What is a Good Domain Description?

Evaluating & Revising Action Theories in Dynamic Logic

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Reasoning About Actions

Problem: describing domains by logical formulas

- Actions and their effects
- Executabilities of actions
- Inexecutabilities of actions
- Domain constraints

Example

- A turkey that walks is alive
- Teasing a turkey makes it walk
- It is always possible to tease a turkey
- A dead turkey remains dead after teasing it

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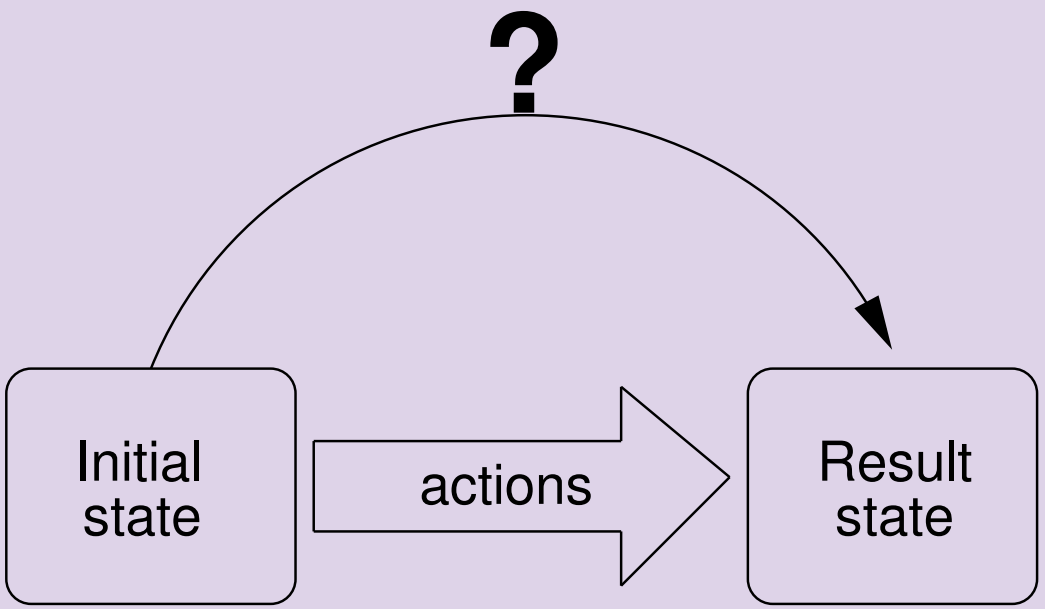
Reasoning About Actions

Goal: inference tasks

- Prediction
- Explanation
- Planning

Reasoning About Actions

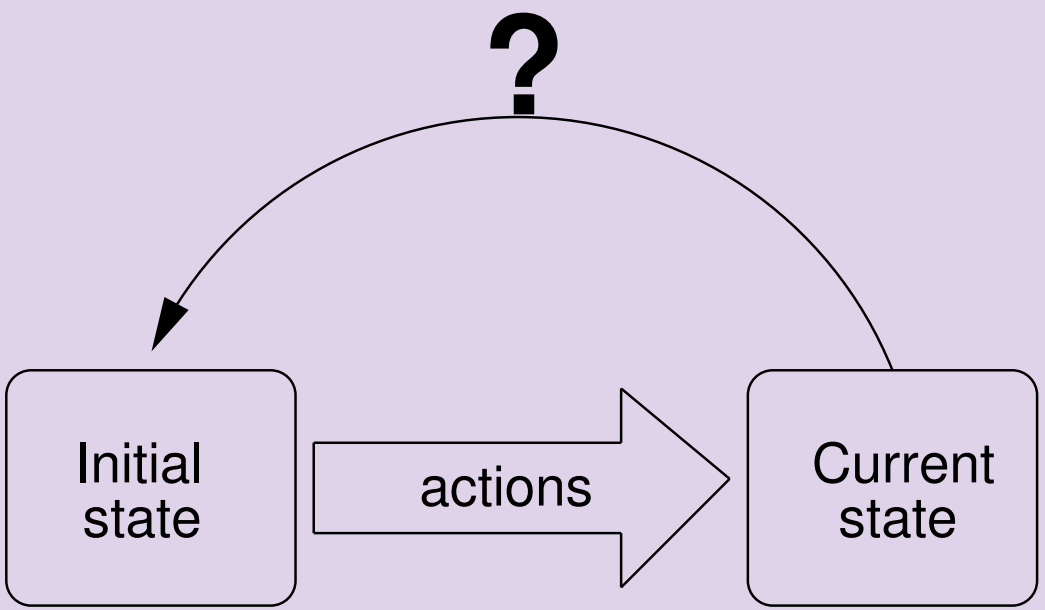
Prediction: reasoning about the future



- After shooting, the turkey stops walking

Reasoning About Actions

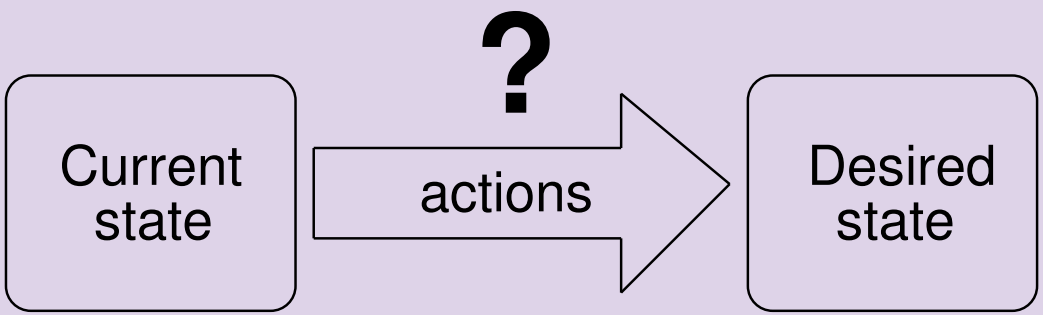
Explanation: reasoning about the past



- After shooting, the turkey is dead: the gun was loaded

Reasoning About Actions

Planning: what to do to achieve a goal



- To have the turkey dead: load the gun, then shoot

Reasoning About Actions

Other important tasks

- Consistency check
- Test of executability/inexecutability
- Theory change
- ...

