

# Geometrical frustration, phase transitions and dynamical order

The  $Tb_2M_2O_7$  compounds ( $M = Ti, Sn$ )

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spsms



# Outline

- 1 Introduction
- 2 The  $Tb_2M_2O_7$  compounds ( $M = Ti, Sn$ )
- 3  $Tb_2Ti_2O_7$  : sample study
- 4 Crystal field levels
- 5 Conclusions

# Outline

## 1 Introduction

- geometrical frustration
- connectivity and degeneracy
- spin ice

## 2 The $Tb_2M_2O_7$ compounds ( $M = Ti, Sn$ )

## 3 $Tb_2Ti_2O_7$ : sample study

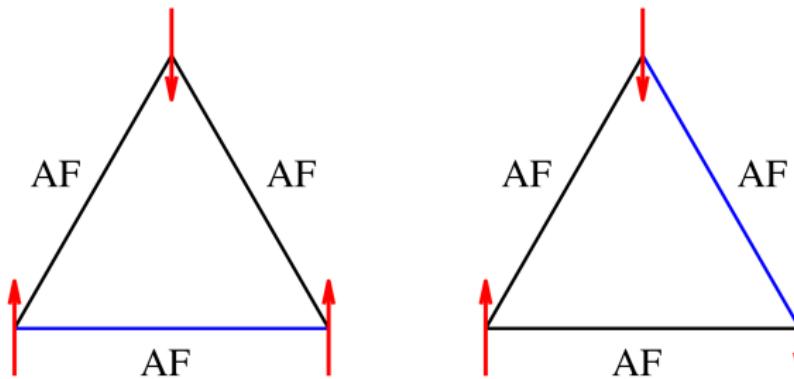
## 4 Crystal field levels

## 5 Conclusions

# Introduction

## geometrical frustration

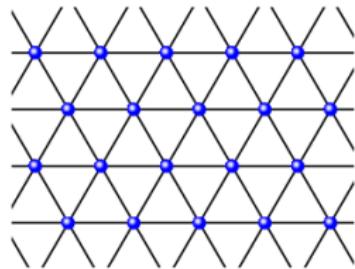
- impossibility to satisfy simultaneously all the magnetic interactions
- frustration index :  $f = |\theta_{CW}|/T_c$ ; frustration if  $f \gtrsim 5$ 
  - $\theta_{CW}$  : Curie-Weiss temperature
  - $T_c$  : transition temperature
- example of 2D Ising antiferromagnets



