Control of Timed Systems

Habilitation à Diriger les Recherches

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Nantes, France

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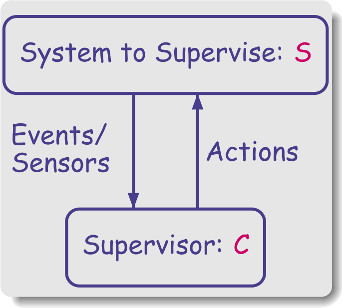
Research Domain: Design of Real-Time Systems



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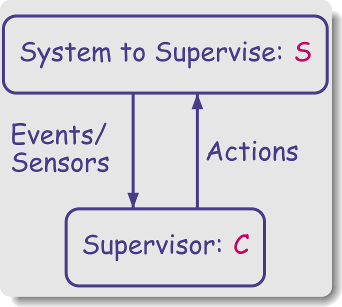
Build Safe Systems



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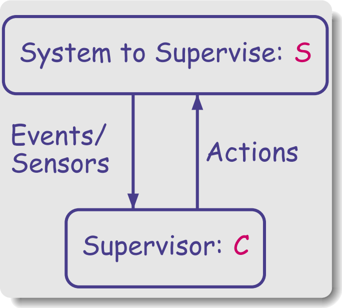

Property φ

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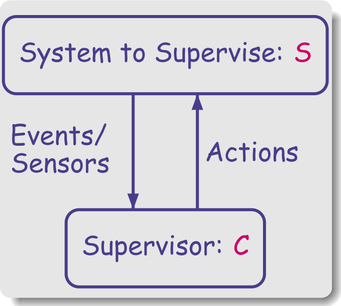
Build Safe Systems

Modeling
- Timed Automata
- Time Petri Nets
- Timed Logics



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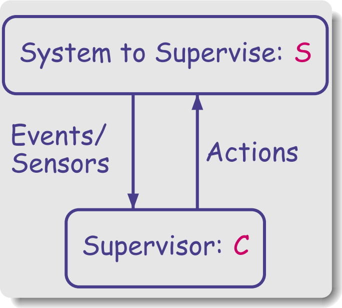

Verification
- Test
- Theorem Proving
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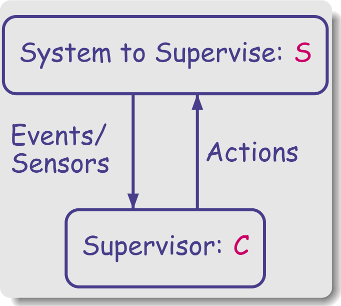

Diagnosis & Control
- Diagnosis
- Control
- Optimal Control

Verification
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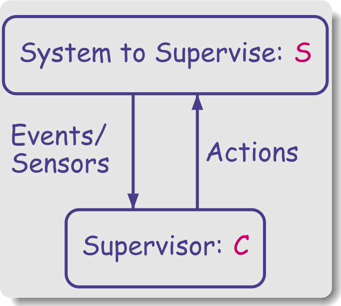

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Implementation
- Digital Supervisors
- Continuous Systems

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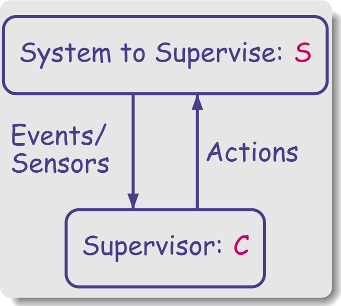

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Outline of the Talk

- Control of Timed Systems: Basics
  - Verification and Control
  - Timed Automata
  - Timed Game Automata
  - Symbolic Algorithms for Timed Game Automata
- Selected Contributions
  - Implementable Controllers
  - Optimal Controllers
- Conclusion & Perspectives

Next:

- Control of Timed Systems: Basics
  - Verification and Control
  - Timed Automata
  - Timed Game Automata
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- Selected Contributions
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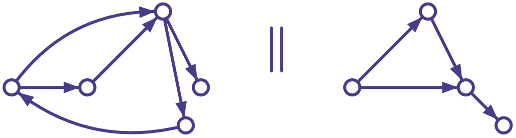
Verification and Control

Verification and Control

Modelling



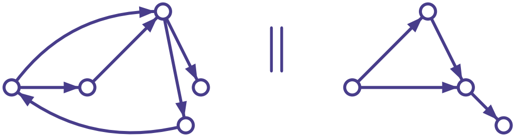
Always (not bad)

$S \quad \| \quad C \quad \models \quad \varphi$

Verification and Control

Does the system meet the specification?

Modelling



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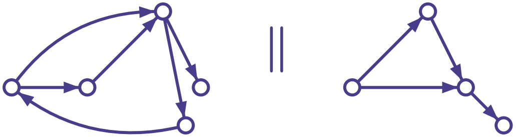

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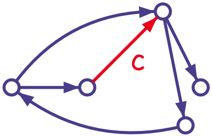
Model Checking Problem

Does the closed system $(S \parallel C)$ satisfy φ ?

Verification and **Control**

Can we enforce the system to meet the specification?

Modelling



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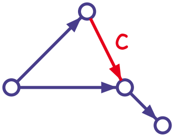

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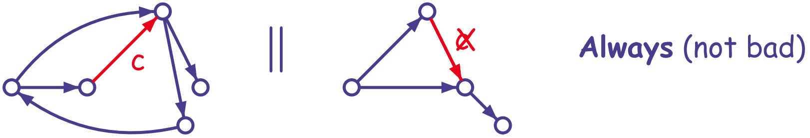

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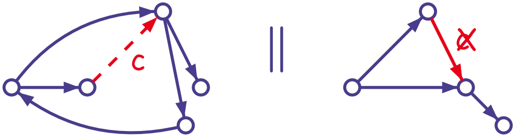

$S \;\|\; C \models \varphi$

Control Problem

Can the open system S be restricted to satisfy φ ?

Is there a Controller C such that $(S \| C) \models \varphi$?

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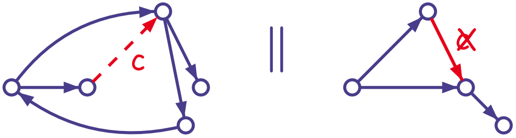

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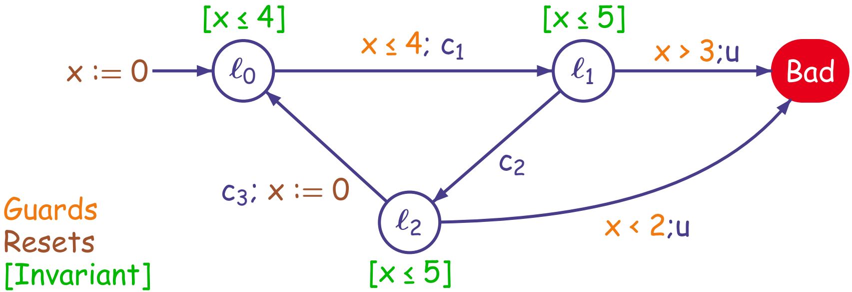

Timed Automaton = Finite Automaton + clock variables

Run = sequence of discrete and time steps

$\rho_1 : \quad (\ell_0, 0) \xrightarrow{1.55} (\ell_0, 1.55) \xrightarrow{c_1} (\ell_1, 1.55) \xrightarrow{1.67} (\ell_1, 3.22) \xrightarrow{u} (Bad, 3.22)$

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Zeno behaviour

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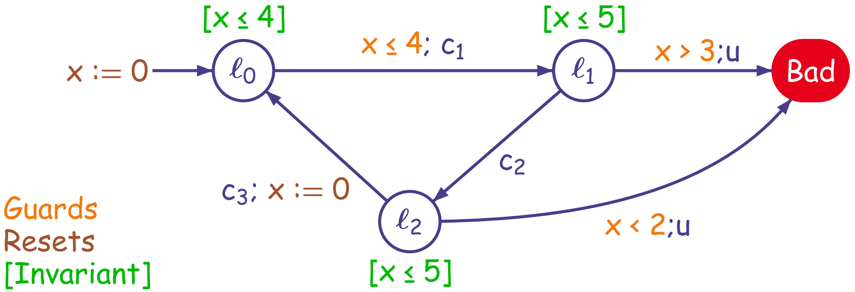

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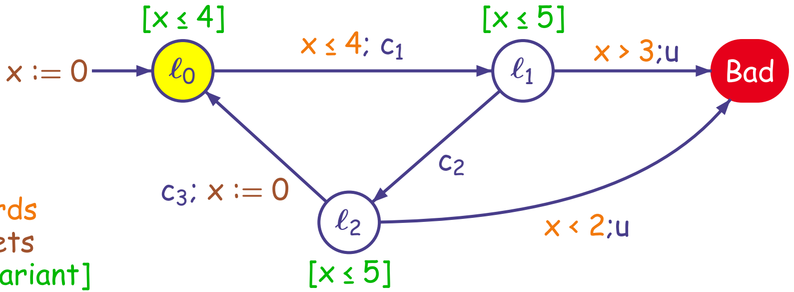

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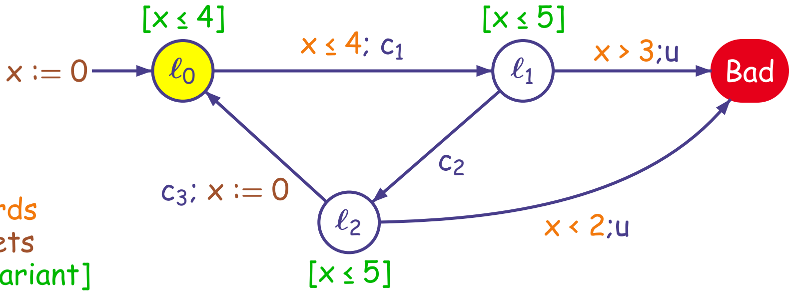

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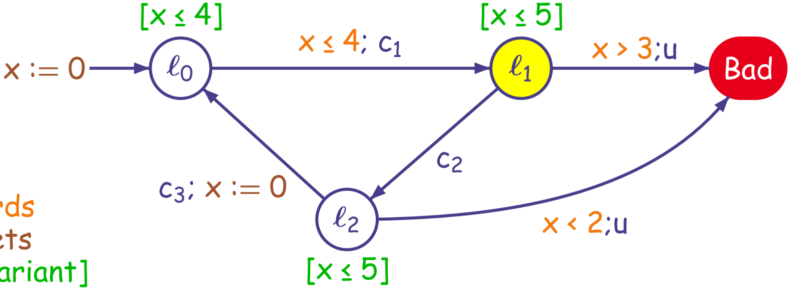

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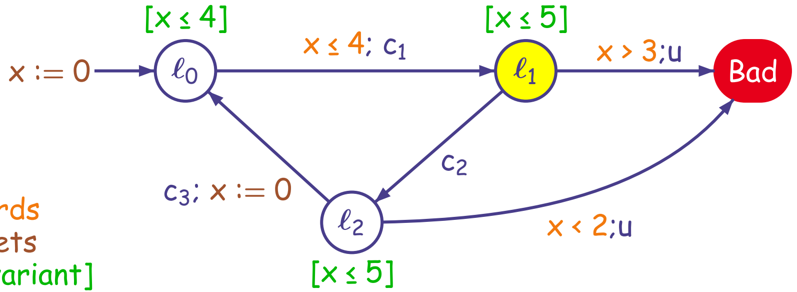

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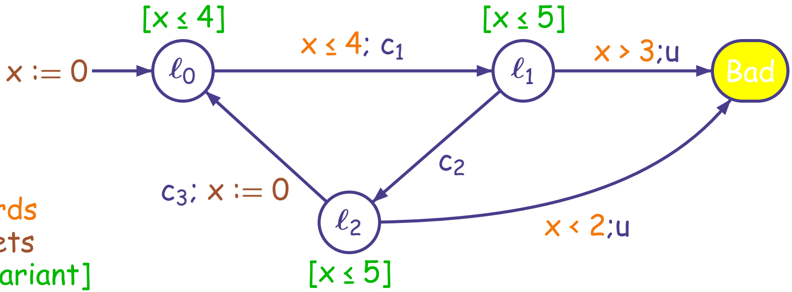

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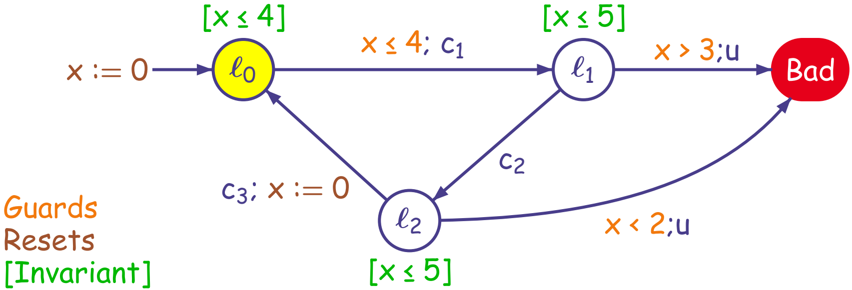

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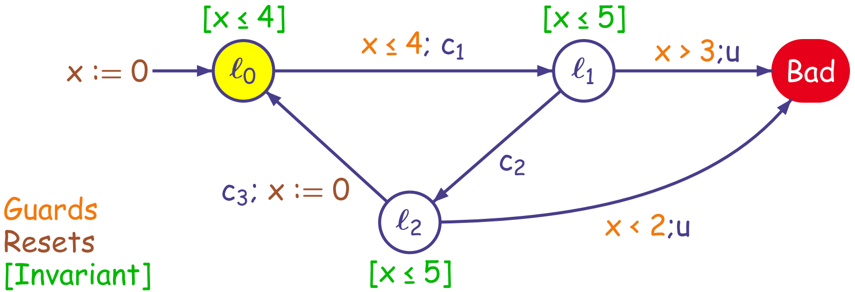

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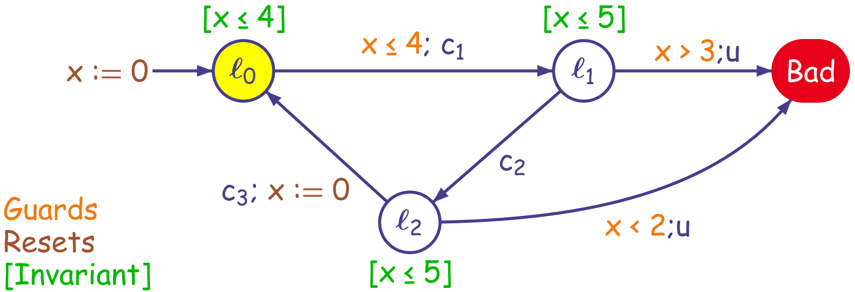

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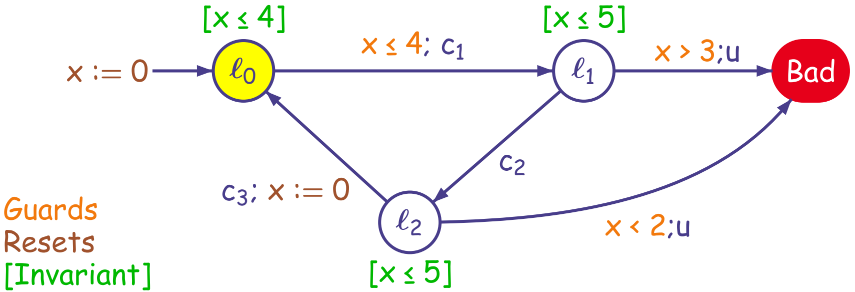

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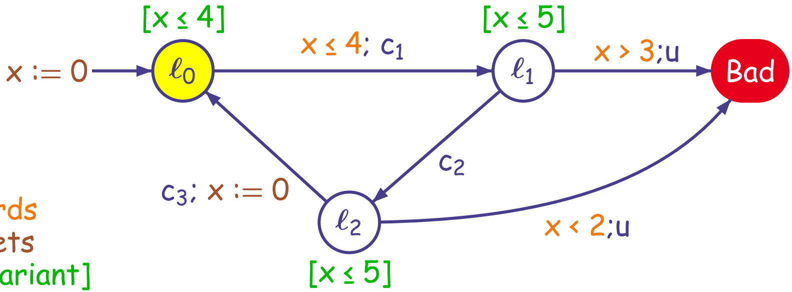

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States & Symbolic States

- $Q = L \times \mathbb{R}_{\geq 0}^{Clock}$ is the set of states of the TGA
  $q = (\ell, v) \in Q$
- Discrete Successors of $X \subseteq Q$ by an action a:
  $$Post^a(X) = \{q' \in Q \mid q \xrightarrow{a} q' \text{ and } q \in X\}$$
- Time Successors of $X \subseteq Q$:
  $$Post^\delta(X) = \{q' \in Q \mid \exists t \geq 0 \mid q \xrightarrow{t} q' \text{ and } q \in X\}$$
- Zone = conjunction of triangular constraints
  $x - y < 3, x \geq 2 \wedge 1 < y - x < 2$
- Symbolic State is defined by a State predicate (SP)
  $P = \cup_{i \in [1..n]}(\ell_{j_i}, Z_i)$, $\ell_{j_i} \in L$, Z_i is a zone
  $(\ell_1, 2 \leq x < 4)$ or $(\ell_0, x < 1 \wedge y - x \geq 2)$ or $(\ell_0, x \leq 2) \cup (\ell_2, x > 0)$

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  $(\ell_1, 2 \leq x < 4)$ or $(\ell_0, x < 1 \wedge y - x \geq 2)$ or $(\ell_0, x \leq 2) \cup (\ell_2, x > 0)$

Effectiveness of *$Post^a$* and *$Post^\delta$*

If P is a SP then $Post^a(P), Post^\delta(P)$ are SP and can be computed effectively. (There is a symbolic version for $Post^a$ and $Post^\delta$.)

