Modelling with X-FEM dynamic propagation and arrest of a cleavage crack in PWR steel

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Outline of the speech

Context and Objectives

I. Numerical tools

II. Experimental support

III. Model of Propagation

step 1: Proposition of a model

step 2: Predictive Simulations

Conclusion and Prospect

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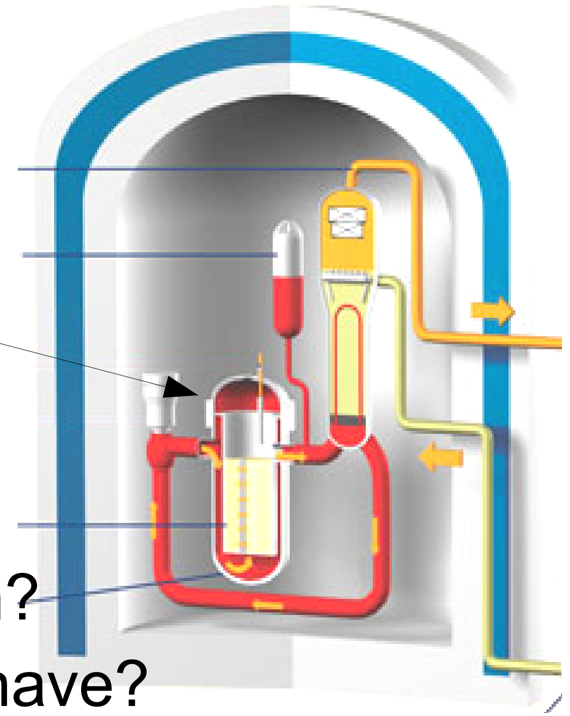
Context and Objectives



- **PWR life extension**

- Case of an **accidental pressured thermal shock** of PWR vessel:

→ Lot of work on brittle fracture
→ Ensure the non-initiation of a hypothetical defect

- What about **brittle crack propagation**?

→ After initiation, how will the crack behave?
→ Consequence for the structure integrity?

- **Thesis goals:**

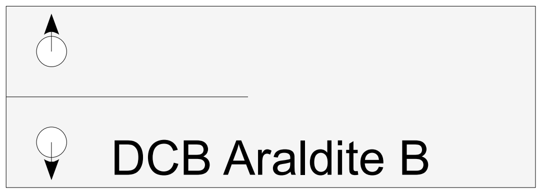
→ Understand phenomena occurring during propagation of a cleavage crack in a PWR steel up to arrest
→ Propose a model of propagation validated by experiments

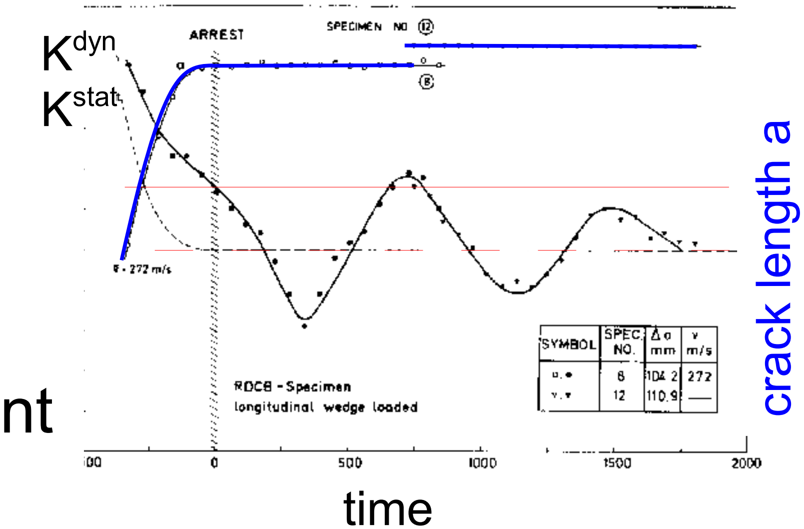
Context and Objectives

How to deal with dynamic crack propagation?

Elastic dynamic analysis?

- **Kalthoff's observations**:

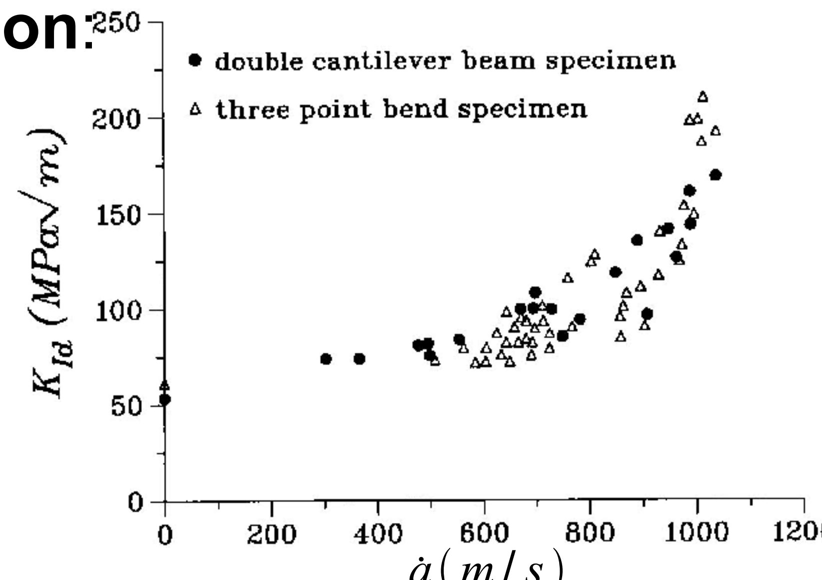




→ Dynamic analysis is relevant

- **Kaninnen's empirical relation**:

$$K^{dyn}(\dot{a}) = \frac{K_A}{1-\left(\dot{a}/v_{lim}\right)^m}$$



→ Simple model for a brittle crack

→ Limited when plasticity is not negligible

Context and Objectives

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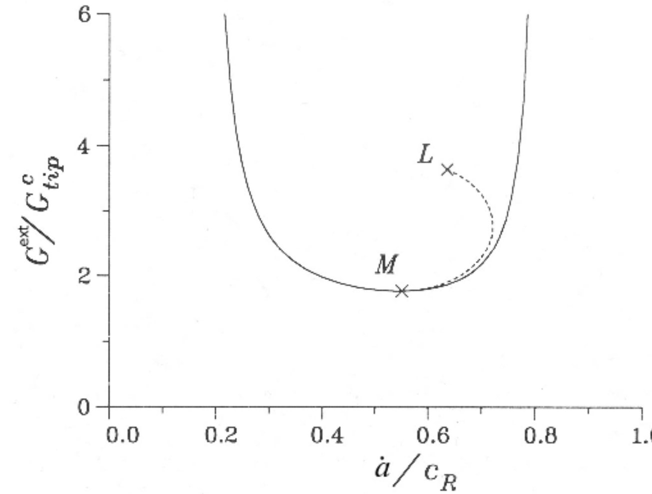
How to deal with dynamic crack propagation?

Elastic-viscoplastic analysis:

- **Freund's theoretical model**:

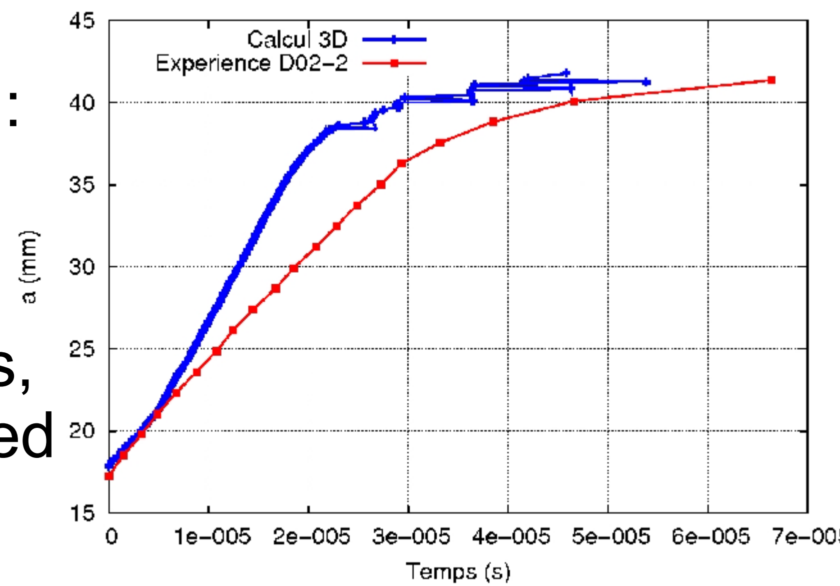
→ Crack speed predicted >> observed



- **RKR model used by Hajjaj**:

$$\sigma_I(r=100\mu m) = \sigma_{Ic}(T)$$

→ Relatively good predictions, but only 1 configuration studied



Context and Objectives

To investigate crack propagation and arrest, one needs to perform **Dynamic analysis** taking into account:

→ Inertial effects on mechanical fields

→ Strain rate dependent constitutive law

• Relevant **Experimental data** should be collected:

→ as crack speed measurements

→ for different configurations (geometries and loadings)

• Efficient **Numerical tool** should be used to model:

→ any crack propagation

→ with minimal effort,

→ accurately,

→ for any constitutive law or any configuration hypothesis

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Modelling: Numerical tools

Difficulties encountered with classical numerical methods for modelling arbitrary crack growth in Finite Element:

- Node release method → suppose to know the crack path
- Deletion element → dissipated energy depend on mesh
- Cohesive zone model → mesh-dependency
- Re-meshing → problem with projection of fields
- ...

Efficiency of methods based on Partition of Unity like:

- The **eXtended Finite Element Method*** :

→ the implicit representation of a crack by level set

→ ad hoc enrichment of the displacement with additional degrees of freedom

► **Implementation of the X-FEM in Cast3M**

* : *[Belytschko 99], [Moes 99], [Gravouil 02], ...*

Modelling: Numerical tools

The eXtended Finite Element Method:

- **Implicit Crack Description**

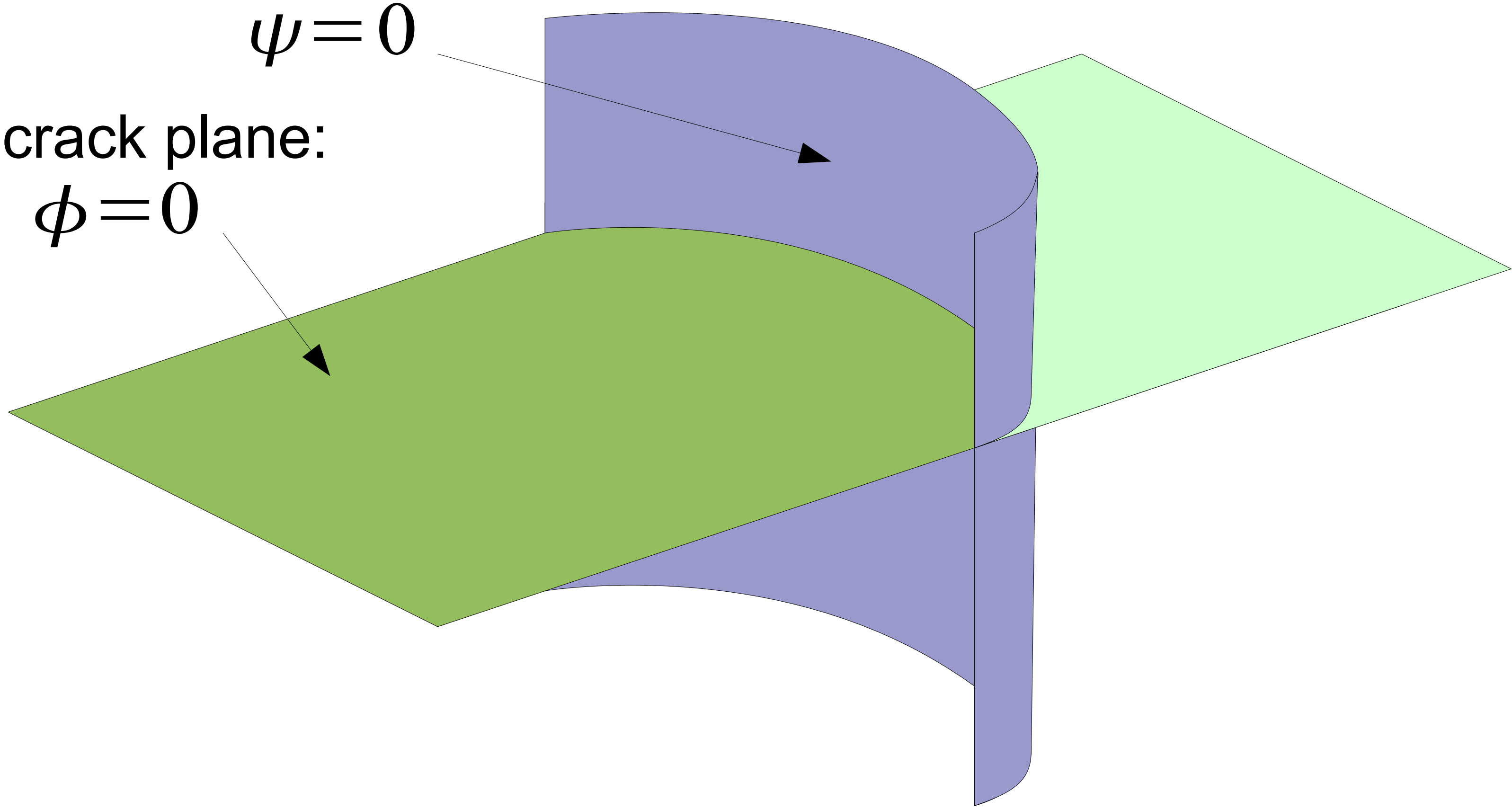
→ by a couple of Level Set (or "distance") functions

the crack front:

$\psi = 0$

the crack plane:

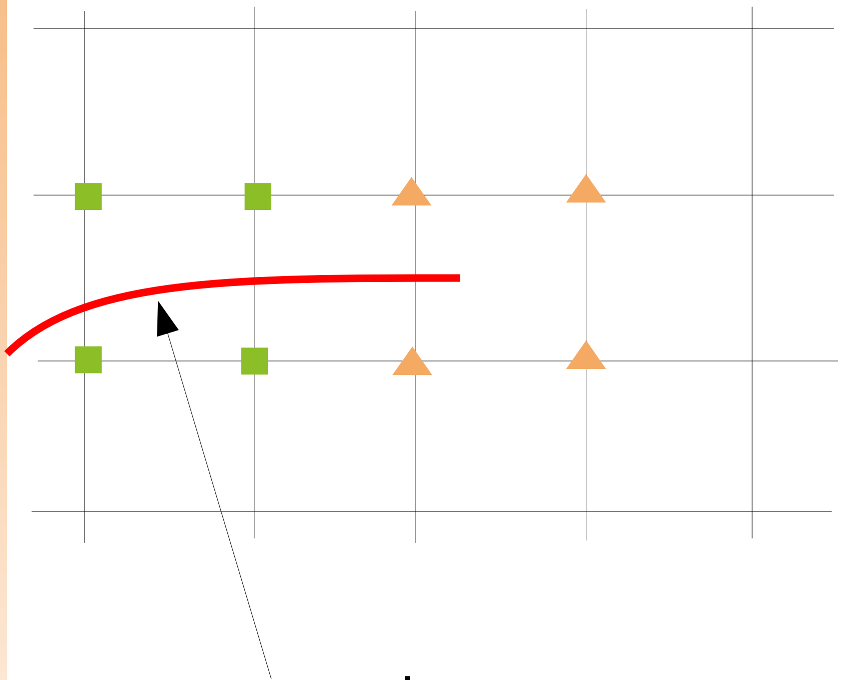
$\phi = 0$



Modelling: Numerical tools

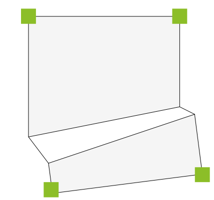
The eXtended Finite Element Method:

- **Enrichment of the displacement approximation**



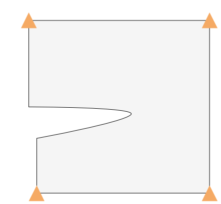
H-enrichment:
→ discontinuous function

$$H(\underline{x}) = \begin{cases} +1 & si\ \phi > 0 \\ -1 & si\ \phi < 0 \end{cases}$$



F-enrichment:
→ asymptotic functions

$$F_j(\underline{x}) = \sqrt{r}\begin{cases} \sin(\theta/2) \\ \sin(\theta/2)\sin(\theta) \\ \cos(\theta/2) \\ \cos(\theta/2)\sin(\theta) \end{cases}$$

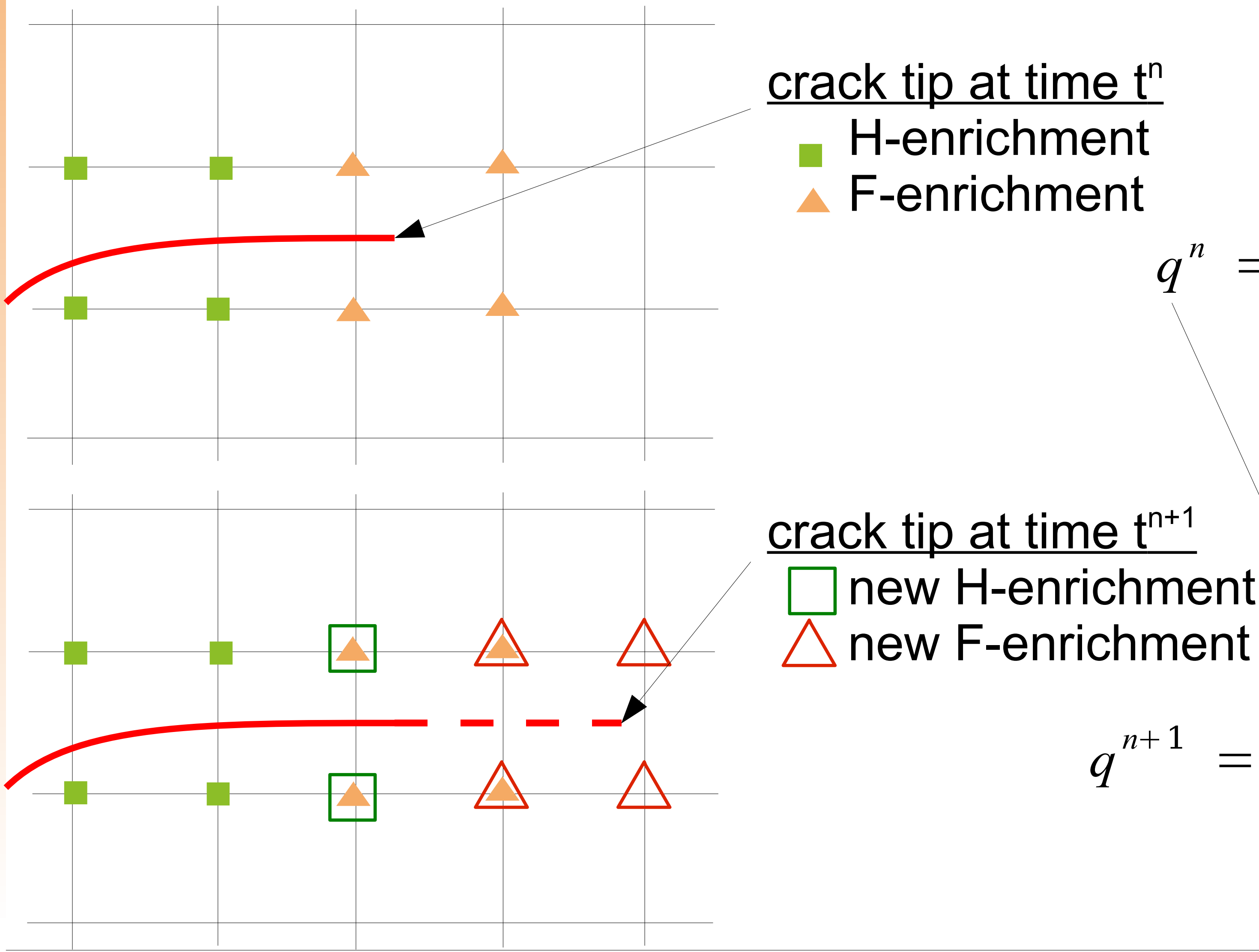


$$u(x) \simeq \sum_i N_i(x)\left[u_i + H(x_i)\, a_i + \left(\sum_{j=1,..4} F_j(x_i)\, b_{i,j}\right)\right]$$

Modelling: Numerical tools

The eXtended Finite Element Method:

