Logics for n-ary Queries in Trees

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Ph.D. Defense, 2008, October

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eXtensible Markup Language

XML

- markup language to represent tree-shaped data
- XML data big bang !
- standard for data exchange and data storage

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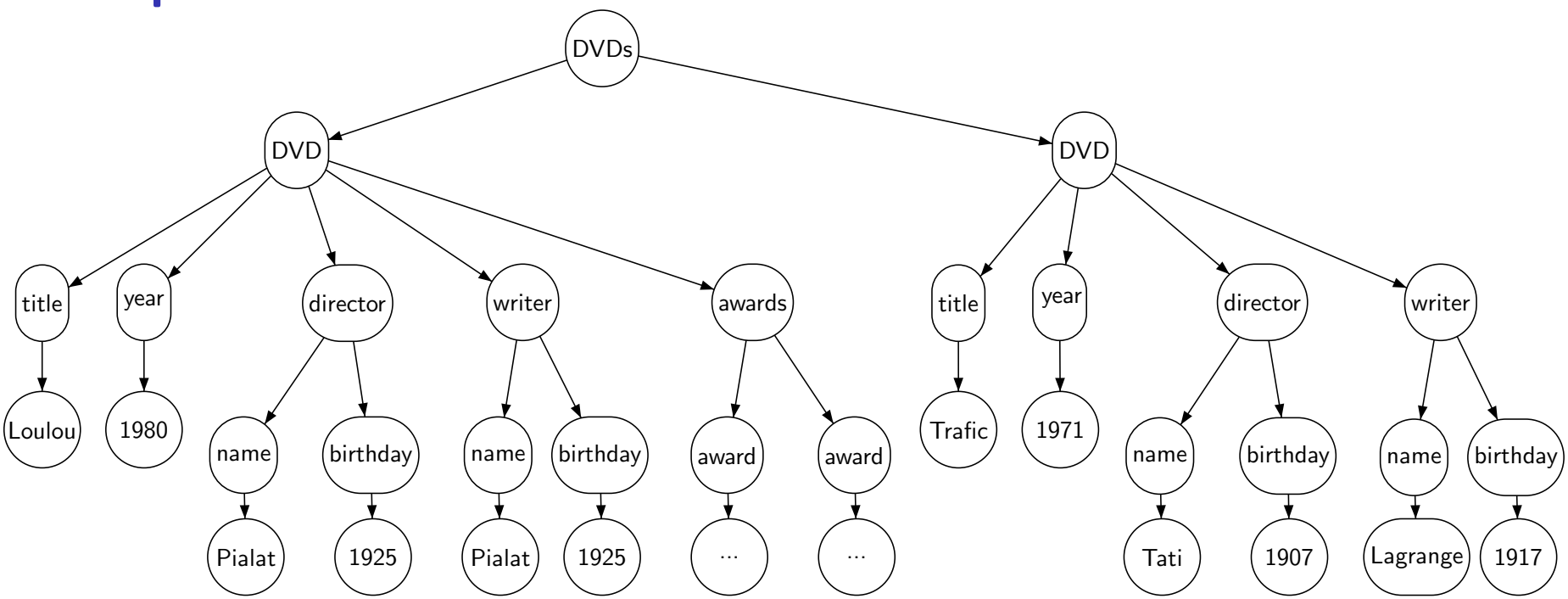
Tree representation



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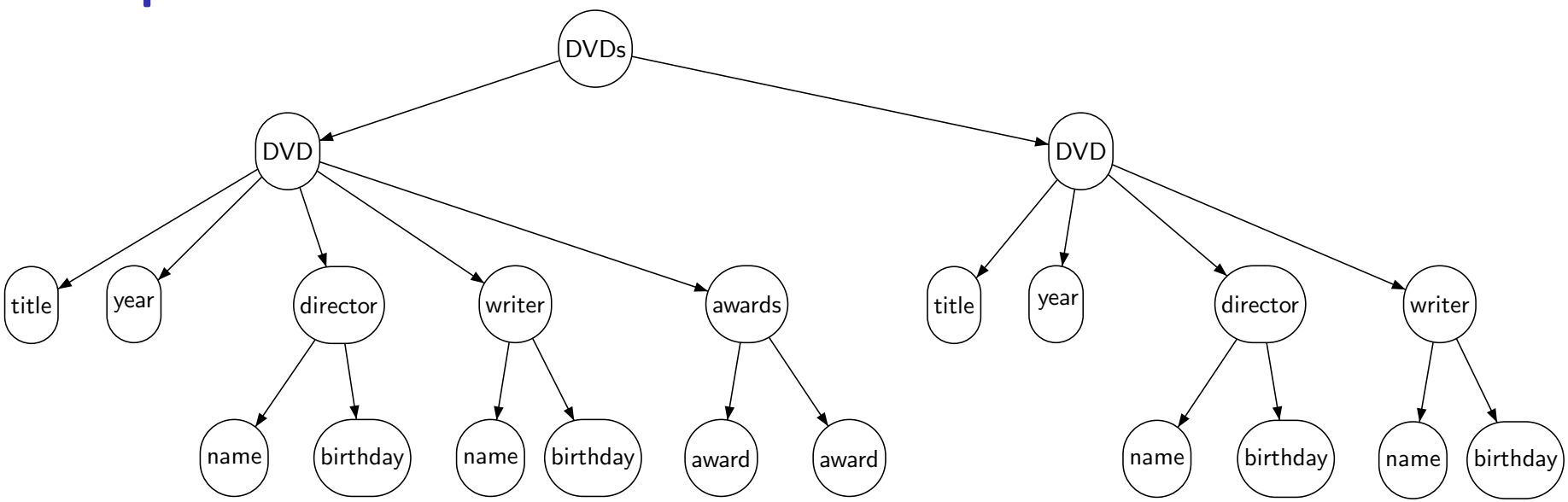

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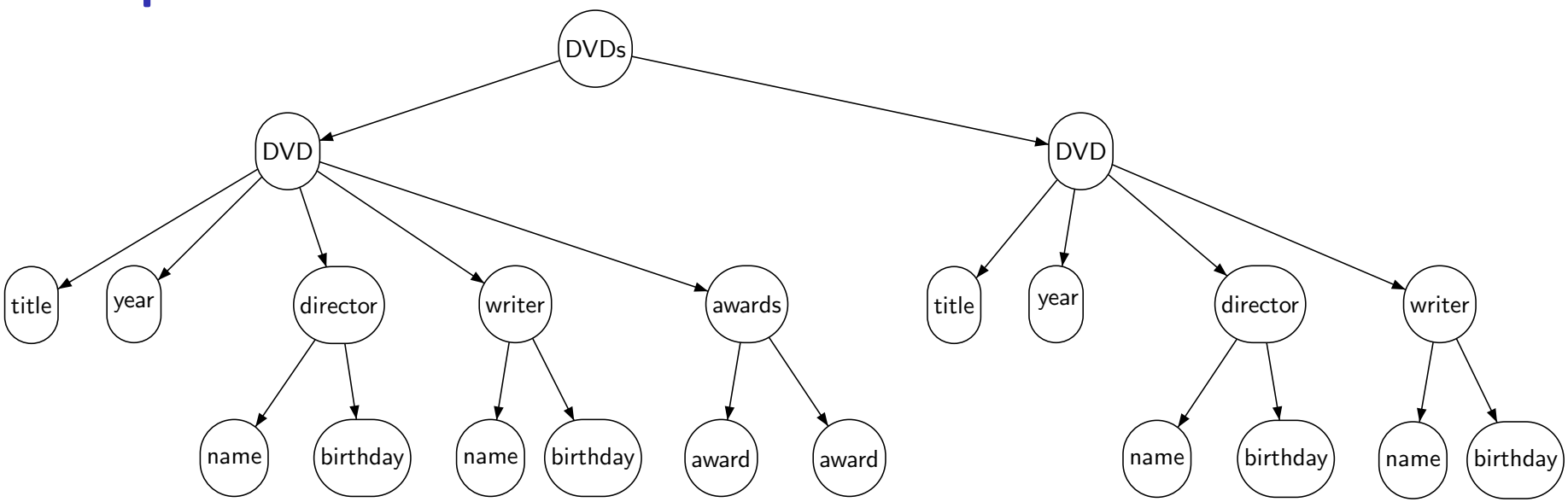

Trees are **ordered** and **unranked**.

XML Queries

Queries

- access XML data, transform XML documents
- node selection in XML trees
- n-**ary** queries select **set of** n-**tuples** of nodes
  - $n = 1$: unary queries
  - $n = 2$: binary queries

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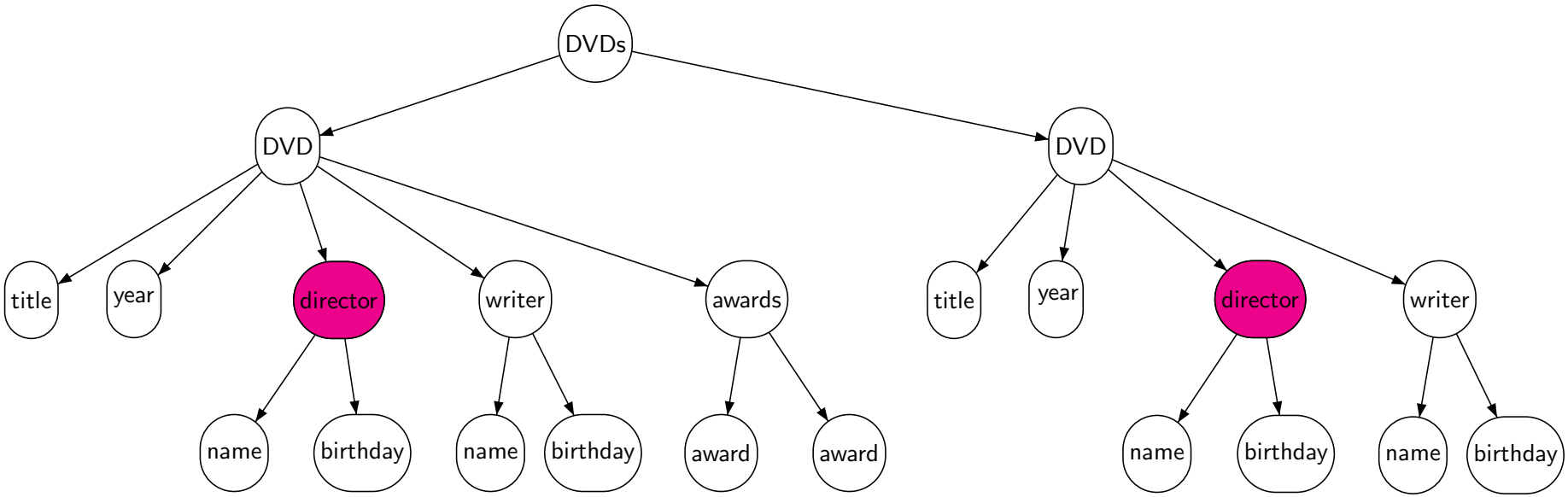
Example (Select all directors)



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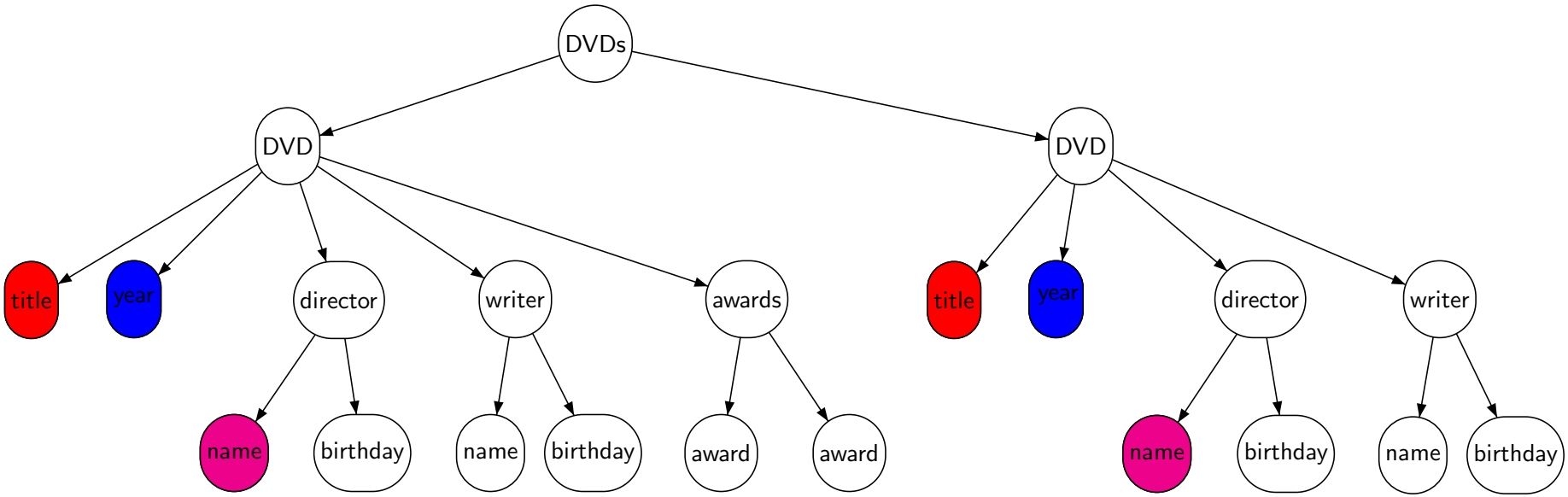
Example (Select all triples (title,year,director name))



Logics and Automata to Query XML Trees

- FO,MSO (yardstick logics but high query evaluation complexity)
- FO-relatives
  - temporal logics (LibkinN03,BarceloL05,ABDGGMR05)
  - navigational language XPath (W3C, GottlobKP02, Marx04, tenCate06,...)
- MSO-relatives
  - μ-calculus (BarceloL05)
  - Monadic Datalog (GottlobK04)
  - query automata (NevenS99)
  - node-selecting automata (Neven00,FrickGK03, NiehrenPTT06)
- Combination Logics (Schwentick00, ArenasBL07)
- pattern-matching approach: XDuce/CDuce (HosoyaP03,BenzakenCF03), Spatial Logic TQL (CardelliG02,BonevaTT05)

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Only a few logics are well-suited to express n-ary queries

Objectives

Two popular approaches:

Navigational Approach

Pattern-matching approach

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How to define a navigation-based n-ary query language?

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Navigational Approach

How to define a navigation-based n-ary query language?

- expressiveness vs query evaluation complexity
- composition language: from binary to n-ary queries
- application to XPath-based n-ary query languages

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- is there an expressive **decidable** TQL fragment that can define n-ary queries?

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Pattern-matching approach

- satisfiability problem
- is there an expressive **decidable** TQL fragment that can define n-ary queries?

- adaptation to ordered trees
- automata-based satisfiability algorithm

Outline

1. Composing Binary Queries
   1. definitions
   2. expressiveness, query evaluation
   3. application to n-ary XPath logics
2. The Spatial Logic TQL
   1. Examples, Definition
   2. Expressiveness, Satisfiability
   3. Tree Automata with Global Constraints
3. Summary and Perspectives

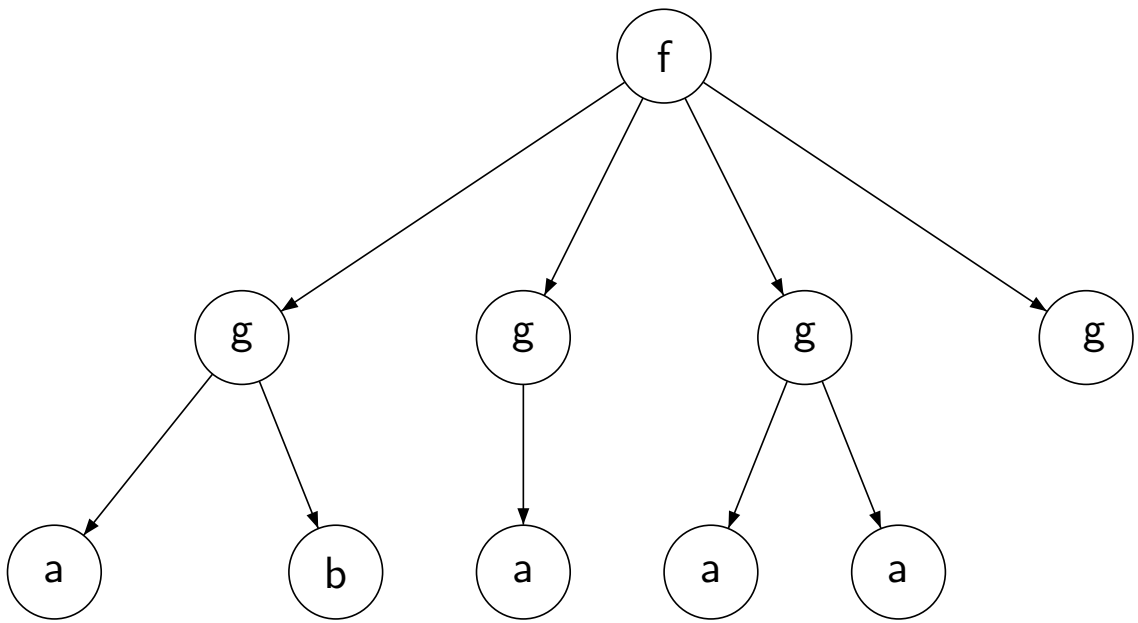
Outline

PART I: Composing Binary Queries

Trees and Queries

Trees

Trees are finite, unranked and ordered over a finite alphabet $\Sigma = f, g, a, b \ldots$.



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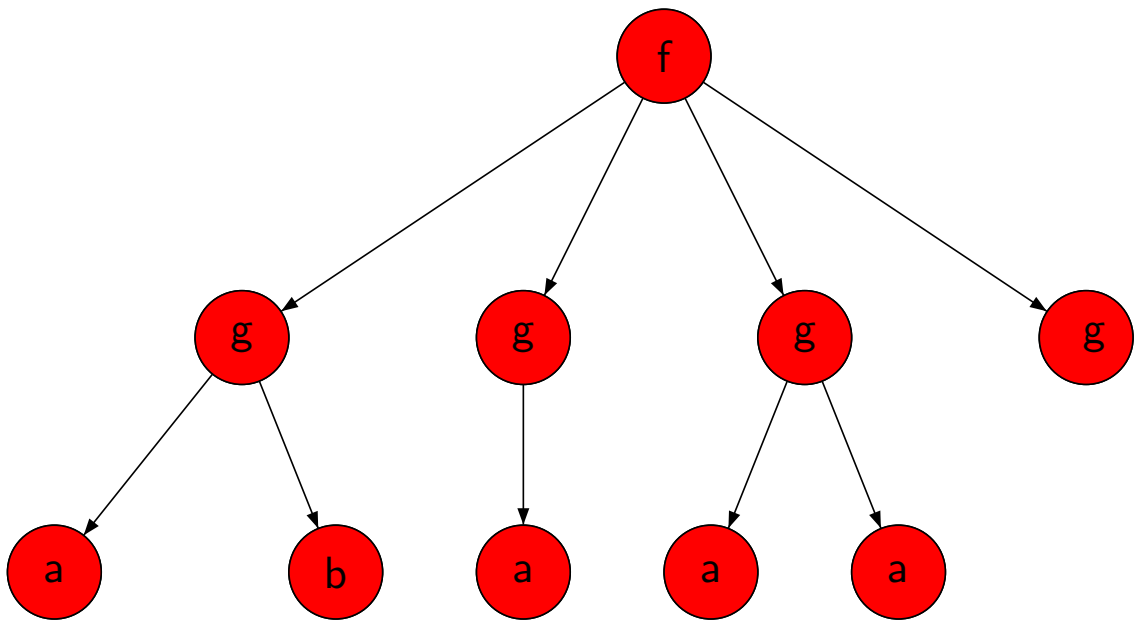

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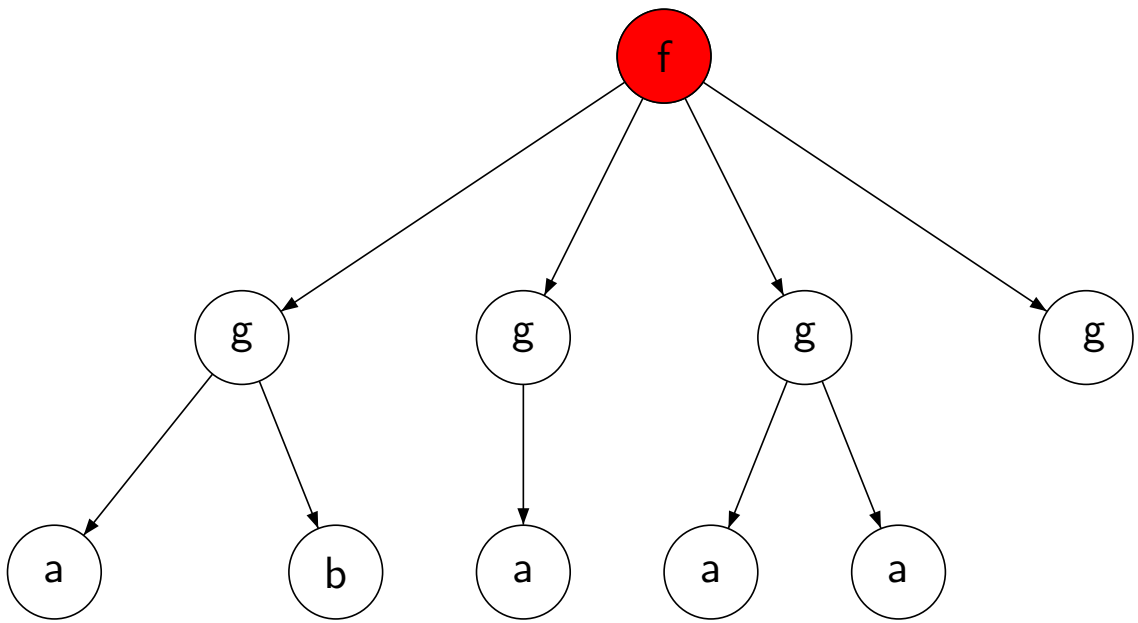
Unary Relations: nodes(t) , root(t)



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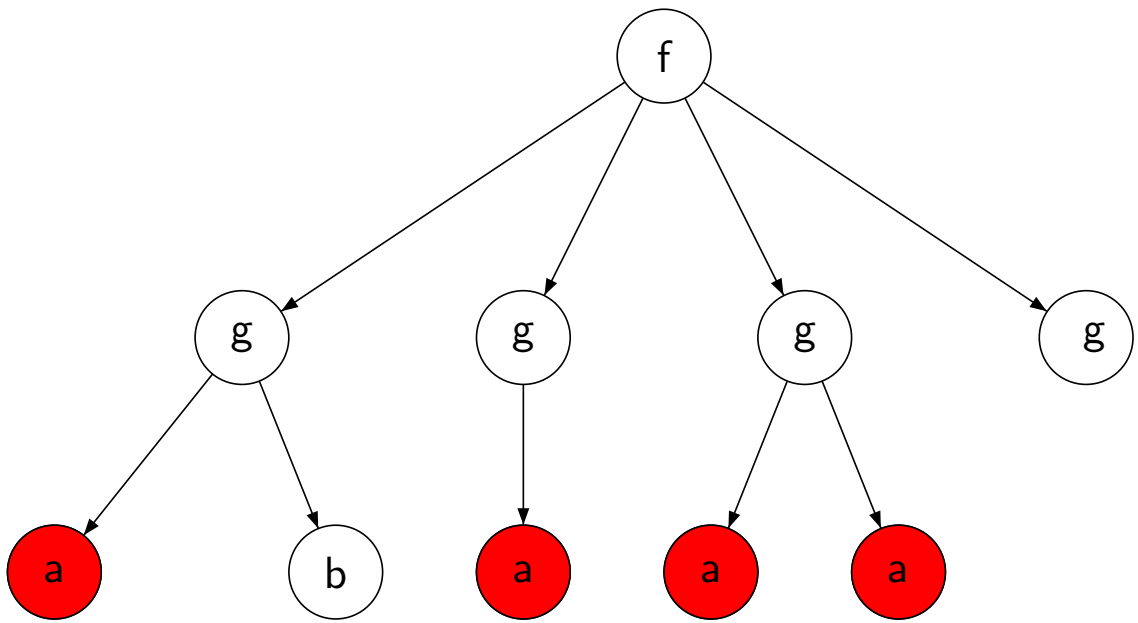
Unary Relations: $\text{nodes}(t)$, $\text{root}(t)$, $(\text{lab}_a(t))_{a \in \Sigma}$