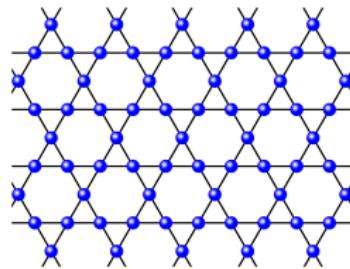
# Introduction

## geometrical frustration

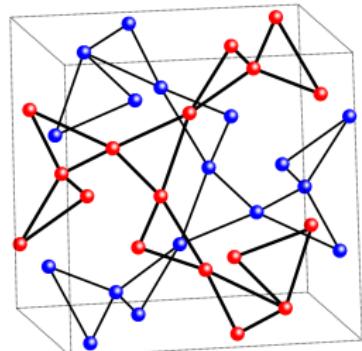
- triangular lattice



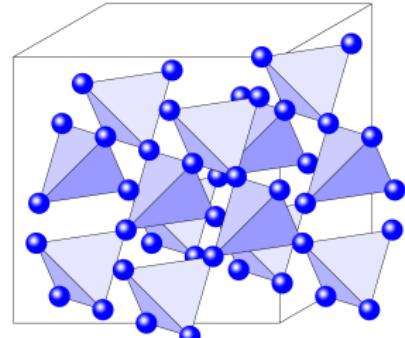
- Kagomé lattice



- garnet lattice

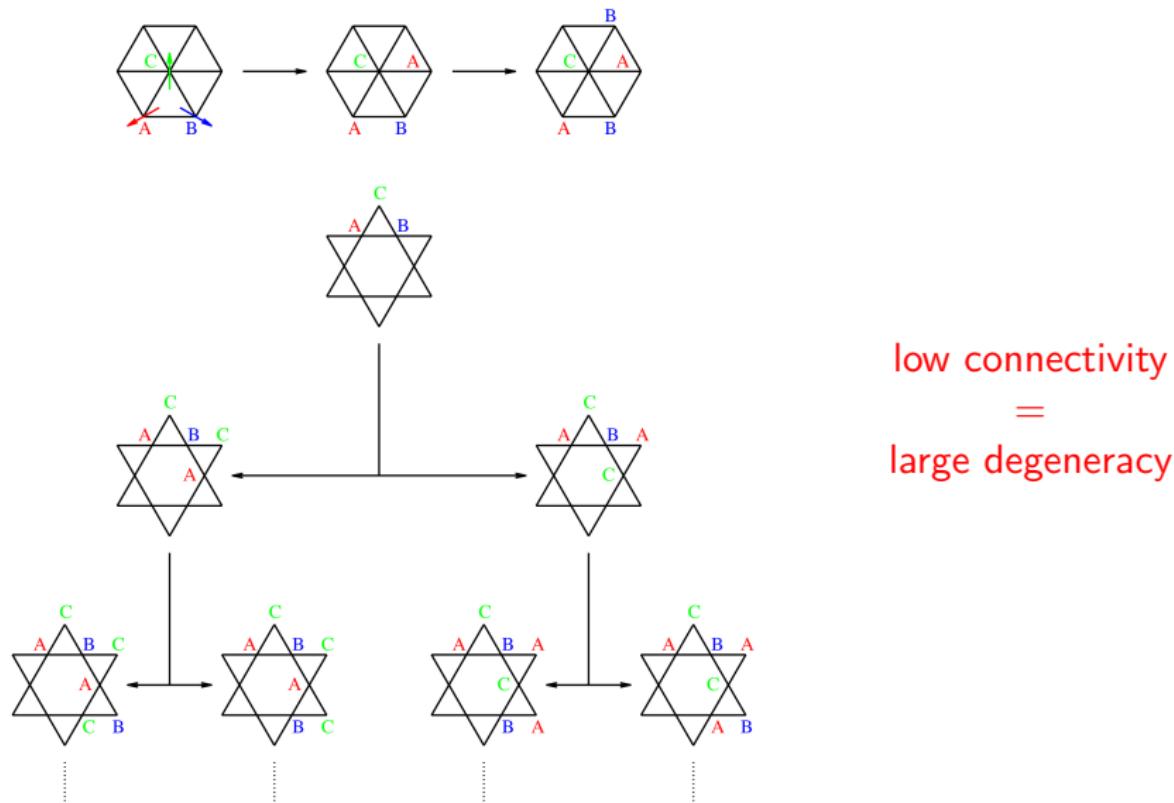


- pyrochlore lattice



# Introduction

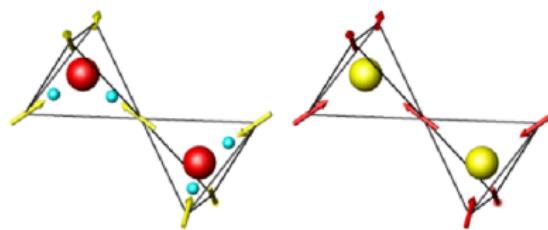
## connectivity and degeneracy



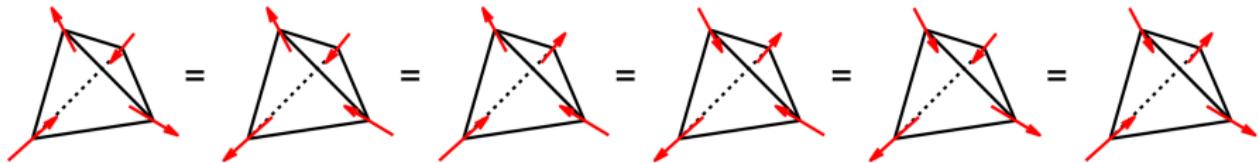
# Introduction

## spin ice

- water ice (on the left) and spin ice (on the right) : analogy between protons displacement vectors and magnetic moments



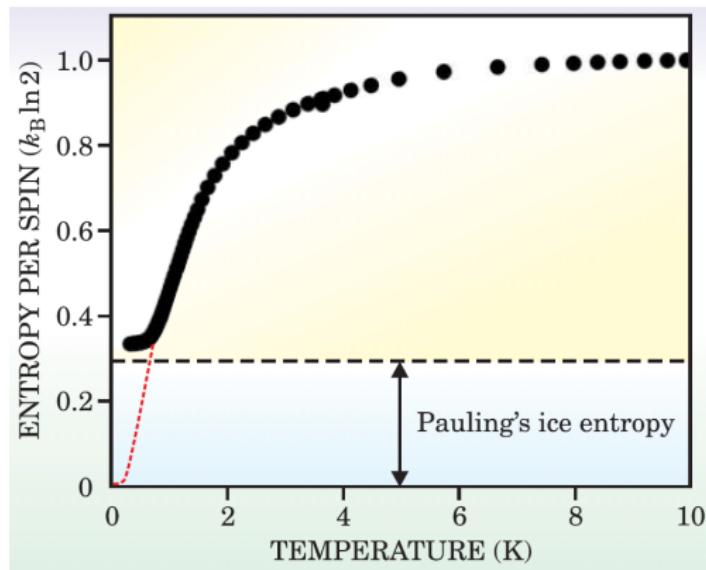
- water ice = each oxygen with two protons close and two protons away (Pauling)
- spin ice = two spins in, two spins out → six equivalent configurations



# Introduction

## spin ice

entropy of the spin ice compound  $Dy_2Ti_2O_7$



source : Moessner and Ramirez, Physics Today 2006

→ non vanishing entropy for  $T \rightarrow 0$

# Outline

## 1 Introduction

## 2 The $Tb_2M_2O_7$ compounds ( $M = Ti, Sn$ )

- the pyrochlore structure
- general points and spin dynamics
- Tb environment
- energy levels

## 3 $Tb_2Ti_2O_7$ : sample study

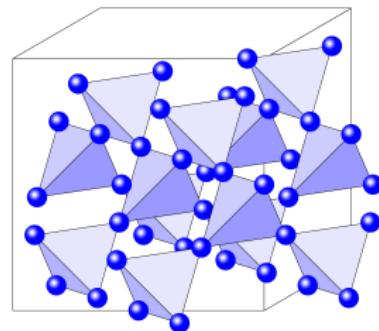
## 4 Crystal field levels

## 5 Conclusions

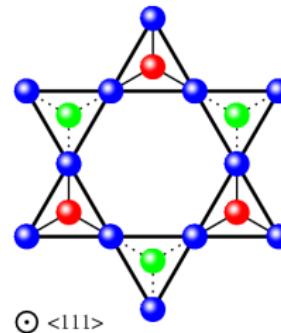
# The $\text{Tb}_2\text{M}_2\text{O}_7$ compounds ( $\text{M} = \text{Ti}, \text{Sn}$ )

the pyrochlore structure

- on the left : pyrochlore structure = three dimensional arrangement of corner-sharing tetrahedra
- on the right : projection of the pyrochlore structure on the (111) plane



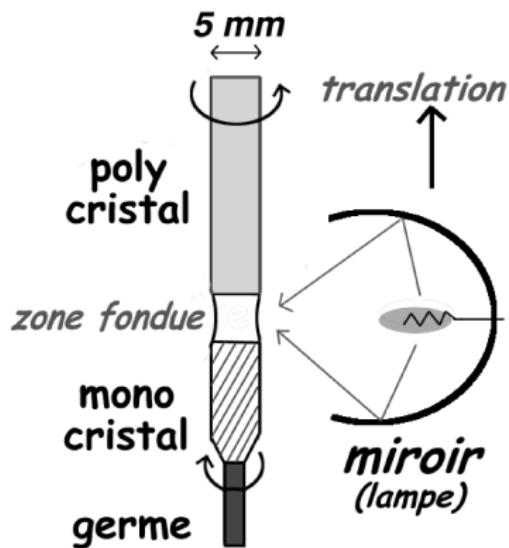
space group  $Fd\bar{3}m$



compounds	lattice parameter
$\text{Tb}_2\text{Ti}_2\text{O}_7$	10.149 Å
$\text{Tb}_2\text{Sn}_2\text{O}_7$	10.426 Å

# The $Tb_2M_2O_7$ compounds ( $M = Ti, Sn$ )

general points :  $Tb_2Ti_2O_7$



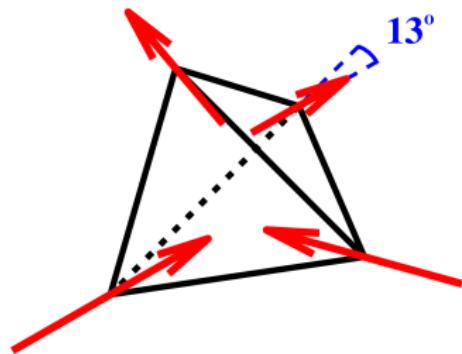
Christophe Marin  
(CEA-Grenoble/Inac/SPSMS)