Outline

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Formalizing Domains

Several base formalisms

- Situation calculus [McCarthy & Hayes, 1969]
  - $\forall s.(Holds(loaded, s) \rightarrow \neg Holds(alive, do(shoot, s)))$
- Languages $\mathcal{A}$, $\mathcal{AR}$, etc. [Lifschitz *et al.*, 90's]
  - *shoot* causes $\neg$*alive* if *loaded*
- Fluent calculus [Thielscher, 1995]
  - $Poss(shoot(tk), s) \rightarrow$
    $State(do(shoot(tk), s)) = State(s) \circ dead(tk) - alive(tk)$
- ...

Formalizing Domains

In this work...

- we have chosen Modal Logic
  - Weak version of Propositional Dynamic Logic (PDL)
  - Simple and decidable
  - With a tableaux-based theorem prover: Lotrec

Logical Preliminaries

Ontology

- Actions: $\mathfrak{Act} = \{a_1, a_2, \ldots\}$
- Atomic propositions: $\mathfrak{Prop} = \{p_1, p_2, \ldots\}$
- Literals: $\mathfrak{Lit} = \mathfrak{Prop} \cup \{\neg p : p \in \mathfrak{Prop}\}$
- Classical formulas: $\mathfrak{Fml} = \{\varphi_1, \varphi_2, \ldots\}$

Action operators

For each $a \in \mathfrak{Act}$, a modal operator $[a]$

- $[a]\varphi$: "after execution of a, φ is true"
- $\langle a \rangle \varphi =_{\text{def}} \neg [a] \neg \varphi$
- Complex formulas: $[a_1; a_2]\varphi$, $[a^*]\varphi$, ...

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Logical Preliminaries

Example

- Actions: *shoot*, *tease*
- Propositions: *loaded*, *alive*, *walking*
- Formulas: $alive \wedge \neg walking$, $\langle tease \rangle \top$, $loaded \rightarrow [shoot] \neg alive$

Semantics

Multimodal logic K [Popkorn 94,Blackburn *et al.* 2001].

Definition

Models $\mathscr{M} = \langle W, R \rangle$ where

- $W \subseteq 2^{\mathfrak{Prop}}$: set of possible worlds (states)
- R: $\mathfrak{Act} \longrightarrow 2^{W \times W}$

Definition

- $\models^{\mathscr{M}}_{w} p$ (p is true at world w of model $\mathscr{M}$) iff $p \in w$
- $\models^{\mathscr{M}}_{w} [a]\Phi$ iff for every w' such that wR_aw', $\models^{\mathscr{M}}_{w'} \Phi$
- the usual truth conditions for the other connectives

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Semantics

Example

If $\mathfrak{Act} = \{a_1, a_2\}$, and $\mathfrak{Prop} = \{p_1, p_2\}$, $\mathscr{M} = \langle W, R \rangle$, where

$$W = \{\{p_1, p_2\}, \{p_1, \neg p_2\}, \{\neg p_1, p_2\}\},$$

$$R(a_1) = \left\{ \begin{array}{c} (\{p_1, p_2\}, \{p_1, \neg p_2\}), (\{p_1, p_2\}, \{\neg p_1, p_2\}), \\ (\{\neg p_1, p_2\}, \{\neg p_1, p_2\}), (\{\neg p_1, p_2\}, \{p_1, \neg p_2\}) \end{array} \right\}$$

$$R(a_2) = \{(\{p_1, p_2\}, \{p_1, \neg p_2\}), (\{p_1, \neg p_2\}, \{p_1, \neg p_2\})\}$$

is a model

Semantics

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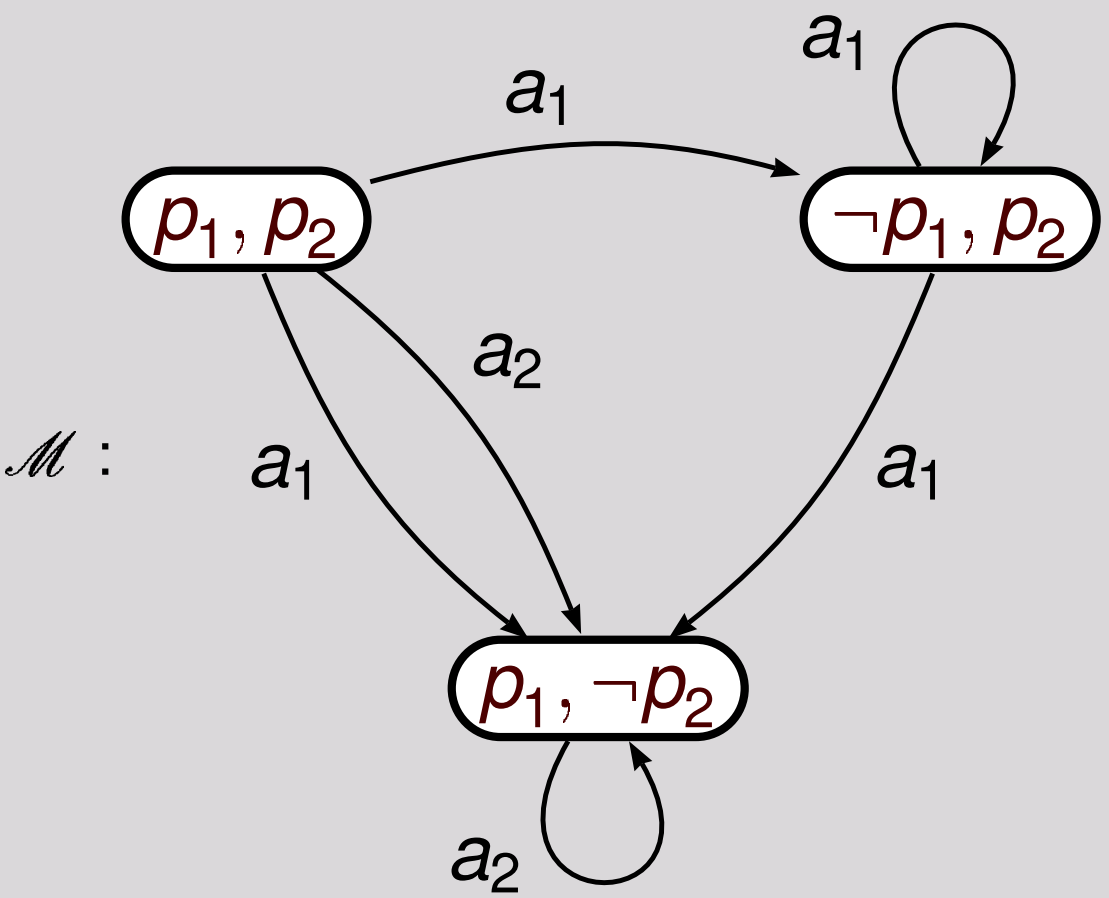

