Automatization of one-loop diagram calculation by unitarity methods: the six-photon amplitudes.

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LAPTH - Université de Savoie

04/07/2008

INTRODUCTION - MOTIVATIONS 1

The Large Hadronic Collider

OBSERVE NEW PHYSICS AND NEW PARTICLES.... by the interaction of two protons...

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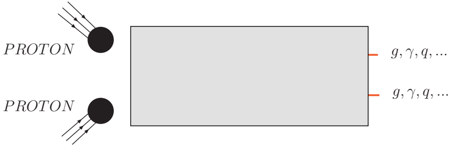
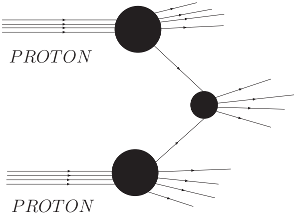


$\Rightarrow$ **Know exactly the background**

INTRODUCTION - MOTIVATIONS 2

To have a good prediction ⇒ NLO calculation of each subprocess.



$$\begin{cases} 2 \rightarrow 2 & \text{all known} \\ 2 \rightarrow 3 & \text{almost all known} \\ 2 \rightarrow 4 & \text{almost unknown} \\ \ldots & \end{cases}$$

INTRODUCTION - MOTIVATIONS 2

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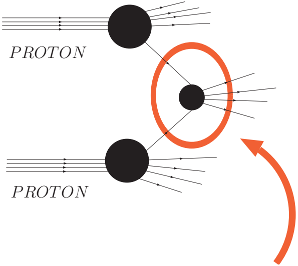

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$$\textbf{PP} \to 4 \text{ jets} = \begin{cases} q\bar{q} \to gggg \\ q\bar{q} \to q\bar{q}gg \\ q\bar{q} \to q\bar{q}q\bar{q} \\ gg \to gggg \ldots \end{cases}$$

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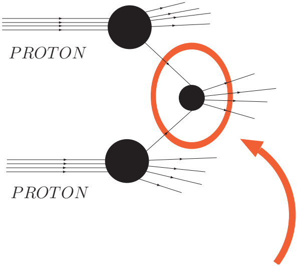

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- ⋆ Study one-loop multi-leg diagrams
- ⋆ Apply to the subprocess: $\gamma\gamma \to \gamma\gamma\gamma\gamma$

PROBLEMS OF THE CALCULATION OF AN ONE LOOP DIAGRAM.

- Calculation of an one loop diagram: two difficulties

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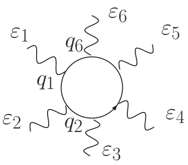

$$tr\left(\varepsilon_1\gamma_{\mu_1}...\varepsilon_6\gamma_{\mu_6}\right) \quad \int d^nq \frac{q_1^{\mu_1}...q_6^{\mu_6}}{(q_1^2+i\lambda)\,...(q_6^2+i\lambda)}$$

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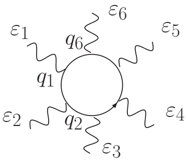

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HOW TO SIMPLIFY ???

HOW TO REDUCE ???

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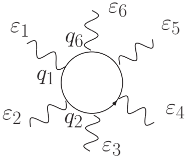

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HOW TO SIMPLIFY ???

HOW TO REDUCE ???

- The solutions:

  COMPACT NOTATION $\Rightarrow$ HELICITY AMPLITUDES METHODS

  REDUCE INTEGRALS $\Rightarrow$ AIM OF MY THESIS

HELICITY AMPLITUDES

Helicity amplitudes - Compact notation

- Helicity amplitude methods:

$$\text{Amplitude} = \sum_{i \in \text{helicity states of external photons}} \text{Amplitude}_i$$

$$(\gamma\gamma \rightarrow \gamma\gamma\gamma\gamma) = \sum_{\sigma_i = \pm} (\gamma^{\sigma_1}\gamma^{\sigma_2} \rightarrow \gamma^{\sigma_3}\gamma^{\sigma_4}\gamma^{\sigma_5}\gamma^{\sigma_6})$$

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- Polarisations vectors of photons (Zhan Xu and al. [*Nuclear Physics* **B291** (1987) 392-428]):

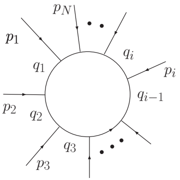
$$\varepsilon_+^\mu(p) = \frac{\langle R - |\gamma^\mu| p - \rangle}{\langle R - | p + \rangle} \quad \varepsilon_-^\mu(p) = \frac{\langle R + |\gamma^\mu| p + \rangle}{\langle p + | R - \rangle}$$

$|R\rangle$ reference light-like vector.

CLASSICAL REDUCTION METHODS

Vectoriel Space of Amplitudes

- General Amplitudes of an one-loop massless diagram:



$$\star\ \mathcal{A}_N = \int d^nQ \frac{\text{Num}(Q)}{D_1^2 ... D_N^2}$$

$$\star\ Q = q + \widehat{q}$$

$$q : \text{ 4 dimensions part} \quad \widehat{q} : \ n-4 \text{ dimensions part}$$

$$\star\ D_i^2 = (Q + p_1 + ... + p_i)^2 + i\lambda$$

- Num(Q): polynome in classical standard gauge theories.
- $(\mathcal{A}_i, +, .)$: vector space.
- Scalar integrals: $I_N^n = \int d^nQ \frac{1}{D_1^2 ... D_N^2}$ well-known.

Classical reduction Methods

- Many Classical reduction methods to reduce $\mathcal{A}_N = \int d^n Q \frac{\mathrm{Num}(Q)}{D_1^2 \ldots D_N^2}$.
- **Principe**: Transform the numerator:

$$\mathrm{Num}(Q) = \sum_i \alpha_i D_i^2 + \sum_{i,j} \beta_i D_i^2 D_j^2 \ldots$$

Therefore $\mathcal{A}_N = \sum_i \gamma_i I_i^n$.

Then gathering and factorisation...

- **Problem**: one diagram of the six-photon amplitudes: 1000000 terms... too many, and factorisation not obvious.