- Curie-Weiss temperature  $\theta_{CW} = -18.4(5)$  K (AF interactions)
- no obvious magnetic transition
- frustration index  $f \rightarrow \infty$

# The $\text{Tb}_2\text{M}_2\text{O}_7$ compounds ( $\text{M} = \text{Ti}, \text{Sn}$ )

general points :  $\text{Tb}_2\text{Sn}_2\text{O}_7$

- Curie-Weiss temperature  $\theta_{\text{CW}} = -12$  K (AF interactions)  
Matsuhira *et al.*, J. Phys. Soc. Jpn. **71** (2002)
- magnetic transition at  $T_c = 0.88(1)$  K
- frustration index  $f \simeq 14$



neutron diffraction :  
→ ordered spin ice  
→ magnetic moments with both ferro and antiferromagnetic components

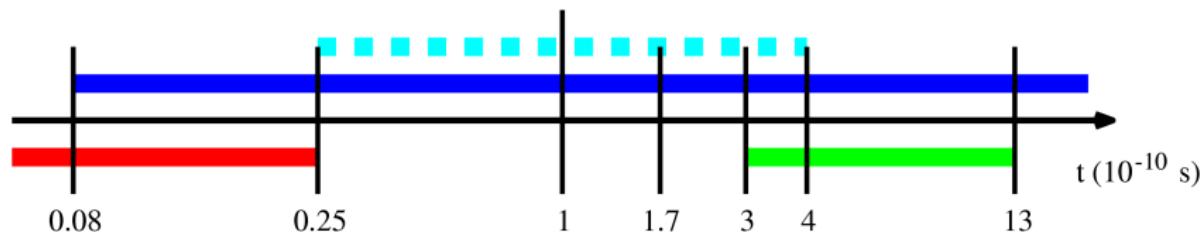
Mirebeau *et al.*, Phys. Rev. Lett. **94** (2005)

Dalmas de Réotier *et al.*, Phys. Rev. Lett. **96** (2006)

# The $\text{Tb}_2\text{M}_2\text{O}_7$ compounds ( $\text{M} = \text{Ti}, \text{Sn}$ )

general points :  $\text{Tb}_2\text{Sn}_2\text{O}_7$

fluctuation times in the ordered phase



— Polarized neutron diffraction - Rule *et al.*

— Neutron diffraction - our data

— Neutron back-scattering - Mirebeau *et al.*

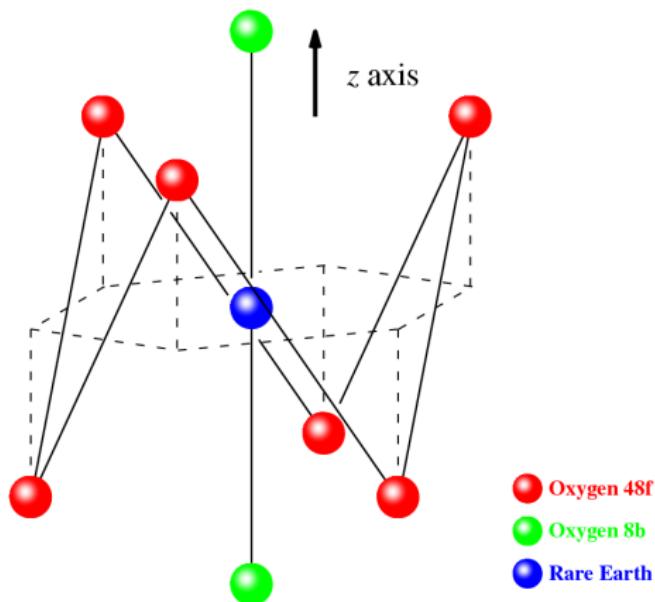
— Muon spin relaxation - our data

+ no relaxation in our **neutron spin echo** data (IN11 at ILL) :  $\tau > 10^{-8}$  s

→ complex spin dynamics mechanism characterized by **different fluctuation times** : static at low  $Q$  and dynamic at high  $Q$  ?

# The $Tb_2M_2O_7$ compounds ( $M = Ti, Sn$ )

## Tb environment



- spin angular momentum :  $S = 3$
- orbital angular momentum :  $L = 3$
- total angular momentum :  $J = 6$
- Landé factor :  $g_J = 3/2$

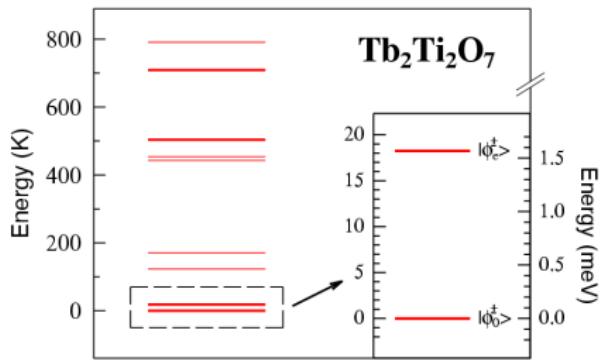
	$d_{8b}$ (Å)	$d_{48f}$ (Å)
$Tb_2Ti_2O_7$	2.197	2.502
$Tb_2Sn_2O_7$	2.257	2.517

with d the distance between the oxygen and the rare earth

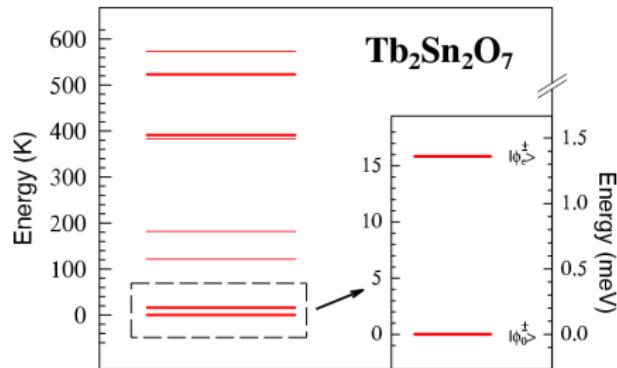
# The $\text{Tb}_2\text{M}_2\text{O}_7$ compounds ( $\text{M} = \text{Ti}, \text{Sn}$ ) energy levels

trigonal symmetry with main axis  $\parallel$  to  $\langle 111 \rangle$  (local symmetry  $\equiv \bar{3}m$ )

$$\mathcal{H}_{tri}^{CF} = B_2^0 O_2^0 + B_4^0 O_4^0 + B_4^3 O_4^3 + B_6^0 O_6^0 + B_6^3 O_6^3 + B_6^6 O_6^6$$



Gingras et al., Phys. Rev. B **62** (2000)