States & Symbolic States

- $Q = L \times \mathbb{R}_{\geq 0}^{Clock}$ is the set of states of the TGA
  $q = (\ell, v) \in Q$
- Discrete Successors of $X \subseteq Q$ by an action a:
  $$Post^a(X) = \{q' \in Q \mid q \xrightarrow{a} q' \text{ and } q \in X\}$$
- Time Successors of $X \subseteq Q$:
  $$Post^\delta(X) = \{q' \in Q \mid \exists t \geq 0 \mid q \xrightarrow{t} q' \text{ and } q \in X\}$$
- Zone = conjunction of triangular constraints
  $x - y < 3, x \geq 2 \wedge 1 < y - x < 2$
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  $P = \cup_{i \in [1..n]}(\ell_{j_i}, Z_i), \ell_{j_i} \in L, Z_i$ is a zone
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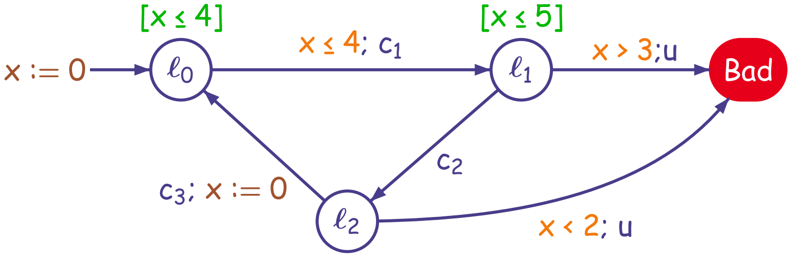
Decidability Result for TA

▸ Region Graph

The Reachability Problem for TA is PSPACE-Complete.
Build a finite abstraction: region automaton

Timed Game Automata



- Introduced by Maler, Pnueli, Sifakis [Maler et al.'95]
- The controller continuously observes the system
  time elapsing and discrete moves are observable
- The controller has the choice between two types of moves:
  - "do nothing" (delay action)
  - "do a controllable action" (among the ones that are possible)
- It can prevent time elapsing by taking a controllable move

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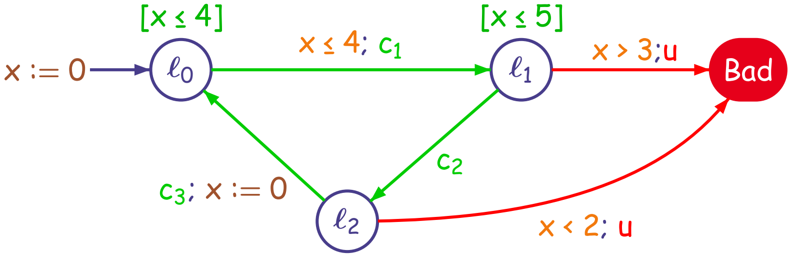

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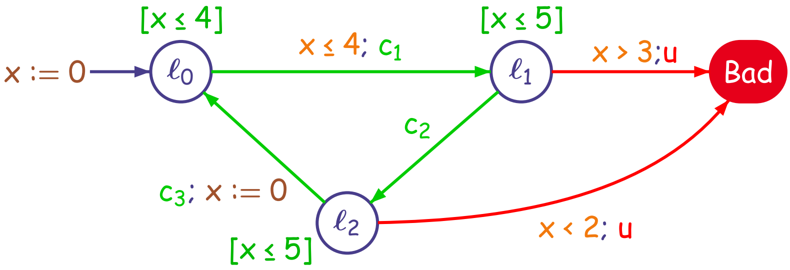
Strategies and Winning States



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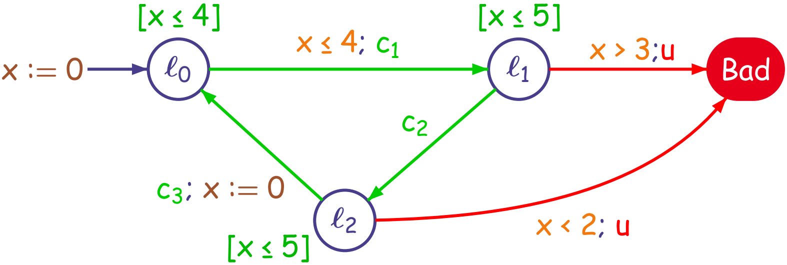

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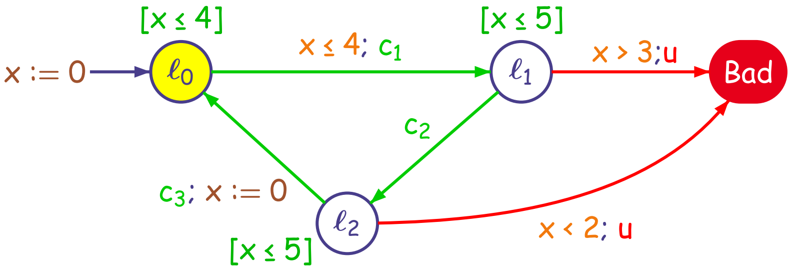

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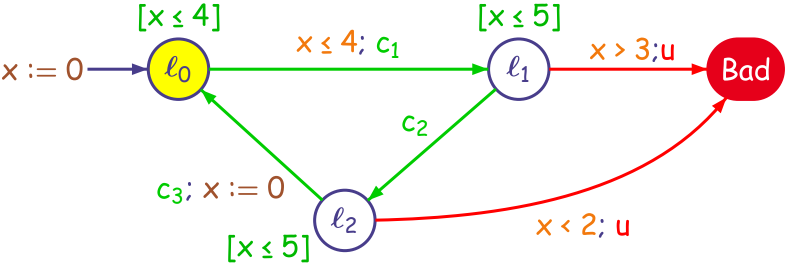

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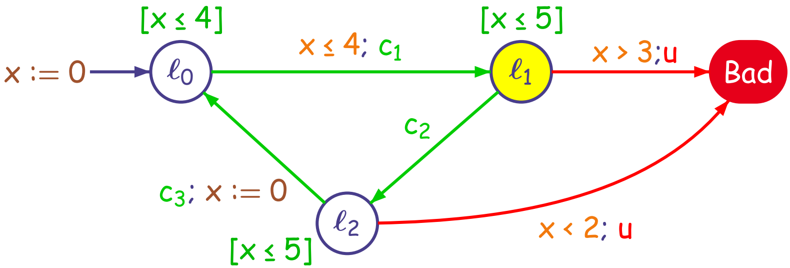

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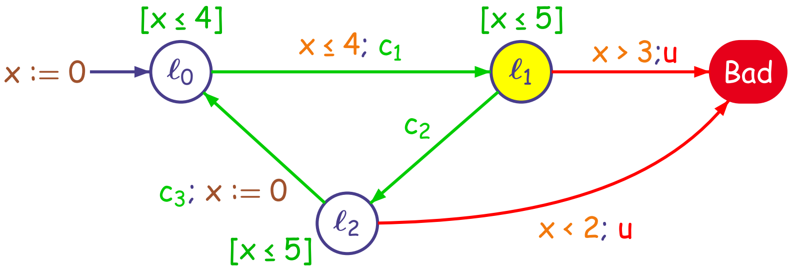

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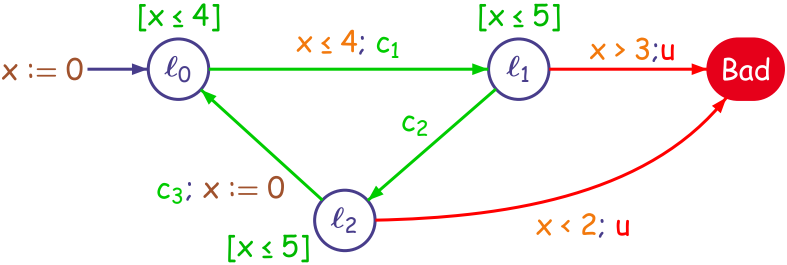

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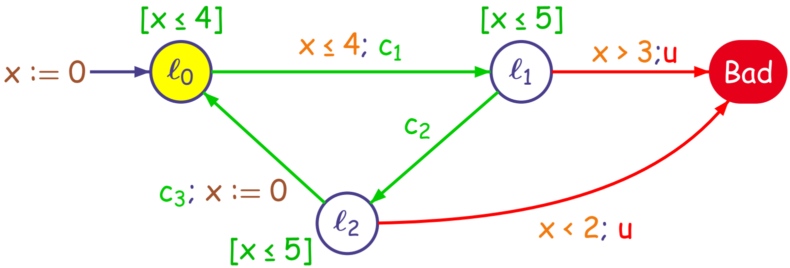

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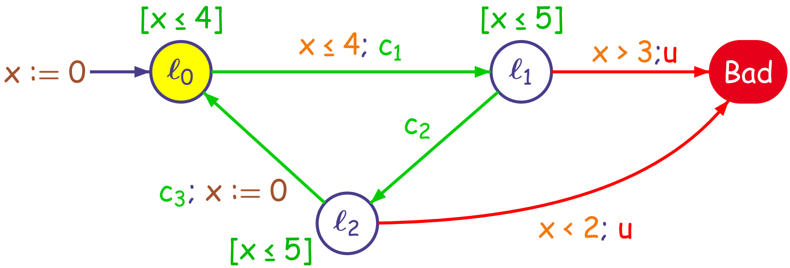

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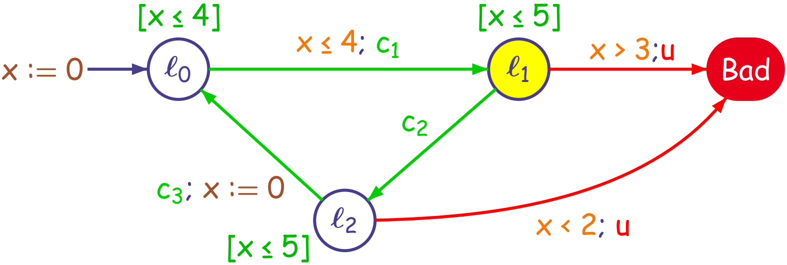

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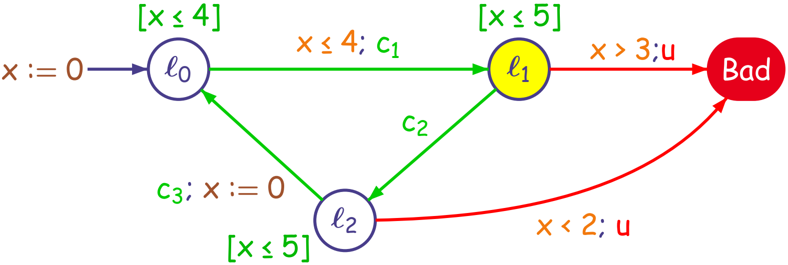

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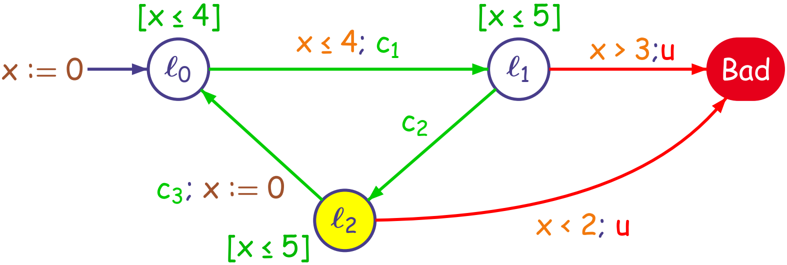

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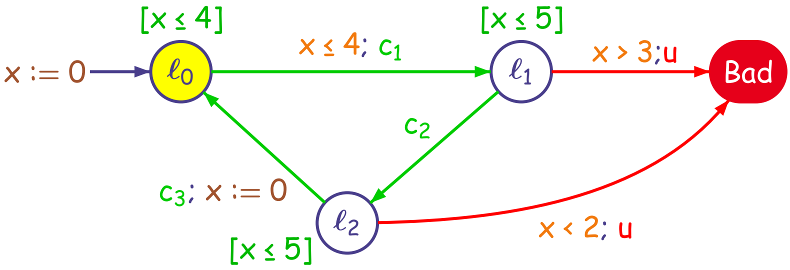

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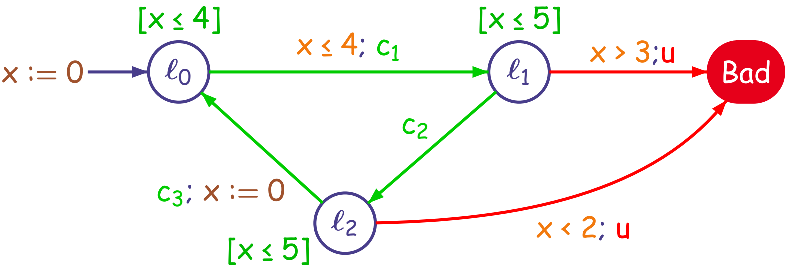

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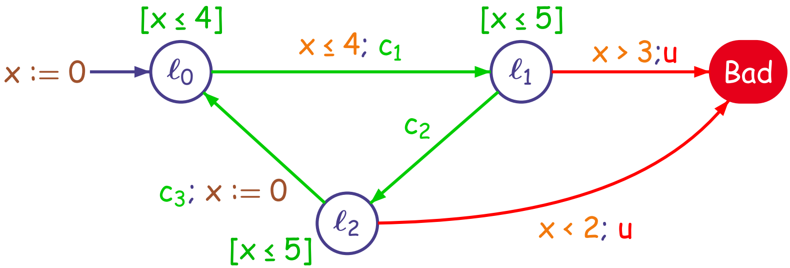

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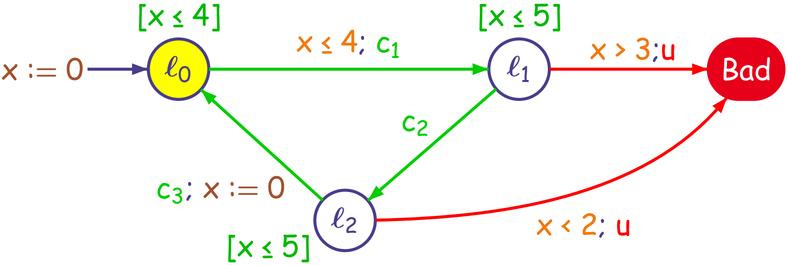

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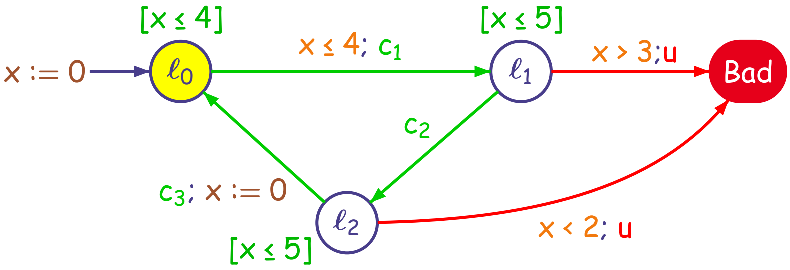

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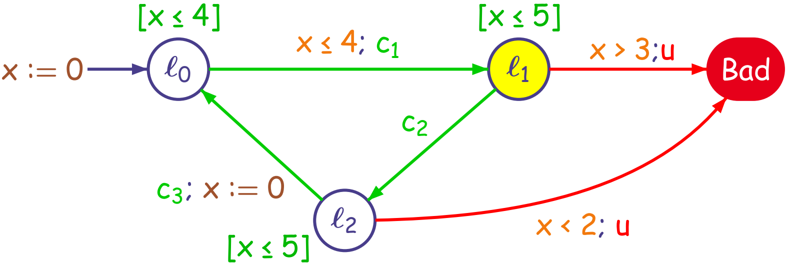

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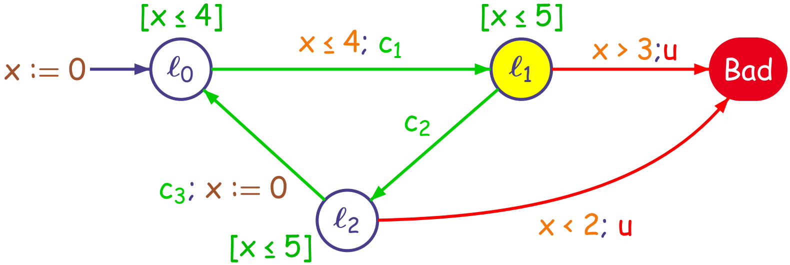