- **Enrichment of the displacement approximation**



$$q^n = \begin{bmatrix} u_i \\ a_i^n \\ b_{i,j}^n \end{bmatrix}$$

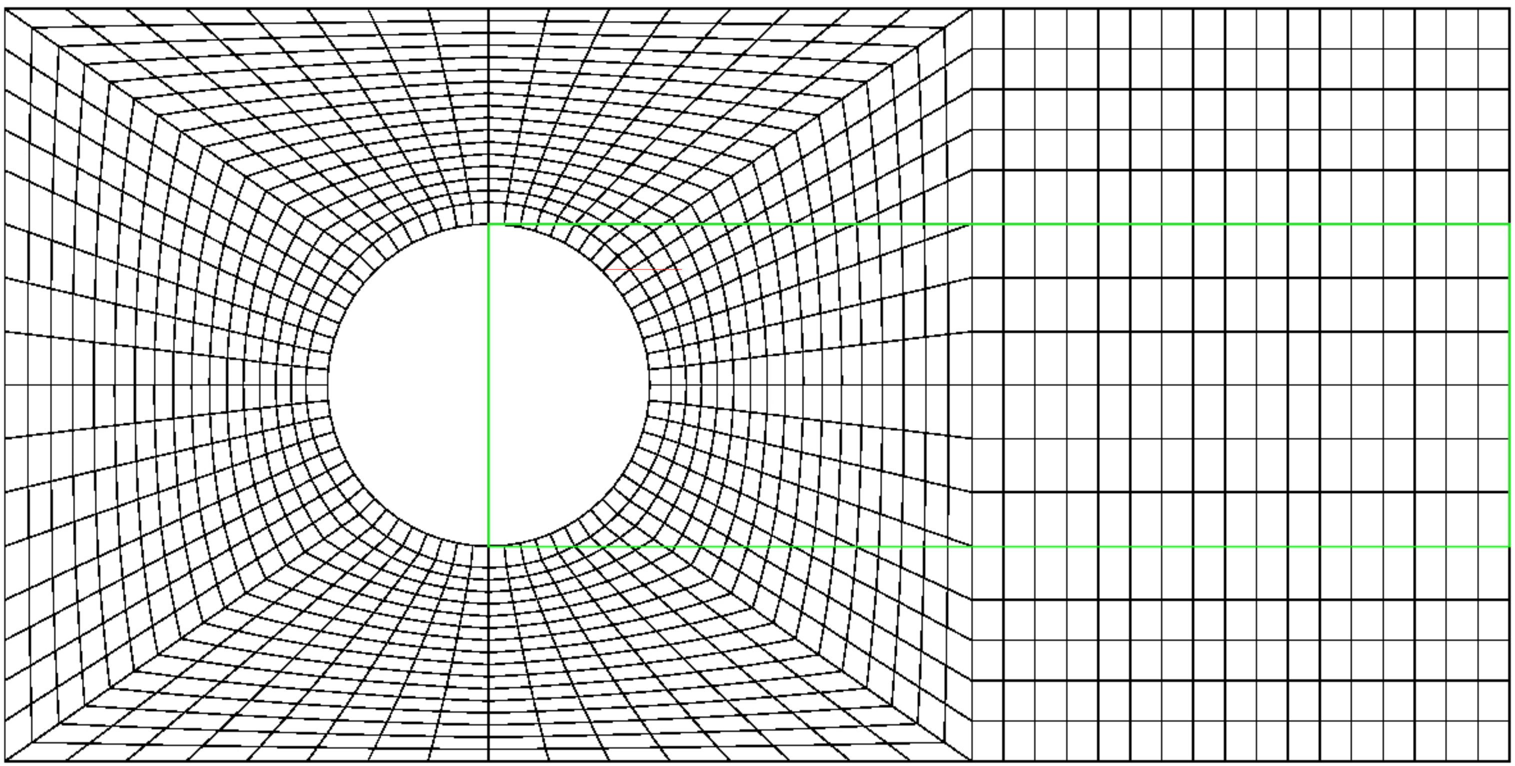
$$q^{n+1} = \begin{bmatrix} q^n \\ a_i^{n+1} \\ b_{i,j}^{n+1} \end{bmatrix}$$

Modelling: Numerical tools

The eXtended Finite Element Method:

• Level Set functions defined on a regular auxiliary grid different from the mechanical mesh

→ **mechanical mesh** (not fitted to level set update)



$$\underline{div}(\underline{\sigma}) = \rho\,\ddot{u} \quad \Rightarrow \quad M\ddot{q} + \int B^T \sigma = F^{ext}$$

Modelling: Numerical tools

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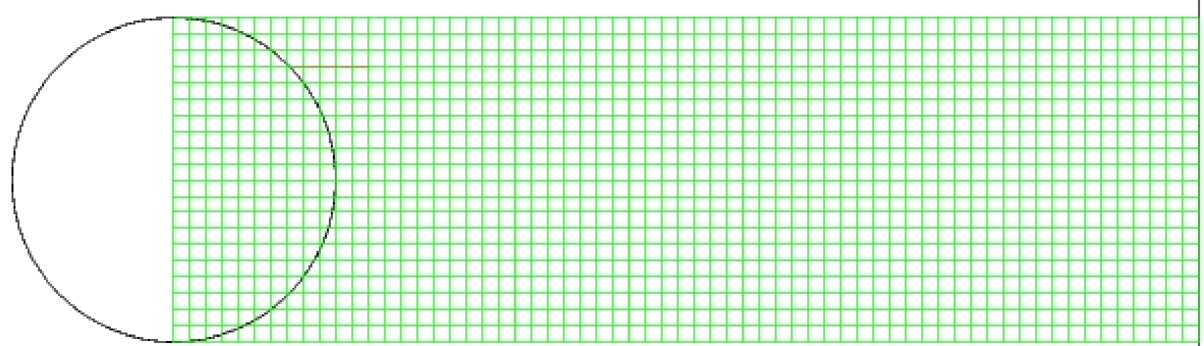
The eXtended Finite Element Method:

• Level Set functions defined on a regular auxiliary grid* different from the mechanical mesh

→ **auxiliary grid** (easy, fast and accurate level set update)



$$\frac{\partial \psi}{\partial t} = \underline{V}_{\psi} \cdot \underline{\nabla} \psi \quad \Rightarrow \quad \frac{\psi_{i,j}^{t+\Delta t} - \psi_{i,j}^{t}}{\Delta t} = H\left(V_{i,j}, \psi_{m,n}\right)$$

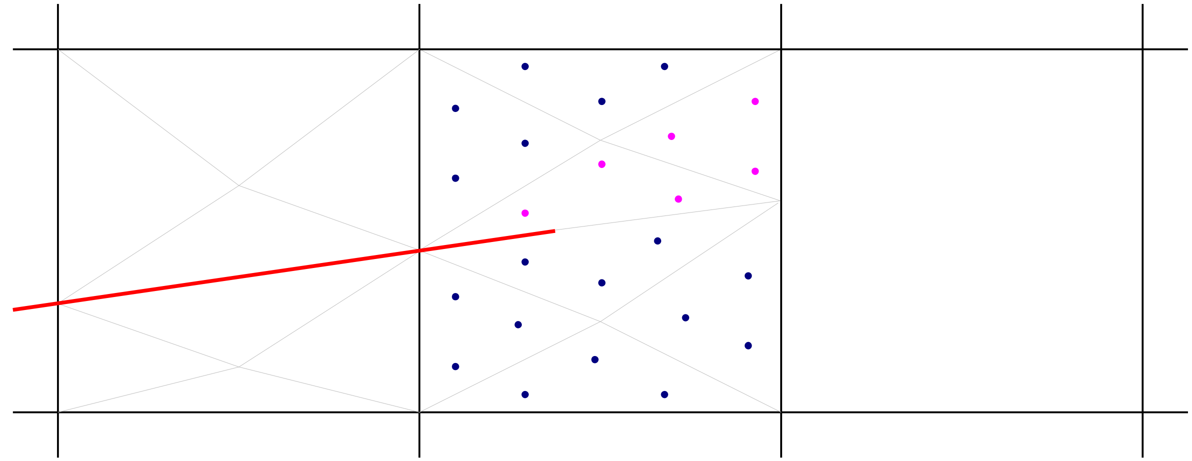
* : *[Prabel et al. 07]*

Modelling: Numerical tools

The eXtended Finite Element Method:

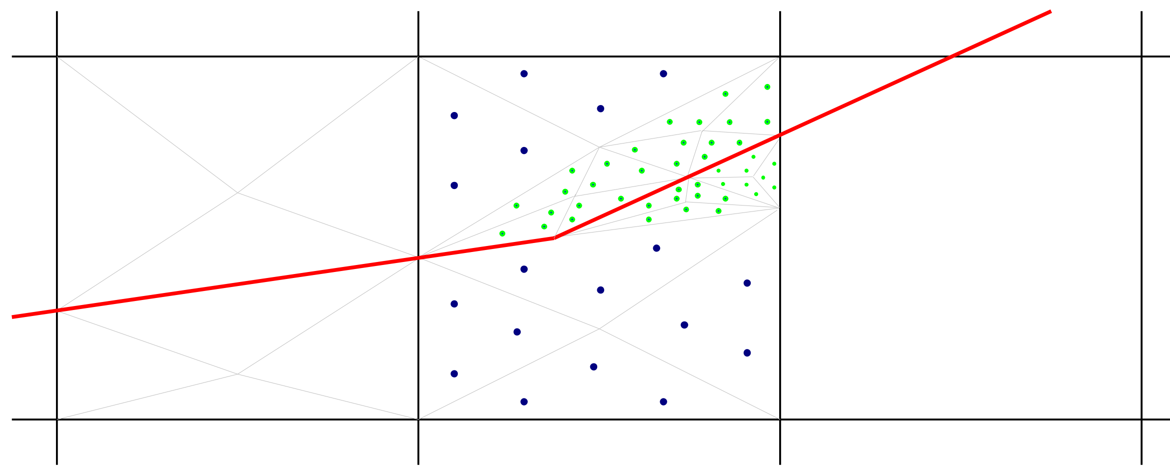
- Standard Numerical **Integration**:

Usually, conformed sub-triangle partitioning



Crack growth → Change in Gauss points location

→ **Projection!**



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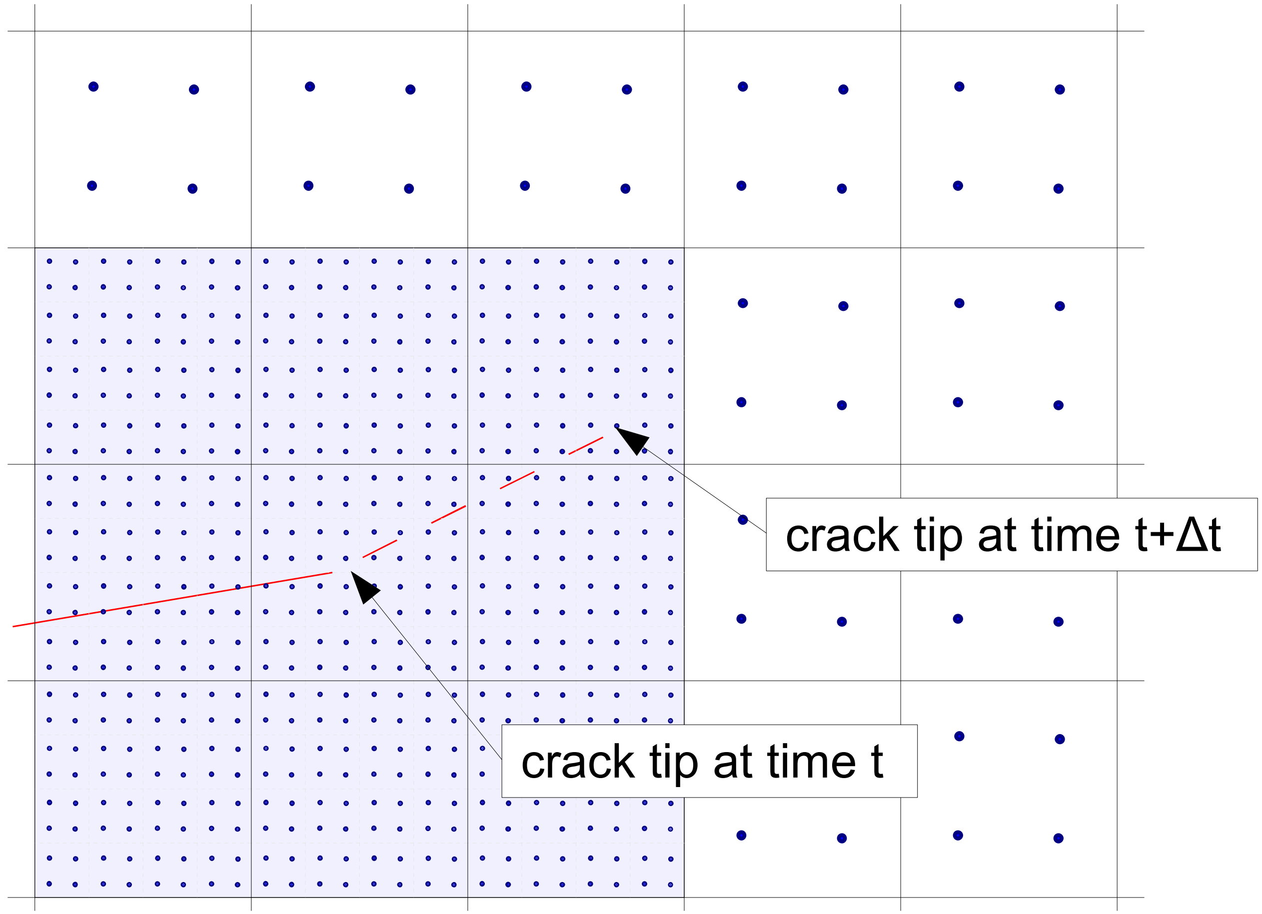
The eXtended Finite Element Method:

- Selected Numerical **Integration**:

Alternative, non-conformed sub-partitioning*



*: *[El guedj 06], [Prabel et al. 07]*

Modelling: Numerical tools

The eXtended Finite Element Method:

- Selected Numerical **Integration**:

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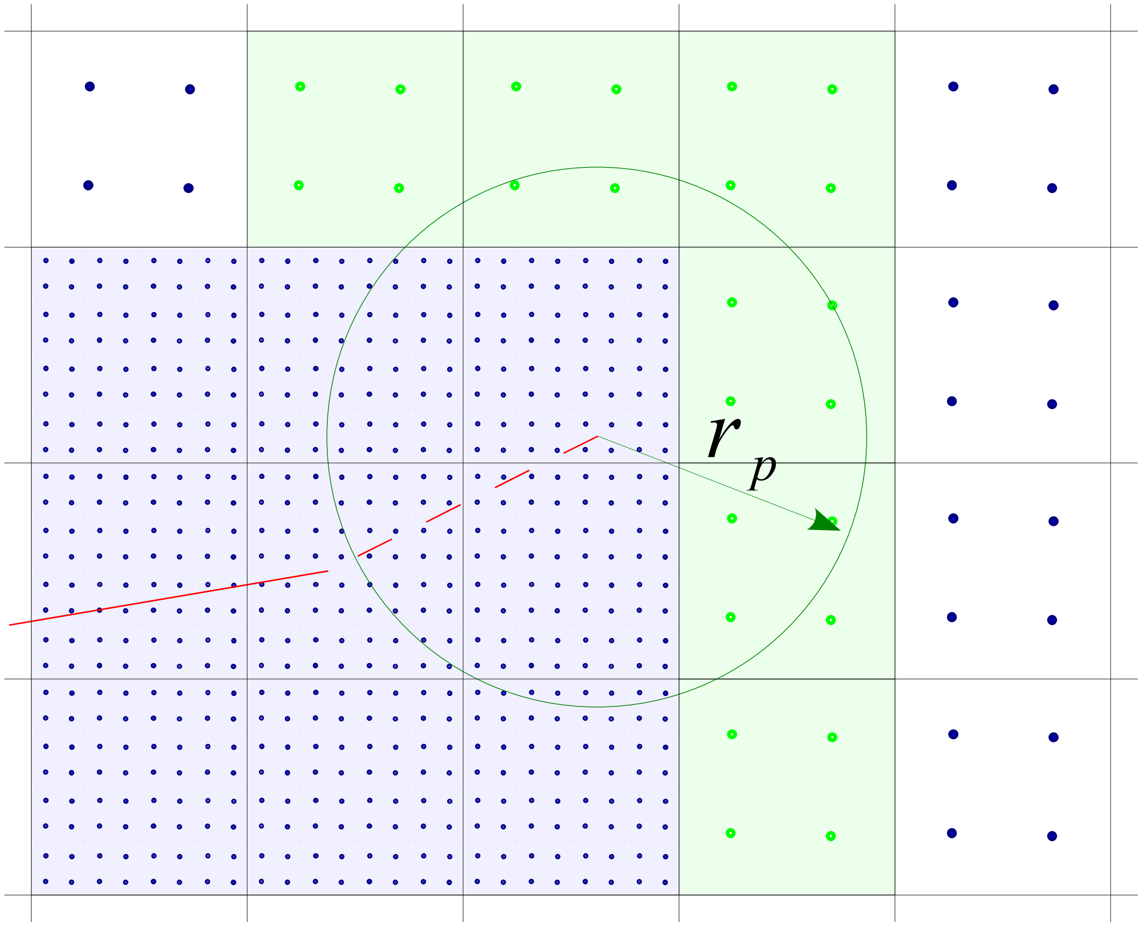


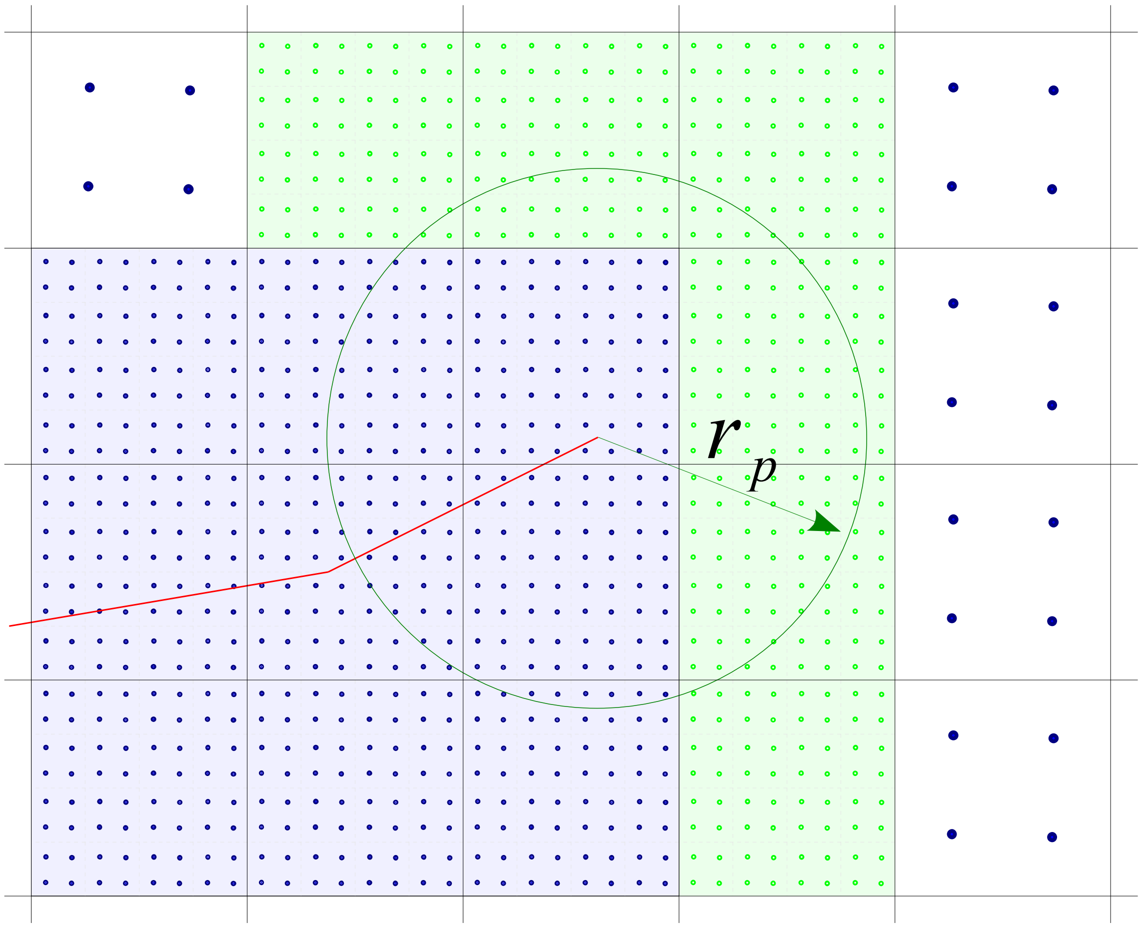
→ **Anticipate** the crack arrival, and the plasticity

Modelling: Numerical tools

The eXtended Finite Element Method:

- Selected Numerical **Integration**:

Alternative, non-conformed sub-partitioning

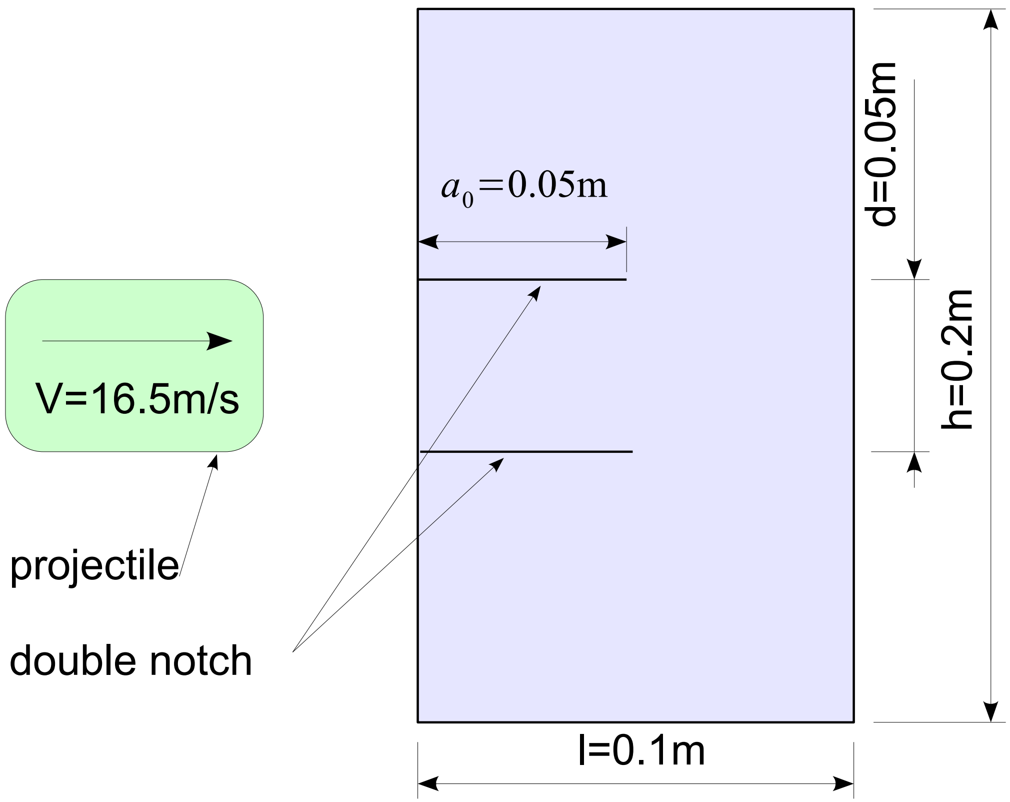


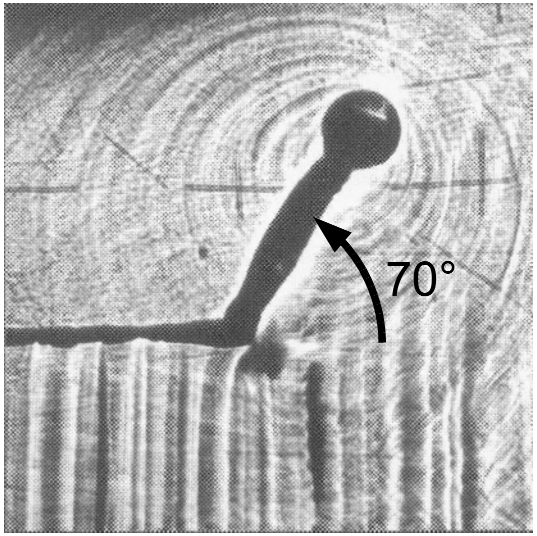
→ **Anticipate** the crack arrival, and the plasticity

Modelling: Numerical tools

The eXtended Finite Element Method:

- Numerical **Application**: → **Kalthoff** experiment



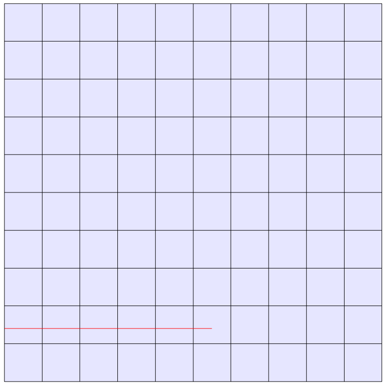


High strength steel

→ LEFM criteria:

$$\dot{a} = c_R\left[1-\frac{K_A}{K_{eq}^{\mathrm{dyn}}}\right] \qquad \theta_c = \{\theta, max(\sigma_{\theta\theta}(r_c))\} = f(K_I, K_{II})$$

→ Easy Mesh :



Modelling: Numerical tools

The eXtended Finite Element Method:

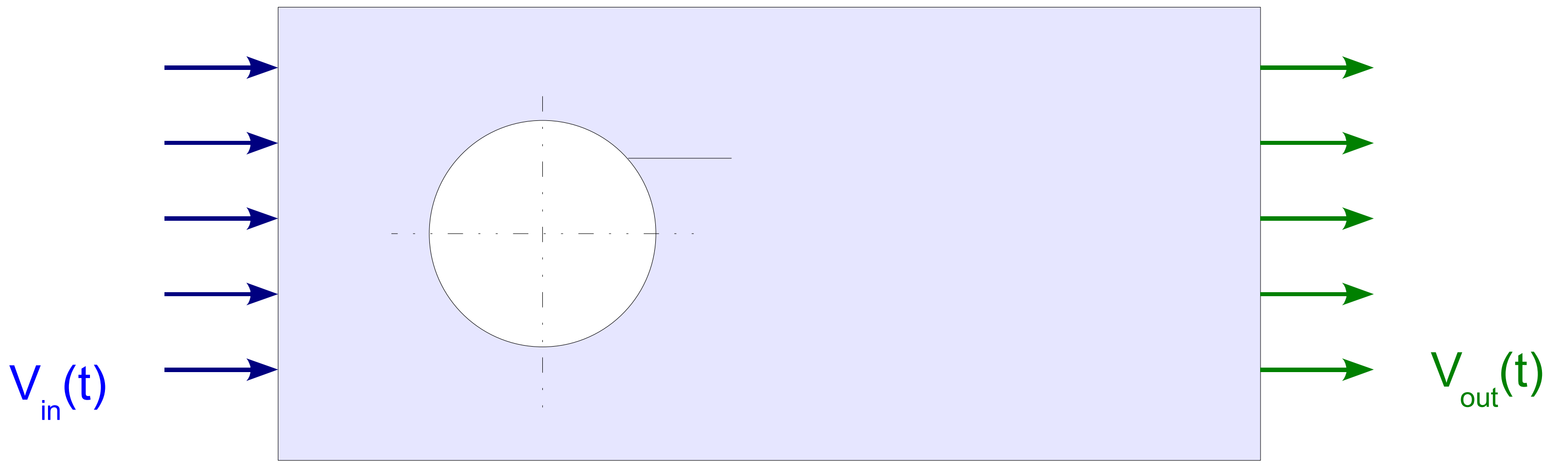
- Numerical **Application**: → **Kalthoff** experiment

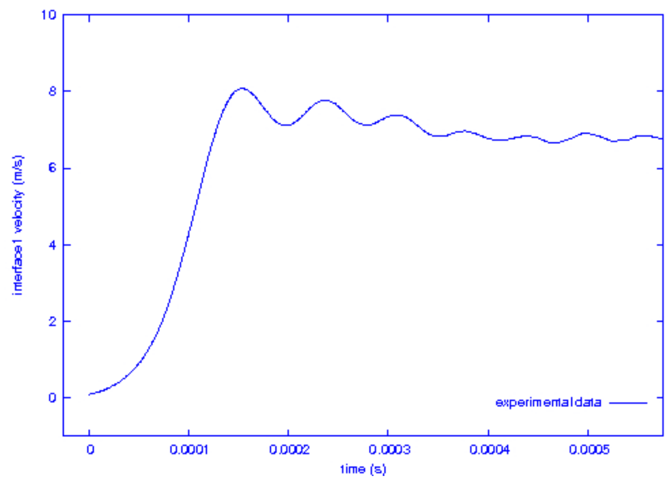
Modelling: Numerical tools

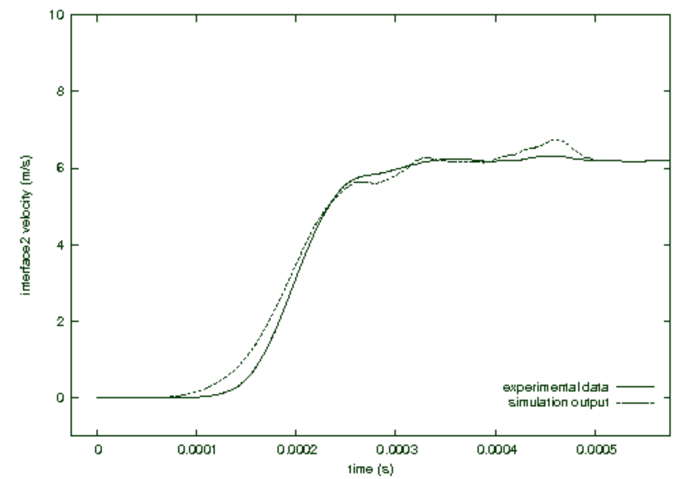
The eXtended Finite Element Method:

- Numerical **Application**: → **Gregoire** experiment*

Notched PMMA specimen between Hopkinson bars







* : *[Grégoire et al. 07]*

Modelling: Numerical tools

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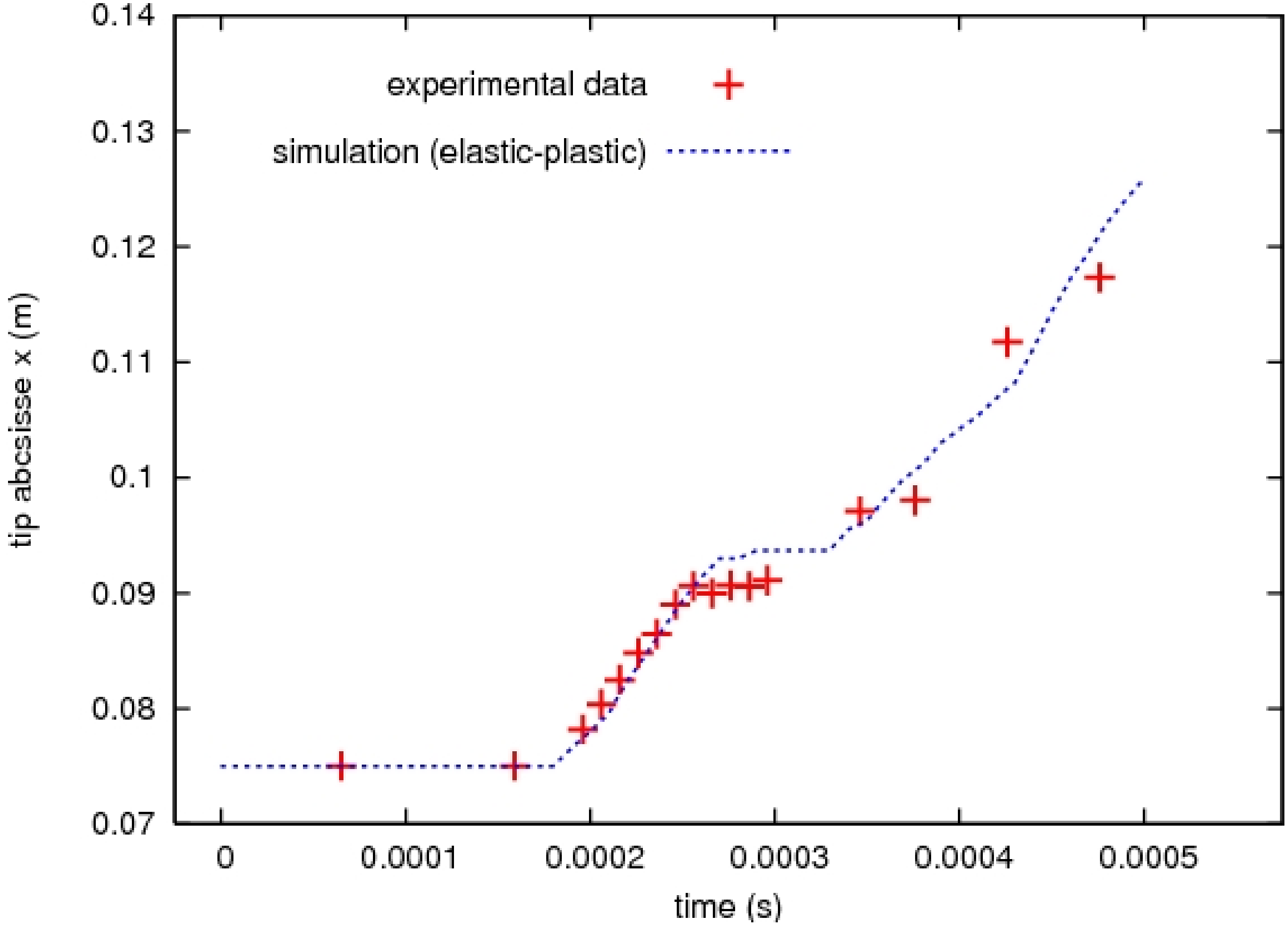
The eXtended Finite Element Method:

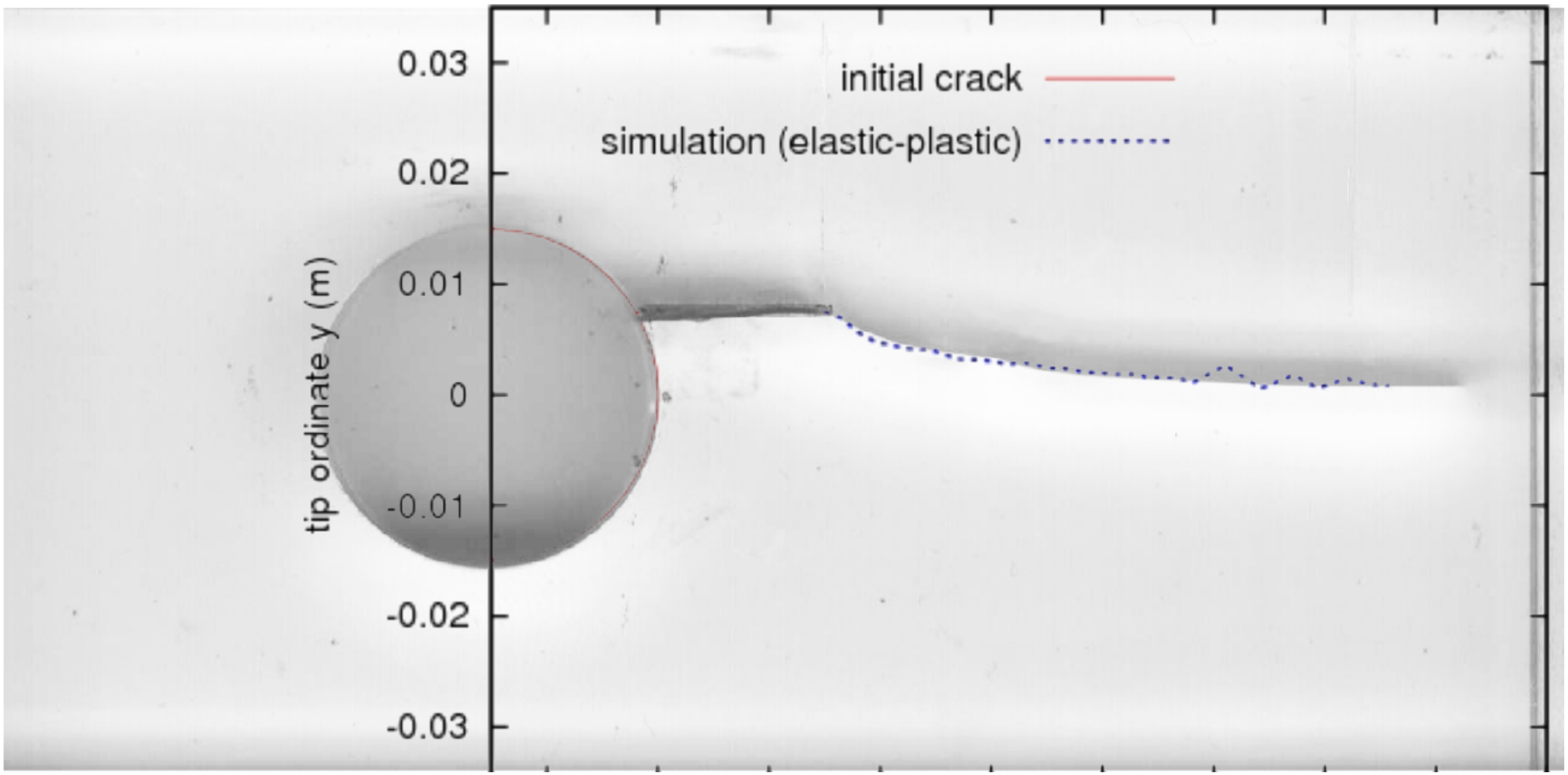
- Numerical **Application**: → **Gregoire** experiment



Modelling: Numerical tools

The eXtended Finite Element Method:

- Numerical **Application**: → **Gregoire** experiment



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Experimental support

Available experimental data for 16MND5 steel:

- Very important material characterization* in term of:
  - → static constitutive law (from -175°C to 25°C)
  - → cleavage initiation (triaxiality, WPS, ...)

Experimental **needs** for dynamic fracture:

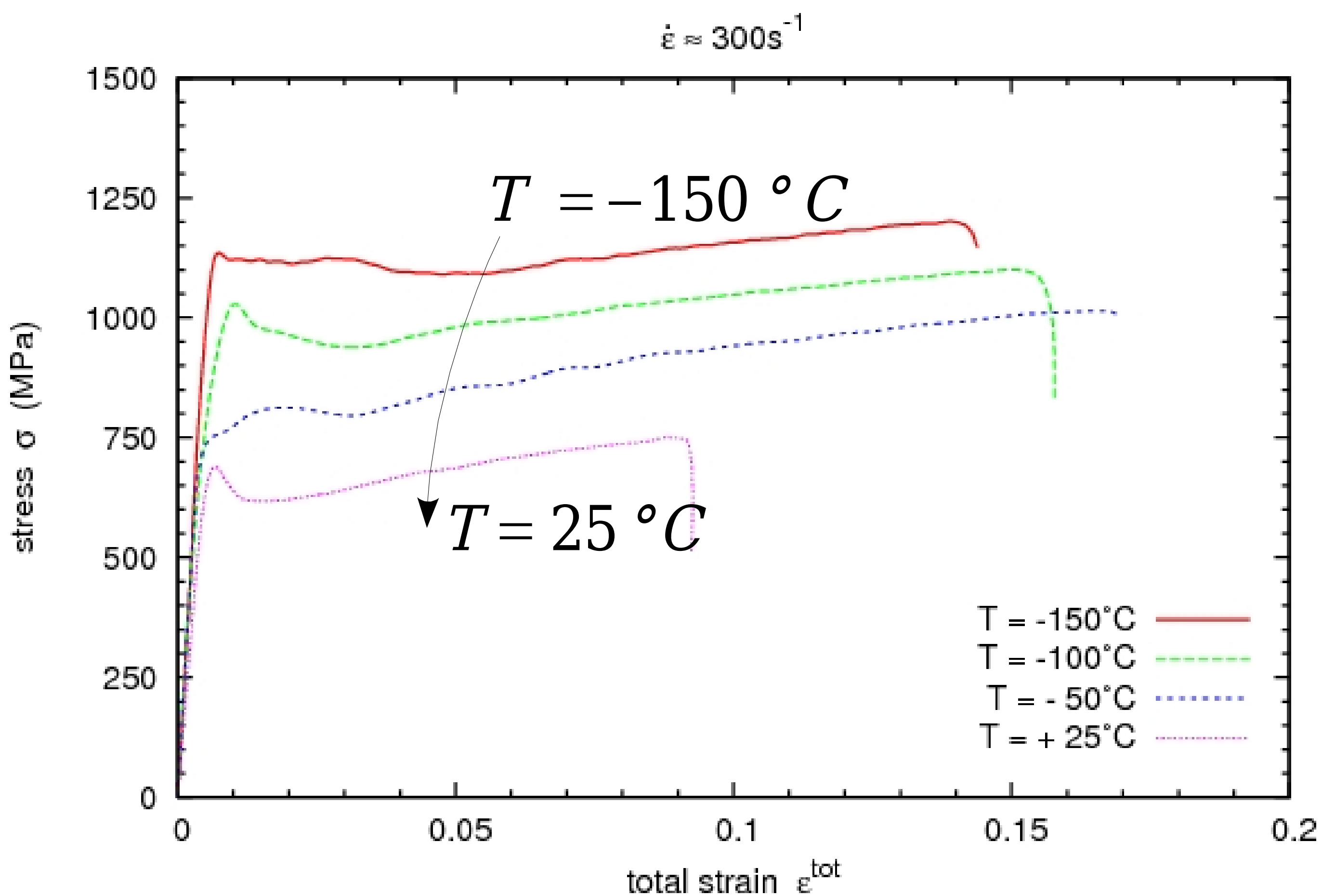
- **Dynamic material characterization**:
  - → strain rate effect (in temperature range)
- **Fracture tests**:
  - → on laboratory specimen (CT)
  - → on analytical specimen (Ring loaded in mode I)
  - → for various configurations (Ring in mixed mode)
- **Fracture surface observations**:
  - → to determine fracture mechanism
  - → for every specimen type,
  - → at every stage of the propagation