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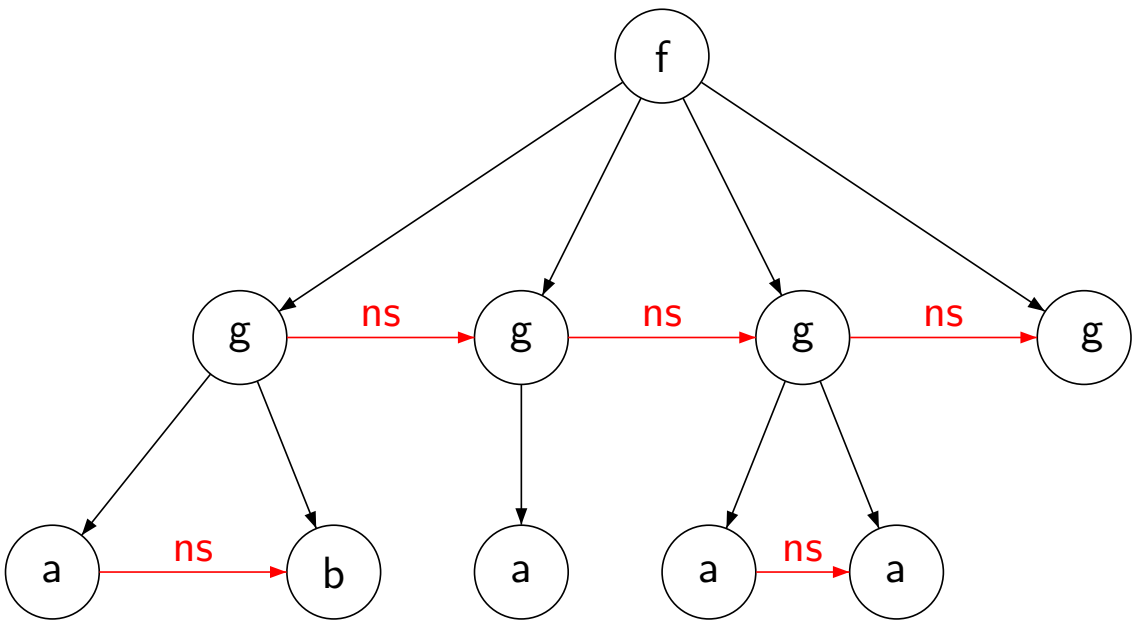
Binary Relations: ns



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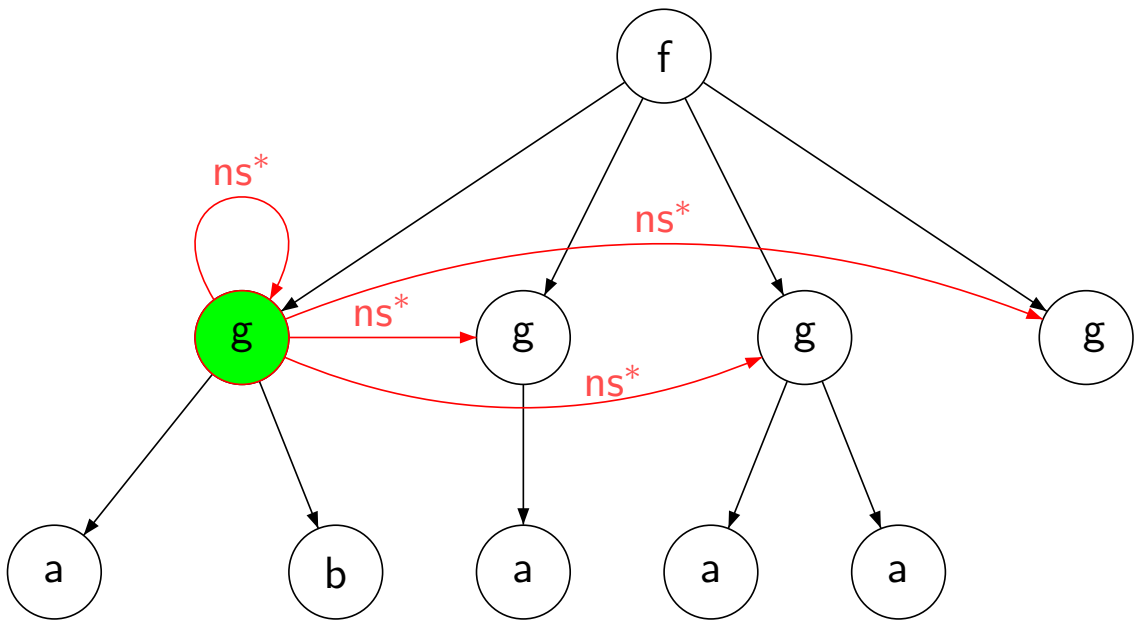

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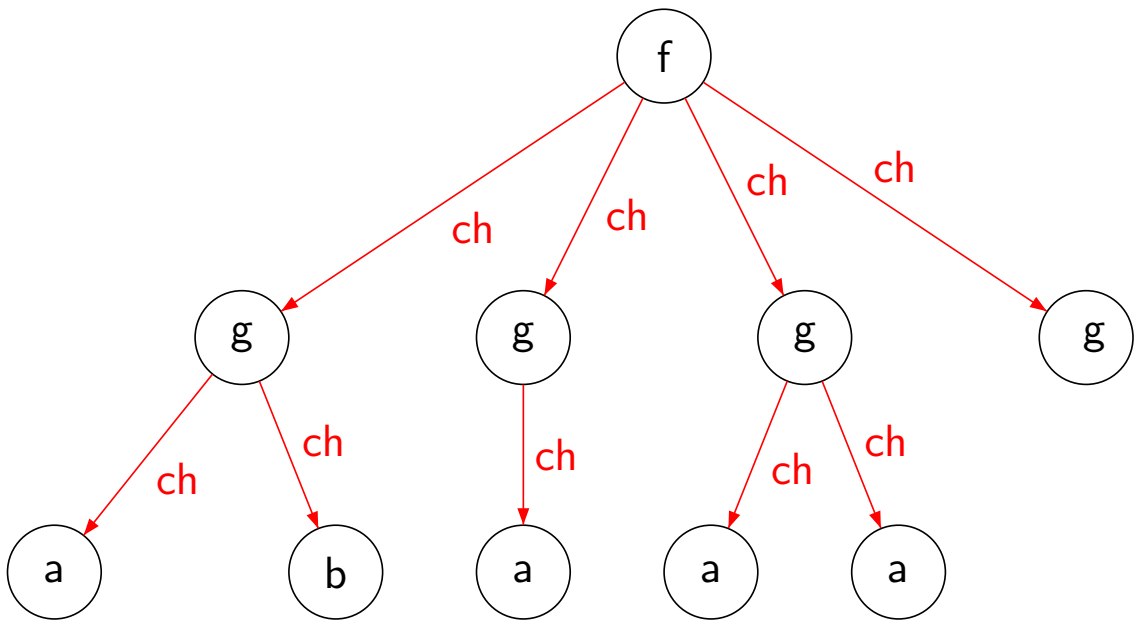

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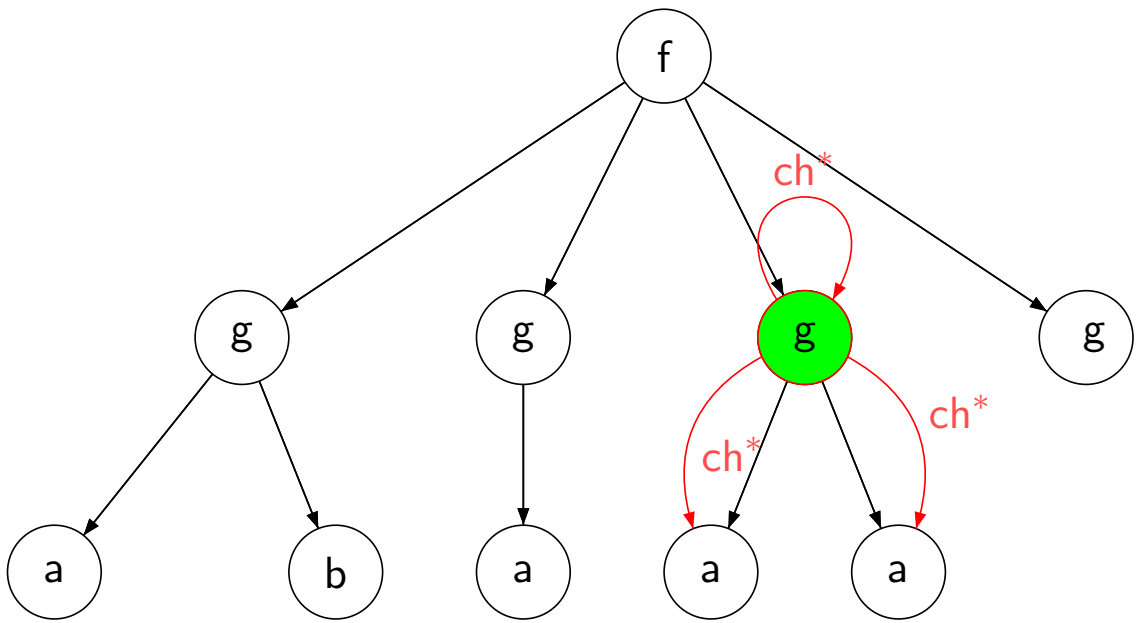

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Queries

Let $n \in \mathbb{N}$. An n-ary query q maps trees t to n-tuples of nodes

$$q(t) \subseteq \text{nodes}(t)^n$$

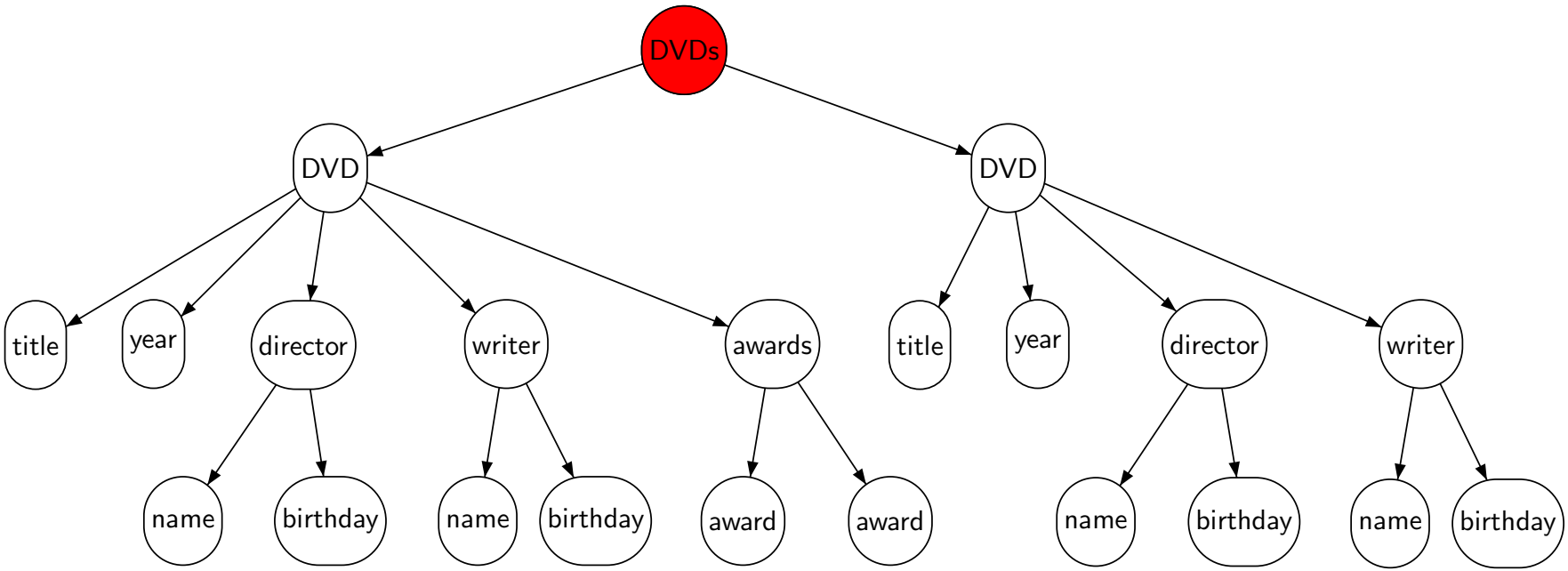
The navigational language XPath

- to navigate and select sets of nodes in XML trees
- by defining path expressions
- complex counting conditions
- *CoreXPath*: navigational core (GottlobKP02)

The navigational language XPath

Example: select all director names

ch :: *DVD*/ch :: *director*/ch :: *name*



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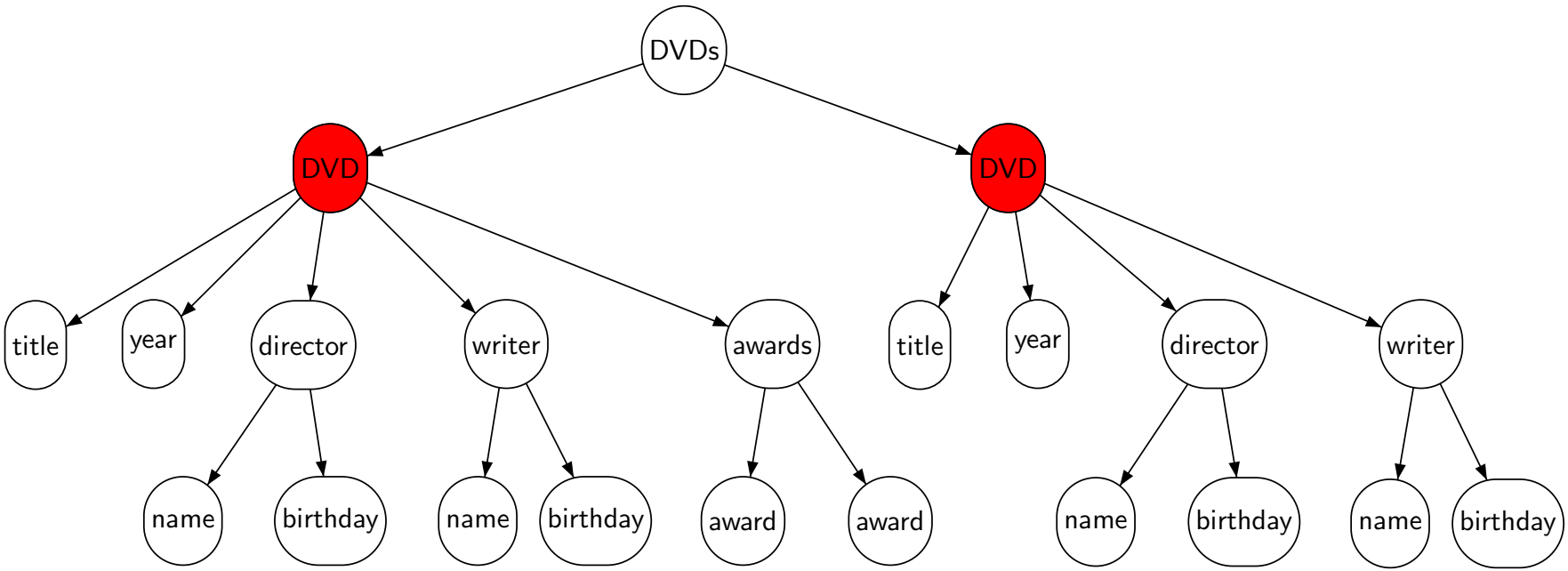

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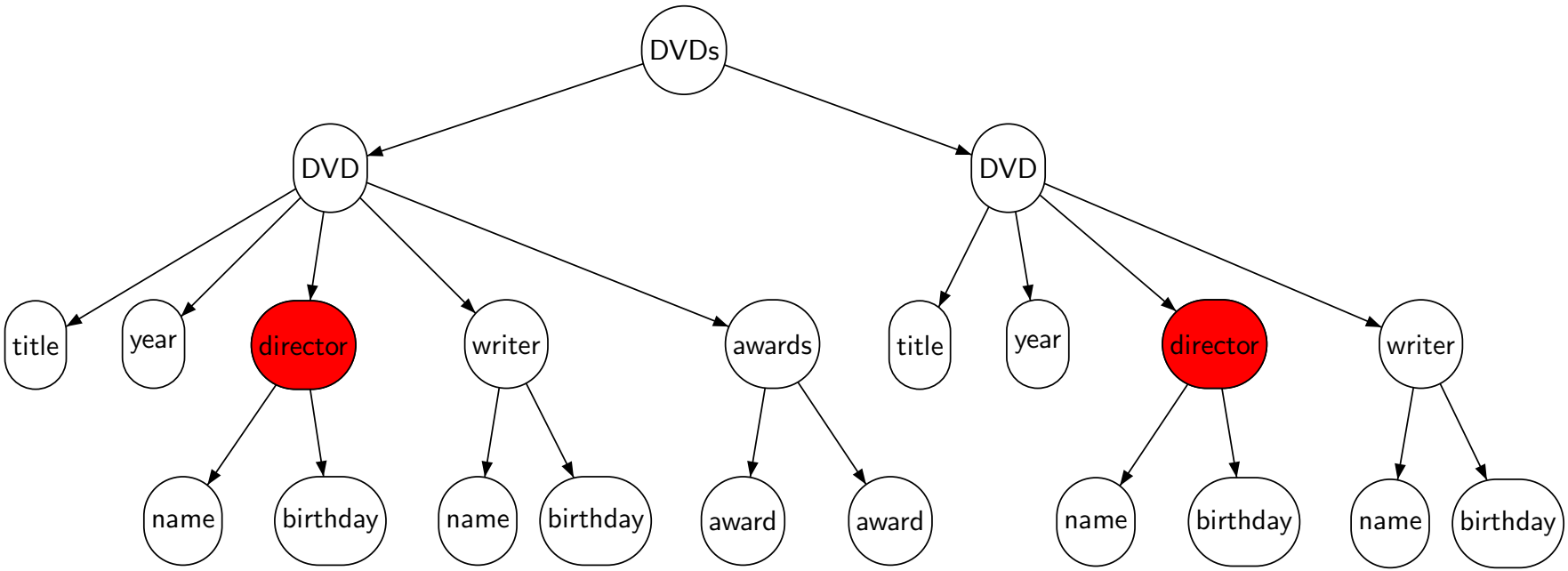

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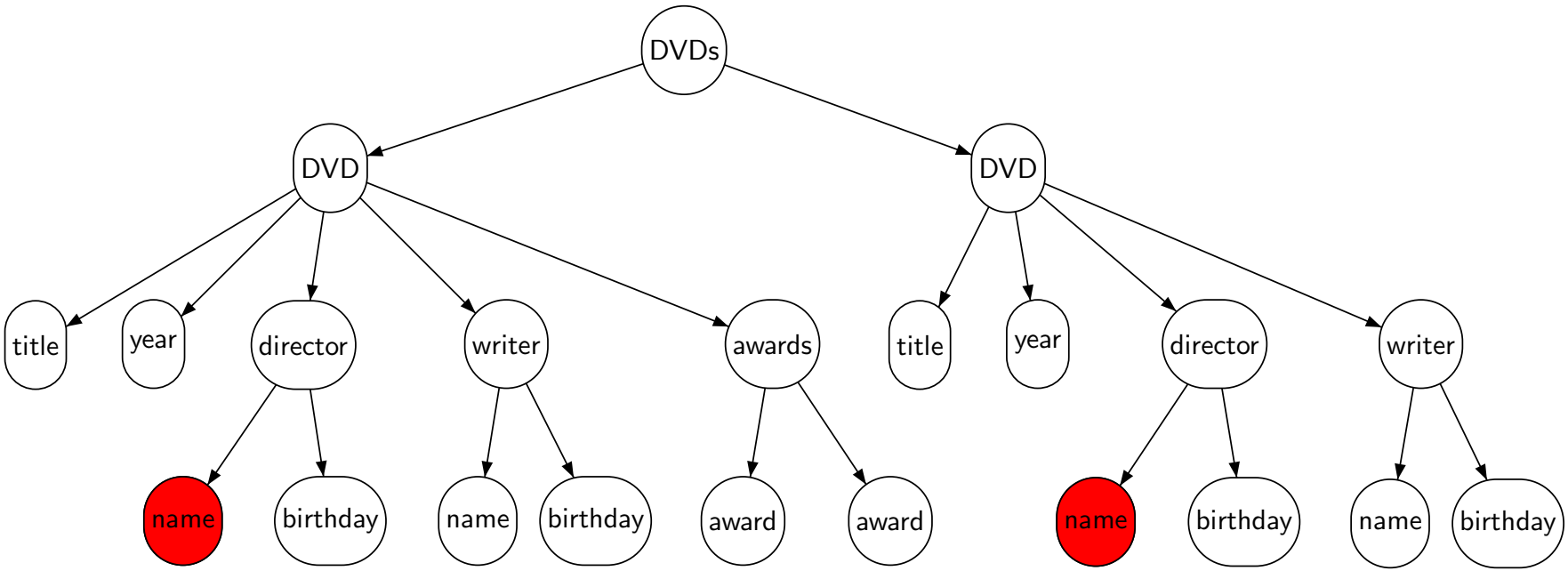
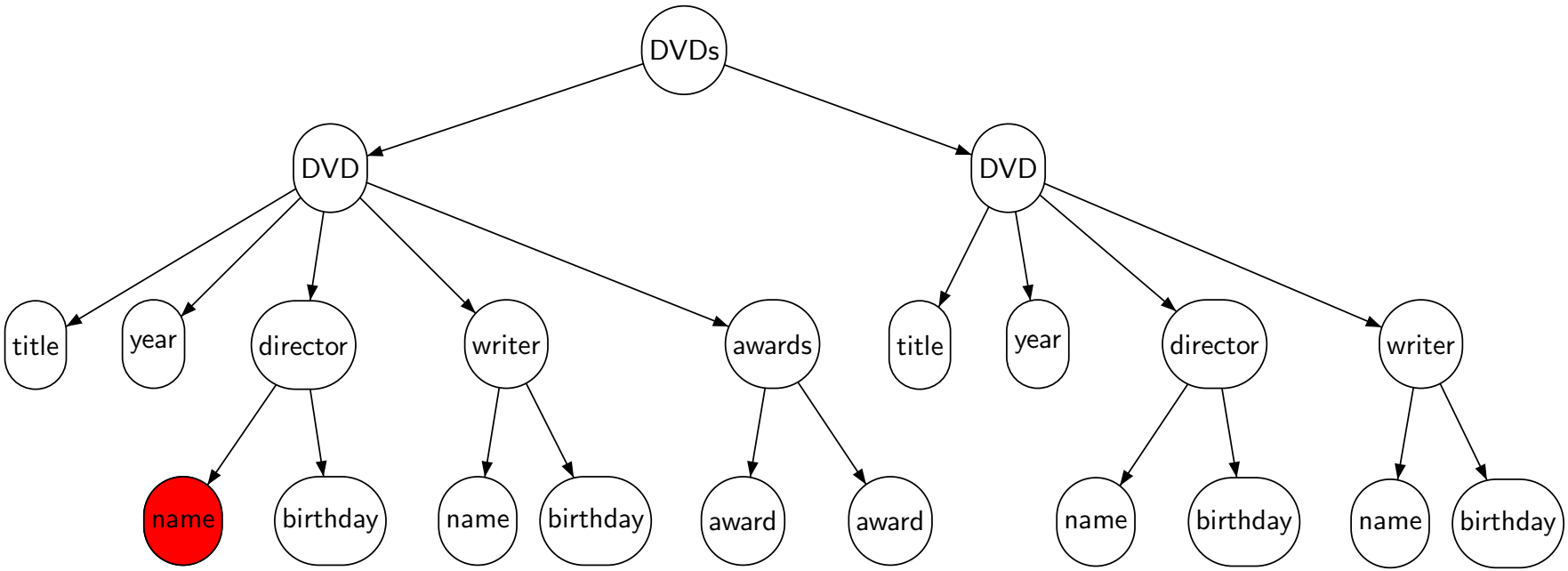


The navigational language XPath

Example: select all awarded director names

ch :: *DVD*[ch :: *awards*]/ch :: *director*/ch :: *name*



Expressions of CoreXPath and their semantics

Axis	$\mathsf{self}, \mathsf{ch}, \mathsf{ch}^+, \mathsf{ns}, \mathsf{ns}^+$
	$\mathsf{ch}^{-1}, (\mathsf{ch}^{-1})^+, \mathsf{ns}^{-1}, (\mathsf{ns}^{-1})^+$
Steps	$Axis::a$
	$Axis::*$
Composition	P_1/P_2
Union	$P_1 \cup P_2$
Tests	$P[\mathcal{T}]$
Path existence	P
Negation	not $\mathcal{T}$
Conjunction	$\mathcal{T}_1$ and $\mathcal{T}_2$