$\models^{\mathscr{M}} p_1 \vee p_2$

$\models^{\mathscr{M}} p_1 \rightarrow [a_2]\neg p_2$

$\models^{\mathscr{M}} \neg p_1 \rightarrow \langle a_1 \rangle \top$

$\models^{\mathscr{M}} \neg p_2 \rightarrow [a_1]\bot$

Semantics

Definition

- $\models^{\mathscr{M}} \Phi$ iff for all $w \in W$, $\models^{\mathscr{M}}_{w} \Phi$
- $\models^{\mathscr{M}} \Sigma$ iff $\models^{\mathscr{M}} \Phi$ for every $\Phi \in \Sigma$

Definition

Φ is a *consequence of the set of global axioms* Σ in all PDL-models (noted $\Sigma \models_{\mathrm{PDL}} \Phi$) iff for every $\mathscr{M}$, if $\models^{\mathscr{M}} \Sigma$, then $\models^{\mathscr{M}} \Phi$.

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- Teasing a turkey makes it to walk: $[tease]walking$
- It is always possible to tease a turkey: $\langle tease \rangle \top$
- A dead turkey remains dead after teasing it
  - $\neg alive \rightarrow [tease]\neg alive$
- If the gun is loaded, shooting kills the turkey
  - $loaded \rightarrow [shoot]\neg alive$
- Teasing does not unload the gun
  - $loaded \rightarrow [tease]loaded$

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$$\left\{\begin{array}{c} \langle tease\rangle\top, \\ walking \rightarrow alive, \\ [tease]walking, \\ loaded \rightarrow [shoot]\neg alive, \\ \neg alive \rightarrow [tease]\neg alive, \\ loaded \rightarrow [tease]loaded \end{array}\right\} \models [tease]alive$$

$$\models \neg alive \rightarrow [tease](alive \wedge \neg alive)$$

$$\models \neg alive \rightarrow [tease]\bot$$

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N.B.: Such a description is consistent

What is the problem?

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Natural Modules in Action Theories

Types of domain laws

Static laws : $walking \rightarrow alive$

Effect laws : $loaded \rightarrow [shoot]\neg alive$

Executability laws : $hasGun \rightarrow \langle shoot \rangle \top$

Inexecutability laws : $\neg hasGun \rightarrow [shoot]\bot$

only formulas of these types

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Natural Modules in Action Theories

Defining modules

- $\mathcal{S}$: set of static laws
- Given $a \in \mathfrak{Act}$
  - $\mathcal{E}^a$: effect laws for a
  - $\mathcal{X}^a$: executability laws for a
  - $\mathcal{I}^a$: inexecutability laws for a
- $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$: domain description for a
- $\mathcal{E} = \bigcup_{a \in \mathfrak{Act}} \mathcal{E}^a$, $\mathcal{X} = \bigcup_{a \in \mathfrak{Act}} \mathcal{X}^a$, and $\mathcal{I} = \bigcup_{a \in \mathfrak{Act}} \mathcal{I}^a$
- $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$: the action theory of a given domain

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What About the Frame Problem?

In our example

- If we had an action *wait*
  - $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \not\models_{\mathsf{PDL}} \mathit{loaded} \rightarrow [\mathit{wait}]\mathit{loaded}$

Definition

Dependence relation [Castilho *et al.* 99]: $\rightsquigarrow \subseteq \mathfrak{Act} \times \mathfrak{Lit}$

Example

- $\mathit{shoot} \rightsquigarrow \neg \mathit{alive}$, $\mathit{tease} \rightsquigarrow \mathit{walking}$, $\mathit{tease} \not\rightsquigarrow \mathit{alive}$
- From $\mathit{wait} \not\rightsquigarrow \neg \mathit{loaded}$ conclude $\mathit{loaded} \rightarrow [\mathit{wait}]\mathit{loaded}$

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Restriction on models

For all wR_aw':

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New logical consequence

- $\models_{\rightsquigarrow}$ instead of $\models_{\mathrm{PDL}}$

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The dependence-based approach...

- solves the frame problem
- subsumes Reiter's regression [Demolombe *et al.* 2003]
- does not entirely solve the ramification problem
  - e.g. $shoot \rightsquigarrow \neg walking$
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Consistency and More

Postulates

PC (*Consistency*): $\mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \not\models_{\rightsquigarrow} \bot$

PS (*No implicit static laws*): if $\mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \models_{\rightsquigarrow} \varphi$, then $\mathcal{S} \models \varphi$

PI (*No implicit inexecutability laws*):

if $\mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \models_{\rightsquigarrow} \varphi \rightarrow [a]\bot$,
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PX (*No implicit executability laws*):

if $\mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \models_{\rightsquigarrow} \varphi \rightarrow \langle a\rangle\top$,
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Motivation

- Better control what is going on

Consistency and More

Postulates

PC (*Consistency*): $\mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \not\models_{\rightsquigarrow} \bot$

PS (*No implicit static laws*): if $\mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \models_{\rightsquigarrow} \varphi$, then $\mathcal{S} \models \varphi$

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Example

$$\mathcal{S} = \{walking \rightarrow alive\},\ \mathcal{E} = \left\{ \begin{array}{c} [tease]walking, \\ loaded \rightarrow [shoot]\neg alive \end{array} \right\},$$

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$$tease \rightsquigarrow walking, shoot \rightsquigarrow \neg alive$$

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- But $\mathcal{S} \not\models alive$