** : Campaign FISTER (2000-2004); Campaign CRITER (2002-...) ~ sponsored by IRSN*

Experimental support

Material:

- French PWR ferritic steel: 16MND5 (A508)
- Temperature characterization:

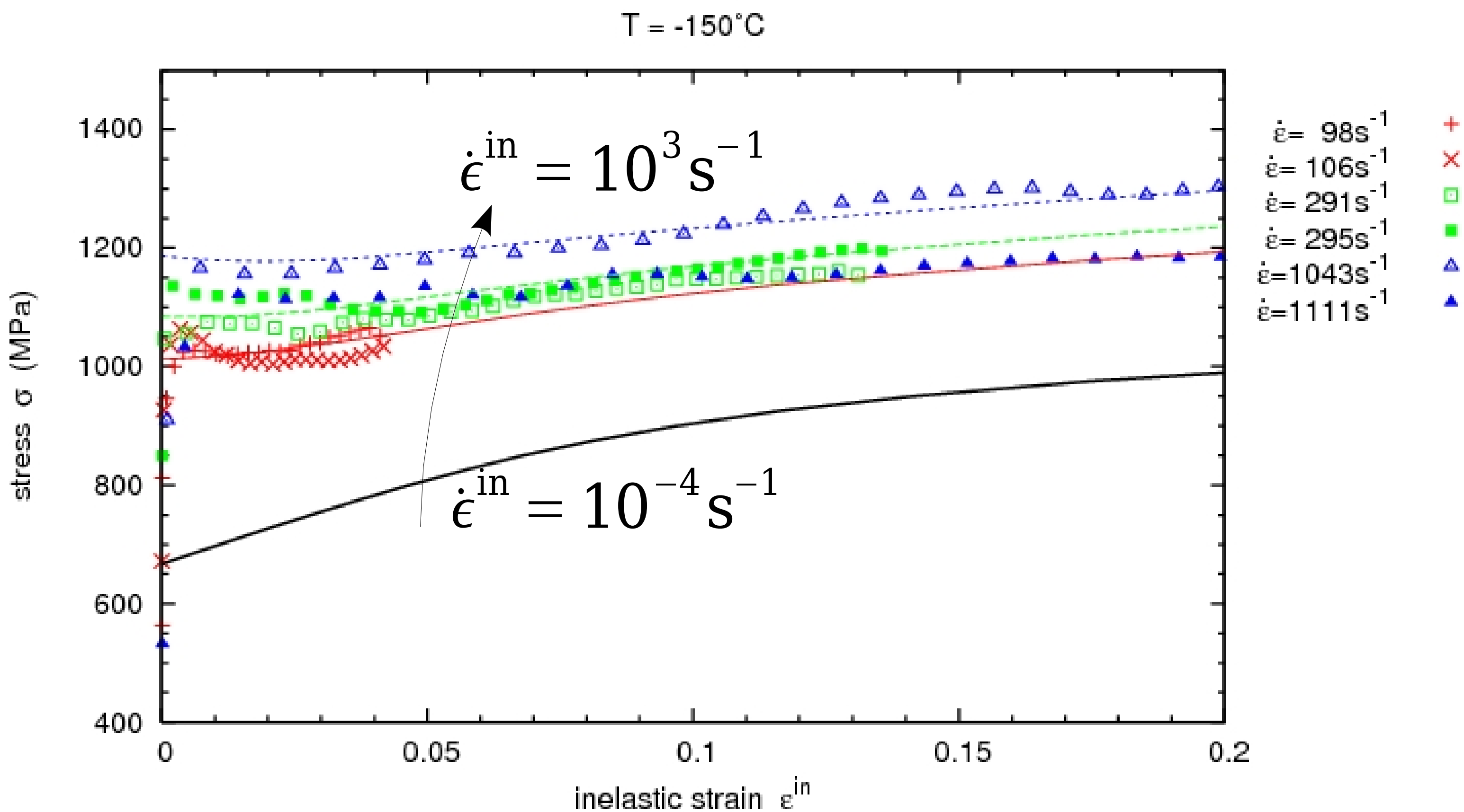


Experimental support

Material:

- **High strain rate characterization** (Split Hopkinson Pressure Bar) → Modified Symonds-Cowper law:

$$\sigma\left(T,\epsilon^{in},\dot{\epsilon}^{in}\right) = \sigma^{stat}\left(T,\epsilon^{in}\right)\cdot\left[1+H\left(T,\epsilon^{in}\right)\,\dot{\epsilon}^{in\;1/p(T)}\right]$$



Experimental support

- **Geometries** investigated:

1. Compact Tension specimen
2. Notched Ring under Compression (mode I)
3. Notched Ring under Compression (mixed mode)

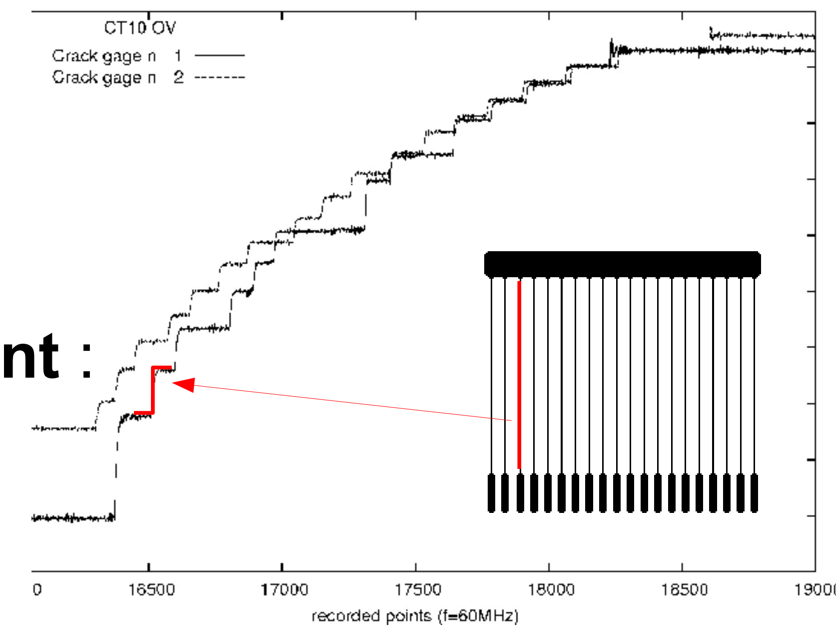
- **Fatigue pre-cracking**

- **Fracture test:**

→ Isothermal (-125°C)

→ Quasi-static loading

→ **Crack speed measurement** : Crack gages + Fast acquisition board (60MHz)



- Arrest front marked by fatigue

Experimental support

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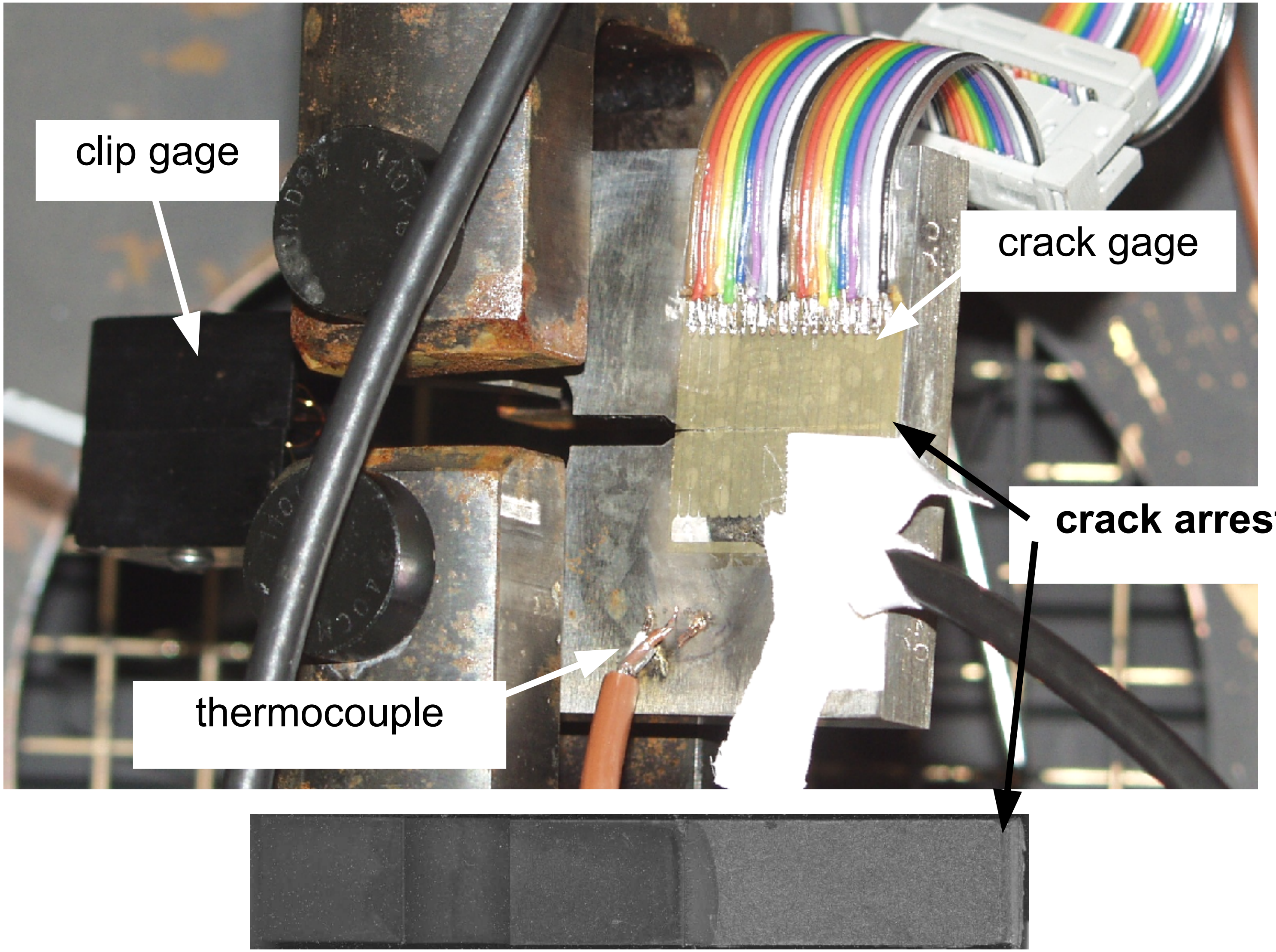
Geometries investigated:

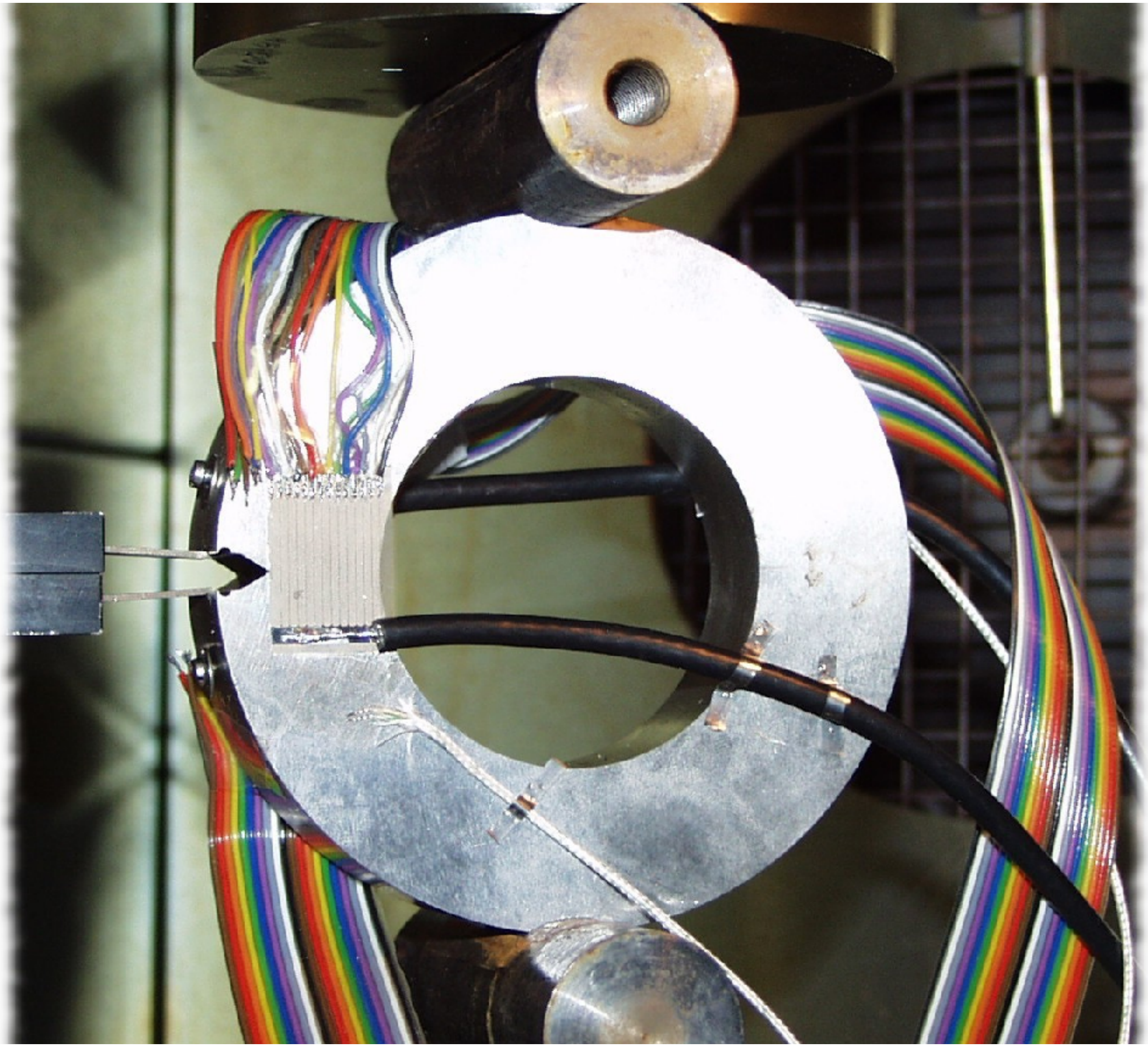
1. Compact Tension specimen:



Experimental support

Geometries investigated:

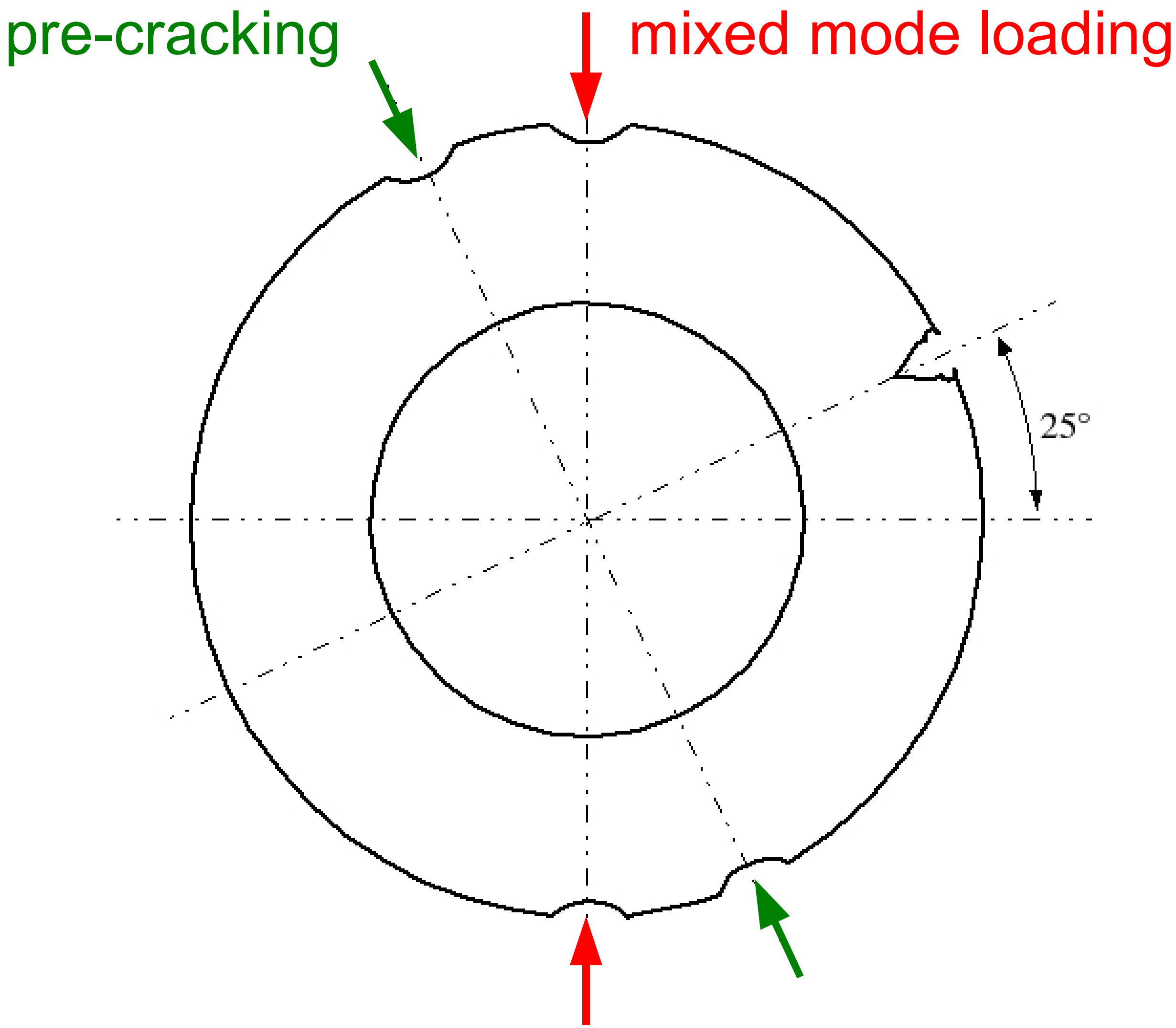
2. Ring under Compression (mode I):



Experimental support

Geometries investigated:

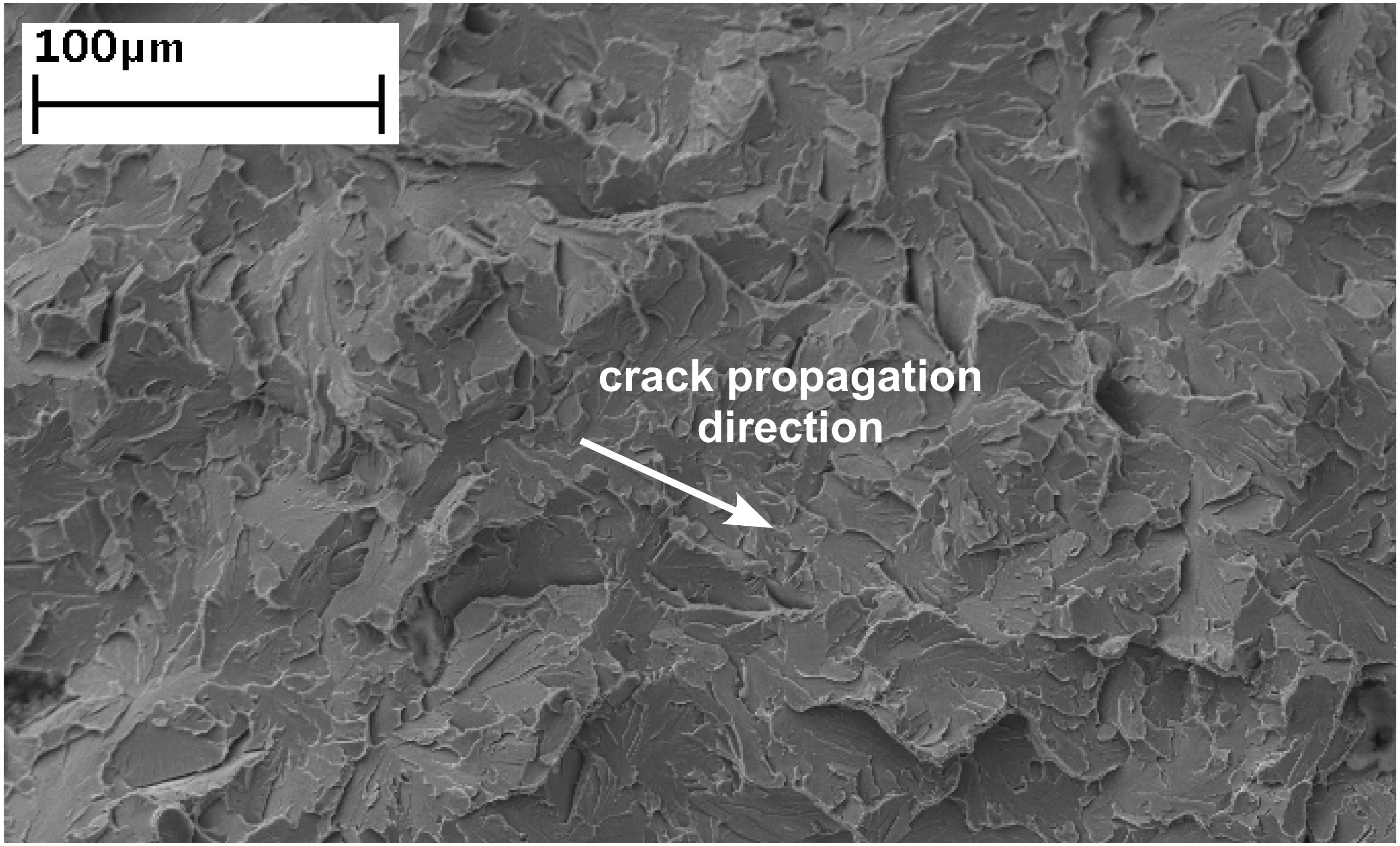
3. Ring under Compression (mixed mode):



Experimental support

Fractography:

- What is the fracture mechanism during propagation?