Mirebeau et al., Phys. Rev. B **76** (2007)

two lowest energy levels  $\equiv$  doublets

# The $\text{Tb}_2\text{M}_2\text{O}_7$ compounds ( $\text{M} = \text{Ti}, \text{Sn}$ )

## energy levels

### $\text{Tb}_2\text{Ti}_2\text{O}_7$

$$\begin{aligned} |\phi_0^\pm\rangle &= -0.958|\pm 4\rangle \pm 0.129|\pm 1\rangle - 0.121|\mp 2\rangle \mp 0.226|\mp 5\rangle \\ |\phi_e^\pm\rangle &= -0.937|\pm 5\rangle \pm 0.241|\pm 2\rangle - 0.078|\mp 1\rangle \mp 0.241|\mp 4\rangle \end{aligned}$$

→ ground state magnetic moment :  $\mu^{\text{CF}} = 5.1 \mu_{\text{B}}$

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### $\text{Tb}_2\text{Sn}_2\text{O}_7$

$$\begin{aligned} |\phi_0^\pm\rangle &= 0.922|\pm 5\rangle \mp 0.243|\pm 2\rangle + 0.016|\mp 1\rangle \mp 0.301|\mp 4\rangle \\ |\phi_e^\pm\rangle &= \pm 0.938|\pm 4\rangle - 0.156|\pm 1\rangle \pm 0.019|\mp 2\rangle - 0.309|\mp 5\rangle \end{aligned}$$

→ ground state magnetic moment :  $\mu^{\text{CF}} = 6.0 \mu_{\text{B}}$

# The $\text{Tb}_2\text{M}_2\text{O}_7$ compounds ( $\text{M} = \text{Ti}, \text{Sn}$ ) energy levels

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→ ground state magnetic moment :  $\mu^{\text{CF}} = 6.0 \mu_{\text{B}}$

swap of  $|m_z\rangle$  state mainly contributing to  $|\phi_0^\pm\rangle$  and  $|\phi_e^\pm\rangle$ , depending on  
the  $B_6^0$  parameter

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1 Introduction

2 The  $Tb_2M_2O_7$  compounds ( $M = Ti, Sn$ )

3  $Tb_2Ti_2O_7$  : sample study

- sample dependence of some properties
- characteristic temperatures

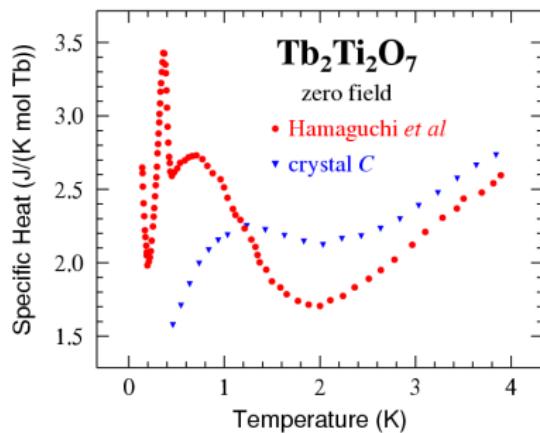
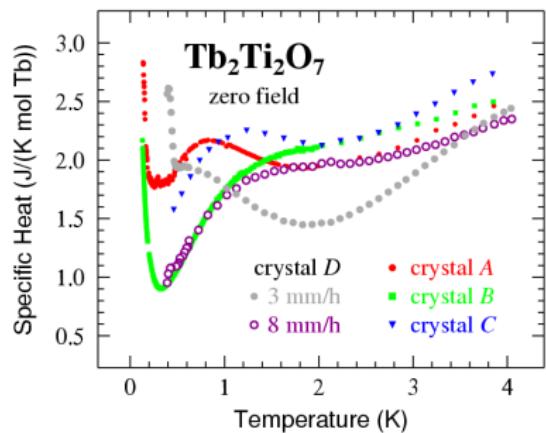
4 Crystal field levels

5 Conclusions

# $Tb_2Ti_2O_7$ : sample study

sample dependence of some properties

crystals	initial powders	growth rate, flow
A, B	$TiO_2 + Tb_4O_7$	8 mm/h, argon
C	$TiO_2 + Tb_2O_3$	7 mm/h, oxygen
D	$TiO_2 + Tb_4O_7$	3 & 8 mm/h, argon



- crystal growth rate dependence of specific heat + possible well defined anomaly at 0.4 K

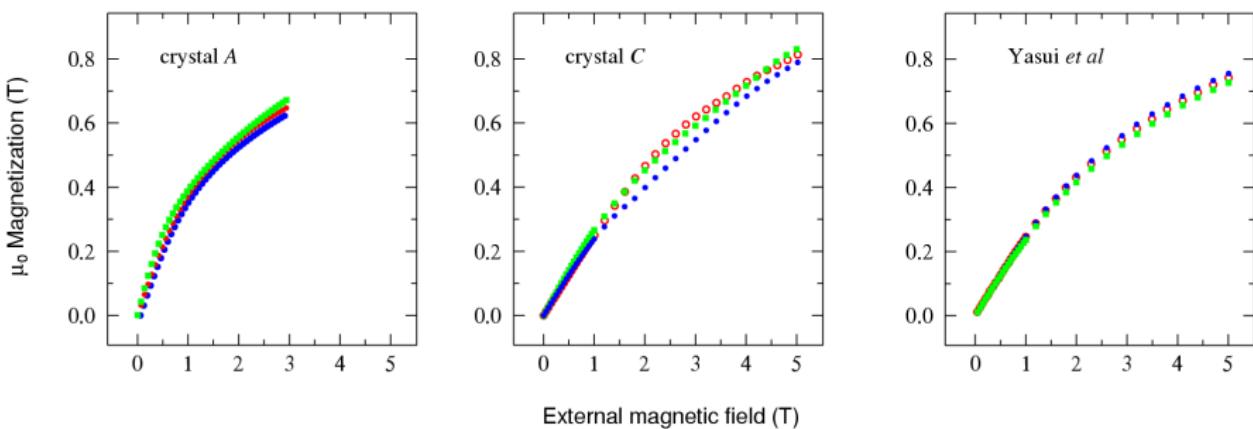
Hamaguchi *et al.*, Phys. Rev. B **69** (2004)

# $\text{Tb}_2\text{Ti}_2\text{O}_7$ : sample study

sample dependence of some properties

$\text{Tb}_2\text{Ti}_2\text{O}_7$   
 $T = 5.0 \text{ K}$

- $\mathbf{B}_{\text{ext}} \parallel [001]$
- $\mathbf{B}_{\text{ext}} \parallel [110]$
- $\mathbf{B}_{\text{ext}} \parallel [111]$

