String theory compactifications with fluxes, and generalized geometry

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LPTENS Paris & Roma “Tor Vergata”

PhD thesis defence

Paris, 4 June 2009

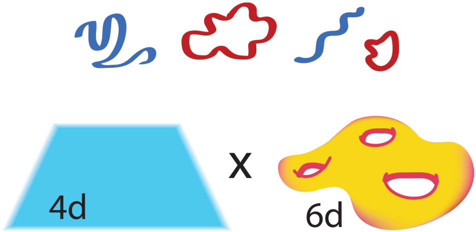
Outline

- Motivations
- Flux compactifications
- Generalized geometry
- Examples: coset spaces

Based on

- DC and A. Bilal, *Effective actions and N=1 vacuum conditions from $SU(3) \times SU(3)$ compactifications* , JHEP **0709** (2007) 076 [arXiv:0707.3125 [hep-th]]
- DC, *Reducing democratic type II supergravity on $SU(3) \times SU(3)$ structures*, JHEP **0806** (2008) 027 [arXiv:0804.0595 [hep-th]]
- DC and A. K. Kashani-Poor, *Exploiting N=2 in consistent coset reductions of type IIA* , Nucl. Phys. B **817** (2009) 25 [arXiv:0901.4251 [hep-th]]

Superstring compactifications



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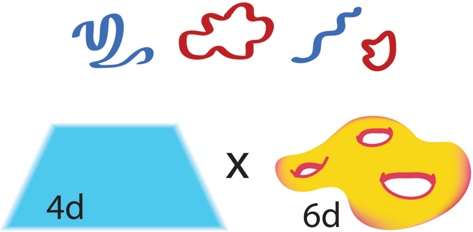

★ Goals ★

- vacuum state of string theory
- low energy effective theory in 4d
- $(N = 1)$ supersymmetry $\rightarrow$ { phenomenology (MSSM); control on the compactification }

Realistic models

Type II scenario :

Realistic models

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- Compact geometry (Calabi-Yau)

Realistic models

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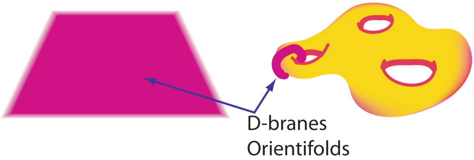

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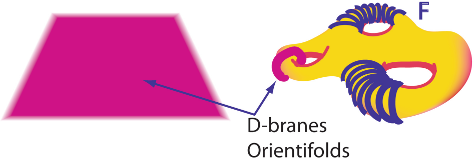

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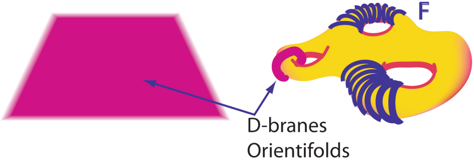

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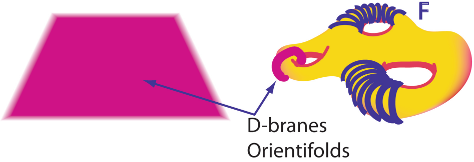

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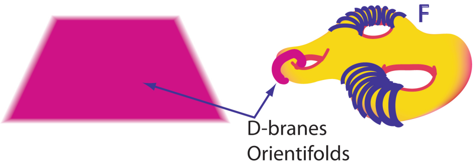

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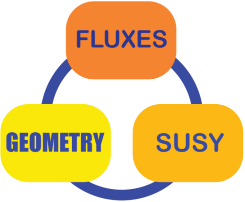
We focus on interplay



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Candelas, Horowitz, Strominger, Witten '85

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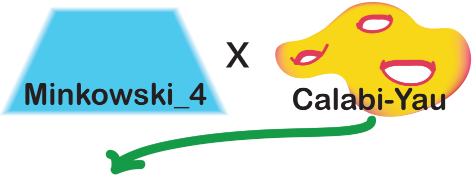

Ricci-flat metric

↓ ↓ ↓

solves 10d Einstein eq.

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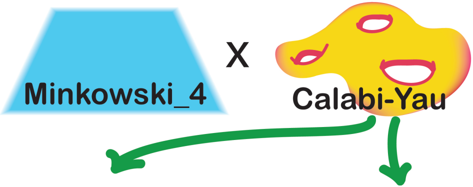

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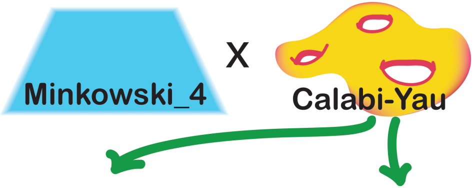

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Type II string theory $\longrightarrow$ $N = 2$ **supergravity**

large number of fields. In particular: massless scalars

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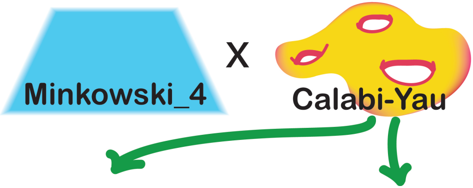

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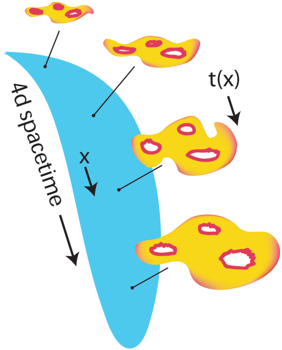
► **MODULI PROBLEM** ◄

Moduli problem



Moduli:

- “shape and size” deformations of the compact manifold
- parameterize degeneracy of 10d vacua
- from 4d viewpoint: propagating massless scalars

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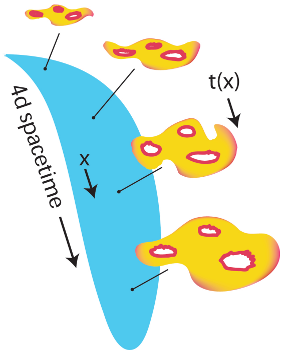

Moduli:

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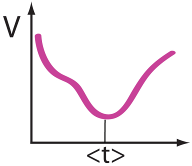
!! problem !!