Expressions of CoreXPath and their semantics

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$[\![.]\!]^t \subseteq \mathsf{nodes}(t) \times \mathsf{nodes}(t)$

Steps $\quad [\![\mathit{Axis}::a]\!]^t = \{(v_1, v_2) \mid v_1 \ \mathit{Axis}\ v_2 \text{ and } v_2 \in \mathsf{lab}_a(t)\}$
$[\![\mathit{Axis}::*]\!]^t = \{(v_1, v_2) \mid v_1 \ \mathit{Axis}\ v_2\}$

Composition $\quad [\![P_1/P_2]\!]^t = [\![P_1]\!]^t \circ [\![P_2]\!]^t$

Union $\quad [\![P_1 \cup P_2]\!]^t = [\![P_1]\!]^t \cup [\![P_2]\!]^t$

Tests $\quad [\![P[\mathcal{T}]]\!]^t = \{(v_1, v_2) \in [\![P]\!]^t \mid v_2 \in [\![\mathcal{T}]\!]^t_{\text{test}}\}$

$[\![.]\!]^t_{\text{test}} \subseteq \mathsf{nodes}(t)$

Path existence $\quad [\![P]\!]^t_{\text{test}} = \{v \mid (v, v') \in [\![P]\!]^t\}$

Negation $\quad [\![\text{not } \mathcal{T}]\!]^t_{\text{test}} = \mathsf{nodes}(t) - [\![\mathcal{T}]\!]^t_{\text{test}}$

Conjunction $\quad [\![\mathcal{T}_1 \text{ and } \mathcal{T}_2]\!]^t_{\text{test}} = [\![\mathcal{T}_1]\!]^t_{\text{test}} \cap [\![\mathcal{T}_2]\!]^t_{\text{test}}$

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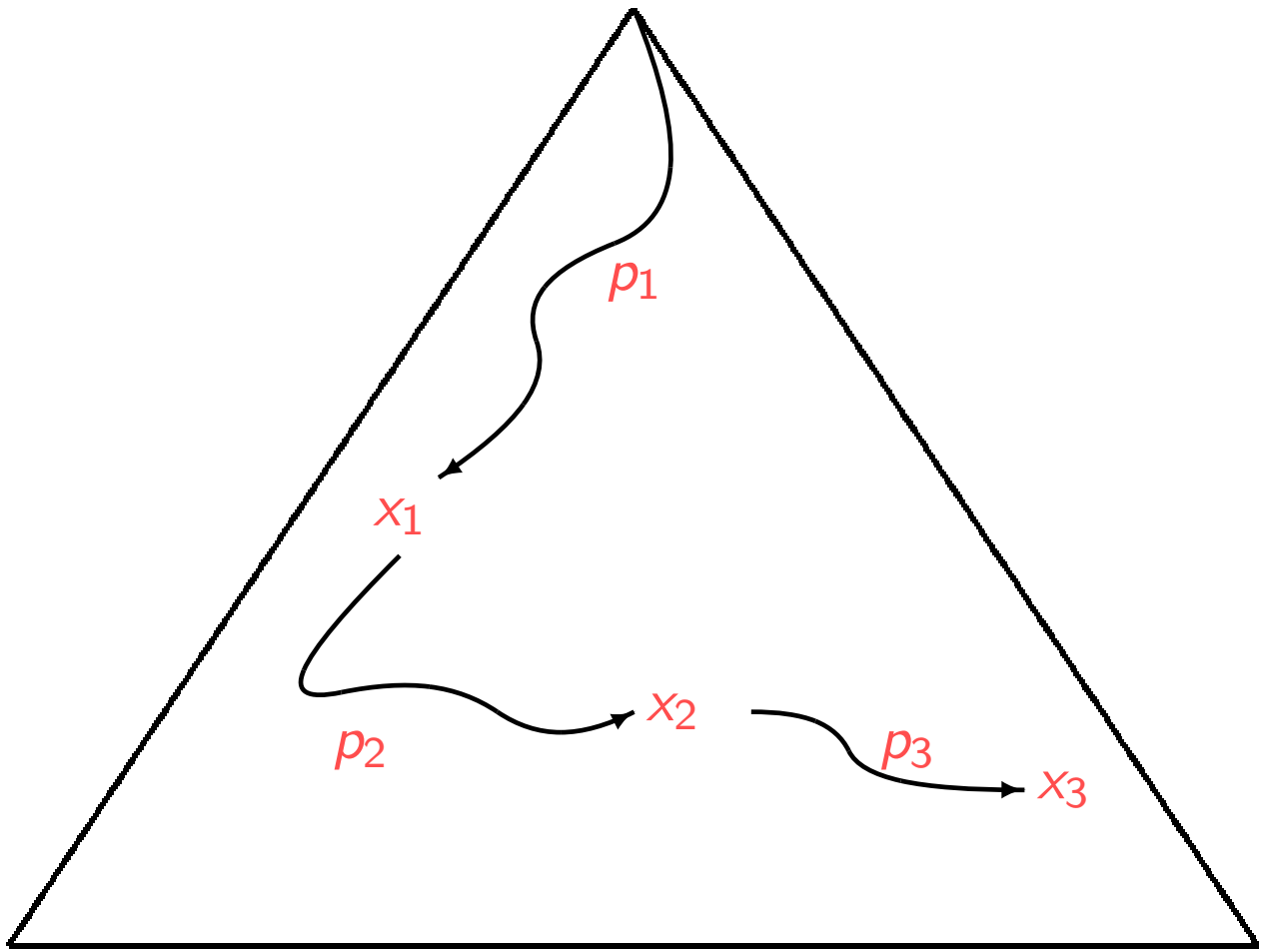
- use path expressions p to navigate
- use node variables $x_1, x_2, \ldots, x_n$ to select n-tuples



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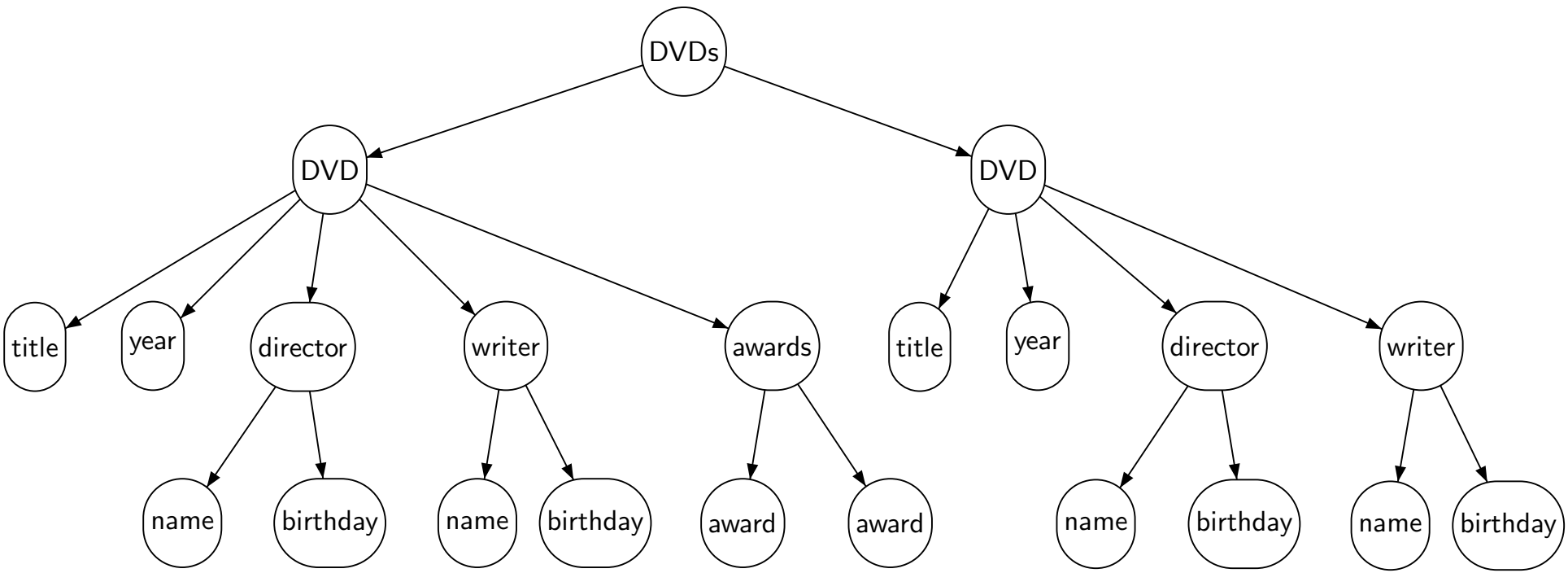
Example (All triples (title,year,director name))

$\phi(x, y, z) \ =$



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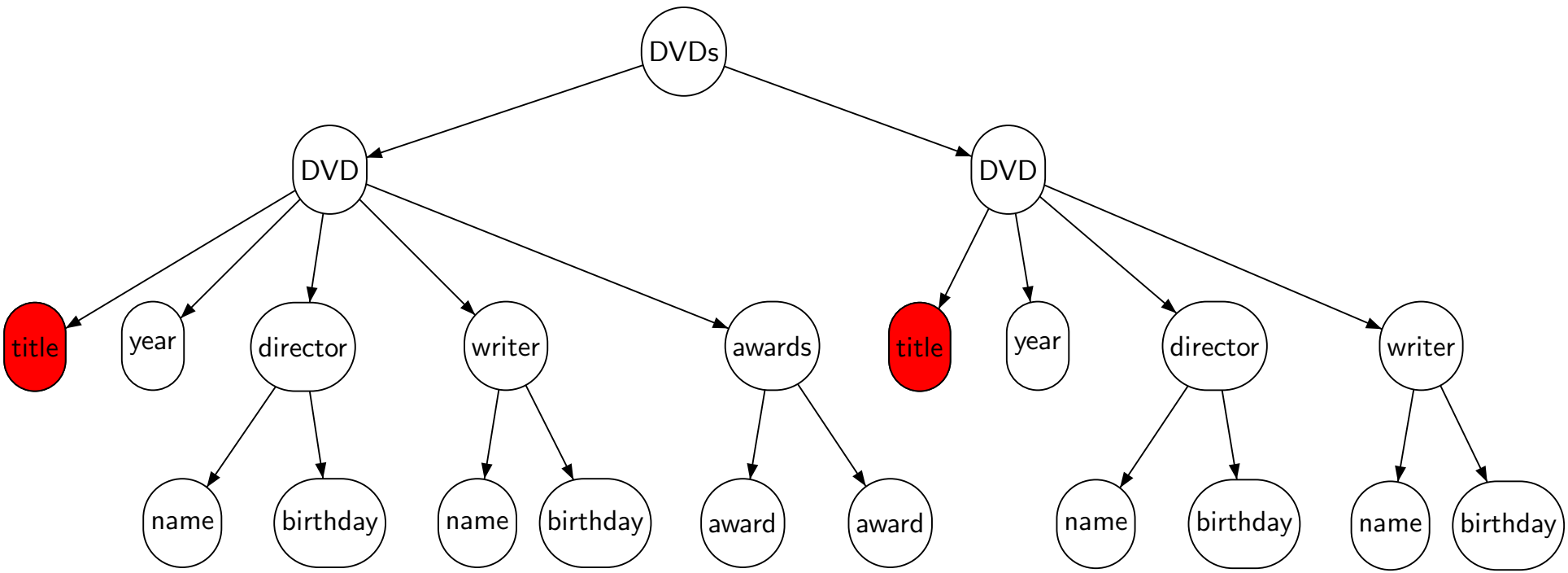

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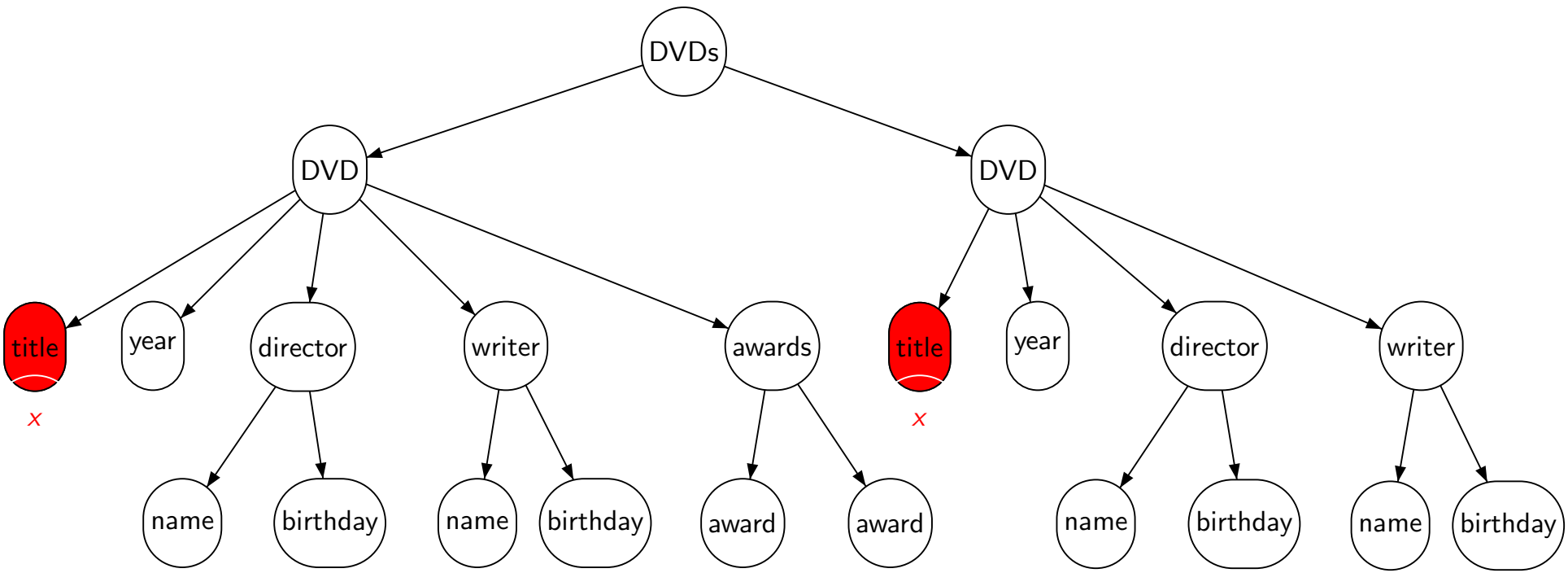

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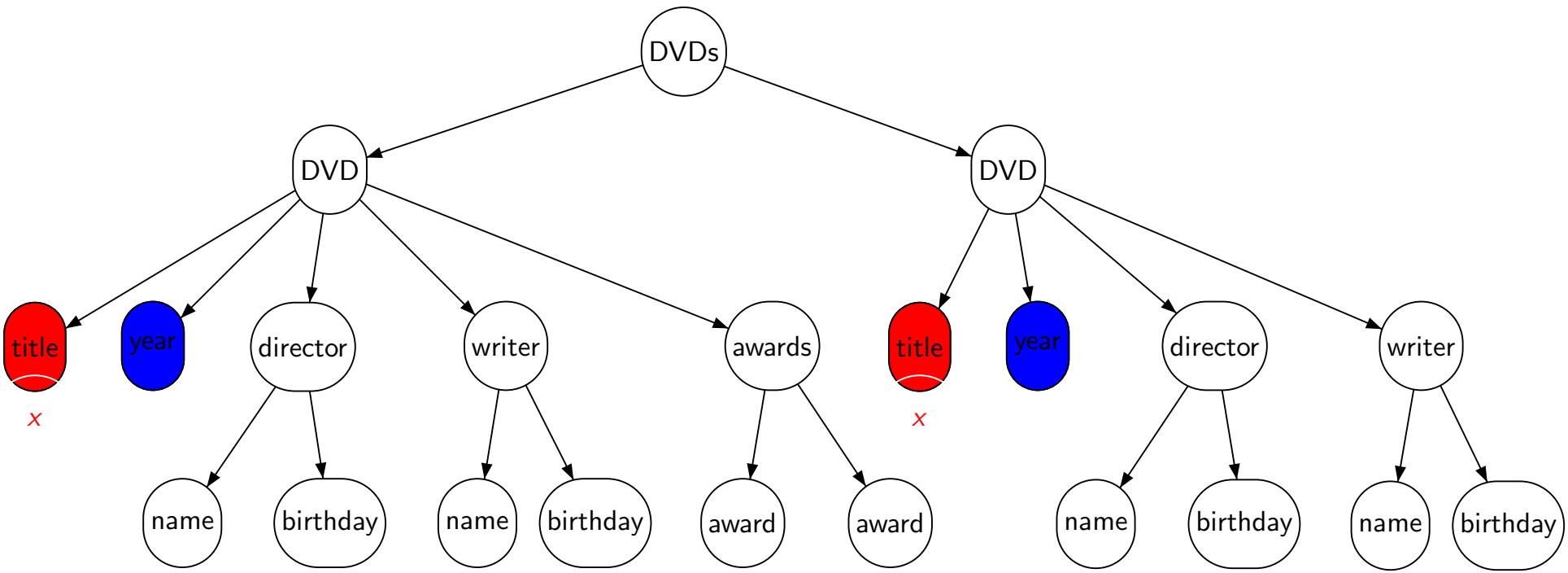

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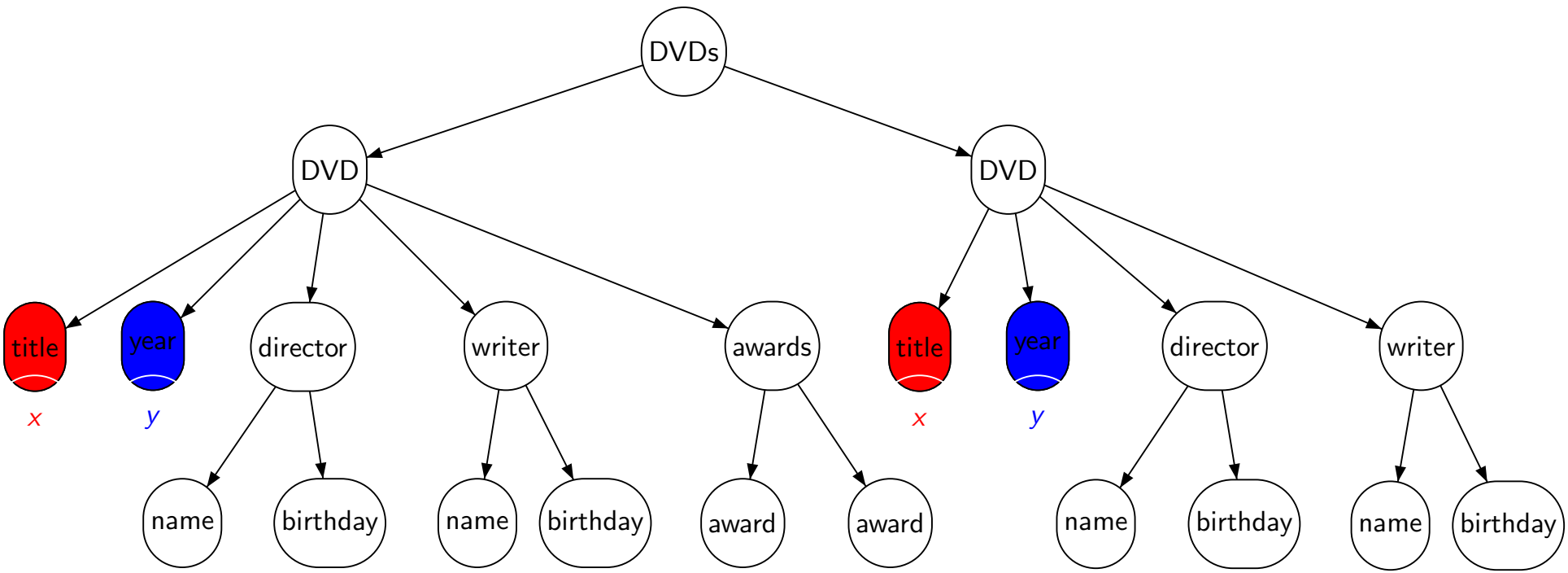

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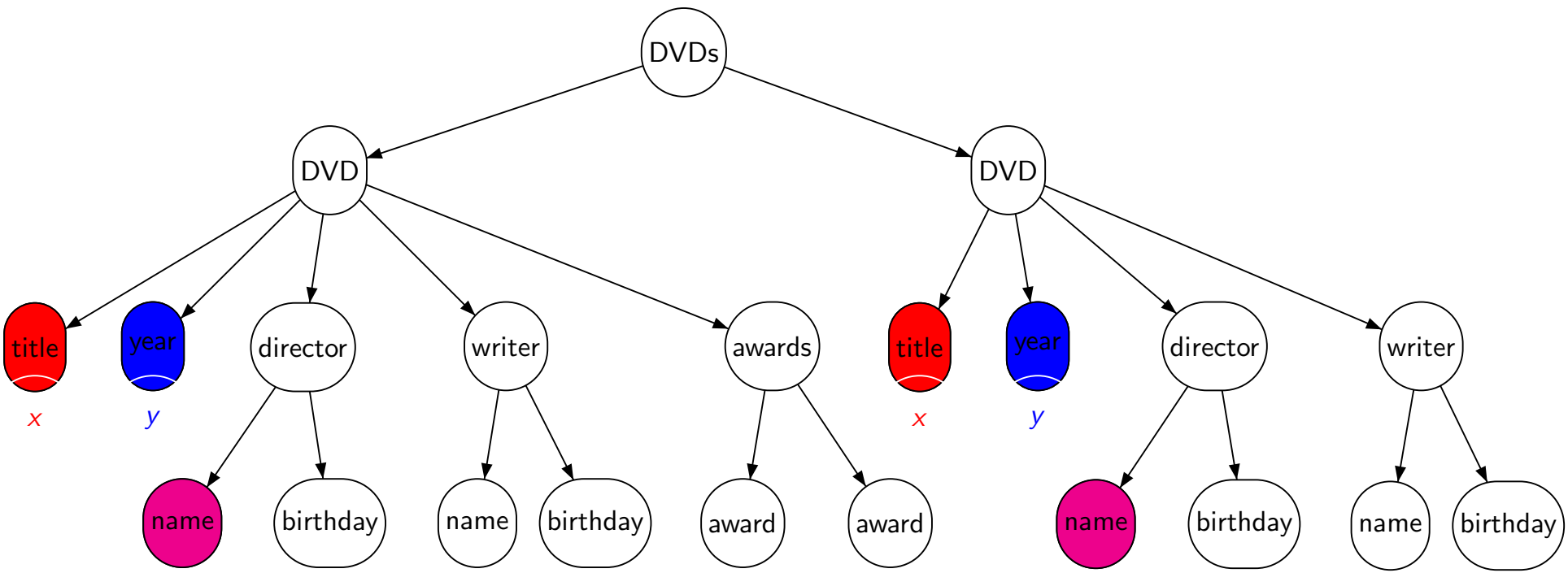

