supergravity (SUGRA). SUGRA: 10d supersymmetric theory, including gravity.
- To lower further the energy: $10d \rightarrow 4d$. Go through R...





Technical procedure: Kaluza-Klein (KK) reduction:

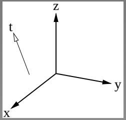
1. find a background (a solution to 10d equations of motion, a vacuum of SUGRA)

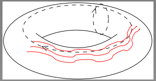
"Zooming-out"

L ← l_{SUSY} — R — l_s → E

SM, Cl. Grav. | MSSM+Gr.? | 10d SUGRA | string theory

- "Zoom-out" from superstring theory: below l_s, consider its low energy effective theory: supergravity (SUGRA). SUGRA: 10d supersymmetric theory, including gravity.
- To lower further the energy: $10d \rightarrow 4d$. Go through R...





Technical procedure: Kaluza-Klein (KK) reduction:

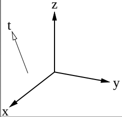
1. find a background (a solution to 10d equations of motion, a vacuum of SUGRA)
2. consider small fluctuations around it (determine light modes, truncate the spectrum to them)

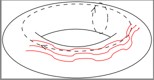
"Zooming-out"

L ← l_{SUSY} — R — l_s → E

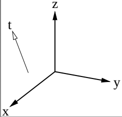
SM, Cl. Grav. | MSSM+Gr.? | 10d SUGRA | string theory

- "Zoom-out" from superstring theory: below l_s, consider its low energy effective theory: supergravity (SUGRA).
  SUGRA: 10d supersymmetric theory, including gravity.
- To lower further the energy: $10d \rightarrow 4d$. Go through R...





$\Rightarrow$



Technical procedure: Kaluza-Klein (KK) reduction:

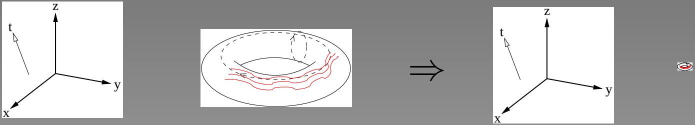
1. find a background (a solution to 10d equations of motion, a vacuum of SUGRA)
2. consider small fluctuations around it (determine light modes, truncate the spectrum to them)
3. "zoom-out" (integrate the lagrangian over 6d dim.)
   $\hookrightarrow$ get a 4d theory.

"Zooming-out"

L ← l_{SUSY} — R — l_s → E

SM, Cl. Grav. | MSSM+Gr.? | $10d$ SUGRA | string theory

- "Zoom-out" from superstring theory: below l_s, consider its low energy effective theory: supergravity (SUGRA). SUGRA: $10d$ supersymmetric theory, including gravity.
- To lower further the energy: $10d \to 4d$. Go through R...



Technical procedure: Kaluza-Klein (KK) reduction:

1. find a background (a solution to $10d$ equations of motion, a vacuum of SUGRA)
2. consider small fluctuations around it (determine light modes, truncate the spectrum to them)
3. "zoom-out" (integrate the lagrangian over $6d$ dim.) $\hookrightarrow$ get a $4d$ theory.

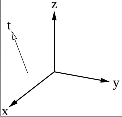
- In this thesis: only the 1$^{\text{st}}$ step:

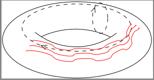
"Zooming-out"

L ←— l_{SUSY} — R — l_s —→ E

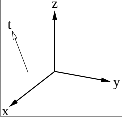
SM, Cl. Grav. | MSSM+Gr.? | $10d$ SUGRA | string theory

- "Zoom-out" from superstring theory: below l_s, consider its low energy effective theory: supergravity (SUGRA).
  SUGRA: $10d$ supersymmetric theory, including gravity.
- To lower further the energy: $10d \to 4d$. Go through R...





$\Rightarrow$



  Technical procedure: Kaluza-Klein (KK) reduction:
  1. find a background (a solution to $10d$ equations of motion, a vacuum of SUGRA)
  2. consider small fluctuations around it (determine light modes, truncate the spectrum to them)
  3. "zoom-out" (integrate the lagrangian over $6d$ dim.)
     $\hookrightarrow$ get a $4d$ theory.
- In this thesis: only the 1st step:

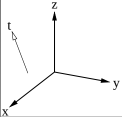
Find $10d$ solutions of SUGRA

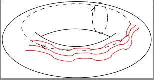
"Zooming-out"

L ← l_{SUSY} — R — l_s → E

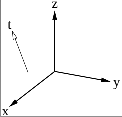
SM, Cl. Grav. | MSSM+Gr.? | $10d$ SUGRA | string theory

- "Zoom-out" from superstring theory: below l_s, consider its low energy effective theory: supergravity (SUGRA). SUGRA: $10d$ supersymmetric theory, including gravity.
- To lower further the energy: $10d \to 4d$. Go through R...





$\Rightarrow$



Technical procedure: Kaluza-Klein (KK) reduction:

1. find a background (a solution to $10d$ equations of motion, a vacuum of SUGRA)
2. consider small fluctuations around it (determine light modes, truncate the spectrum to them)
3. "zoom-out" (integrate the lagrangian over $6d$ dim.) $\hookrightarrow$ get a $4d$ theory.

- In this thesis: only the 1$^{\text{st}}$ step:

Find $10d$ solutions of SUGRA on $4d + 6d$, and preserve some SUSY...

$10d$ solutions of SUGRA

Type IIA/B SUGRA: $\mathcal{N}_{10d} = 2$ SUSY

$10d$ solutions of SUGRA

Type IIA/B SUGRA: $\mathcal{N}_{10d} = 2$ SUSY
Spectrum:

- bosons of NSNS sector: $g_{MN}, \phi, B_{(2)},\ M = 0 \dots 9$, $H = \mathrm{d}B$ flux
- bosons of RR sector: C_{p-1}, p even/odd in IIA/B, $F_p \sim \mathrm{d}C_{p-1}$ flux
- fermions: doublets $\psi_M^{1,2}, \tilde{\lambda}^{1,2}$

$10d$ solutions of SUGRA

Type IIA/B SUGRA: $\mathcal{N}_{10d} = 2$ SUSY
Spectrum:

- bosons of NSNS sector: $g_{MN}, \phi, B_{(2)},\ M = 0 \dots 9$, $H = \mathrm{d}B$ flux
- bosons of RR sector: C_{p-1}, p even/odd in IIA/B, $F_p \sim \mathrm{d}C_{p-1}$ flux
- fermions: doublets $\psi_M^{1,2}, \tilde{\lambda}^{1,2}$

$10d$ solutions on a max. symmetric $4d$ space-time $\otimes\ \mathcal{M}$
$\hookrightarrow$ ansatz for the fields, preserve the $4d$ max. symmetry

$10d$ solutions of SUGRA

Type IIA/B SUGRA: $\mathcal{N}_{10d} = 2$ SUSY
Spectrum:

- bosons of NSNS sector: $g_{MN}, \phi, B_{(2)},\ M = 0 \dots 9$, $H = \mathrm{d}B$ flux
- bosons of RR sector: C_{p-1}, p even/odd in IIA/B, $F_p \sim \mathrm{d}C_{p-1}$ flux
- fermions: doublets $\psi_M^{1,2}, \tilde{\lambda}^{1,2}$

$10d$ solutions on a max. symmetric $4d$ space-time $\otimes\ \mathcal{M}$
$\hookrightarrow$ ansatz for the fields, preserve the $4d$ max. symmetry

- Metric Ansatz : $ds^2_{(10)} = e^{2A(y)}\ g_{\mu\nu}\, dx^\mu\, dx^\nu + g_{mn}(y)\, dy^m\, dy^n$

$10d$ solutions of SUGRA

Type IIA/B SUGRA: $\mathcal{N}_{10d} = 2$ SUSY
Spectrum:

- bosons of NSNS sector: $g_{MN}, \phi, B_{(2)},\ M = 0 \dots 9$, $H = \mathrm{d}B$ flux
- bosons of RR sector: C_{p-1}, p even/odd in IIA/B, $F_p \sim \mathrm{d}C_{p-1}$ flux
- fermions: doublets $\psi_M^{1,2}, \tilde{\lambda}^{1,2}$

$10d$ solutions on a max. symmetric $4d$ space-time $\otimes\ \mathcal{M}$
$\hookrightarrow$ ansatz for the fields, preserve the $4d$ max. symmetry

- Metric Ansatz : $ds^2_{(10)} = e^{2A(y)}\ g_{\mu\nu}\, dx^\mu\, dx^\nu + g_{mn}(y)\, dy^m\, dy^n$
- Fluxes: non-trivial only on $\mathcal{M}$.

$10d$ solutions of SUGRA

Type IIA/B SUGRA: $\mathcal{N}_{10d} = 2$ SUSY
Spectrum:

- bosons of NSNS sector: $g_{MN}, \phi, B_{(2)},\ M = 0 \dots 9$, $H = \mathrm{d}B$ flux
- bosons of RR sector: C_{p-1}, p even/odd in IIA/B, $F_p \sim \mathrm{d}C_{p-1}$ flux
- fermions: doublets $\psi_M^{1,2}, \tilde{\lambda}^{1,2}$

$10d$ solutions on a max. symmetric $4d$ space-time $\otimes\ \mathcal{M}$
$\hookrightarrow$ ansatz for the fields, preserve the $4d$ max. symmetry

- Metric Ansatz : $ds^2_{(10)} = e^{2A(y)}\ g_{\mu\nu}\, dx^\mu\, dx^\nu + g_{mn}(y)\, dy^m\, dy^n$
- Fluxes: non-trivial only on $\mathcal{M}$.
- No fermions in the vacuum.

$10d$ solutions of SUGRA

Type IIA/B SUGRA: $\mathcal{N}_{10d} = 2$ SUSY
Spectrum:

- bosons of NSNS sector: $g_{MN}, \phi, B_{(2)}, \ M = 0 \dots 9,$ $H = \mathrm{d}B$ flux
- bosons of RR sector: C_{p-1}, p even/odd in IIA/B, $F_p \sim \mathrm{d}C_{p-1}$ flux
- fermions: doublets $\psi_M^{1,2}, \tilde{\lambda}^{1,2}$

$10d$ solutions on a max. symmetric $4d$ space-time $\otimes\ \mathcal{M}$
$\hookrightarrow$ ansatz for the fields, preserve the $4d$ max. symmetry

- Metric Ansatz : $ds^2_{(10)} = e^{2A(y)}\ g_{\mu\nu}\, dx^\mu\, dx^\nu + g_{mn}(y)\, dy^m\, dy^n$
- Fluxes: non-trivial only on $\mathcal{M}$.
- No fermions in the vacuum.

To get a solution:
- solve equations of motion: Einstein, ϕ, fluxes (F, H).
- solve the Bianchi identities (BI) of the fluxes (sources...).

Supersymmetric solutions and choice of $\mathcal{M}$

Preserve SUSY in the vacuum$\Rightarrow$ SUSY conditions.

Supersymmetric solutions and choice of $\mathcal{M}$

Preserve SUSY in the vacuum$\Rightarrow$ SUSY conditions.
For $4d$ Minkowski space-time, one can show:

SUSY conditions + BI $\Rightarrow$ e.o.m.

Supersymmetric solutions and choice of $\mathcal{M}$

Preserve SUSY in the vacuum$\Rightarrow$ SUSY conditions.
For $4d$ Minkowski space-time, one can show:

SUSY conditions + BI $\Rightarrow$ e.o.m.

Simpler because 1^{st} order equations instead of 2^{nd} order.

Supersymmetric solutions and choice of $\mathcal{M}$

Preserve SUSY in the vacuum$\Rightarrow$ SUSY conditions.
For $4d$ Minkowski space-time, one can show:

SUSY conditions + BI $\Rightarrow$ e.o.m.

Simpler because 1^{st} order equations instead of 2^{nd} order.

SUSY conditions

Supersymmetric solutions and choice of $\mathcal{M}$

Preserve SUSY in the vacuum$\Rightarrow$ SUSY conditions.
For $4d$ Minkowski space-time, one can show:

SUSY conditions + BI $\Rightarrow$ e.o.m.

Simpler because 1$^{\text{st}}$ order equations instead of 2$^{\text{nd}}$ order.

SUSY conditions:
annihilation of the fermionic SUSY variations:

$$0 = \delta\psi_M \quad , \quad 0 = \delta\tilde{\lambda}$$

Supersymmetric solutions and choice of $\mathcal{M}$

Preserve SUSY in the vacuum$\Rightarrow$ SUSY conditions.
For $4d$ Minkowski space-time, one can show:

SUSY conditions + BI $\Rightarrow$ e.o.m.

Simpler because 1^{st} order equations instead of 2^{nd} order.

SUSY conditions:
annihilation of the fermionic SUSY variations:

$$0 = \delta\psi_M \quad , \quad 0 = \delta\tilde{\lambda}$$

$\hookrightarrow$ constraints on $\mathcal{M}$

Supersymmetric solutions and choice of $\mathcal{M}$

Preserve SUSY in the vacuum $\Rightarrow$ SUSY conditions.
For $4d$ Minkowski space-time, one can show:

SUSY conditions + BI $\Rightarrow$ e.o.m.

Simpler because 1$^{\text{st}}$ order equations instead of 2$^{\text{nd}}$ order.

SUSY conditions:
annihilation of the fermionic SUSY variations:

$$0 = \delta\psi_M \quad , \quad 0 = \delta\tilde{\lambda}$$

$\hookrightarrow$ constraints on $\mathcal{M}$, depends if fluxes are present.

Supersymmetric solutions and choice of $\mathcal{M}$

Preserve SUSY in the vacuum$\Rightarrow$ SUSY conditions.
For $4d$ Minkowski space-time, one can show:

SUSY conditions + BI $\Rightarrow$ e.o.m.

Simpler because 1$^{\text{st}}$ order equations instead of 2$^{\text{nd}}$ order.

SUSY conditions:
annihilation of the fermionic SUSY variations:

$$0 = \delta\psi_M \quad , \quad 0 = \delta\tilde{\lambda}$$

$\hookrightarrow$ constraints on $\mathcal{M}$, depends if fluxes are present.

$10d$ SUSY solutions on $4d$ Mink. + $6d$ $\mathcal{M}$

Supersymmetric solutions and choice of $\mathcal{M}$

Preserve SUSY in the vacuum$\Rightarrow$ SUSY conditions.
For $4d$ Minkowski space-time, one can show:

SUSY conditions + BI $\Rightarrow$ e.o.m.

Simpler because 1$^{\text{st}}$ order equations instead of 2$^{\text{nd}}$ order.

SUSY conditions:
annihilation of the fermionic SUSY variations:

$$0 = \delta\psi_M \quad , \quad 0 = \delta\tilde{\lambda}$$

$\hookrightarrow$ constraints on $\mathcal{M}$, depends if fluxes are present.

$10d$ SUSY solutions on $4d$ Mink. + $6d$ $\mathcal{M}$
Without fluxes $\Rightarrow$ $\mathcal{M}$: CY.

Supersymmetric solutions and choice of $\mathcal{M}$

Preserve SUSY in the vacuum$\Rightarrow$ SUSY conditions.
For $4d$ Minkowski space-time, one can show:

SUSY conditions + BI $\Rightarrow$ e.o.m.

Simpler because 1$^{\text{st}}$ order equations instead of 2$^{\text{nd}}$ order.