- ▶ long range scalar interactions never detected
- ▶ loss of predictive power (vevs $\leftrightarrow$ 4d couplings)

Flux compactifications

possible solution to moduli problem: generate a potential



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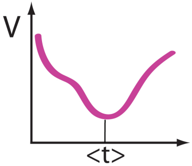

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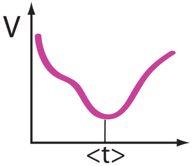

$\Rightarrow$ { stabilizes vevs / yields mass to fluctuations }

mechanism for a potential : **FLUXES**

p-form field-strengths F_p of 10d sugra

$\langle F_p \rangle \neq 0$ along M_6

$\int F_p = n \neq 0$

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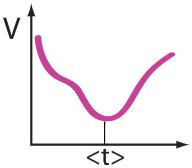

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10d sugra : $\quad \cdots - \int_{M_4} \cdots \underbrace{\int_{M_6} d^6y \sqrt{g}\, g^{m_1 n_1} \cdots g^{m_p n_p} (F_p)_{m_1 \ldots m_p} (F_p)_{n_1 \ldots n_p}}_{V(t)}$

$g(y,t)$: metric on M_6

Fluxes & 4d gauged sugra

Type II sugra on CY_3

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

ungauged $N = 2$ sugra in 4d

$\downarrow$

no scalar potential

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gauged $N = 2$ sugra in 4d

$\downarrow$

nontrivial scalar potential

$\downarrow$

stabilizes (part of) the moduli

Flux compactifications

However, ‘Calabi-Yau with fluxes’ background :

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- Not consistent with the (pure sugra) EoM

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10d level $\longrightarrow$ Fluxes backreact on the geometry

$F_p \quad \longrightarrow \quad$ en.-mom. tensor $T_{MN} \quad \longrightarrow \quad R_{MN} - \frac{1}{2} g_{MN} R \;\sim\; T_{MN}$

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- Gaugings are limited

  ?? embed more general 4d supergravities in 10d **??**

Program

M_6 other than Calabi-Yau. Still preserve a fraction of susy

- **General study**
  - Flux compactifications of type II leading to $N = 2$ sugra in 4d
  - How $N = 2$ data are determined by the compact geometry

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- Coset spaces G/H

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Concrete examples

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Tools

- 6d : generalized geometry (Hitchin)
- 4d : gauged $N = 2$ supergravity

Type II sugra and SU(3)×SU(3) structures

To preserve 8 supercharges:

$$10d \downarrow \qquad 4d \downarrow \qquad \downarrow 6d$$

$$\epsilon^1 = \varepsilon^1 \otimes \eta^1 + c.c.$$

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a pair of spinors η^1, η^2 on M_6

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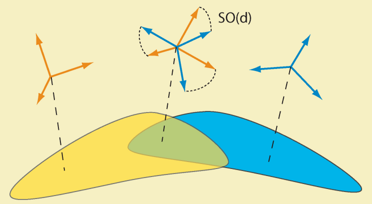
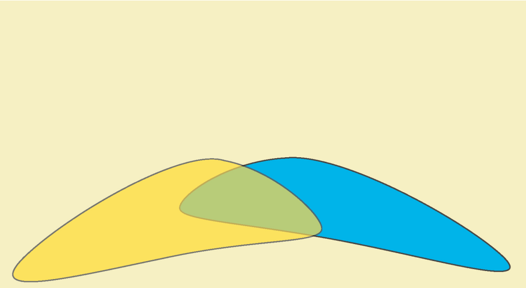


↑ *structure group*

Type II sugra and SU(3)×SU(3) structures

Reduction of the structure group:



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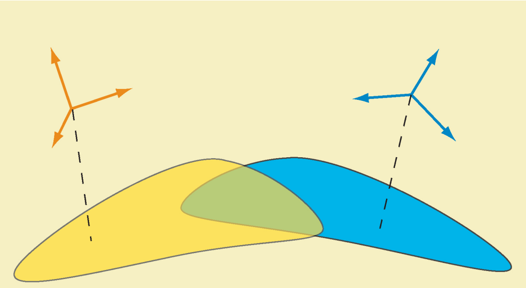

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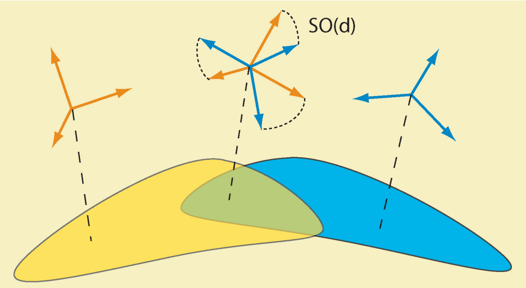