$\Rightarrow$ **FIND ANOTHER METHOD**

Properties of amplitudes

- $(\mathcal{A}_i, +, .)$ vectoriel space, use it algebric structure:

$$\Rightarrow \quad \begin{cases} \textbf{FIND A GOOD BASE} \\ \textbf{FIND A SCALAR PRODUCT} \end{cases}$$

Properties of amplitudes

- $(\mathcal{A}_i, +, .)$ vectoriel space, use it algebric structure:

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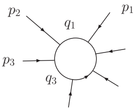
- Analytical Structure of an Amplitude $\mathcal{A}_N = \int d^n Q \frac{\mathrm{Num}(Q)}{D_1^2 \ldots D_N^2}$:

$$\frac{\mathrm{Num}(Q)}{D_1^2 \ldots D_3^2 \ldots D_N^2} \quad \text{Almost Analytical Function}$$



$$D_i^2 = Q_i^2 + i\lambda$$

$$\Rightarrow \{ \text{ CAUCHY's THEOREM}$$

Properties of amplitudes

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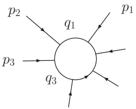

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$$\Rightarrow \left\{ \textbf{CAUCHY's THEOREM} \right.$$

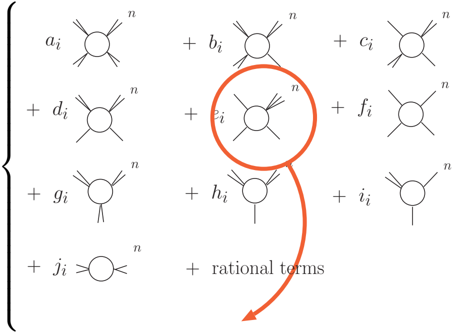
- Gather diagrams not to break the gauge symetry.

THE REDUCTION METHOD

The Standard Basis of Reduction

- Decomposition of a tensor integral:

- $$\mathcal{A}_N = \int d^n Q \frac{\mathrm{Num}(Q)}{D_1^2 \ldots D_N^2} =$$



scalar integral $\int d^n Q \frac{1}{D_1^2 D_2^2 D_3^2 D_4^2}$

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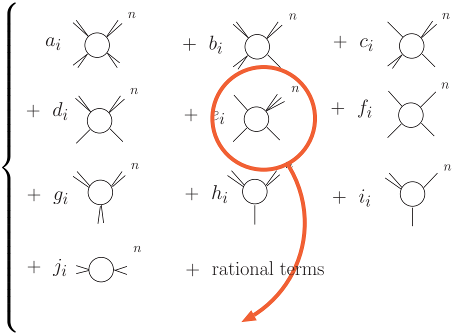

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- Two problems:
  - ⋆ Don't separate the UV/IF/finite structures
  - ⋆ Spurious Gram determinant problems.

$\Rightarrow$ **NOT THE BEST BASE.**

The good basis of decomposition

- I create a new basis of decomposition with many properties. C. Bernicot "Basis of decomposition of one-loop integral", in progress.

- Two problems:

  ⋆ Don't separate the UV/IF/finite structures

  ⋆ Gram determinant problems.

- Solutions: Linear relations:

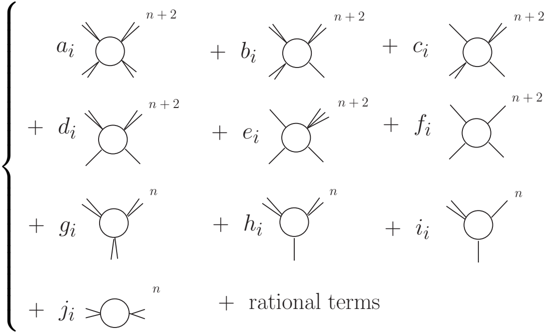
$$I_4^n = \sum_{i=1}^{4} \alpha_i I_3^n - \frac{\det(G)}{\det(S)} I_4^{n+2}$$

⋆ Separate the IR/finite structures in the four-point functions

⋆ Solve some Gram determinant problems.

A Better Basis of Decomposition

- $\mathcal{A}_N = \sum_{i \in \sigma}$



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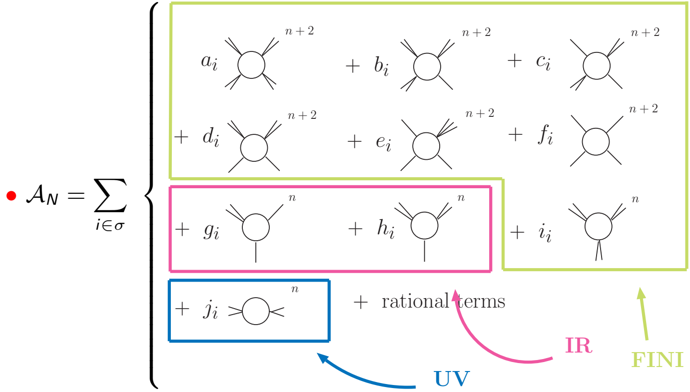

- Separate the UV/IF/finite structures
- Eliminate some Gram determinant problems.

$\Rightarrow$ **CALCULATE THE TEN COEFFICIENTS AND THE RATIONAL TERMS WITH SCALAR PRODUCT**

How calculate the coefficients ?

- Analytical Structure of an Amplitude $\mathcal{A}_N = \int d^n Q \frac{\mathrm{Num}(Q)}{D_1^2 \ldots D_N^2}$:

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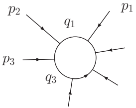
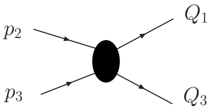
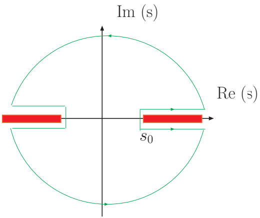


$$D_i^2 = Q_i^2 + i\lambda$$

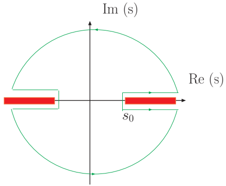
- Consider the invariant $s = (p_2 + p_3)^2$. Analycity of $\mathcal{A}_N(s)$. Physical region where $Q_1^2, Q_3^2 \longrightarrow 0$:





How calculate coefficient 2 ?