SUSY conditions:
annihilation of the fermionic SUSY variations:

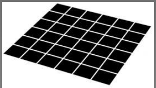
$$0 = \delta\psi_M \quad , \quad 0 = \delta\tilde{\lambda}$$

$\hookrightarrow$ constraints on $\mathcal{M}$, depends if fluxes are present.

$10d$ SUSY solutions on $4d$ Mink. + $6d$ $\mathcal{M}$
Without fluxes $\Rightarrow$ $\mathcal{M}$: CY.
With fluxes $\Rightarrow$ $\mathcal{M}$: GCY.

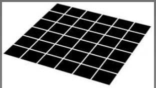
Choice of the internal manifold $\mathcal{M}$

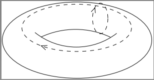
CY is Ricci flat.



Choice of the internal manifold $\mathcal{M}$

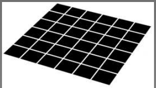
CY is Ricci flat.

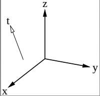


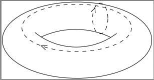


Choice of the internal manifold $\mathcal{M}$

CY is Ricci flat.
In absence of fluxes,
$\mathbb{R}^{3,1} \times T^6$ is a simple solution.

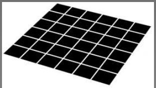


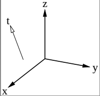


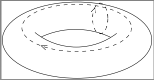


Choice of the internal manifold $\mathcal{M}$

CY is Ricci flat.
In absence of fluxes,
$\mathbb{R}^{3,1} \times T^6$ is a simple
solution.



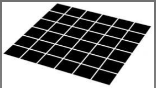


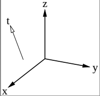


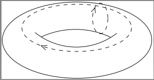
In presence of fluxes

Choice of the internal manifold $\mathcal{M}$

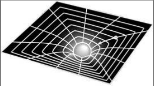
CY is Ricci flat.
In absence of fluxes, $\mathbb{R}^{3,1} \times T^6$ is a simple solution.





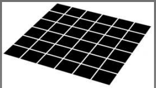


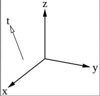
In presence of fluxes: they backreact on $\mathcal{M}$
$\hookrightarrow$ a priori $\mathcal{M}$ not flat anymore... GCY!

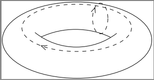


Choice of the internal manifold $\mathcal{M}$

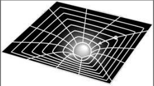
CY is Ricci flat.
In absence of fluxes, $\mathbb{R}^{3,1} \times T^6$ is a simple solution.

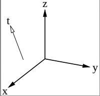


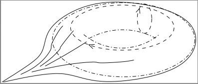




In presence of fluxes: they backreact on $\mathcal{M}$
$\hookrightarrow$ a priori $\mathcal{M}$ not flat anymore... GCY!



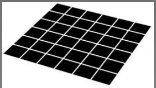


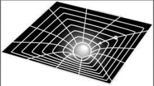


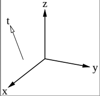
Choice of the internal manifold $\mathcal{M}$

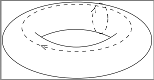
CY is Ricci flat.
In absence of fluxes, $\mathbb{R}^{3,1} \times T^6$ is a simple solution.

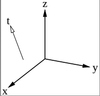
In presence of fluxes: they backreact on $\mathcal{M}$
$\hookrightarrow$ a priori $\mathcal{M}$ not flat anymore... GCY!

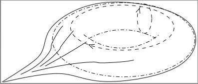










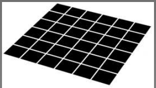


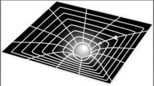
The backreaction can be more dramatic: change of topology...

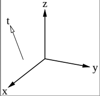
Choice of the internal manifold $\mathcal{M}$

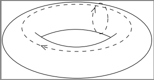
CY is Ricci flat.
In absence of fluxes, $\mathbb{R}^{3,1} \times T^6$ is a simple solution.

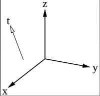
In presence of fluxes: they backreact on $\mathcal{M}$
$\hookrightarrow$ a priori $\mathcal{M}$ not flat anymore... GCY!

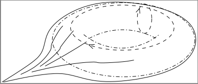




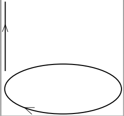








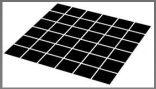
The backreaction can be more dramatic: change of topology...
Example: a twist

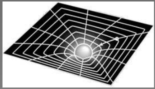


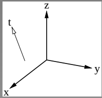
Choice of the internal manifold $\mathcal{M}$

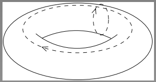
CY is Ricci flat.
In absence of fluxes,
$\mathbb{R}^{3,1} \times T^6$ is a simple solution.

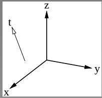
In presence of fluxes: they backreact on $\mathcal{M}$
$\hookrightarrow$ a priori $\mathcal{M}$ not flat anymore... GCY!

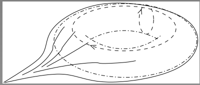




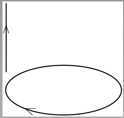








The backreaction can be more dramatic: change of topology...
Example: a twist

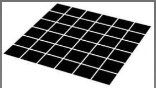


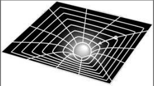
$\Rightarrow$

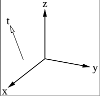
Choice of the internal manifold $\mathcal{M}$

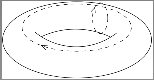
CY is Ricci flat.
In absence of fluxes, $\mathbb{R}^{3,1} \times T^6$ is a simple solution.

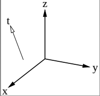
In presence of fluxes: they backreact on $\mathcal{M}$
$\hookrightarrow$ a priori $\mathcal{M}$ not flat anymore... GCY!

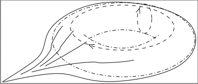




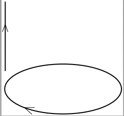




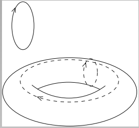




The backreaction can be more dramatic: change of topology...
Example: a twist



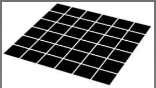
$\Rightarrow$

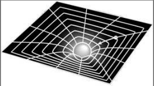


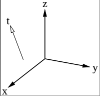
Choice of the internal manifold $\mathcal{M}$

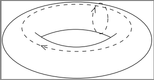
CY is Ricci flat.
In absence of fluxes,
$\mathbb{R}^{3,1} \times T^6$ is a simple
solution.

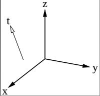
In presence of fluxes: they
backreact on $\mathcal{M}$
$\hookrightarrow$ a priori $\mathcal{M}$ not flat
anymore... GCY!

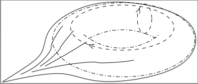




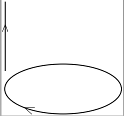




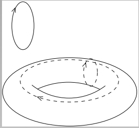




The backreaction can be more dramatic: change of topology...
Example: a twist



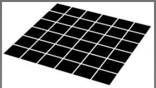
$\Rightarrow$

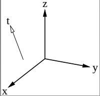


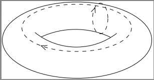
$\Rightarrow$ Twisted torus

Choice of the internal manifold $\mathcal{M}$

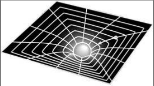
CY is Ricci flat.
In absence of fluxes,
$\mathbb{R}^{3,1} \times T^6$ is a simple solution.

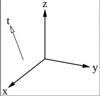


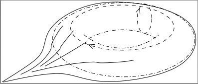




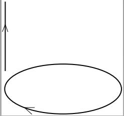
In presence of fluxes: they backreact on $\mathcal{M}$
$\hookrightarrow$ a priori $\mathcal{M}$ not flat anymore... GCY!



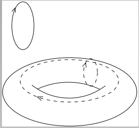




The backreaction can be more dramatic: change of topology...
Example: a twist



$\Rightarrow$



$\Rightarrow$

Twisted torus
Some are proved to be GCY.

Fluxes $\neq 0$: they are necessary, for the moduli problem.

Fluxes $\neq 0$: they are necessary, for the moduli problem.
KK reduction on a CY
$\Rightarrow$ moduli: massless scalar fields... Not observed!

Fluxes $\neq 0$: they are necessary, for the moduli problem.
KK reduction on a CY
$\Rightarrow$ moduli: massless scalar fields... Not observed!
Need a potential to fix them (give them a mass).

Fluxes $\neq 0$: they are necessary, for the moduli problem.
KK reduction on a CY
$\Rightarrow$ moduli: massless scalar fields... Not observed!
Need a potential to fix them (give them a mass).
Fluxes generate a potential...

Fluxes $\neq 0$: they are necessary, for the moduli problem.
KK reduction on a CY
$\Rightarrow$ moduli: massless scalar fields... Not observed!
Need a potential to fix them (give them a mass).
Fluxes generate a potential...

SUSY solutions of $10d$ SUGRA

Fluxes $\neq 0$: they are necessary, for the moduli problem.
KK reduction on a CY
$\Rightarrow$ moduli: massless scalar fields... Not observed!
Need a potential to fix them (give them a mass).
Fluxes generate a potential...

SUSY solutions of $10d$ SUGRA
with fluxes, on $\mathcal{M}$ = GCY

Fluxes $\neq 0$: they are necessary, for the moduli problem.
KK reduction on a CY
$\Rightarrow$ moduli: massless scalar fields... Not observed!
Need a potential to fix them (give them a mass).
Fluxes generate a potential...

SUSY solutions of $10d$ SUGRA with fluxes, on $\mathcal{M} =$ GCY

- SUSY conditions.
  SUSY solutions on twisted tori.

Fluxes $\neq 0$: they are necessary, for the moduli problem.
KK reduction on a CY
$\Rightarrow$ moduli: massless scalar fields... Not observed!
Need a potential to fix them (give them a mass).
Fluxes generate a potential...

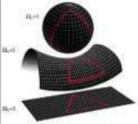
SUSY solutions of 10d SUGRA with fluxes, on $\mathcal{M}$ = GCY

- SUSY conditions.
  SUSY solutions on twisted tori.
- Twisted tori as solvmanifolds.
  Twist transformation
  $\hookrightarrow$ can relate and generate solutions.
  Generalized Complex Geometry (GCG).

Fluxes $\neq 0$: they are necessary, for the moduli problem.
KK reduction on a CY
$\Rightarrow$ moduli: massless scalar fields... Not observed!
Need a potential to fix them (give them a mass).
Fluxes generate a potential...

SUSY solutions of $10d$ SUGRA with fluxes, on $\mathcal{M} = \text{GCY}$

- SUSY conditions.
  SUSY solutions on twisted tori.
- Twisted tori as solvmanifolds.
  Twist transformation
  $\hookrightarrow$ can relate and generate solutions.
  Generalized Complex Geometry (GCG).
- De Sitter solutions: cosmology.



SUSY solutions of IIA/B SUGRA

The SUSY conditions

SUSY solutions of IIA/B SUGRA

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions:

SUSY solutions of IIA/B SUGRA

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions:

$$0 = \delta\psi_M$$

$$0 = \delta\tilde{\lambda}$$

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions:

$$0 = \delta\psi_M = D_M\epsilon + \frac{1}{4}H_M\mathcal{P}\epsilon + \frac{1}{16}e^{\phi}\sum_n \not{F}_{2n}\gamma_M\mathcal{P}_n\epsilon$$

$$0 = \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2}\not{H}\mathcal{P}\right)\epsilon + \frac{1}{8}e^{\phi}\sum_n(-1)^{2n}(5-2n)\not{F}_{2n}\mathcal{P}_n\epsilon$$

$10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions:

$$0 = \delta\psi_M = D_M\epsilon + \frac{1}{4}H_M\mathcal{P}\epsilon + \frac{1}{16}e^{\phi}\sum_n \not{F}_{2n}\gamma_M\mathcal{P}_n\epsilon$$

$$0 = \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2}\not{H}\mathcal{P}\right)\epsilon + \frac{1}{8}e^{\phi}\sum_n(-1)^{2n}(5-2n)\,\not{F}_{2n}\mathcal{P}_n\epsilon$$

  $10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.
  Fluxes in the SUSY conditions

SUSY solutions of IIA/B SUGRA

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions: CY condition: $D_m \eta = 0$.

$$0 = \delta\psi_M = D_M \epsilon$$

$$0 = \delta\tilde{\lambda} = \left(\partial\!\!\!/ \phi \qquad\qquad \right)\epsilon$$

$10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.

Fluxes in the SUSY conditions

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions:

$$\begin{aligned} 0 &= \delta\psi_M = D_M\epsilon + \frac{1}{4} H_M \mathcal{P}\epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}_{2n}\gamma_M \mathcal{P}_n \epsilon \\ 0 &= \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2}\not{H}\mathcal{P}\right)\epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n}(5-2n)\,\not{F}_{2n}\mathcal{P}_n\epsilon \end{aligned}$$

  $10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.
  Fluxes in the SUSY conditions $\Rightarrow$ GCG rewriting.

SUSY solutions of IIA/B SUGRA

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions: CY condition: $D_m \eta = 0$.

$$0 = \delta\psi_M = D_M\epsilon + \frac{1}{4} H_M \mathcal{P}\epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}_{2n} \gamma_M \mathcal{P}_n \epsilon$$

$$0 = \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2}\not{H}\mathcal{P}\right)\epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n} (5-2n)\, \not{F}_{2n} \mathcal{P}_n \epsilon$$

  $10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.
  Fluxes in the SUSY conditions $\Rightarrow$ GCG rewriting.
- Decompose according to $SO(1,3) \times SO(6)$: $\epsilon^{i=1,2} \longrightarrow \eta_{\pm}^{i=1,2}$.
  $\hookrightarrow$ SUSY conditions on $\mathcal{M}$.

SUSY solutions of IIA/B SUGRA

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions: CY condition: $D_m \eta = 0$.

$$0 = \delta\psi_M = D_M \epsilon + \frac{1}{4} H_M \mathcal{P} \epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}_{2n} \gamma_M \mathcal{P}_n \epsilon$$

$$0 = \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2} \not{H} \mathcal{P} \right) \epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n} (5 - 2n) \not{F}_{2n} \mathcal{P}_n \epsilon$$

  $10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.
  Fluxes in the SUSY conditions $\Rightarrow$ GCG rewriting.