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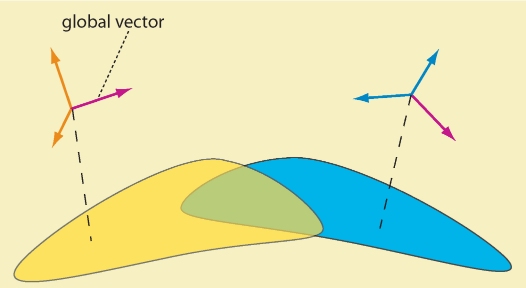

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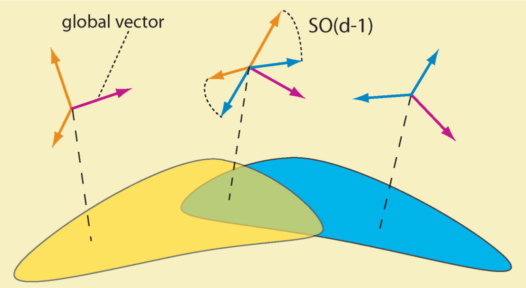

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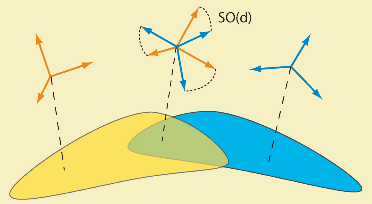

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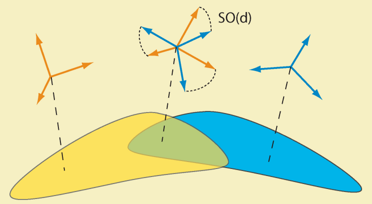

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Best seen as an SU(3)×SU(3) structure on $TM_6 \oplus T^*M_6$

$\rightsquigarrow$ Generalized Geometry

Graña,Louis,Waldram '05,'06

Hitchin '02, Gualtieri'04

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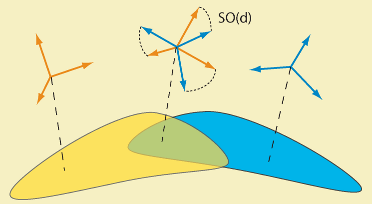

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SU(3) structure on M_6 $\longrightarrow$ relevant for our cosets G/H

Type II sugra and SU(3)$\times$SU(3) structures

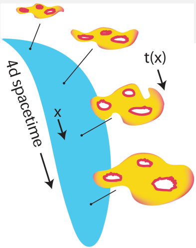
Basic objects: $O(6,6)$ pure spinors Φ_+ and Φ_-

- polyforms : $\Phi_+ \in \wedge^{\text{even}} T^* M_6$, $\Phi_- \in \wedge^{\text{odd}} T^* M_6$
- generalize J and Ω of a CY
- encode the whole *internal* NSNS sector $(g_{mn},\ B_{mn},\ \phi)$
- $\Phi_\pm$ can be built as $e^{-B}(\eta^1_+ \otimes \eta^{2\dagger}_\pm)$

Graña,Minasian, Petrini,Tomasiello'04'05

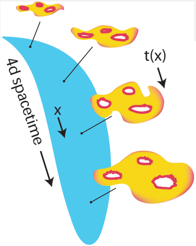
$\hookrightarrow$ polyforms via fierzing

Moduli space of CY manifolds



CY_3 characterized by $\begin{cases} \text{holomorphic (3,0)-form } \Omega \\ \text{Kähler form } J \end{cases}$

Moduli space of CY manifolds



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δg_{mn}

$\delta\Omega$ — complex structure moduli

δJ — Kähler structure moduli

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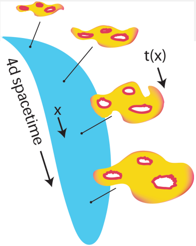

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Special Kähler | **Special Kähler**

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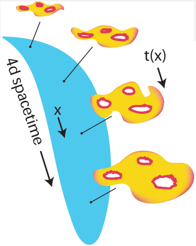
$$K_+ = -\log \tfrac{4}{3} \int J \wedge J \wedge J$$

Moduli space of CY manifolds



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this fits into 4d, $N = 2$ sugra

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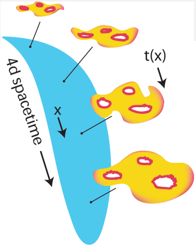

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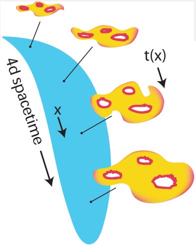
More generic situations?

Deformations of SU(3)×SU(3) structures



SU(3)×SU(3) structure Φ_+ , Φ_-

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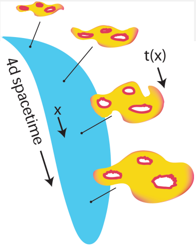


SU(3)×SU(3) structure Φ_+ , Φ_-

$\delta\Phi_+$, $\delta\Phi_-$ *at a point* of M_6

$\Updownarrow$

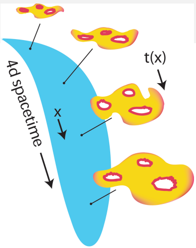
Special Kähler geometries

Hitchin'02

Graña,Louis,Waldram

Kähler potentials : $K_\pm = -\log i \int \langle \Phi_\pm, \overline{\Phi}_\pm \rangle$