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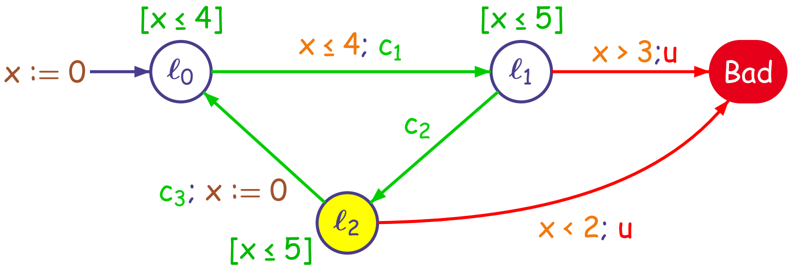

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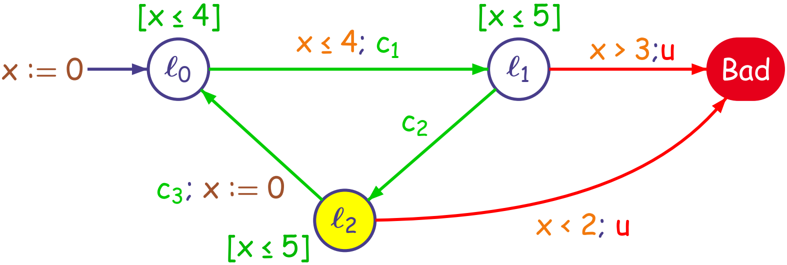

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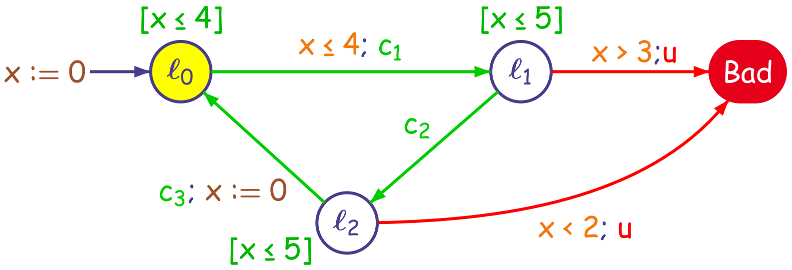

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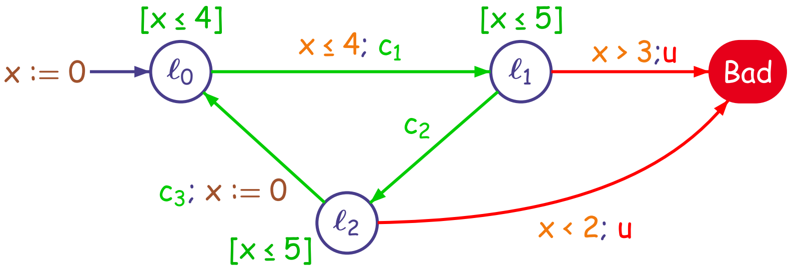

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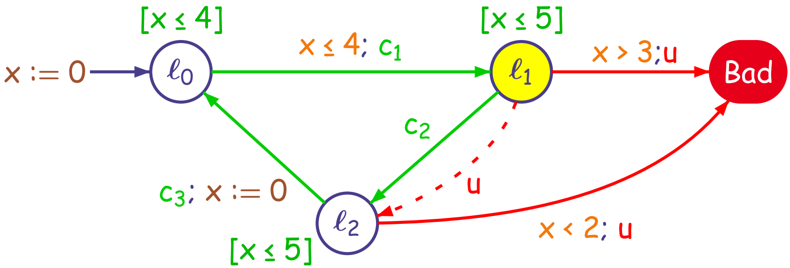

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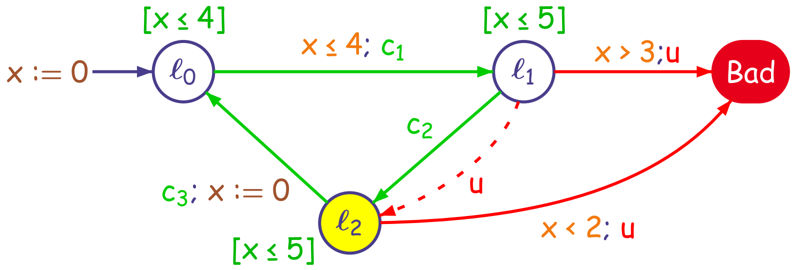

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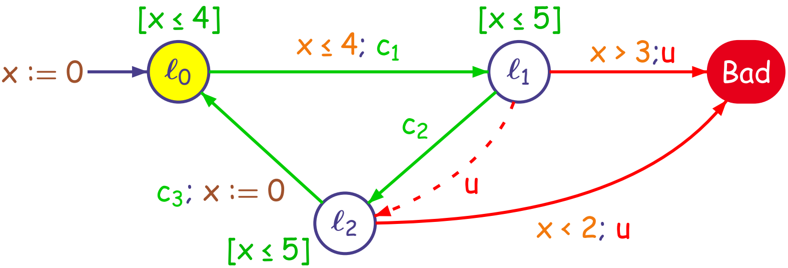
Strategies and Winning States



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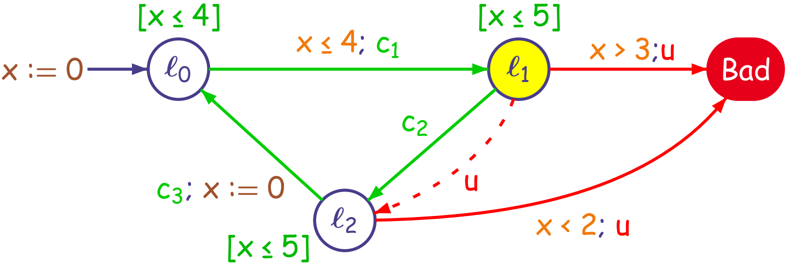

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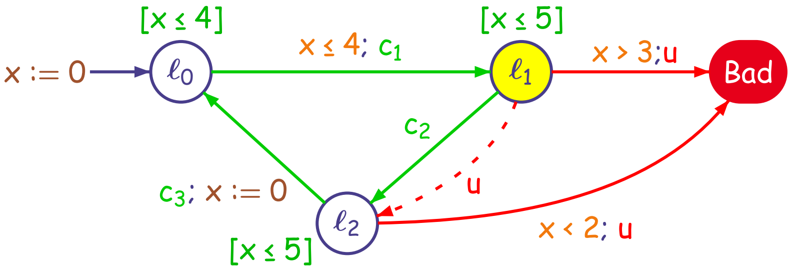

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ρ : $(\ell_0, 0) \xrightarrow{2} (\ell_0, 2) \xrightarrow{c_1} (\ell_1, 2) \xrightarrow{u \text{ at } \delta \leq 0.5} (\ell_2, 2 + \delta) \xrightarrow{c_3 \text{ at } 2-\delta} (\ell_0, 0)$

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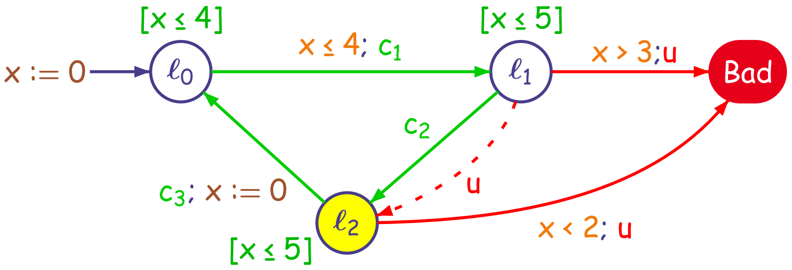
Strategies and Winning States



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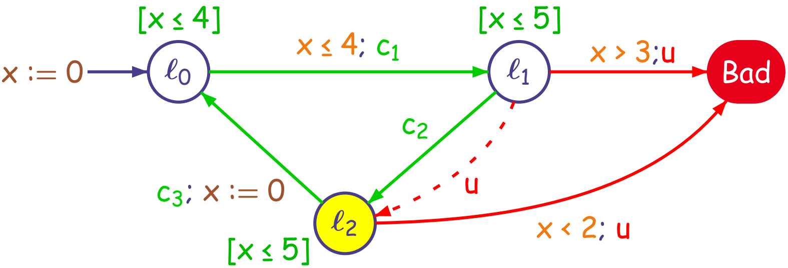

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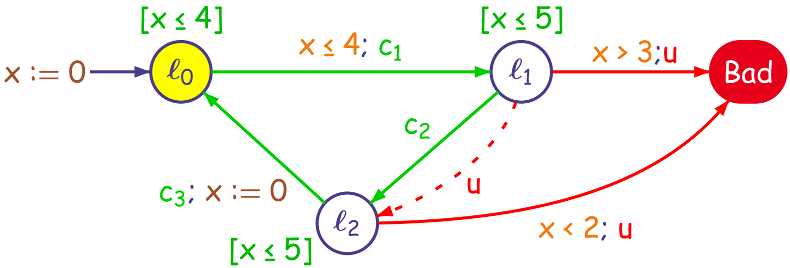
Strategies and Winning States



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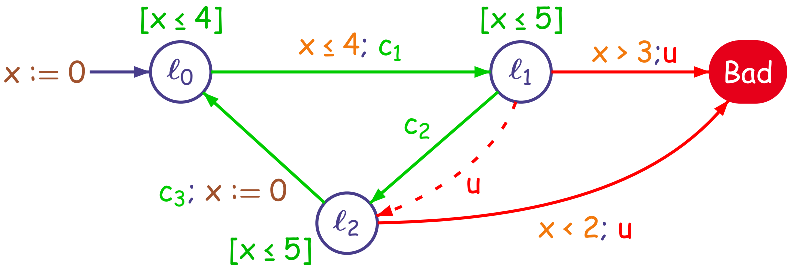

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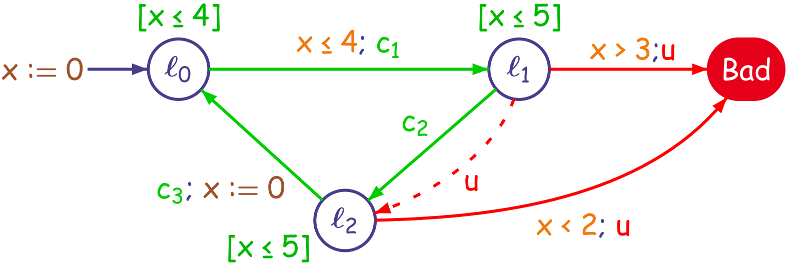
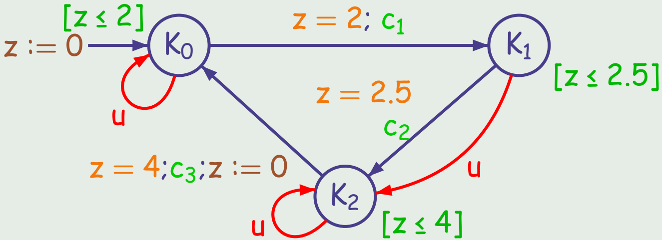
Strategies and Winning States

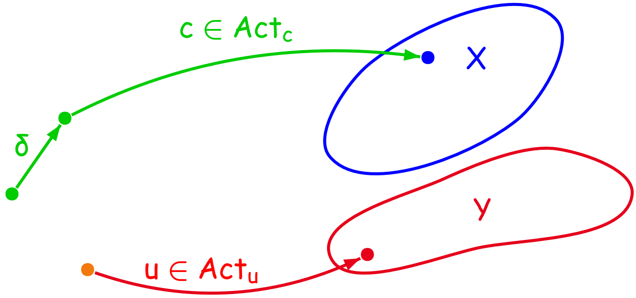


The Strategy f′ as a Timed Automaton



Controllable Predecessors

$\pi(X, Y) =$ states from which one can enforce X and avoid Y by: time elapsing followed by a controllable action



Fixpoint Characterization of Winning States for Safety Games:

1. Let φ be a set of safe (good) states and G a game
2. Let W^* be the greatest fixpoint of $h(X) = \varphi \cap \pi(X, \overline{X})$
3. W^* is the set of winning states for (G, φ)

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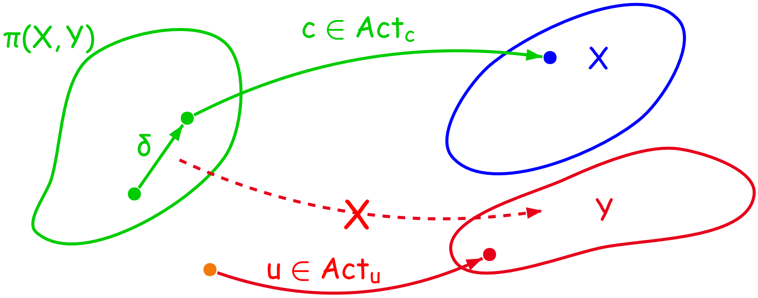

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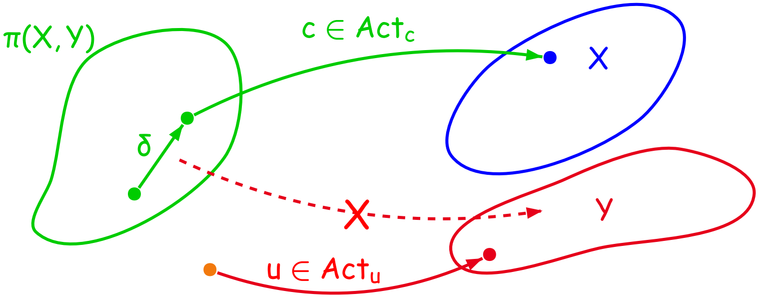

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Symbolic Algorithms for Safety Control

[Maler et al.'95, De Alfaro et al.'01]

Details & Example

1. There is a symbolic version for $\pi(X, Y)$
2. $\Longrightarrow$ there is a symbolic version for $h(X)$

Symbolic Algorithms for Safety Control

[Maler et al.'95, De Alfaro et al.'01]

▸ Details & Example

1. There is a symbolic version for $\pi(X, Y)$
2. $\Longrightarrow$ there is a symbolic version for $h(X)$

- Control Problem (CP): check that $(\ell_0, 0) \in W^*$
- Control Synthesis Problem (CSP): by definition of π there is a strategy

Symbolic Algorithms for Safety Control

[Maler et al.'95, De Alfaro et al.'01]

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Theorem (Termination)

The iterative computation of W^* terminates for (G, φ) with G a timed game automaton φ a ω-regular winning condition.

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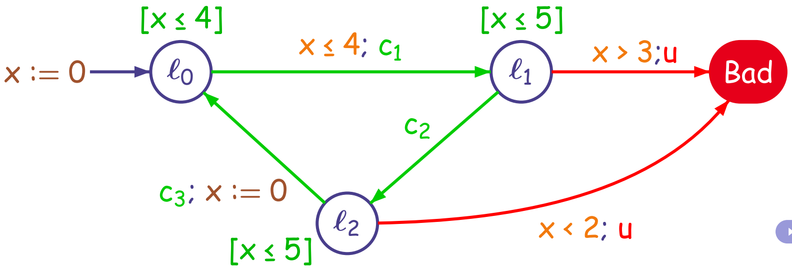
Theorem (Decidability of CP for Timed Game Automata)

The (Safety) Control Problem is decidable.

Theorem (Effectiveness of CSP)

If $(\ell_0, 0) \in W^*$ we can compute the most permissive positional winning strategy.

Result of the Computation for the Example



Skip

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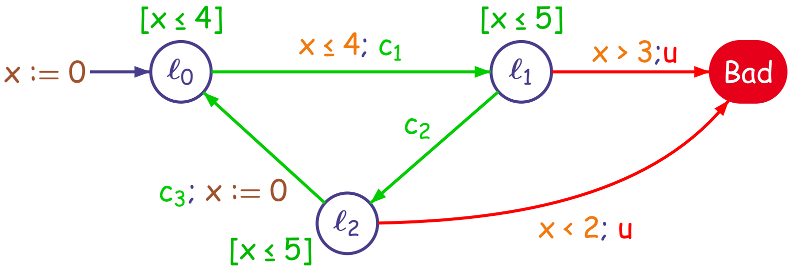
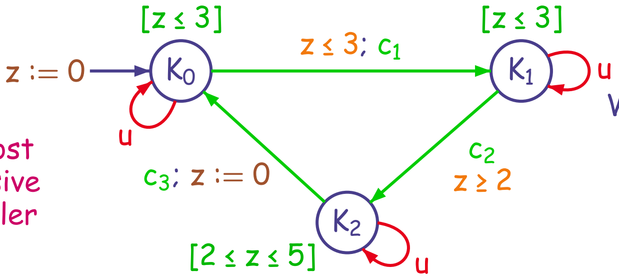




The Most Permissive Controller

Winning States

$(\ell_0, 0 \leq x \leq 3)$
$(\ell_1, 0 \leq x \leq 3)$
$(\ell_2, 2 \leq x \leq 5)$

Next:

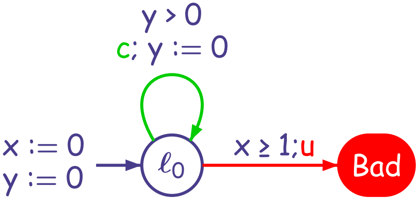
- Control of Timed Systems: Basics
- Selected Contributions
  - Implementable Controllers
  - Optimal Controllers
- Conclusion & Perspectives

Selection 1
Implementable Controllers

Joint work with Tom Henzinger and Jean-François Raskin

[HSCC'02]

Problems with Dense-Time Control (1)



The System

The Controller is Zeno !!!