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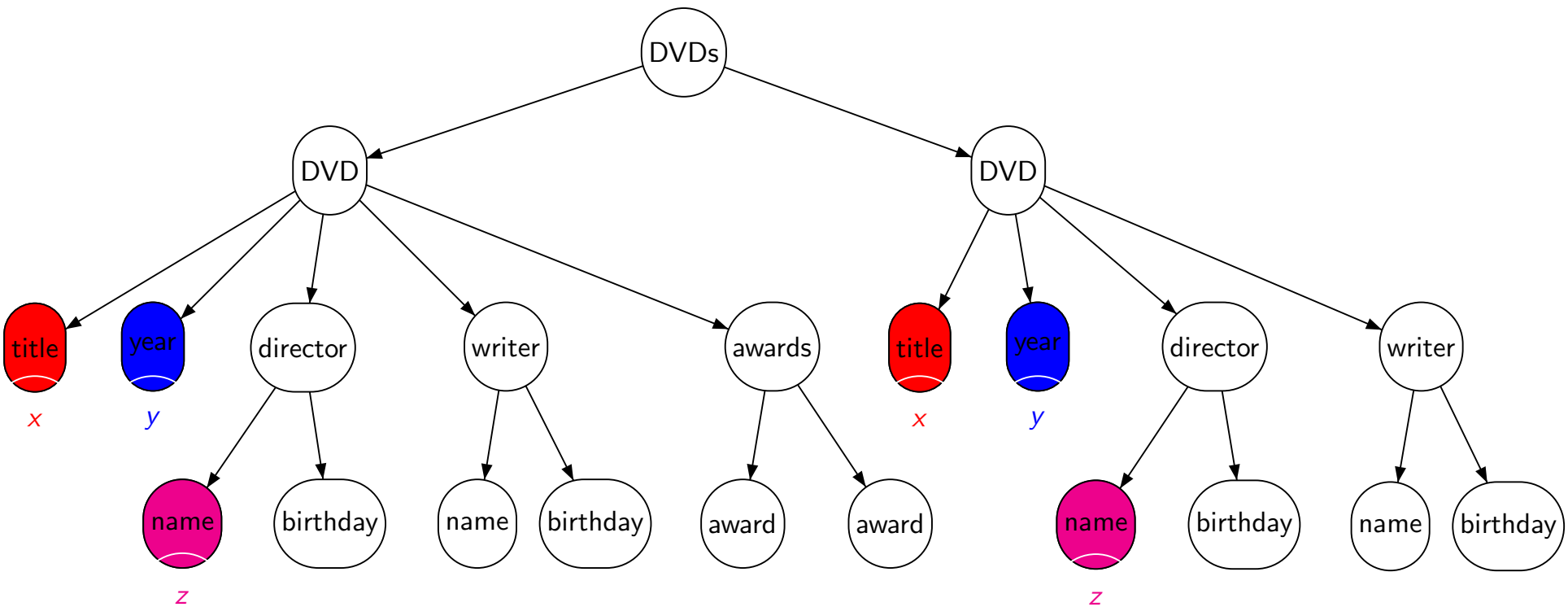

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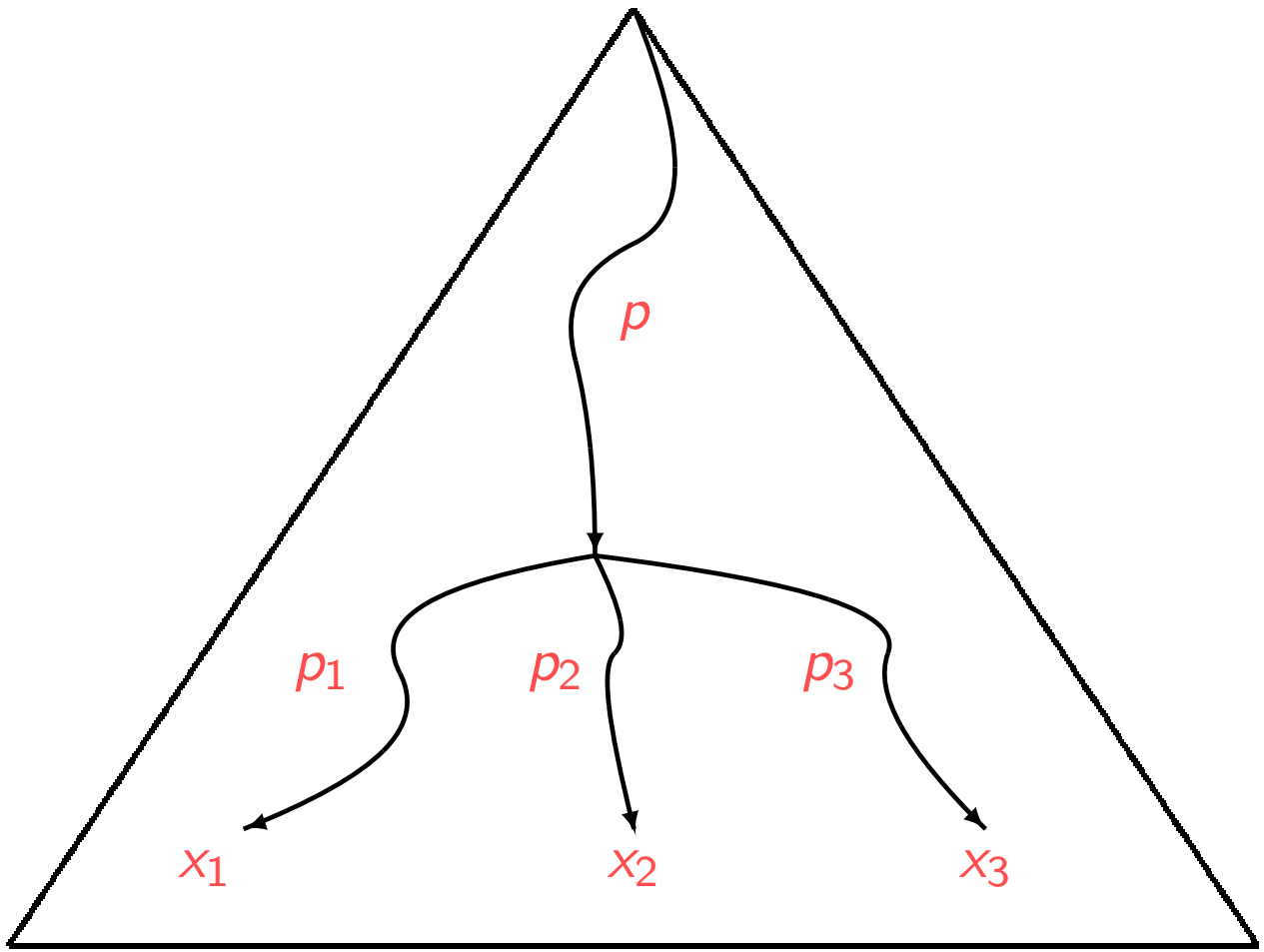

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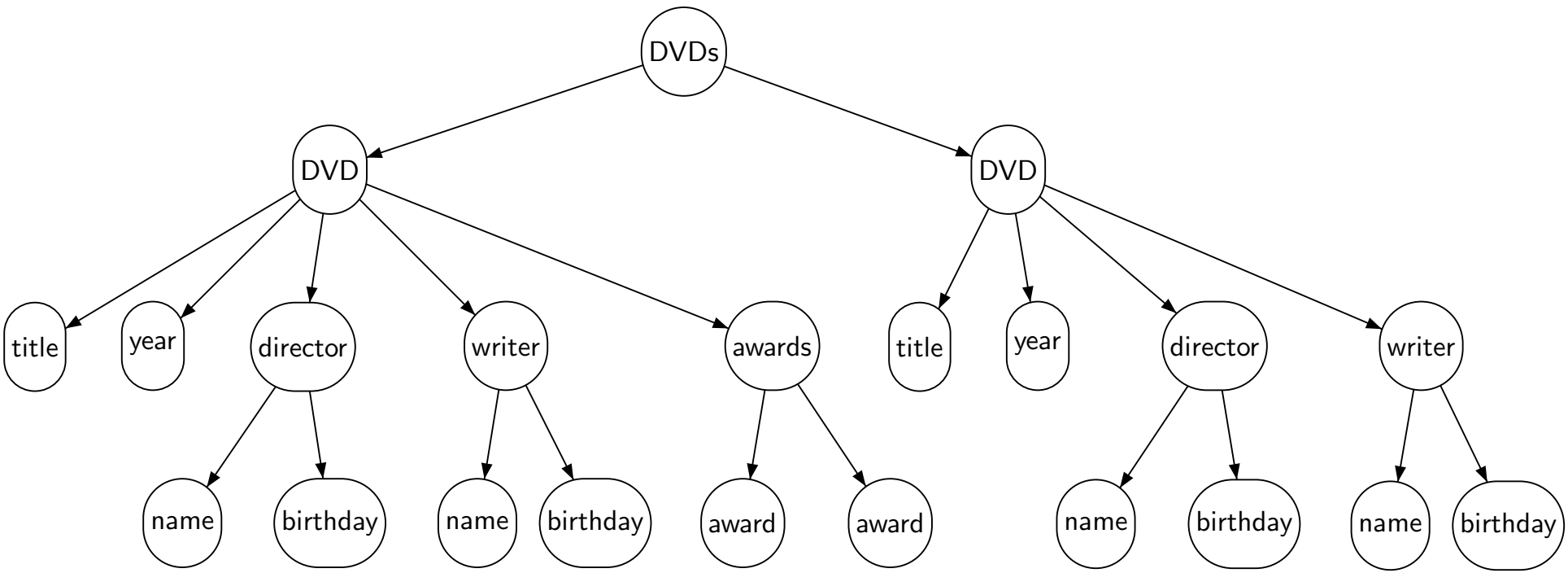

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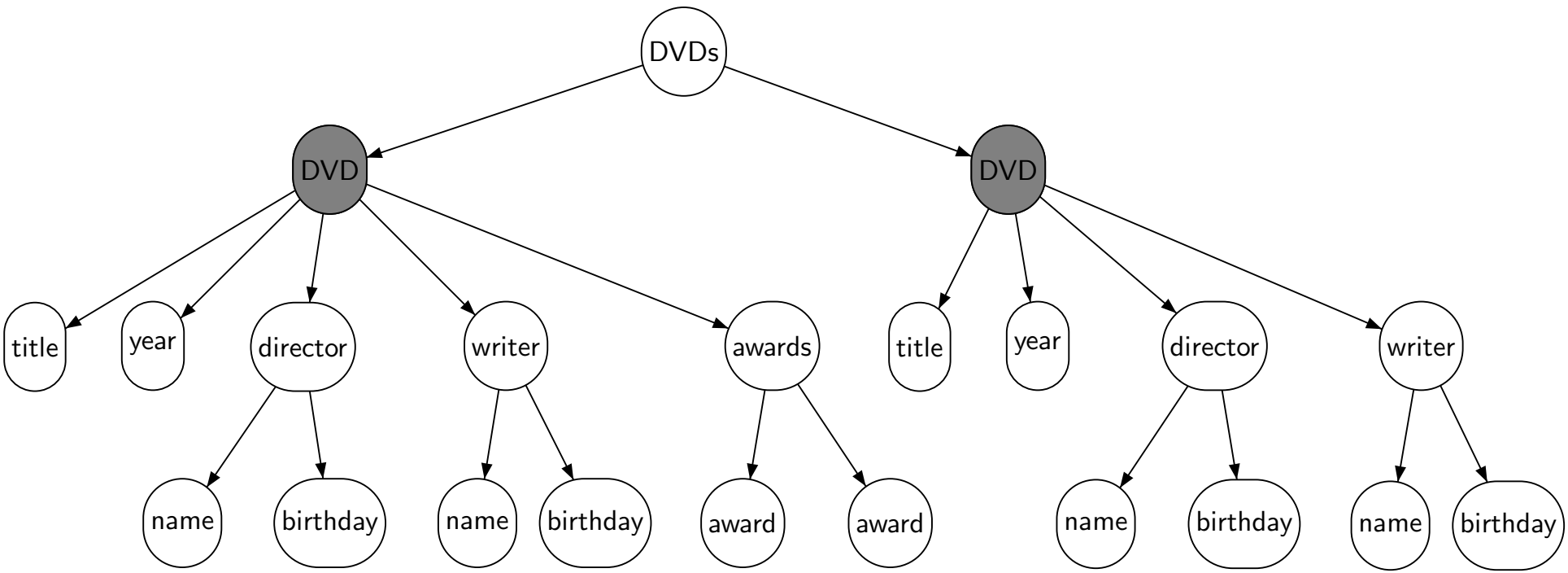

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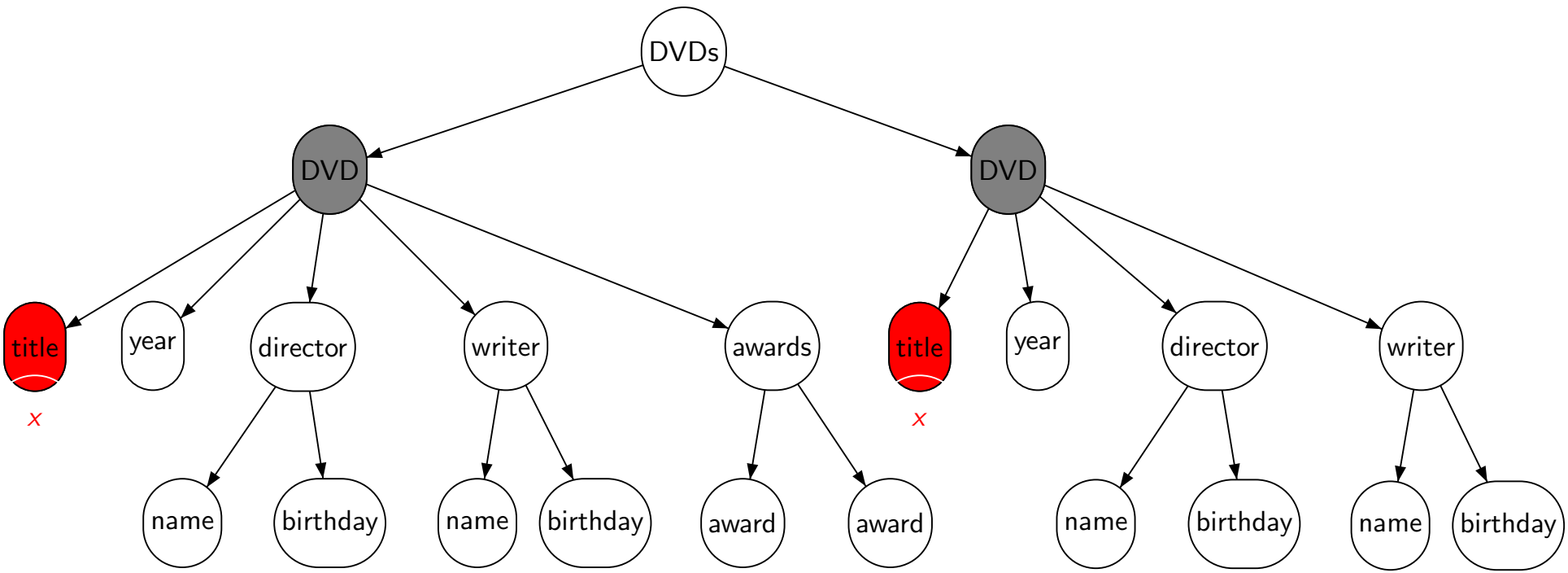

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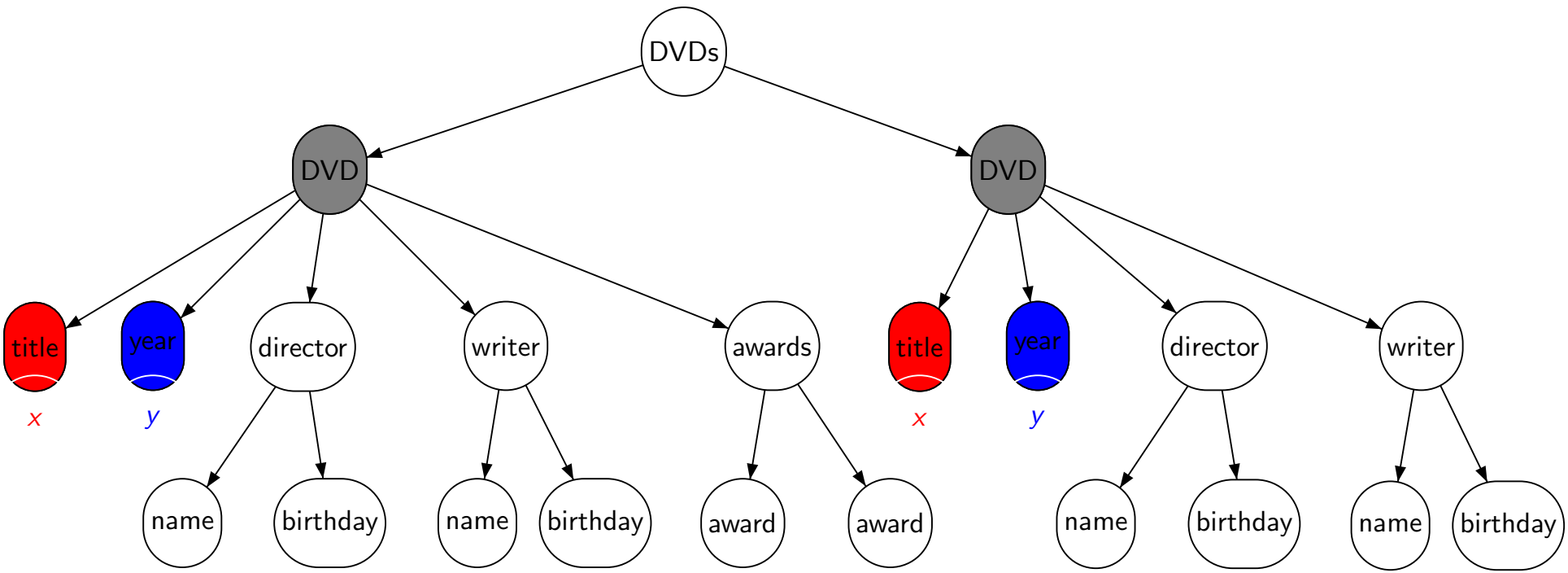
$\phi(x, y, z) = \mathrm{ch} :: DVD[\mathrm{ch} :: title/x][\mathrm{ch} :: year/y]$



How to turn XPath into an n-ary query language?

- use path expressions to navigate
- use node variables $x_1, x_2, \ldots, x_n$ to select n-tuples

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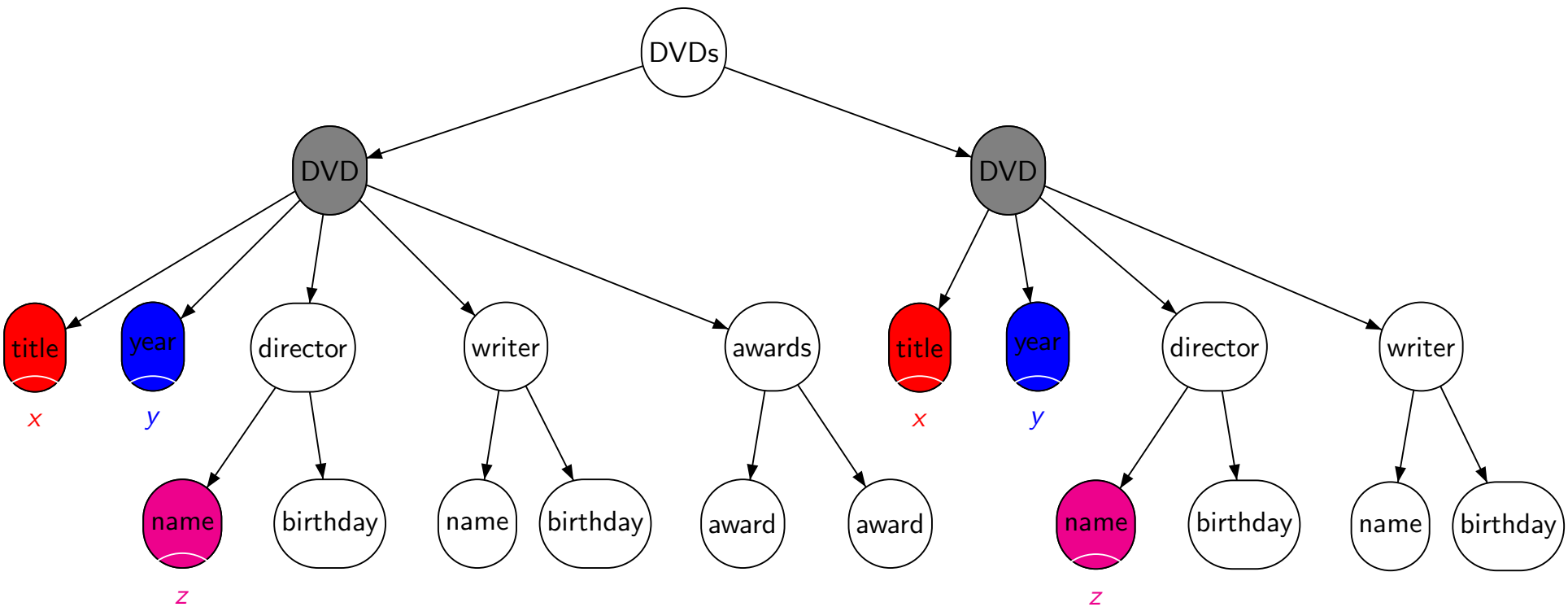

Idea of the composition language

- use ~~path expressions~~ to navigate
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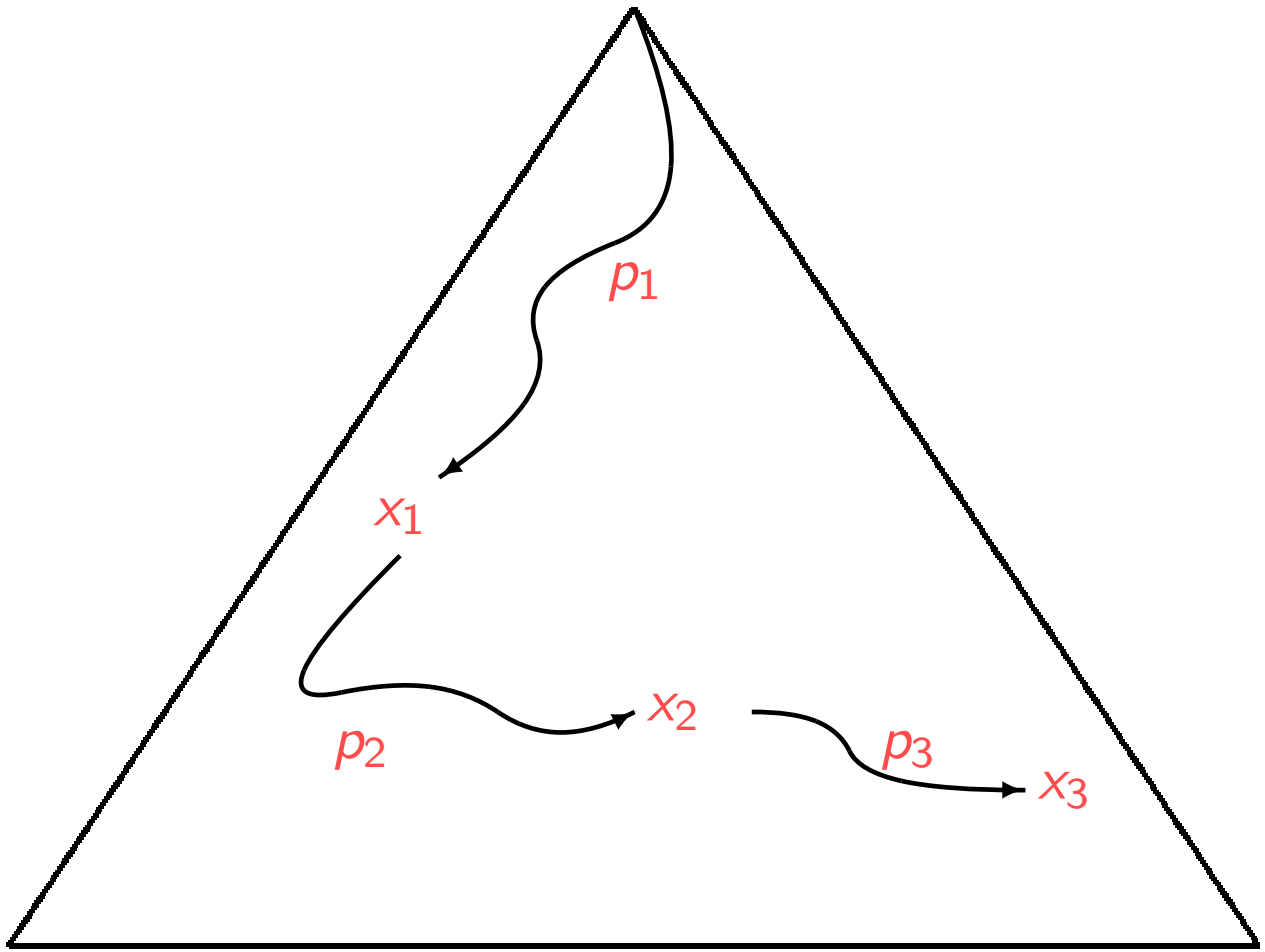
- composition operator $\circ$ to compose queries