Deformations of SU(3)×SU(3) structures



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Kähler potentials : $K_\pm = -\log i \int \langle \Phi_\pm, \overline{\Phi}_\pm \rangle$

We computed:

$$\underbrace{\frac{e^{2\varphi}}{8} \int vol_6 e^{-2\phi} g^{mn} g^{pq} (\delta g_{mp} \delta g_{nq} + \delta B_{mp} \delta B_{nq})}_{\text{metric on space of } g_{mn} \text{ and } B_{mn} \text{ deform.}} = \underbrace{\delta^{\text{holo}} \delta^{\text{anti}} K_- + \delta^{\text{holo}} \delta^{\text{anti}} K_+}_{\text{sp. Kähler metrics for } \Phi_+ \text{ and } \Phi_- \text{ def.}}$$

$\Downarrow\Downarrow$

4d scalar kinetic terms

DC,Bilal '07

Generalized diamond

Complex polyforms decompose in reps of SU(3)×SU(3) :

$$
\begin{array}{ccccccc}
 & & & \mathbf{1},\overline{\mathbf{1}} & & & \\
 & & \mathbf{1},\mathbf{3} & & \overline{\mathbf{3}},\overline{\mathbf{1}} & & \\
 & \mathbf{1},\overline{\mathbf{3}} & & \overline{\mathbf{3}},\mathbf{3} & & \mathbf{3},\overline{\mathbf{1}} & \\
\mathbf{1},\mathbf{1} & & \overline{\mathbf{3}},\overline{\mathbf{3}} & & \mathbf{3},\mathbf{3} & & \overline{\mathbf{1}},\overline{\mathbf{1}} \\
 & \overline{\mathbf{3}},\mathbf{1} & & \mathbf{3},\overline{\mathbf{3}} & & \overline{\mathbf{1}},\mathbf{3} & \\
 & & \mathbf{3},\mathbf{1} & & \overline{\mathbf{1}},\overline{\mathbf{3}} & & \\
 & & & \overline{\mathbf{1}},\mathbf{1} & & &
\end{array}
$$

Generalized diamond

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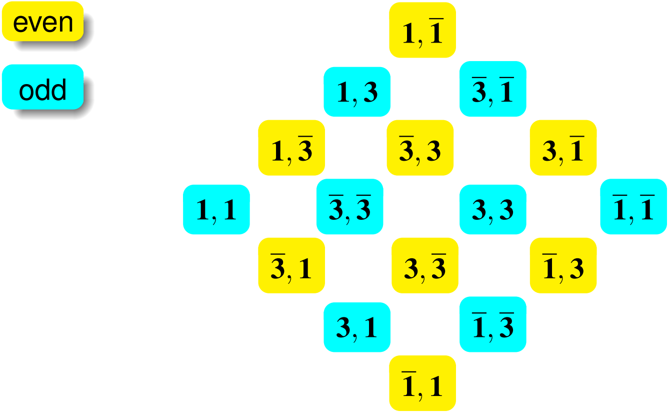

Generalized diamond

SU(3)×SU(3) invariant polyforms :

$$
\begin{array}{ccccccc}
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\mathbf{1},\mathbf{1} & & \overline{\mathbf{3}},\overline{\mathbf{3}} & & \mathbf{3},\mathbf{3} & & \overline{\mathbf{1}},\overline{\mathbf{1}} \\
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\end{array}
$$

Generalized diamond

SU(3)$\times$SU(3) invariant polyforms :

$$\Phi_+$$

$$\mathbf{1},\mathbf{3} \qquad \bar{\mathbf{3}},\bar{\mathbf{1}}$$

$$\mathbf{1},\bar{\mathbf{3}} \qquad \bar{\mathbf{3}},\mathbf{3} \qquad \mathbf{3},\bar{\mathbf{1}}$$

$$\mathbf{1},\mathbf{1} \qquad \bar{\mathbf{3}},\bar{\mathbf{3}} \qquad \mathbf{3},\mathbf{3} \qquad \bar{\mathbf{1}},\bar{\mathbf{1}}$$

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$$\Phi_- \qquad \bar{\mathbf{3}},\bar{\mathbf{3}} \qquad \mathbf{3},\mathbf{3} \qquad \bar{\mathbf{1}},\bar{\mathbf{1}}$$

$$\bar{\mathbf{3}},\mathbf{1} \qquad \mathbf{3},\bar{\mathbf{3}} \qquad \bar{\mathbf{1}},\mathbf{3}$$

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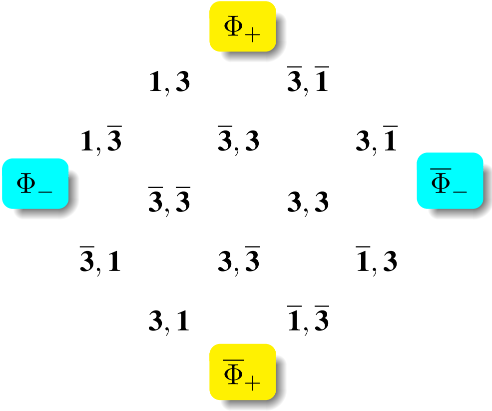