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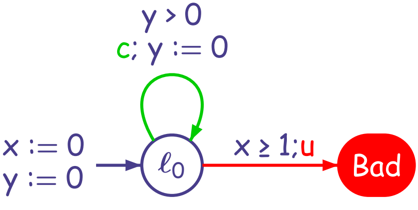

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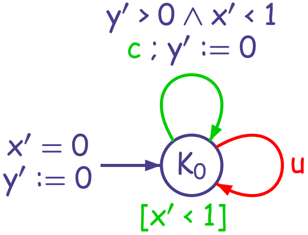

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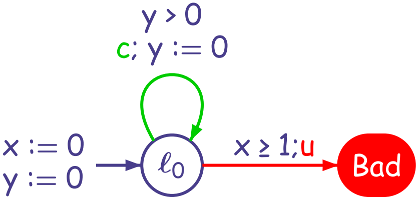

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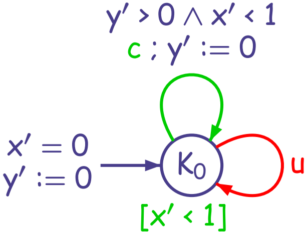

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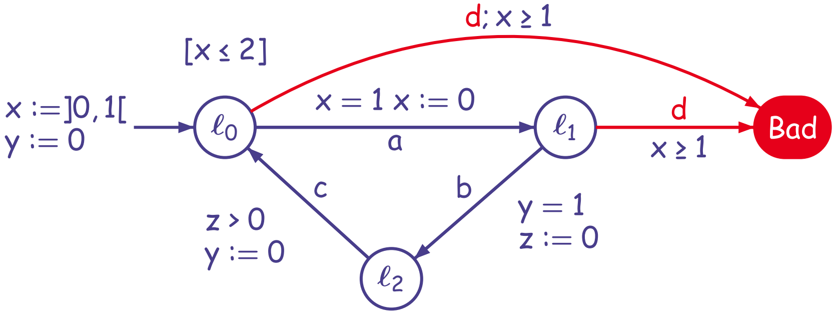
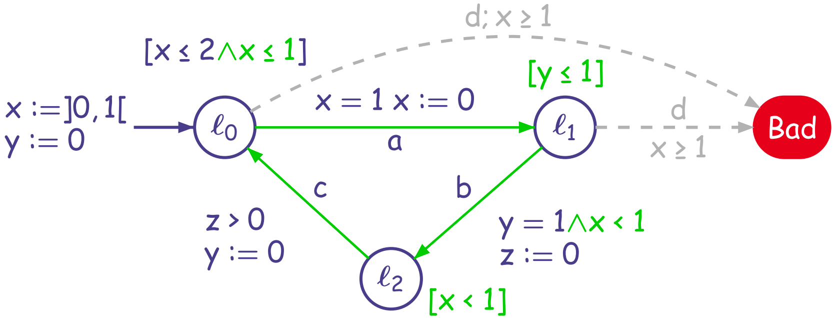
Problems with Dense-Time Control (2)



- The controller is Non-Zeno; One untimed behavior: $(\ell_0\ell_1\ell_2)^\omega$
- Let Δ_k = time spent in ℓ_2 in the k-th loop from ℓ_0 to ℓ_0
- It implies: $\forall k, \sum_{i=1}^{k} \Delta_i < 1 - x_0$, with $\forall i, \Delta_i > 0$

Problems with Dense-Time Control (2)



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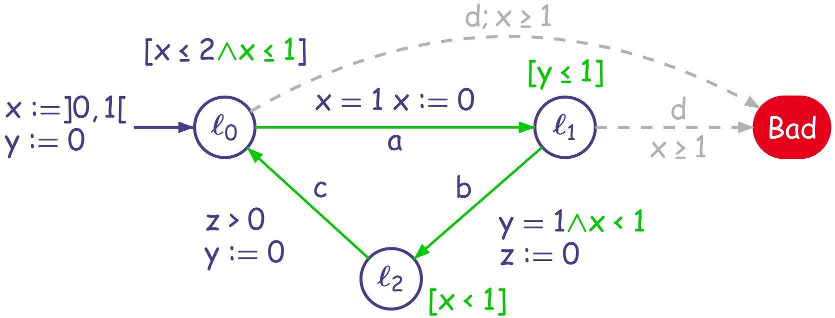

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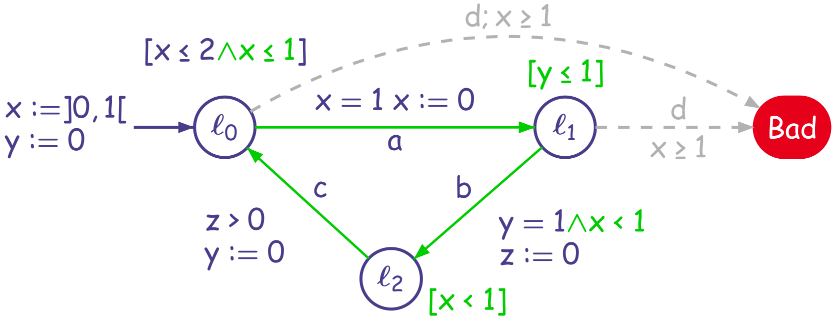

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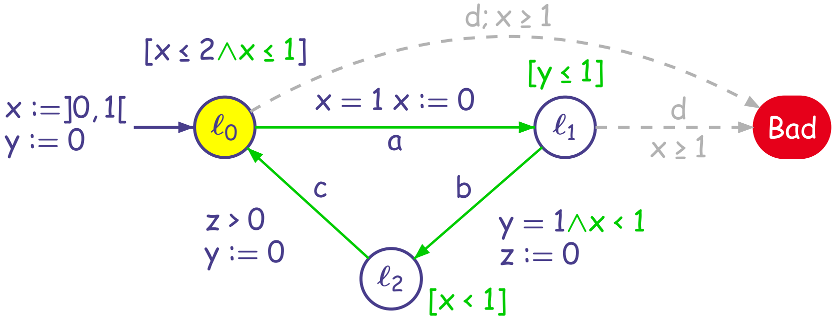

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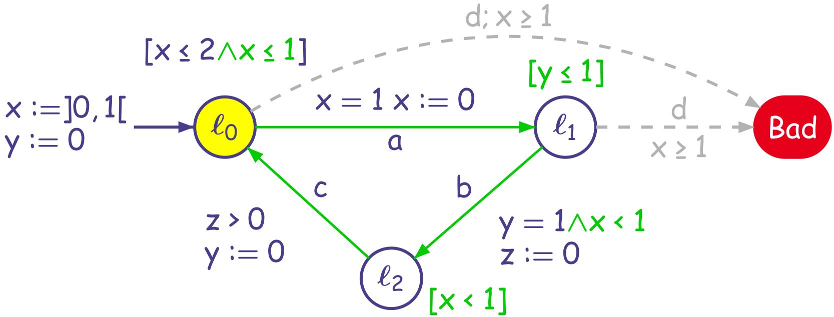
	ℓ_0
x:	x_0
y:	0

Problems with Dense-Time Control (2)



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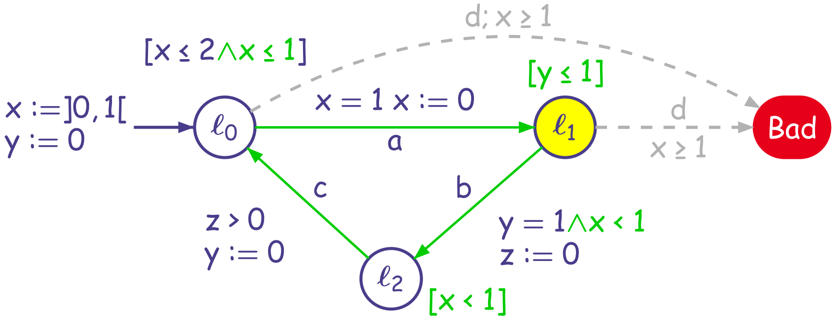
	ℓ_0		
x:	x_0	$\rightsquigarrow$	1
y:	0		$1 - x_0$

Problems with Dense-Time Control (2)



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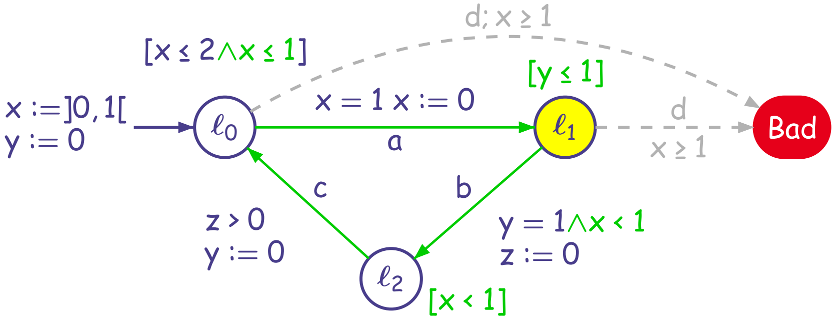
	ℓ_0				ℓ_1
x:	x_0	$\rightsquigarrow$	1	$\xrightarrow{a}$	0
y:	0		$1 - x_0$		$1 - x_0$

Problems with Dense-Time Control (2)



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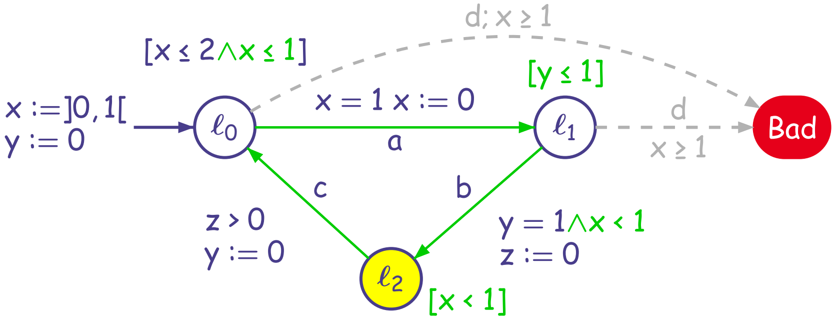
	ℓ_0				ℓ_1		
x:	x_0	$\rightsquigarrow$	1	$\xrightarrow{a}$	0	$\rightsquigarrow$	x_0
y:	0		$1 - x_0$		$1 - x_0$		1

Problems with Dense-Time Control (2)



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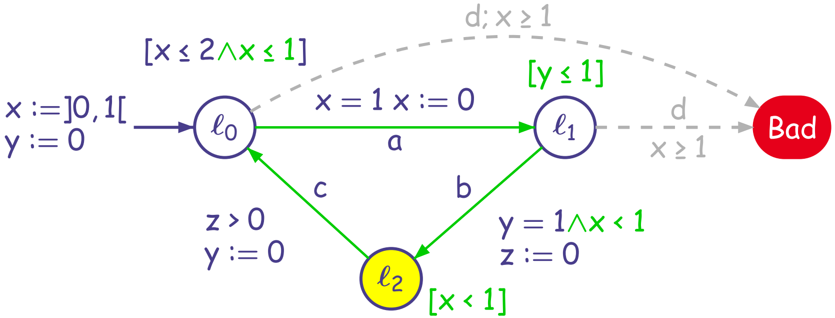

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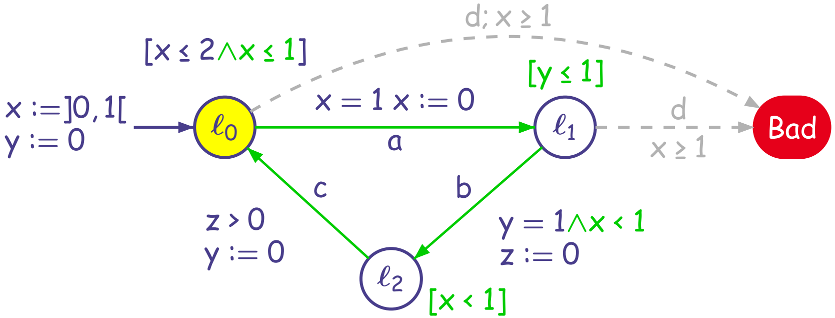

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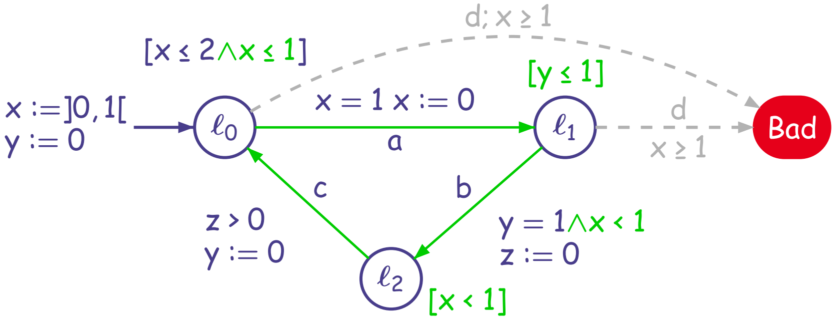

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$$
\begin{array}{lccccccccccc}
 & \ell_0 & & & & \ell_1 & & & & \ell_2 & & & & \ell_0 \\
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y: & 0 & & 1 - x_0 & & 1 - x_0 & & 1 & & 1 & & 1 + \Delta_1 & & 0
\end{array}
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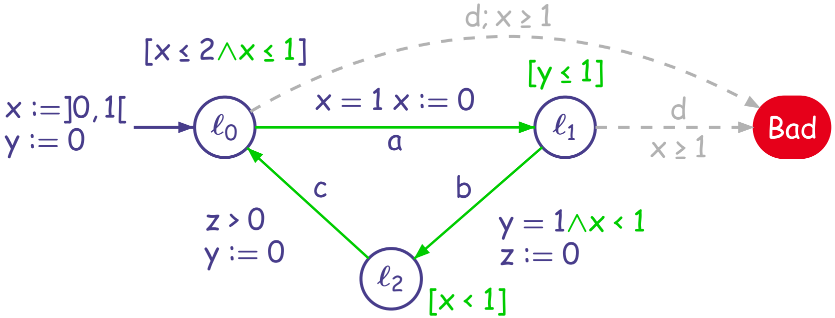
Problems with Dense-Time Control (2)



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- Must hold for ever: $\sum_{k=1}^{k=+\infty} \Delta_k < 1 - x_0$ with $\forall k, \Delta_k > 0$

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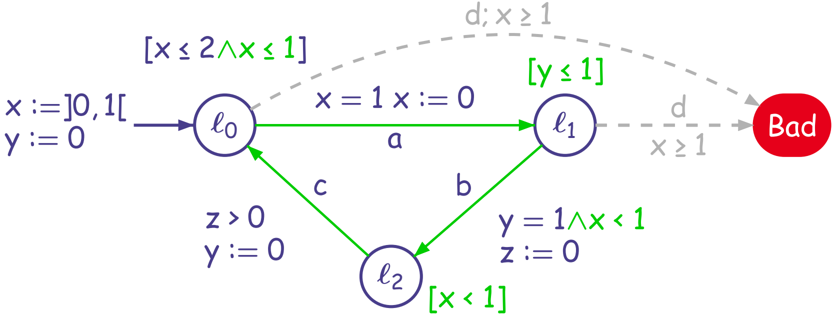

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- Let Δ_k = time spent in ℓ_2 in the k-th loop from ℓ_0 to ℓ_0
- It implies: $\forall k, \sum_{i=1}^{k} \Delta_i < 1 - x_0$, with $\forall i, \Delta_i > 0$
- Must hold for ever: $\sum_{k=1}^{k=+\infty} \Delta_k < 1 - x_0$ with $\forall k, \Delta_k > 0$

The Controller is Non-Zeno but not Implementable !!!

Sampling Control

- Let $a \in \mathbb{Q}^*$ be a sampling rate
- An a-controller is a controller that can do actions only at $k \cdot a, k \geq 1$ and $k \in \mathbb{N}$

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Known Sampling Rate Control Problem (KSR)

Input: $a \in \mathbb{Q}^*$, Bad (states), G a TGA
Problem: Is there a a-controller for G that avoids Bad ?