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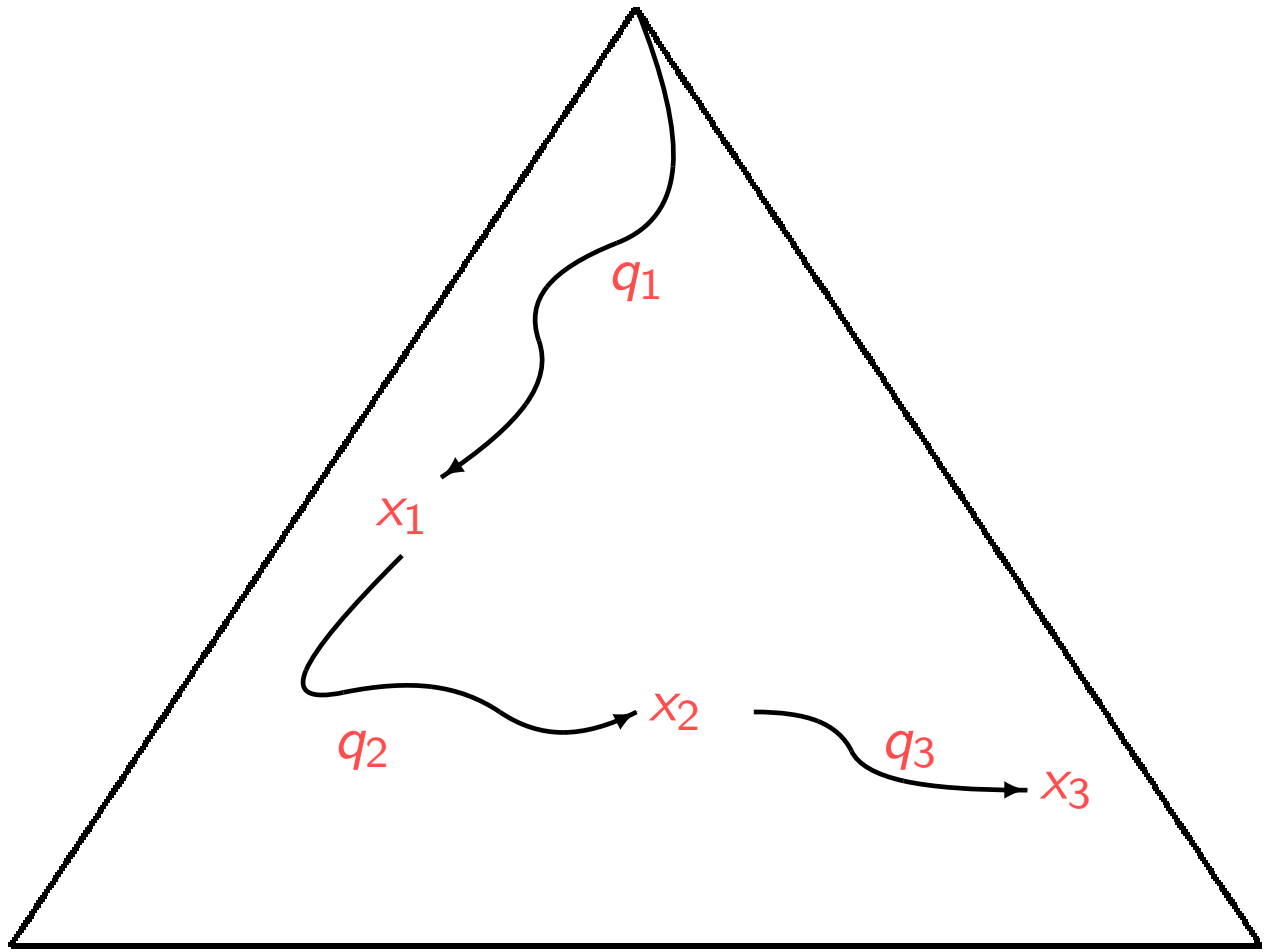

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Example (Composition of CoreXPath expressions)

$$\underbrace{\mathsf{ch}^*::\mathit{title}}_{\downarrow\ q_1} \;/\; x \;/\; \underbrace{\mathsf{ns}::\mathit{year}}_{\downarrow\ q_2} \;/\; y \;/\; \underbrace{\mathsf{ns}::\mathit{director}/\mathsf{ch}::\mathit{name}}_{\downarrow\ q_3} \;/\; z$$

$$q_1 \circ x \circ q_2 \circ y \circ q_3 \circ z$$

where $q_1, q_2, q_3 \in \mathit{CoreXPath}$.

The composition language Comp(L)

Syntax of composition formulas Comp(L)

We start from L a binary query language, and Var a set of variables.

$$
\begin{array}{rcll}
\phi & := & q & q \in L \\
 & | & x & \textit{variable} \\
 & | & \phi \circ \phi & \textit{composition} \\
 & | & [\phi] & \textit{test} \\
 & | & \phi \vee \phi & \textit{disjunction}
\end{array}
$$

- thanks to variables, you can define n-ary queries
- $\mathrm{Ans}(\phi, t)$: set of answers.

Query Evaluation

Query evaluation problem

- **Input**: a tree t, a formula $\phi(x_1, \ldots, x_n) \in \mathrm{Comp}(L)$
- **Output**: $\mathrm{Ans}(\phi, t)$

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Polynomial-time query evaluation

- The number of n-tuples of nodes is exponential in $|t|$ and:

$$|\mathrm{Ans}(\phi, t)| \ << \ |t|^n$$

- one needs **polynomial-time** query evaluation:

$$poly(|t|, |\phi|, |\mathrm{Ans}(\phi, t)|)$$

Query Evaluation Algorithm for $\mathrm{Comp}^{\mathrm{nvs}}(L)$

Non-variable sharing fragment

- variable sharing: $q \circ x \circ q' \circ y \circ q'' \circ x$
- disallow variable sharing: $\phi_1 \circ \phi_2 \rightarrow \mathit{Var}(\phi_1) \cap \mathit{Var}(\phi_2) = \varnothing$
- $\mathrm{Comp}^{\mathrm{nvs}}(L) = \mathrm{Comp}(L) \ + \$ non-variable sharing
- related to acyclicity of conjunctive queries (Yannakakis81)

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Query evaluation for $\mathrm{Comp}^{\mathrm{nvs}}(L)$ is in PTIME if query evaluation for L is in PTIME.

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Theorem

Query evaluation for $\mathrm{Comp}^{\mathrm{nvs}}(L)$ is in PTIME if query evaluation for L is in PTIME.

Idea (Yannakakis81): process the formula recursively:

1. at each step, check if there is a solution $\rightarrow$ remain linear in $|\mathrm{Ans}(\phi, t)|$
2. use memoization to avoid redundant calculus

Expressiveness

Two yardstick logics, FO and MSO

- MSO$=$ FO$+$ set quantification
- formulas $\psi(x_1, \ldots, x_n) \in$ FO (MSO) define n-ary queries
- FO_n $=$ n-ary FO queries
- MSO_n $=$ n-ary MSO queries

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Theorem

$$\begin{aligned} FO_n &= Comp^{nvs}(FO_2) \\ MSO_n &= Comp^{nvs}(MSO_2) \end{aligned}$$

Remark

It uses folklore result from finite model theory based on the Shelah's decomposition method. (Schwentick'00 or Marx'05 for instance).

n-ary XPath Extensions (I)

Conditional XPath (Marx'04)

- extends *CoreXPath* with a "while" operator $(\text{axis} :: l[\text{test}])^+$
- CXPath $= FO_2$
- query evaluation of a path expression p is in $O(|p|.|t|)$

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- linear-time back and forth translations into $\text{Comp}^{\text{nvs}}(CXPath)$
- $\longrightarrow$ captures FO_n
- $\longrightarrow$ query evaluation in time $O(|p|.|t|^2.(1 + |\text{Ans}(p, t)|))$

Remark: query evaluation of FO 0-ary queries is PSPACE-complete

n-ary XPath Extensions (II)

XPath 2.0

- extends XPath (1.0) with:

path intersection	$p_1 \cap p_2$
path complement	$\mathrm{compl}(p)$
variables	x
quantification	for x in p_1 return p_2

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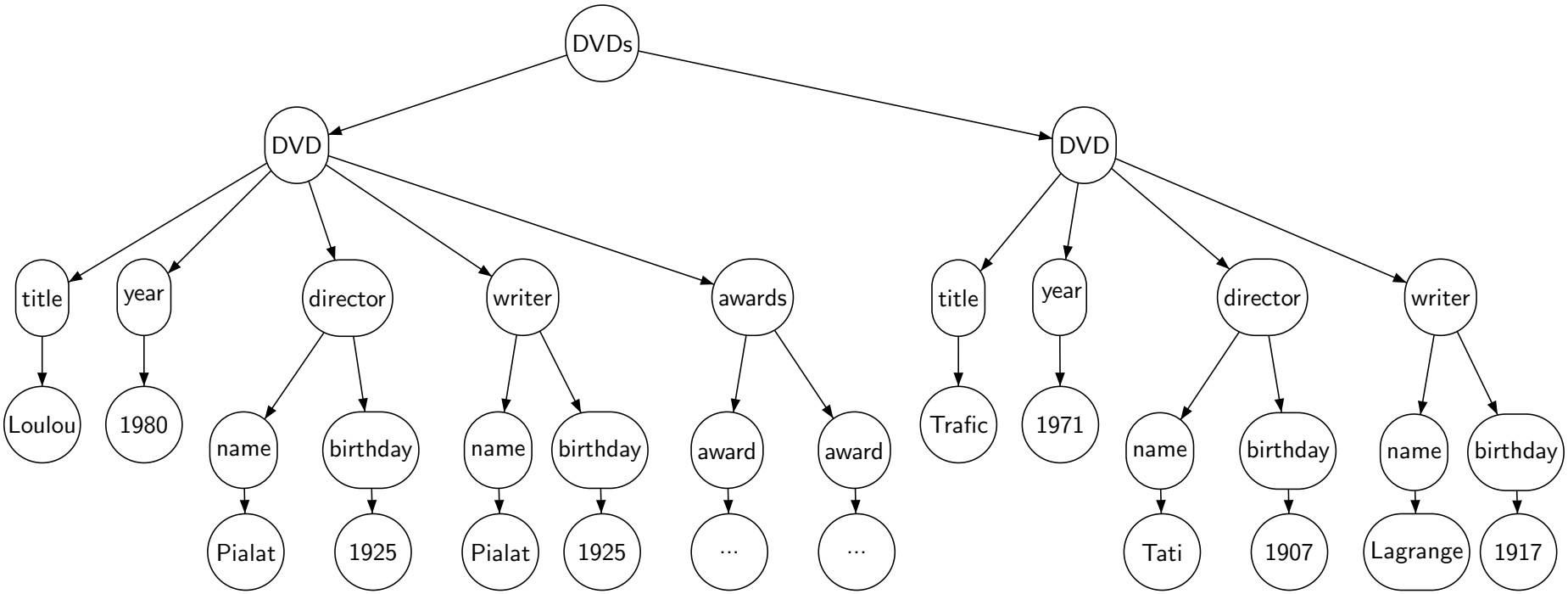
Application of the composition language

- to define a syntactic fragment of *CoreXPath2.0*
- FO_n-expressive
- with query evaluation problem in $O(|p|.|t|^3 + |p|.|t^2|.|\mathrm{Ans}(p,t)|)$

Outline

PART II: The Spatial Logic TQL

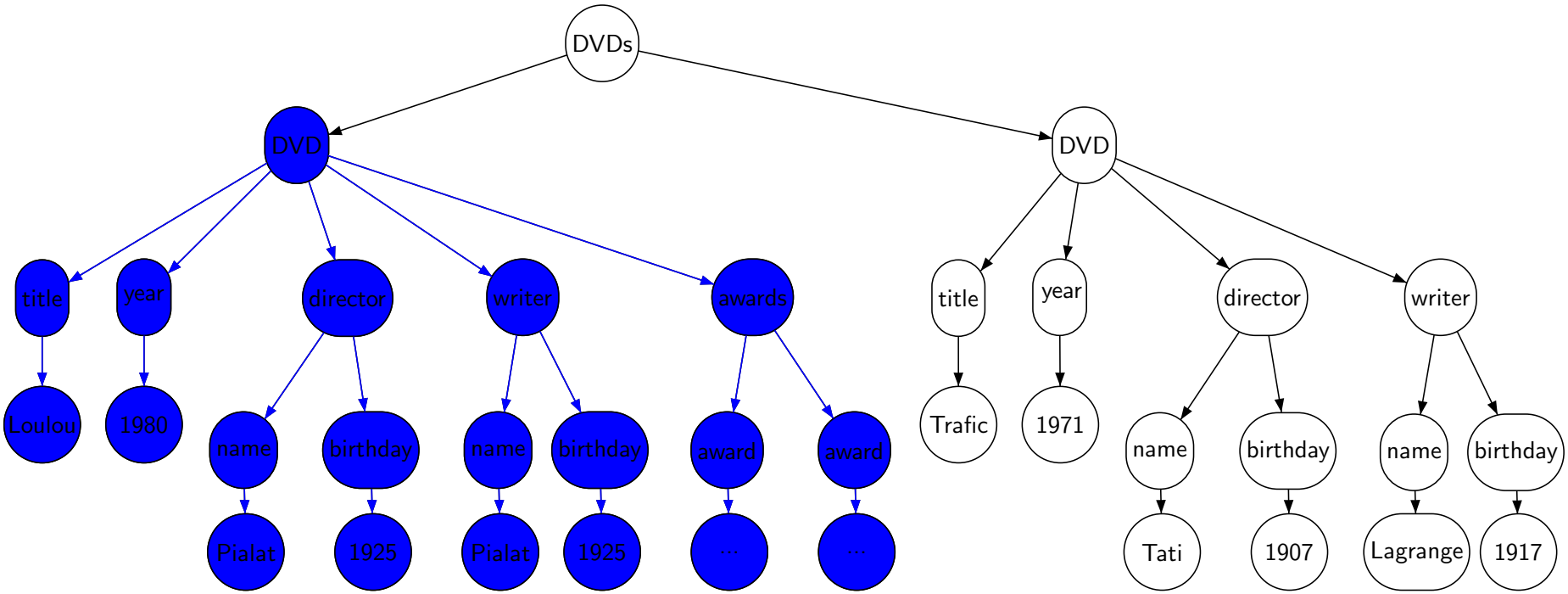
TQL Examples



Example (Check if there is an awarded movie)

$$DVDs[\_ \mid DVD[\_ \mid awards[\_]] \mid \_]$$

TQL Examples



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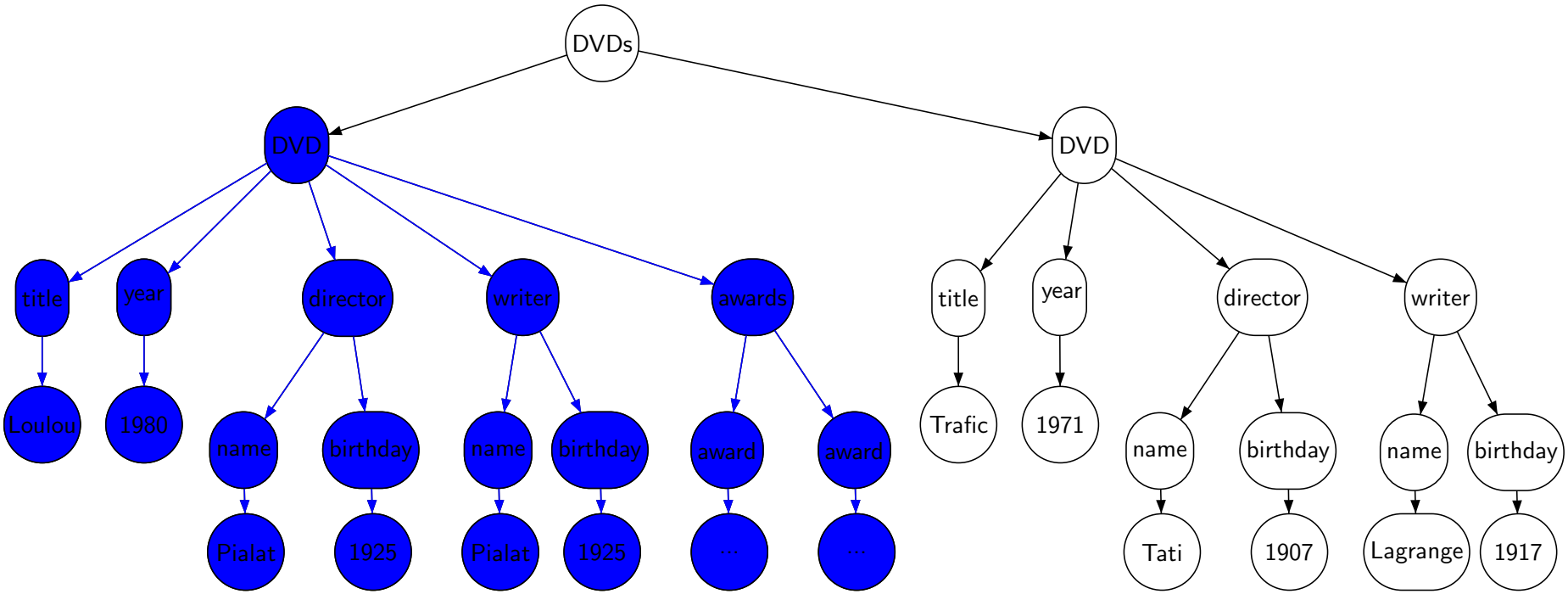


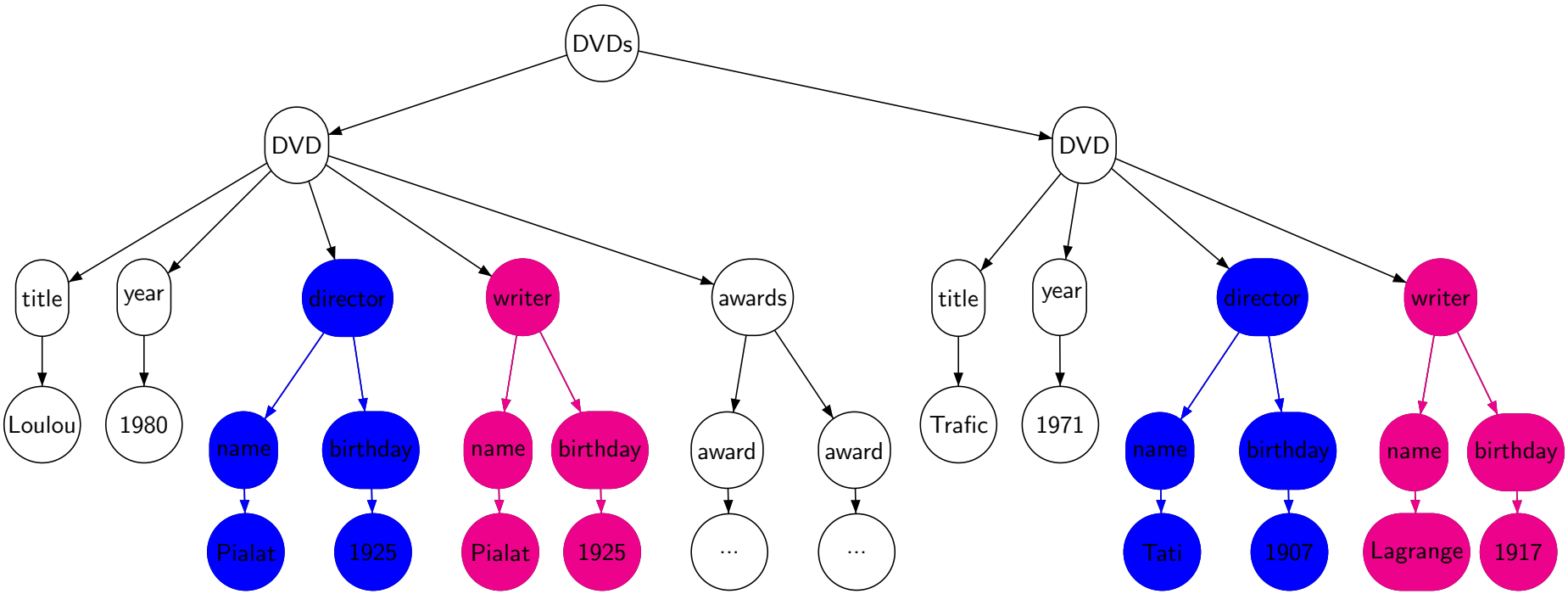
Example (Select all awarded movies)

$$\phi(X) \;=\; DVDs[\_ \mid X \wedge DVD[\_ \mid awards[\_]] \mid \_]$$

$\downarrow$

tree variable

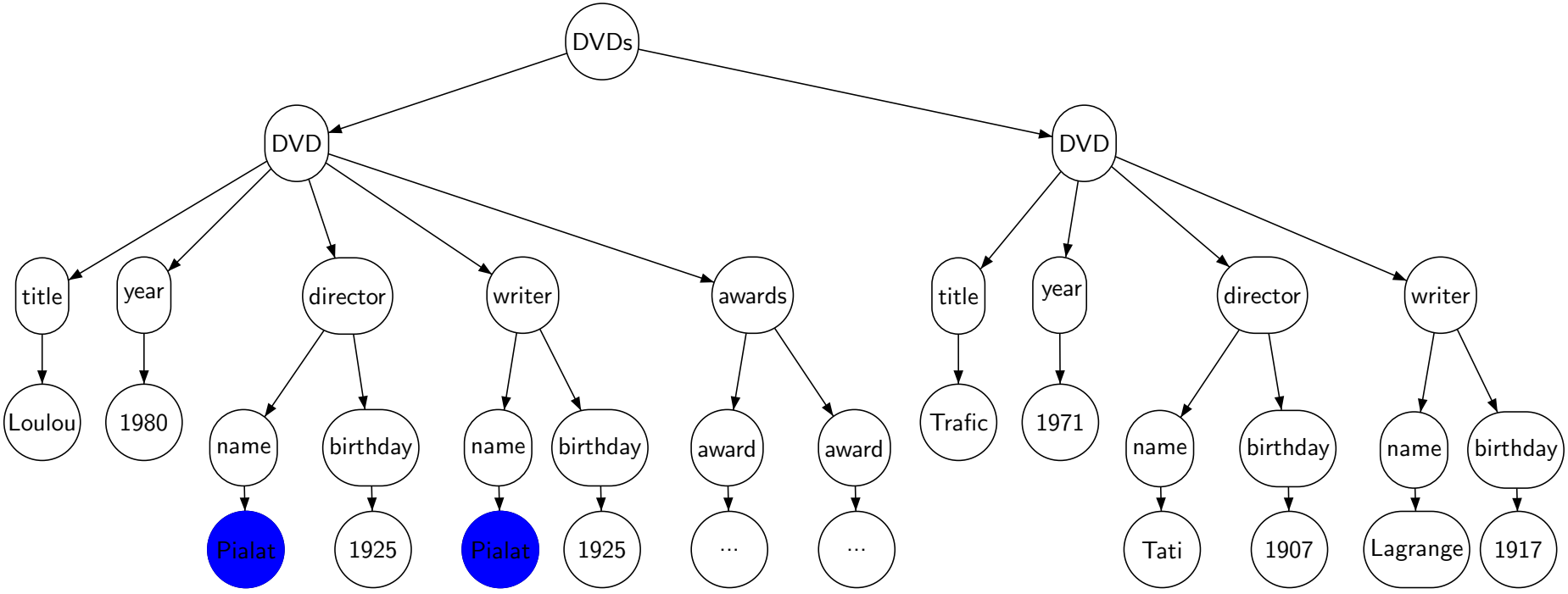
TQL Examples



Example (Select all pairs of (*director*, *writer*))

$$\phi(X, Y) \;=\; DVDs[\_ \mid DVD[\_|year[\_]|X|Y|\_]|\_]$$

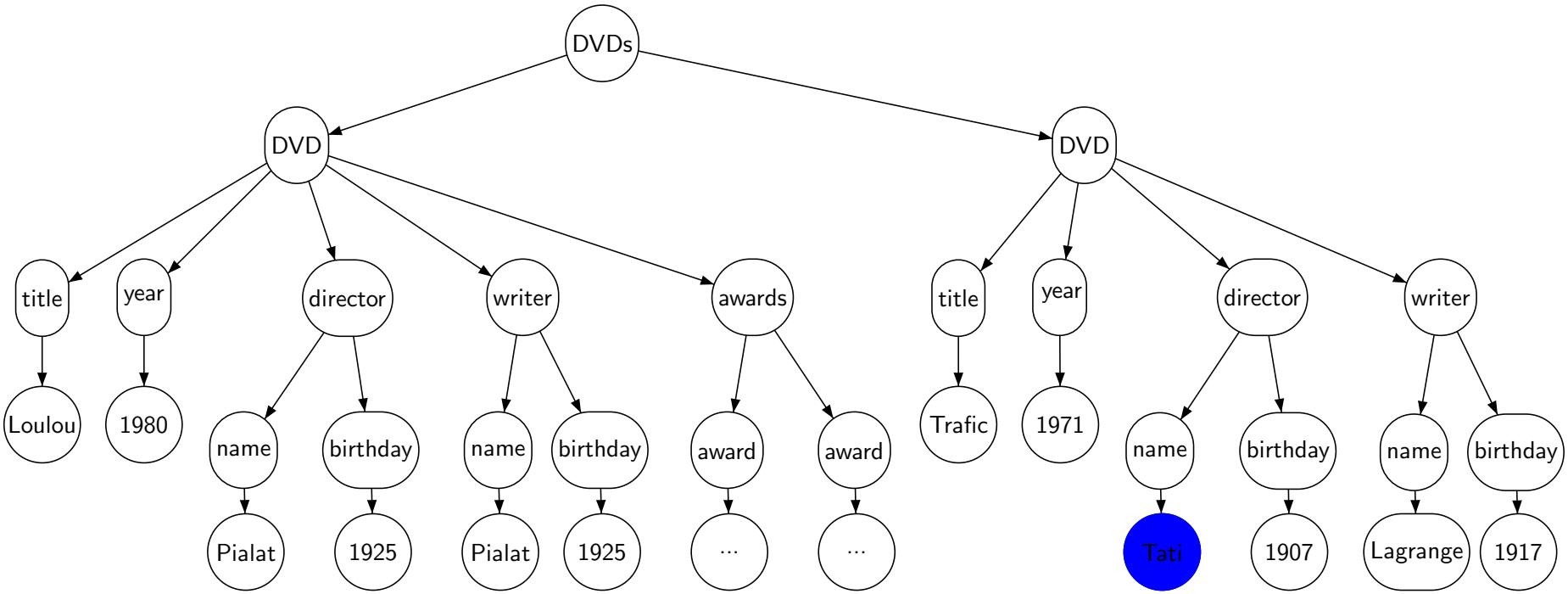
TQL Examples



Example (Select all names of persons who are both director and writer)

$$\phi(X) \;=\; DVDs[\_ \mid DVD[\_|director[name[X]|\_]|writer[name[X]|\_]|\_]|\_]$$