- Decompose according to $SO(1,3) \times SO(6)$: $\epsilon^{i=1,2} \longrightarrow \eta_{\pm}^{i=1,2}$.
  $\hookrightarrow$ SUSY conditions on $\mathcal{M}$. GCG rewriting:

$$(\mathrm{d} - H\wedge)(e^{2A-\phi} \Phi_1) = 0$$
$$(\mathrm{d} - H\wedge)(e^{A-\phi} \,\mathrm{Re}(\Phi_2)) = 0$$
$$(\mathrm{d} - H\wedge)(e^{3A-\phi} \,\mathrm{Im}(\Phi_2)) = \frac{e^{4A}}{8} * \lambda(\textstyle\sum_p F_p)$$

  $\Phi_{\pm} = \eta_+^1 \otimes \eta_{\pm}^{2\dagger}$

SUSY solutions of IIA/B SUGRA

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions: CY condition: $D_m \eta = 0$.

$$0 = \delta\psi_M = D_M \epsilon + \frac{1}{4} H_M \mathcal{P} \epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}_{2n} \gamma_M \mathcal{P}_n \epsilon$$

$$0 = \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2} \not{H} \mathcal{P} \right) \epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n} (5 - 2n) \not{F}_{2n} \mathcal{P}_n \epsilon$$

  $10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.
  Fluxes in the SUSY conditions $\Rightarrow$ GCG rewriting.
- Decompose according to $SO(1,3) \times SO(6)$: $\epsilon^{i=1,2} \longrightarrow \eta_{\pm}^{i=1,2}$.
  $\hookrightarrow$ SUSY conditions on $\mathcal{M}$. GCG rewriting:

$$(\mathrm{d} - H\wedge)(e^{2A-\phi} \Phi_1) = 0$$
$$(\mathrm{d} - H\wedge)(e^{A-\phi} \mathrm{Re}(\Phi_2)) = 0$$
$$(\mathrm{d} - H\wedge)(e^{3A-\phi} \mathrm{Im}(\Phi_2)) = \frac{e^{4A}}{8} * \lambda(\textstyle\sum_p F_p)$$

  $\Phi_{\pm} = \eta_+^1 \otimes \eta_{\pm}^{2\dagger}$, spinors on $TM \oplus T^*M$ (GCG)

SUSY solutions of IIA/B SUGRA

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions: CY condition: $D_m \eta = 0$.

$$0 = \delta\psi_M = D_M \epsilon + \frac{1}{4} H_M \mathcal{P} \epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}_{2n} \gamma_M \mathcal{P}_n \epsilon$$

$$0 = \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2}\not{H}\mathcal{P}\right)\epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n}(5-2n)\, \not{F}_{2n} \mathcal{P}_n \epsilon$$

  $10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.
  Fluxes in the SUSY conditions ⇒ GCG rewriting.
- Decompose according to $SO(1,3) \times SO(6)$: $\epsilon^{i=1,2} \longrightarrow \eta_{\pm}^{i=1,2}$.
  ↪ SUSY conditions on $\mathcal{M}$. GCG rewriting:

$$(\mathrm{d} - H\wedge)(e^{2A-\phi}\Phi_1) = 0$$
$$(\mathrm{d} - H\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi_2)) = 0$$
$$(\mathrm{d} - H\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi_2)) = \frac{e^{4A}}{8} * \lambda(\textstyle\sum_p F_p)$$

  $\Phi_{\pm} = \eta_+^1 \otimes \eta_{\pm}^{2\dagger}$, spinors on $TM \oplus T^*M$ (GCG) ⇒ $\mathcal{M}$: GCY.

SUSY solutions of IIA/B SUGRA

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions: CY condition: $D_m \eta = 0$.

$$0 = \delta\psi_M = D_M \epsilon + \frac{1}{4} H_M \mathcal{P} \epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}_{2n} \gamma_M \mathcal{P}_n \epsilon$$

$$0 = \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2} \not{H} \mathcal{P} \right) \epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n} (5 - 2n) \not{F}_{2n} \mathcal{P}_n \epsilon$$

  $10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.
  Fluxes in the SUSY conditions $\Rightarrow$ GCG rewriting.
- Decompose according to $SO(1,3) \times SO(6)$: $\epsilon^{i=1,2} \longrightarrow \eta_{\pm}^{i=1,2}$.
  $\hookrightarrow$ SUSY conditions on $\mathcal{M}$. GCG rewriting:

$$(\mathrm{d} - H\wedge)(e^{2A-\phi} \Phi_1) = 0$$
$$(\mathrm{d} - H\wedge)(e^{A-\phi}\, \mathrm{Re}(\Phi_2)) = 0$$
$$(\mathrm{d} - H\wedge)(e^{3A-\phi}\, \mathrm{Im}(\Phi_2)) = \frac{e^{4A}}{8} * \lambda(\textstyle\sum_p F_p)$$

  $\Phi_\pm = \eta^1_+ \otimes \eta^{2\dagger}_\pm$, spinors on $TM \oplus T^*M$ (GCG) $\Rightarrow \mathcal{M}$: GCY.
  $\Phi_\pm$ are also polyforms.

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions: CY condition: $D_m \eta = 0$.

$$\begin{aligned} 0 &= \delta\psi_M = D_M \epsilon + \frac{1}{4} H_M \mathcal{P} \epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}_{2n} \gamma_M \mathcal{P}_n \epsilon \\ 0 &= \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2} \not{H} \mathcal{P} \right) \epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n} (5-2n) \not{F}_{2n} \mathcal{P}_n \epsilon \end{aligned}$$

  $10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.
  Fluxes in the SUSY conditions ⇒ GCG rewriting.
- Decompose according to $SO(1,3) \times SO(6)$: $\epsilon^{i=1,2} \longrightarrow \eta_{\pm}^{i=1,2}$.
  ↪ SUSY conditions on $\mathcal{M}$. GCG rewriting:

$$\begin{aligned} &(\mathrm{d} - H\wedge)(e^{2A-\phi}\Phi_1) = 0 \\ &(\mathrm{d} - H\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi_2)) = 0 \\ &(\mathrm{d} - H\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi_2)) = \frac{e^{4A}}{8} * \lambda(\textstyle\sum_p F_p) \end{aligned}$$

  $\Phi_{\pm} = \eta^1_+ \otimes \eta^{2\dagger}_{\pm}$, spinors on $TM \oplus T^*M$ (GCG) ⇒ $\mathcal{M}$: GCY.
  $\Phi_{\pm}$ are also polyforms. For an $SU(3)$ structure ($\eta^1_+ = \eta^2_+$):

$$\begin{aligned} \Phi_+/N_+ &= e^{i\theta}\left(1 - iJ - \tfrac{1}{2} J\wedge J + \tfrac{i}{3!} J\wedge J\wedge J\right) \\ \Phi_-/N_- &= -i\ \Omega_3 \end{aligned}$$

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions: CY condition: $D_m \eta = 0$.
$$0 = \delta\psi_M = D_M\epsilon + \frac{1}{4} H_M \mathcal{P}\epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}_{2n} \gamma_M \mathcal{P}_n \epsilon$$
$$0 = \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2}\not{H}\mathcal{P}\right)\epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n}(5-2n)\, \not{F}_{2n} \mathcal{P}_n \epsilon$$
$10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.
Fluxes in the SUSY conditions $\Rightarrow$ GCG rewriting.
- Decompose according to $SO(1,3) \times SO(6)$: $\epsilon^{i=1,2} \longrightarrow \eta_{\pm}^{i=1,2}$.
$\hookrightarrow$ SUSY conditions on $\mathcal{M}$. GCG rewriting:
$$(\mathrm{d} - H\wedge)(e^{2A-\phi}\Phi_1) = 0$$
$$(\mathrm{d} - H\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi_2)) = 0$$
$$(\mathrm{d} - H\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi_2)) = \frac{e^{4A}}{8} * \lambda\left(\sum_p F_p\right)$$
$\Phi_{\pm} = \eta_+^1 \otimes \eta_{\pm}^{2\dagger}$, spinors on $TM \oplus T^*M$ (GCG) $\Rightarrow \mathcal{M}$: GCY.
$\Phi_{\pm}$ are also polyforms. For an $SU(3)$ structure ($\eta_+^1 = \eta_+^2$):
$$\Phi_+/N_+ = e^{i\theta}\, e^{-iJ}$$
$$\Phi_-/N_- = -i\ \Omega_3$$

SUSY solutions of IIA/B SUGRA

The SUSY conditions

- For $\mathbb{R}^{3,1} \times \mathcal{M}$, SUSY conditions: CY condition: $D_m \eta = 0$.

$$0 = \delta\psi_M = D_M \epsilon + \frac{1}{4} H_M \mathcal{P} \epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}_{2n} \gamma_M \mathcal{P}_n \epsilon$$

$$0 = \delta\tilde{\lambda} = \left(\not{\partial}\phi + \frac{1}{2} \not{H} \mathcal{P} \right) \epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n} (5 - 2n) \not{F}_{2n} \mathcal{P}_n \epsilon$$

  $10d$ SUSY parameters: $\epsilon = (\epsilon^1, \epsilon^2)$.
  Fluxes in the SUSY conditions $\Rightarrow$ GCG rewriting.

- Decompose according to $SO(1,3) \times SO(6)$: $\epsilon^{i=1,2} \longrightarrow \eta_{\pm}^{i=1,2}$.
  $\hookrightarrow$ SUSY conditions on $\mathcal{M}$. GCG rewriting:

$$(\mathrm{d} - H\wedge)(e^{2A-\phi}\Phi_1) = 0$$
$$(\mathrm{d} - H\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi_2)) = 0$$
$$(\mathrm{d} - H\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi_2)) = \frac{e^{4A}}{8} * \lambda(\textstyle\sum_p F_p)$$

  $\Phi_{\pm} = \eta_+^1 \otimes \eta_{\pm}^{2\dagger}$, spinors on $TM \oplus T^*M$ (GCG) $\Rightarrow \mathcal{M}$: GCY.
  $\Phi_{\pm}$ are also polyforms. For an $SU(3)$ structure $(\eta_+^1 = \eta_+^2)$:

$$\Phi_+/N_+ = e^{i\theta}\, e^{-iJ}$$
$$\Phi_-/N_- = -i\ \Omega_3$$

SUSY conditions rewritten with polyforms, $\mathcal{M}$ = GCY.

SUSY solutions

- Explicit flux solutions on non-CY?

SUSY solutions

- Explicit flux solutions on non-CY? Via T-duality...

SUSY solutions

- Explicit flux solutions on non-CY? Via T-duality...
  IIB solution: warped T^6 + O3-plane + fluxes (H, F_3, F_5)

SUSY solutions

- Explicit flux solutions on non-CY? Via T-duality...
  IIB solution: warped T^6 + O3-plane + fluxes (H, F_3, F_5)
  Two T-dualities along $T^2 \to$ flux solution on twisted torus.

$$T^6 = T^2 \times T^4 \quad \xrightarrow{T-d.} \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

SUSY solutions

- Explicit flux solutions on non-CY? Via T-duality...
  IIB solution: warped T^6 + O3-plane + fluxes (H, F_3, F_5)
  Two T-dualities along $T^2 \rightarrow$ flux solution on twisted torus.

$$T^6 = T^2 \times T^4 \quad \xrightarrow{T-d.} \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

- SUSY conditions + BI $\Rightarrow$ e.o.m.
  SUSY conditions are more tractable in terms of GCG.

SUSY solutions

- Explicit flux solutions on non-CY? Via T-duality...
  IIB solution: warped T^6 + O3-plane + fluxes (H, F_3, F_5)
  Two T-dualities along $T^2 \to$ flux solution on twisted torus.

$$T^6 = T^2 \times T^4 \quad \xrightarrow{T-d.} \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

- SUSY conditions + BI $\Rightarrow$ e.o.m.
  SUSY conditions are more tractable in terms of GCG.
  $\hookrightarrow$ systematic resolution method to get solutions.

SUSY solutions

- Explicit flux solutions on non-CY? Via T-duality...
  IIB solution: warped T^6 + O3-plane + fluxes (H, F_3, F_5)
  Two T-dualities along $T^2 \to$ flux solution on twisted torus.

$$T^6 = T^2 \times T^4 \quad \xrightarrow{T-d.} \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

- SUSY conditions + BI $\Rightarrow$ e.o.m.
  SUSY conditions are more tractable in terms of GCG.
  $\hookrightarrow$ systematic resolution method to get solutions.
- $\Rightarrow$ a whole list of solutions: ex. in IIB with O5:

	Algebras	O5	
		$SU(3)$	$\perp SU(2)$
n 3.14	$(0,0,0,12,23,14-35)$	45 + 26	
n 4.4	$(0,0,0,0,12,14+23)$	56	56
n 4.5	$(0,0,0,0,12,34)$	56	56
n 4.6	$(0,0,0,0,12,13)$	56	56
n 4.7	$(0,0,0,0,13+42,14+23)$	56	56
n 5.1	$(0,0,0,0,0,12+34)$	56	56
s 2.5	$(25,-15,r45,-r35,0,0)$	13 + 24	13 + 24

SUSY solutions

- Explicit flux solutions on non-CY? Via T-duality...
  IIB solution: warped T^6 + O3-plane + fluxes (H, F_3, F_5)
  Two T-dualities along $T^2 \rightarrow$ flux solution on twisted torus.

$$T^6 = T^2 \times T^4 \quad \xrightarrow{T-d.} \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

- SUSY conditions + BI $\Rightarrow$ e.o.m.
  SUSY conditions are more tractable in terms of GCG.
  $\hookrightarrow$ systematic resolution method to get solutions.
- $\Rightarrow$ a whole list of solutions: ex. in IIB with O5:

	Algebras	O5	
		$SU(3)$	$\perp$ $SU(2)$
n 3.14	$(0,0,0,12,23,14-35)$	45 + 26	
n 4.4	$(0,0,0,0,12,14+23)$	56	56
n 4.5	$(0,0,0,0,12,34)$	56	56
n 4.6	$(0,0,0,0,12,13)$	56	56
n 4.7	$(0,0,0,0,13+42,14+23)$	56	56
n 5.1	$(0,0,0,0,0,12+34)$	56	56
s 2.5	$(25,-15,r45,-r35,0,0)$	13 + 24	13 + 24

Some solutions are T-dual to T^6, others are not T-dual!

SUSY solutions

- Explicit flux solutions on non-CY? Via T-duality...
  IIB solution: warped T^6 + O3-plane + fluxes (H, F_3, F_5)
  Two T-dualities along $T^2 \to$ flux solution on twisted torus.

$$T^6 = T^2 \times T^4 \quad \xrightarrow{T-d.} \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

- SUSY conditions + BI $\Rightarrow$ e.o.m.
  SUSY conditions are more tractable in terms of GCG.
  $\hookrightarrow$ systematic resolution method to get solutions.
- $\Rightarrow$ a whole list of solutions: ex. in IIB with O5:

	Algebras	O5	
		$SU(3)$	$\perp SU(2)$
n 3.14	$(0,0,0,12,23,14-35)$	45 + 26	
n 4.4	$(0,0,0,0,12,14+23)$	56	56
n 4.5	$(0,0,0,0,12,34)$	56	56
n 4.6	$(0,0,0,0,12,13)$	56	56
n 4.7	$(0,0,0,0,13+42,14+23)$	56	56
n 5.1	$(0,0,0,0,0,12+34)$	56	56
s 2.5	$(25,-15,r45,-r35,0,0)$	13 + 24	13 + 24

Some solutions are T-dual to T^6, others are not T-dual!
Find non T-dual intermediate $SU(2)$ str. solutions.