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Known Sampling Rate Control Problem (KSR)

Input: $a \in \mathbb{Q}^*$, Bad (states), G a TGA
Problem: Is there a a-controller for G that avoids Bad ?

Theorem ([Henzinger & Kopke'99])

The Known Sampling Rate Control Problem is decidable.

Sampling Control

- Let $a \in \mathbb{Q}^*$ be a sampling rate
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Unknown Sampling Rate Control Problem (USR)

Input: Bad (states), G a TGA
Problem: Is there a sampling rate $a \in \mathbb{Q}^*$ such that there is a a-controller for G that avoids Bad ?

Sampling Control

- Let $a \in \mathbb{Q}^*$ be a sampling rate
- An a-controller is a controller that can do actions only at $k \cdot a, k \geq 1$ and $k \in \mathbb{N}$

Unknown Sampling Rate Control Problem (USR)

Input: Bad (states), G a TGA
Problem: Is there a sampling rate $a \in \mathbb{Q}^*$ such that there is a a-controller for G that avoids Bad ?

Theorem ([HSCC'02])

The Unknown Sampling Rate Control Problem is undecidable.

Summary of the Results

Decidability results for the safety control problem on LHA:

	Known Switch Cond.	Unknown Switch Cond.
Timed Auto.	√ [Maler et al.'95]	√ [Maler et al.'95]
Init. Rect. Auto	√ [Henzinger et al.'99]	× [Henzinger et al.'95]
Rect. Auto.	× [Henzinger et al.'99]	× [Henzinger et al.'99]

	Known Sampling Rate	Unknown SR
Timed Auto.	√ [Hoffmann & Wong-Toi'92]	× [HSCC'02]
Init. Rect. Auto.	√ [Henzinger & Kopke'97]	× [HSCC'02]
Rect. Auto.	√ [Henzinger & Kopke'97]	× [HSCC'02]

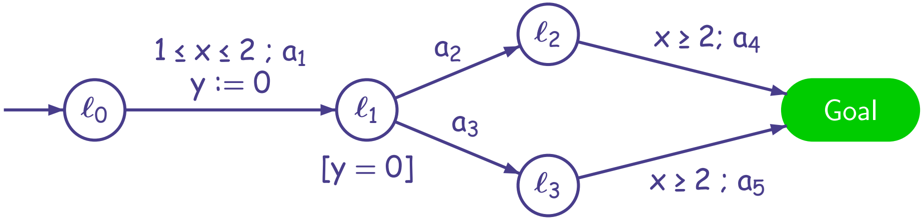
√: Decidable ×: Undecidable

Recent result [Bouyer et al.'06]
The reachability USC-CP is decidable for o-minimal automata.
Results on implementation of Timed Automata
[De Wulf et al.'04b, De Wulf et al.'04a, De Wulf et al.'05b]

Selection 2
Optimal Controllers

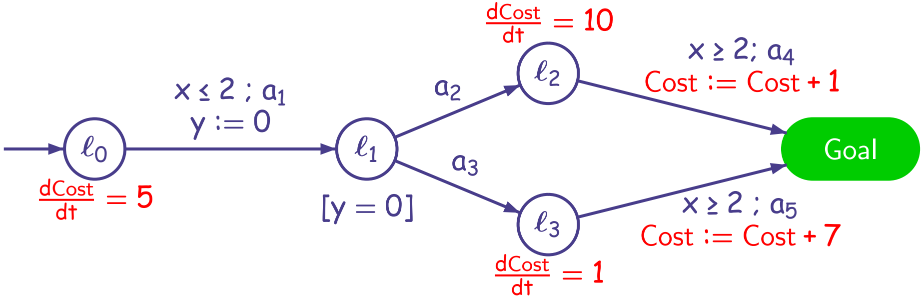
Joint work with Patricia Bouyer, Emmanuel Fleury and Kim G. Larsen
[FSTTCS'04, GDV'04]

Optimal Reachability for Timed Automata



- Reachability for Timed Automata [Alur & Dill'94]

Optimal Reachability for Timed Automata

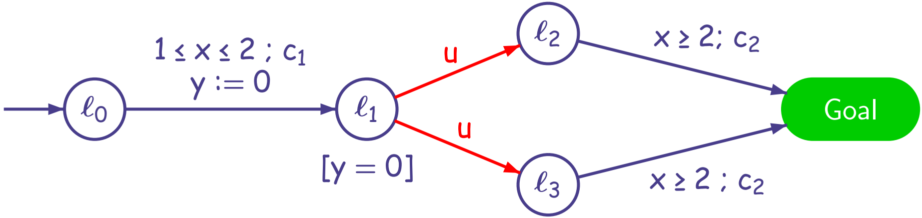


- Reachability for Timed Automata [Alur & Dill'94]
- Optimal Reachability for Priced (or Weighted) Timed Automata [Larsen et al.'01, Alur et al.'01]

$$(\ell_0,0,0) \xrightarrow{1} (\ell_0,1,1) \xrightarrow{a_1\ a_2} (\ell_2,1,0) \xrightarrow{3} (\ell_2,4,3) \xrightarrow{a_4} (Goal,4,3)$$

$$Cost = 1 \cdot 5 + 3 \cdot 10 + 1 = 36$$

Optimal Reachability for Timed Automata



- Reachability for Timed Automata [Alur & Dill'94]
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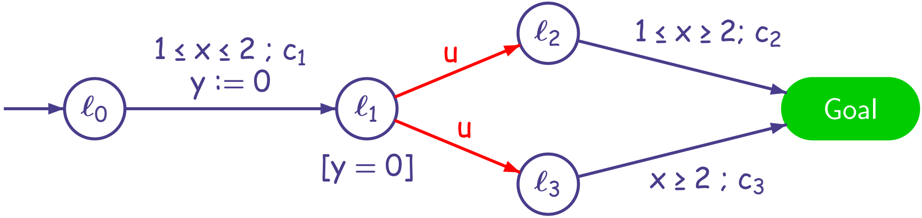
- Control for Timed *Game* Automata [Maler et al.'95]

Optimal Reachability for Timed Automata



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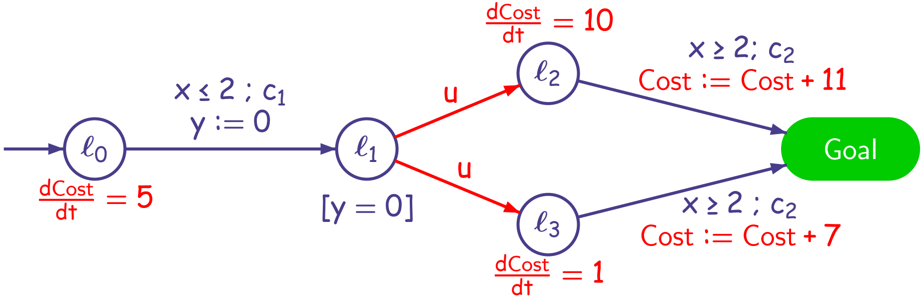

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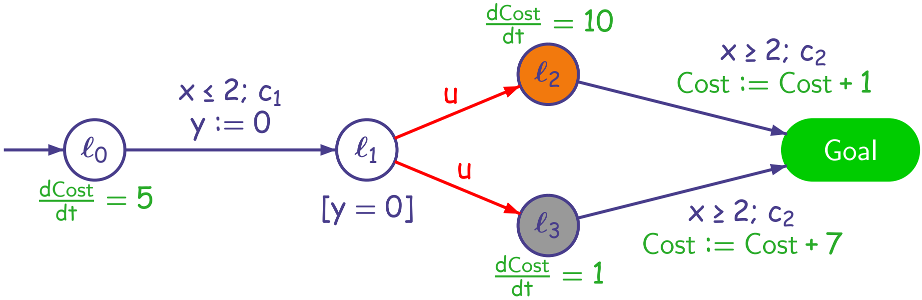
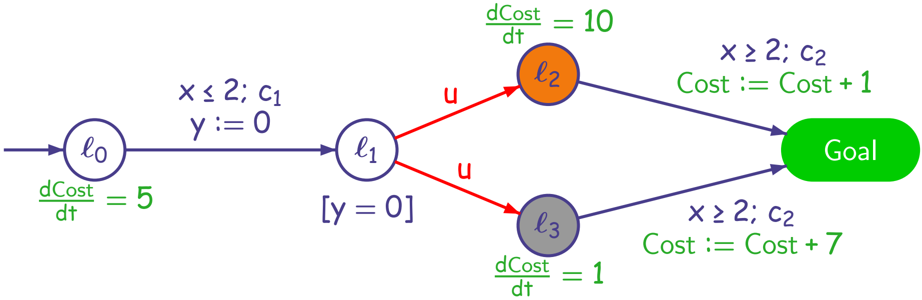
Optimal Control for Priced Timed Game Automata ?

A Small Example



- What is the **best** cost **whatever** the environment does ?

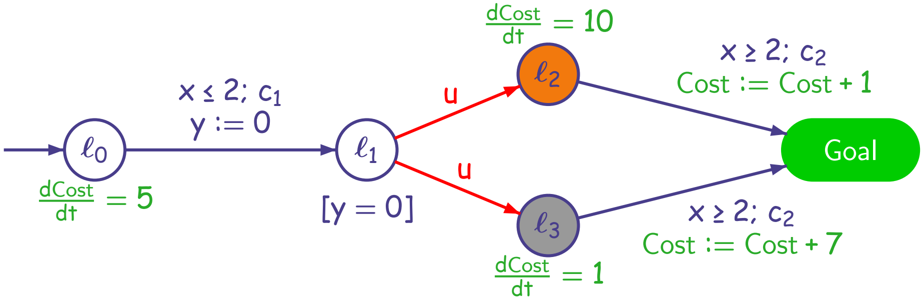
A Small Example



- What is the best cost whatever the environment does ?

$$\inf_{0 \leq t \leq 2} \max\{5t + 10(2-t) + 1, 5t + (2-t) + 7\} = 14 + \frac{1}{3}$$

A Small Example



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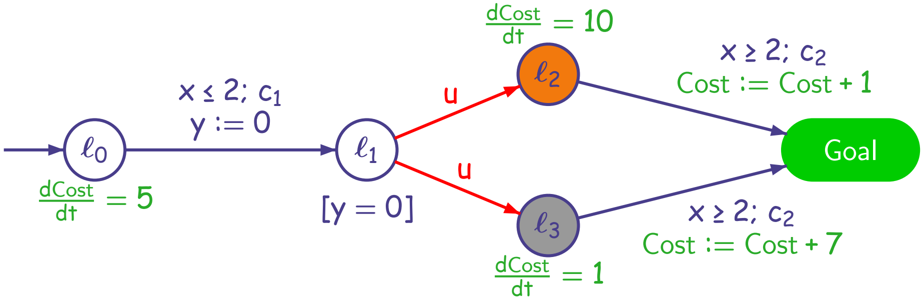

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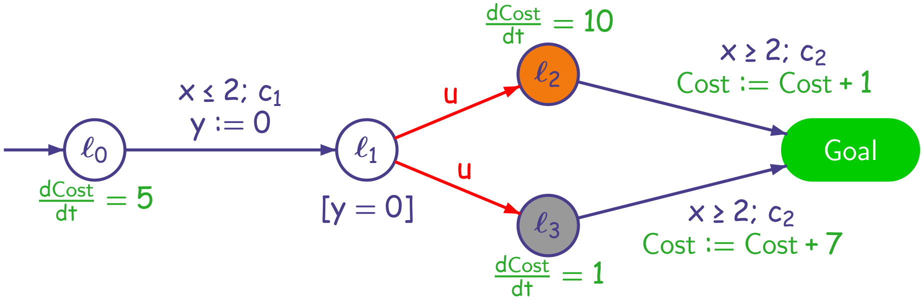

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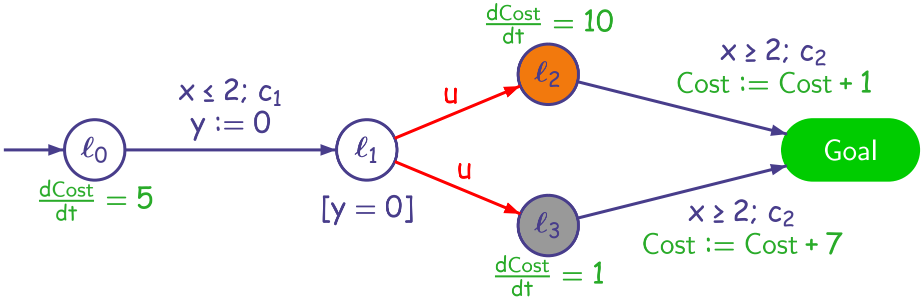

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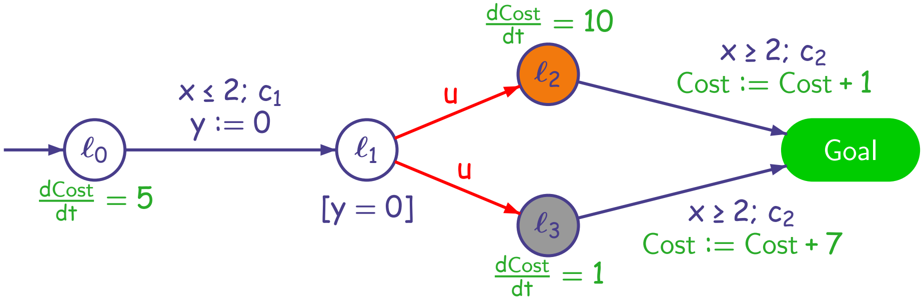

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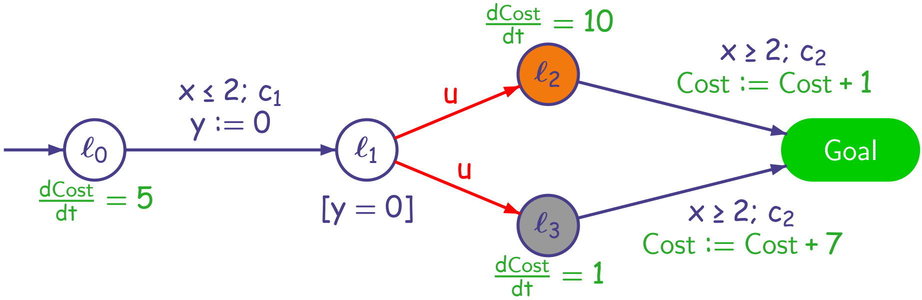
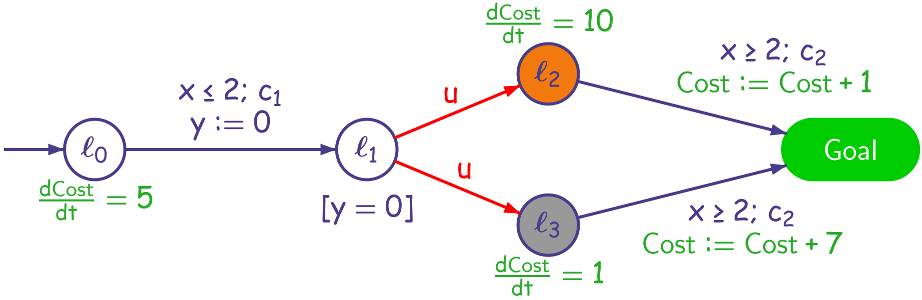


- What is the best cost whatever the environment does ?
- Is there a strategy to achieve this optimal cost ?
  Yes: wait in ℓ_0 until $t = \frac{4}{3}$ and then fire c_1
- Can we compute such a strategy ?
  Yes: but need memory sometimes

Optimal Control Problems

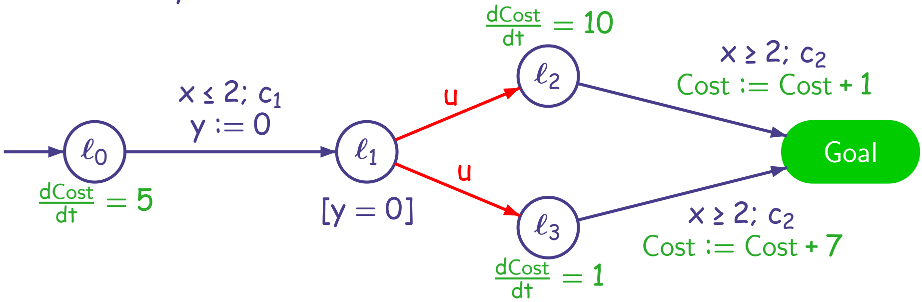


Can we find algorithms for these problems on PTGA ?