Postulate **PS** violated

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Idea of algorithm

For each $\varphi \rightarrow \langle a \rangle \top$

1. find $\varphi' \rightarrow [a]\bot$ entailed by the theory
2. if $\varphi \wedge \varphi'$ is consistent with $\mathcal{S}$
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Result: the set of all implicit static laws

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calls: $NewCons_\varphi(\psi) = Pl(\varphi \wedge \psi) \setminus Pl(\varphi)$

$\mathcal{S}_{imp^*} := \emptyset$
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        if $\mathcal{S} \cup \mathcal{S}_{imp^*} \cup \{\varphi, \varphi_{\mathcal{C}^{a\prime}}, \neg\chi\} \nvdash \bot$ **and** $\forall l_i \in \chi,\ a \not\rightsquigarrow l_i$ **then**
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Example

$$\mathcal{S} = \{walking \rightarrow alive\},\ \mathcal{E} = \left\{ \begin{array}{c} [tease]walking, \\ loaded \rightarrow [shoot]\neg alive \end{array} \right\},$$

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For $\langle tease\rangle\top$ and $[tease]walking$:

- $NewCons_{\mathcal{S}}(walking) = alive$: $[tease]alive$
- $tease \not\rightsquigarrow alive$: $\neg alive \rightarrow [tease]\neg alive$
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Example (cont.)

Alternatives for repairing:

- $\mathcal{S} := \mathcal{S} \cup \{\mathit{alive}\}$
- add $\mathit{tease} \rightsquigarrow \mathit{alive}$
- weaken $[\mathit{tease}]\mathit{walking}$: $\mathit{alive} \rightarrow [\mathit{tease}]\mathit{walking}$
- weaken $\langle \mathit{tease} \rangle \top$: $\mathit{alive} \rightarrow \langle \mathit{tease} \rangle \top$
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Theorem

$\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ and $\rightsquigarrow$ satisfy Postulate **PS** *iff $\mathcal{S}_{imp^*} = \emptyset$.*

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Let $\mathcal{S}_{imp^*}$ be the output of Algorithm 1 on input $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ and $\rightsquigarrow$. Then

- *$\langle \mathcal{S} \cup \mathcal{S}_{imp^*}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ has no implicit static law.*
- *$\mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \models_{\rightsquigarrow} \bigwedge \mathcal{S}_{imp^*}$.*

Corollary

For all $\varphi \in \mathfrak{Fml}$, $\mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \models_{\rightsquigarrow} \varphi$ iff $\mathcal{S} \cup \mathcal{S}_{imp^*} \models \varphi$.

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Let $\mathcal{S}_{imp^*}$ *be the output of Algorithm 1 on input* $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ *and* $\rightsquigarrow$. *Then*

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No Implicit Inexecutability Laws

Example

$$\mathcal{S} = \{walking \rightarrow alive\},\ \mathcal{E} = \left\{ \begin{array}{c} [tease]walking, \\ loaded \rightarrow [shoot]\neg alive \end{array} \right\},$$

$$\mathcal{X} = \mathcal{I} = \emptyset,\ tease \rightsquigarrow walking, shoot \rightsquigarrow \neg alive$$

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- Thus $\mathcal{S}, \mathcal{E}^{tease}, \mathcal{X}^{tease}, \mathcal{I}^{tease} \models_{\rightsquigarrow} \neg alive \rightarrow [tease]\bot$
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Postulate **PI** violated

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No Implicit Inexecutability Laws

Idea of algorithm

For each combination of effect laws

1. find inconsistent consequents
2. mark it as an implicit inexecutability

Result: the set of all implicit inexecutabilities

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Algorithm 2: Finding implicit inexecutability laws for a

input: $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{I}^a \rangle$ and $\rightsquigarrow$

output: $\mathcal{I}^a_{imp}$, the set of implicit inexecutability laws for a

calls: $NewCons_\varphi(\psi) = Pl(\varphi \wedge \psi) \setminus Pl(\varphi)$

$\mathcal{I}^a_{imp} := \emptyset$

for all $\mathcal{E}^{a\prime} \subseteq \mathcal{E}^a$ **do**

$\varphi_{\mathcal{E}^{a\prime}} := \bigwedge\{\varphi_i : \varphi_i \rightarrow [a]\psi_i \in \mathcal{E}^{a\prime}\}$

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Example

$$\mathcal{S} = \{walking \rightarrow alive\},\ \mathcal{E} = \left\{ \begin{array}{c} [tease]walking, \\ loaded \rightarrow [shoot]\neg alive \end{array} \right\},$$

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No Implicit Inexecutability Laws

Theorem

If $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ *and* $\rightsquigarrow$ *satisfy Postulate* **PS**, *then* $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ *and* $\rightsquigarrow$ *satisfy Postulate* **PI** *iff* $\mathcal{I}_{imp} = \emptyset$.

Generalizing the Postulates

Postulate

- **PS*** (**No implicit static laws**):
  if $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \models_{\rightsquigarrow} \varphi$, then $\mathcal{S} \models_{\text{PDL}} \varphi$

Theorem

$\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$ and $\rightsquigarrow$ satisfy **PS*** *iff $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ and $\rightsquigarrow$ satisfy* **PS** *for all $a \in \mathfrak{Act}$.*

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  if $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \models_{\rightsquigarrow} \varphi$, then $\mathcal{S} \models_{\text{PDL}} \varphi$

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$\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$ and $\rightsquigarrow$ satisfy **PS*** *iff $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ and $\rightsquigarrow$ satisfy* **PS** *for all $a \in \mathfrak{Act}$.*

Generalizing the Postulates

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- **PC*** (**Logical consistency**): $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \not\models_{\rightsquigarrow} \bot$

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If $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$ *and* $\rightsquigarrow$ *satisfy* **PS***, *then* $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$ *and* $\rightsquigarrow$ *satisfy* **PC*** *iff* $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ *and* $\rightsquigarrow$ *satisfies* **PC** *for all* $a \in \mathfrak{Act}$.