- Cauchy theorem for an analytic function except on the real axis:



$$A(s) = A(s_0) + \frac{s - s_0}{\pi} \int dS \frac{\text{Im}(A(S))}{(S - s + i\lambda)(S - s_0 + i\lambda)}$$

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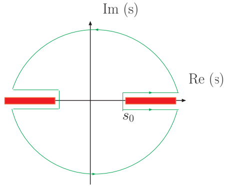

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Rationals terms

Reconstruction

Analytic structure

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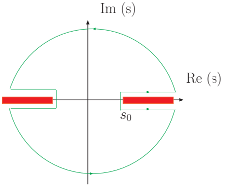

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Rationals terms

Reconstruction

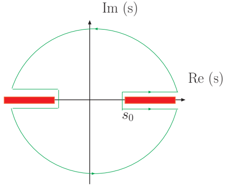
Analytic structure

-

$\Rightarrow$
- $\star$ Calculate the imaginary part
- $\star$ Avoid the reconstruction
- $\star$ Calculate the rational terms

Imaginary part: Concept of "Discontinuity"

- Cauchy theorem for an analytic function except on the real axis:



$$A(s) = A(s_0) + \frac{s - s_0}{\pi} \int dS \frac{\mathrm{Im}(A(S))}{(S - s + i\lambda)(S - s_0 + i\lambda)}$$

- Imaginary part:

$$2\ i\ \mathrm{Im}\left(\mathcal{A}_N\right) = \sum_{i \in invariant} \int d^n Q \frac{\mathrm{Num}(Q)}{D_2^2 \ldots D_4^2 \ldots D_N^2} \delta\left(Q_1^2\right) \delta\left(Q_3^2\right)$$

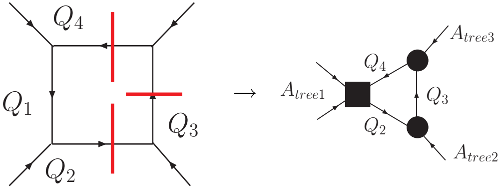
- Calculation of discontinuities with two cuts (Cutkowsky rules 1960), based on unitarity:

$\Rightarrow$ **EXTENSIONS**

Extension of discontinuity

- The discontinuity with M cuts is the limit when M propagators go on-shell. (Z.Bern, L. Dixon, D. Kosower, *Nucl.Phys.* B **513** (1998) 3-86).

- The three-cuts discontinuity of an amplitude:



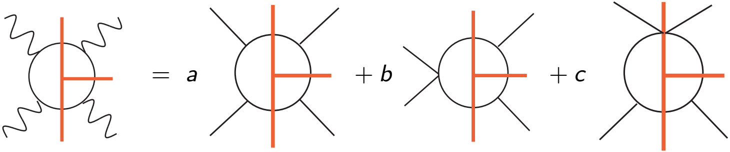
$$
\begin{aligned}
\mathrm{Disc}_3\,(\mathcal{A}_4) &= \int d^n Q\, \frac{\mathrm{Num}(Q)}{D_1} \delta(q_2^2)\delta(q_3^2)\delta(q_4^2) \\
&= \int d^n Q\; A_{tree1} A_{tree2} A_{tree3}\; \delta(q_2^2)\delta(q_3^2)\delta(q_4^2)
\end{aligned}
$$

Calculation of a discontinuity 1

- Calculation of the discontinuity:

$$\mathrm{Disc}_3\left(\mathcal{A}_4\right) = \int d^n Q \; A_{tree1} A_{tree2} A_{tree3} \; \delta(q_2^2)\delta(q_3^2)\delta(q_4^2) = \sum_i a_i \mathrm{Disc}_3\left(I_i^n\right).$$

- Calculation of a three-cut discontinuity of a four-photon one-loop diagram:



-

$\Rightarrow$ **AVOID THE RECONSTRUCTION...**

BUT Only the coefficients "a" and "b" with this discontinuity:

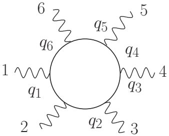
$\Rightarrow$ **NEED OTHER DISCONTINUITIES...**

$\Rightarrow$ **WAY TO CALCULATE EACH COEFFICIENT...**

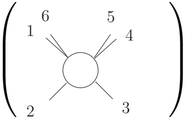
- Cannot have rational terms:

$\Rightarrow$ **EXTENTION TO THE RATIONAL TERMS...**

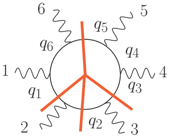
Direct calculation of a coefficient

- Example: 

$$\int d^nQ \frac{\mathrm{Num}\,(Q)}{(Q_1^2+i\lambda)\,(Q_2^2+i\lambda)\ldots(Q_6^2+i\lambda)}.$$

- The coefficient α in front of:  4 cuts (R.Britto,

F.Cachazo, B.Feng, *Nucl.Phys.* B**725** (2005) 275-305):



$$\alpha = \lim_{q_1^2,q_2^2,q_3^2 q_5^2\to 0} \frac{\mathrm{Num}\,(q)}{(q_4^2+i\lambda)\,(q_6^2+i\lambda)}$$

$\Rightarrow$ ONE COEFFICIENT = ONE RESIDUS

Origin of the rational terms

- Dispersive relation if $\mathcal{A}_N(s)$ analytic function:

$$\mathcal{A}_N(s) = \mathcal{A}_N(s_0) + \frac{s - s_0}{\pi} \int dS \frac{\mathrm{Im}(\mathcal{A}_N(S))}{(S - s + i\lambda)(S - s_0 + i\lambda)}$$

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- Dispersive relation if $\mathcal{A}_N(s)$ analytic function and $\mathcal{A}_N(s) \xrightarrow[s \to +\infty]{} 0$:

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NO RATIONAL TERMS

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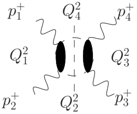
NO RATIONAL TERMS

- The rational terms link to the ultraviolet divergences.
- Regularization in n dimensions $\Rightarrow$ cut in n dimensions.