Generalized diamond

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$$\eta^1_+\eta^{2\dagger}_+$$

$$\mathbf{1},\mathbf{3} \qquad \bar{\mathbf{3}},\bar{\mathbf{1}}$$

$$\mathbf{1},\bar{\mathbf{3}} \qquad \bar{\mathbf{3}},\mathbf{3} \qquad \mathbf{3},\bar{\mathbf{1}}$$

$$\eta^1_+\eta^{2\dagger}_- \qquad \bar{\mathbf{3}},\bar{\mathbf{3}} \qquad \mathbf{3},\mathbf{3} \qquad \eta^1_-\eta^{2\dagger}_+$$

$$\bar{\mathbf{3}},\mathbf{1} \qquad \mathbf{3},\bar{\mathbf{3}} \qquad \bar{\mathbf{1}},\mathbf{3}$$

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$$\eta^1_-\eta^{2\dagger}_-$$

act with (anti)holomorphic γ matrices $\longrightarrow$ build a basis for the repr space

Graña,Minasian,Petrini,Tomasiello'05

Deformations of SU(3)×SU(3) structures

Deformations of Φ_+ (analogous for Φ_-) :

$$\Phi_+$$

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			$\delta\Phi_+$			
		$\mathbf{1},\mathbf{3}$		$\bar{\mathbf{3}},\bar{\mathbf{1}}$		
	$\delta\Phi_+$		$\delta\Phi_+$		$\delta\Phi_+$	
$\mathbf{1},\mathbf{1}$		$\bar{\mathbf{3}},\bar{\mathbf{3}}$		$\mathbf{3},\mathbf{3}$		$\bar{\mathbf{1}},\bar{\mathbf{1}}$
	$\bar{\mathbf{3}},\mathbf{1}$		$\mathbf{3},\bar{\mathbf{3}}$		$\bar{\mathbf{1}},\mathbf{3}$	
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? relation with δg_{mn} , δB_{mn} ?

Deformations of SU(3)×SU(3) structures

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	$\delta\Phi_+$		$\delta\chi_+$		$\delta\Phi_+$	
$\mathbf{1},\mathbf{1}$		$\bar{\mathbf{3}},\bar{\mathbf{3}}$		$\mathbf{3},\mathbf{3}$		$\bar{\mathbf{1}},\bar{\mathbf{1}}$
	$\bar{\mathbf{3}},\mathbf{1}$		$\mathbf{3},\bar{\mathbf{3}}$		$\bar{\mathbf{1}},\mathbf{3}$	
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Deformations of SU(3)×SU(3) structures

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- we arrive at:

$$\frac{e^{2\varphi}}{8}\int e^{-2\phi} vol_6 g^{mn} g^{pq} (\delta g_{mp}\delta g_{nq} + \delta B_{mp}\delta B_{nq}) = -\frac{\int\langle\delta\chi_-, \delta\bar{\chi}_-\rangle}{\int\langle\Phi_-, \bar{\Phi}_-\rangle} - \frac{\int\langle\delta\chi_+, \delta\bar{\chi}_+\rangle}{\int\langle\Phi_+, \bar{\Phi}_+\rangle}$$

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- contributions of $\delta\chi_+$ & $\delta\chi_-$ are $\begin{cases} \text{independent (ok } N=2 \text{ in 4d)} \\ \text{symmetric} \end{cases}$

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- contributions of $\delta\chi_+$ & $\delta\chi_-$ are { independent (ok $N=2$ in 4d) / symmetric }

- Recall : $K_\pm = -\log i\int\langle\Phi_\pm,\overline{\Phi}_\pm\rangle$. Then :

$$\delta^{\text{holo}}\delta^{\text{anti}}K_- + \delta^{\text{holo}}\delta^{\text{anti}}K_+ = -\frac{\int\langle\delta\chi_-,\delta\bar{\chi}_-\rangle}{\int\langle\Phi_-,\bar{\Phi}_-\rangle} - \frac{\int\langle\delta\chi_+,\delta\bar{\chi}_+\rangle}{\int\langle\Phi_+,\bar{\Phi}_+\rangle} + \spadesuit\spadesuit$$

Scalar potential

NSNS sector$\longrightarrow$ $V_{\rm NS} \sim \int_{M_6} vol_6 e^{-2\phi}\left(R_6 + 4\partial_m\phi\partial^m\phi - \frac{1}{12}H_{mnp}H^{mnp}\right)$

Scalar potential

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Recast in generalized geometry language

(4d variables from NSNS sector are encoded in $\Phi_\pm$)

- $[D_m, D_n]\,\eta \;\sim R_{mnpq}\gamma^{pq}\eta$
- derive formula relating R_6 and $\Phi_\pm \sim \eta^1_+ \otimes \eta^{2\dagger}_\pm$
- 'dress' it with ϕ and B

Scalar potential

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$$V_{\rm NS} = \frac{e^{4\varphi}}{4}\int \langle\, d\Phi_+, *_B(d\overline{\Phi}_+)\,\rangle + \langle\, d\Phi_-, *_B(d\overline{\Phi}_-)\,\rangle$$

$$- e^{4\varphi}\int \frac{\left|\langle d\Phi_+, \Phi_-\rangle\right|^2 + \left|\langle d\Phi_+, \overline{\Phi}_-\rangle\right|^2}{i\langle\Phi, \overline{\Phi}\rangle}$$

DC '08

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$$V = V_{\rm NS} + V_{\rm RR} \;=\; \frac{e^{4\varphi}}{4}\int \langle\, d\Phi_+, *_B(d\overline{\Phi}_+)\,\rangle + \langle\, d\Phi_-, *_B(d\overline{\Phi}_-)\,\rangle$$