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Generalizing the Postulates

Postulate

- **PI*** (**No implicit inexecutability laws**):
  if $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \models_{\rightsquigarrow} \varphi \rightarrow [a]\bot$, then $\mathcal{S}, \mathcal{I} \models_{\mathrm{PDL}} \varphi \rightarrow [a]\bot$

Theorem

Let $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$ and $\rightsquigarrow$ satisfy **PS***. $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$ and $\rightsquigarrow$ satisfy* **PI*** *iff $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ and $\rightsquigarrow$ satisfy* **PI** *for all $a \in \mathfrak{Act}$.*

Generalizing the Postulates

Postulate

- **PI*** (**No implicit inexecutability laws**):
  if $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \models_{\rightsquigarrow} \varphi \rightarrow [a]\bot$, then $\mathcal{S}, \mathcal{I} \models_{\mathsf{PDL}} \varphi \rightarrow [a]\bot$

Theorem

Let $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$ *and* $\rightsquigarrow$ *satisfy* **PS***. $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$ *and* $\rightsquigarrow$ *satisfy* **PI*** *iff* $\langle \mathcal{S}, \mathcal{E}^a, \mathcal{X}^a, \mathcal{I}^a \rangle$ *and* $\rightsquigarrow$ *satisfy* **PI** *for all* $a \in \mathfrak{Act}$.

Outline

Reasoning Modularly

If $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$ and $\sim\!\!>$ satisfy Postulate **PS***, then

Theorem

$\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \mid\!\!\sim_{\sim\!\!>} \bot$ *iff* $\mathcal{S} \models \bot$.

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Corollary

PX *is a consequence of* **PS**.

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If $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \rangle$ and $\rightsquigarrow$ satisfy Postulate **PS***, then

Theorem

$\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \models_{\overrightarrow{\sim}} \varphi \rightarrow [a_1, \ldots, a_n]\psi$ *iff*
$\mathcal{S}, \mathcal{E}^{a_1,\ldots,a_n}, \overrightarrow{\mathcal{I}}^{a_1,\ldots,a_n} \models_{\rightsquigarrow} \varphi \rightarrow [a_1, \ldots, a_n]\psi.$

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Outline

Another Tale

Example

- If the switch is up, the room is lit up
  - $up \rightarrow light$
- Toggling the switch changes its position
  - $\neg up \rightarrow [toggle]up$
  - $up \rightarrow [toggle]\neg up$
- It is always possible to toggle the switch
  - $\langle toggle \rangle \top$

The Need for Theory Change

You observe that...

- even if the switch is up the light is off.
- in a blackout, you do not succeed to switch the light on.
- despite your efforts you do not manage to toggle the switch.

Contraction: Motivation

Contracting by a static law

- You observe that even if the switch is up the light is off
- Static law $up \rightarrow light$ must be given up
- Can we just contract the static laws of $\mathcal{S}$?
  - May not be enough: side effects!
    - Conflict with $\mathcal{X}$
    - The contracted law may be an implicit one

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Contraction: Motivation

Contracting by an effect law

- During a blackout you do not succeed to switch the light on
- Effect law $\neg up \rightarrow [toggle]light$ must be given up
- Important issue: give up as few as possible

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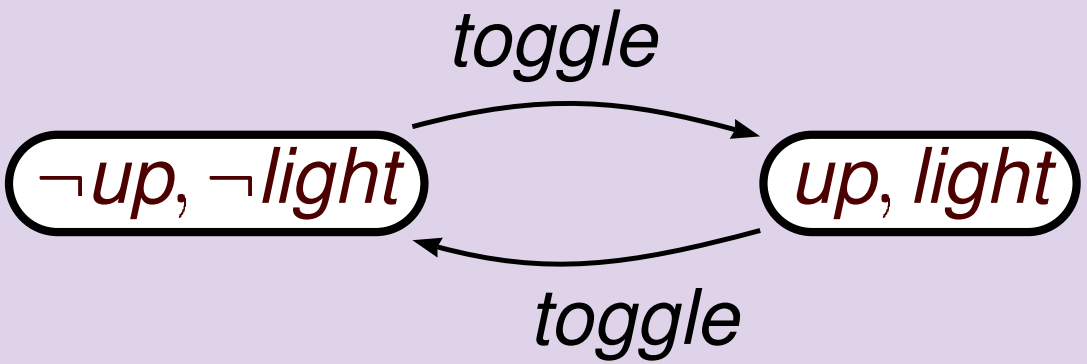
Contraction: Motivation

Contracting by an executability law

- Despite your efforts you do not manage to toggle the switch
- Executability law $up \rightarrow \langle toggle \rangle \top$ must be given up
- Side effects?

Contraction: Semantics

Playing with models



- Semantical contraction produces a set of models:

$$\langle W, R\rangle^{-}_{\Phi} = \{\langle W', R'\rangle \ : \ \ldots\}$$

Contraction: Semantics

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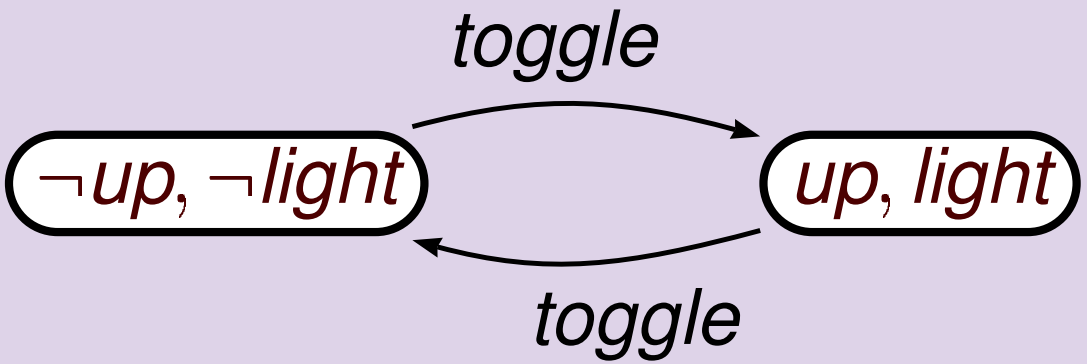