How to calculate the rational terms

$$\mathrm{Im}\ \mathcal{A}_4(++++) =$$



$$= \int d^n Q\ \delta(Q_2^2)\delta(Q_4^2) A_{tree1} A_{tree2}$$

The two trees are on-shell

How to calculate the rational terms

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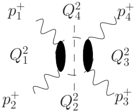

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- Cuts in four dimensions $A_{tree1} = A_{tree2} = 0$ $\qquad \mathcal{A}_4(++++) = 0$

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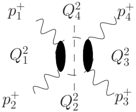

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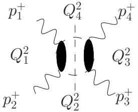
- Cuts in four dimensions $A_{tree1} = A_{tree2} = 0$ $\qquad \mathcal{A}_4(++++) = 0$
- Cuts in n dimensions : $Q = q + \widehat{q}$ $\qquad A_{tree1} \propto -\frac{\widehat{q}^2}{Q_1^2 + i\lambda}$

How to calculate the rational terms

$$\text{Im } \mathcal{A}_4(++++) =$$



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The two trees are on-shell

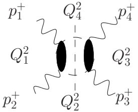
- Cuts in four dimensions $A_{tree1} = A_{tree2} = 0 \qquad \mathcal{A}_4(++++) = 0$
- Cuts in n dimensions : $Q \; = \; q + \widehat{q} \qquad A_{tree1} \propto -\frac{\widehat{q}^2}{Q_1^2 + i\lambda}$

$$\text{Im } \mathcal{A}_4(++++) \; = \; \int d^nQ \; \delta(Q_2^2)\delta(Q_4^2) \frac{\widehat{q}^2}{D_1^2} \frac{\widehat{q}^2}{D_3^2}$$

$$\Rightarrow \mathcal{A}_4(++++) \; = \; \int d^nQ \; \frac{\widehat{q}^4}{D_1^2 D_2^2 D_3^2 D_4^2} \; = \; -\frac{1}{6}$$

How to calculate the rational terms

$$\mathrm{Im}\ \mathcal{A}_4(++++) =$$



$$= \int d^n Q\ \delta(Q_2^2)\delta(Q_4^2) A_{tree1} A_{tree2}$$

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$$\Rightarrow \mathcal{A}_4(++++) = \int d^n Q\ \frac{\widehat{q}^4}{D_1^2 D_2^2 D_3^2 D_4^2} = -\frac{1}{6}$$

- $\Rightarrow$ **RATIONAL TERMS DESCRIDED BY EXTRA SCALAR INTEGRALS** $\int d^n Q \frac{\widehat{q}^{2r}}{D_1..D_N}$.

Summary

- $(\mathcal{A}_i, +, .)$ vector space:

$$\mathcal{A}_N = \sum_i a_i I_i^n$$

$$\mathcal{A}_N = a_i I_4^{n+2} + b_i I_3^n + c_i I_2^n + \text{rational terms.}$$

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- $\mathcal{A}_N$ almost analytic function:

$$\mathrm{a}_i, \mathrm{b}_i, \mathrm{c}_i = \text{Discontinuity, residus, in 4 dimensions}$$
$$\text{rational terms} = \text{Discontinuity, residus, in n-4 dimensions}$$

Summary

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$$\text{rational terms} = \text{Discontinuity, residus, in n-4 dimensions}$$

- Gather diagrams to restore the gauge invariance $\Rightarrow \mathrm{a}_i \ldots$ gauge invariant.

THE SIX PHOTON AMPLITUDES

Computation of the Six-photon Amplitudes.

- $\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 \to \mathbf{0}$ in massless loop.

- In the past:

  G.Mahlon [arXiv:hep-ph/9311213]
  Z.Nagy,D.E.Soper [arXiv:hep-ph/0610028]
  T.Binoth,T.Gehrmann, G.Heinrich,P.Mastrolia [arXiv:hep-ph/0703311]
  G.Ossola, C.G.Papadopoulos,R.Pittau [arXiv:0704.1271[hep-ph]]

  $\Rightarrow$ **NUMERICAL OR/AND ANALYTICAL CALCULATION IN QED**

- My work:

$$3\ \text{QED} : \begin{cases} \text{QED} \\ \text{scalar QED} \\ \text{QED}^{\mathcal{N}=1} \end{cases}$$

$\Rightarrow$ **THE MOST COMPACT RESULTS**

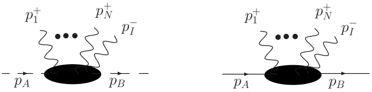
- C.Bernicot, J.Ph.Guillet [arXiv:hep-ph/0711.4713]
  Les Houches 2007: The NLO multileg working group: [arXiv:hep-ph/0803.0494]
  C.Bernicot,"The six-photon amplitude", Moriond 2008, [arXiv:hep-ph/0803.0494]

HOW I CALCULATE 1

- Linear relation between the three amplitudes:

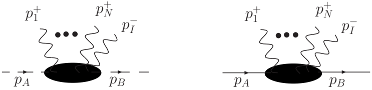
$$\mathcal{A}_6^{spinor} = -2\mathcal{A}_6^{scalar} + \mathcal{A}_6^{\mathcal{N}=1}$$

- Calculate many trees in scalar QED by recursive relations:



$$\mathcal{A}_{tree}^{scalar}, \quad \mathcal{A}_{tree}^{spinor} = \frac{\langle IA\rangle^2 + \langle IB\rangle^2}{\langle IA\rangle\langle IB\rangle}\mathcal{A}_{tree}^{scalar} = \left\{\frac{(\langle IA\rangle - \langle IB\rangle)^2}{\langle IA\rangle\langle IB\rangle} + 2\right\}\mathcal{A}_{tree}^{scalar}$$

HOW I CALCULATE 2



$$\mathcal{A}_{tree}^{scalar}, \quad \mathcal{A}_{tree}^{spinor} = \frac{\langle IA\rangle^2 + \langle IB\rangle^2}{\langle IA\rangle\langle IB\rangle}\mathcal{A}_{tree}^{scalar} = \left\{\frac{(\langle IA\rangle - \langle IB\rangle)^2}{\langle IA\rangle\langle IB\rangle} + 2\right\}\mathcal{A}_{tree}^{scalar}$$

- First I calculate $\mathcal{A}_6^{scalar}$.
- Then with few spinor manipulation calculation of discontinuity:

$$\mathrm{Disc}_2\left(\mathcal{A}_6^{spinor}\right) = -\int d^nQ\,\delta\left(q_i^2\right)\delta\left(q_j^2\right)\mathcal{A}_{tree1}^{spinor}\mathcal{A}_{tree2}^{spinor} = \mathcal{C}\ \mathrm{Disc}_2\left(\mathcal{A}_6^{scalar}\right),$$

- and:

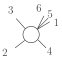
$$\mathrm{Disc}_2\left(\mathcal{A}_6^{spinor}\right) = -2\ \mathrm{Disc}_2\left(\mathcal{A}_6^{scalar}\right) + \mathcal{C}'\ \mathrm{Disc}_2\left(\mathcal{A}_6^{scalar}\right)$$
$$\mathcal{A}_6^{spinor} = -2\ \mathcal{A}_6^{scalar} + \mathcal{A}_6^{\mathcal{N}=1}$$