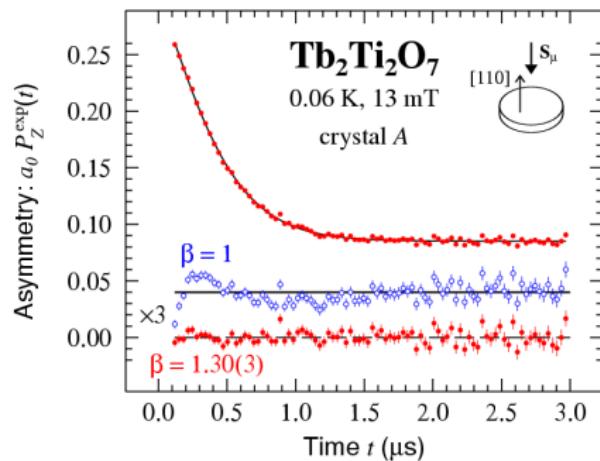
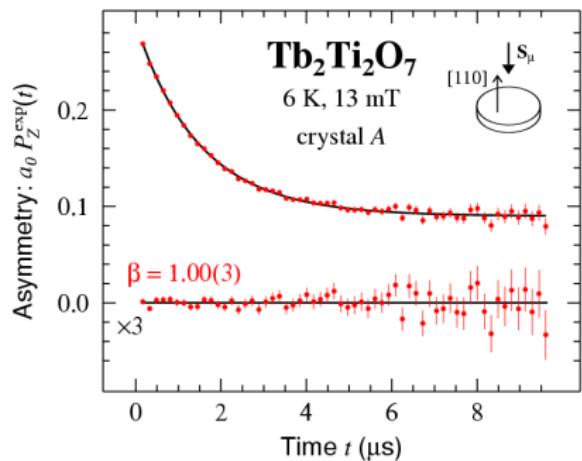


- same sample as Hamaguchi *et al.* Yasui *et al.*, J. Phys. Soc. Jpn. **71** (2002)
- some weak differences between samples

# $\text{Tb}_2\text{Ti}_2\text{O}_7$ : sample study

sample dependence of some properties

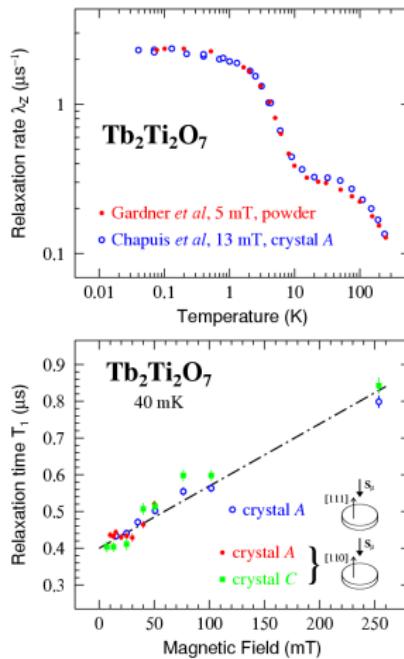
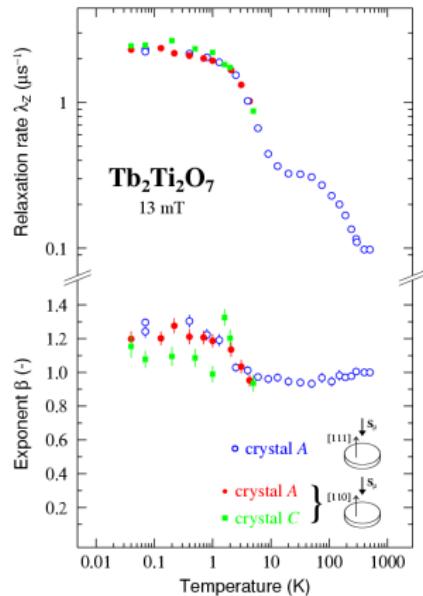
- muon = magnetic local probe ( $\mu$ SR measurements from ISIS)



- counts rate :  $N(t) = N_0 \exp(-t/\tau_\mu) [1 + a_0 P_Z(t)]$
- asymmetry :  $a_0 P_Z(t) = a_s \exp[-(\lambda_Z t)^\beta] + a_{\text{bg}}$
- at 6 K :  $\beta = 1.00(3)$ ; and at 0.06 K :  $\beta = 1.30(2)$

# $Tb_2Ti_2O_7$ : sample study

## sample dependence of some properties

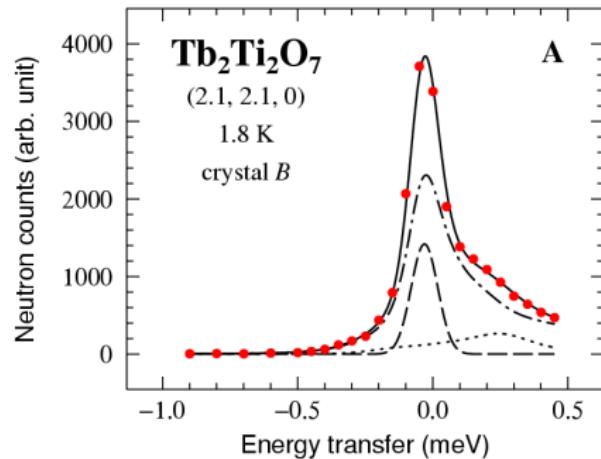
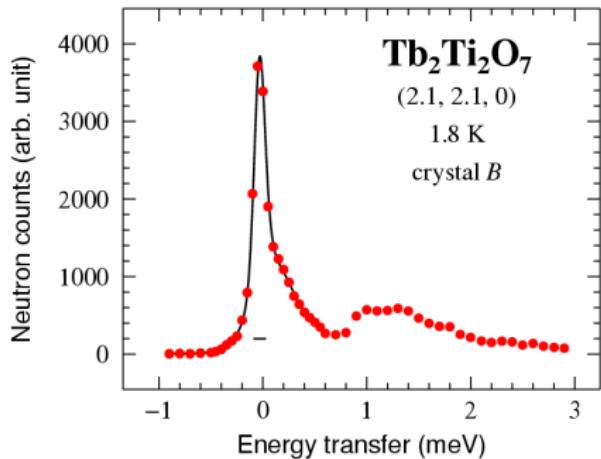


- no sample dependence of the relaxation rate  $\lambda_Z$
- sample dependence of the exponent  $\beta$  needs to be checked