→ 100% **Cleavage fracture** (no large ductile ligament)

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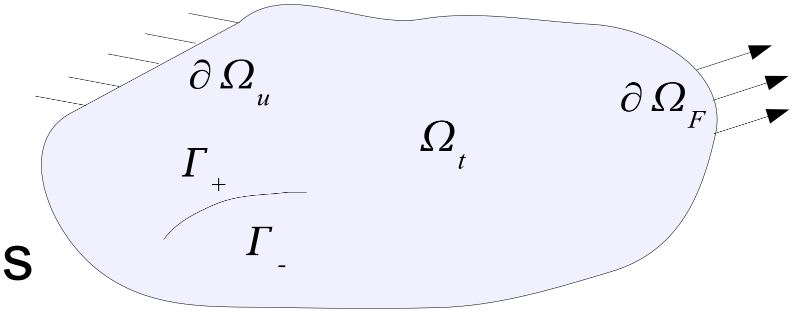
Model of Propagation: A part of the mechanical model

Let us consider a body $\Omega(t)$ with a crack of length $a(t)$



- Dynamic equilibrium:

$$\underline{\mathrm{div}}(\underline{\underline{\sigma}}) = \rho\,\underline{\ddot{u}}$$

- Boundary and initial conditions

$$\underline{\underline{\sigma}}\cdot\underline{n} = F^{ext} \quad \text{on } \partial\Omega_F \quad , \ \ldots$$

- Elastic-viscoplastic constitutive equations:

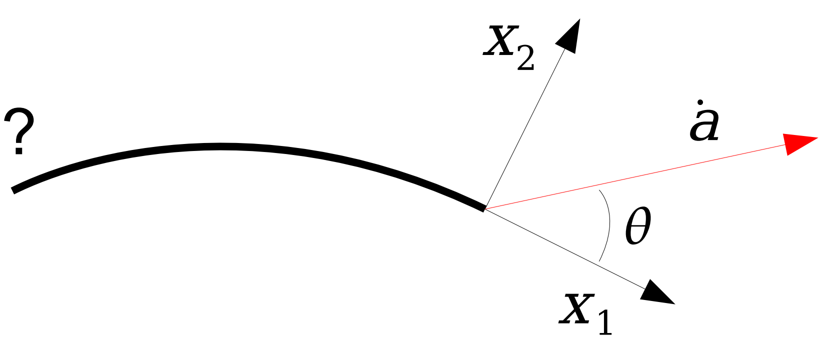
$$\underline{\underline{\epsilon}} = \underline{\underline{\epsilon}}^{el} + \underline{\underline{\epsilon}}^{in}$$

$$\underline{\underline{\epsilon}}^{el} = \boldsymbol{D}^{-1}\underline{\underline{\sigma}}$$

$$\underline{\underline{\dot{\epsilon}}}^{in} = \begin{cases} 0 & \text{if } \Phi(\sigma^{eq}, \epsilon^{in}, \dot{\epsilon}^{in}) < 0 \\ \dot{p}\dfrac{\partial\Phi}{\partial\underline{\underline{\sigma}}} & \text{if } \Phi(\sigma^{eq}, \epsilon^{in}, \dot{\epsilon}^{in}) = 0 \end{cases}$$

- **Crack Propagation Model** to determine:

→ does the crack grow?
→ if it does, at which speed?
→ and in which direction?



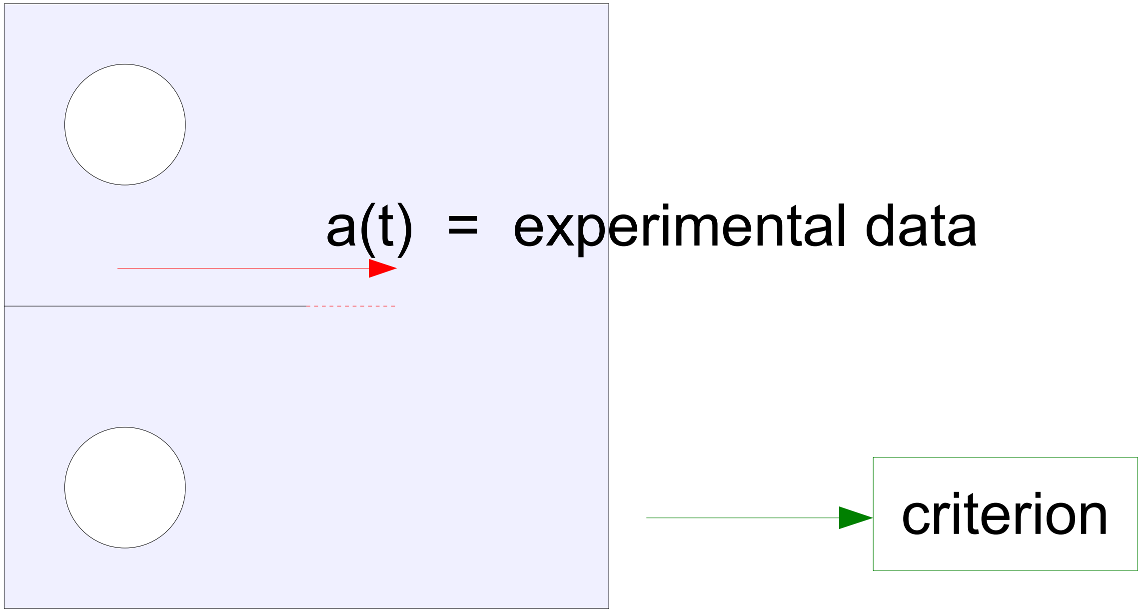
Model of Propagation: Methodology

Goal: Propose a relevant **model to predict crack propagation and arrest** in PWR reactor steel.

1st Step:

Modelling the CT experiment

- **Crack speed imposed** equal to the experimental data
- **Proposition** of a relevant crack propagation criteria



Model of Propagation: Methodology

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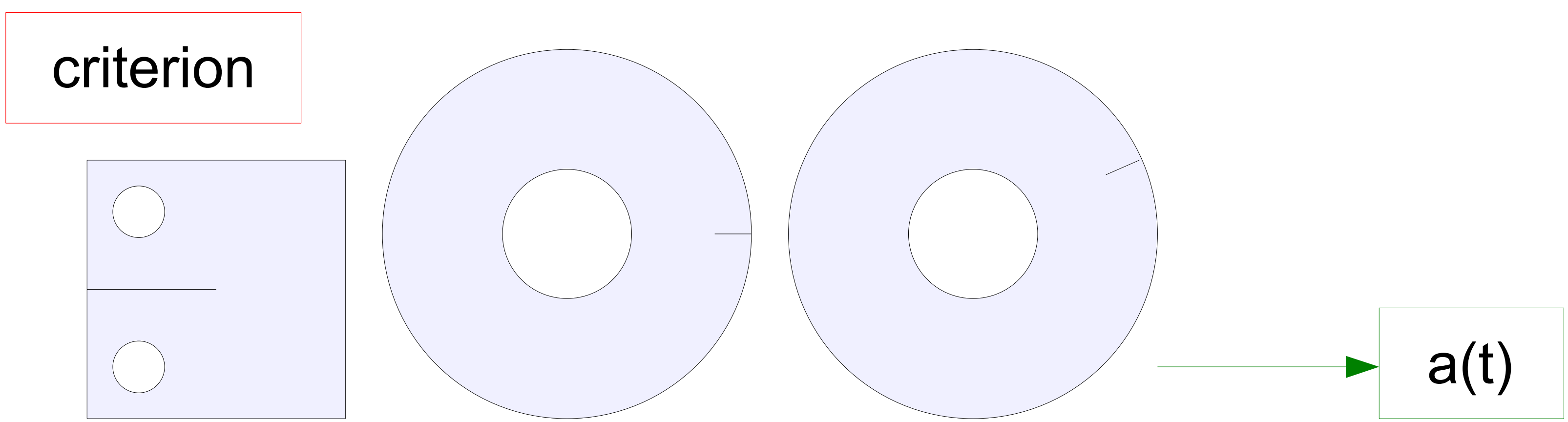
Goal: Propose a relevant **model to predict crack propagation and arrest** in PWR reactor steel.

2nd Step:

Application of the criteria in a **Predictive** way to:

1. CT
2. Ring under compression in mode I
3. Ring under compression in mixed mode

and... **Comparison with experimental** measurements



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Model of Propagation – Step 1: Proposition of a model

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Modelling the CT experiment:

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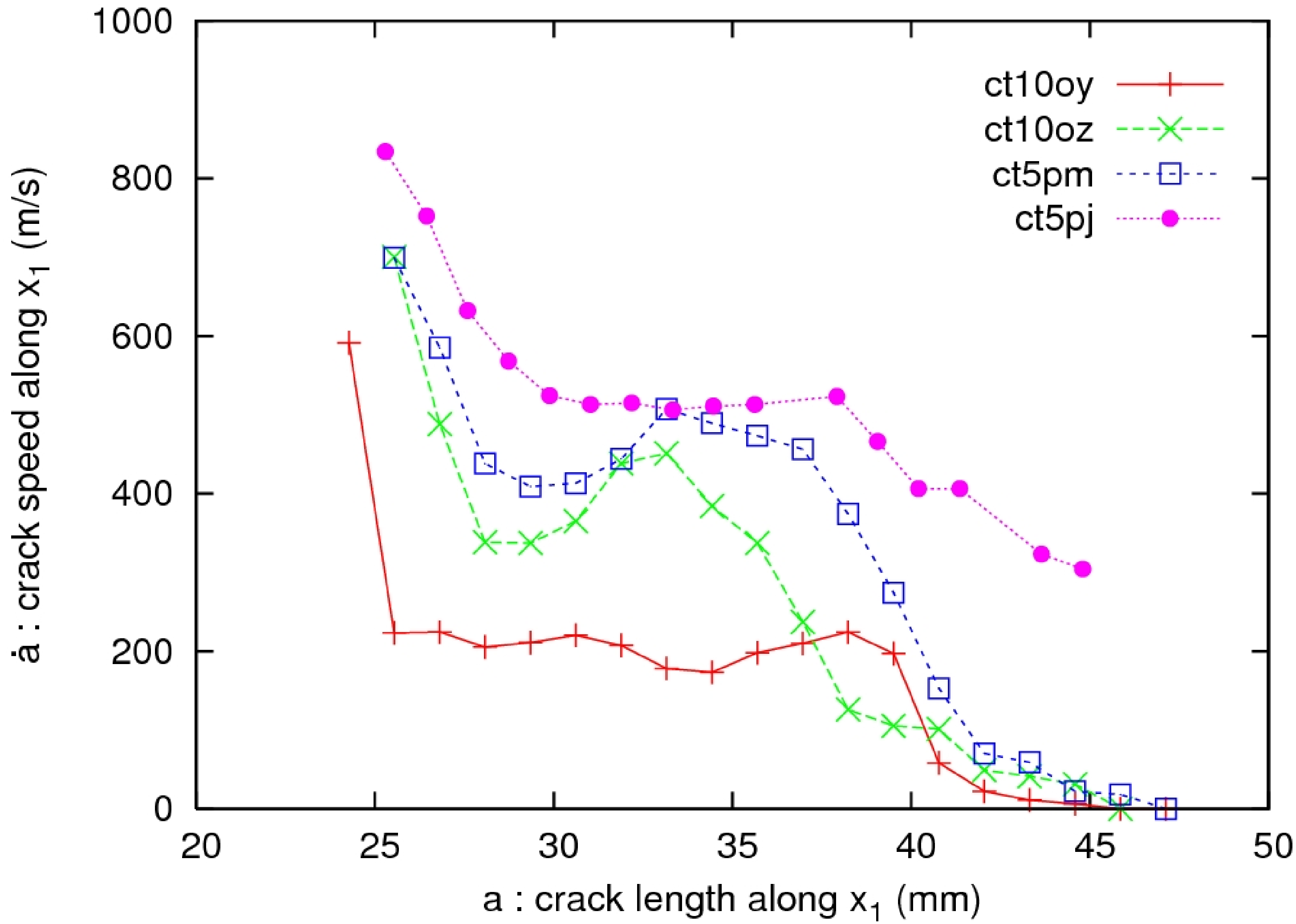

Model of Propagation – Step 1: Proposition of a model

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Modelling the CT experiment:

- Proposition of crack propagation criteria

Global concepts as J^{dyn}:

→ difficult to dissociate cleavage dissipation (G^{tip}) and plasticity (G^{plas})

$$J^{dyn} = \frac{\partial W^{dissipée}}{\partial a} = G^{tip} + G^{plas}$$

Local models of Cleavage:

→ based on the Maximum Principal Stress σ_I

Model of Propagation – Step 1: Proposition of a model

Modelling the CT experiment:

• Proposition of crack propagation criteria

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Local models of Cleavage:

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Model of Propagation – Step 1: Proposition of a model

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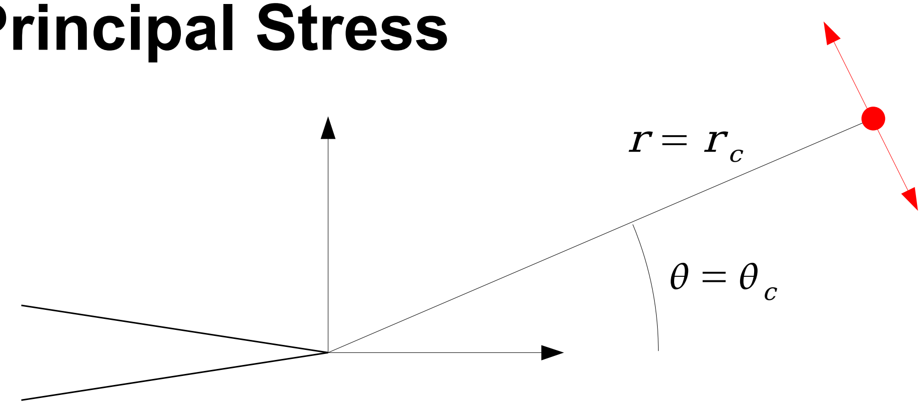
Modelling the CT experiment:

- Proposition of crack propagation criteria based on the **Maximum Principal Stress**

→ **RKR** model:

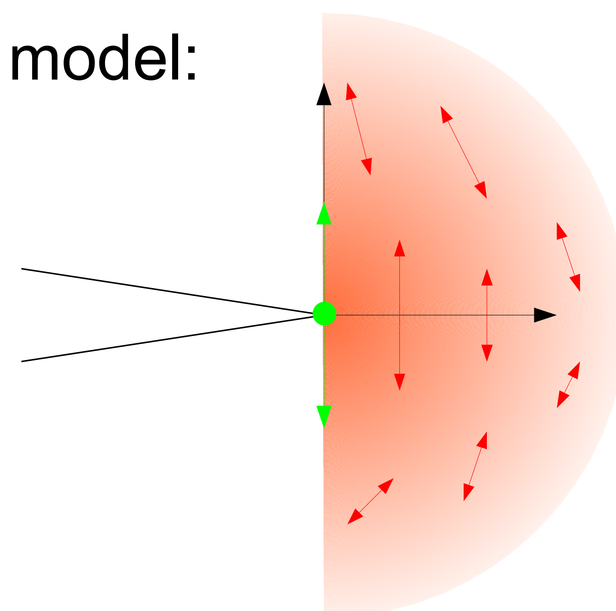
$$\sigma_I(r=r_c,\ \theta=\theta_c) \ = \ \sigma_c$$



→ **Half Disc** average stress tensor model:

$$\tilde{\sigma}_I \equiv \left[\frac{\int w\,\underline{\underline{\sigma}}\,d\Omega}{\int w\,d\Omega}\right]_I = \sigma_c$$

$$w = H(\psi)\exp\left(\frac{-r^2}{2l^2}\right)$$



Model of Propagation – Step 1: Proposition of a model

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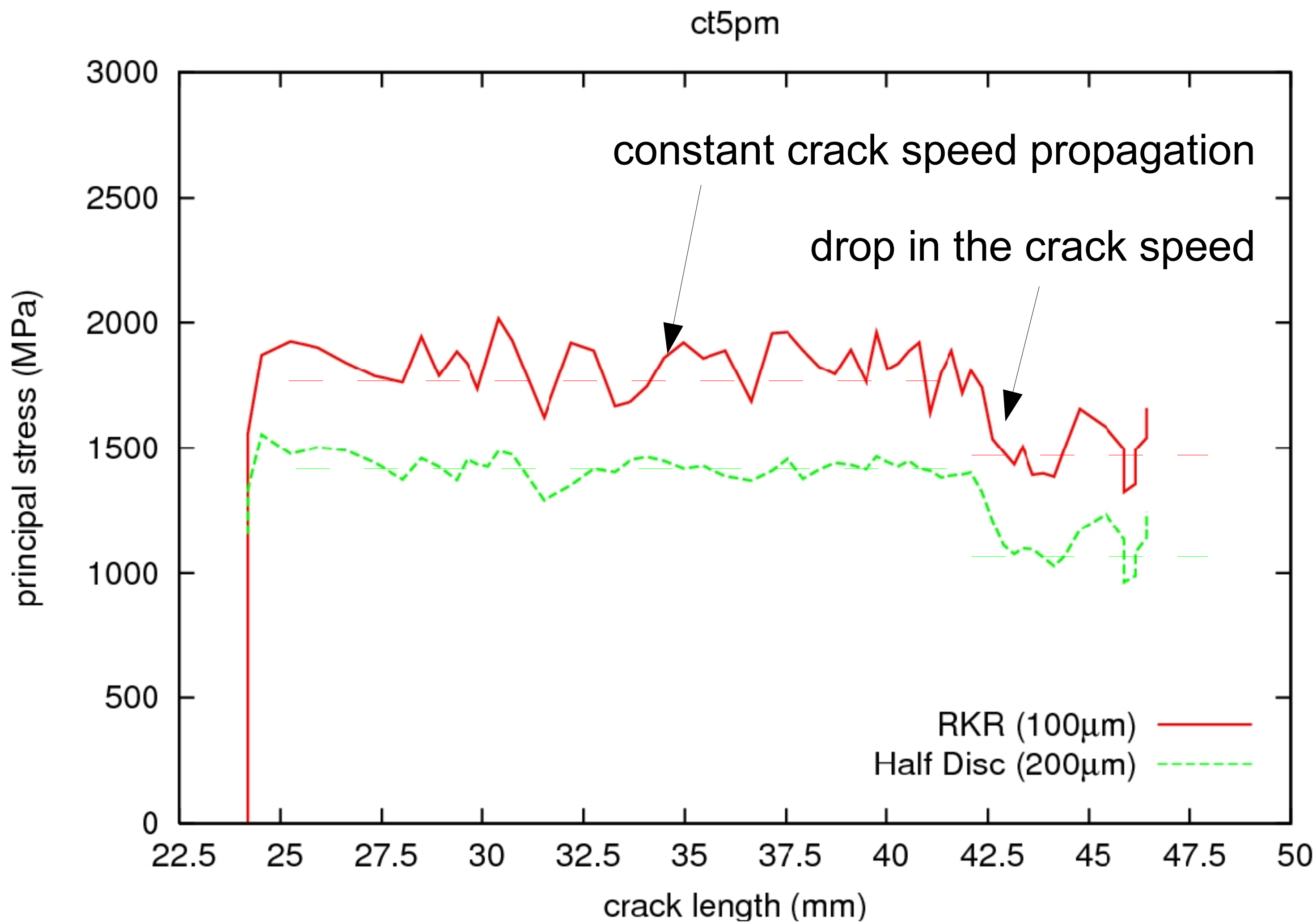
Modelling the CT experiment:

• Proposition of crack propagation criteria based on the **Maximum Principal Stress**