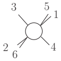
⇒ **CALCULATION OF ONLY ONE AMPLITUDE**

⇒ **THE TWO OTHER ARE STRAIGHTFORWARD**

Results for A_{--++++}

$$\mathcal{A}^{scalar/spinor/\mathcal{N}=1}_{--++++} = \frac{i\ e^6}{2\pi^2} \sum_{\sigma(1,2)} \sum_{\sigma(3..6)} R^{scalar/spinor/\mathcal{N}=1} \left([\text{diagram}]^{n+2} - [\text{diagram}]^{n+2} \right)$$





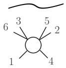
$$R^{scalar} = -\frac{\langle 13\rangle\langle 41\rangle\langle 23\rangle\langle 42\rangle}{\langle 35\rangle\langle 45\rangle\langle 36\rangle\langle 46\rangle} \frac{\langle 34\rangle}{[34]}$$

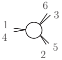
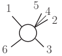
$$R^{scalar} = 2\,\frac{\langle 13\rangle\langle 42\rangle\langle 13\rangle\langle 42\rangle}{\langle 35\rangle\langle 45\rangle\langle 36\rangle\langle 46\rangle} \frac{\langle 34\rangle}{[34]}$$

$$R^{\mathcal{N}=1} = -\frac{s_{34}\langle 12\rangle^2}{\langle 35\rangle\langle 45\rangle\langle 36\rangle\langle 46\rangle}$$

$$\langle ABCD\rangle = \langle p_A - |p_B+\rangle\langle p_B + |p_C-\rangle\langle p_C - |p_D+\rangle$$

Results for $A^{scalar/fermion}_{---+++}$.

$$\mathcal{A}^{scalar/spinor}_{---+++} = \frac{i\ e^6}{\pi^2} \sum_{\sigma(1,2,3)} \sum_{\sigma(4,5,6)} \Bigg(d^{scalar/spinor} \Big($$  $$\Big)^{n+2}$$

$$+ \frac{g^{scalar/spinor}}{12} \Big($$  $$\Big)^{n} + \frac{e^{scalar/spinor}}{4} \Big($$  $$\Big)^{n+2} + h.c. \Bigg)$$

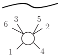
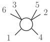
$$d^{scalar} = -\frac{\langle 24\rangle[16]}{\langle 45\rangle[31]} \frac{[1P_{425}2][6P_{425}4]}{[1P_{425}5][3P_{425}4]} \frac{\langle 1P_{425}4\rangle}{[1P_{425}4]}$$

$$e^{scalar} = -\frac{\langle 2P_{425}1\rangle\langle 2P_{425}3\rangle[36][16]s_{425}}{\langle 4P_{425}1\rangle\langle 5P_{425}3\rangle\langle 5P_{425}1\rangle\langle 4P_{425}3\rangle} \frac{\langle 31\rangle}{[31]}$$

$$g^{scalar} = \frac{[4P_{25}1]}{[1P_{25}4]} \frac{[5P_{14}2]}{[2P_{14}5]} \frac{[6P_{25}3]}{[3P_{25}6]} \sum_{\gamma_\pm} \frac{[1K^b_2 1]}{[4K^b_2 4]} \frac{[2K^b_2 2]}{[5K^b_2 5]} \frac{[3K^b_2 3]}{[6K^b_2 6]}$$

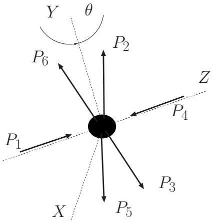
$$K_2^{b\,\mu} = \gamma_\pm\, (-P_{25})^\mu - s_{25}\, (P_{14})^\mu$$

$$\gamma_\pm = -P_{25}.P_{14} \pm \sqrt{\Delta} \qquad \Delta = (P_{25}.P_{14})^2 - P_{14}^2 P_{25}^2$$

 $$^{n+2} = \text{“Finite part”} \Big($$  $$\Big)^{n}$$

A Plot of those Amplitudes ...

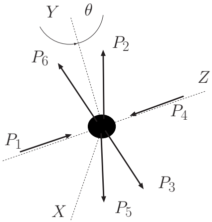
Nagy-Soper kinematical configuration (*Phys.Rev.* D **74** (2006) 093006)



$$\left\{ \begin{array}{l} \overrightarrow{p_2} = (-33.5, -15.9, -25.0) \\ \overrightarrow{p_3} = (11.0, 13.2, 22.0) \\ \overrightarrow{p_5} = (12.5, -15.3, -0.3) \\ \overrightarrow{p_6} = (10.0, 18.0, 3.3) \end{array} \right.$$

A Plot of those Amplitudes ...

Nagy-Soper kinematical configuration (*Phys.Rev.* D **74** (2006) 093006)



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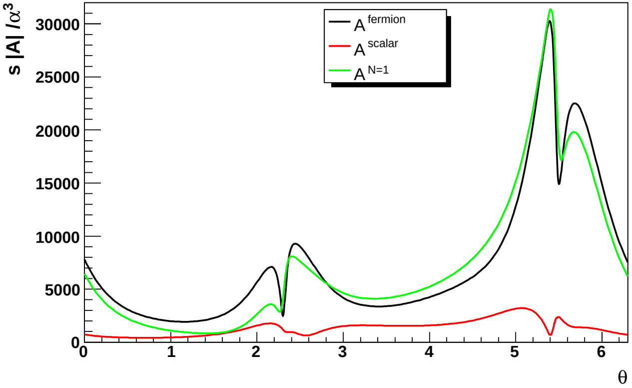
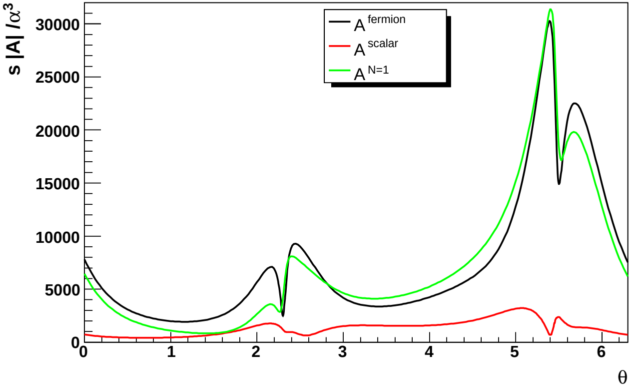
rotation of the final state around the Y-axis

The $A^{scalar/fermion}_{---+++}$ amplitudes

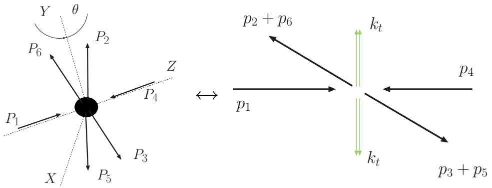


The $A^{scalar/fermion}_{---+++}$ amplitudes

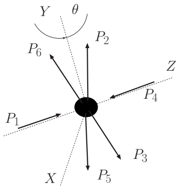


WHAT ARE THOSE DIPS ???