# $\text{Tb}_2\text{Ti}_2\text{O}_7$ : sample study

## characteristic temperatures

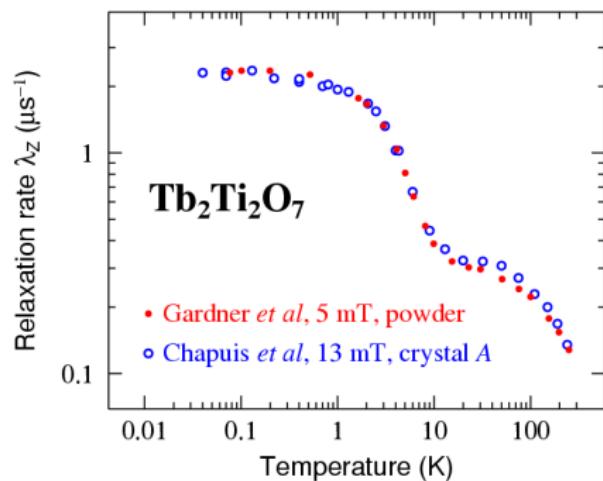
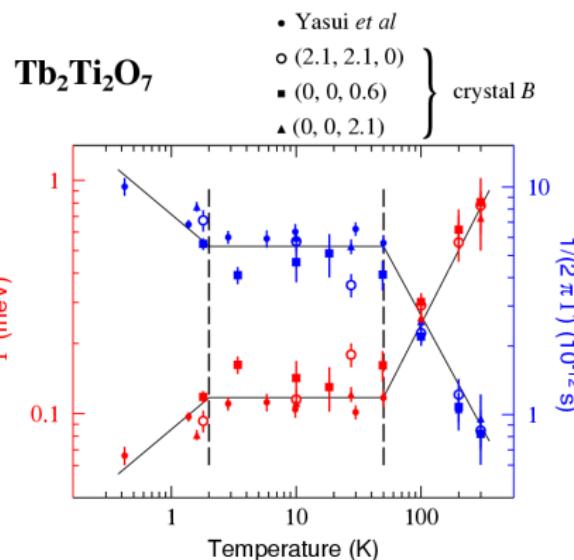
- inelastic neutron scattering measurements from IN12 at ILL



- $I(Q, \omega) = |F(Q)|^2 R \otimes \left[ \frac{\hbar\omega}{1-\exp(-\hbar\omega/k_B T)} \left\{ L_q(\omega, \omega_0, \Gamma, A_L) + L_{\text{inel}}(\omega, \omega_0, \delta, \sigma_{\text{inel}}, A_{\text{inel}}) \right\} \right] + G_{\text{inc}}(\omega, \omega_0, \sigma_{\text{inc}}, A_{\text{inc}}) + C_{\text{bg}}$
- weakly inelastic contribution only for  $T \leq 10$  K and relative high  $Q$

# $\text{Tb}_2\text{Ti}_2\text{O}_7$ : sample study

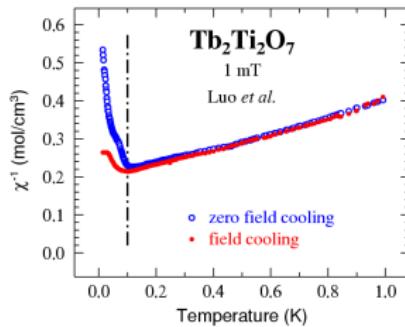
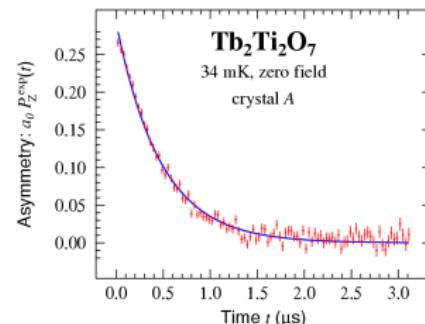
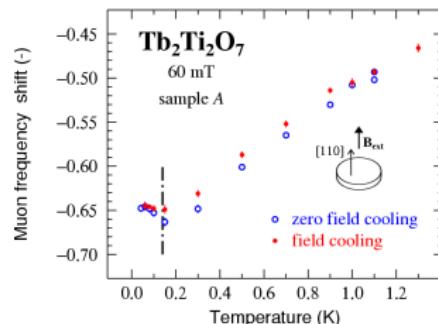
## characteristic temperatures



- $\lambda_Z \propto T \int \mathcal{A}(Q) \frac{\chi(Q)}{\Gamma(Q)} d^3 Q$
- from inelastic neutron scattering and muon spin relaxation : two characteristic temperatures = **2 and 50 K**

# $Tb_2Ti_2O_7$ : sample study

## characteristic temperatures



Luo *et al.*, Phys. Lett. A 291 (2001)

- low temperature  $\mu$ SR spectra (from PSI) : no oscillation

- susceptibility : anomaly at around 0.1 K

→ magnetic transition or not ?  
 → nature of the lowest temperature phase ?

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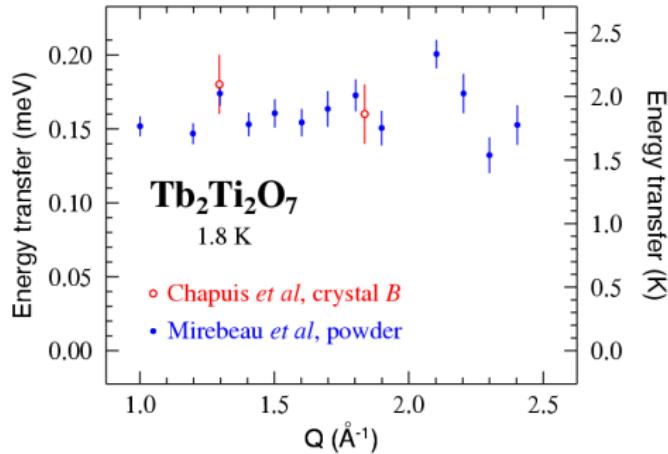
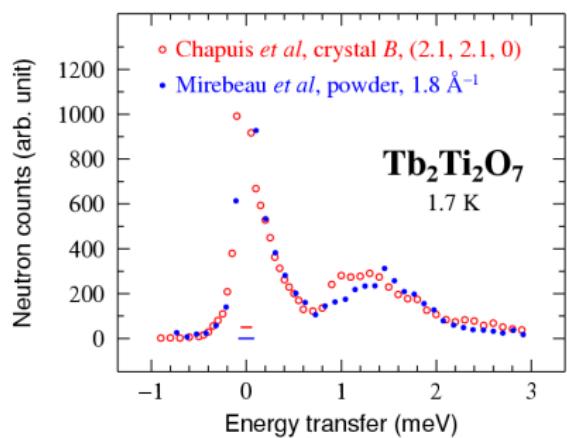
3  $Tb_2Ti_2O_7$  : sample study