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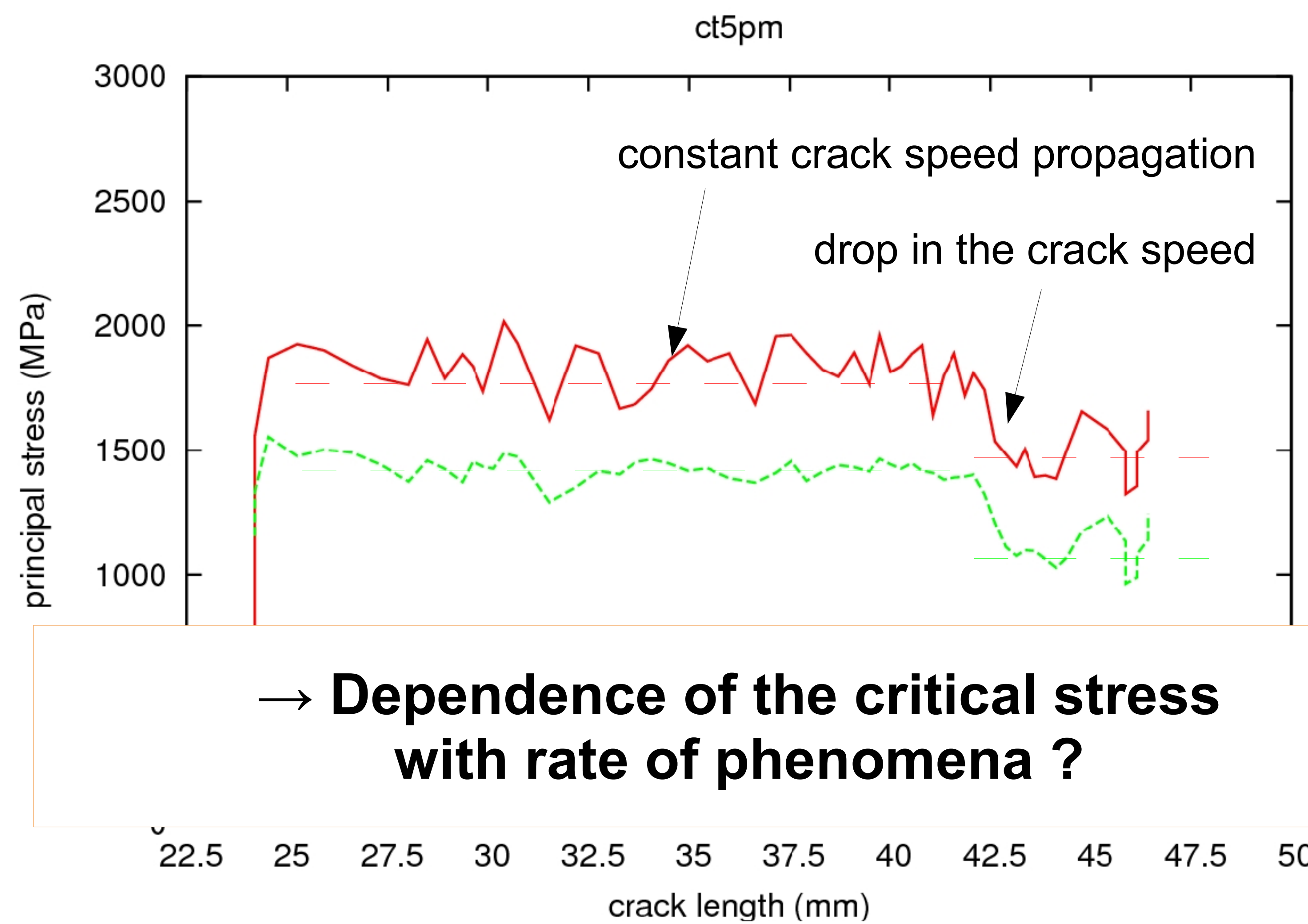


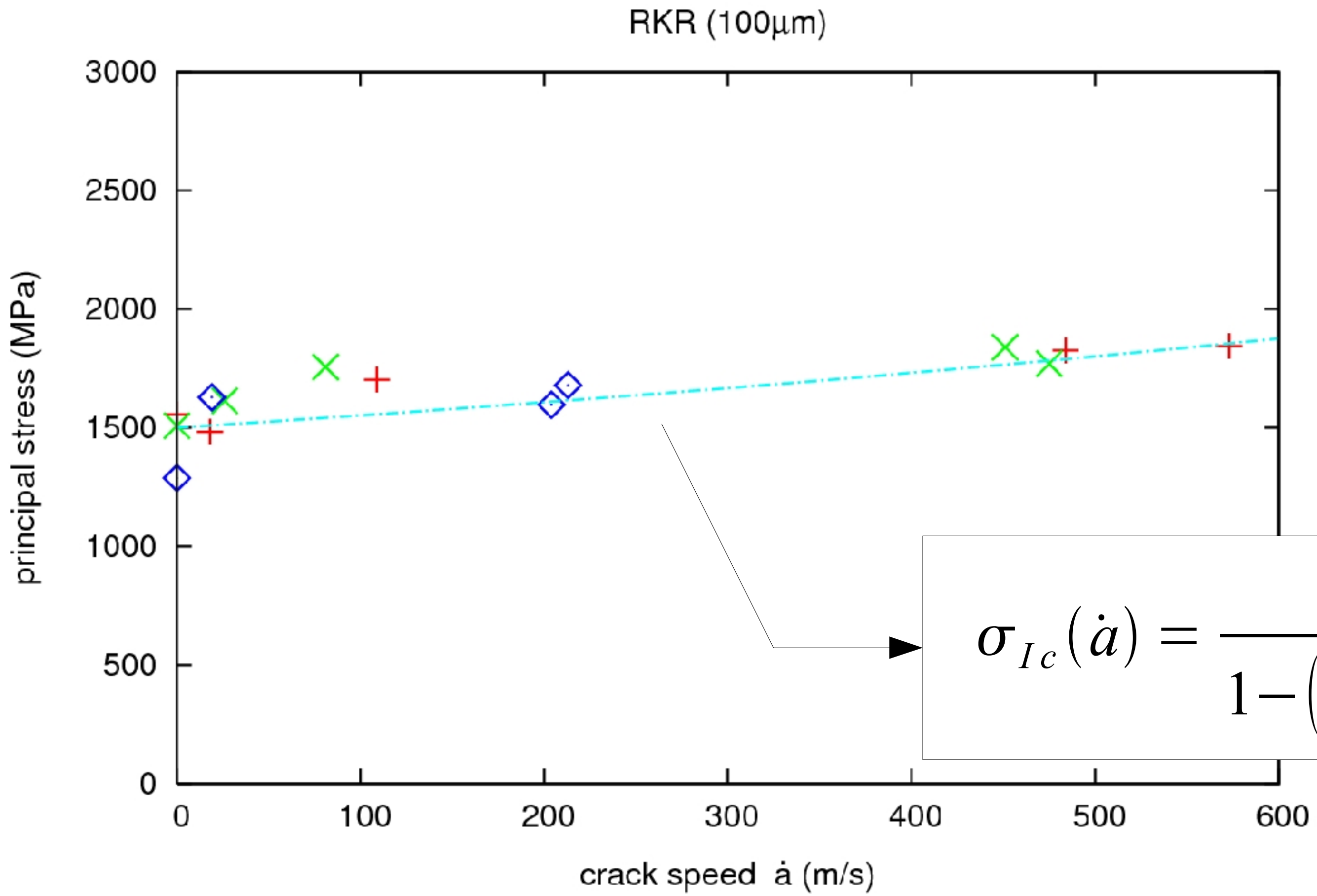
→ Dependence of the critical stress with rate of phenomena ?

Model of Propagation – Step 1: Proposition of a model

Modelling the CT experiment:

• Proposition of crack propagation criteria

→ **Dependence of the critical stress with crack speed**



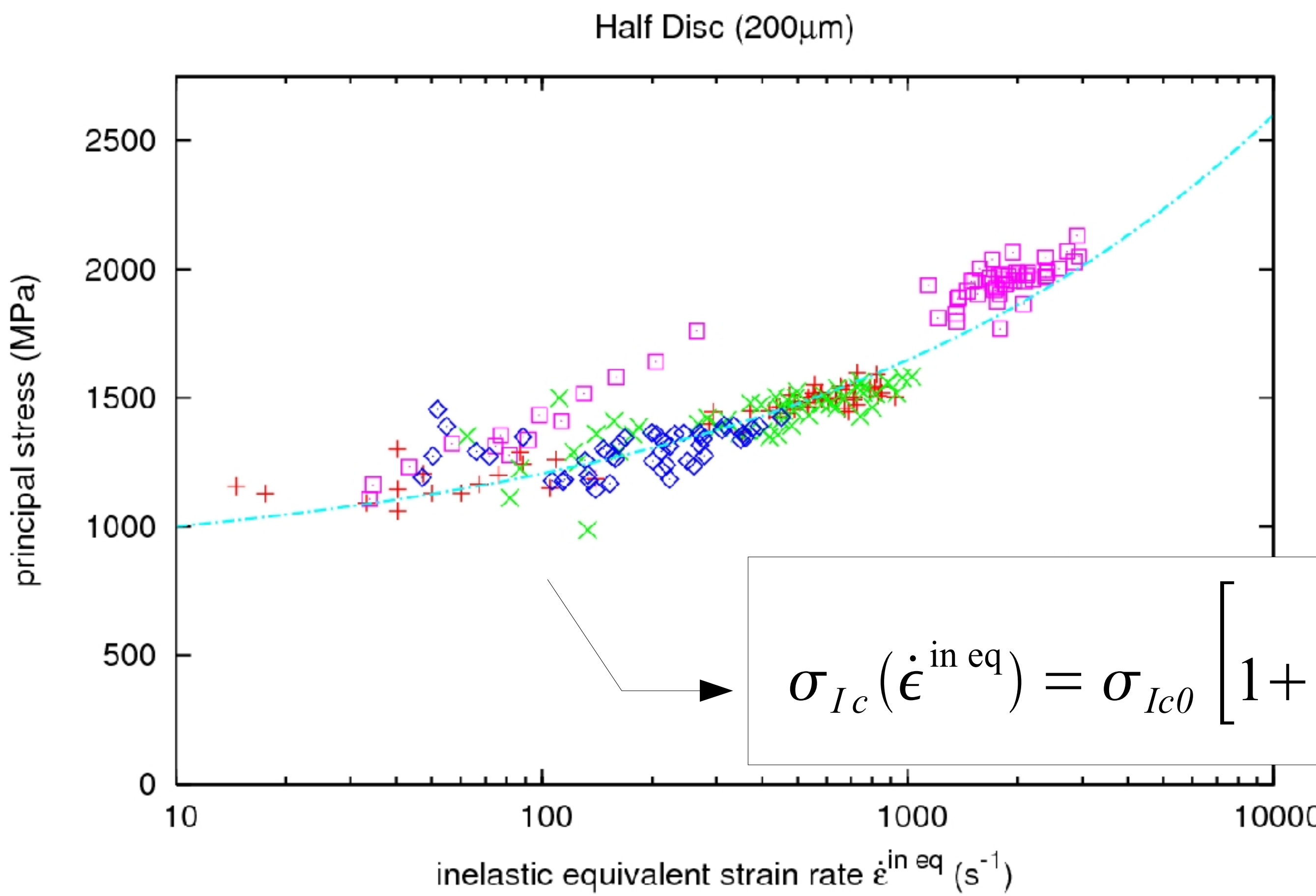
$$\sigma_{Ic}(\dot{a}) = \frac{\sigma_A}{1-\left(\dot{a}/v_{\lim}\right)^m}$$

Model of Propagation – Step 1: Proposition of a model

Modelling the CT experiment:

- Proposition of crack propagation criteria

→ **Dependence of the critical stress with strain rate**



$$\sigma_{Ic}(\dot{\epsilon}^{\text{in eq}}) = \sigma_{Ic0}\left[1 + C_1(\dot{\epsilon}^{\text{in eq}})^{\frac{1}{C_2}}\right]$$

Model of Propagation – Step 1: Proposition of a model

Modelling the CT experiment:

Review of crack propagation models identified in step 1

- Quantify the intensity of the **Maximum Principal Stress:**

→ **RKR** model $\sigma_I(r=r_c, \theta=\theta_c) = \sigma_c$

→ **Half Disc** average stress tensor

$$\tilde{\sigma}_I \equiv \left[\frac{\int w \underline{\underline{\sigma}} \, d\Omega}{\int w \, d\Omega} \right]_I = \sigma_c$$

- Critical stress dependence with:

→ **crack speed** $\sigma_{Ic}(\dot{a})$

→ **strain rate** $\sigma_{Ic}(\dot{\epsilon}^{\text{in eq}})$

Outline of the speech

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step 2: Predictive Simulations

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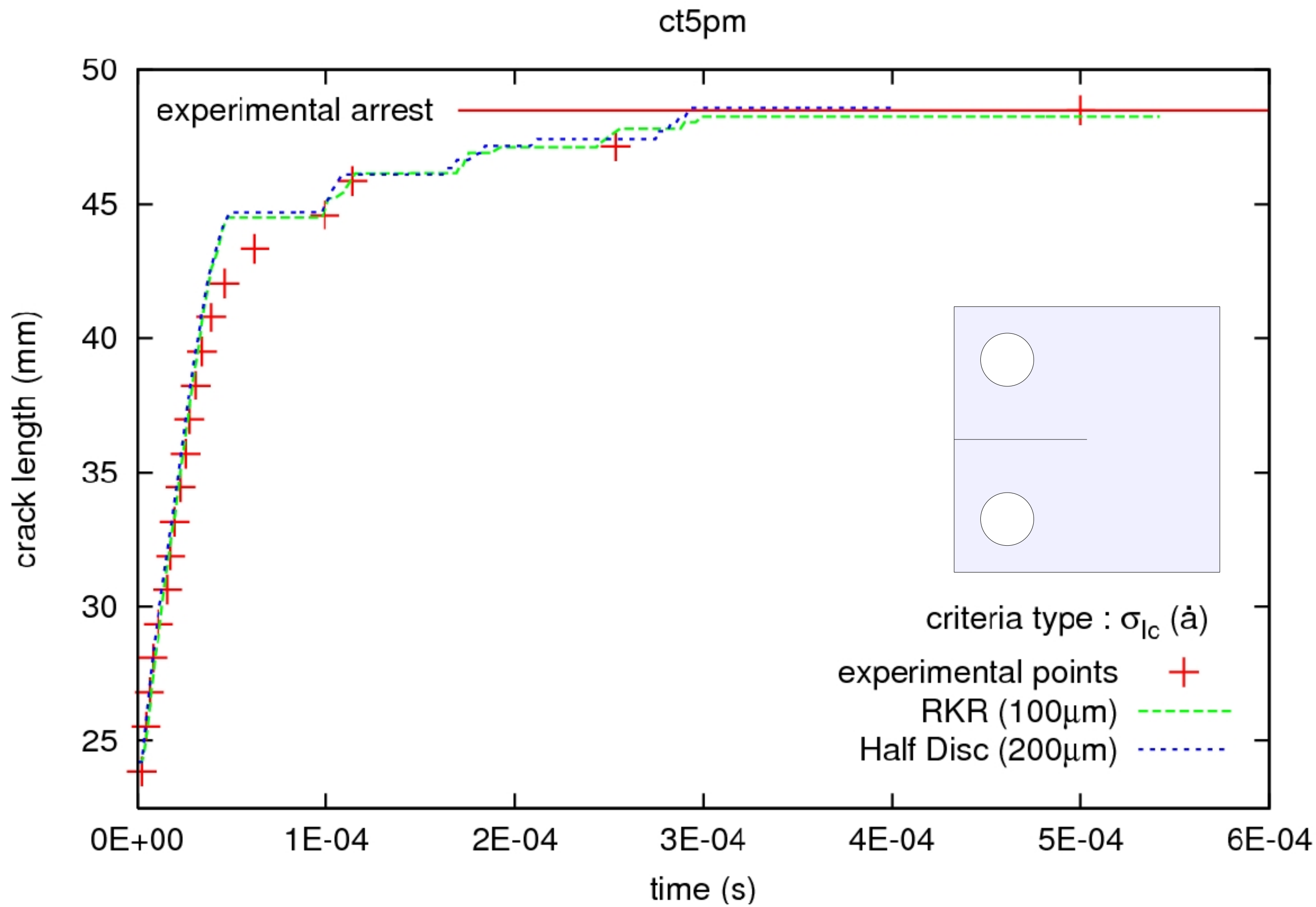
Model of Propagation – Step 2: Predictive Simulations

Application of the criteria in a **predictive** way to:

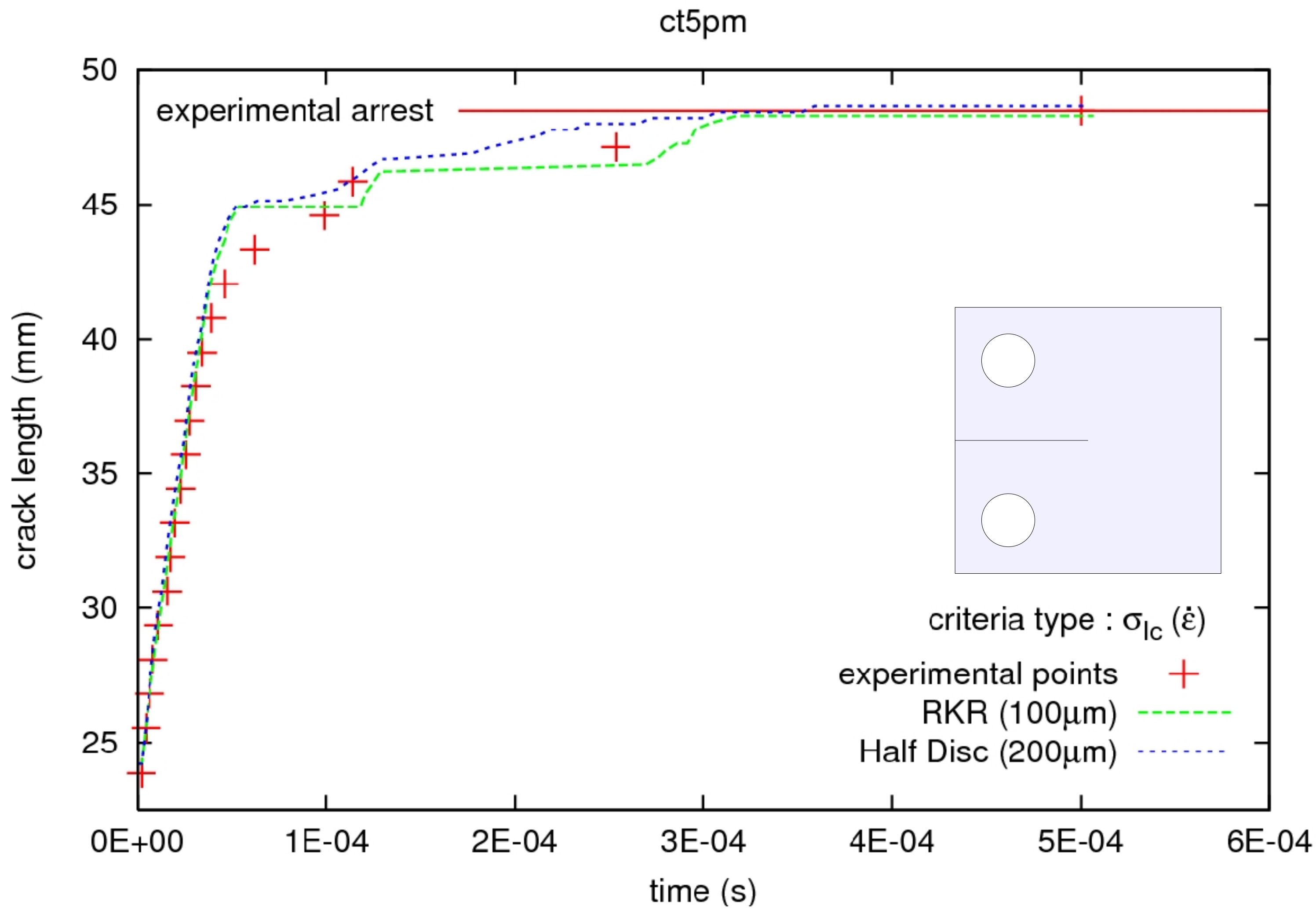
1. **CT**



Model of Propagation – Step 2: Predictive Simulations

Application of the criteria in a **predictive** way to:

1. **CT**



Model of Propagation – Step 2: Predictive Simulations

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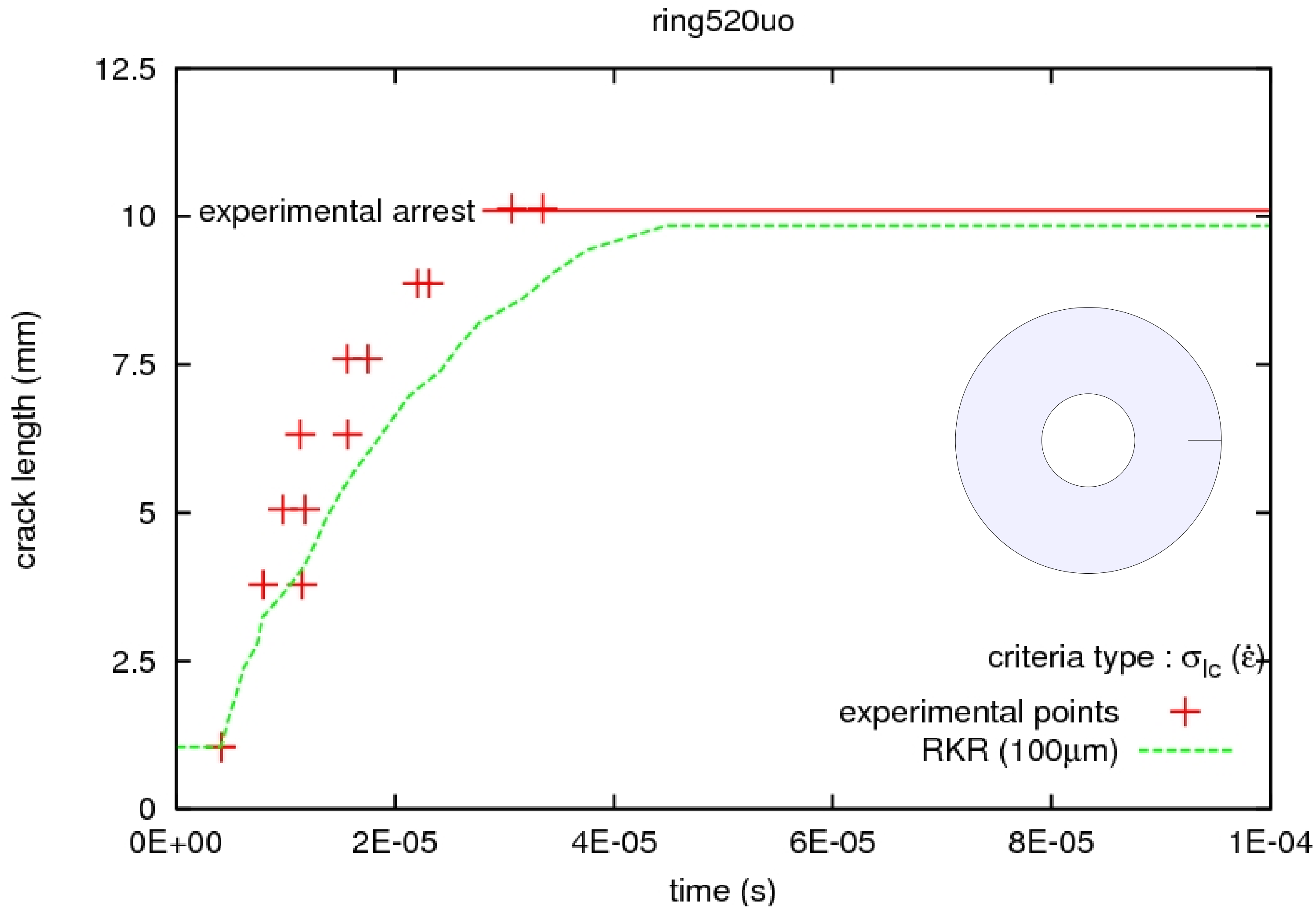
Application of the criteria in a **predictive** way to:

2. **Ring** under compression in **mode I**



Model of Propagation – Step 2: Predictive Simulations

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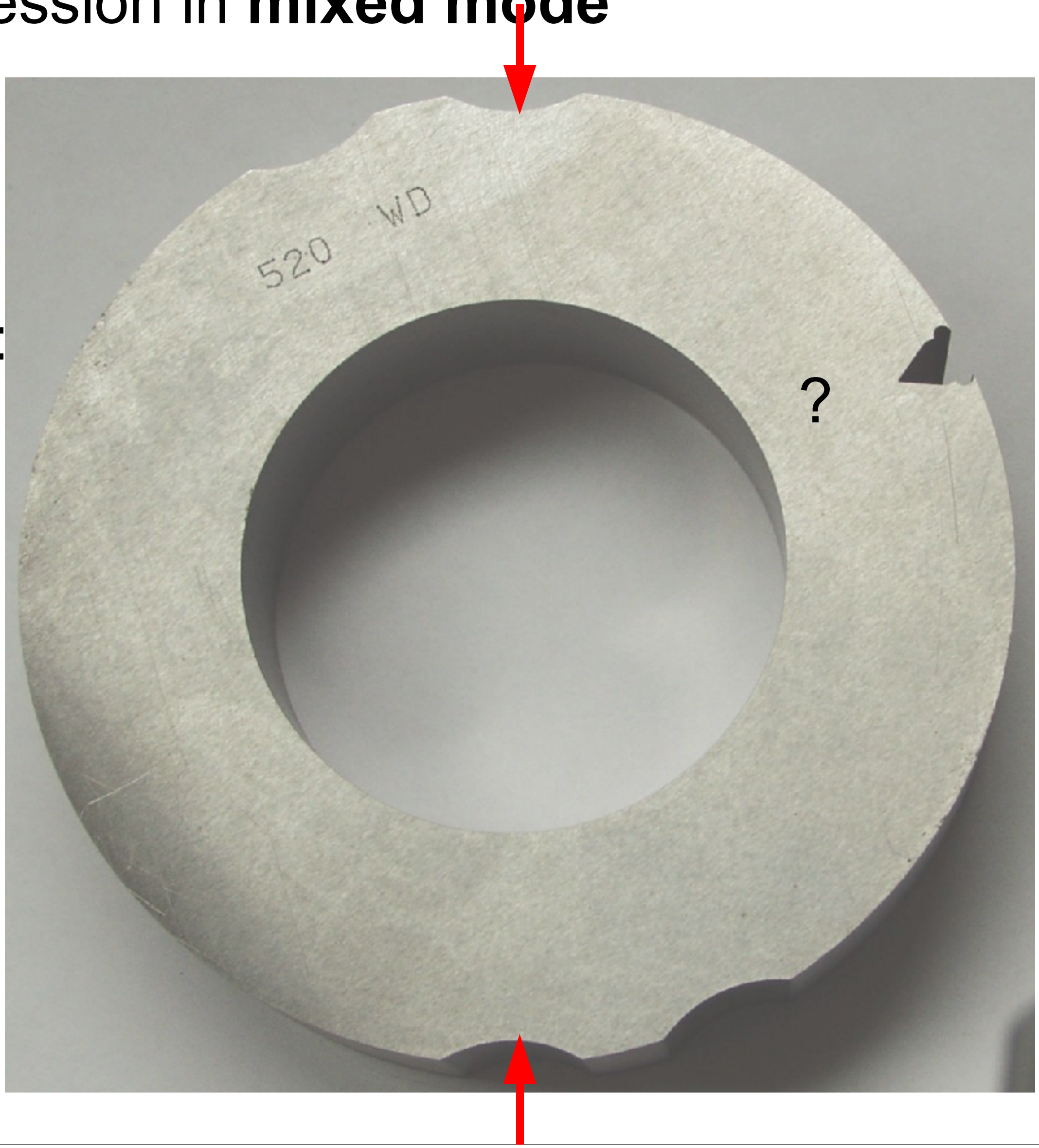
Application of the criteria in a **predictive** way to:
3. **Ring** under compression in **mixed mode**

- Will the crack stop?

- Subsidiary question: **Where will the crack grow?**



Model of Propagation – Step 2: Predictive Simulations

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Application of the criteria in a **predictive** way to:
3. **Ring** under compression in **mixed mode**

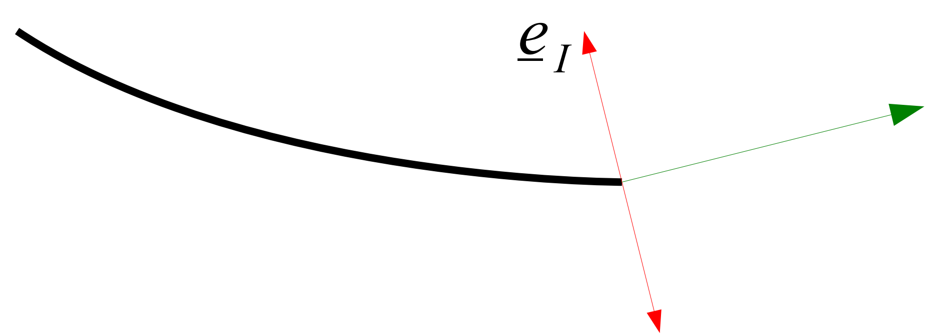
- Subsidiary question: **Where will the crack grow?**

One can use:

→ Direction of the **maximum hoop stress** $\sigma_{\theta\theta}$

→ Direction **perpendicular to the principal stress** of the **average stress tensor**

$$\underline{\underline{\tilde{\sigma}}} = \tilde{\sigma}_I \left(\underline{e}_I \times \underline{e}_I\right) + \dots$$



Model of Propagation – Step 2: Predictive Simulations

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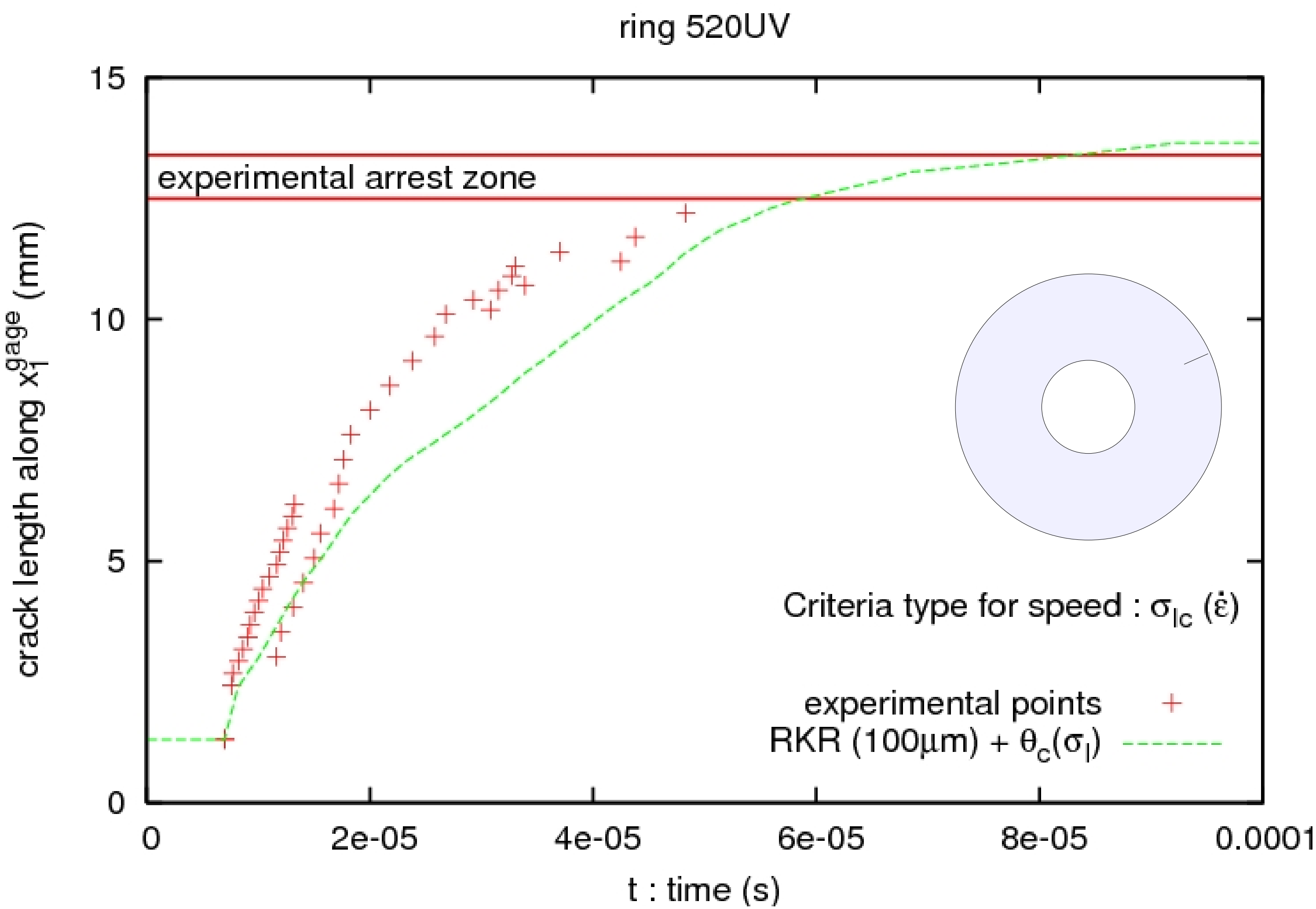
Application of the criteria in a **predictive** way to:
3. **Ring** under compression in **mixed mode**



Model of Propagation – Step 2: Predictive Simulations

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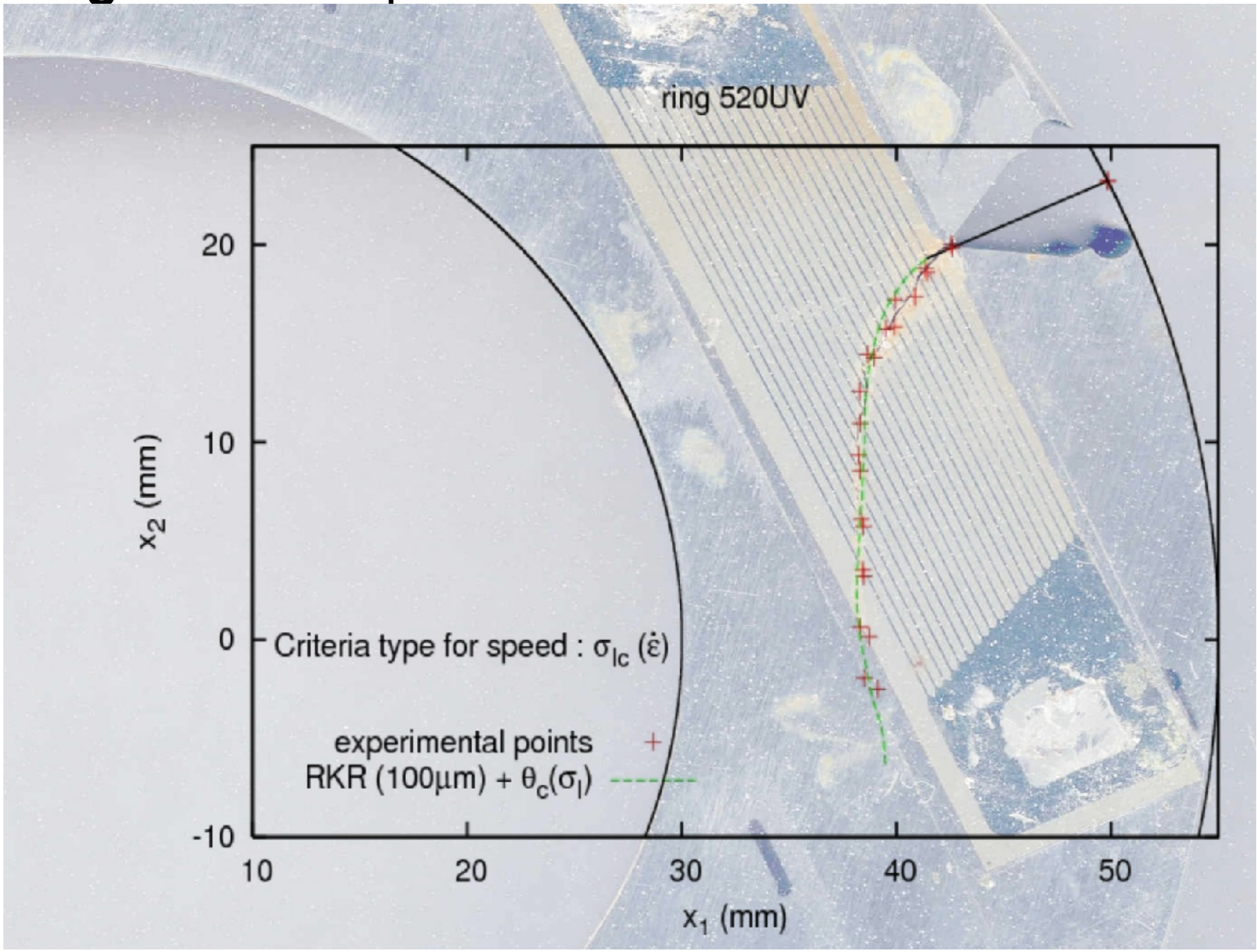
Application of the criteria in a **predictive** way to:

3. **Ring** under compression in **mixed mode**



Model of Propagation – Step 2: Predictive Simulations

Application of the criteria in a **predictive** way to:

3. **Ring** under compression in **mixed mode**

→ Hydrostatic stress σ_H

Outline of the speech

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step 2: Predictive Simulations

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Conclusion and prospect

Conclusion:

Experimental work has been realized:

- Material behavior has been identified at high strain rate
- **3** distinct isothermal fracture tests were realized
- **Cleavage** was clearly identified from fracture surface observations

Numerical development have lead to:

- The implementation of the X-FEM in Cast3m
- A level set update performed on a auxiliary grid
- A non conforming partitioning technique which enables numerical integration without projection

Conclusion and prospect

Conclusion:

Crack propagation model have been proposed:

- Based on the evaluation of the **intensity** of **principal stress** at crack tip
- Critical stress **dependence with rate of phenomena** must be taken into account
- **Good predictive results** were found in term of speed, length at arrest, and orientation of the crack

Conclusion and prospect

Context and Objectives

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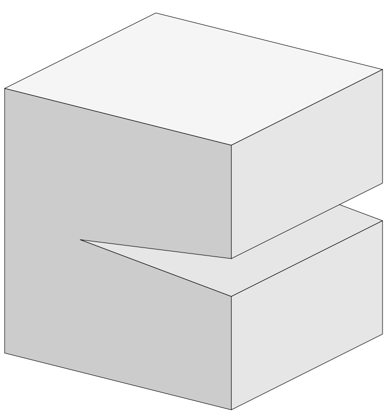
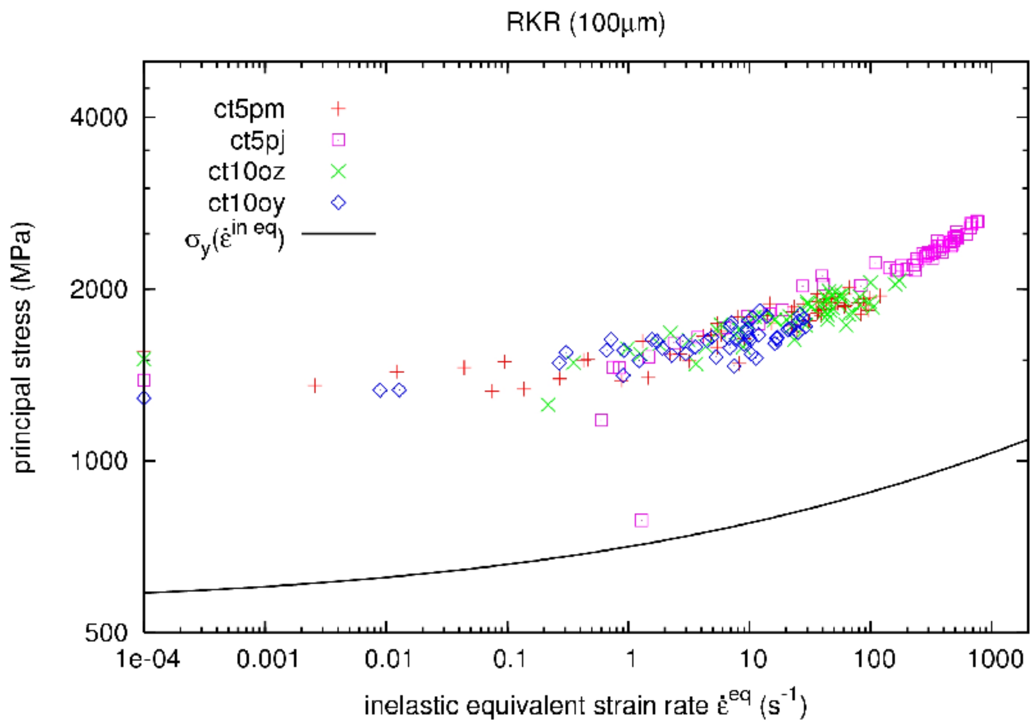
Prospect:

- Complete the implantation of X-FEM in Cast3m (In particular 3D element)
- **Confirm** the **critical stress dependence** with strain rate
- Possible link with the **yield stress dependence**?