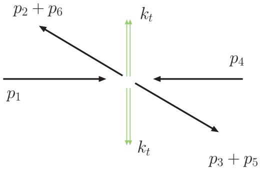
... What is it ???

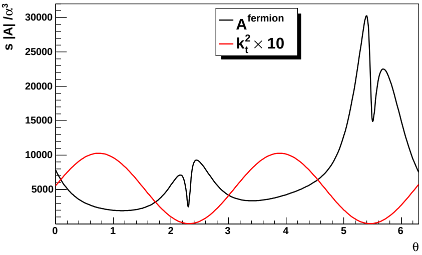


… What is it ???



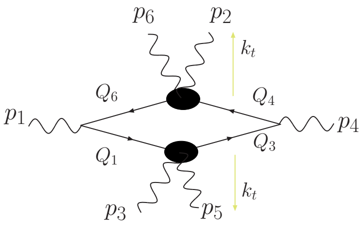
↔





⇒ **DOUBLE PARTON SCATTERING**

… Double Parton Scattering …



DOUBLE PARTON SCATTERING:

$$\text{Configuration such as } \begin{cases} Q_1 \propto Q_6 \propto p_1 \\ Q_3 \propto Q_4 \propto p_4 \end{cases}$$

$$\Leftrightarrow k_t = 0$$

$\Rightarrow$ **A LANDAU SINGULARITY !**

The Landau Singularities.

- Loop amplitude:

$$\mathcal{A}_N = \int d^n Q \, \frac{\mathrm{Num}\,(Q)}{D_1^2 \dots D_N^2}$$

The Landau Singularities.

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$$\mathcal{A}_N = \int d^n Q \, \frac{\mathrm{Num}\,(Q)}{D_1^2 \ldots D_N^2}$$

- Definition of a Landau Singularity (1957):
  = Kinematical configuration such as $\frac{\mathrm{Num}(Q)}{D_1^2 \ldots D_N^2}$ not analytic.

The Landau Singularities.

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- Definition of a Landau Singularity (1957):
  = Kinematical configuration such as $\frac{\mathrm{Num}(Q)}{D_1^2 \ldots D_N^2}$ not analytic.

- Definition of a Divergence:
  = Kinematical configuration such as $\mathcal{A}_N \to +\infty$.

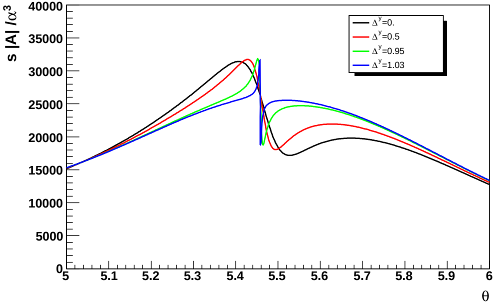
$\Rightarrow$ **In the case of six-photon : DIVERGENCE OR NOT ?**

Divergence or Not ???

Change Nagy-Soper configuration:

$$\begin{cases} \overrightarrow{p_2} = (-33.5, -15.9 - \Delta^y, -25.0) \\ \overrightarrow{p_3} = (11.0, 13.2 + \Delta^y, 22.0) \\ \overrightarrow{p_5} = (12.5, -15.3 + \Delta^y, -0.3) \\ \overrightarrow{p_6} = (10.0, 18.0 - \Delta^y, 3.3) \end{cases}$$

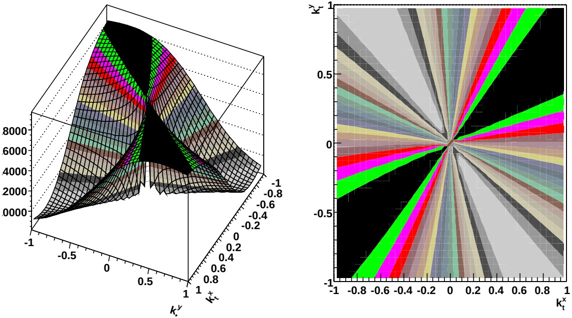
Singularity reached for $\Delta^y = 1,05$ GeV



$\Rightarrow$ **NO DIVERGENCE !**

Around the singularity ...

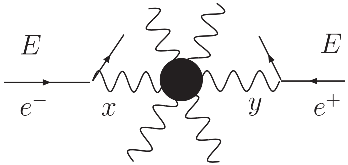
Plot the QED amplitude around the singularity:



⇒ **SADDLE POINT !**

Cross Section... 1

- Cross section of the physical process:



$$E = 100 GeV$$

- Structure fonction is a Weisacker-Williams.

Cross Section... 2

- Detection of four photons:

$$\text{Cuts in } p_t > 1 GeV$$
$$\text{Cuts in collinearity} > 0.3$$

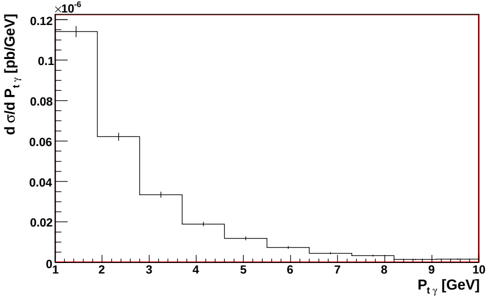
- Integration with BASES and GOLEM.

$$\sigma \sim 2.1\ 10^{-9}\text{pb} \pm 0.9\%$$

very stable result, no integration problem !!!

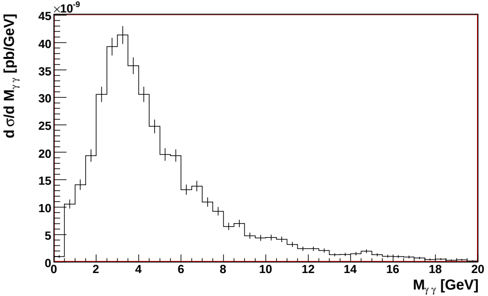
Differential Cross Section... 1

- Differential cross section versus the p_t of one photon.