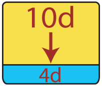
$$- \,e^{4\varphi}\int \frac{\left|\langle d\Phi_+, \Phi_-\rangle\right|^2 + \left|\langle d\Phi_+, \overline{\Phi}_-\rangle\right|^2}{i\langle \Phi, \overline{\Phi}\rangle}$$

$$+ \,\frac{e^{4\varphi}}{2}\int \langle G, *_B G\rangle$$

DC '08

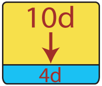
G: sum of internal RR fields

Reducing to 4d

When reducing 

$\Rightarrow$ need to truncate to a finite set of modes

Reducing to 4d

When reducing 

$\Rightarrow$ need to truncate to a finite set of modes

- Truncation specified by a finite basis of (poly)forms

$$\Sigma_+ = \begin{pmatrix} \tilde{\omega}^A \\ \omega_A \end{pmatrix} \quad , \quad \Sigma_- = \begin{pmatrix} \beta^I \\ \alpha_I \end{pmatrix}$$

to be used in expansions like :

$$\Phi_+ = X^A \omega_A - \mathcal{F}_A \tilde{\omega}^A \quad , \quad \Phi_- = Z^I \alpha_I - \mathcal{G}_I \beta^I$$

Graña,Louis,Waldram

- for a CY : $\Phi_+ = e^{B+iJ}$, $\Phi_- = \Omega$ and the forms span $H^\bullet(M_6)$

Reducing to 4d

- $d\Phi_\pm \neq 0 \;\; \Rightarrow$ in general $\Sigma_\pm$ are not closed :

$$d\Sigma_- = \mathbb{Q}\Sigma_+$$

$\mathbb{Q}$: 'geometric fluxes' $\longrightarrow$ more gaugings than CY with fluxes

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- Postulate this system of expansion forms

(satisfying a set of constraints)

- derive the full bosonic action of $N = 2$ gauged sugra

Summary : Comparison with Calabi-Yau

	CY & no fluxes	SU(3)$\times$SU(3) + fluxes
4d action	$N=2$ ungauged sugra	$N=2$ gauged sugra charges: RR, NSNS-fluxes $d\Sigma_- = \mathbb{Q}\Sigma_+$
Geometric moduli δg_{mn}	$\delta J\ ,\ \delta\Omega$	$\delta\Phi_+\ ,\ \delta\Phi_-$ (include $\delta B\ ,\ \delta\phi$)
Kähler potentials	$K_+ \sim \log\int J\wedge J\wedge J$ $K_- \sim \log i\int \Omega\wedge\overline{\Omega}$	$K_\pm = \log i\int\langle\Phi_\pm,\overline{\Phi}_\pm\rangle$
Scalar potential	$V=0$	$V = V(d\Phi_\pm,$ fluxes$)$
Susy vacua	trivially $N=2$	nontrivial $N=1$ conditions.

Lifting $N = 1$ vacua

Graña, Minasian, Petrini, Tomasiello'05

10d IIA sugra → N=1 conditions :

$$d\Phi_+ = -2\bar{\mu}\,\mathrm{Re}(\Phi_-)$$

$$d\Phi_- = -3i\,\mathrm{Im}(\mu\Phi_+) + \tfrac{i}{2}e^{\phi} *_B G$$

with $|\mu|^2 \sim -\Lambda$

↓ dimensional reduction

4d N=2 gauged sugra → N=1 conditions :

$\langle\delta_\varepsilon \text{gravitini}\rangle = 0$

$\langle\delta_\varepsilon \text{gaugini}\rangle = 0$

$\langle\delta_\varepsilon \text{hyperini}\rangle = 0$

under a single susy ε

↓ N=2 → N=1 truncation

4d N=1 sugra → N=1 conditions :

F-flatness

D-flatness

map

Cosets with SU(3) structure

DC, Kashani-Poor '09

Concrete examples of M_6 :

$$\frac{G_2}{SU(3)} \qquad \frac{Sp(2)}{S(U(2) \times U(1))} \qquad \frac{SU(3)}{U(1) \times U(1)}$$

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↑

S^6

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$\uparrow$ S^6 $\qquad$ $\uparrow$ $\mathbb{CP}^3$

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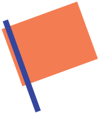
$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$

S^6 $\qquad$ $\mathbb{CP}^3$ $\qquad$ Flag $\mathbb{F}(1,2;3)$



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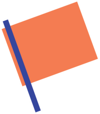

- Admit SU(3) structure not SU(2)

Cosets with SU(3) structure

DC, Kashani-Poor '09

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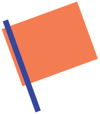

- Admit SU(3) structure not SU(2)
- Group action simplifies the problem
- Support $\mathcal{N}=1$ AdS_4 vacua of massive type IIA

Behrndt,Cvetic'04, Tomasiello'07, Koerber,Lüst,Tsimpis'08

The basis forms

Expansion basis $=$ { Left-invariant forms }

$$\frac{\mathbf{SU(3)}}{\mathbf{U(1)\times U(1)}}$$

Left-invariant metric :

$$g_{mn} = \text{diag}(v_1, v_1, v_2, v_2, v_3, v_3) \qquad v_a > 0 \quad : \quad \text{geometric moduli}$$