4 Crystal field levels

- inelastic neutron scattering
- entropy variation study

5 Conclusions

# Crystal field levels inelastic neutron scattering



- for  $\text{Tb}_2\text{Ti}_2\text{O}_7$ , our data are consistent with the ones of Mirebeau *et al.* and Yasui *et al.*

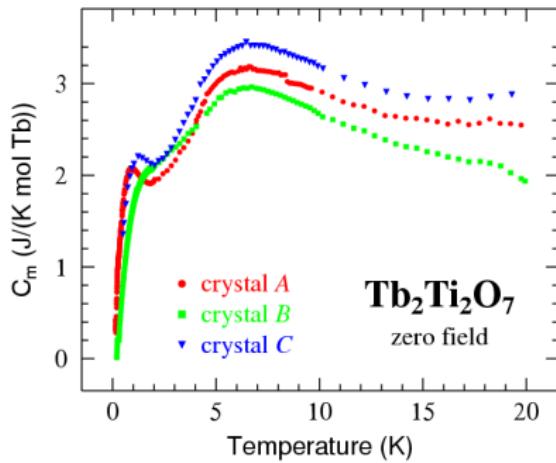
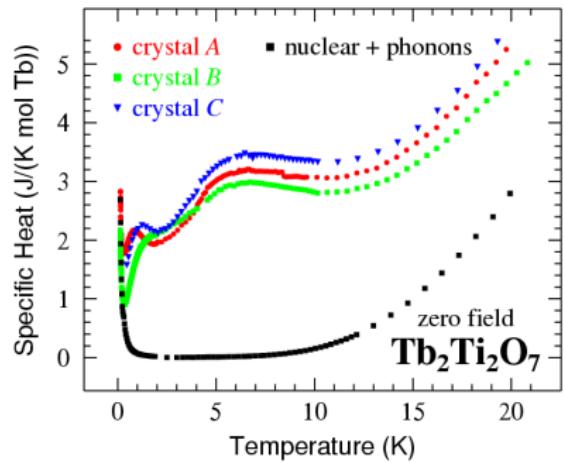
Mirebeau *et al.*, Phys. Rev. B **76** (2007)

Yasui *et al.*, J. Phys. Soc. Jpn. **71** (2002)

- weakly inelastic contribution** : energy level at  $\simeq 2 \text{ K}$
- similar results for  $\text{Tb}_2\text{Sn}_2\text{O}_7$  are obtained by Mirebeau *et al.*

# Crystal field levels

## entropy variation study



- on the left, total specific heat, nuclear and phonons contributions
  - nuclear contribution from Zeeman splitting ( $\Delta_N = 82(5) \text{ mK}$ )  
→  $\mu_{\text{SH}} = 4.5(7) \mu_B$
  - phonons from non magnetic and isostructural compound
- on the right, magnetic specific heat

# Crystal field levels

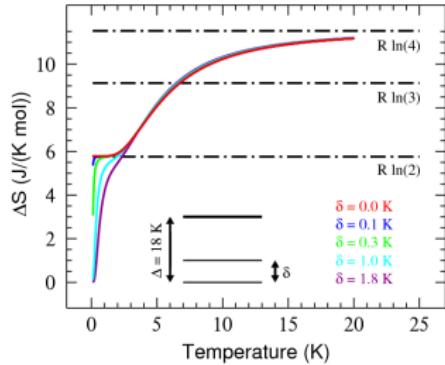
## entropy variation study

2 doublets

$$\langle E \rangle = \frac{0 + 2\Delta \exp(-\beta\Delta)}{2 + 2 \exp(-\beta\Delta)} = \frac{\Delta \exp(-\beta\Delta)}{1 + \exp(-\beta\Delta)}$$

2 singlets and 1 doublet

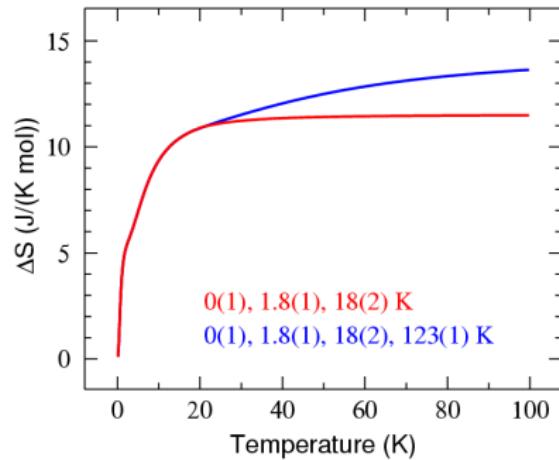
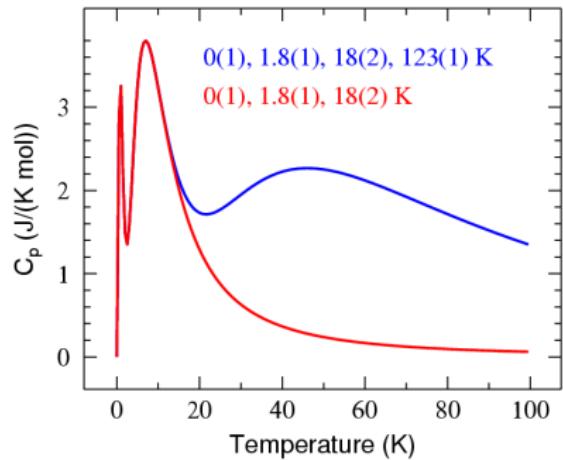
$$\langle E \rangle = \frac{0 + \delta \exp(-\beta\delta) + 2\Delta \exp(-\beta\Delta)}{1 + \exp(-\beta\delta) + 2 \exp(-\beta\Delta)}$$



- $C_v = d\langle E \rangle / dT$
- $\Delta S = \int \frac{C_v}{T} dT$
- $\Delta S$  calculated between 200 K down to 0.13 K

# Crystal field levels

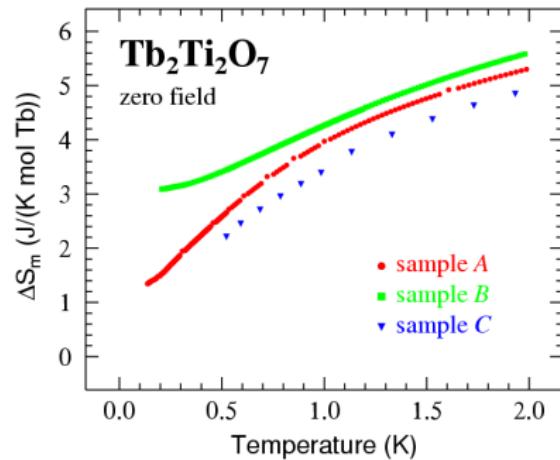
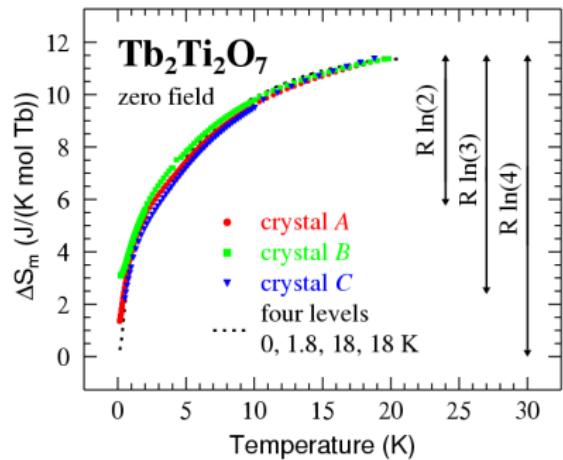
## entropy variation study