SUSY solutions

- Explicit flux solutions on non-CY? Via T-duality...
  IIB solution: warped T^6 + O3-plane + fluxes (H, F_3, F_5)
  Two T-dualities along $T^2 \rightarrow$ flux solution on twisted torus.

$$T^6 = T^2 \times T^4 \quad \xrightarrow{T-d.} \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

- SUSY conditions + BI $\Rightarrow$ e.o.m.
  SUSY conditions are more tractable in terms of GCG.
  $\hookrightarrow$ systematic resolution method to get solutions.
- $\Rightarrow$ a whole list of solutions: ex. in IIB with O5:

	Algebras	O5	
		$SU(3)$	$\perp$ $SU(2)$
n 3.14	$(0,0,0,12,23,14-35)$	45 + 26	
n 4.4	$(0,0,0,0,12,14+23)$	56	56
n 4.5	$(0,0,0,0,12,34)$	56	56
n 4.6	$(0,0,0,0,12,13)$	56	56
n 4.7	$(0,0,0,0,13+42,14+23)$	56	56
n 5.1	$(0,0,0,0,0,12+34)$	56	56
s 2.5	$(25,-15,r45,-r35,0,0)$	13 + 24	13 + 24

  Some solutions are T-dual to T^6, others are not T-dual!
  Find non T-dual intermediate $SU(2)$ str. solutions.
- Can all solutions be related a transformation?
  (no isolated solution)

SUSY solutions

- Explicit flux solutions on non-CY? Via T-duality...
  IIB solution: warped T^6 + O3-plane + fluxes (H, F_3, F_5)
  Two T-dualities along $T^2 \to$ flux solution on twisted torus.

$$T^6 = T^2 \times T^4 \quad \xrightarrow{T-d.} \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

- SUSY conditions + BI $\Rightarrow$ e.o.m.
  SUSY conditions are more tractable in terms of GCG.
  $\hookrightarrow$ systematic resolution method to get solutions.
- $\Rightarrow$ a whole list of solutions: ex. in IIB with O5:

	Algebras	O5	
		$SU(3)$	$\perp$ $SU(2)$
n 3.14	$(0,0,0,12,23,14-35)$	45 + 26	
n 4.4	$(0,0,0,0,12,14+23)$	56	56
n 4.5	$(0,0,0,0,12,34)$	56	56
n 4.6	$(0,0,0,0,12,13)$	56	56
n 4.7	$(0,0,0,0,13+42,14+23)$	56	56
n 5.1	$(0,0,0,0,0,12+34)$	56	56
s 2.5	$(25,-15,r45,-r35,0,0)$	13 + 24	13 + 24

  Some solutions are T-dual to T^6, others are not T-dual!
  Find non T-dual intermediate $SU(2)$ str. solutions.
- Can all solutions be related a transformation?
  (no isolated solution)
  $\hookrightarrow$ Twist transf.: relate/generate solutions on twisted tori.

Twist transformation

Twisted tori: nil- and solvmanifolds

Twist transformation

Twisted tori: nil- and solvmanifolds

- Built out of Lie groups G nilpotent or solvable.

Twist transformation

Twisted tori: nil- and solvmanifolds

- Built out of Lie groups G nilpotent or solvable. Nilmanifolds $\subset$ solvmanifolds (nilpotent $\subset$ solvable).

Twist transformation

Twisted tori: nil- and solvmanifolds

- Built out of Lie groups G nilpotent or solvable.
  Nilmanifolds $\subset$ solvmanifolds (nilpotent $\subset$ solvable).
  All nilmanifolds are GCY $\Rightarrow$ interesting candidates for $\mathcal{M}$.

Twist transformation

Twisted tori: nil- and solvmanifolds

- Built out of Lie groups G nilpotent or solvable.
  Nilmanifolds $\subset$ solvmanifolds (nilpotent $\subset$ solvable).
  All nilmanifolds are GCY $\Rightarrow$ interesting candidates for $\mathcal{M}$.

  General nilmanifold :

$$\begin{pmatrix} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{pmatrix} = N/\Gamma_N$$

Twist transformation

Twisted tori: nil- and solvmanifolds

- Built out of Lie groups G nilpotent or solvable.
  Nilmanifolds $\subset$ solvmanifolds (nilpotent $\subset$ solvable).
  All nilmanifolds are GCY $\Rightarrow$ interesting candidates for $\mathcal{M}$.

  General solvmanifold:

$$\left(\begin{array}{ccc} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{array}\right) \;=\; N/\Gamma_N \quad \hookrightarrow \quad \begin{array}{c} \mathcal{M} = G/\Gamma \\ \downarrow \\ T^k \end{array}$$

Twist transformation

Twisted tori: nil- and solvmanifolds

- Built out of Lie groups G nilpotent or solvable.
  Nilmanifolds $\subset$ solvmanifolds (nilpotent $\subset$ solvable).
  All nilmanifolds are GCY $\Rightarrow$ interesting candidates for $\mathcal{M}$.

  General solvmanifold:

$$\begin{pmatrix} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{pmatrix} \quad = \quad N/\Gamma_N \quad \hookrightarrow \quad \begin{array}{c} \mathcal{M} = G/\Gamma \\ \downarrow \\ T^k \end{array}$$

- The algebra gives the Maurer-Cartan (MC) equation:

$$[E_i, E_j] = f^k{}_{ij}\ E_k \ \Leftrightarrow\ \mathrm{d}e^k = -\sum_{i<j} f^k{}_{ij}\ e^i \wedge e^j\ .$$

  $E_i \in \mathfrak{g}$: vector, $e^i \in \mathfrak{g}^*$: dual 1-form, $f^k{}_{ij}$: struct. constants.

Twist transformation

Twisted tori: nil- and solvmanifolds

- Built out of Lie groups G nilpotent or solvable.
  Nilmanifolds $\subset$ solvmanifolds (nilpotent $\subset$ solvable).
  All nilmanifolds are GCY $\Rightarrow$ interesting candidates for $\mathcal{M}$.

  General solvmanifold:

$$\left(\begin{array}{ccc} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{array}\right) \quad = \; N/\Gamma_N \quad \hookrightarrow \quad \begin{array}{c} \mathcal{M} = G/\Gamma \\ \downarrow \\ T^k \end{array}$$

- The algebra gives the Maurer-Cartan (MC) equation:

$$[E_i, E_j] = f^k{}_{ij}\ E_k \ \Leftrightarrow\ \mathrm{d}e^k = -\sum_{i<j} f^k{}_{ij}\ e^i \wedge e^j\ .$$

  $E_i \in \mathfrak{g}$: vector, $e^i \in \mathfrak{g}^*$: dual 1-form, $f^k{}_{ij}$: struct. constants.

  Out of the MC equation, one can read the topology.
  Topological properties encoded in the MC 1-forms.

Twist transformation

Twisted tori: nil- and solvmanifolds

- Built out of Lie groups G nilpotent or solvable.
  Nilmanifolds $\subset$ solvmanifolds (nilpotent $\subset$ solvable).
  All nilmanifolds are GCY $\Rightarrow$ interesting candidates for $\mathcal{M}$.

  General solvmanifold:

$$\begin{pmatrix} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{pmatrix} = N/\Gamma_N \quad \hookrightarrow \quad \begin{array}{c} \mathcal{M} = G/\Gamma \\ \downarrow \\ T^k \end{array}$$

- The algebra gives the Maurer-Cartan (MC) equation:

$$[E_i, E_j] = f^k{}_{ij}\ E_k \ \Leftrightarrow\ \mathrm{d}e^k = -\sum_{i<j} f^k{}_{ij}\ e^i \wedge e^j \ .$$

  $E_i \in \mathfrak{g}$: vector, $e^i \in \mathfrak{g}^*$: dual 1-form, $f^k{}_{ij}$: struct. constants.

  Out of the MC equation, one can read the topology.
  Topological properties encoded in the MC 1-forms.
  $\hookrightarrow$ twist transformation uses this idea.

Twist transformation

$\Rightarrow$

To reproduce a change of topology, we transform the 1-forms.

Twist transformation

$\Rightarrow$

To reproduce a change of topology, we transform the 1-forms.
Here: obtain 1-forms of the solvmanifold out of those of T^6:

$$A \begin{pmatrix} \mathrm{d}x^1 \\ \vdots \\ \mathrm{d}x^6 \end{pmatrix} = \begin{pmatrix} e^1 \\ \vdots \\ e^6 \end{pmatrix} .$$

Twist transformation

 $\Rightarrow$ 

To reproduce a change of topology, we transform the 1-forms. Here: obtain 1-forms of the solvmanifold out of those of T^6:

$$A \begin{pmatrix} \mathrm{d}x^1 \\ \vdots \\ \mathrm{d}x^6 \end{pmatrix} = \begin{pmatrix} e^1 \\ \vdots \\ e^6 \end{pmatrix} .$$

A encodes the topology.

Twist transformation

$\Rightarrow$

To reproduce a change of topology, we transform the 1-forms.
Here: obtain 1-forms of the solvmanifold out of those of T^6:

$$A \begin{pmatrix} \mathrm{d}x^1 \\ \vdots \\ \mathrm{d}x^6 \end{pmatrix} = \begin{pmatrix} e^1 \\ \vdots \\ e^6 \end{pmatrix} .$$

A encodes the topology. Given the Mostow bundle

$$\begin{pmatrix} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{pmatrix} = N/\Gamma_N \quad \hookrightarrow \quad \begin{array}{c} \mathcal{M} = G/\Gamma \\ \downarrow \\ T^k \end{array}$$

Twist transformation

$\Rightarrow$

To reproduce a change of topology, we transform the 1-forms.
Here: obtain 1-forms of the solvmanifold out of those of T^6:

$$A \begin{pmatrix} \mathrm{d}x^1 \\ \vdots \\ \mathrm{d}x^6 \end{pmatrix} = \begin{pmatrix} e^1 \\ \vdots \\ e^6 \end{pmatrix} .$$

A encodes the topology. Given the Mostow bundle, we take

$$\begin{pmatrix} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{pmatrix} = N/\Gamma_N \quad \hookrightarrow \quad \begin{array}{c} \mathcal{M} = G/\Gamma \\ \downarrow \\ T^k \end{array}$$

$$A = \left(\begin{array}{c|c} A_N & 0 \\ \hline 0 & 1_k \end{array} \right) \left(\begin{array}{c|c} A_M & 0 \\ \hline 0 & 1_k \end{array} \right) , \; A_N = A_p \dots A_1 \; .$$

Twist transformation

$\Rightarrow$

To reproduce a change of topology, we transform the 1-forms.
Here: obtain 1-forms of the solvmanifold out of those of T^6:

$$A \begin{pmatrix} \mathrm{d}x^1 \\ \vdots \\ \mathrm{d}x^6 \end{pmatrix} = \begin{pmatrix} e^1 \\ \vdots \\ e^6 \end{pmatrix} .$$

A encodes the topology. Given the Mostow bundle, we take

$$\begin{pmatrix} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{pmatrix} = N/\Gamma_N \quad \hookrightarrow \quad \begin{array}{c} \mathcal{M} = G/\Gamma \\ \downarrow \\ T^k \end{array}$$

$$A = \left(\begin{array}{c|c} A_N & 0 \\ \hline 0 & 1_k \end{array} \right) \left(\begin{array}{c|c} A_M & 0 \\ \hline 0 & 1_k \end{array} \right) , \; A_N = A_p \dots A_1 .$$

Twist transformation

$\Rightarrow$

To reproduce a change of topology, we transform the 1-forms.
Here: obtain 1-forms of the solvmanifold out of those of T^6:

$$A \begin{pmatrix} dx^1 \\ \vdots \\ dx^6 \end{pmatrix} = \begin{pmatrix} e^1 \\ \vdots \\ e^6 \end{pmatrix} .$$

A encodes the topology. Given the Mostow bundle, we take

$$\begin{pmatrix} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{pmatrix} = N/\Gamma_N \quad \hookrightarrow \quad \begin{array}{c} \mathcal{M} = G/\Gamma \\ \downarrow \\ T^k \end{array}$$

$$A = \left(\begin{array}{c|c} A_N & 0 \\ \hline 0 & 1_k \end{array} \right) \left(\begin{array}{c|c} A_M & 0 \\ \hline 0 & 1_k \end{array} \right) , \; A_N = A_p \dots A_1 \ .$$

Twist transformation

 $\Rightarrow$ 

To reproduce a change of topology, we transform the 1-forms.
Here: obtain 1-forms of the solvmanifold out of those of T^6:

$$A \begin{pmatrix} \mathrm{d}x^1 \\ \vdots \\ \mathrm{d}x^6 \end{pmatrix} = \begin{pmatrix} e^1 \\ \vdots \\ e^6 \end{pmatrix} .$$

A encodes the topology. Given the Mostow bundle, we take

$$\begin{pmatrix} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{pmatrix} = N/\Gamma_N \quad \hookrightarrow \quad \begin{array}{c} \mathcal{M} = G/\Gamma \\ \downarrow \\ T^k \end{array}$$

$$A = \left(\begin{array}{c|c} A_N & 0 \\ \hline 0 & 1_k \end{array} \right) \left(\begin{array}{c|c} A_M & 0 \\ \hline 0 & 1_k \end{array} \right) , \ A_N = A_p \dots A_1 \ .$$

$A_i = \begin{pmatrix} 1 & 0 \\ \mathcal{A}_i(x_{\mathcal{B}^i}) & 1 \end{pmatrix}$, $A_M =$ rotation.

Twist transformation

 $\Rightarrow$ 

To reproduce a change of topology, we transform the 1-forms.
Here: obtain 1-forms of the solvmanifold out of those of T^6:

$$A \begin{pmatrix} dx^1 \\ \vdots \\ dx^6 \end{pmatrix} = \begin{pmatrix} e^1 \\ \vdots \\ e^6 \end{pmatrix} .$$

A encodes the topology. Given the Mostow bundle, we take

$$\begin{pmatrix} \mathcal{F}^p & \hookrightarrow & \mathcal{M}^p \\ & & \downarrow \\ & & \vdots \\ & & \downarrow \\ \mathcal{F}^1 & \hookrightarrow & \mathcal{M}^1 \\ & & \downarrow \\ & & \mathcal{B}^1 \end{pmatrix} = N/\Gamma_N \quad \hookrightarrow \quad \begin{matrix} \mathcal{M} = G/\Gamma \\ \downarrow \\ T^k \end{matrix}$$

$$A = \left(\begin{array}{c|c} A_N & 0 \\ \hline 0 & 1_k \end{array} \right) \left(\begin{array}{c|c} A_M & 0 \\ \hline 0 & 1_k \end{array} \right) , \ A_N = A_p \dots A_1 \ .$$

$A_i = \begin{pmatrix} 1 & 0 \\ \mathcal{A}_i(x_{\mathcal{B}^i}) & 1 \end{pmatrix}$, $A_M =$ rotation.

$A_i,\ A_M$ are given by adjoint actions...

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

$$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array}$$

E: the generalized tangent bundle.

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

$$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array}$$

E: the generalized tangent bundle.

Locally: $TM \oplus T^*M$.

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

$$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array}$$

  E: the generalized tangent bundle.

  Locally: $TM \oplus T^*M$. Sections: generalized vectors:

$$X = v + \xi = \begin{pmatrix} v \\ \xi \end{pmatrix} \ , \ v \in TM \ , \ \xi \in T^*M \ .$$

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

$$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array} \qquad E\text{: the generalized tangent bundle.}$$

Locally: $TM \oplus T^*M$. Sections: generalized vectors:

$$X = v + \xi = \begin{pmatrix} v \\ \xi \end{pmatrix} \ , \ v \in TM \ , \ \xi \in T^*M \ .$$

Natural $O(d,d)$ action on $TM \oplus T^*M$.

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

$$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array}$$

  E: the generalized tangent bundle.

  Locally: $TM \oplus T^*M$. Sections: generalized vectors:

$$X = v + \xi = \begin{pmatrix} v \\ \xi \end{pmatrix} \ , \ v \in TM \ , \ \xi \in T^*M \ .$$

  Natural $O(d,d)$ action on $TM \oplus T^*M$. One element:

$$\left(\begin{array}{c|c} A & 0_d \\ \hline 0_d & A^{-T} \end{array}\right)$$

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

$$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array}$$

  E: the generalized tangent bundle.