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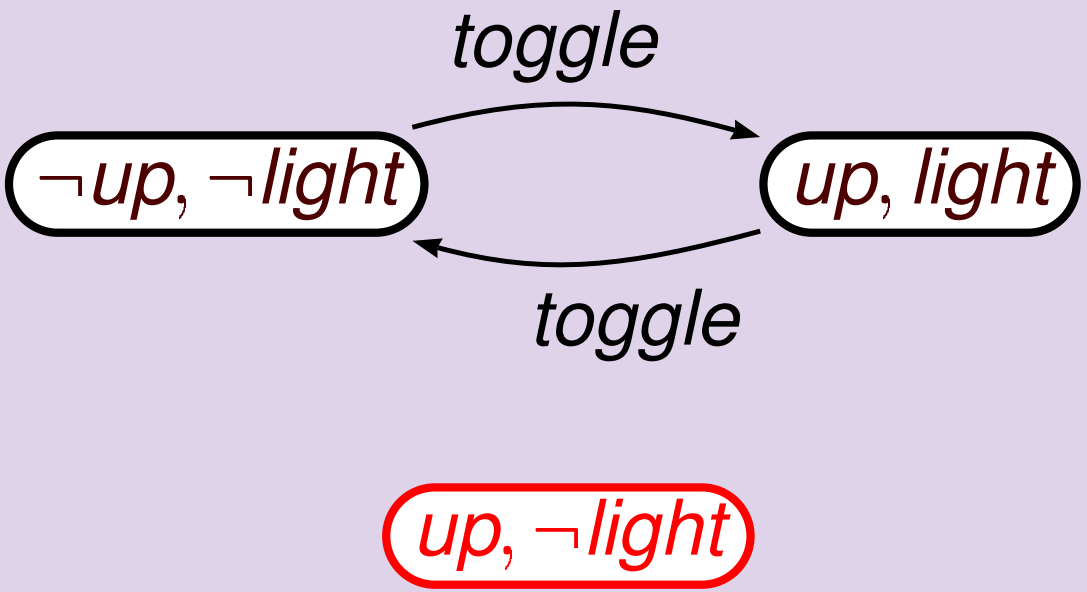
Contraction: Semantics

Contracting static laws

- $\langle W, R \rangle^{-}_{up \to light} = ?$
- Intuition: add some $up \wedge \neg light$-worlds to W



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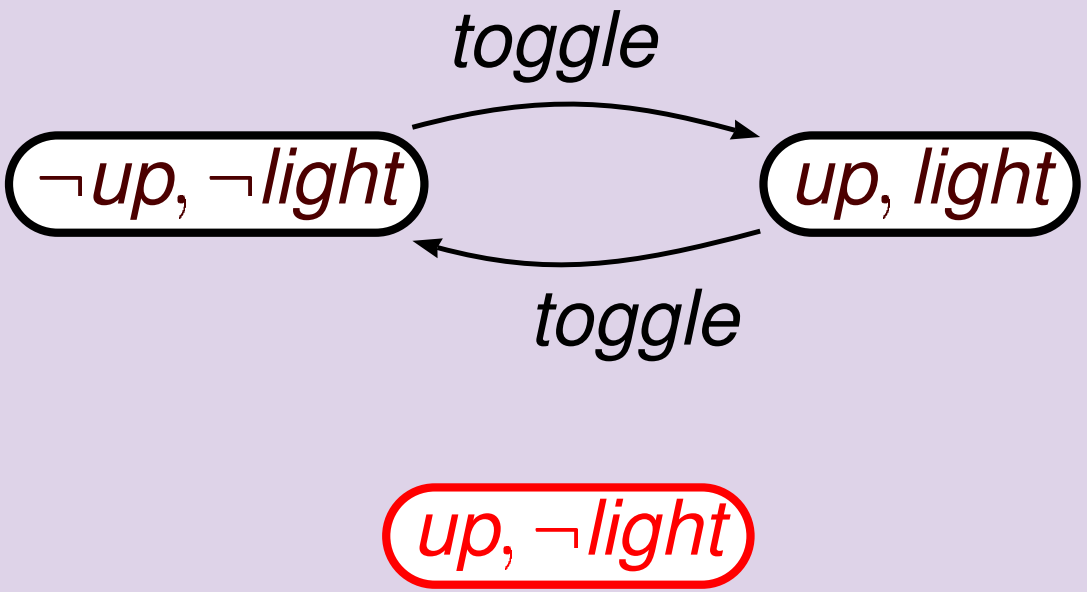

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Contraction: Semantics

Contracting static laws

- Rely on any belief change operator for classical logic
  - Say PMA, ...
- $\langle W, R\rangle^{-}_{up \to light} = \{\langle W', R'\rangle\}$, where
  - $W' = W -_{PMA} up \to light$
  - $R' = R$
- N.B.: $\langle W', R'\rangle \not\models \langle toggle\rangle \top$
  - Executability laws to be weakened!

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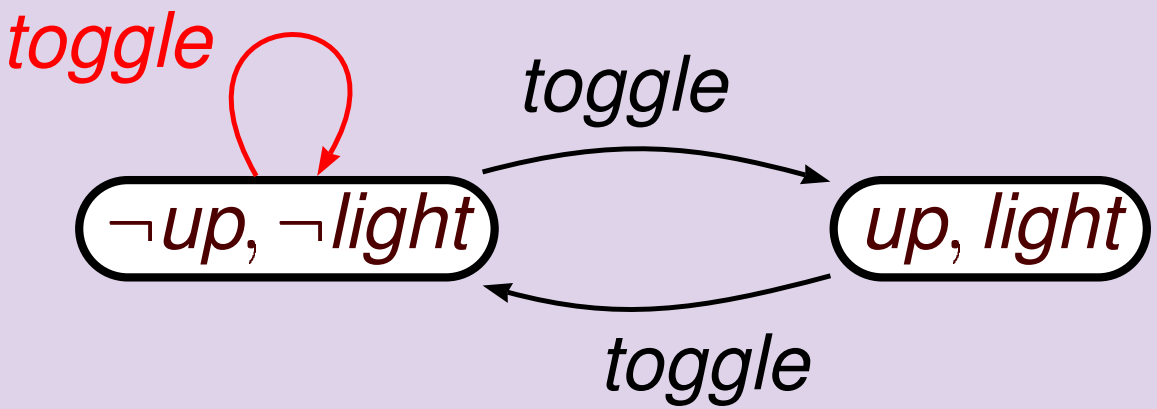
Contraction: Semantics

