REPORT ON THE PH.D THESIS BY D. BORRELLO

"INTERACTING PARTICLE SYSTEMS: STOCHASTIC ORDER, ATTRACTIVENESS AND RANDOM WALKS ON SMALL WORLD GRAPHS."

Interacting particle systems form a well defined class of Markov processes on probability spaces of the form $\Omega = S^E$ where typically S is some regular lattice like $\mathbb{Z}^d$ and $E \subset \mathbb{N}$, that are more and more studied in probability with applications to statistical physics, theoretical computer science, chemical processes and population dynamics. The evolution of the process is ruled by some transition rates which usually are assumned to be local: at each time t each coordinate $\eta_t(x) \in E$, $x \in S$, (or, more generally, a finite group of coordinates) attempts to jump to another value in E with a rate that depends on the current values of the coordinates $\{\eta_t(y)\}$ with y in a neighborhood of x. Under suitable and general conditions the existence and uniqueness of these processes is well known. Quite non trivial is instead the question of the existence and uniqueness/non-uniqueness of non trivial invariant measures (*i.e.* not concentrated on adsorbing states). In the reversible case, which is typical of the MCMC models of statistical physics or of the randomized algorithms approach in theoretical computer science, the problem is much easier but in the non reversible case, which is typical for biological applications, the difficulties can be quite substantial.

In the first part of his thesis D. Borrello addresses this question using a very natural and well known approach based on coupling and monotonicity. Using coupling he establishes a new sharp monotonicity criterium for a very general class of interacting particle models with birth, deaths and jumps of more than one particle per time which also tells exactly when a process in this class is attractive or not. Once attractivity is available then the problem of ergodicity becomes more tractable since one can reduce oneself to check if the maximal and minimal initial distribution eventually couple. Overall this first part is well explained (although sometimes is difficult to follow all the details of the various models) and the results are interesting. As an aside remark I would like to add that coupling has also been quite developed also in the context of randomized algorithms theory with powerful result like "path coupling" (see Bubbley and Dyer, D. Randall, Y. Peres...) which says that in order to prove the contraction property of section 1.2.2 it is enough to consider only two configurations with minimal positive distance.

In the second and quite different part of his thesis D. Borrello begin to study in details a different problem, namely that of the precise asymptotics of the coalescing time between two independent random walks on certain special graphs G known as *small world graphs*. These graphs form a particular class of random graphs in which, depending on the details of the model, some random long range connections are added to an existing non-random graph structure (e.g. a finite periodic grid of side L on $\mathbb{Z}^d$). The main motivation comes from the analysis of coalescing random walk on G (*i.e.* n particles performing independent random walks plus coalescence when particles meet). Following Durrett such an analysis is done in the last section where convergence in law to the Kingman's coalescing model is proved.

I found this second part interesting (I was not familiar with these kind of problems) and well presented with some nice new results about the role of the random long range connections albeit obtained according to some preestablished path. My impression from the scheme of the proofs and from the various arguments is that of someone who has a very good control of the problem, of the known results and has a good working knowledge of several non trivial techniques.

In conclusion my opinion is that D. Borrello has done a very good job, he clearly shows a nice diversification of his probabilistic interests and quite likely more will come in a future.

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