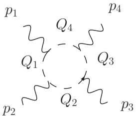
Differential Cross Section... 2

- Differential cross section versus the invariant mass of two photons.



EXTENSION TO A MASSIVE LOOP

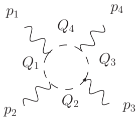
The origin of the "massive" terms

- 

$$\Rightarrow \int d^n Q \frac{\varepsilon_1.Q_1 \; \varepsilon_2.Q_2 \; \varepsilon_3.Q_3 \; \varepsilon_4.Q_4}{(Q_1^2 - m^2 + i\lambda)\ldots(Q_4^2 - m^2 + i\lambda)}$$

Q n-dimensional momentum : $\quad Q \; = \; q + \widehat{q}$

The origin of the “massive” terms

- 

$$\Rightarrow \int d^n Q \frac{\varepsilon_1.Q_1 \ \varepsilon_2.Q_2 \ \varepsilon_3.Q_3 \ \varepsilon_4.Q_4}{(Q_1^2 - m^2 + i\lambda)\ldots(Q_4^2 - m^2 + i\lambda)}$$

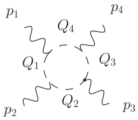
$$Q \ n\text{-dimensional momentum :} \quad Q \ = \ q + \widehat{q}$$

- Four-dimensional helicity scheme

$$\int d^n Q \frac{\varepsilon_1.Q_1 \ \varepsilon_2.Q_2 \ \varepsilon_3.Q_3 \ \varepsilon_4.Q_4}{(Q_1^2 - m^2 + i\lambda)\ldots(Q_4^2 - m^2 + i\lambda)}$$

$$\Rightarrow \int d^n Q \frac{\varepsilon_1.\mathbf{q}_1 \ \varepsilon_2.\mathbf{q}_2 \ \varepsilon_3.\mathbf{q}_3 \ \varepsilon_4.\mathbf{q}_4}{(Q_1^2 - m^2 + i\lambda)\ldots(Q_4^2 - m^2 + i\lambda)}$$

The origin of the "massive" terms

- 

$$\Rightarrow \int d^nQ \frac{\varepsilon_1.Q_1\ \varepsilon_2.Q_2\ \varepsilon_3.Q_3\ \varepsilon_4.Q_4}{(Q_1^2-m^2+i\lambda)\dots(Q_4^2-m^2+i\lambda)}$$

$$Q\ n\text{-dimensional momentum}: \quad Q = q+\widehat{q}$$

- Four-dimensional helicity scheme

$$\int d^nQ \frac{\varepsilon_1.Q_1\ \varepsilon_2.Q_2\ \varepsilon_3.Q_3\ \varepsilon_4.Q_4}{(Q_1^2-m^2+i\lambda)\dots(Q_4^2-m^2+i\lambda)} \Rightarrow \int d^nQ \frac{\varepsilon_1.\mathbf{q}_1\ \varepsilon_2.\mathbf{q}_2\ \varepsilon_3.\mathbf{q}_3\ \varepsilon_4.\mathbf{q}_4}{(Q_1^2-m^2+i\lambda)\dots(Q_4^2-m^2+i\lambda)}$$

- Reduce denominators:

$$q_1^2 \rightarrow Q_1^2 - m^2 + (m^2+\widehat{q}^2)$$

- Introduce extension of extra scalar integrals:

$$K_4^n = \int d^n Q \frac{\left(m^2 + \widehat{q}^2\right)^2}{(Q_1^2 + i\lambda)\dots(Q_4^2 + i\lambda)}$$

$$J_4^n = \int d^n Q \frac{m^2 + \widehat{q}^2}{(Q_1^2 + i\lambda)\dots(Q_4^2 + i\lambda)}$$

$$J_3^n = \int d^n Q \frac{m^2 + \widehat{q}^2}{(Q_1^2 + i\lambda)\dots(Q_3^2 + i\lambda)}$$

- Massive terms directly link to rational terms.

- C.Bernicot, "Light-Light amplitude from generalized unitarity in massive QED", PRD, [arXiv:hep-ph/0804.1315]

- In all order of ϵ.
-

$$\mathcal{A}_4^{scalar}{}_{(++++)} = i\frac{4e^4}{(4\pi)^{n/2}} \sum_{\sigma(2,3,4)} \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} K_4^n$$

- In all order of ϵ.
- $$\mathcal{A}_4^{scalar}{}_{(++++)} = i\frac{4e^4}{(4\pi)^{n/2}} \sum_{\sigma(2,3,4)} \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} K_4^n$$
- $$\mathcal{A}_4^{scalar}{}_{(-+++)} = i\frac{4e^4}{(4\pi)^{n/2}} \sum_{\sigma(2,3,4)} \frac{[34][231]}{\langle 34\rangle\langle 231\rangle} \left(K_4^n - \frac{st}{2u} J_4^n + \left(\frac{s^2+t^2+u^2}{2tu}\right) J_3^n \right)$$

- In all order of ϵ.
- $$\mathcal{A}_4^{scalar}{}_{(++++)} = i\frac{4e^4}{(4\pi)^{n/2}} \sum_{\sigma(2,3,4)} \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} K_4^n$$
- $$\mathcal{A}_4^{scalar}{}_{(-+++)} = i\frac{4e^4}{(4\pi)^{n/2}} \sum_{\sigma(2,3,4)} \frac{[34][231]}{\langle 34\rangle\langle 231\rangle} \left(K_4^n - \frac{st}{2u} J_4^n + \left(\frac{s^2+t^2+u^2}{2tu}\right) J_3^n \right)$$
- $$\mathcal{A}_4^{scalar}{}_{(--++)} = i\frac{4e^4}{(4\pi)^{n/2}} \frac{\langle 12\rangle}{[12]} \frac{[34]}{\langle 34\rangle} \left(-\frac{2tu}{s} I_4^{n+2} + \sum_{\sigma(1,2)} \left(\frac{t-u}{s} I_2^n + 4\frac{u}{s} J_3^n \right) + \sum_{\sigma(2,3,4)} K_4^n \right)$$