1. Compute the optimal cost
2. Decide if there is an optimal strategy
3. Compute an optimal strategy (if one exists)

From Optimal Control to Control

A Reachability TGA $\mathcal{A}$

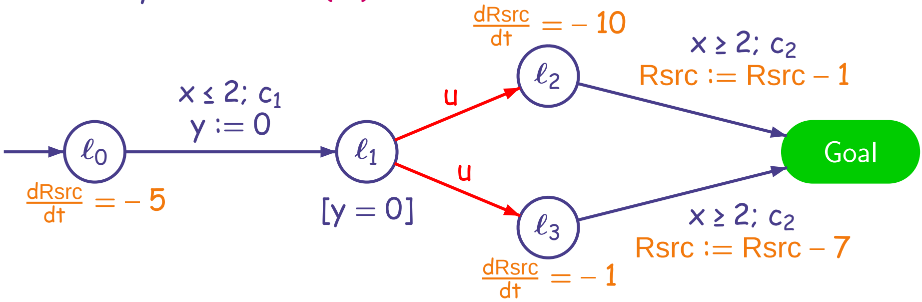


- Transform $\mathcal{A}$ in Linear Hybrid Game Automaton $H(\mathcal{A})$
- Define the reachability game for $H(\mathcal{A})$ with goal: $Goal \wedge Rsrc \geq 0$

Optimal Control for $\mathcal{A}$ $\Longleftrightarrow$ Reachability Control for $H(\mathcal{A})$

From Optimal Control to Control

A Linear Hybrid Game $H(\mathcal{A})$



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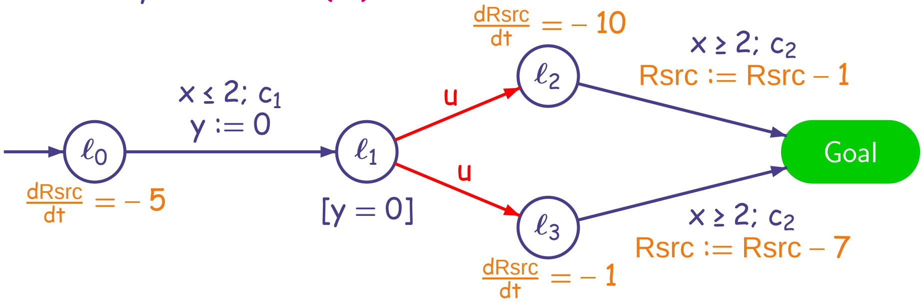

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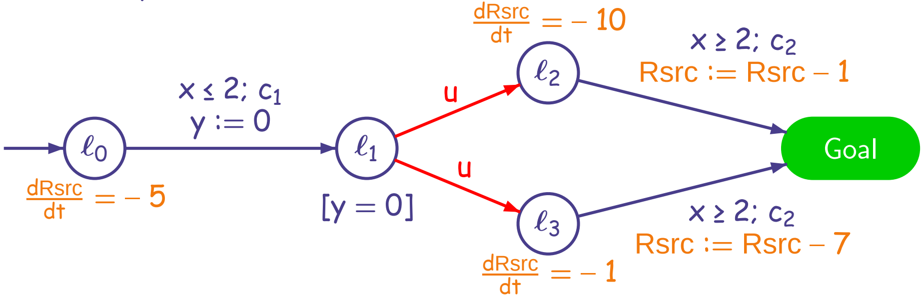

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Optimal Control for $\mathcal{A} \iff$ Reachability Control for $H(\mathcal{A})$

Results for Optimal Control

Theorem (Reachability Control for LHA)

There is a semi-algorithm CompWin that computes the set of winning states for LHA.
Uses polyhedra instead of zones.

Results for Optimal Control

Let A be a Reachability Priced Timed Game Automaton such that:

- A is cost non-zeno *i.e.* $\exists\kappa$ s.t. every cycle in the region automaton of A has cost at least κ
- A is bounded *i.e.* all clocks in A are bounded

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Theorem (Termination for Non-Zeno Cost)

The algorithm CompWin terminates for $H(A)$.

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Theorem ([Brihaye et al.'05])

Non-Zeno Cost is a necessary assumption.

Summary of the Results

What's decidable about optimal control?

- Non-Zeno Cost [FSTTCS'04]
- O-minimal automata [Bouyer et al.'07]
- 1-clock PTGA (3EXPTIME) [Bouyer et al.'06a]

What's undecidable about optimal control?

- 5-clock Zeno PTGA [Brihaye et al.'05]
- 3-clock Zeno PTGA [Bouyer et al.'06b]

What's decidable for infinite schedules (safety) ?

- Mean Cost decidable for PTA [Bouyer et al.'04a]

What's open?

Optimal Mean Cost for PTGA

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Next:

- Control of Timed Systems: Basics
- Selected Contributions
- **Conclusion & Perspectives**

Conclusion

- Other Recent Research Results:
  - Efficient algorithms for solving Timed Games
  - Expressiveness of timed automata vs. timed Petri nets
  - Fault Diagnosis
- Ongoing Collaborations:
  - In France:
    LSV (Cachan),
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    Funded Projects CHRONO, CORTOS, DOTS
  - Abroad:
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Research Perspectives

- Efficient Algorithms for Safety, Büchi games
- Concurrent Semantics (unfoldings) for Network of Timed Automata
- Applications of Control Theory to Other Domain
  Non-Interference
- Application of theories and tools to real systems
  e.g. L4 based-technology developped at NICTA/Sydney

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Timed Automata

[Alur & Dill'94]

A Timed Automaton $\mathcal{A}$ is a tuple $(L, \ell_0, Act, X, \text{inv}, \longrightarrow)$ where:

- L is a finite set of locations
- ℓ_0 is the initial location
- X is a finite set of clocks
- Act is a finite set of actions
- $\longrightarrow$ is a set of transitions of the form $\ell \xrightarrow{g,a,R} \ell'$ with:
  - $\ell, \ell' \in L$,
  - $a \in Act$
  - a guard g which is a clock constraint over X
  - a reset set R which is the set of clocks to be reset to 0

Clock constraints are boolean combinations of $x \sim k$ with $x \in C$ and $k \in \mathbb{Z}$ and $\sim \in \{\leq, <\}$.

Semantics of Timed Automata

Let $\mathcal{A} = (L, \ell_0, Act, X, inv, \longrightarrow)$ be a Timed Automaton.

A state (ℓ, v) of $\mathcal{A}$ is in $L \times \mathbb{R}_{\geq 0}^X$

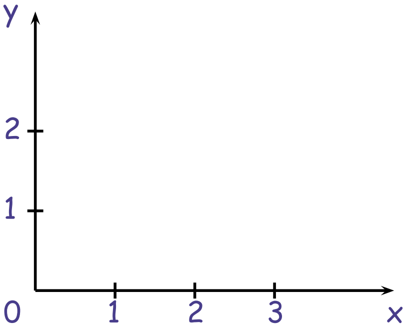
The semantics of $\mathcal{A}$ is a Timed Transition System $S_{\mathcal{A}} = (Q, q_0, Act \cup \mathbb{R}_{\geq 0}, \longrightarrow)$ with:

- $Q = L \times \mathbb{R}_{\geq 0}^X$
- $q_0 = (\ell_0, \overline{0})$
- $\longrightarrow$ consists in:

discrete transition: $(\ell, v) \xrightarrow{a} (\ell', v') \iff \begin{cases} \exists\, \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A} \\ v \models g \\ v' = v[r \leftarrow 0] \\ v' \models inv(\ell') \end{cases}$

delay transition: $(\ell, v) \xrightarrow{d} (\ell, v + d) \iff d \in \mathbb{R}_{\geq 0} \land v + d \models inv(\ell)$

The Region Abstraction [Alur & Dill'94]



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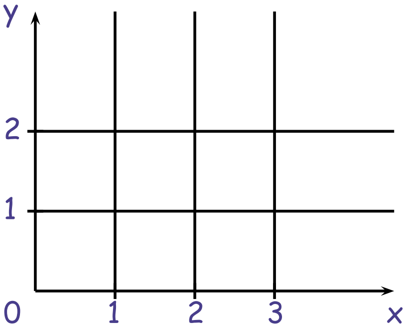

Build an equivalence relation which is of finite index and is:

- "compatible" with clock constraints ($g ::= x \sim c \quad g \wedge g$)

  $r, r' \in R \implies \forall$ constraints $g, \quad r \models g \iff r' \models g$

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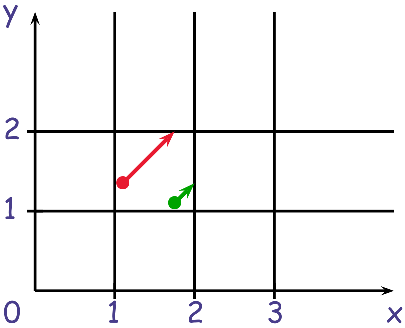

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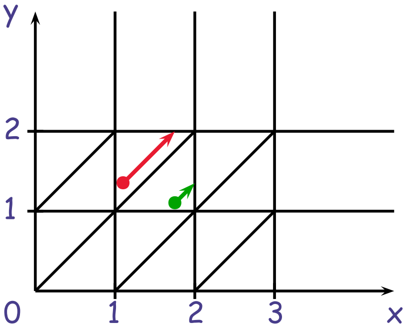

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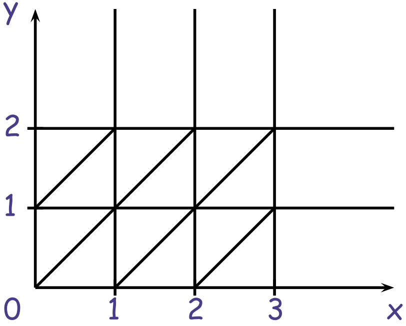

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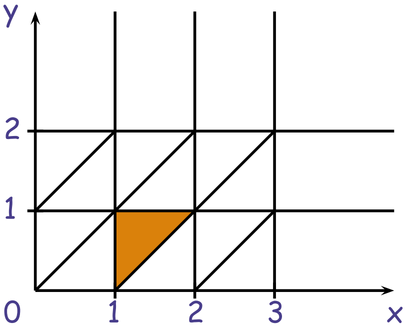

region defined by

$I_x =]1;2[\quad I_y =]0;1[$

$\{x\} < \{y\}$

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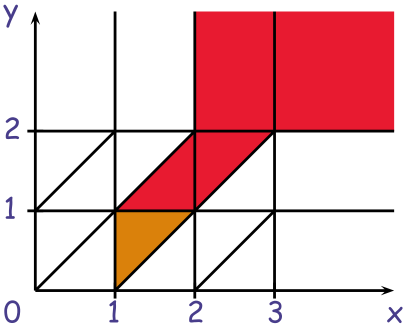

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delay successors

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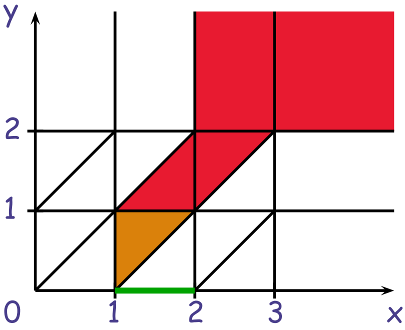

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delay successors

successor by reset

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The Region Automaton

- For each transition $\ell \xrightarrow{g,a,C:=0} \ell'$ of the TA
- Build transitions in the region automaton RA: $(\ell,R) \xrightarrow{a} (\ell',R')$ if:
  - there exists R″ a delay successor of R s.t.
  - R″ satisfies the guard g ($R'' \subseteq [\![g]\!]$)
  - $R''[C \leftarrow 0]$ is included in R′

a TA and its region automaton RA are time-abstract bisimilar

- The region automaton is finite
- Language accepted by the RA = untimed language accepted by the TA

  a timed word $w = (a,1.2)(b,3.4)(a,6.256)$; untimed$(w) = aba$
- Language Emptyness can be decided on the RA

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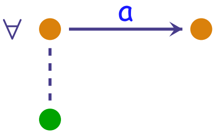

$$
\begin{array}{ccccccc}
(\ell_0, v_0) & \xrightarrow{a_1, t_1} & (\ell_1, v_1) & \xrightarrow{a_2, t_2} & (\ell_2, v_2) & \xrightarrow{a_3, t_3} & \cdots \\
\updownarrow & & \updownarrow & & \updownarrow & & \\
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with $v_i \in R_i$ for all i.

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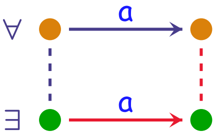

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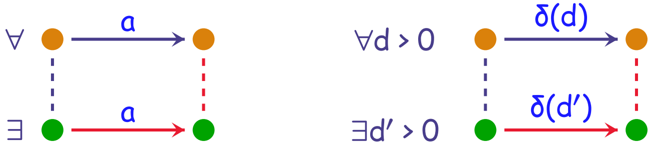

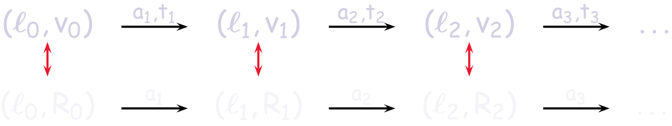

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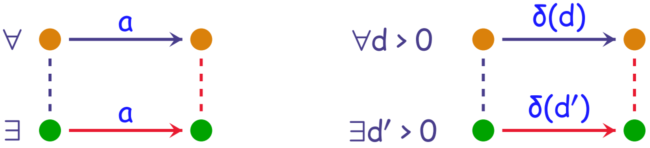

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$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$

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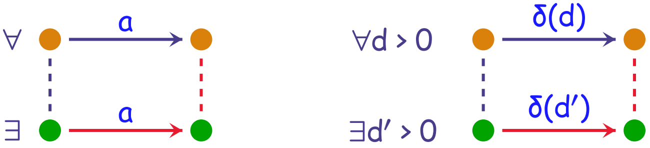

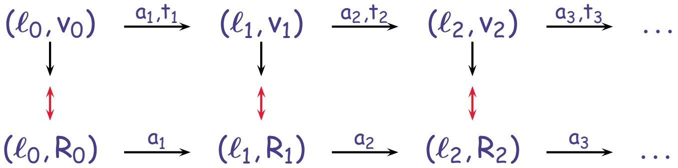

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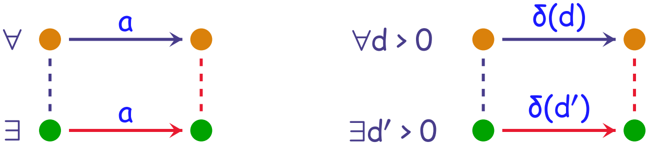

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Definition (Outcome in Timed Games)

Let $G = (L, \ell_0, Act, X, E, inv)$ be a TGA and f a strategy over G. The **outcome** $Outcome((\ell, v), f)$ of f from configuration (ℓ, v) in G is the subset of $Runs((\ell, v), G)$ defined inductively by:

- $(\ell, v) \in Outcome((\ell, v), f)$,
- if $\rho \in Outcome((\ell, v), f)$ then $\rho' = \rho \xrightarrow{e} (\ell', v') \in Outcome((\ell, v), f)$ if $\rho' \in Runs((\ell, v), G)$ and one of the following three conditions hold:
  1. $e \in Act_u$,
  2. $e \in Act_c$ and $e = f(\rho)$,
  3. $e \in \mathbb{R}_{\geq 0}$ and $\forall 0 \leq e' < e, \exists (\ell'', v'') \in (L \times \mathbb{R}^X_{\geq 0})$ s.t. $last(\rho) \xrightarrow{e'} (\ell'', v'') \wedge f(\rho \xrightarrow{e'} (\ell'', v'')) = \lambda$.
- an infinite run ρ is in $\in Outcome((\ell, v), f)$ if all the finite prefixes of ρ are in $Outcome((\ell, v), f)$.

States & Symbolic States

- $Q = L \times \mathbb{R}_{\geq 0}^{Clock}$ is the set of states of the TGA
  $$q = (\ell, v) \in Q$$
- Discrete predecessors of $X \subseteq Q$ by an action a:
  $$\mathrm{Pred}^a(X) = \{q \in Q \mid q \xrightarrow{a} q' \text{ and } q' \in X\}$$
- Time predecessors of $X \subseteq Q$:
  $$\mathrm{Pred}^\delta(X) = \{q \in Q \mid \exists t \geq 0 \mid q \xrightarrow{t} q' \text{ and } q' \in X\}$$
- Zone = conjunction of triangular constraints
  $x - y < 3, x \geq 2 \wedge 1 < y - x < 2$
- Symbolic State is defined by a State predicate (SP)
  $P = \cup_{i \in [1..n]}(\ell_{j_i}, Z_i)$, $\ell_{j_i} \in L$, Z_i is a zone
  $(\ell_1, 2 \leq x < 4)$ or $(\ell_0, x < 1 \wedge y - x \geq 2)$ or $(\ell_0, x \leq 2) \cup (\ell_2, x > 0)$

Effectiveness of Pred^a and Pred^δ

If P is a SP then $\mathrm{Pred}^a(P), \mathrm{Pred}^\delta(P)$ are SP and can be computed effectively. (There is a symbolic version for Pred^a and Pred^δ.)