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Basis of left-invariant forms :

$$\omega_0 = 1 \quad , \quad \omega_1 = -e^{12} \quad , \quad \omega_2 = e^{34} \quad , \quad \omega_3 = -e^{56} \, ,$$

$$\alpha = \frac{1}{2}(e^{135} + e^{146} - e^{236} + e^{245}) \quad , \quad \beta = \frac{1}{2}(-e^{136} + e^{145} - e^{235} - e^{246}) \, ,$$

$$\tilde{\omega}^0 = e^{123456} \quad , \quad \tilde{\omega}^1 = e^{3456} \quad , \quad \tilde{\omega}^2 = -e^{1256} \quad , \quad \tilde{\omega}^3 = e^{1234} \, .$$

Starting from type IIA, we derive the full 4d bosonic action

Scalar potential V

$$V = V_{\mathrm{NS}} + V_{\mathrm{RR}}$$

Scalar potential V

$$V = V_{\text{NS}} + V_{\text{RR}} = e^{2\varphi}(\ldots) + e^{4\varphi}(\ldots)$$

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- 4d version of Maldacena-Nunez no-go thm.

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- 4d version of Maldacena-Nunez no-go thm.
- Holds for any SU(3) structure compactification
- Interesting for AdS_4/CFT_3 ABJM '08
- Mechanism for dS?

Idea: modify $V(\varphi)$ by including string loop corrections

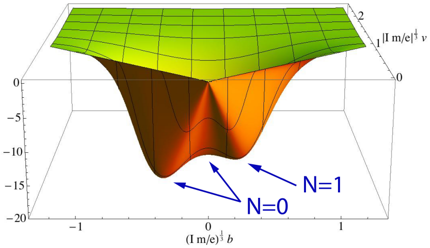
Establish the corrected V

♠ Don't find any dS ♠

Extremizing V

In the Nearly-Kähler limit (v^a all equal) :

given a choice of flux $\longrightarrow$ 3 extrema



$\longleftarrow$ V for $\frac{G_2}{SU(3)}$

Moduli are fixed

$$N=1: \quad v = \frac{\sqrt{15}}{2}\left(\frac{1}{20}\left|\frac{e}{m}\right|\right)^{1/3}, \quad b = \frac{1}{2}\left(\frac{1}{20}\,\frac{e}{m}\right)^{1/3}, \quad \tilde{\xi} = \frac{24mb^2}{q}\,, \quad e^{2\varphi} = \frac{5q^2}{48m^2v^4}$$

Extremizing V

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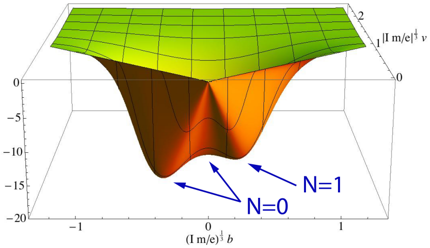

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Stability

Breitenlohner-Freedman bound $m^2_{\text{tachyonic}} \geq -\frac{3}{4}\langle V\rangle$

all extrema are stable

Consistency of the coset reduction

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We analyze the EoM of massive type IIA

$\downarrow\downarrow\downarrow$

precisely recover 4d N=2 gauged sugra EoM

Conclusions

- Flux compactifications demand new mathematical tools

  For $N = 2$ in 4d from type II:

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- Interplay between 10d and 4d

Fluxes & 4d gauged sugra

Type II sugra on CY_3

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

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4d fields are defined
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E.g. in IIA:

$$C_3 = \xi^I \alpha_I - \tilde{\xi}_I \beta^I + \dots \qquad {\scriptstyle I=0,1,\dots,h^{2,1}}$$

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If no H fluxes, then:

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scalar potential arises from $\int_{M_6} H \wedge *H$

Deformations of SU(3)×SU(3) structures

compatible $\Phi_+ \ , \ \Phi_-$

$\downarrow\downarrow\downarrow$

$$\mathcal{J}_{\pm}{}^{\Lambda}{}_{\Sigma} \;=\; 4i\frac{\langle\, \text{Re}\,\Phi_\pm \;,\, \Gamma^{\Lambda}{}_{\Sigma}\text{Re}\,\Phi_\pm \,\rangle}{\langle\, \Phi_\pm \;,\, \overline{\Phi}_\pm \,\rangle} \qquad \text{with} \qquad [\mathcal{J}_+, \mathcal{J}_-] = 0$$

$$\mathcal{J}_\pm \ : T \oplus T^* \to T \oplus T^* \quad , \quad (\mathcal{J}_\pm)^2 = -id_{T\oplus T^*}$$

generalized almost complex structure

where $\Gamma^\Lambda = \begin{pmatrix} dx^m\wedge \\ \iota_{\partial_m} \end{pmatrix}$: $O(6,6)$ gamma matrices

Metric on $T \oplus T^*$: $\mathcal{G} = -\mathcal{J}_+\mathcal{J}_- = \begin{pmatrix} g^{-1}B & g^{-1} \\ g - Bg^{-1}B & -Bg^{-1} \end{pmatrix}$

Deformations :

$$g^{mn}g^{pq}(\delta g_{mp}\delta g_{nq} + \delta B_{mp}\delta B_{nq}) \;=\; -\tfrac{1}{2}\text{Tr}\big[\delta\mathcal{G}\delta\mathcal{G}\big]$$

use : $\delta\mathcal{G} = -\delta\mathcal{J}_+\mathcal{J}_- - \mathcal{J}_+(\delta\mathcal{J}_-)$