SIX-PHOTON AMPLITUDE IN MASSIVE LOOP

- The first helicity amplitude $\mathcal{A}_6(++++++)$:

$$\mathcal{A}_6^{scalar} = -i\frac{(e\sqrt{2})^6 m^6}{96\pi^2}\sum_{\sigma(1,2,3,4,5,6)}\frac{[12][34][56]}{\langle 12\rangle\langle 34\rangle\langle 56\rangle}I_6^n$$
$$+i\frac{(e\sqrt{2})^6 m^4}{96\pi^2}\sum_{\sigma(1,2,3,4,5,6)}\frac{[3(1+2)6(5+4)3]}{2\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}I_{4,2B}^n(s_{12},s_{45})$$
$$+i\frac{(e\sqrt{2})^6 m^4}{96\pi^2}\sum_{\sigma(1,2,3,4,5,6)}\frac{\langle 54(1+2+3)65\rangle}{2\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}I_{4,1}^n(s_{123})$$

-

$$\mathcal{A}_6^{spinor} = -2\,\mathcal{A}_6^{scalar}$$
$$\mathcal{A}_6^{\mathcal{N}=1} = 0$$

Conclusion.

- Reduce one-loop diagram:

  ⋆ Helicity Amplitude Method

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  - ⋆ Basis of decomposition I_4^{n+2}, I_3^n, I_2^n.
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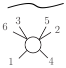
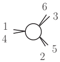
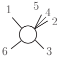
- Future prospect ... collaboration to calculate $ee \to ee\gamma$ in massive loop ...

$$A^{scalar/spinor}_{---+++} = \frac{i\ e^6}{\pi^2} \sum_{\sigma(1,2,3)} \sum_{\sigma(4,5,6)} \Bigg(d^{scalar/spinor} \Bigg($$ 

$$^{n+2} \Bigg)$$

$$+ \frac{g^{scalar/spinor}}{12} \Bigg($$ 

$$^{n} \Bigg) + \frac{e^{scalar/spinor}}{4} \Bigg($$ 

$$^{n+2} \Bigg) + h.c. \Bigg)$$

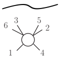
$$d^{scalar} = -\frac{\langle 24\rangle[16]}{\langle 45\rangle[31]} \frac{[1P_{425}2][6P_{425}4]}{[1P_{425}5][3P_{425}4]} \frac{\langle 1P_{425}4\rangle}{[1P_{425}4]}$$

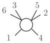
$$e^{scalar} = -\frac{\langle 2P_{425}1\rangle\langle 2P_{425}3\rangle[36][16]s_{425}}{\langle 4P_{425}1\rangle\langle 5P_{425}3\rangle\langle 5P_{425}1\rangle\langle 4P_{425}3\rangle} \frac{\langle 31\rangle}{[31]}$$

$$g^{scalar} = \frac{[4P_{25}1]}{[1P_{25}4]} \frac{[5P_{14}2]}{[2P_{14}5]} \frac{[6P_{25}3]}{[3P_{25}6]} \sum_{\gamma\pm} \frac{[1K_2^b1]}{[4K_2^b4]} \frac{[2K_2^b2]}{[5K_2^b5]} \frac{[3K_2^b3]}{[6K_2^b6]}$$

$$K_2^{b\mu} = \gamma_\pm\,(-P_{25})^\mu - s_{25}\,(P_{14})^\mu$$

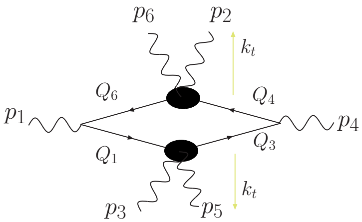
$$\gamma_\pm = -P_{25}.P_{14} \pm \sqrt{\Delta} \qquad \Delta = (P_{25}.P_{14})^2 - P_{14}^2 P_{25}^2$$



$$^{n+2} = \text{“Finite part”} \Bigg($$ 

$$^{n} \Bigg)$$

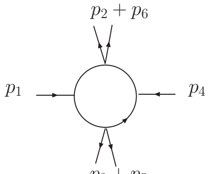
... Double Parton Scattering...



DOUBLE PARTON SCATTERING:

$$\text{Configuration such as} \left\{ \begin{array}{ccccc} Q_1 & \propto & Q_6 \propto & p_1 \\ Q_3 & \propto & Q_4 & \propto & p_4 \end{array} \right. \Leftrightarrow \quad k_t = 0 \Leftrightarrow \det(S) = 0$$

$S_{ij} = (q_i - q_j)^2 - m_i^2 - m_j^2$ kinematical matrix of :



$$A = \sum_i a_i I^n_{4,i} + \sum_i b_i I^n_{3,i} + \sum_i c_i I^n_{2,i} + \text{rational terms}$$

But we have

$$I^n_4 = \sum_i \beta_i I^n_{3,i} - \frac{\det(G)}{\det(S)} I^{n+2}_4$$

So,

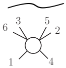
$$A = -\sum_i a_i \frac{\det(G)}{\det(S)} I^{n+2}_{4,i} + \sum_i (b_i + a_i \beta_i) I^n_{3,i} + \sum_i c_i I^n_{2,i} + \text{rational terms}$$

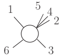
With the three cut technics it is very easy to calculate the coefficient $b_i + a_i\beta_i$.

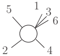
Results for $A^{\mathcal{N}=1}_{---+++}$

$$A^{\mathcal{N}=1}_{---+++} = \frac{i\ e^6}{\pi^2} \sum_{\sigma(1,2,3)} \sum_{\sigma(4,5,6)} d^{\mathcal{N}=1}$$

$$\left(\langle 1 P_{425} 4 P_{425} 1 \rangle \left(\text{[diagram 1]} \right)^{n+2} + \frac{s_{13} \left(\text{[diagram 2]} \right)^{n+2} + s_{45} \left(\text{[diagram 3]} \right)^{n+2}}{2} \right)$$







$$d^{\mathcal{N}=1} = \frac{[6 P_{425} 2]^2}{[31]\langle 45 \rangle [1 P_{425} 5][3 P_{425} 4]}$$

NO TRIANGLE

Origin of the rational terms

- Dispersive relation if $\mathcal{A}(s)$ analytic function:

$$A(s) = A(s_0) + \frac{s - s_0}{\pi} \int dS \frac{\mathrm{Im}(A(S))}{(S - s + i\lambda)(S - s_0 + i\lambda)}$$

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NO RATIONAL TERMS

- The rational terms link to the ultraviolet divergences.
- Regularization in n dimension $\Rightarrow$ cut in n dimensions.