- for  $0 \text{ K} \leq T \leq 20 \text{ K}$ , no influence of higher energy levels
- similar results for  $Tb_2Ti_2O_7$  and  $Tb_2Sn_2O_7$

# Crystal field levels

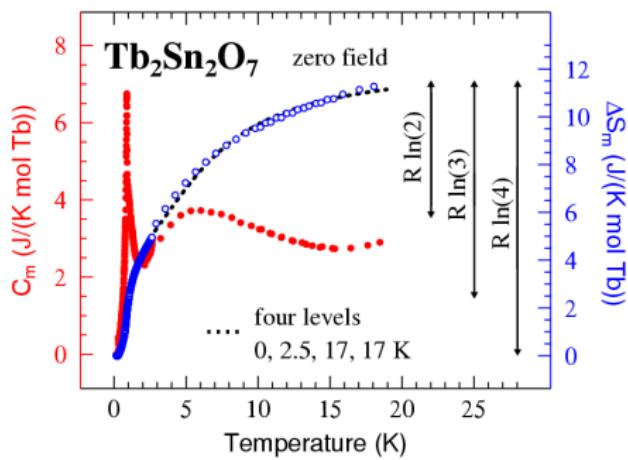
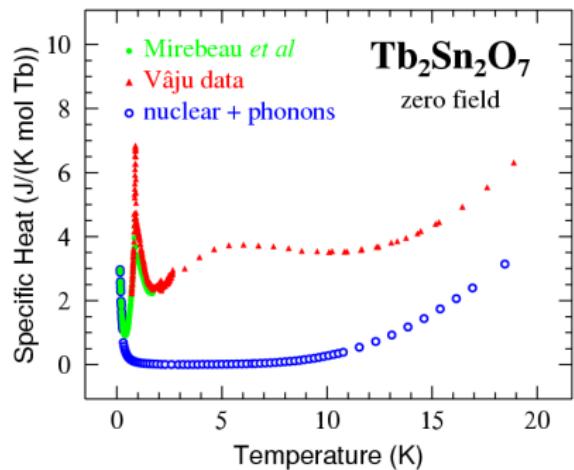
## entropy variation study



- on the left, magnetic entropy variation :  $\Delta S_m(0 \rightarrow 20K) > R \ln(2)$   
 $\rightarrow$  ground state splitting
- on the right, at low temperature :  $\Delta S_m(T = 0K) = R \ln(4)$  ?

# Crystal field levels

## entropy variation study



- $\Delta S_m$  reaches  $R \ln(4)$  at 20 K
- splitting observed by Mirebeau *et al.* with inelastic neutron scattering

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- summary
- outlook

# Conclusions

## summary

### Tb<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub>

- sample dependence of specific heat, magnetization and  $\mu$ SR (?) → a detailed structural study is required
- but sample independence of inelastic neutron scattering results
- characteristic temperatures  $T \simeq 2\text{ K}$  and  $50\text{ K}$  :  $\mu$ SR and inelastic neutron scattering are consistent
- from muon frequency shift and susceptibility, possible transition at  $\simeq 0.1\text{ K}$  : what is the nature of the lowest temperature phase ?

# Conclusions

## summary

### Tb<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub>

- sample dependence of specific heat, magnetization and  $\mu$ SR (?) → a detailed structural study is required
- but sample independence of inelastic neutron scattering results
- characteristic temperatures  $T \simeq 2\text{ K}$  and  $50\text{ K}$  :  $\mu$ SR and inelastic neutron scattering are consistent
- from muon frequency shift and susceptibility, possible transition at  $\simeq 0.1\text{ K}$  : what is the nature of the lowest temperature phase ?

### Tb<sub>2</sub>Sn<sub>2</sub>O<sub>7</sub>

- low temperature spin dynamics characterized by different fluctuation times
- order of magnitude depending on  $Q$  :  $10^{-11}\text{ s} / 10^{-9}\text{ s}$  and  $> 10^{-8}\text{ s}$

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Tb<sub>2</sub>M<sub>2</sub>O<sub>7</sub>

$\simeq 2$  K splitting of the ground state : deduced from inelastic neutron scattering ; consistent with the tetragonal distortion along [001] and now, confirmed by entropy variation study

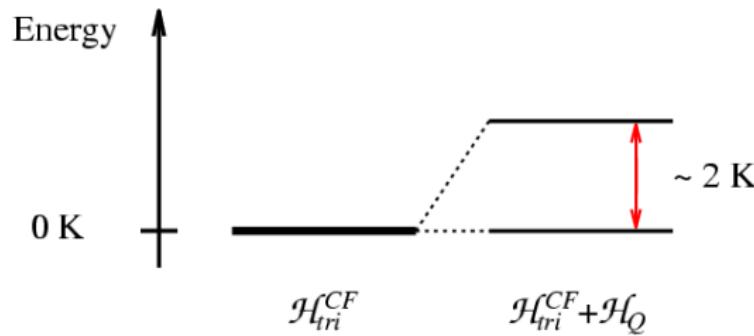
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new energy levels scheme



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## outlook

### Tb<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub>

- $\mu$ SR measurements on sample C to know the behaviour of the exponent  $\beta$  versus the temperature
- specific heat measurements in dilution on sample C to compare with other samples :  $\Delta S \rightarrow R \ln(4)$  ?
- understand X-Ray and neutron diffraction patterns

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### Tb<sub>2</sub>Sn<sub>2</sub>O<sub>7</sub>

- the magnetic moment has both ferromagnetic and antiferromagnetic components : what are consequences on the spin dynamics ?
- static at low  $Q$  and dynamic at high  $Q$  ?
- why no oscillations have been observed in  $\mu$ SR spectra at low temperature ?

# Thanks

## the jury

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- P. Dalmas de Réotier

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- J. F. Jacquot (magnetization)
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And thank you for your attention...