Important ingredient : $\mathcal{G}_I = \mathcal{M}_{IJ} Z^J \quad , \quad D\mathcal{G}_I = \overline{\mathcal{M}}_{IJ} D Z^J$

↖ period matrix ↗

$\mathcal{R}$=Re$\mathcal{M}$, $\mathcal{I}$=Im$\mathcal{M}$

$$\mathbb{M} \equiv \begin{pmatrix} \mathcal{I} + \mathcal{R}\mathcal{I}^{-1}\mathcal{R} & -\mathcal{R}\mathcal{I}^{-1} \\ -\mathcal{I}^{-1}\mathcal{R} & \mathcal{I}^{-1} \end{pmatrix} = \begin{pmatrix} -\int\langle\alpha, *_B\alpha\rangle & \int\langle\alpha, *_B\beta\rangle \\ \int\langle\beta, *_B\alpha\rangle & -\int\langle\beta, *_B\beta\rangle \end{pmatrix}$$

- uses $*_B \bullet := e^{-B} * \lambda(e^B \bullet)$
- generalizes a result valid for the harmonic 3-forms of CY
- parallel expression for even forms $\longrightarrow \mathcal{N}$ & $\mathbb{N}$
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Special Kähler Geometry

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  (valid for CY as well)
- In e.g. IIA:
  - Im$\mathcal{N}$ and Re$\mathcal{N}$ define kinetic & top. terms for gauge fields
  - $\mathbb{M}$ enters in the hyperscalar kinetic terms
  - Both $\mathbb{M}$ and $\mathbb{N}$ appear in the scalar potential

The basis forms

Closed system under d :

$$d\omega_a = q_a \alpha$$

$$d\alpha = 0 \qquad d\beta = q_a \tilde{\omega}^a$$

$$d\tilde{\omega}^a = 0$$

q_a : geometric fluxes $\longrightarrow$ new charges w.r.t. the CY case

Also closed under the Hodge $*$:

$$*\alpha = \beta \quad , \quad *\tilde{\omega}^0 \sim \omega_0 \quad , \quad *\tilde{\omega}^a \sim \omega_a$$

Most general left-invariant SU(3) structure:

$$J = v^a \omega_a \qquad \Omega \sim \alpha + i\beta$$

The 4d theory

Starting from type IIA, we derive the full 4d bosonic action

$N = 2$ gauged sugra with :

- gravitational multiplet $(g_{\mu\nu} \, , \, A^0)$
- at most 3 vector multiplets $(b^a + i v^a \, , \, A^a)$
- just the universal hypermultiplet $(B_{\mu\nu} \, , \, \varphi \, , \, \xi \, , \, \tilde{\xi})$
  (actually, tensor multiplet)

Fluxes $\longrightarrow$ Gaugings

scalar field	electric ch. under A^0, A^a	provided by	magnetic ch. under A^0, A^a	provided by
ξ	q_a	$d\omega_a = q_a \alpha$	–	–
dual of $B_{\mu\nu}$	e_0, e_a	$\underbrace{G_4, G_6}_{\text{RR field-str.}}$	m^0, m^a	$\underbrace{G_0, G_2}_{\text{RR field-str.}}$

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$$\hat{R}_{MN} + 2\hat{\nabla}_M \partial_N \phi - \frac{1}{2}\iota_M \hat{H} \lrcorner \iota_N \hat{H} - \frac{e^{2\phi}}{4}\sum_{k=0}^{10} \iota_M \hat{F}_{(k)} \lrcorner \iota_N \hat{F}_{(k)} \; = \; 0$$

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left-invariance $\Rightarrow$ all terms are constant along M_6

$$d\hat{s}^2 = e^{2\varphi(x)} g_{\mu\nu}(x) dx^\mu \otimes dx^\nu + g_{mn}(x) e^m(y) \otimes e^n(y)$$

$$g_{mn} = diag(v_1, v_1, v_2, v_2, v_3, v_3)$$

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$$\hat{R}_{mn} + 2\hat{\nabla}_m \partial_n \phi \;=\; \underbrace{R_{mn}}_{\downarrow\; \partial_{v^a} V_{\mathrm{NS}}} + \underbrace{\tfrac{1}{2} e^{-2\varphi}\left(g^{pq}\partial_\mu g_{mp} \partial^\mu g_{nq} - \nabla^2 g_{mn}\right)}_{\downarrow\; \left[\partial_{v^a} - \nabla_\mu \frac{\partial}{\partial(\partial_\mu v^a)}\right] \mathcal{G}_{bc}(v) \partial_\mu v^b \partial^\mu v^c}$$

$\Rightarrow$ 4d EoM for the internal metric moduli ...etc...

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Strominger'97, Antoniadis,Minasian,Theisen,Vanhove'03

1. one-loop correction to the 10d action : R^4 terms

   by dim.red. : $$\int d^4x\sqrt{g}\left(e^{-2\varphi} - \frac{4\zeta(2)}{(2\pi)^3}\chi(M_6)\right)R_4 \; + \; \ldots$$

2. Quaternionic metric

   universal hypermultiplet, 3 isometries $\Big\} \longrightarrow$ Calderbank-Pedersen: just one parameter c

   two possibilities : $c = 0$ or $c \sim \chi(M_6)$

3. Loop-corrected scalar potential follows by $N = 2$

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- Destiny tree-level AdS Nearly-Kähler vacua?

  Numerically : they are still there.

  For $N = 1$ vacuum: analytic study