  Locally: $TM \oplus T^*M$. Sections: generalized vectors:

$$X = v + \xi = \begin{pmatrix} v \\ \xi \end{pmatrix} \ , \ v \in TM \ , \ \xi \in T^*M \ .$$

  Natural $O(d,d)$ action on $TM \oplus T^*M$. One element:

$$\left(\begin{array}{c|c} A & 0_d \\ \hline 0_d & A^{-T} \end{array} \right)$$

  $GL(d)$ transf.: on vectors and 1-forms of $TM \oplus T^*M$.

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

$$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array}$$

  E: the generalized tangent bundle.

  Locally: $TM \oplus T^*M$. Sections: generalized vectors:

$$X = v + \xi = \begin{pmatrix} v \\ \xi \end{pmatrix} \ , \ v \in TM \ , \ \xi \in T^*M \ .$$

  Natural $O(d,d)$ action on $TM \oplus T^*M$. One element:

$$\left(\begin{array}{c|c} A & 0_d \\ \hline 0_d & A^{-T} \end{array} \right)$$

  $GL(d)$ transf.: on vectors and 1-forms of $TM \oplus T^*M$.
  Twist acting on 1-forms $\Rightarrow$ embed it in $O(d,d)$.

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

$$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array} \qquad E\text{: the generalized tangent bundle.}$$

  Locally: $TM \oplus T^*M$. Sections: generalized vectors:

$$X = v + \xi = \begin{pmatrix} v \\ \xi \end{pmatrix} \ , \ v \in TM \ , \ \xi \in T^*M \ .$$

  Natural $O(d,d)$ action on $TM \oplus T^*M$. One element:

$$\left(\begin{array}{c|c} A & 0_d \\ \hline 0_d & A^{-T} \end{array} \right)$$

  $GL(d)$ transf.: on vectors and 1-forms of $TM \oplus T^*M$.
  Twist acting on 1-forms $\Rightarrow$ embed it in $O(d,d)$.
  Extend it with other $O(d,d)$ elements...

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

  $$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array} \qquad E\text{: the generalized tangent bundle.}$$

  Locally: $TM \oplus T^*M$. Sections: generalized vectors:

  $$X = v + \xi = \begin{pmatrix} v \\ \xi \end{pmatrix} \ , \ v \in TM \ , \ \xi \in T^*M \ .$$

  Natural $O(d,d)$ action on $TM \oplus T^*M$. One element:

  $$\left(\begin{array}{c|c} A & 0_d \\ \hline 0_d & A^{-T} \end{array} \right)$$

  $GL(d)$ transf.: on vectors and 1-forms of $TM \oplus T^*M$.
  Twist acting on 1-forms $\Rightarrow$ embed it in $O(d,d)$.
  Extend it with other $O(d,d)$ elements...
  $\hookrightarrow$ Twist transformation: local $O(d,d)$ changing topology.

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:
  $$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array} \qquad E\text{: the generalized tangent bundle.}$$
  Locally: $TM \oplus T^*M$. Sections: generalized vectors:
  $$X = v + \xi = \begin{pmatrix} v \\ \xi \end{pmatrix} \ , \ v \in TM \ , \ \xi \in T^*M \ .$$
  Natural $O(d,d)$ action on $TM \oplus T^*M$. One element:
  $$\left(\begin{array}{c|c} A & 0_d \\ \hline 0_d & A^{-T} \end{array}\right)$$
  $GL(d)$ transf.: on vectors and 1-forms of $TM \oplus T^*M$.
  Twist acting on 1-forms $\Rightarrow$ embed it in $O(d,d)$.
  Extend it with other $O(d,d)$ elements...
  $\hookrightarrow$ Twist transformation: local $O(d,d)$ changing topology.
- Spinorial representation of $O(d,d)$: $(S)pin(d,d)$
  $\Rightarrow$ Majorana-Weyl spinors on $TM \oplus T^*M$

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

$$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array} \qquad E\text{: the generalized tangent bundle.}$$

  Locally: $TM \oplus T^*M$. Sections: generalized vectors:

$$X = v + \xi = \begin{pmatrix} v \\ \xi \end{pmatrix} \ , \ v \in TM \ , \ \xi \in T^*M \ .$$

  Natural $O(d,d)$ action on $TM \oplus T^*M$. One element:

$$\left(\begin{array}{c|c} A & 0_d \\ \hline 0_d & A^{-T} \end{array} \right)$$

  $GL(d)$ transf.: on vectors and 1-forms of $TM \oplus T^*M$.
  Twist acting on 1-forms $\Rightarrow$ embed it in $O(d,d)$.
  Extend it with other $O(d,d)$ elements...
  $\hookrightarrow$ Twist transformation: local $O(d,d)$ changing topology.
- Spinorial representation of $O(d,d)$: $(S)pin(d,d)$
  $\Rightarrow$ Majorana-Weyl spinors on $TM \oplus T^*M$: $\Phi_\pm = \eta^1_+ \otimes \eta^{2\dagger}_\pm$.

Twist and GCG

- Transformation relating 1-forms of T^6 to 1-forms of a twisted torus, reproduces the change of topology.
  $\Rightarrow$ extend the twist in GCG...
- GCG considers the fibration:

  $$\begin{array}{ccc} T^*M & \hookrightarrow & E \\ & & \downarrow \\ & & TM \end{array} \qquad E\text{: the generalized tangent bundle.}$$

  Locally: $TM \oplus T^*M$. Sections: generalized vectors:

  $$X = v + \xi = \begin{pmatrix} v \\ \xi \end{pmatrix} \ , \ v \in TM \ , \ \xi \in T^*M \ .$$

  Natural $O(d,d)$ action on $TM \oplus T^*M$. One element:

  $$\left(\begin{array}{c|c} A & 0_d \\ \hline 0_d & A^{-T} \end{array}\right)$$

  $GL(d)$ transf.: on vectors and 1-forms of $TM \oplus T^*M$.
  Twist acting on 1-forms $\Rightarrow$ embed it in $O(d,d)$.
  Extend it with other $O(d,d)$ elements...
  $\hookrightarrow$ Twist transformation: local $O(d,d)$ changing topology.
- Spinorial representation of $O(d,d)$: $(S)pin(d,d)$
  $\Rightarrow$ Majorana-Weyl spinors on $TM \oplus T^*M$: $\Phi_\pm = \eta^1_+ \otimes \eta^{2\dagger}_\pm$.
  They transform: $\Phi \mapsto \Phi' = O \cdot \Phi$.

- Solutions on torus and on twisted tori.

- Solutions on torus and on twisted tori.
  Our solutions are given in terms of the polyforms $\Phi_{\pm}$.

- Solutions on torus and on twisted tori.
  Our solutions are given in terms of the polyforms $\Phi_{\pm}$.
  $\Phi_{\pm}$ can encode a solution (NSNS, RR via SUSY)

- Solutions on torus and on twisted tori.
  Our solutions are given in terms of the polyforms $\Phi_\pm$.
  $\Phi_\pm$ can encode a solution (NSNS, RR via SUSY)
  $\hookrightarrow$ a transformation (T-duality, Twist...) could lead to another solution...

- Solutions on torus and on twisted tori.
  Our solutions are given in terms of the polyforms $\Phi_\pm$.
  $\Phi_\pm$ can encode a solution (NSNS, RR via SUSY)
  $\hookrightarrow$ a transformation (T-duality, Twist...) could lead to another solution...
  $\hookrightarrow$ relate solutions with the twist?

- Solutions on torus and on twisted tori.
  Our solutions are given in terms of the polyforms $\Phi_\pm$.
  $\Phi_\pm$ can encode a solution (NSNS, RR via SUSY)
  $\hookrightarrow$ a transformation (T-duality, Twist...) could lead to another solution...
  $\hookrightarrow$ relate solutions with the twist?

- Consider a solution of the SUSY conditions.

$$(\mathrm{d} - H\wedge)(e^{2A-\phi}\Phi_1) = 0$$
$$(\mathrm{d} - H\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi_2)) = 0$$
$$(\mathrm{d} - H\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi_2)) = R$$

- Solutions on torus and on twisted tori.
  Our solutions are given in terms of the polyforms $\Phi_\pm$.
  $\Phi_\pm$ can encode a solution (NSNS, RR via SUSY)
  $\hookrightarrow$ a transformation (T-duality, Twist...) could lead to another solution...
  $\hookrightarrow$ relate solutions with the twist?

- Consider a solution of the SUSY conditions.
  Perform a Twist: $\Phi'_\pm = O \cdot \Phi_\pm$

$$(\mathrm{d} - H\wedge)(e^{2A-\phi}\Phi_1) = 0$$
$$(\mathrm{d} - H\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi_2)) = 0$$
$$(\mathrm{d} - H\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi_2)) = R$$

- Solutions on torus and on twisted tori.
  Our solutions are given in terms of the polyforms $\Phi_{\pm}$.
  $\Phi_{\pm}$ can encode a solution (NSNS, RR via SUSY)
  $\hookrightarrow$ a transformation (T-duality, Twist...) could lead to another solution...
  $\hookrightarrow$ relate solutions with the twist?

- Consider a solution of the SUSY conditions.
  Perform a Twist: $\Phi'_{\pm} = O \cdot \Phi_{\pm} \Rightarrow$ Get a new solution?

$$\begin{array}{lcl} (\mathrm{d} - H\wedge)(e^{2A-\phi}\Phi_1) = 0 & & (\mathrm{d} - H'\wedge)(e^{2A-\phi}\Phi'_1) = 0 \\ (\mathrm{d} - H\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi_2)) = 0 & \Rightarrow & (\mathrm{d} - H'\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi'_2)) = 0 \\ (\mathrm{d} - H\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi_2)) = R & & (\mathrm{d} - H'\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi'_2)) = R' \end{array}$$

- Solutions on torus and on twisted tori.
  Our solutions are given in terms of the polyforms $\Phi_\pm$.
  $\Phi_\pm$ can encode a solution (NSNS, RR via SUSY)
  $\hookrightarrow$ a transformation (T-duality, Twist...) could lead to another solution...
  $\hookrightarrow$ relate solutions with the twist?

- Consider a solution of the SUSY conditions.
  Perform a Twist: $\Phi'_\pm = O \cdot \Phi_\pm \Rightarrow$ Get a new solution?

$$\begin{aligned} &(\mathrm{d} - H\wedge)(e^{2A-\phi}\Phi_1) = 0 & & (\mathrm{d} - H'\wedge)(e^{2A-\phi}\Phi'_1) = 0 \\ &(\mathrm{d} - H\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi_2)) = 0 & \Rightarrow \quad & (\mathrm{d} - H'\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi'_2)) = 0 \\ &(\mathrm{d} - H\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi_2)) = R & & (\mathrm{d} - H'\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi'_2)) = R' \end{aligned}$$

Expand in terms of $\Phi_{1,2}, R, O \Rightarrow$ constraints on the twist with respect to the first solution.

- Solutions on torus and on twisted tori.
  Our solutions are given in terms of the polyforms $\Phi_\pm$.
  $\Phi_\pm$ can encode a solution (NSNS, RR via SUSY)
  $\hookrightarrow$ a transformation (T-duality, Twist...) could lead to another solution...
  $\hookrightarrow$ relate solutions with the twist?

- Consider a solution of the SUSY conditions.
  Perform a Twist: $\Phi'_\pm = O \cdot \Phi_\pm \Rightarrow$ Get a new solution?

$$\begin{array}{lcl} (\mathrm{d} - H\wedge)(e^{2A-\phi}\Phi_1) = 0 & & (\mathrm{d} - H'\wedge)(e^{2A-\phi}\Phi'_1) = 0 \\ (\mathrm{d} - H\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi_2)) = 0 & \Rightarrow & (\mathrm{d} - H'\wedge)(e^{A-\phi}\,\mathrm{Re}(\Phi'_2)) = 0 \\ (\mathrm{d} - H\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi_2)) = R & & (\mathrm{d} - H'\wedge)(e^{3A-\phi}\,\mathrm{Im}(\Phi'_2)) = R' \end{array}$$

Expand in terms of $\Phi_{1,2}, R, O \Rightarrow$ constraints on the twist with respect to the first solution. For instance:

$$\mathrm{d}(O) \cdot \Phi_1 = 0 \ .$$

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta} \ e^{-iJ} \quad , \quad \Phi_- = -N_- i \ \Omega_3 \ .$$

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta}\ e^{-iJ} \quad , \quad \Phi_- = -N_- i\ \Omega_3 \ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \quad \Rightarrow \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta}\ e^{-iJ} \quad , \quad \Phi_- = -N_- i\ \Omega_3\ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \quad \Rightarrow \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad (\mathrm{d}z+\alpha, \mathrm{d}\overline{z}+\overline{\alpha})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta}\ e^{-iJ} \quad , \quad \Phi_- = -N_- i\ \Omega_3\ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \quad \Rightarrow \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad\qquad (\mathrm{d}z + \alpha, \mathrm{d}\overline{z} + \overline{\alpha})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0\ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0\ .$$

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta}\ e^{-iJ} \quad , \quad \Phi_- = -N_- i\ \Omega_3\ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \quad \Rightarrow \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad\qquad (\mathrm{d}z + \alpha, \mathrm{d}\overline{z} + \overline{\alpha})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0\ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0\ .$$

Among the 34 nilmanifolds, 5 have this topology.

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta} \ e^{-iJ} \quad , \quad \Phi_- = -N_- i \ \Omega_3 \ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \qquad \Rightarrow \qquad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z}) \ , \ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad\qquad (\mathrm{d}z + \alpha, \mathrm{d}\overline{z} + \overline{\alpha}) \ , \ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0 \ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0 \ .$$

Among the 34 nilmanifolds, 5 have this topology.
$\mathrm{d}\alpha$ is the curvature, precisely defined, related to ${f^k}_{ij}$.

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta} \ e^{-iJ} \quad , \quad \Phi_- = -N_- i \ \Omega_3 \ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \qquad \Rightarrow \qquad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z}) \ , \ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad\qquad (\mathrm{d}z + \alpha, \mathrm{d}\overline{z} + \overline{\alpha}) \ , \ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0 \ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0 \ .$$

Among the 34 nilmanifolds, 5 have this topology.
$\mathrm{d}\alpha$ is the curvature, precisely defined, related to ${f^k}_{ij}$.
Constraints can be satisfied on all these $\mathcal{M} \Rightarrow$ solutions!

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta}\ e^{-iJ} \quad , \quad \Phi_- = -N_- i\ \Omega_3 \ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \quad \Rightarrow \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad\qquad (\mathrm{d}z+\alpha, \mathrm{d}\overline{z}+\overline{\alpha})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0\ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0\ .$$

Among the 34 nilmanifolds, 5 have this topology.
$\mathrm{d}\alpha$ is the curvature, precisely defined, related to ${f^k}_{ij}$.
Constraints can be satisfied on all these $\mathcal{M}$ $\Rightarrow$ solutions!

	Algebras	O5	
		$SU(3)$	$\perp$ $SU(2)$
n 3.14	$(0,0,0,12,23,14-35)$	45 + 26	
n 4.4	$(0,0,0,0,12,14+23)$	56	56
n 4.5	$(0,0,0,0,12,34)$	56	56
n 4.6	$(0,0,0,0,12,13)$	56	56
n 4.7	$(0,0,0,0,13+42,14+23)$	56	56
n 5.1	$(0,0,0,0,0,12+34)$	56	56
s 2.5	$(25,-15,r45,-r35,0,0)$	13 + 24	13 + 24

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta}\ e^{-iJ} \quad , \quad \Phi_- = -N_- i\ \Omega_3\ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \quad \Rightarrow \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots 4} \qquad\qquad (\mathrm{d}z+\alpha, \mathrm{d}\overline{z}+\overline{\alpha})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots 4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0\ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0\ .$$

Among the 34 nilmanifolds, 5 have this topology.
$\mathrm{d}\alpha$ is the curvature, precisely defined, related to ${f^k}_{ij}$.
Constraints can be satisfied on all these $\mathcal{M}$ $\Rightarrow$ solutions!