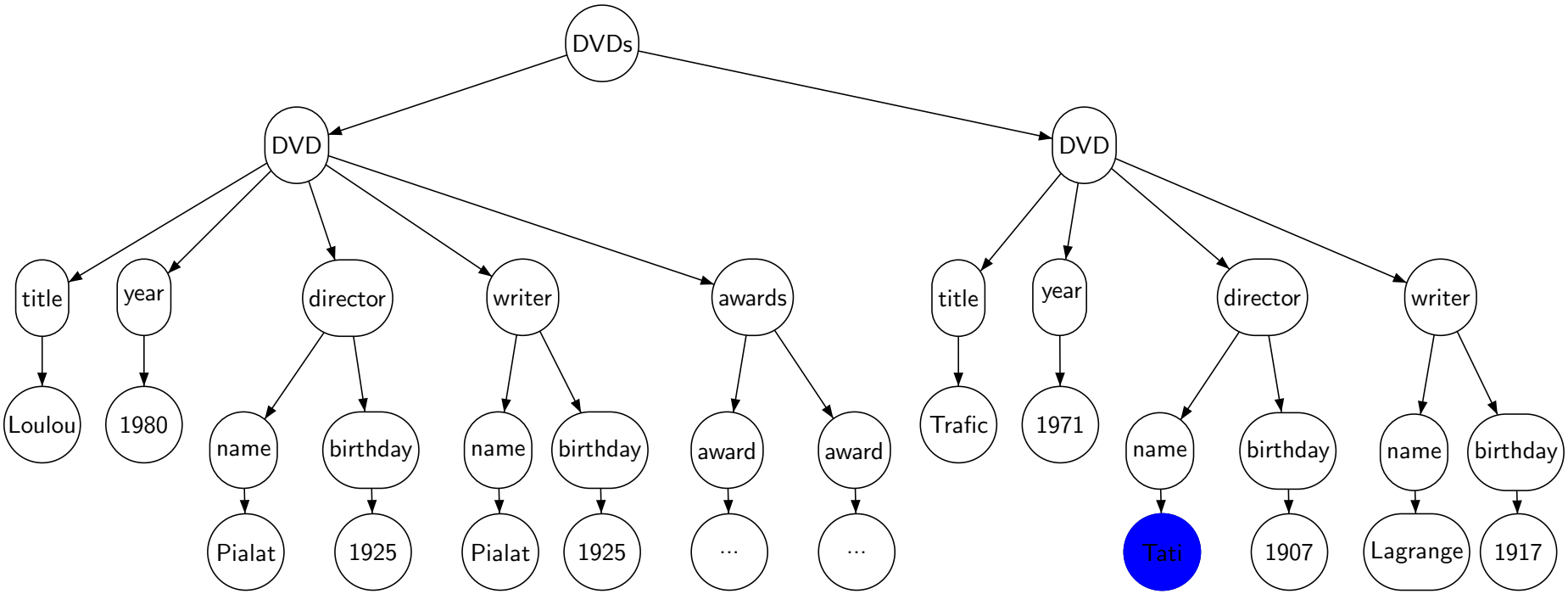
TQL Examples



Example (Select all director names who is not a writer)

$$\phi(X) \;=\; DVDs[\_ \mid DVD[\_|director[name[X]|\_]|writer[name[\neg X]|\_]|\_]|\_]$$

TQL Examples



Tree (dis)equality tests

- main difficulty of TQL satisfiability problem
- incomparable to FO fragments with data-value comparison (BojanczykDMSS06)

Hedge Algebra H_Λ

- Λ: countable set of labels
- hedge = ordered sequence of unranked trees

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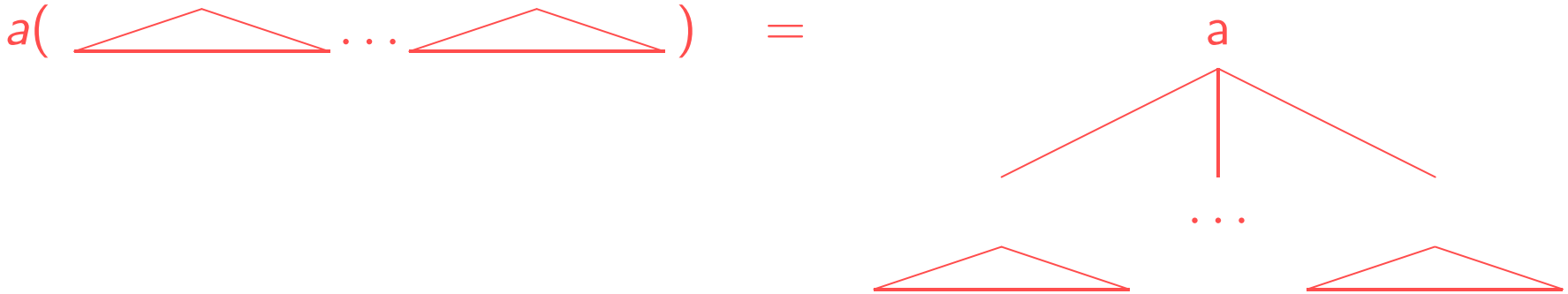

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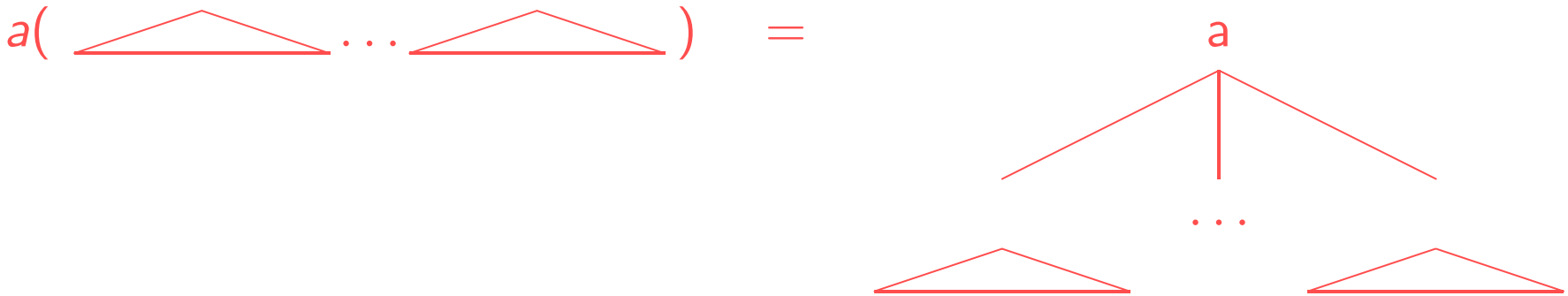

- binary symbol $|$

TQL: Syntax and Semantics

empty hedge	0		
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concatenation	$\phi \vert \phi'$		

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concatenation	$[\![\phi\|\phi']\!]_{\rho,\delta}$	$=$	$\{h\|h' \mid h \in [\![\phi]\!]_{\rho,\delta}, h' \in [\![\phi']\!]_{\rho,\delta}\}$

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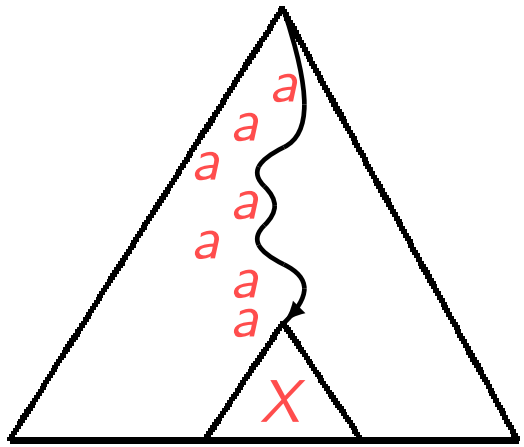
tree variable	$[\![X]\!]_{\rho,\delta}$	$=$	$\{\rho(X)\}$
recursion variable	$[\![\xi]\!]_{\rho,\delta}$	$=$	$\delta(\xi)$
least fixpoint	$[\![\mu\xi.\phi]\!]_{\rho,\delta}$	$=$	$\bigcap\{S \subseteq H_\Lambda \mid [\![\phi]\!]_{\rho,\delta[\xi \mapsto S]} \subseteq S\}$

Examples with fixpoint

Example (Select all subtrees reachable from the root by following an 'a'-path)

$$\phi(X) \;=\; \mu\xi.(\; a[\_|\xi|\_] \;\vee X)$$



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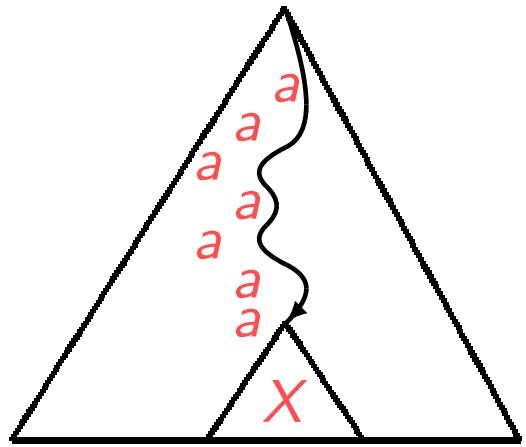

Example ($a^n b^n$)

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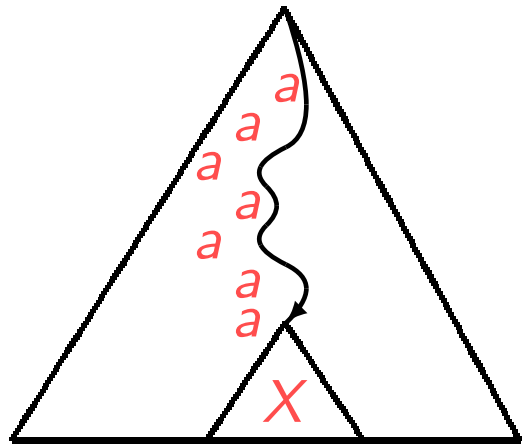

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- vertical recursion $\longrightarrow$ regular tree languages
- horizontal recursion $\longrightarrow$ context-free word languages

A Decidable Fragment: Bounded TQL

Satisfiability problem

Input: TQL formula ϕ **Output:** $\exists h \exists \rho \exists \delta,\ h \in [\![\phi]\!]_{\rho,\delta}$?

A Decidable Fragment: Bounded TQL

Proposition

Satisfiability of TQL formulas is undecidable.

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Bounded TQL

- recursion variables are **guarded** by some $\alpha[.]$

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$$\mu\xi.(\ a[\_|\xi|\_]\ \vee X) \quad \rightarrow \quad \textbf{guarded}$$
$$\mu\xi.(a[0]|\xi|b[0]\ \vee\ 0) \quad \rightarrow \quad \textbf{not guarded}$$

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$$[\![\phi^*]\!]_\rho \;=\; 0 \;\cup\; \bigcup_{i>0} \underbrace{[\![\phi]\!]_\rho | \cdots | [\![\phi]\!]_\rho}_{i \text{ times}}$$

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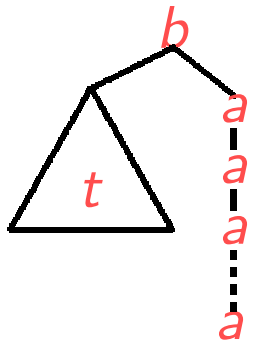
$b[X \mid \mu\xi.(\neg X \wedge a[\xi] \ \vee \ 0)] \rightarrow$ **not bounded**



$(\neg X)^* \mid X \mid (\neg X)^* \rightarrow$ **bounded**

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- restriction negative variables: only a **bounded number of disequality tests** along the paths
- no **negative** occurences of Kleene star and $|$

Expressiveness and Satisfiability of Bounded TQL

Theorem

1. *Bounded TQL sentences capture MSO.*
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The proof is by reduction to emptiness of bounded TAGEDs.

Bottom-up Tree Automata for Binary Trees

- Σ: finite alphabet
- Q: set of states
- $F \subseteq Q$: set of final states
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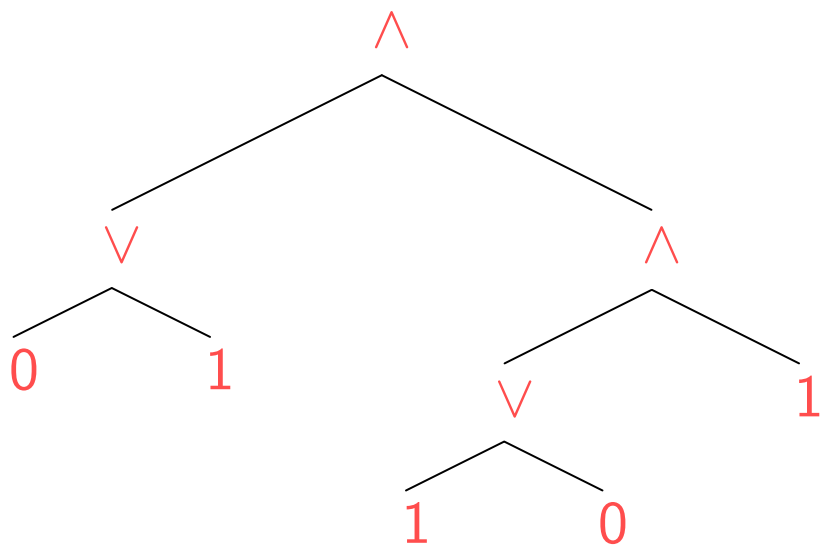

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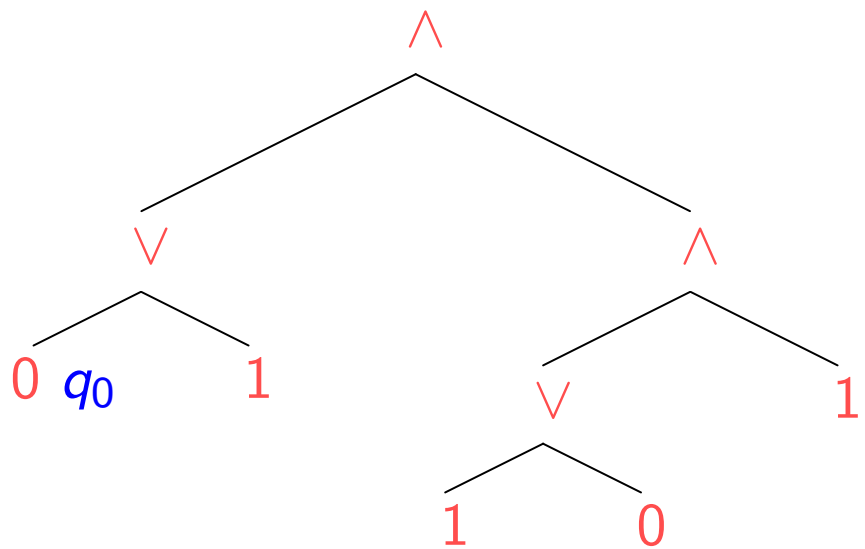

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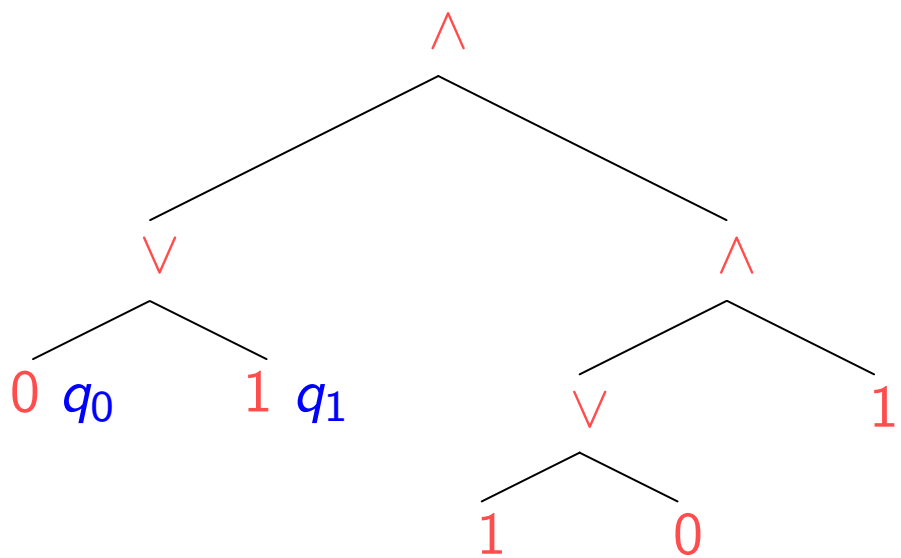

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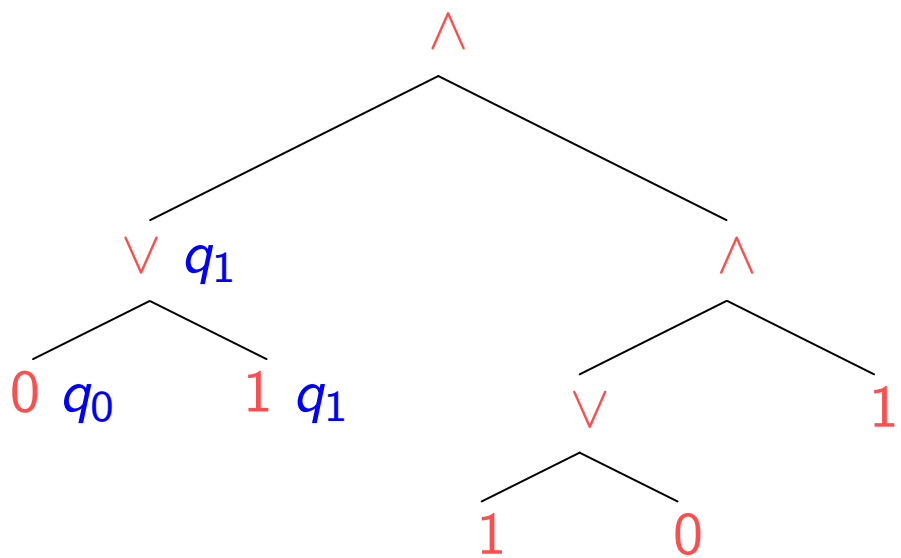

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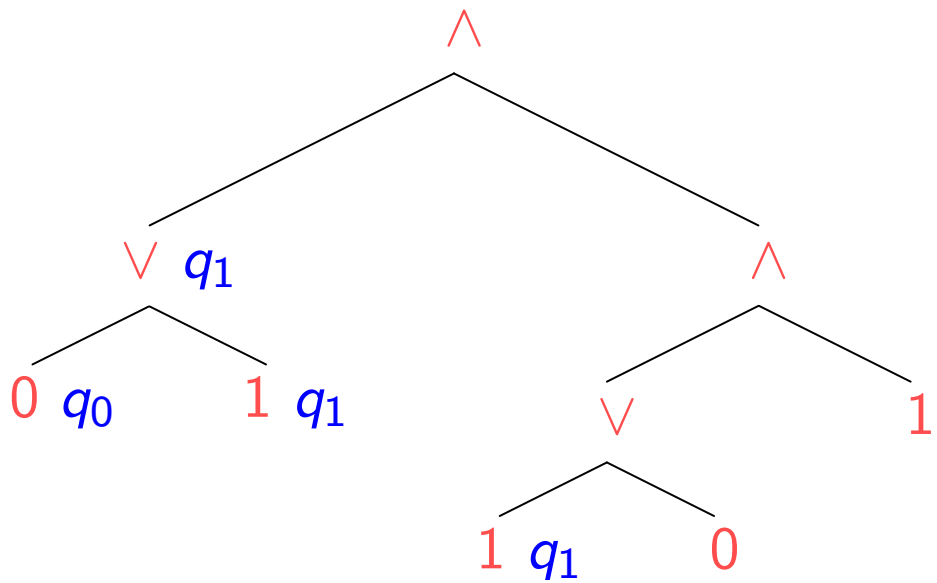

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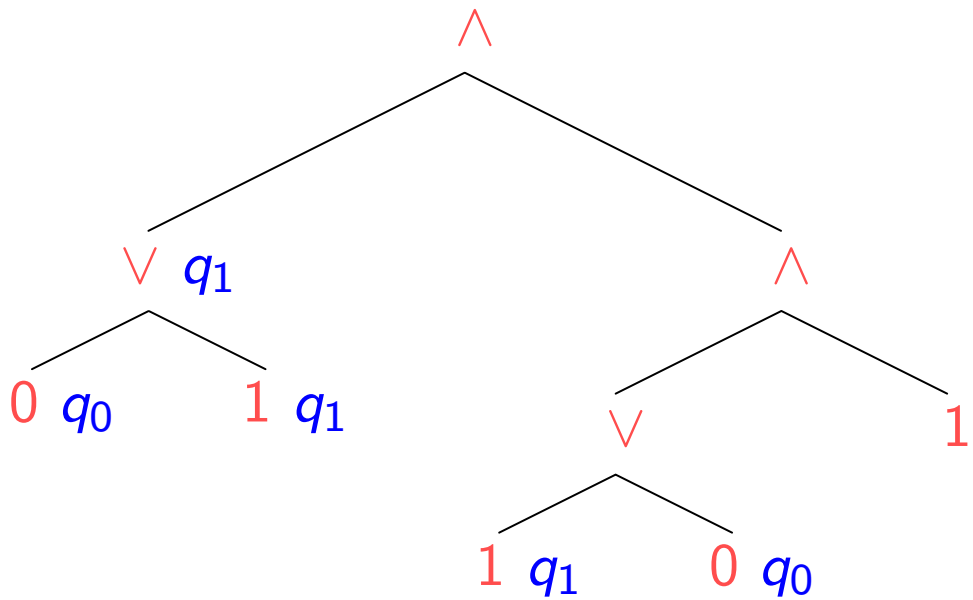