Contracting effect laws

- $\langle W, R \rangle^{-}_{\neg up \rightarrow [toggle] light} = ?$
- Intuition: add some arrows from $\neg up$-worlds to $\neg light$-worlds



Contraction: Semantics

Contracting effect laws

- $\langle W, R \rangle^{-}_{\neg up \rightarrow [toggle] light} =$

$$\{\langle W, R \cup R'_a \rangle \ : \ R'_a \subseteq \{(w, w') \ : \ w \models \neg up\}\}$$

- Problems:
  - Don't link *light*-worlds
  - Don't link all $\neg$*light*-worlds

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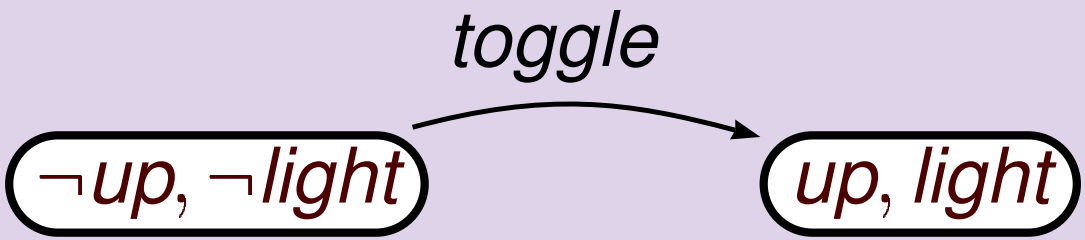
Contraction: Semantics

Contracting executability laws

- $\langle W, R \rangle^{-}_{up \rightarrow \langle toggle \rangle \top} = ?$
- Intuition: delete some arrows leaving *up*-worlds



Contraction: Semantics

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- $\langle W, R \rangle^{-}_{up \rightarrow \langle toggle \rangle \top} =$

$$\{\langle W, R \setminus R'_a \rangle \ : \ R'_a \subseteq \{(w, w') \ : \ wR_aw' \text{ and } w \models up\}\}$$

- N.B.: if there is no *up*-world, then contraction is not successful!

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Contraction: Syntax

Domain descriptions

- Simplification: $\langle \mathcal{S}, \mathcal{E}, \mathcal{X} \rangle$
- Resulting action theory
  - $\langle \mathcal{S}, \mathcal{E}, \mathcal{X} \rangle^{-}_{\Phi} = \langle \mathcal{S}', \mathcal{E}', \mathcal{X}' \rangle$

Contraction: Syntax

Contracting static laws

- $\langle \mathcal{S}, \mathcal{E}, \mathcal{X} \rangle^{-}_{up \to light} = \langle \mathcal{S}', \mathcal{E}, \mathcal{X}' \rangle$, where
  - $\mathcal{S}' = \mathcal{S} -_{PMA} up \to light = \{light \to up\}$
  - $\mathcal{X}' =$

$$\{((\neg up \vee light) \wedge \varphi) \to \langle toggle \rangle \top \ : \ \varphi \to \langle toggle \rangle \top \in \mathcal{X}\}$$

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Contraction: Syntax

Contracting effect laws

- $\langle \mathcal{S}, \mathcal{E}, \mathcal{X} \rangle^{-}_{\neg up \rightarrow [toggle] light} = \langle \mathcal{S}, \mathcal{E}', \mathcal{X} \rangle$, where
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Contraction: Syntax

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- $\langle \mathcal{S}, \mathcal{E}, \mathcal{X} \rangle^{-}_{up \to \langle toggle \rangle \top} = \langle \mathcal{S}, \mathcal{E}, \mathcal{X}' \rangle$, where
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Soundness

Theorem

If $\langle W, R\rangle \models \mathcal{S} \wedge \mathcal{E} \wedge \mathcal{X}$, *then* $\langle W, R\rangle^{-}_{\Phi} \models \langle \mathcal{S}, \mathcal{E}, \mathcal{X}\rangle^{-}_{\Phi}$.

Incompleteness

Example

- $\mathcal{S} = \emptyset, \mathcal{E} = \{p \rightarrow [a]\bot\}, \mathcal{X} = \{\langle a\rangle\top\}$
- Unique model: $\langle W, R\rangle = \langle\{\{\neg p\}\}, \{(\{\neg p\}, \{\neg p\})\}\rangle$
- $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}\rangle^{-}_{p\rightarrow\langle a\rangle\top} = \langle\emptyset, \{p \rightarrow [a]\bot\}, \{\neg p \rightarrow \langle a\rangle\top\}\rangle$.
- Syntactically: successful, as $\mathcal{S}, \mathcal{E}, \mathcal{X}' \nvdash_{\mathrm{PDL}} p \rightarrow \langle a\rangle\top$.
- Semantically: contraction is unsuccessful!

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Completeness: Modularity

Theorem

If $\langle \mathcal{S}, \mathcal{E}, \mathcal{X} \rangle$ *and* $\rightsquigarrow$ *satisfy Postulate* **PS***, *then* $\langle \mathcal{S}, \mathcal{E}, \mathcal{X} \rangle^{-}_{\Phi} \models \Psi$ *iff* $\langle W, R \rangle^{-}_{\Phi} \models \Psi$, *for every* $\langle W, R \rangle$ *such that* $\langle W, R \rangle \models \mathcal{S} \wedge \mathcal{E} \wedge \mathcal{X}$.