	Algebras	O5	
		$SU(3)$	$\perp\ SU(2)$
n 3.14	$(0, 0, 0, 12, 23, 14-35)$	45 + 26	
n 4.4	$(0, 0, 0, 0, 12, 14+23)$	56	56
n 4.5	$(0, 0, 0, 0, 12, 34)$	56	56
n 4.6	$(0, 0, 0, 0, 12, 13)$	56	56
n 4.7	$(0, 0, 0, 0, 13+42, 14+23)$	56	56
n 5.1	$(0, 0, 0, 0, 0, 12+34)$	56	56
s 2.5	$(25, -15, r45, -r35, 0, 0)$	13 + 24	13 + 24

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta}\ e^{-iJ} \quad , \quad \Phi_- = -N_- i\ \Omega_3\ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \quad \Rightarrow \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad (\mathrm{d}z+\alpha, \mathrm{d}\overline{z}+\overline{\alpha})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0\ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0\ .$$

Among the 34 nilmanifolds, 5 have this topology.
$\mathrm{d}\alpha$ is the curvature, precisely defined, related to ${f^k}_{ij}$.
Constraints can be satisfied on all these $\mathcal{M} \Rightarrow$ solutions!

	Algebras	O5	
		$SU(3)$	$\perp$ $SU(2)$
n 3.14	$(0, 0, 0, 12, 23, 14-35)$	45 + 26	
n 4.4	$(0, 0, 0, 0, 12, 14+23)$	56	56
n 4.5	$(0, 0, 0, 0, 12, 34)$	56	56
n 4.6	$(0, 0, 0, 0, 12, 13)$	56	56
n 4.7	$(0, 0, 0, 0, 13+42, 14+23)$	56	56
n 5.1	$(0, 0, 0, 0, 0, 12+34)$	56	56
s 2.5	$(25, -15, r45, -r35, 0, 0)$	13 + 24	13 + 24

- Solvmanifolds (no T^6 T-dual)

s 2.5

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta} \ e^{-iJ} \quad , \quad \Phi_- = -N_- i \ \Omega_3 \ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \quad \Rightarrow \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z}) \ , \ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad\qquad (\mathrm{d}z + \alpha, \mathrm{d}\overline{z} + \overline{\alpha}) \ , \ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0 \ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0 \ .$$

Among the 34 nilmanifolds, 5 have this topology.
$\mathrm{d}\alpha$ is the curvature, precisely defined, related to ${f^k}_{ij}$.
Constraints can be satisfied on all these $\mathcal{M} \Rightarrow$ solutions!

	Algebras	O5	
		$SU(3)$	$\perp \ SU(2)$
n 3.14	$(0,0,0,12,23,14-35)$	45 + 26	
n 4.4	$(0,0,0,0,12,14+23)$	56	56
n 4.5	$(0,0,0,0,12,34)$	56	56
n 4.6	$(0,0,0,0,12,13)$	56	56
n 4.7	$(0,0,0,0,13+42,14+23)$	56	56
n 5.1	$(0,0,0,0,0,12+34)$	56	56
s 2.5	$(25,-15,r45,-r35,0,0)$	13 + 24	13 + 24

- Solvmanifolds (no T^6 T-dual)

s 2.5

$$\mathrm{d}J = 0$$

$$\mathrm{d}(\mathrm{Im}\,\Omega_3) = 0$$

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta}\ e^{-iJ} \quad , \quad \Phi_- = -N_- i\ \Omega_3\ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \quad \Rightarrow \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad (\mathrm{d}z+\alpha, \mathrm{d}\overline{z}+\overline{\alpha})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0\ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0\ .$$

Among the 34 nilmanifolds, 5 have this topology.
$\mathrm{d}\alpha$ is the curvature, precisely defined, related to $f^k{}_{ij}$.
Constraints can be satisfied on all these $\mathcal{M} \Rightarrow$ solutions!

	Algebras	O5	
		$SU(3)$	$\perp\ SU(2)$
n 3.14	$(0, 0, 0, 12, 23, 14-35)$	$45+26$	
n 4.4	$(0, 0, 0, 0, 12, 14+23)$	56	56
n 4.5	$(0, 0, 0, 0, 12, 34)$	56	56
n 4.6	$(0, 0, 0, 0, 12, 13)$	56	56
n 4.7	$(0, 0, 0, 0, 13+42, 14+23)$	56	56
n 5.1	$(0, 0, 0, 0, 0, 12+34)$	56	56
s 2.5	$(25, -15, r45, -r35, 0, 0)$	$13+24$	$13+24$

- Solvmanifolds (no T^6 T-dual)

$$s\ 2.5 \quad \Rightarrow \quad \mathfrak{g}_{5.17}^{p,-p,r} \approx s\ 2.5 + p\ \mathfrak{g}_{5.7}^{1,-1,-1}$$

$$\mathrm{d}J = 0$$

$$\mathrm{d}(\mathrm{Im}\,\Omega_3) = 0$$

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta} \ e^{-iJ} \quad , \quad \Phi_- = -N_- i \ \Omega_3 \ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \quad \Rightarrow \quad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z}) \ , \ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad\qquad (\mathrm{d}z+\alpha, \mathrm{d}\overline{z}+\overline{\alpha}) \ , \ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0 \ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0 \ .$$

Among the 34 nilmanifolds, 5 have this topology.
$\mathrm{d}\alpha$ is the curvature, precisely defined, related to ${f^k}_{ij}$.
Constraints can be satisfied on all these $\mathcal{M} \Rightarrow$ solutions!

	Algebras	O5	
		$SU(3)$	$\perp\ SU(2)$
n 3.14	$(0,0,0,12,23,14-35)$	$45+26$	
n 4.4	$(0,0,0,0,12,14+23)$	56	56
n 4.5	$(0,0,0,0,12,34)$	56	56
n 4.6	$(0,0,0,0,12,13)$	56	56
n 4.7	$(0,0,0,0,13+42,14+23)$	56	56
n 5.1	$(0,0,0,0,0,12+34)$	56	56
s 2.5	$(25,-15,r45,-r35,0,0)$	$13+24$	$13+24$

- Solvmanifolds (no T^6 T-dual)

$$\begin{array}{ccc} s\ 2.5 & \Rightarrow & \mathfrak{g}_{5.17}^{p,-p,r} \approx s\ 2.5 + p\ \mathfrak{g}_{5.7}^{1,-1,-1} \\ \mathrm{d}J = 0 & & \mathrm{d}J' = p(\lambda-1)\ \dots\ \text{forms} \\ \mathrm{d}(\mathrm{Im}\,\Omega_3) = 0 & & \mathrm{d}(\mathrm{Im}\,\Omega_3') = p(\lambda-1)\ \dots\ \text{forms} \end{array}$$

Examples of constraints and solutions, with $SU(3)$ structure:

$$\Phi_+ = N_+ e^{i\theta}\ e^{-iJ} \quad , \quad \Phi_- = -N_- i\ \Omega_3\ .$$

- Nilmanifolds

$$T^6 = T^2 \times T^4 \qquad \Rightarrow \qquad \begin{array}{ccc} T^2 & \hookrightarrow & \mathcal{M} \\ & & \downarrow \\ & & T^4 \end{array}$$

$$(\mathrm{d}z, \mathrm{d}\overline{z})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4} \qquad\qquad (\mathrm{d}z + \alpha, \mathrm{d}\overline{z} + \overline{\alpha})\ ,\ \mathrm{d}x_{\mathcal{B}}^{i=1\dots4}$$

Constraint on the twist of parameter α:

$$\mathrm{d}\alpha \wedge J_{\mathcal{B}} = 0\ , \qquad \mathrm{d}\alpha \wedge \Omega_{\mathcal{B}} = 0\ .$$

Among the 34 nilmanifolds, 5 have this topology.
$\mathrm{d}\alpha$ is the curvature, precisely defined, related to ${f^k}_{ij}$.
Constraints can be satisfied on all these $\mathcal{M}$ $\Rightarrow$ solutions!

	Algebras	O5	
		$SU(3)$	$\perp\ SU(2)$
n 3.14	$(0,0,0,12,23,14-35)$	$45+26$	
n 4.4	$(0,0,0,0,12,14+23)$	56	56
n 4.5	$(0,0,0,0,12,34)$	56	56
n 4.6	$(0,0,0,0,12,13)$	56	56
n 4.7	$(0,0,0,0,13+42,14+23)$	56	56
n 5.1	$(0,0,0,0,0,12+34)$	56	56
s 2.5	$(25,-15,r45,-r35,0,0)$	$13+24$	$13+24$

- Solvmanifolds (no T^6 T-dual)

$$s\ 2.5 \qquad \Rightarrow \qquad \mathfrak{g}_{5.17}^{p,-p,r} \approx s\ 2.5 + p\ \mathfrak{g}_{5.7}^{1,-1,-1}$$

$$\mathrm{d}J = 0 \qquad\qquad \mathrm{d}J' = p(\lambda-1)\ \dots \text{ forms}$$

$$\mathrm{d}(\mathrm{Im}\,\Omega_3) = 0 \qquad\qquad \mathrm{d}(\mathrm{Im}\,\Omega_3') = p(\lambda-1)\ \dots \text{ forms}$$

New SUSY solution on $\mathfrak{g}_{5.17}^{p,-p,r}$ for $\lambda = 1$.

De Sitter solutions and SUSY breaking sources

Finding de Sitter solutions

$4d$ = de Sitter, cosmological interest.

De Sitter solutions and SUSY breaking sources

Finding de Sitter solutions

$4d$ = de Sitter, cosmological interest. Get solutions: difficult!

De Sitter solutions and SUSY breaking sources

Finding de Sitter solutions

$4d$ = de Sitter, cosmological interest. Get solutions: difficult!
Technical difficulty: non-SUSY vacua
$\hookrightarrow$ no use of the previous methods...

De Sitter solutions and SUSY breaking sources

Finding de Sitter solutions

$4d$ = de Sitter, cosmological interest. Get solutions: difficult!
Technical difficulty: non-SUSY vacua
$\hookrightarrow$ no use of the previous methods...

	$10d$	$4d$
S	$\frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{\lvert g_{10}\rvert}\ e^{-2\phi}R_{10}$ $+4e^{-2\phi}\lvert\nabla\phi\rvert^2+$ fluxes $+$ sources	$M_p^2\int \mathrm{d}^4x\sqrt{\lvert g_4\rvert}\Big(R_4+...-\frac{1}{M_p^2}V\Big)$

De Sitter solutions and SUSY breaking sources

Finding de Sitter solutions

$4d$ = de Sitter, cosmological interest. Get solutions: difficult!
Technical difficulty: non-SUSY vacua
$\hookrightarrow$ no use of the previous methods...

	$10d$	$4d$
S	$\frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{\lvert g_{10}\rvert}\ e^{-2\phi}R_{10}$ $+4e^{-2\phi}\lvert\nabla\phi\rvert^2 +$ fluxes $+$ sources	$M_p^2\int \mathrm{d}^4x\sqrt{\lvert g_4\rvert}\Big(R_4 + ... - \frac{1}{M_p^2}V\Big)$
1.	Solve e.o.m.	$\partial_\varphi V = 0$

De Sitter solutions and SUSY breaking sources

Finding de Sitter solutions

$4d$ = de Sitter, cosmological interest. Get solutions: difficult!
Technical difficulty: non-SUSY vacua
$\hookrightarrow$ no use of the previous methods...

	$10d$	$4d$
S	$\frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{\lvert g_{10}\rvert}\ e^{-2\phi}R_{10}$ $+4e^{-2\phi}\lvert\nabla\phi\rvert^2 +$ fluxes $+$ sources	$M_p^2\int \mathrm{d}^4x\sqrt{\lvert g_4\rvert}\Big(R_4 + ... - \frac{1}{M_p^2}V\Big)$
1.	Solve e.o.m.	$\partial_\varphi V = 0$

1. $10d$: without SUSY, technically difficult!
   $4d$: coupled equations $\Rightarrow$ numerics...

De Sitter solutions and SUSY breaking sources

Finding de Sitter solutions

$4d$ = de Sitter, cosmological interest. Get solutions: difficult!
Technical difficulty: non-SUSY vacua
$\hookrightarrow$ no use of the previous methods...

	$10d$	$4d$
S	$\frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{\lvert g_{10}\rvert}\ e^{-2\phi}R_{10}$ $+4e^{-2\phi}\lvert\nabla\phi\rvert^2 +$ fluxes $+$ sources	$M_p^2\int \mathrm{d}^4x\sqrt{\lvert g_4\rvert}\Big(R_4 + ... - \frac{1}{M_p^2}V\Big)$
1.	Solve e.o.m.	$\partial_\varphi V = 0$
2.	$\Lambda = \frac{1}{4}R_4 > 0$	$\Lambda = \frac{1}{2M_p^2}V\vert_0 > 0$

1. $10d$: without SUSY, technically difficult!
   $4d$: coupled equations $\Rightarrow$ numerics...

Finding de Sitter solutions

$4d$ = de Sitter, cosmological interest. Get solutions: difficult!
Technical difficulty: non-SUSY vacua
$\hookrightarrow$ no use of the previous methods...

	$10d$	$4d$
S	$\frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{\lvert g_{10}\rvert}\ e^{-2\phi}R_{10}$ $+4e^{-2\phi}\lvert\nabla\phi\rvert^2 +$ fluxes $+$ sources	$M_p^2\int \mathrm{d}^4x\sqrt{\lvert g_4\rvert}\Big(R_4 + ... - \frac{1}{M_p^2}\,V\Big)$
1.	Solve e.o.m.	$\partial_\varphi V = 0$
2.	$\Lambda = \frac{1}{4}R_4 > 0$	$\Lambda = \frac{1}{2M_p^2}\,V\vert_0 > 0$

1. $10d$: without SUSY, technically difficult!
   $4d$: coupled equations $\Rightarrow$ numerics...
2. $\Lambda > 0$: difficult for classical SUGRA backgrounds...

De Sitter solutions and SUSY breaking sources

Finding de Sitter solutions

$4d$ = de Sitter, cosmological interest. Get solutions: difficult!
Technical difficulty: non-SUSY vacua
$\hookrightarrow$ no use of the previous methods...

	$10d$	$4d$
S	$\frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{\|g_{10}\|}\ e^{-2\phi}R_{10}$ $+4e^{-2\phi}\|\nabla\phi\|^2 +$ fluxes $+$ sources	$M_p^2\int \mathrm{d}^4x\sqrt{\|g_4\|}\Big(R_4 + ... - \frac{1}{M_p^2}V\Big)$
1.	Solve e.o.m.	$\partial_\varphi V = 0$
2.	$\Lambda = \frac{1}{4}R_4 > 0$	$\Lambda = \frac{1}{2M_p^2}V\|_0 > 0$
3.	...	$4d$ stability: $\partial_\varphi V\|_0 > 0$

1. $10d$: without SUSY, technically difficult!
   $4d$: coupled equations $\Rightarrow$ numerics...
2. $\Lambda > 0$: difficult for classical SUGRA backgrounds...