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If P is a SP then $\text{Pred}^a(P), \text{Pred}^\delta(P)$ are SP and can be computed effectively. (There is a symbolic version for Pred^a and Pred^δ.)

States & Symbolic States

- $Q = L \times \mathbb{R}_{\geq 0}^{Clock}$ is the set of states of the TGA
$$q = (\ell, v) \in Q$$
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Symbolic Computation For Timed Games

X is a state predicate

- $\mathsf{cPred}(X) = \bigcup_{c \in Act_c} \mathsf{Pred}^c(X)$ $\qquad$ $\mathsf{uPred}(X) = \bigcup_{u \in Act_u} \mathsf{Pred}^u(X)$
  cPred and uPred are effectively computable
- $\mathsf{Pred}_\delta(X, Y)$: Time controllable predecessors of X avoiding Y:

q $\qquad$ $q' \in X$

$\mathsf{Pred}_\delta(X, Y)$ is effectively computable for state predicates X, Y

- Controllable Predecessors Operator for Timed Games

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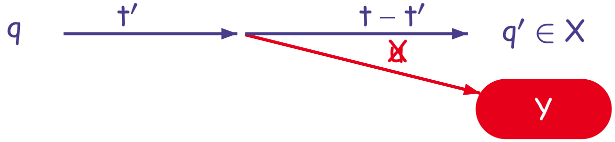

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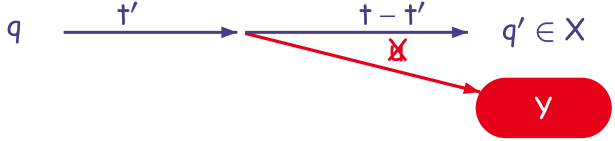

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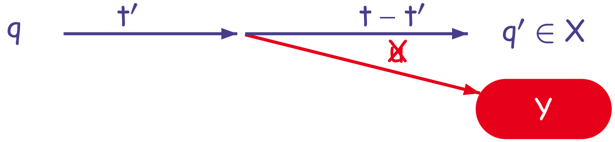

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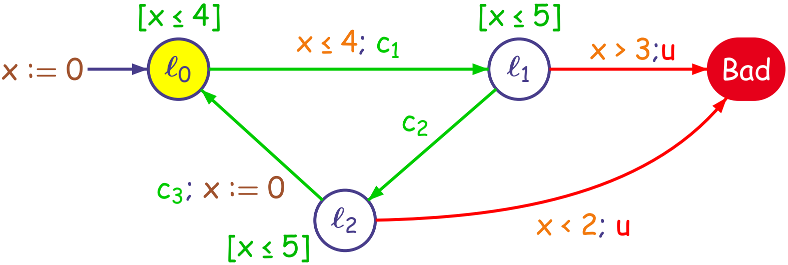
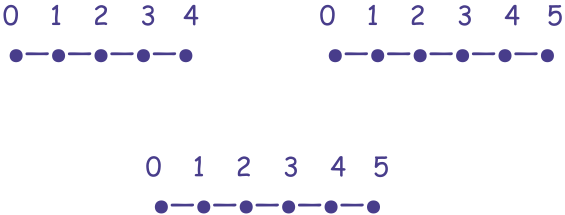
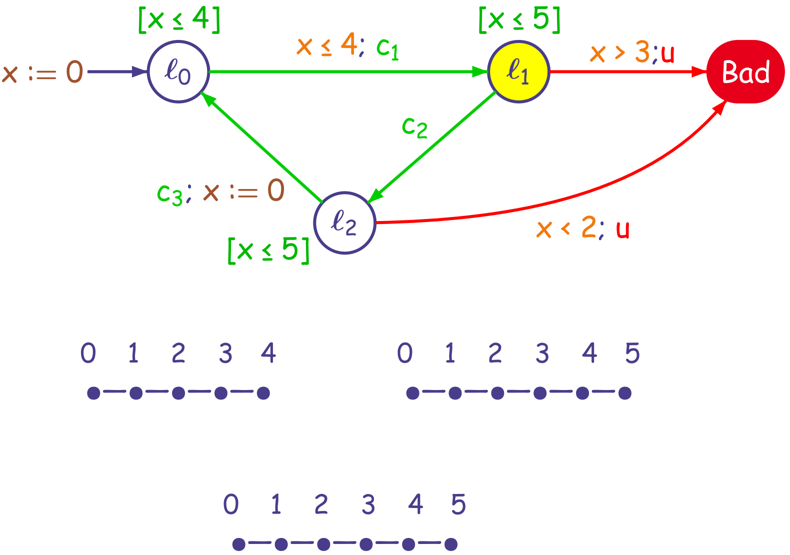
Example of Computation for Safety Games





Skip

Example of Computation for Safety Games



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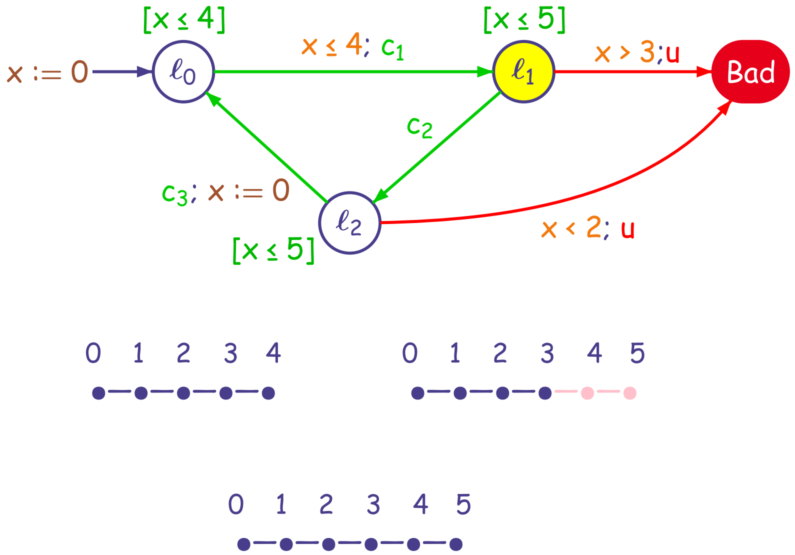

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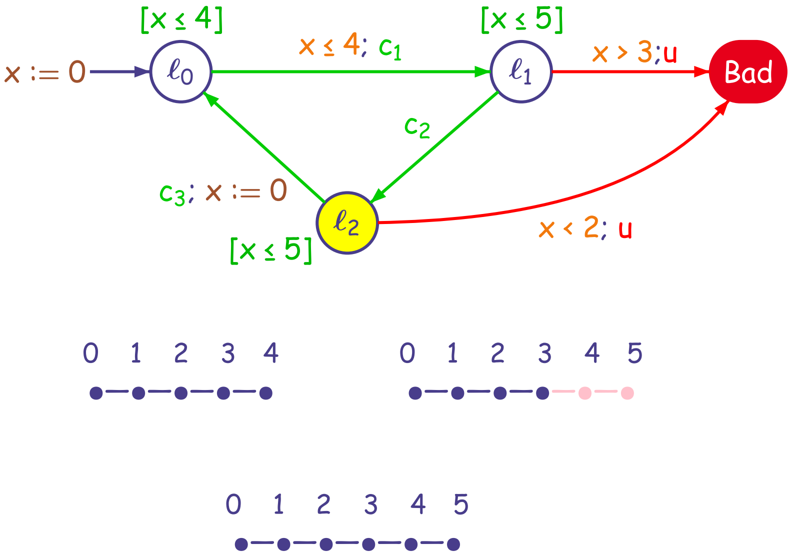

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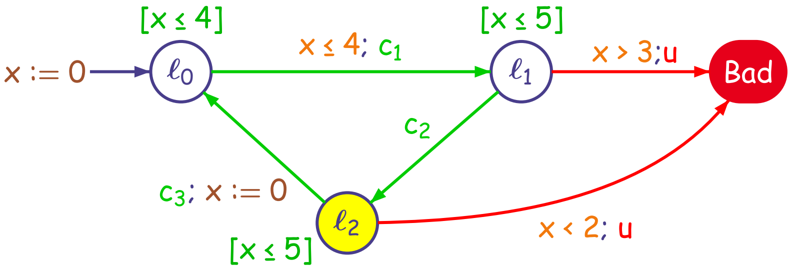

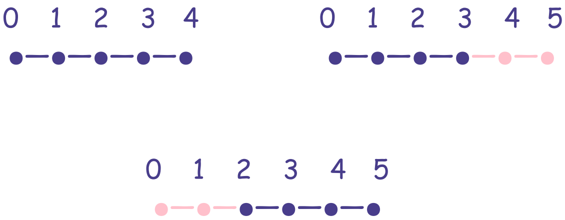

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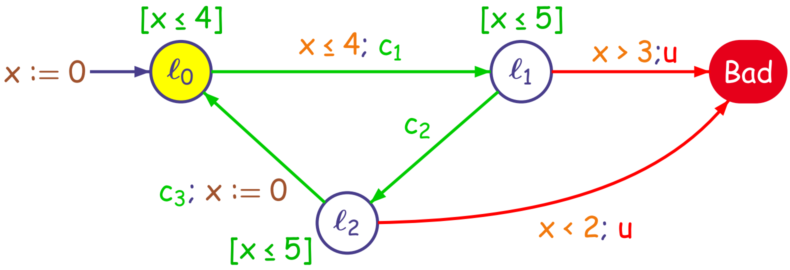

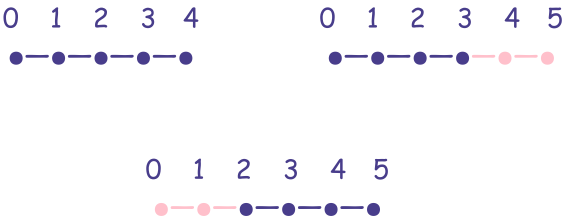

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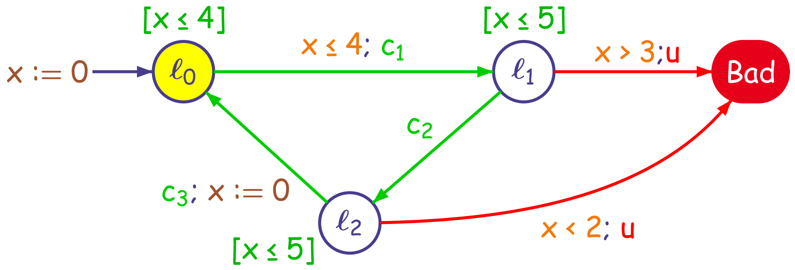

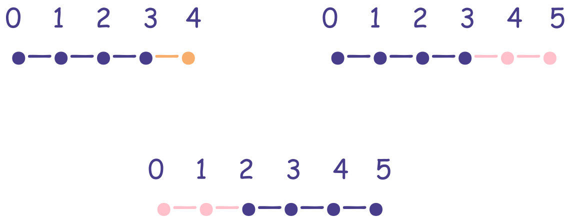

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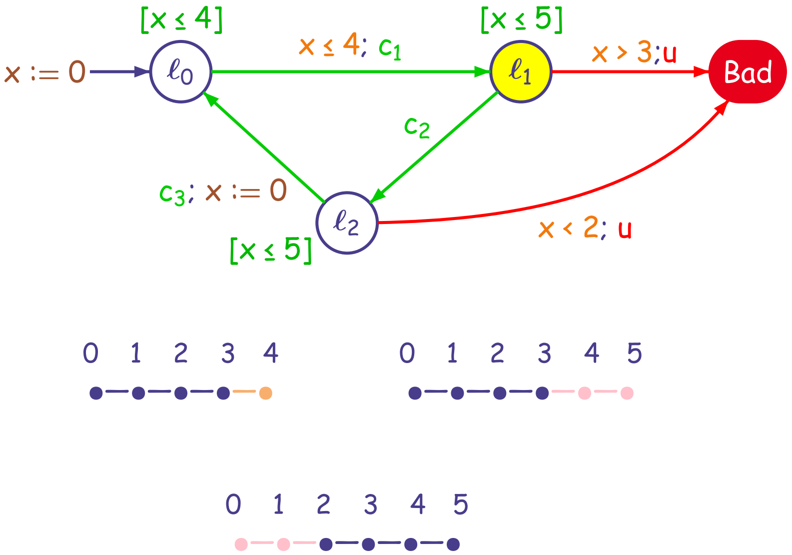

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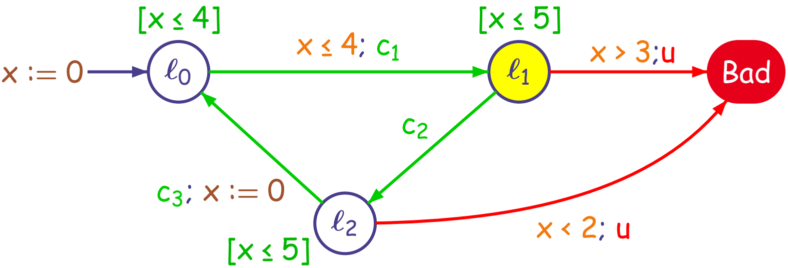

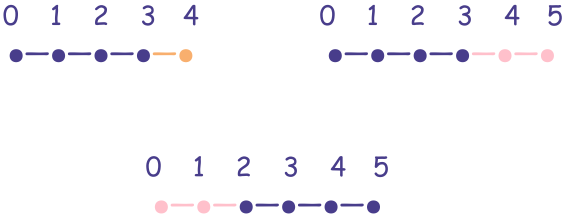

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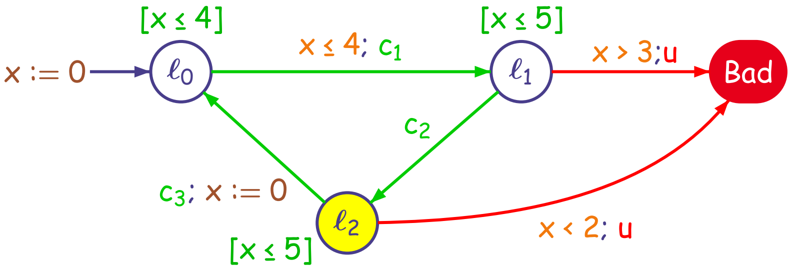

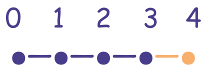

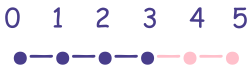

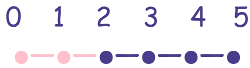

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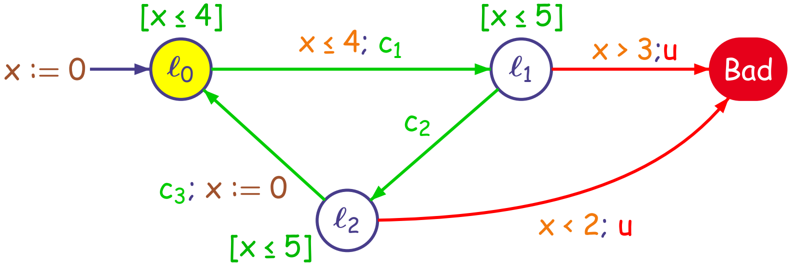

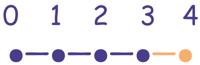

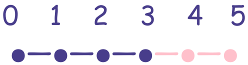

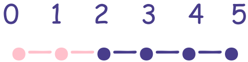

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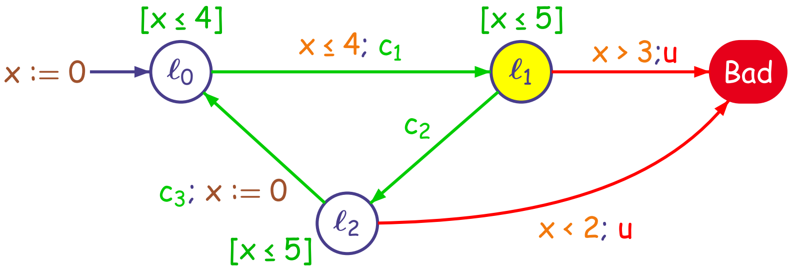

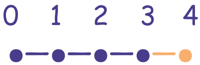

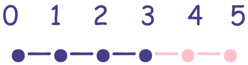

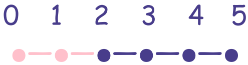

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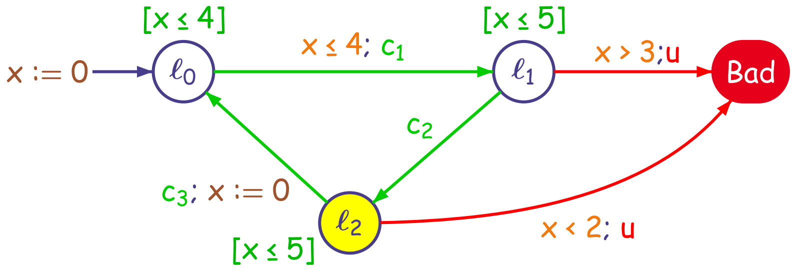