Outlook: Semantics of Revision

Levi identity

- Revise by $glued \rightarrow [toggle]\bot$ amounts to
  1. Contract by $\neg(glued \rightarrow [toggle]\bot)$
  2. Expand by $glued \rightarrow [toggle]\bot$
- Problem: we can contract by domain laws only
  - $\neg(glued \rightarrow [toggle]\bot) \leftrightarrow (glued \wedge \langle toggle\rangle\top)$
  - $\langle \mathcal{S}, \mathcal{E}, \mathcal{X}\rangle^{-}_{glued \wedge \langle toggle\rangle\top}$ not defined
  - What is the negation of
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Related Work

Modularity

- [Pirri & Reiter 1999]: deterministic actions without ramifications in Situation Calculus
- [Amir 2000]: object-oriented concepts in Situation Calculus
- [Zhang *et al.* 2002]: EPDL approach/normal form
- [Lang *et al.* 2003]: computational complexity
- [Kakas *et al.* 2005]: elaboration tolerance, concurrent actions
- [Ghilardi, Lutz & Wolter, KR'06]: uniform interpolation and conservative extensions in $\mathcal{ALC}$

Related Work

Theory change

- [Li& Pereira 1996]: motivations
- [Liberatore 2000]: meta-results
- [Eiter *et al.* 2005/06]: update in action languages

Summary

Claim

- Consistency is not enough to evaluate a domain description
- The dynamic part of an action theory should not influence the non-dynamic one (otherwise: problems)

Contribution

- Fine-grained postulates of modularity
- Algorithms to check/give hints on modularity
- Satisfaction of modularity
  - More efficient reasoning
  - Important for updating theories [Herzig *et al.* ECAI'06]
- Our results apply to every approach allowing for $\mathcal{S}$, $\mathcal{E}$, $\mathcal{X}$ and $\mathcal{I}$

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Summary

Contribution (cont.)

- Semantics of action theory contraction
  - Domain-independent
  - Does not require extra information (preferences/epistemic entrenchment relation/. . .)
  - Fully automatic
- Completeness result: highlights importance of modularity

Summary

Modularity is also fruitful. . .

- for theories in general [Herzig & Varzinczak AiML'04]
- in the Situation Calculus [Herzig & Varzinczak IJCAI'05]
- in Description Logics [Herzig & Varzinczak JELIA'06]
  - (See next slide)

Future work

- Fine tune contraction of effect laws
- Contract by any formulas (not just laws)
- Postulates about effect laws? about causation?

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Outlook: Modularity in Description Logics

Example

- Suppose a passport control system in an airport
- Such a system is composed of many software components
- One of them an *ontology* (knowledge base) about passengers
- All passengers must be controlled

Outlook: Modularity in Description Logics

Example (Ontology)

- A passenger has a passport
- European citizens have European passports
- Foreigners have non-European passports
- Someone with double citizenship is a foreigner and a European

Outlook: Modularity in Description Logics

Example (The ontology in DL)

- Terminology:
  - $\text{Passenger} \sqsubseteq \exists \text{passport}.\top$
  - $\text{EUcitizen} \sqsubseteq \forall \text{passport}.\text{EU}$
  - $\text{Foreigner} \sqsubseteq \forall \text{passport}.\neg\text{EU}$
  - $\text{2Citizen} \sqsubseteq \text{Foreigner} \sqcap \text{EUcitizen}$
- Assertions:
  - EU(POLAND)
  - EUcitizen(JAN)
  - passport(JAN, POLAND)

Outlook: Modularity in Description Logics

Nevertheless

$$\left\{\begin{array}{c} \mathsf{Passenger} \sqsubseteq \exists \mathsf{passport}.\top, \\ \mathsf{EUcitizen} \sqsubseteq \forall \mathsf{passport}.\mathsf{EU}, \\ \mathsf{Foreigner} \sqsubseteq \forall \mathsf{passport}.\neg\mathsf{EU}, \\ \mathsf{2Citizen} \sqsubseteq \mathsf{Foreigner} \sqcap \mathsf{EUcitizen} \end{array}\right\} \models \mathsf{2Citizen} \sqsubseteq \forall \mathsf{passport}.(\mathsf{EU} \sqcap \neg\mathsf{EU})$$

$$\models \mathsf{2Citizen} \sqsubseteq \forall \mathsf{passport}.\bot$$

$$\models \mathsf{2Citizen} \sqsubseteq \neg\mathsf{Passenger}$$

Someone with double citizenship is not a passenger

Hence...

if we have 2Citizen(BINLADEN),
this individual is not obliged to be controlled!

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Our results. . .

can be applied in DL, too

Thank you!

Merci beaucoup !

Danke schön!

Choukran!

¡Muchas gracias!

Muito obrigado!

Can We Ask for More?

Postulate about effects

- **PE** (**No implicit effect laws**):
  if $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \models_{\rightsquigarrow} \varphi \rightarrow [a]\psi$ and $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \not\models_{\rightsquigarrow} \varphi \rightarrow [a]\bot$,
  then $\mathcal{S}, \mathcal{E} \models_{\rightsquigarrow} \varphi \rightarrow [a]\psi$

Can We Ask for More?

Example

$$\mathcal{S} = \emptyset,\ \mathcal{E} = \left\{ \begin{array}{c} \mathit{loaded} \rightarrow [\mathit{shoot}]\neg \mathit{alive}, \\ (\neg \mathit{loaded} \wedge \mathit{alive}) \rightarrow [\mathit{shoot}]\mathit{alive} \end{array} \right\}$$

$$\mathcal{X} = \{\mathit{hasGun} \rightarrow \langle \mathit{shoot} \rangle \top\},\ \mathcal{I} = \{\neg \mathit{hasGun} \rightarrow [\mathit{shoot}]\bot\}$$

$$\mathit{shoot} \rightsquigarrow \neg \mathit{alive}$$

- $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \models_{\rightsquigarrow} \neg \mathit{hasGun} \vee \mathit{loaded} \rightarrow [\mathit{shoot}]\neg \mathit{alive}$
- $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \not\models_{\rightsquigarrow} \neg \mathit{hasGun} \vee \mathit{loaded} \rightarrow [\mathit{shoot}]\bot$
- but $\mathcal{S}, \mathcal{E} \not\models_{\rightsquigarrow} \neg \mathit{hasGun} \vee \mathit{loaded} \rightarrow [\mathit{shoot}]\neg \mathit{alive}$

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Can We Ask for More?

Postulate about effects

- **P**$\bot$ (**No unattainable effects**):
  if $\mathcal{S}, \mathcal{E} \models_{\rightsquigarrow} \varphi \rightarrow [a]\psi$, then $\mathcal{S}, \mathcal{E}, \mathcal{X}, \mathcal{I} \not\models_{\rightsquigarrow} \varphi \rightarrow [a]\bot$

Can We Ask for More?

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- $\mathcal{E} \models_{\rightsquigarrow} (\neg \mathit{hasGun} \wedge \mathit{loaded}) \rightarrow [\mathit{shoot}]\neg \mathit{alive}$
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