De Sitter solutions and SUSY breaking sources

Finding de Sitter solutions

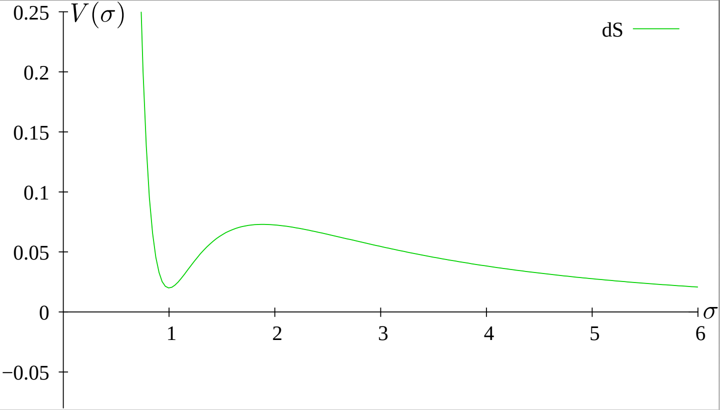
$4d$ = de Sitter, cosmological interest. Get solutions: difficult!
Technical difficulty: non-SUSY vacua
$\hookrightarrow$ no use of the previous methods...

	$10d$	$4d$
S	$\frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{\lvert g_{10}\rvert}\ e^{-2\phi}R_{10}$ $+4e^{-2\phi}\lvert\nabla\phi\rvert^2 +$ fluxes $+$ sources	$M_p^2\int \mathrm{d}^4x\sqrt{\lvert g_4\rvert}\Big(R_4 + ... - \frac{1}{M_p^2}V\Big)$
1.	Solve e.o.m.	$\partial_\varphi V = 0$
2.	$\Lambda = \frac{1}{4}R_4 > 0$	$\Lambda = \frac{1}{2M_p^2}V\rvert_0 > 0$
3.	...	$4d$ stability: $\partial_\varphi V\rvert_0 > 0$

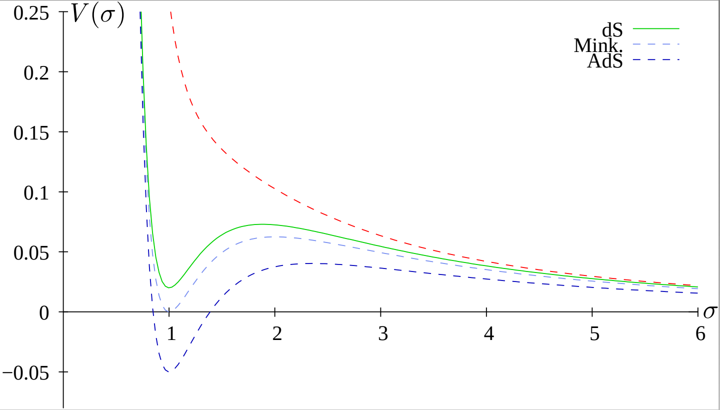
1. $10d$: without SUSY, technically difficult!
   $4d$: coupled equations $\Rightarrow$ numerics...
2. $\Lambda > 0$: difficult for classical SUGRA backgrounds...
3. Difficult! Up to date: no stable dS solution with classical $10d$ SUGRA ingredients.

	$10d$	$4d$
S	$\frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{\lvert g_{10}\rvert}\ e^{-2\phi}R_{10}$ $+4e^{-2\phi}\lvert\nabla\phi\rvert^2 +$ fluxes $+$ sources	$M_p^2\int \mathrm{d}^4x\sqrt{\lvert g_4\rvert}\Big(R_4 + ... - \frac{1}{M_p^2}V\Big)$
1.	Solve e.o.m.	$\partial_\varphi V = 0$
2.	$\Lambda = \frac{1}{4}R_4 > 0$	$\Lambda = \frac{1}{2M_p^2}V\vert_0 > 0$
3.	...	$4d$ stability: $\partial_\varphi V\vert_0 > 0$

	$10d$	$4d$
S	$\frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{\lvert g_{10}\rvert}\ e^{-2\phi}R_{10}$ $+4e^{-2\phi}\lvert\nabla\phi\rvert^2 +$ fluxes $+$ sources	$M_p^2\int \mathrm{d}^4x\sqrt{\lvert g_4\rvert}\Big(R_4 + ... - \frac{1}{M_p^2}V\Big)$
1.	Solve e.o.m.	$\partial_\varphi V = 0$
2.	$\Lambda = \frac{1}{4}R_4 > 0$	$\Lambda = \frac{1}{2M_p^2}V\rvert_0 > 0$
3.	...	$4d$ stability: $\partial_\varphi V\rvert_0 > 0$



	$10d$	$4d$
S	$\frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{\lvert g_{10}\rvert}\ e^{-2\phi} R_{10}$ $+4e^{-2\phi}\lvert\nabla\phi\rvert^2 +$ fluxes $+$ sources	$M_p^2 \int \mathrm{d}^4x\sqrt{\lvert g_4\rvert}\Big(R_4 + \ldots - \frac{1}{M_p^2} V\Big)$
1.	Solve e.o.m.	$\partial_\varphi V = 0$
2.	$\Lambda = \frac{1}{4} R_4 > 0$	$\Lambda = \frac{1}{2M_p^2} V\vert_0 > 0$
3.	$\ldots$	$4d$ stability: $\partial_\varphi V\vert_0 > 0$



Several no-go theorems and ways of circumventing them:

Several no-go theorems and ways of circumventing them:

- need O-planes (negatively charged RR sources)
  $\Rightarrow$ we take O6/D6 in IIA.

Several no-go theorems and ways of circumventing them:

- need O-planes (negatively charged RR sources)
  $\Rightarrow$ we take O6/D6 in IIA.
  Intersecting sources $\Rightarrow$ smeared sources and constant dilaton $e^{\phi} = g_s$.

Several no-go theorems and ways of circumventing them:

- need O-planes (negatively charged RR sources)
  $\Rightarrow$ we take O6/D6 in IIA.
  Intersecting sources $\Rightarrow$ smeared sources and constant dilaton $e^{\phi} = g_s$.
- With O6, $\mathcal{M}$ must have $R_6 < 0$ and $F_0 \neq 0$
  $\hookrightarrow$ we consider F_0, F_2 and H.

Several no-go theorems and ways of circumventing them:

- need O-planes (negatively charged RR sources)
  $\Rightarrow$ we take O6/D6 in IIA.
  Intersecting sources $\Rightarrow$ smeared sources and constant dilaton $e^{\phi} = g_s$.
- With O6, $\mathcal{M}$ must have $R_6 < 0$ and $F_0 \neq 0$
  $\hookrightarrow$ we consider F_0, F_2 and H.

In presence of SUSY (calibrated) sources, combine e.o.m.

Several no-go theorems and ways of circumventing them:

- need O-planes (negatively charged RR sources)
$\Rightarrow$ we take O6/D6 in IIA.
Intersecting sources $\Rightarrow$ smeared sources and constant dilaton $e^{\phi} = g_s$.
- With O6, $\mathcal{M}$ must have $R_6 < 0$ and $F_0 \neq 0$
$\hookrightarrow$ we consider F_0, F_2 and H.

In presence of SUSY (calibrated) sources, combine e.o.m.:

$$R_4 = \frac{2}{3}(g_s^2|F_0|^2 - |H|^2)$$

$$R_6 + \frac{1}{2}g_s^2|F_2|^2 + \frac{3}{2}(g_s^2|F_0|^2 - |H|^2) = 0$$

Several no-go theorems and ways of circumventing them:

- need O-planes (negatively charged RR sources)
  $\Rightarrow$ we take O6/D6 in IIA.
  Intersecting sources $\Rightarrow$ smeared sources and constant dilaton $e^{\phi} = g_s$.
- With O6, $\mathcal{M}$ must have $R_6 < 0$ and $F_0 \neq 0$
  $\hookrightarrow$ we consider F_0, F_2 and H.

In presence of SUSY (calibrated) sources, combine e.o.m.:

$$R_4 = \frac{2}{3}(g_s^2|F_0|^2 - |H|^2)$$
$$R_6 + \frac{1}{2}g_s^2|F_2|^2 + \frac{3}{2}(g_s^2|F_0|^2 - |H|^2) = 0$$

$\hookrightarrow F_0 \neq 0,\ R_6 < 0.$

Several no-go theorems and ways of circumventing them:

- need O-planes (negatively charged RR sources)
  $\Rightarrow$ we take O6/D6 in IIA.
  Intersecting sources $\Rightarrow$ smeared sources and constant dilaton $e^{\phi} = g_s$.
- With O6, $\mathcal{M}$ must have $R_6 < 0$ and $F_0 \neq 0$
  $\hookrightarrow$ we consider F_0, F_2 and H.

In presence of SUSY (calibrated) sources, combine e.o.m.:

$$R_4 = \frac{2}{3}(g_s^2|F_0|^2 - |H|^2)$$

$$R_6 + \frac{1}{2}g_s^2|F_2|^2 + \frac{3}{2}(g_s^2|F_0|^2 - |H|^2) = 0$$

$\hookrightarrow F_0 \neq 0,\ R_6 < 0.$
F_0 alone to balance H...

Several no-go theorems and ways of circumventing them:

- need O-planes (negatively charged RR sources)
  $\Rightarrow$ we take O6/D6 in IIA.
  Intersecting sources $\Rightarrow$ smeared sources and constant dilaton $e^{\phi} = g_s$.
- With O6, $\mathcal{M}$ must have $R_6 < 0$ and $F_0 \neq 0$
  $\hookrightarrow$ we consider F_0, F_2 and H.

In presence of SUSY (calibrated) sources, combine e.o.m.:

$$R_4 = \frac{2}{3}(g_s^2|F_0|^2 - |H|^2)$$

$$R_6 + \frac{1}{2}g_s^2|F_2|^2 + \frac{3}{2}(g_s^2|F_0|^2 - |H|^2) = 0$$

$\hookrightarrow F_0 \neq 0,\ R_6 < 0.$
F_0 alone to balance H... More RR fluxes does not help.

Several no-go theorems and ways of circumventing them:

- need O-planes (negatively charged RR sources)
  $\Rightarrow$ we take O6/D6 in IIA.
  Intersecting sources $\Rightarrow$ smeared sources and constant dilaton $e^{\phi} = g_s$.
- With O6, $\mathcal{M}$ must have $R_6 < 0$ and $F_0 \neq 0$
  $\hookrightarrow$ we consider F_0, F_2 and H.

In presence of SUSY (calibrated) sources, combine e.o.m.:

$$R_4 = \frac{2}{3}(g_s^2|F_0|^2 - |H|^2)$$

$$R_6 + \frac{1}{2}g_s^2|F_2|^2 + \frac{3}{2}(g_s^2|F_0|^2 - |H|^2) = 0$$

$\hookrightarrow F_0 \neq 0,\ R_6 < 0.$
F_0 alone to balance H... More RR fluxes does not help.
$\hookrightarrow$ classical $10d$ SUGRA dS vacuum is difficult!

Several no-go theorems and ways of circumventing them:

- need O-planes (negatively charged RR sources)
  $\Rightarrow$ we take O6/D6 in IIA.
  Intersecting sources $\Rightarrow$ smeared sources and constant dilaton $e^{\phi} = g_s$.
- With O6, $\mathcal{M}$ must have $R_6 < 0$ and $F_0 \neq 0$
  $\hookrightarrow$ we consider F_0, F_2 and H.

In presence of SUSY (calibrated) sources, combine e.o.m.:

$$R_4 = \frac{2}{3}(g_s^2|F_0|^2 - |H|^2)$$

$$R_6 + \frac{1}{2}g_s^2|F_2|^2 + \frac{3}{2}(g_s^2|F_0|^2 - |H|^2) = 0$$

$\hookrightarrow F_0 \neq 0,\ R_6 < 0.$
F_0 alone to balance H... More RR fluxes does not help.
$\hookrightarrow$ classical $10d$ SUGRA dS vacuum is difficult!
Additional ingredients: KK monopoles and Wilson lines, non-geometric fluxes, α' corrections...

SUSY breaking sources

- Proposal: ansatz for SUSY breaking sources.
  Non-SUSY vacua.

SUSY breaking sources

- Proposal: ansatz for SUSY breaking sources. Non-SUSY vacua.
  S_{sources}? $\Rightarrow$ T_{MN}, Einstein equations.
  $$\frac{1}{\sqrt{|g_{10}|}} \frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2}\, T_{MN}$$

SUSY breaking sources

- Proposal: ansatz for SUSY breaking sources.
  Non-SUSY vacua.
  S_{sources}? $\Rightarrow$ T_{MN}, Einstein equations.
  $$\frac{1}{\sqrt{|g_{10}|}}\frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2}\,T_{MN}$$
- SUSY case: p-dimensional source space-filling (IIA).

SUSY breaking sources

- Proposal: ansatz for SUSY breaking sources.
  Non-SUSY vacua.
  S_{sources}? $\Rightarrow T_{MN}$, Einstein equations.
  $$\frac{1}{\sqrt{|g_{10}|}}\frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2}\, T_{MN}$$
- SUSY case: p-dimensional source space-filling (IIA).
  Internal κ-sym. cond. $(\eta_{\pm}^{1,2})$: brane preserves bulk SUSY.

SUSY breaking sources

- Proposal: ansatz for SUSY breaking sources.
  Non-SUSY vacua.
  S_{sources}? $\Rightarrow$ T_{MN}, Einstein equations.
  $$\frac{1}{\sqrt{|g_{10}|}} \frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2} T_{MN}$$
- SUSY case: p-dimensional source space-filling (IIA).
  Internal κ-sym. cond. $(\eta_{\pm}^{1,2})$: brane preserves bulk SUSY.
  Rewritten in terms of GCG (Φ_-) as
  $$(i^*[\Phi_-] \wedge e^{\mathcal{F}})_\Sigma = \frac{i||\Phi_-||}{8} \sqrt{|i^*[g] + \mathcal{F}|}\, \mathrm{d}^\Sigma \sigma \;\Rightarrow\; S_{\text{DBI}}(\Phi_-)...$$

SUSY breaking sources

- Proposal: ansatz for SUSY breaking sources.
  Non-SUSY vacua.
  S_{sources}? $\Rightarrow$ T_{MN}, Einstein equations.
  $$\frac{1}{\sqrt{|g_{10}|}} \frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2} T_{MN}$$
- SUSY case: p-dimensional source space-filling (IIA).
  Internal κ-sym. cond. $(\eta_{\pm}^{1,2})$: brane preserves bulk SUSY.
  Rewritten in terms of GCG (Φ_-) as
  $$(i^*[\Phi_-] \wedge e^{\mathcal{F}})_\Sigma = \frac{i||\Phi_-||}{8} \sqrt{|i^*[g] + \mathcal{F}|}\, \mathrm{d}^\Sigma \sigma \;\Rightarrow\; S_{\text{DBI}}(\Phi_-)...$$
- Non-SUSY case: consider a general expansion:
  $\eta_+, \eta_-, \gamma^m \eta_+, \gamma^m \eta_- \Rightarrow \Phi_-, \gamma^m \Phi_+, \Phi_+ \gamma^m, \gamma^m \Phi_- \gamma^n, \dots$