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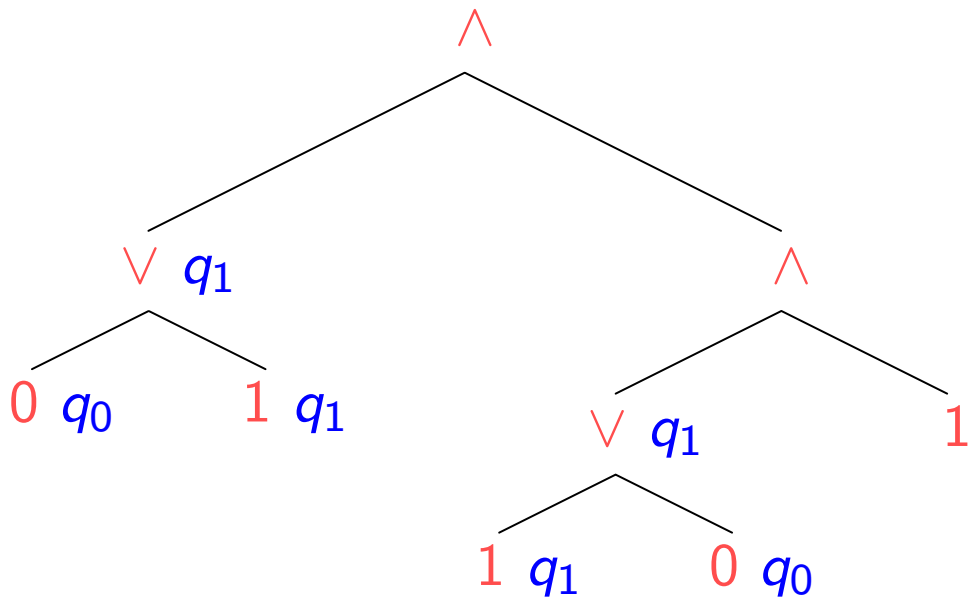

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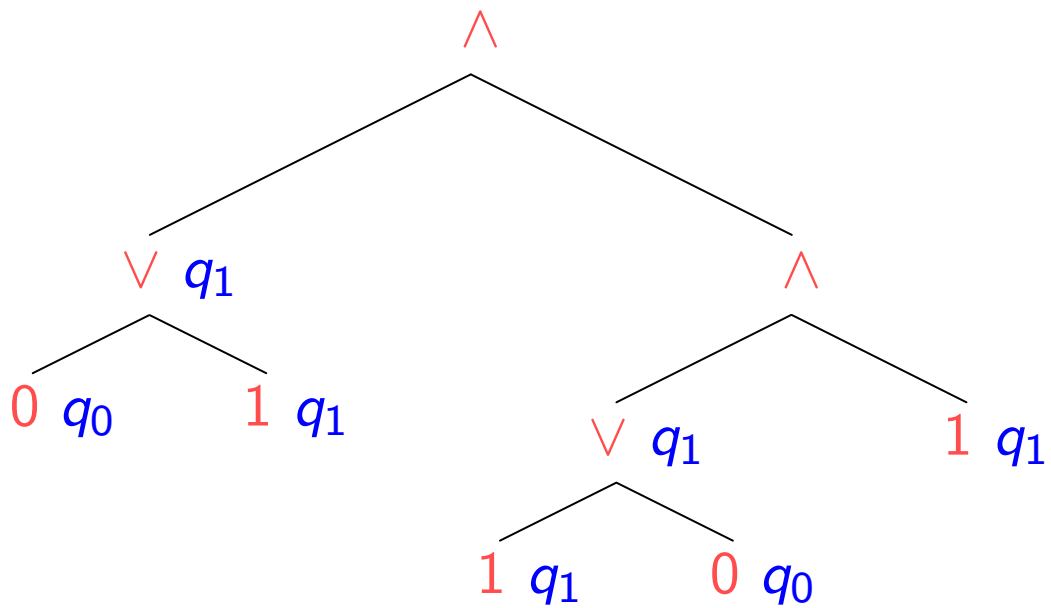

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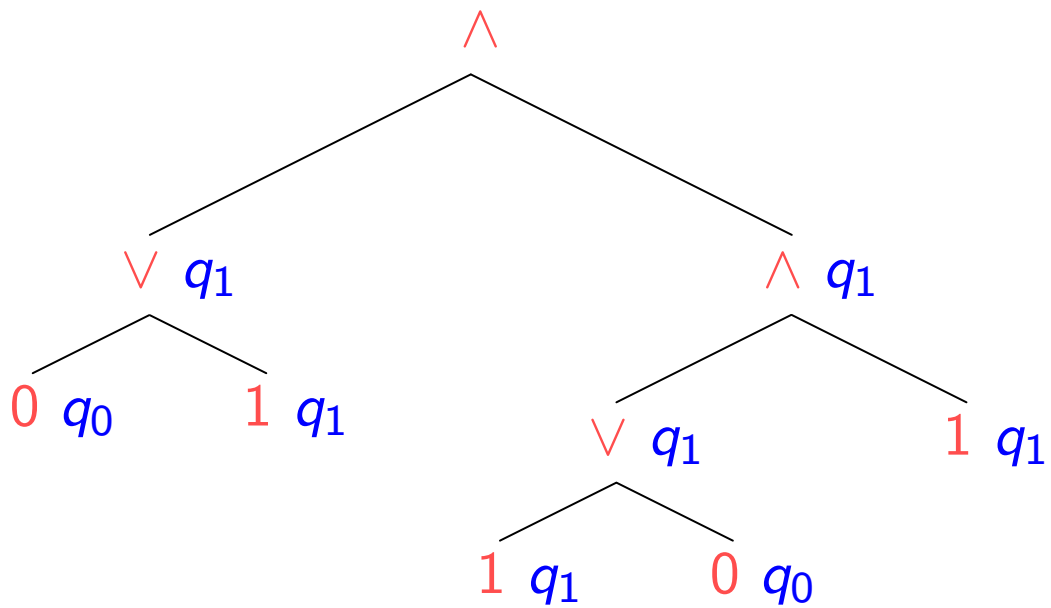

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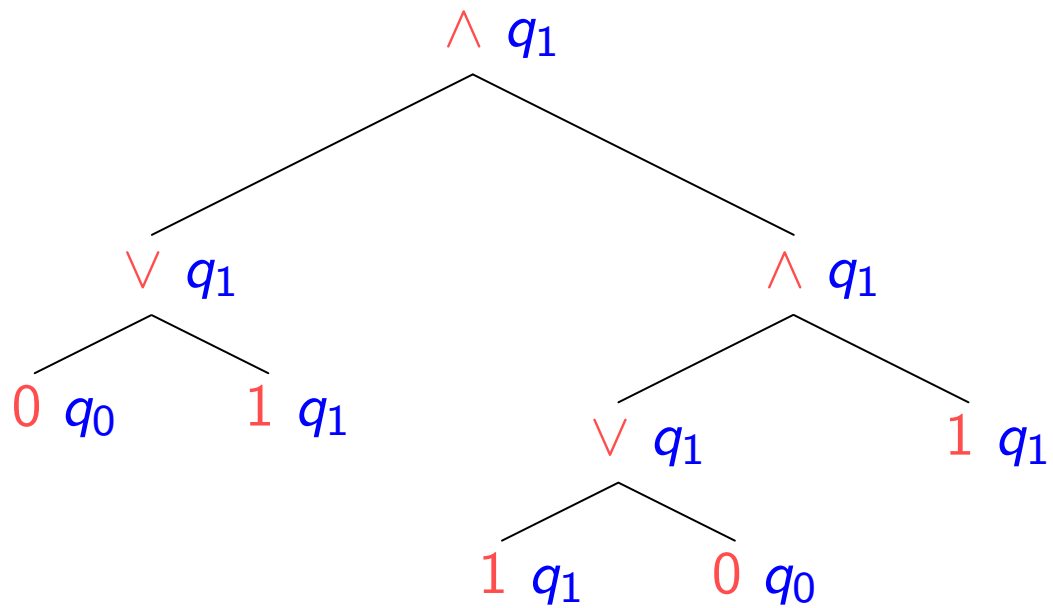

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Tree Automata with Global Equalities and Disequalities

A **tree automaton A with global equalities and disequalities** (TAGED) is given by:

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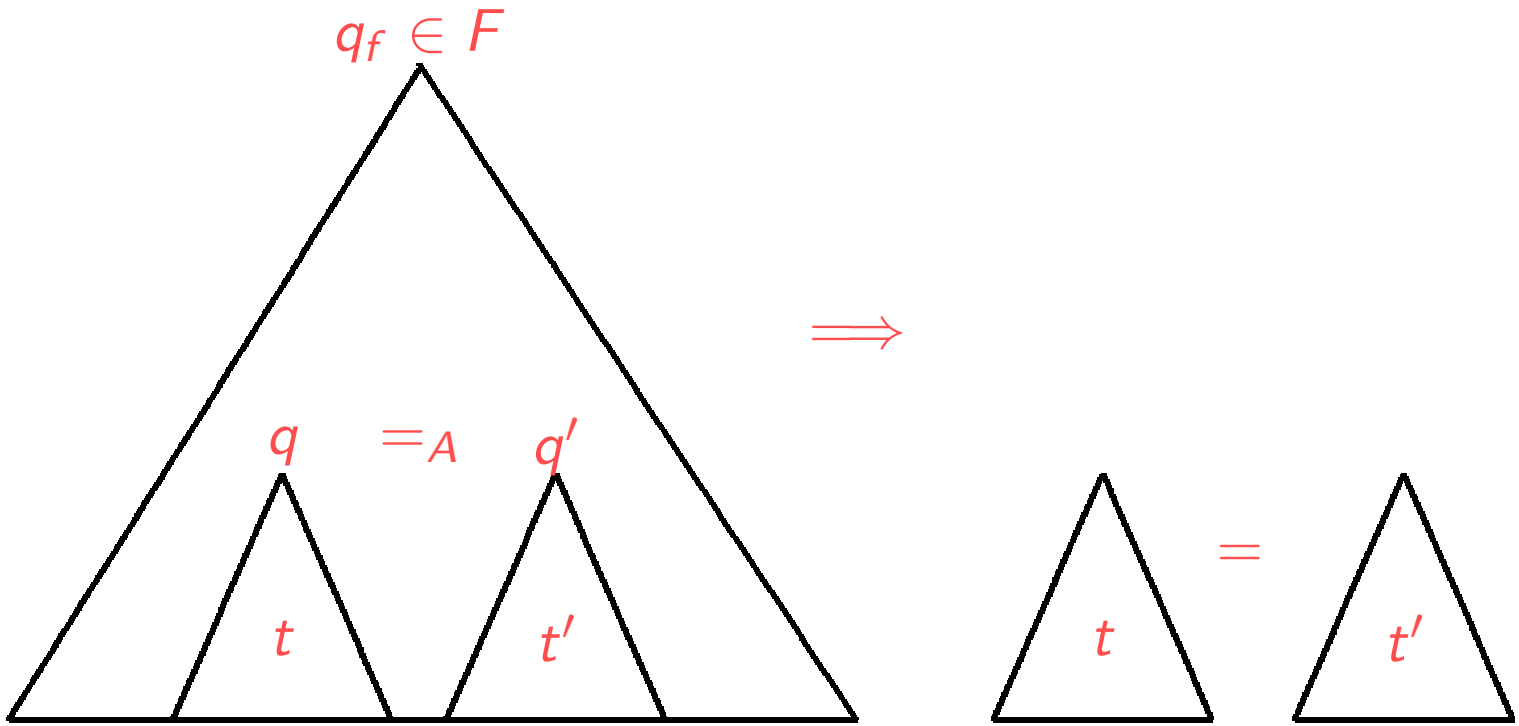
$=_A \subseteq Q^2$ reflexive and symmetric relation on a **subset** of Q

$\neq_A \subseteq Q^2$ irreflexive and symmetric relation

Successful Runs



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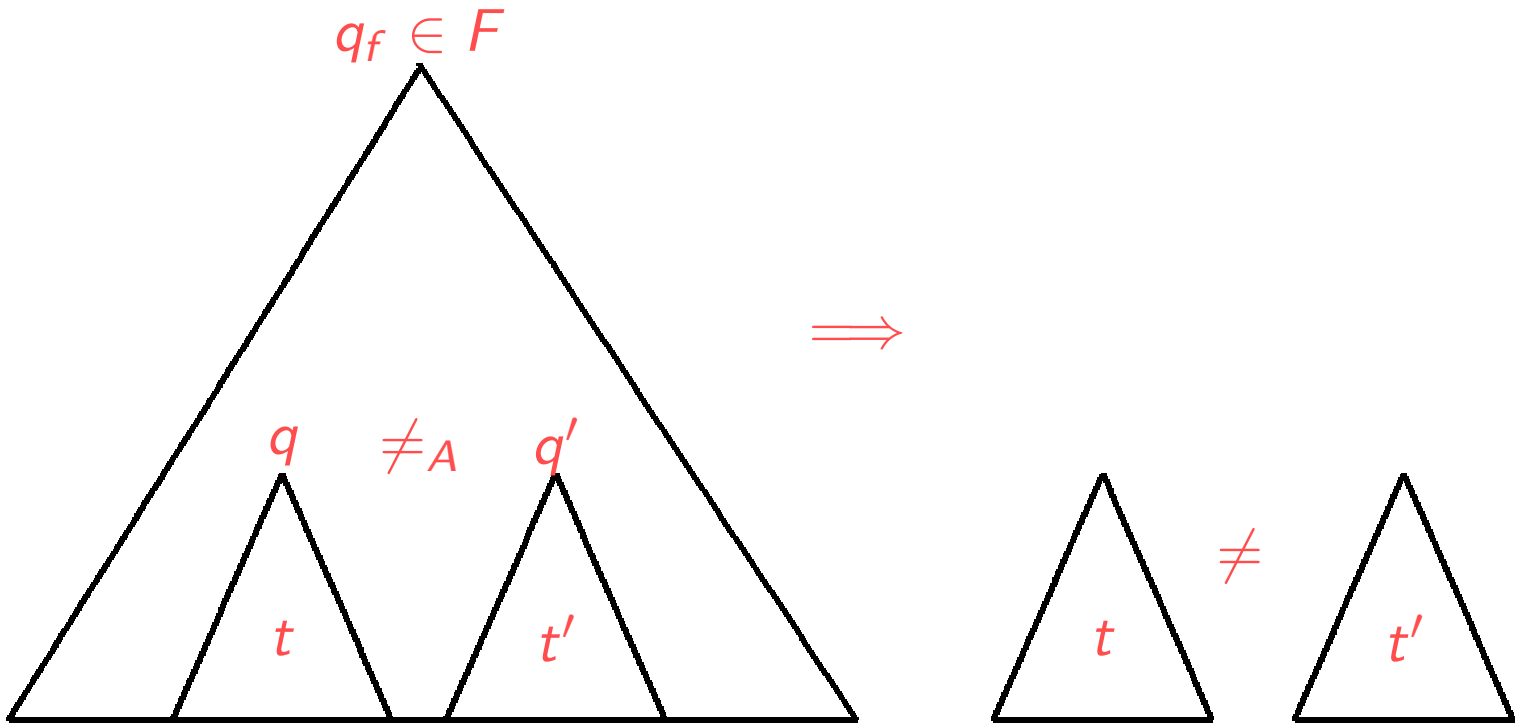

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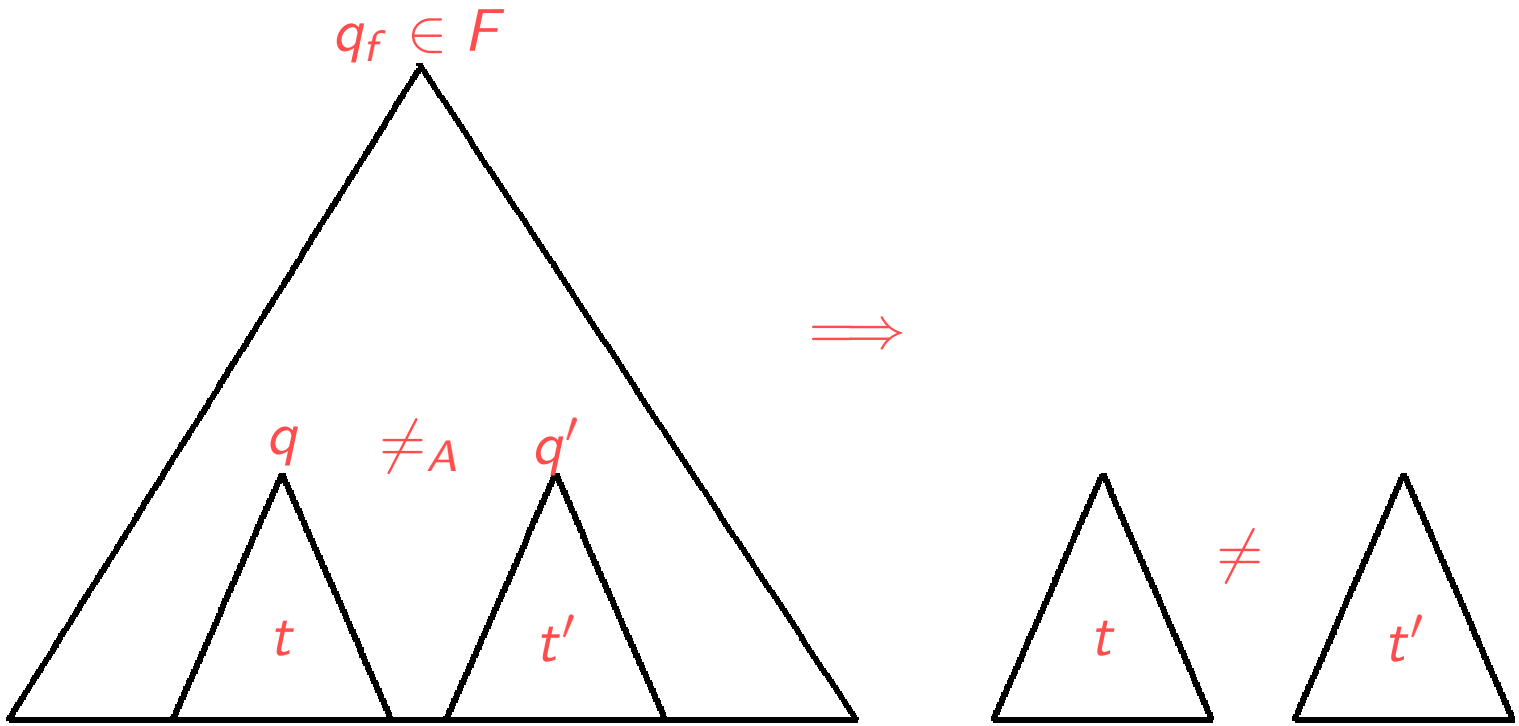
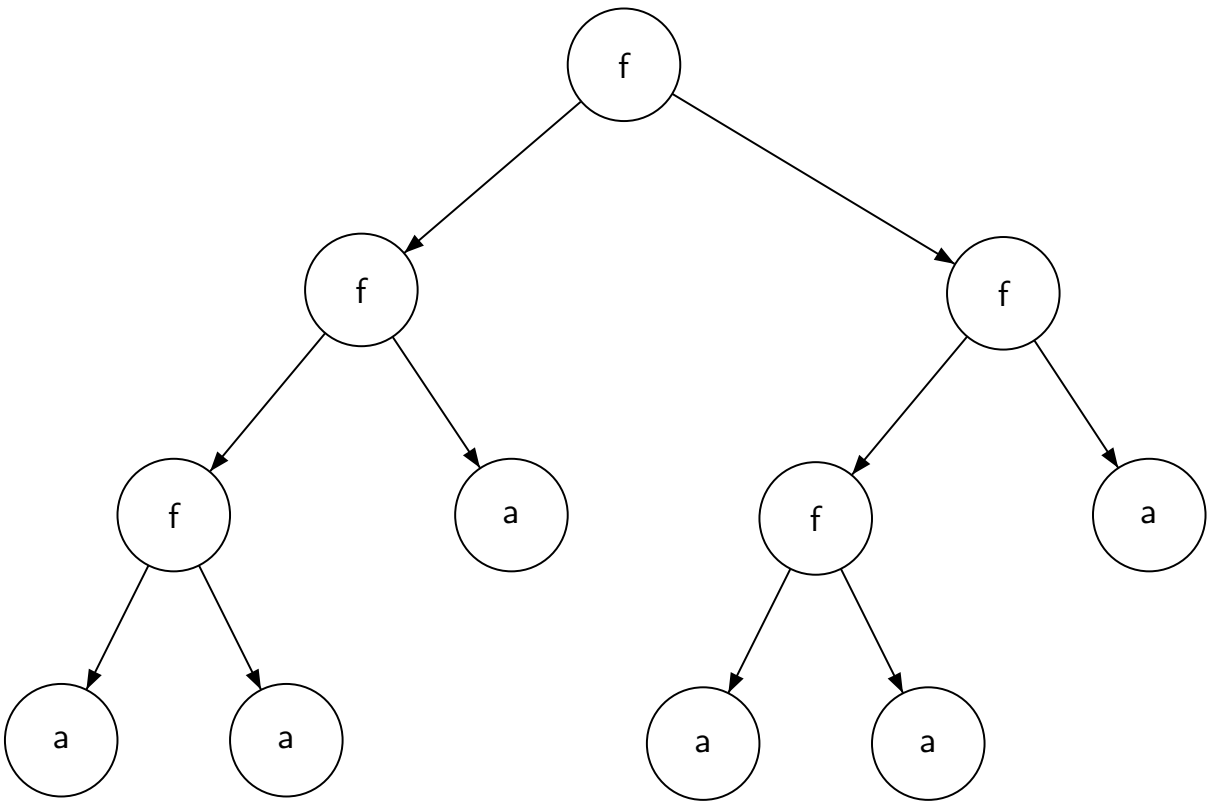


- equalities and disequalities can be tested arbitrarily faraway
- different from usual **Automata with Constraints** where tests are **local** (BogaertT92, DauchetCC95, KariantoL07)

Example: $\{f(t,t) \mid t \in T_\Sigma\}$



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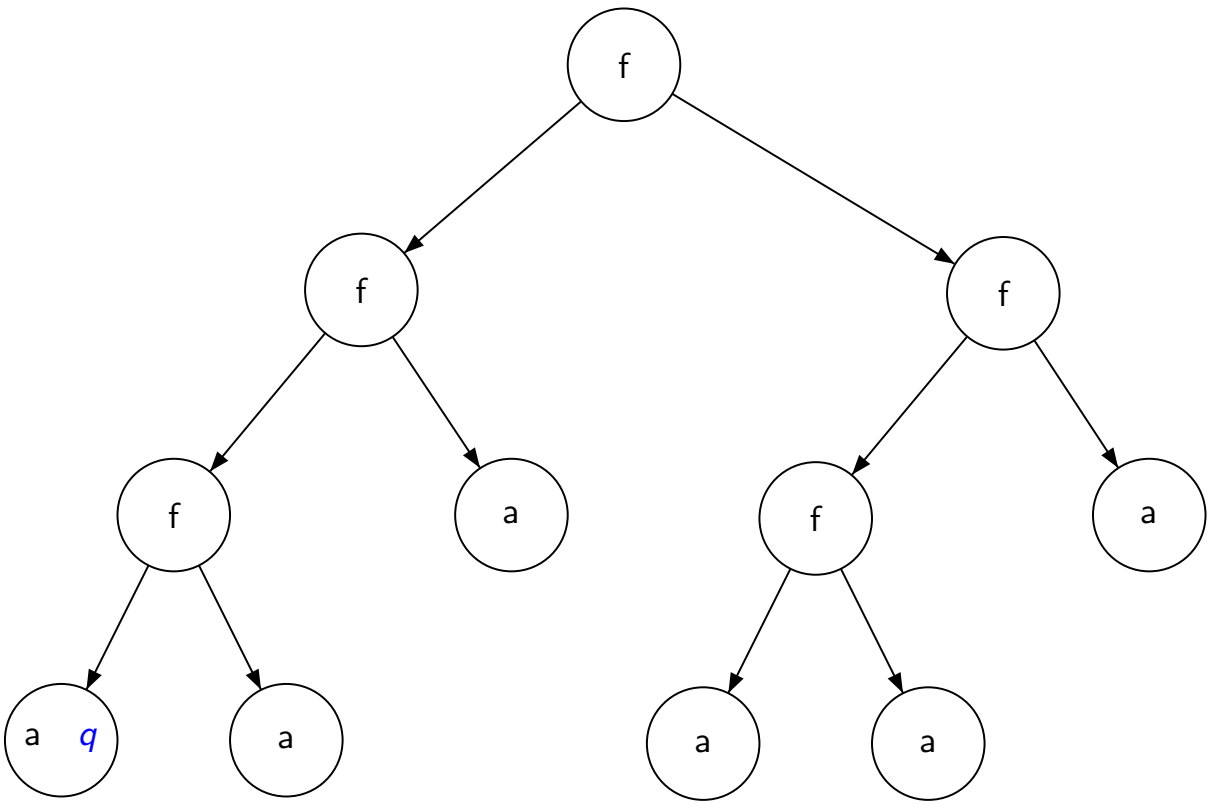