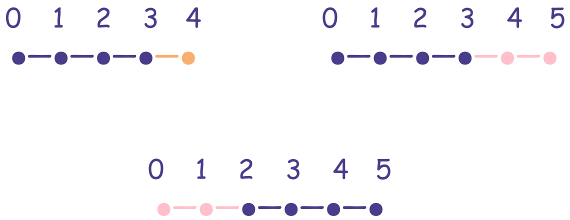

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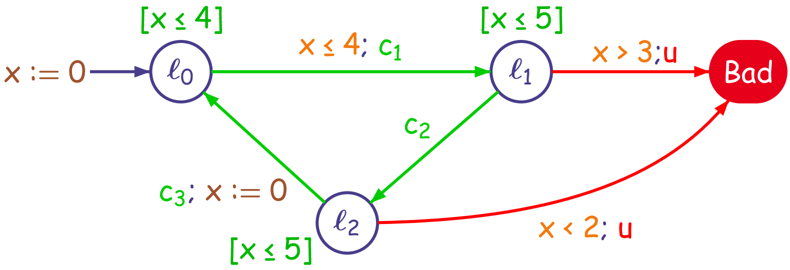

Winning States

$(\ell_0, 0 \leq x \leq 3)$
$(\ell_1, 0 \leq x \leq 3)$
$(\ell_2, 2 \leq x \leq 5)$

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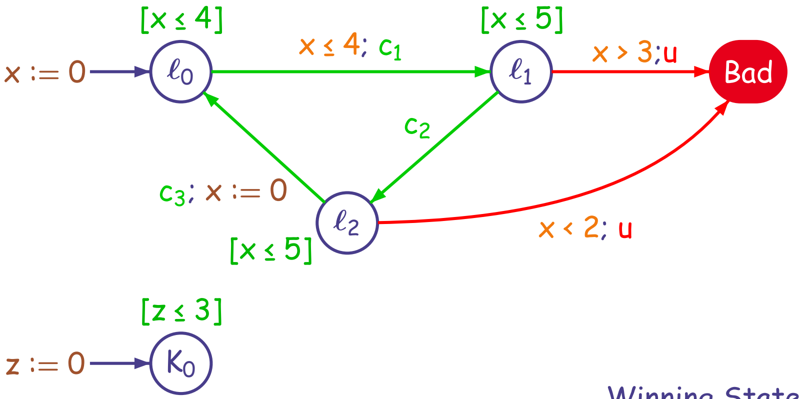

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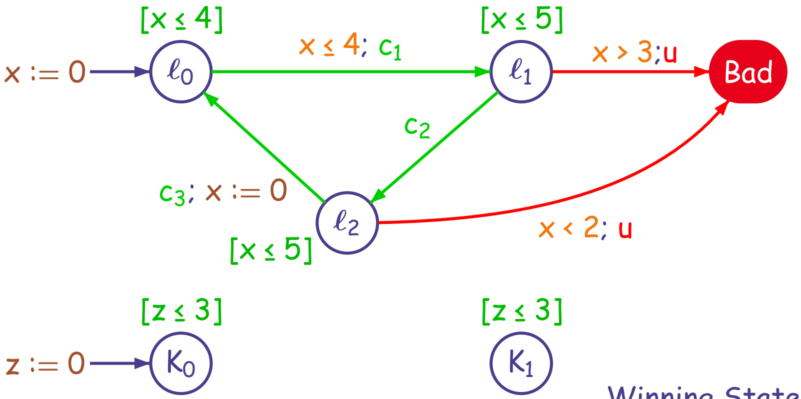

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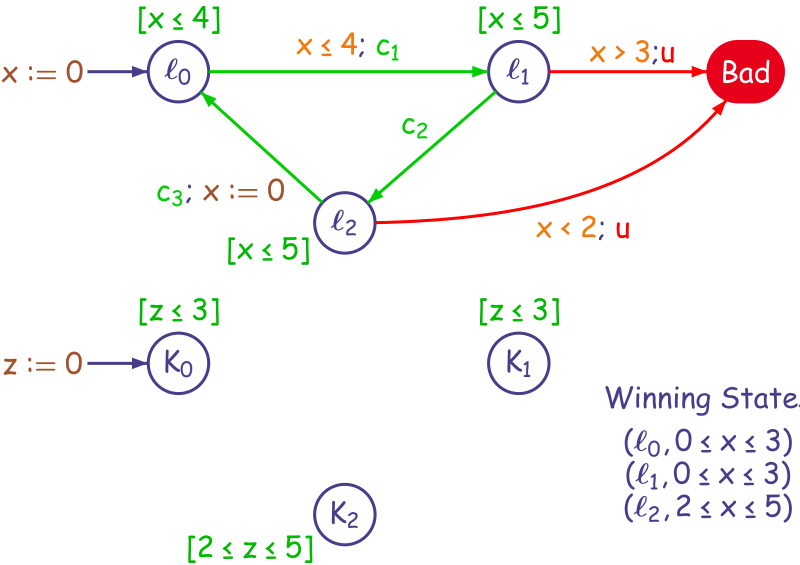

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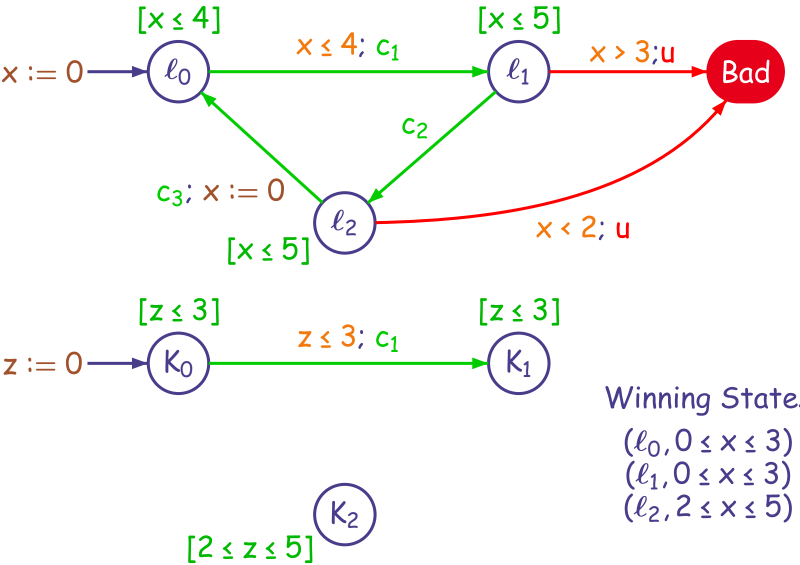

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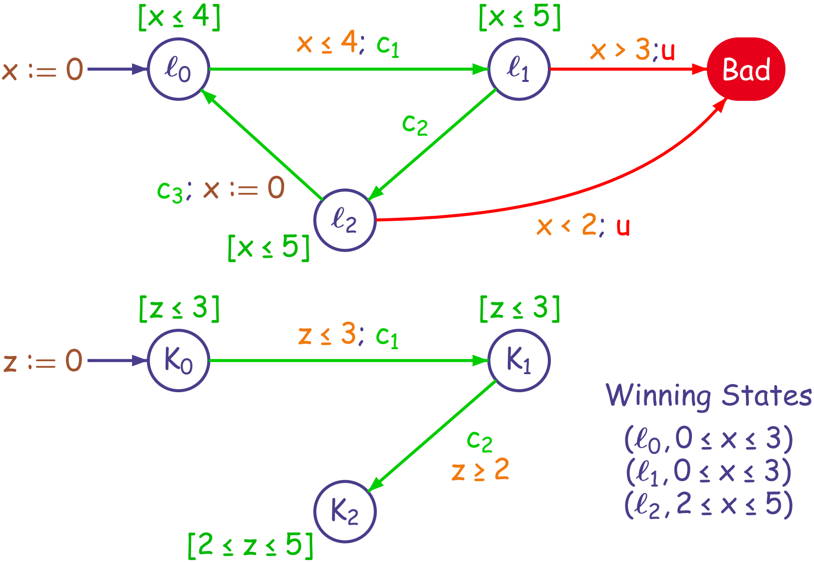

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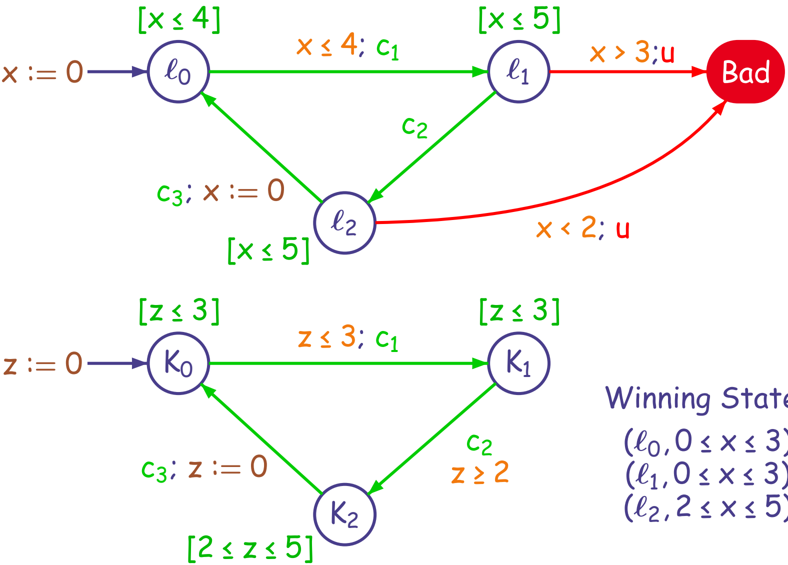

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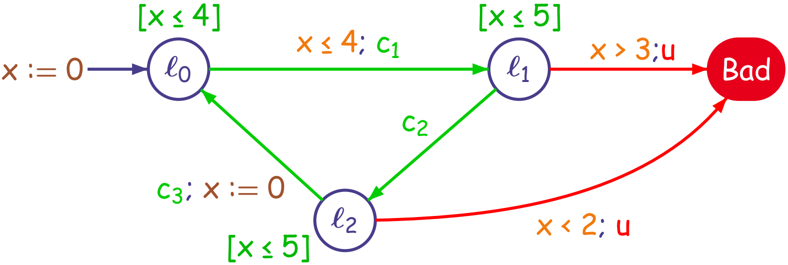

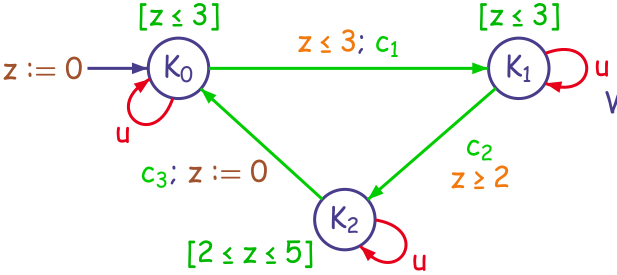

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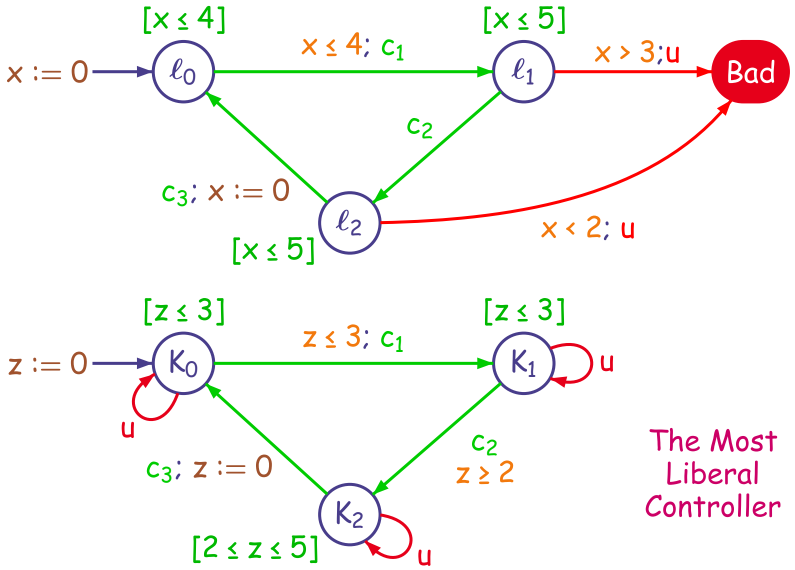
$(\ell_2, 2 \leq x \leq 5)$

Example of Computation for Safety Games



The Most Liberal Controller

Existence of Cost Independent Strategies

Let A be a RPTGA such that:

- guards of u actions are strict
- guards on c actions are large

There is an optimal cost independent strategy

Is it necessary ?

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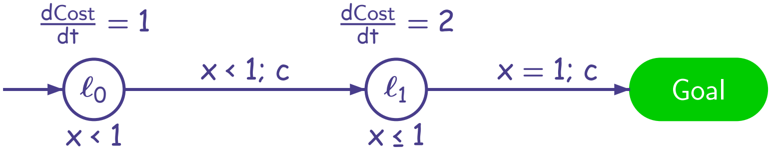
No Optimal Strategy

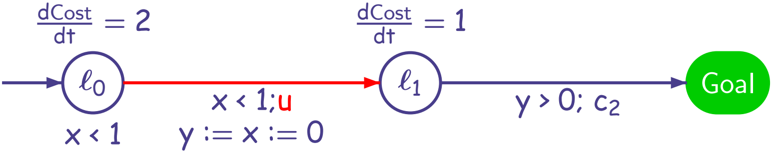


- define f_ε with $0 < \varepsilon < 1$ by:
  in ℓ_0: $f(\ell_0, x < 1 - \varepsilon) = \lambda$, $f(\ell_0, 1 - \varepsilon \leq x < 1) = c$
  in ℓ_1: $f(\ell_1, x < 1) = \lambda$, $f(\ell_1, x = 1) = c$
  $Cost(f_\varepsilon) = (1 - \varepsilon) + 2.\varepsilon = 1 + \varepsilon$ and $OptCost = 1$.
- given $\varepsilon > 0$, there is a sub-optimal strategy f_ε such that

$$|Cost((\ell_0, \vec{0}), f_\varepsilon) - OptCost((\ell_0, \vec{0}), G)| < \varepsilon$$

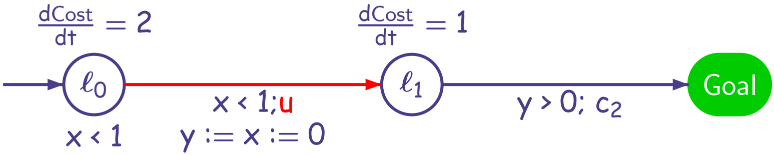
- New problem: given ε, compute such an f_ε strategy.

No Optimal Cost-Independent Strategy



- Optimal cost is 2

No Optimal Cost-Independent Strategy

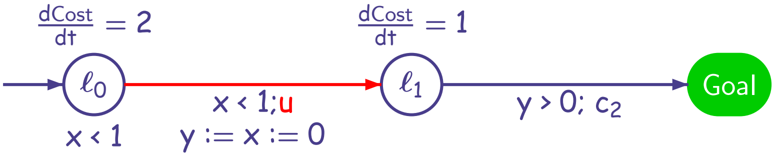


- Optimal cost is 2
- An optimal winning cost-dependent strategy f: $f(\ell_1, -, cost < 2) = \lambda$ and $f(\ell_1, -, cost = 2) = c_2$ assume u taken at time $(1 - \delta_0)$:

$$Cost(f, (\ell_0, 0)) = 2 \cdot (1 - \delta_0) + \delta_1 = 2$$

because according to f we have $\delta_1 = 2 \cdot \delta_0$

No Optimal Cost-Independent Strategy



- Optimal cost is 2
- assume $\exists f^*$ cost-independent: f^* must wait in ℓ_1 at least ε
  assume u taken at time $(1 - \delta)$:

$$\text{Cost}(f^*, (\ell_0, 0)) = 2 \cdot (1 - \delta) + \varepsilon$$

Take $\delta = \frac{\varepsilon}{4}$: $\text{Cost}(f^*, (\ell_0, 0)) = 2 + \frac{\varepsilon}{2}$ and $\text{OptCost}(f^*) = 2 + \varepsilon$

Related Work for Optimal Control

- [La Torre et al.'02]
  - Acyclic Priced Timed Game Automata
  - Recursive definition of optimal cost
  - Computation of the infimum of the optimal cost
    i.e. OptCost = 2 could mean that it is 2 or $2 + \varepsilon$
  - No strategy synthesis
- [Alur et al.'04] (ICALP'04)
  - Bounded optimality: optimal cost within k steps
  - Complexity bound: exponential in k and #states of the PTGA
  - No bound for the more general optimal problem
  - Computation of the infimum of the optimal cost
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- Our work [FSTTCS'04]:
  - Run-based definition of optimal cost
  - We can decide whether ∃ an optimal strategy
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