SUSY breaking sources

- Proposal: ansatz for SUSY breaking sources.
  Non-SUSY vacua.
  S_{sources}? $\Rightarrow$ T_{MN}, Einstein equations.
  $$\frac{1}{\sqrt{|g_{10}|}}\frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2}\, T_{MN}$$
- SUSY case: p-dimensional source space-filling (IIA).
  Internal κ-sym. cond. $(\eta^{1,2}_{\pm})$: brane preserves bulk SUSY.
  Rewritten in terms of GCG (Φ_-) as
  $$(i^*[\Phi_-]\wedge e^{\mathcal{F}})_\Sigma = \frac{i||\Phi_-||}{8}\ \sqrt{|i^*[g]+\mathcal{F}|}\,\mathrm{d}^\Sigma\sigma \ \Rightarrow\ S_{\text{DBI}}(\Phi_-)...$$
- Non-SUSY case: consider a general expansion:
  $\eta_+, \eta_-, \gamma^m\eta_+, \gamma^m\eta_- \Rightarrow \Phi_-, \gamma^m\Phi_+, \Phi_+\gamma^m, \gamma^m\Phi_-\gamma^n, \ldots$
  $$X_- = \frac{8}{||\Phi_-||}\Big(\alpha_0\Phi_- + \widetilde{\alpha}_0\overline{\Phi}_- + \alpha_{mn}\gamma^m\Phi_-\gamma^n + \widetilde{\alpha}_{mn}\gamma^m\overline{\Phi}_-\gamma^n$$
  $$+\alpha^L_m\gamma^m\Phi_+ + \widetilde{\alpha}^L_m\gamma^m\overline{\Phi}_+ + \alpha^R_n\Phi_+\gamma^n + \widetilde{\alpha}^R_n\overline{\Phi}_+\gamma^n\Big)$$

SUSY breaking sources

- Proposal: ansatz for SUSY breaking sources.
  Non-SUSY vacua.
  $S_{\rm sources}$? $\Rightarrow$ T_{MN}, Einstein equations.
  $$\frac{1}{\sqrt{|g_{10}|}}\frac{\delta S_{\rm sources}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2}\,T_{MN}$$
- SUSY case: p-dimensional source space-filling (IIA).
  Internal κ-sym. cond. $(\eta_\pm^{1,2})$: brane preserves bulk SUSY.
  Rewritten in terms of GCG (Φ_-) as
  $$(i^*[\Phi_-]\wedge e^{\mathcal{F}})_\Sigma = \frac{i||\Phi_-||}{8}\,\sqrt{|i^*[g]+\mathcal{F}|}\;{\rm d}^\Sigma\sigma \;\Rightarrow\; S_{\rm DBI}(\Phi_-)...$$
- Non-SUSY case: consider a general expansion:
  $\eta_+,\eta_-,\gamma^m\eta_+,\gamma^m\eta_- \Rightarrow \Phi_-,\gamma^m\Phi_+,\Phi_+\gamma^m,\gamma^m\Phi_-\gamma^n,\dots$
  $$X_- = \frac{8}{||\Phi_-||}\Big(\alpha_0\Phi_- + \widetilde{\alpha}_0\overline{\Phi}_- + \alpha_{mn}\gamma^m\Phi_-\gamma^n + \widetilde{\alpha}_{mn}\gamma^m\overline{\Phi}_-\gamma^n$$
  $$+\alpha^L_m\gamma^m\Phi_+ + \widetilde{\alpha}^L_m\gamma^m\overline{\Phi}_+ + \alpha^R_n\Phi_+\gamma^n + \widetilde{\alpha}^R_n\overline{\Phi}_+\gamma^n\Big)$$

  $$\text{Proposal}: (i^*[X_-]\wedge e^{\mathcal{F}})_\Sigma = i\,\sqrt{|i^*[g]+\mathcal{F}|}\;{\rm d}^\Sigma\sigma$$

SUSY breaking sources

- Proposal: ansatz for SUSY breaking sources.
  Non-SUSY vacua.
  S_{sources}? $\Rightarrow$ T_{MN}, Einstein equations.
  $$\frac{1}{\sqrt{|g_{10}|}} \frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2} T_{MN}$$
- SUSY case: p-dimensional source space-filling (IIA).
  Internal κ-sym. cond. $(\eta_{\pm}^{1,2})$: brane preserves bulk SUSY.
  Rewritten in terms of GCG (Φ_-) as
  $$(i^*[\Phi_-] \wedge e^{\mathcal{F}})_\Sigma = \frac{i||\Phi_-||}{8} \sqrt{|i^*[g]+\mathcal{F}|}\, \mathrm{d}^\Sigma\sigma \;\Rightarrow\; S_{\text{DBI}}(\Phi_-)...$$
- Non-SUSY case: consider a general expansion:
  $\eta_+, \eta_-, \gamma^m\eta_+, \gamma^m\eta_- \Rightarrow \Phi_-, \gamma^m\Phi_+, \Phi_+\gamma^m, \gamma^m\Phi_-\gamma^n, \ldots$
  $$X_- = \frac{8}{||\Phi_-||}\Big(\alpha_0\Phi_- + \widetilde{\alpha}_0\overline{\Phi}_- + \alpha_{mn}\gamma^m\Phi_-\gamma^n + \widetilde{\alpha}_{mn}\gamma^m\overline{\Phi}_-\gamma^n + \alpha_m^L\gamma^m\Phi_+ + \widetilde{\alpha}_m^L\gamma^m\overline{\Phi}_+ + \alpha_n^R\Phi_+\gamma^n + \widetilde{\alpha}_n^R\overline{\Phi}_+\gamma^n\Big)$$
  $$\text{Proposal}: (i^*[X_-] \wedge e^{\mathcal{F}})_\Sigma = i\sqrt{|i^*[g]+\mathcal{F}|}\, \mathrm{d}^\Sigma\sigma \;\Rightarrow\; S_{\text{DBI}}(X_-)...$$

SUSY breaking sources

- Proposal: ansatz for SUSY breaking sources.
  Non-SUSY vacua.
  $S_{\rm sources}$? $\Rightarrow$ T_{MN}, Einstein equations.
  $$\frac{1}{\sqrt{|g_{10}|}} \frac{\delta S_{\rm sources}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2} T_{MN}$$
- SUSY case: p-dimensional source space-filling (IIA).
  Internal κ-sym. cond. $(\eta_\pm^{1,2})$: brane preserves bulk SUSY.
  Rewritten in terms of GCG (Φ_-) as
  $$(i^*[\Phi_-] \wedge e^{\mathcal{F}})_\Sigma = \frac{i||\Phi_-||}{8} \sqrt{|i^*[g] + \mathcal{F}|}\, \mathrm{d}^\Sigma\sigma \;\Rightarrow\; S_{\rm DBI}(\Phi_-)...$$
- Non-SUSY case: consider a general expansion:
  $\eta_+, \eta_-, \gamma^m\eta_+, \gamma^m\eta_- \Rightarrow \Phi_-, \gamma^m\Phi_+, \Phi_+\gamma^m, \gamma^m\Phi_-\gamma^n, \ldots$
  $$X_- = \frac{8}{||\Phi_-||}\Big(\alpha_0\Phi_- + \widetilde{\alpha}_0\overline{\Phi}_- + \alpha_{mn}\gamma^m\Phi_-\gamma^n + \widetilde{\alpha}_{mn}\gamma^m\overline{\Phi}_-\gamma^n$$
  $$+\alpha_m^L\gamma^m\Phi_+ + \widetilde{\alpha}_m^L\gamma^m\overline{\Phi}_+ + \alpha_n^R\Phi_+\gamma^n + \widetilde{\alpha}_n^R\overline{\Phi}_+\gamma^n\Big)$$
  $$\text{Proposal}: (i^*[X_-] \wedge e^{\mathcal{F}})_\Sigma = i\sqrt{|i^*[g] + \mathcal{F}|}\, \mathrm{d}^\Sigma\sigma \;\Rightarrow\; S_{\rm DBI}(X_-)...$$
- Go further: replace Φ_- with X_- in the SUSY conditions.
  $\hookrightarrow$ first order formalism...

- Consequences:

- Consequences: $\neq$ metric dependence, new T_{MN}

$$\frac{1}{\sqrt{|g_{10}|}} \frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2} T_{MN}$$

- Consequences: $\neq$ metric dependence, new T_{MN}

$$\frac{1}{\sqrt{|g_{10}|}}\frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2}\, T_{MN}$$

Einstein equations easier to solve

- Consequences: $\neq$ metric dependence, new T_{MN}

$$\frac{1}{\sqrt{|g_{10}|}}\frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2}\,T_{MN}$$

Einstein equations easier to solve, and new term in R_4:

$$R_4 = \frac{2}{3}\left(\frac{g_s}{2}(T_0 - T) + g_s^2|F_0|^2 - |H|^2\right)$$

$T = g^{MN}T_{MN},\ T_0$ SUSY trace.

- Consequences: $\neq$ metric dependence, new T_{MN}

$$\frac{1}{\sqrt{|g_{10}|}}\frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2}\,T_{MN}$$

Einstein equations easier to solve, and new term in R_4:

$$R_4 = \frac{2}{3}\left(\frac{g_s}{2}(T_0 - T) + g_s^2|F_0|^2 - |H|^2\right)$$

$T = g^{MN}T_{MN}$, T_0 SUSY trace. $T_0 > 0$. For us: $T_0 - T > 0$.

- Consequences: $\neq$ metric dependence, new T_{MN}

$$\frac{1}{\sqrt{|g_{10}|}} \frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2} T_{MN}$$

  Einstein equations easier to solve, and new term in R_4:

$$R_4 = \frac{2}{3}\left(\frac{g_s}{2}(T_0 - T) + g_s^2|F_0|^2 - |H|^2\right)$$

  $T = g^{MN} T_{MN}$, T_0 SUSY trace. $T_0 > 0$. For us: $T_0 - T > 0$.
- De Sitter solution

- Consequences: $\neq$ metric dependence, new T_{MN}

$$\frac{1}{\sqrt{|g_{10}|}}\frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2}\,T_{MN}$$

  Einstein equations easier to solve, and new term in R_4:

$$R_4 = \frac{2}{3}\left(\frac{g_s}{2}(T_0 - T) + g_s^2|F_0|^2 - |H|^2\right)$$

  $T = g^{MN}T_{MN}$, T_0 SUSY trace. $T_0 > 0$. For us: $T_0 - T > 0$.

- De Sitter solution, on $\mathfrak{g}_{5.17}^{p,-p,r} \approx s\ 2.5 + p\ \mathfrak{g}_{5.7}^{1,-1,-1}$

$$\mathrm{d}J' = p(\lambda-1)\ \text{... forms}\ ,\ \mathrm{d}(\mathrm{Im}\,\Omega_3') = p(\lambda-1)\ \text{... forms}$$

- Consequences: $\neq$ metric dependence, new T_{MN}

$$\frac{1}{\sqrt{|g_{10}|}} \frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2} T_{MN}$$

Einstein equations easier to solve, and new term in R_4:

$$R_4 = \frac{2}{3}\left(\frac{g_s}{2}(T_0 - T) + g_s^2|F_0|^2 - |H|^2\right)$$

$T = g^{MN} T_{MN}$, T_0 SUSY trace. $T_0 > 0$. For us: $T_0 - T > 0$.

- De Sitter solution, on $\mathfrak{g}_{5.17}^{p,-p,r} \approx s\ 2.5 + p\ \mathfrak{g}_{5.7}^{1,-1,-1}$

$$\mathrm{d}J' = p(\lambda - 1) \ \text{... forms} \ , \ \mathrm{d}(\mathrm{Im}\,\Omega_3') = p(\lambda - 1) \ \text{... forms}$$

$p \neq 0$, $\lambda \neq 1$: deviation from SUSY case.
Starting ansatz for our solution. F_0, F_2, H, O6/D6 sources.

- Consequences: $\neq$ metric dependence, new T_{MN}
$$\frac{1}{\sqrt{|g_{10}|}}\frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2}\,T_{MN}$$
Einstein equations easier to solve, and new term in R_4:
$$R_4 = \frac{2}{3}\left(\frac{g_s}{2}(T_0 - T) + g_s^2|F_0|^2 - |H|^2\right)$$
$T = g^{MN}T_{MN}$, T_0 SUSY trace. $T_0 > 0$. For us: $T_0 - T > 0$.
- De Sitter solution, on $\mathfrak{g}_{5.17}^{p,-p,r} \approx s\ 2.5 + p\ \mathfrak{g}_{5.7}^{1,-1,-1}$
$$\mathrm{d}J' = p(\lambda - 1)\ \text{... forms}\ ,\ \mathrm{d}(\mathrm{Im}\,\Omega_3') = p(\lambda - 1)\ \text{... forms}$$
$p \neq 0$, $\lambda \neq 1$: deviation from SUSY case.
Starting ansatz for our solution. F_0, F_2, H, O6/D6 sources.
Initial intuition: $R_6 \ll 0$ for $p \nearrow$ or $(\lambda - 1)^2 \nearrow$.

- Consequences: $\neq$ metric dependence, new T_{MN}

$$\frac{1}{\sqrt{|g_{10}|}} \frac{\delta S_{\text{sources}}}{\delta g^{MN}} = -\frac{e^{-\phi}}{4\kappa^2} T_{MN}$$

Einstein equations easier to solve, and new term in R_4:

$$R_4 = \frac{2}{3}\left(\frac{g_s}{2}(T_0 - T) + g_s^2|F_0|^2 - |H|^2\right)$$

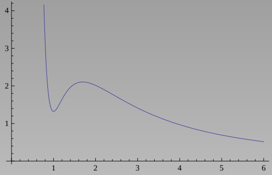
$T = g^{MN}T_{MN}$, T_0 SUSY trace. $T_0 > 0$. For us: $T_0 - T > 0$.

- De Sitter solution, on $\mathfrak{g}_{5.17}^{p,-p,r} \approx s\ 2.5 + p\ \mathfrak{g}_{5.7}^{1,-1,-1}$

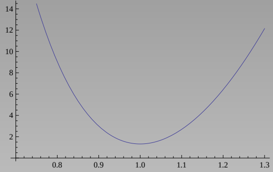
$$\mathrm{d}J' = p(\lambda - 1)\ \ldots\ \text{forms}\ ,\ \mathrm{d}(\mathrm{Im}\,\Omega_3') = p(\lambda - 1)\ \ldots\ \text{forms}$$

$p \neq 0$, $\lambda \neq 1$: deviation from SUSY case.
Starting ansatz for our solution. F_0, F_2, H, O6/D6 sources.
Initial intuition: $R_6 \ll 0$ for $p \nearrow$ or $(\lambda - 1)^2 \nearrow$.

$4d$ stability: in dilaton σ and volume ρ:



$\frac{1}{M_p^2} V(\sigma, \rho = 1)$



$\frac{1}{M_p^2} V(\sigma = 1, \rho)$

Conclusion

Conclusion

- SUSY $10d$ solutions of SUGRA,
  on $4d$ Minkowski and $\mathcal{M}$ = solvmanifold.
  Twist transformation: relates and generates such solutions,
  local $O(d, d)$ transformation in GCG.
  De Sitter solutions, proposal for SUSY breaking sources.

Conclusion

- SUSY $10d$ solutions of SUGRA,
  on $4d$ Minkowski and $\mathcal{M}$ = solvmanifold.
  Twist transformation: relates and generates such solutions,
  local $O(d,d)$ transformation in GCG.
  De Sitter solutions, proposal for SUSY breaking sources.

- Understand the twist transformation...
  Better justification for sources ansatz.
  Calibration, stability?
  1^{st} order formalism.
- Heterotic string: twist transformation and GCG there.
  Dualities relating flux vacua of het. to those of type II.
- Non-geometry.
- KK reduction: effective actions, model building.