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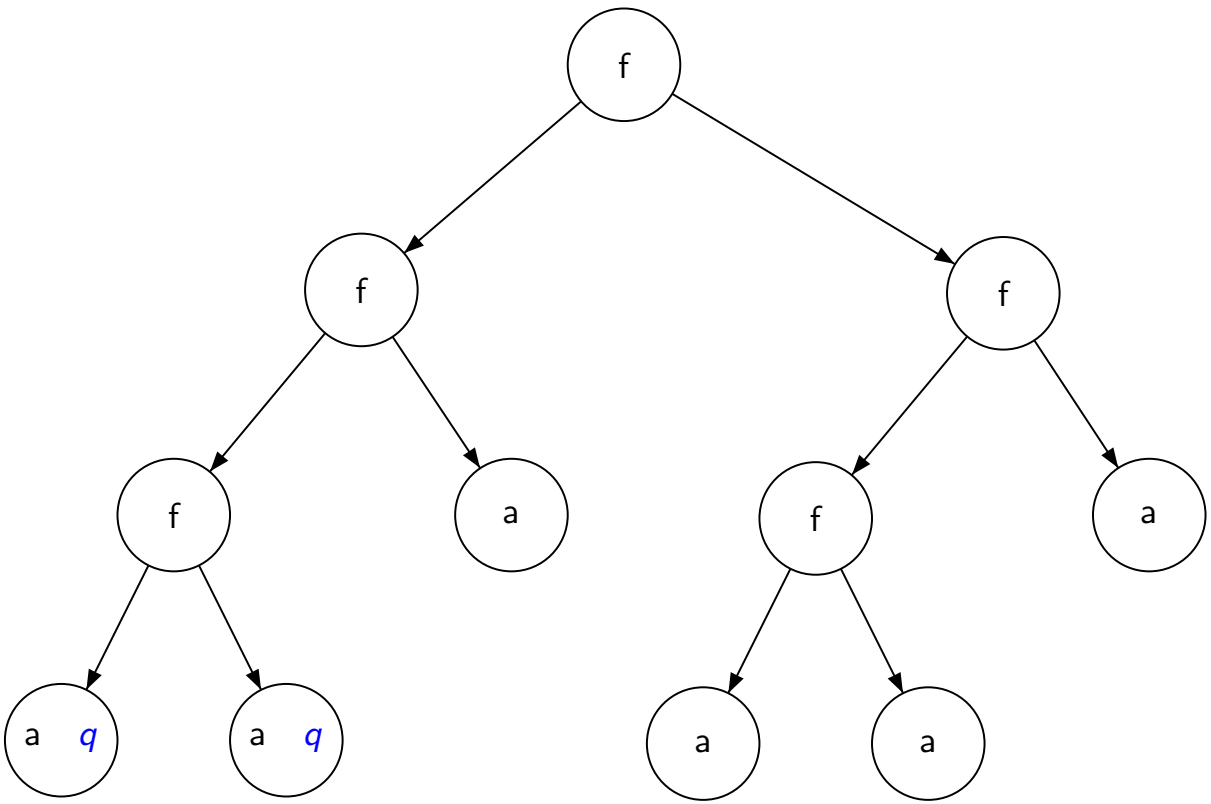

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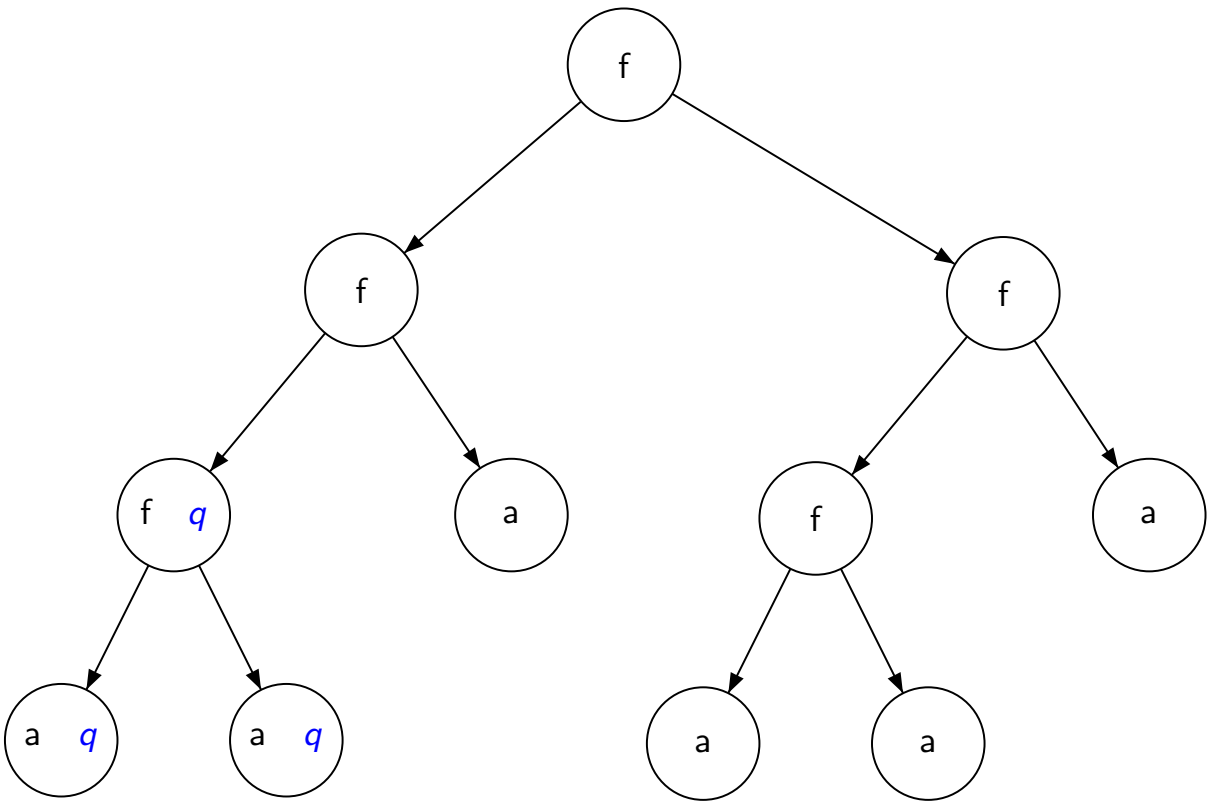

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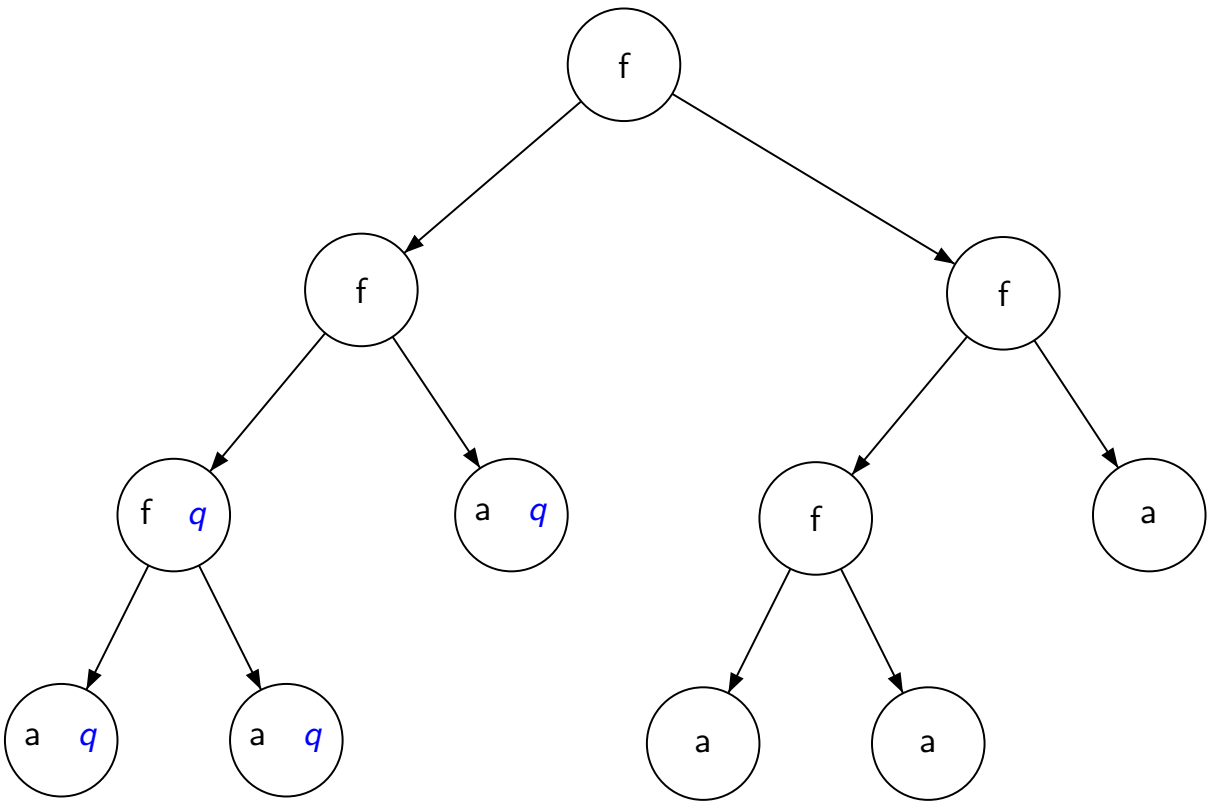

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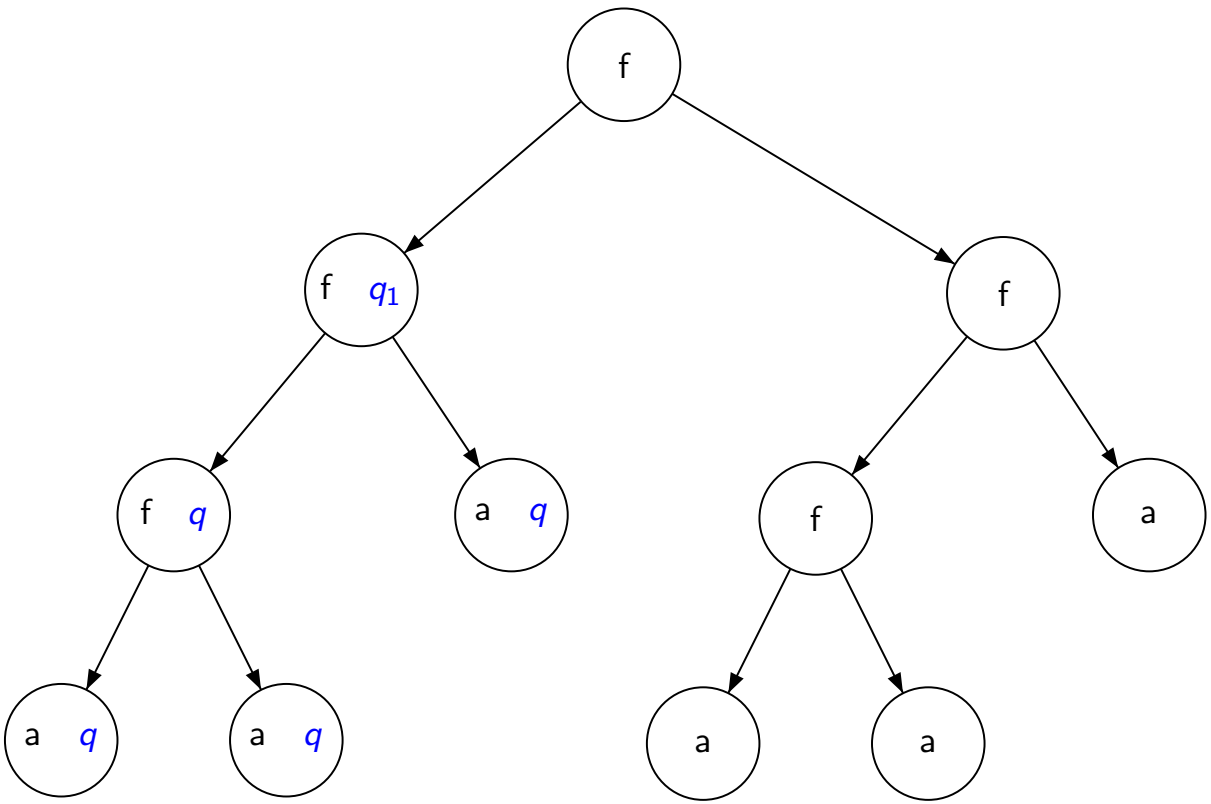

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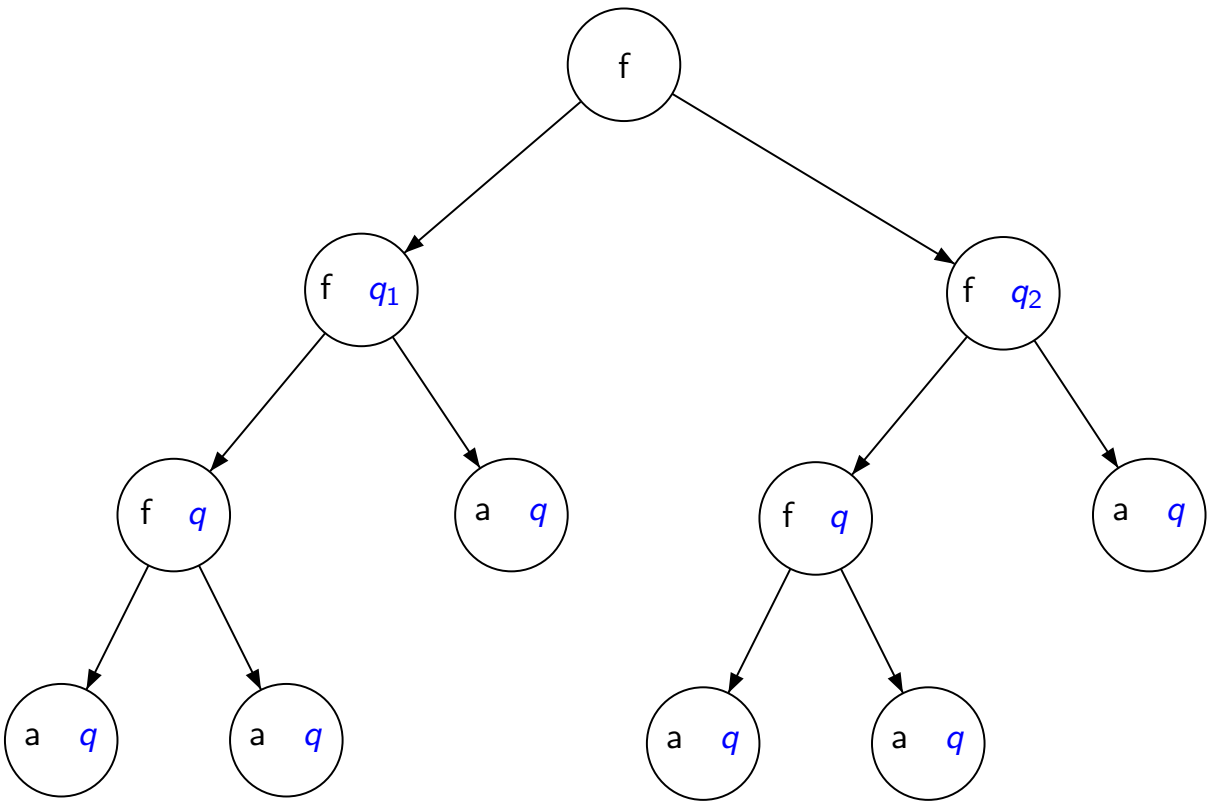

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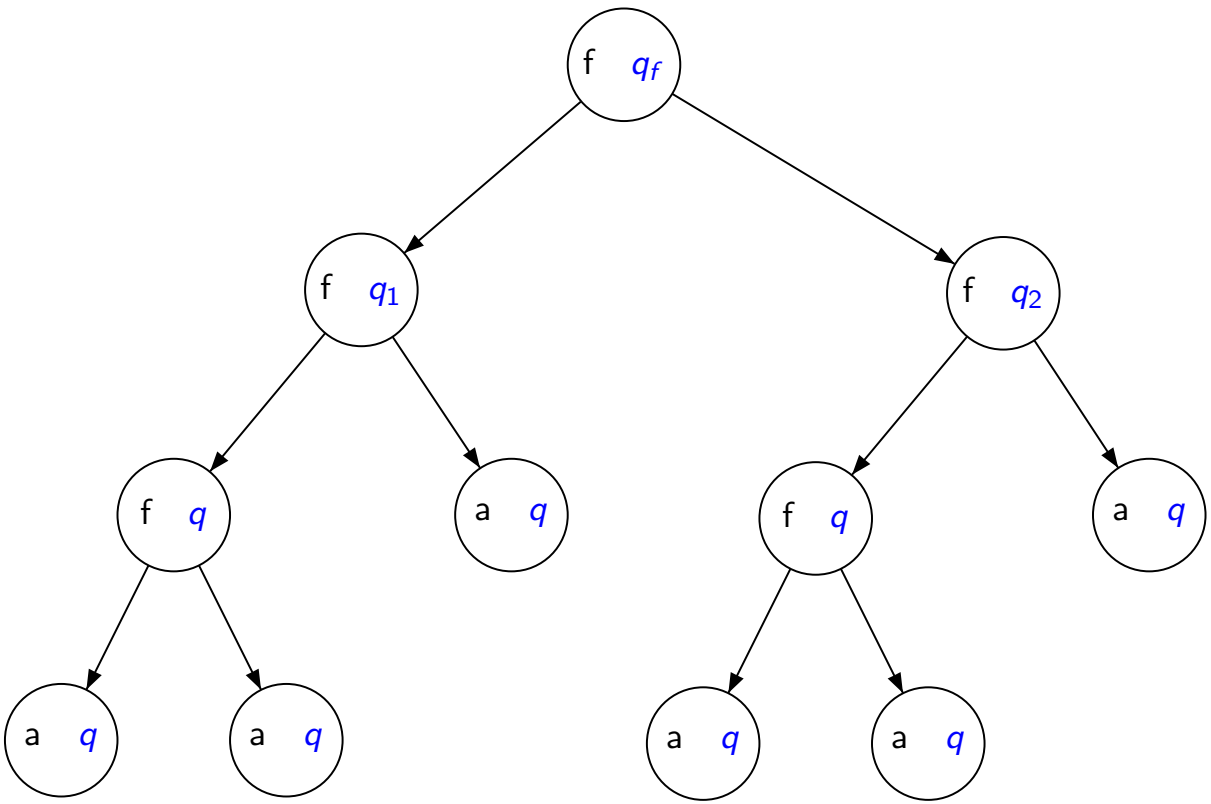

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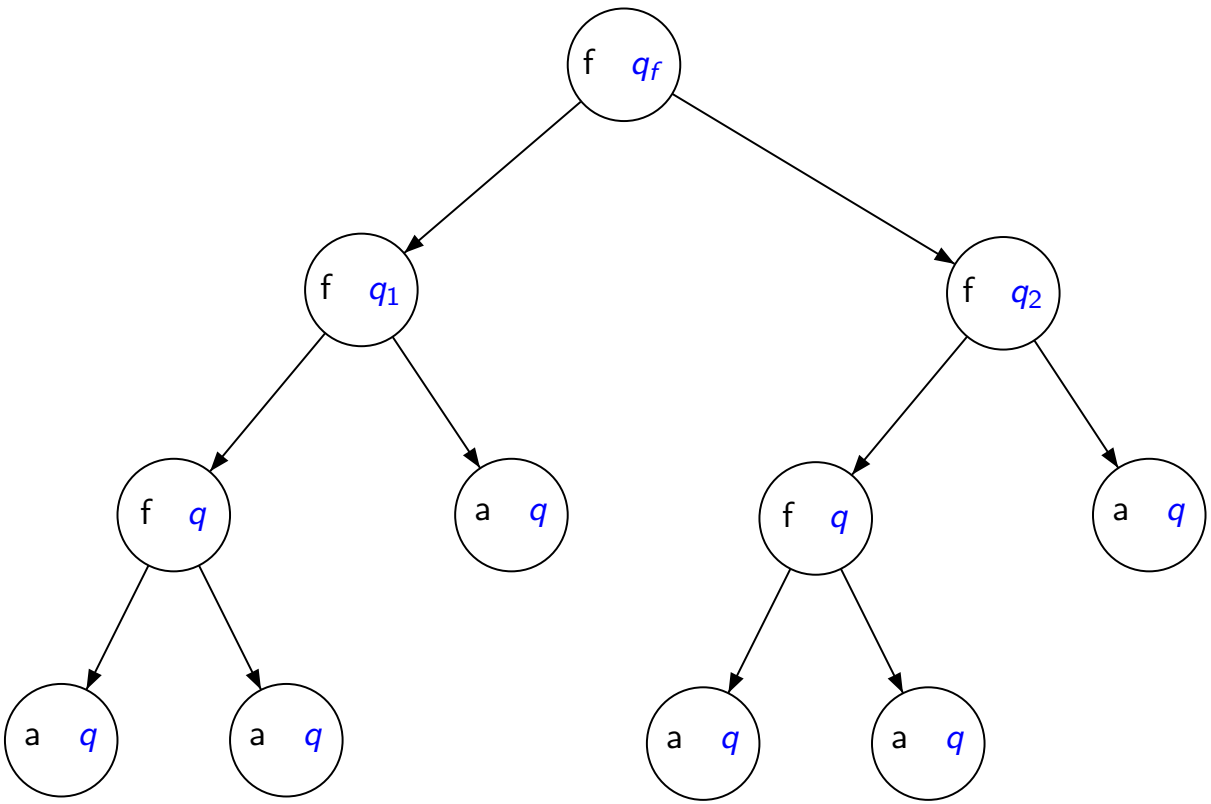

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Some properties of TAGEDs

Proposition

- *TAGED-recognizable languages are closed by union and intersection, but not by complement;*
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Emptiness Problem

Input: a TAGED A **Output**: $L(A) \neq \varnothing$?

Theorem

Emptiness is:

- *EXPTIME-complete for positive TAGED* ($\neq_A = \varnothing$)
- *decidable in NEXPTIME for negative TAGED* ($=_A = \varnothing$)
- *decidable in linear-time for positive TAGED such that* $=_A \subseteq id_Q$

Bounded TAGEDs

Definition

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By using a pumping technique one can show that:

Theorem

Emptiness of bounded TAGEDs is decidable in 2NEXPTIME.

Emptiness of bounded TAGED: Sketch of Proof

Idea: if the automaton accepts a tree t then t is not too big (its size is bounded in $|A|$).

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Termination

If the automaton accepts a tree, then it accepts a **repairable** tree satisfying the contraints from $=_A$ whose size is **exponential** in $|A|$ and k.

TQL to TAGED

TAGED for hedges over an infinite alphabet

- extends hedge automata (Murata'99) with global tests;
- transitititions $\alpha(L) \longrightarrow q$ where $L \subseteq Q^*$;
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Bounded TQL $\rightarrow$ Bounded TAGED

- new construction;
- **two difficulties**: variables and hedge operations;
- states: sets of subformulas $\alpha[\phi],\ X, \neg X$;
- variables are added non-deterministically to the states;
- hedge operations are interpreted as operations on state languages
- $\{\ldots, X, \ldots\} =_A \{\ldots, X, \ldots\}$
- $\{\ldots, X, \ldots\} \neq_A \{\ldots, \neg X, \ldots\}$

Outline

Conclusion

Summary of the contributions

Query composition (FNTT, PODS'07)

- extends the navigational XPath paradigm to n-ary queries
- simple acyclicity notion
- FO-complete and polynomial n-ary XPath languages

Summary of the contributions

Query composition (FNTT, PODS'07)

- extends the navigational XPath paradigm to n-ary queries
- simple acyclicity notion
- FO-complete and polynomial n-ary XPath languages

TQL (FTT,CSL'07)

- tree pattern language for hedges
- decidable fragment with tree variables
- by reduction to TAGED (FTT, DLT'08)
- new automaton construction

Some Perspectives

Query composition

- query answering algorithms specific to $Comp(\mathsf{ch}, \mathsf{ch}^*, \mathsf{lab}_a)$
- streaming (GauwinCNT08), enumeration (collaboration with O.Gauwin, A.Durand, ANR Enum)

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- lower bounds (TQL + TAGED)
- guarded fragment

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- ... or at least, a decidable fragment closed by negation
- query inclusion $\forall \overline{x}\ (\phi(\overline{x}) \rightarrow \psi(\overline{x}))$ iff **not** $\exists \overline{x},\ \phi(\overline{x}) \wedge \neg\psi(\overline{x})$.

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- emptiness of full TAGED
- application to security protocols (C.Vacher,F.Jacquemard, F.Klay)

Outline

Thank You