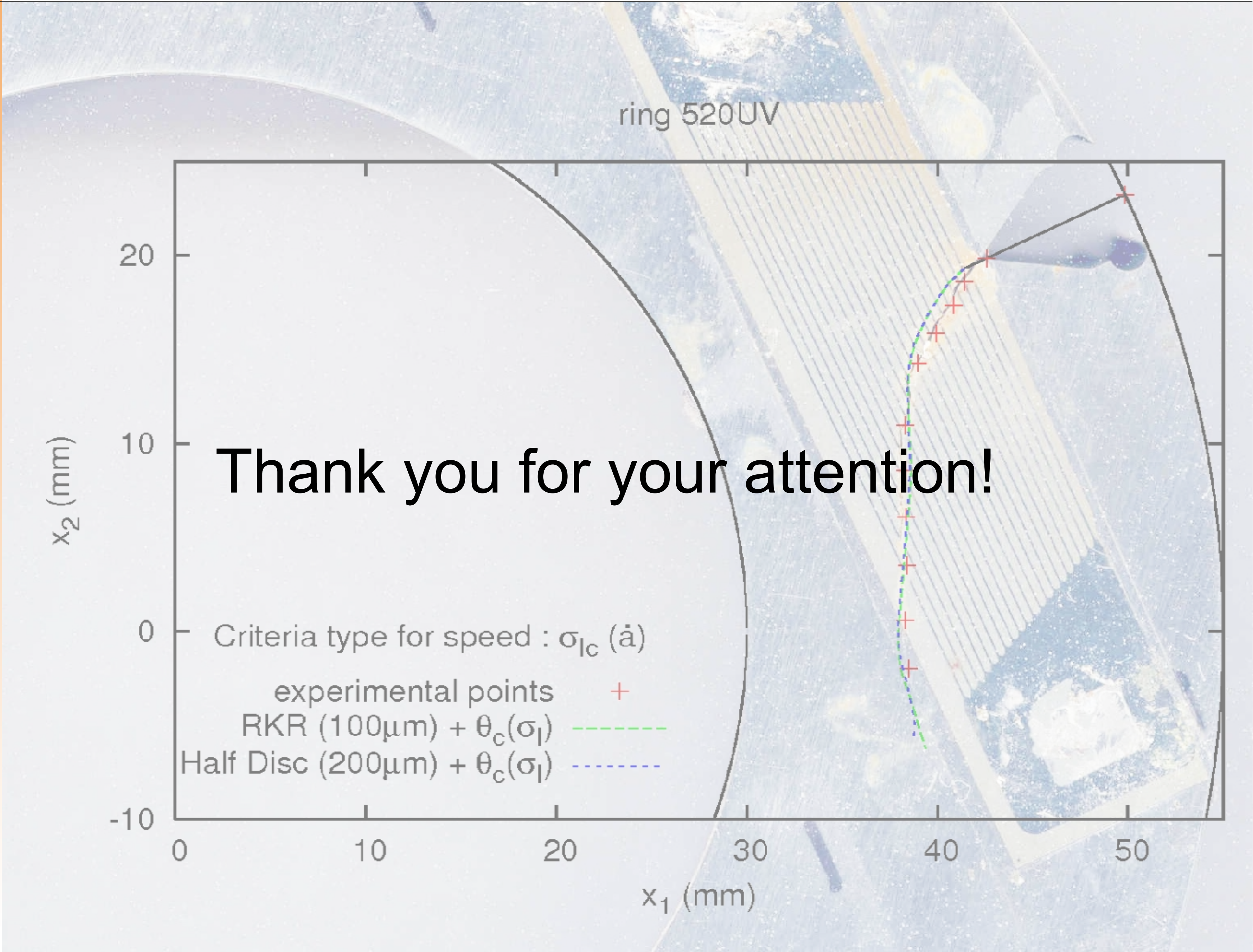


Conclusion and prospect

Prospect:

- Perform other isothermal tests at different temperature
  → Investigate temperature dependence of σ_{Ic}
- Interpretation of the **thermal shock problem**

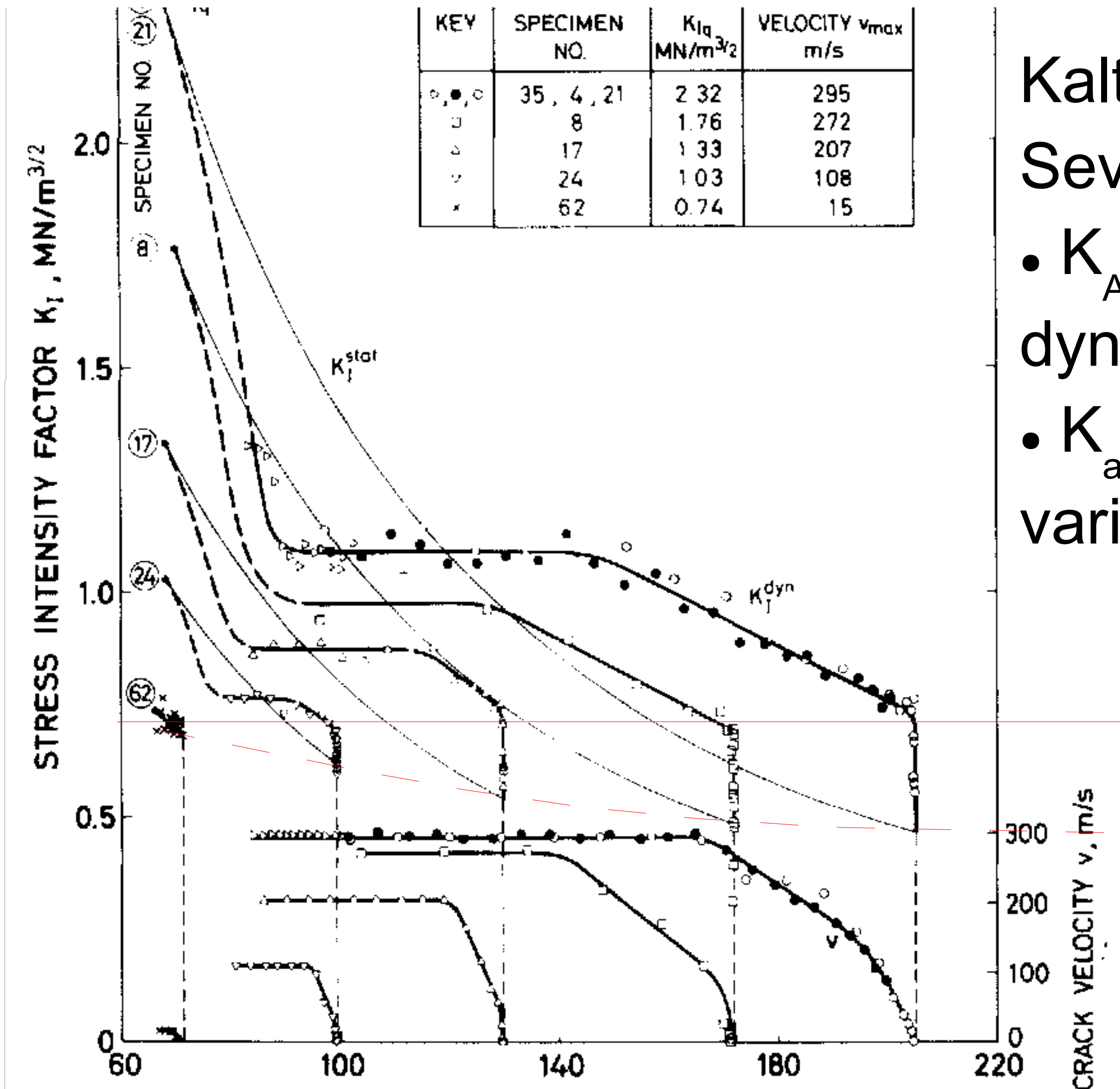
The end



Thank you for your attention!

Context and Objectives

Dynamic analysis is relevant to study crack arrest:



crack length (mm)

Kalthoff's observations:

Several experiments

- K_A is constant when dynamic is considered
- K_a (static analysis) is variable

K_A *(dynamic)*

K_a *(static)*

Experimental support

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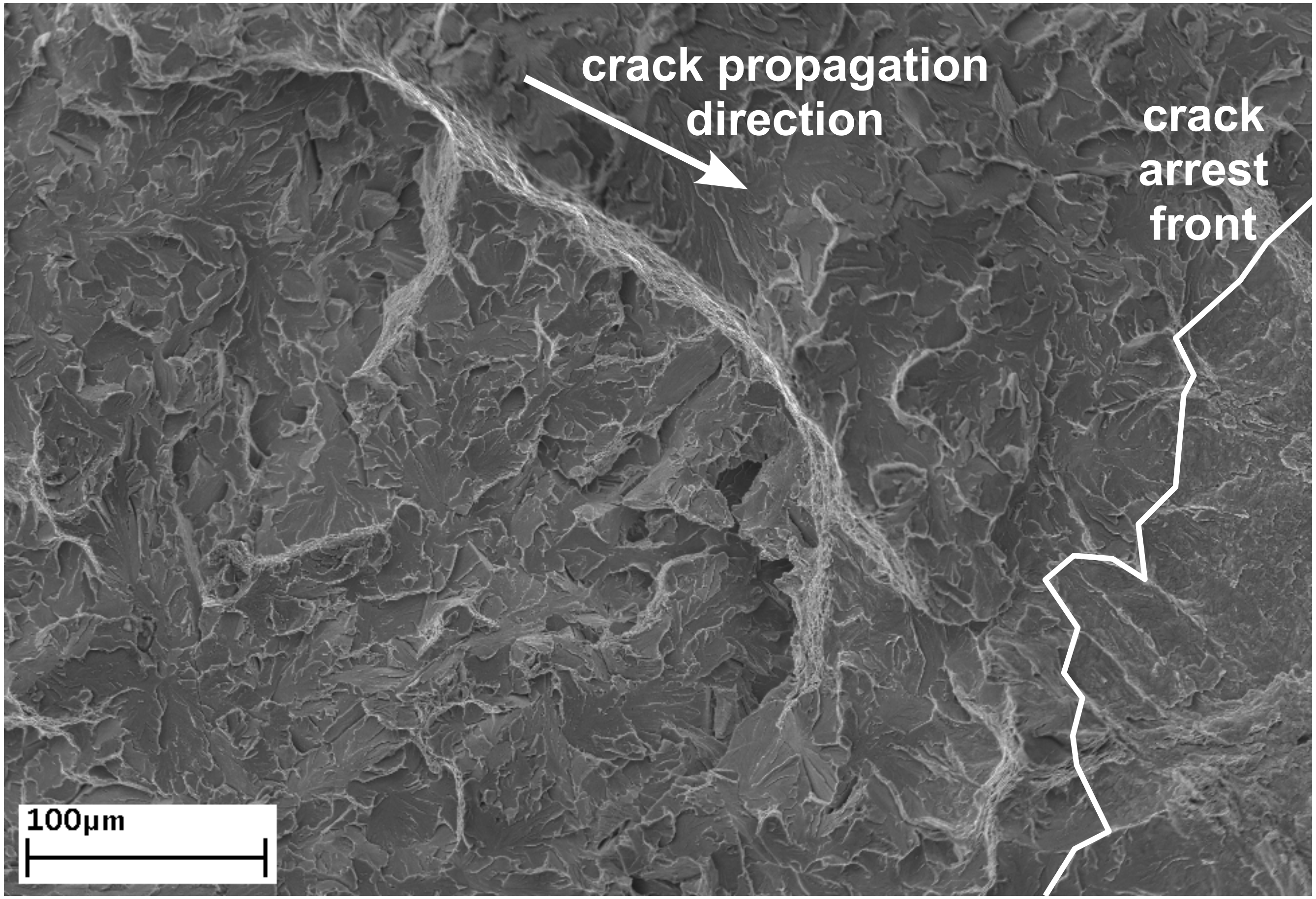
Fractography:

- Is there any particularity of the arrest front?



→ Cleavage surface with larger shear step due to loss of energy of the crack

Model of Propagation – Step1: Proposition

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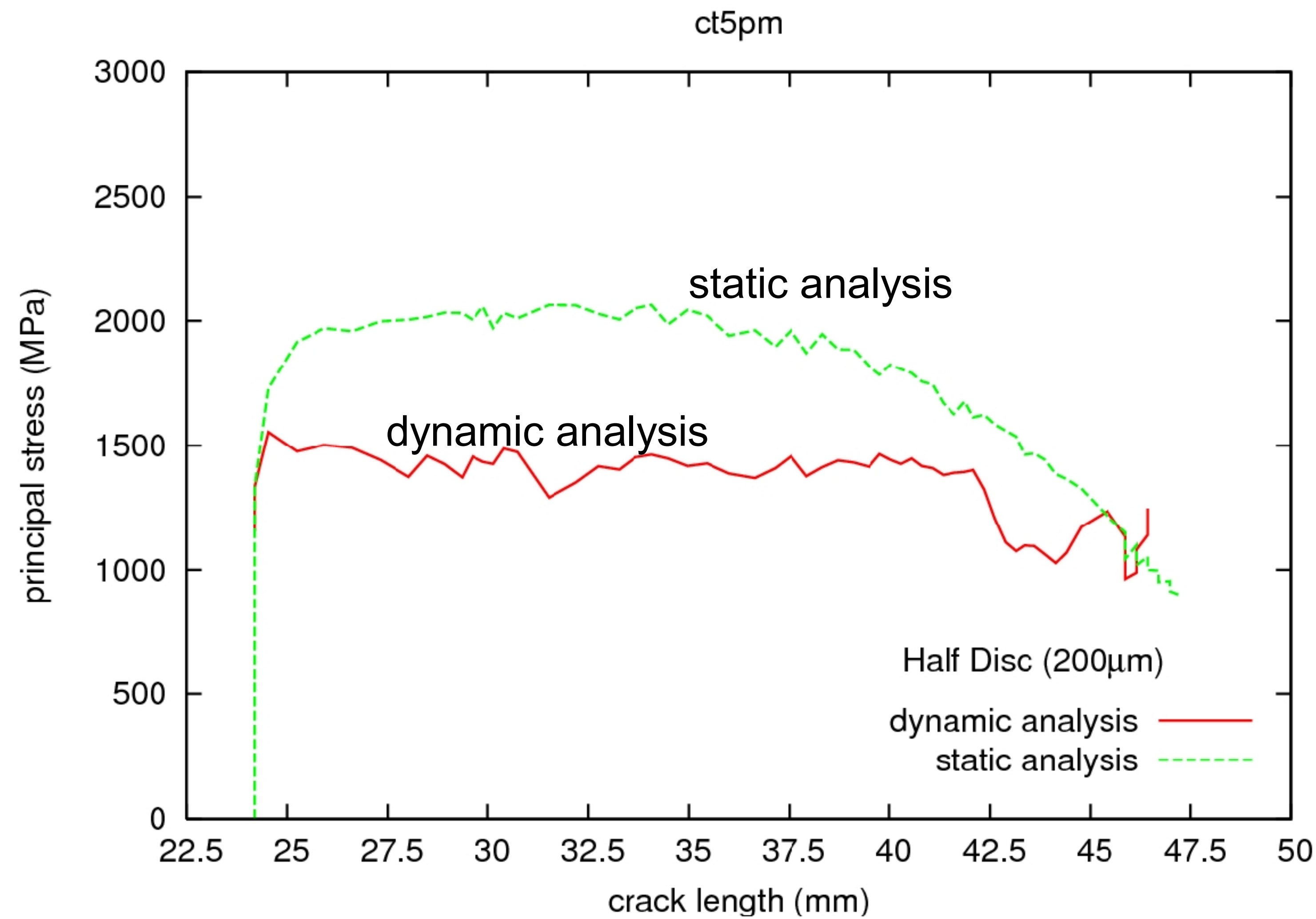
Modelling the CT experiment:

• Proposition of a crack propagation criteria based on the **Maximum Principal Stress**



Model of Propagation – Step 2: Predictive Simulations

Algorithms for crack propagation:

Critical stress dependence with **crack speed:** $\sigma_{Ic}(\dot{a})$

- **Euler**-like scheme

→ resolution at time t_n: $M\ddot{u}_n + \int B^T \sigma_n = F_n^{ext}$

→ criterion calculation: $\dot{a}_n = \sigma_{Ic}^{-1}(\underline{\underline{\sigma}}_n)$

→ crack propagation: $a_{n+1} = a_n + \dot{a}_n \Delta t$

$n \leftarrow n+1$

Critical stress dependence with **strain rate:** $\sigma_{Ic}(\dot{\epsilon}^{\text{in eq}})$

- **Iterative scheme**

→ crack propagation: $a_{n+1} = a_n + \Delta a$

→ resolution at: $t_{n+1}^{(i)} = t_n + \Delta t^{(i)}$ $M\ddot{u}_{n+1}^{(i)} + \int B^T \sigma_n^{(i)} = F_n^{ext}$

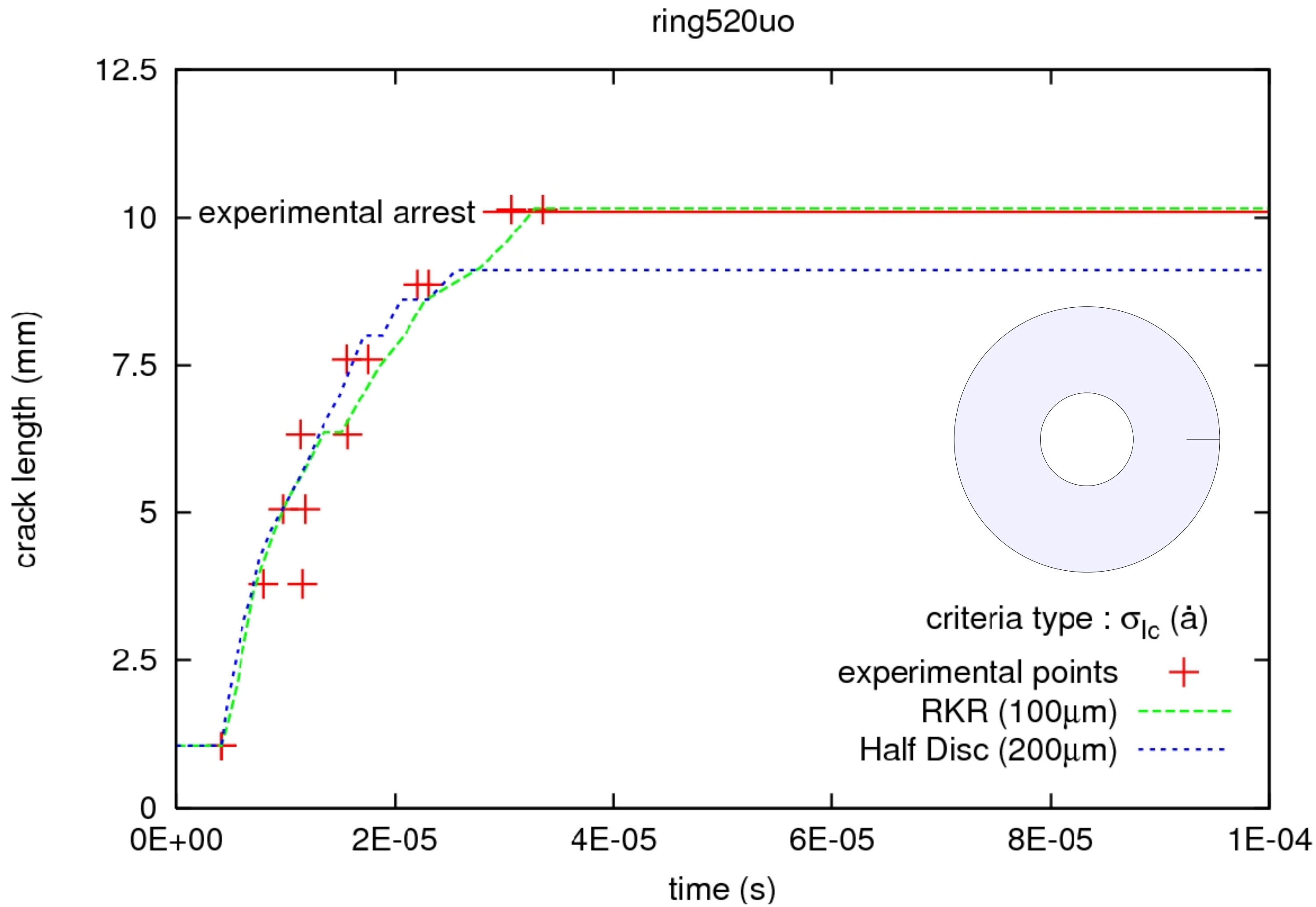
→ criterion calculation: $\sigma_{I\,n}^{(i)} - \sigma_{Ic}(\dot{\epsilon}_n^{\text{in eq}\,(i)}) = 0\,?$

No: $i \leftarrow i+1$

Yes: $n \leftarrow n+1$

Model of Propagation – Step 2: Predictive Simulations

Application of the criteria in a **predictive** way to:
2. **Ring** under compression in **mode I**



Model of Propagation – Step 2: Predictive Simulations

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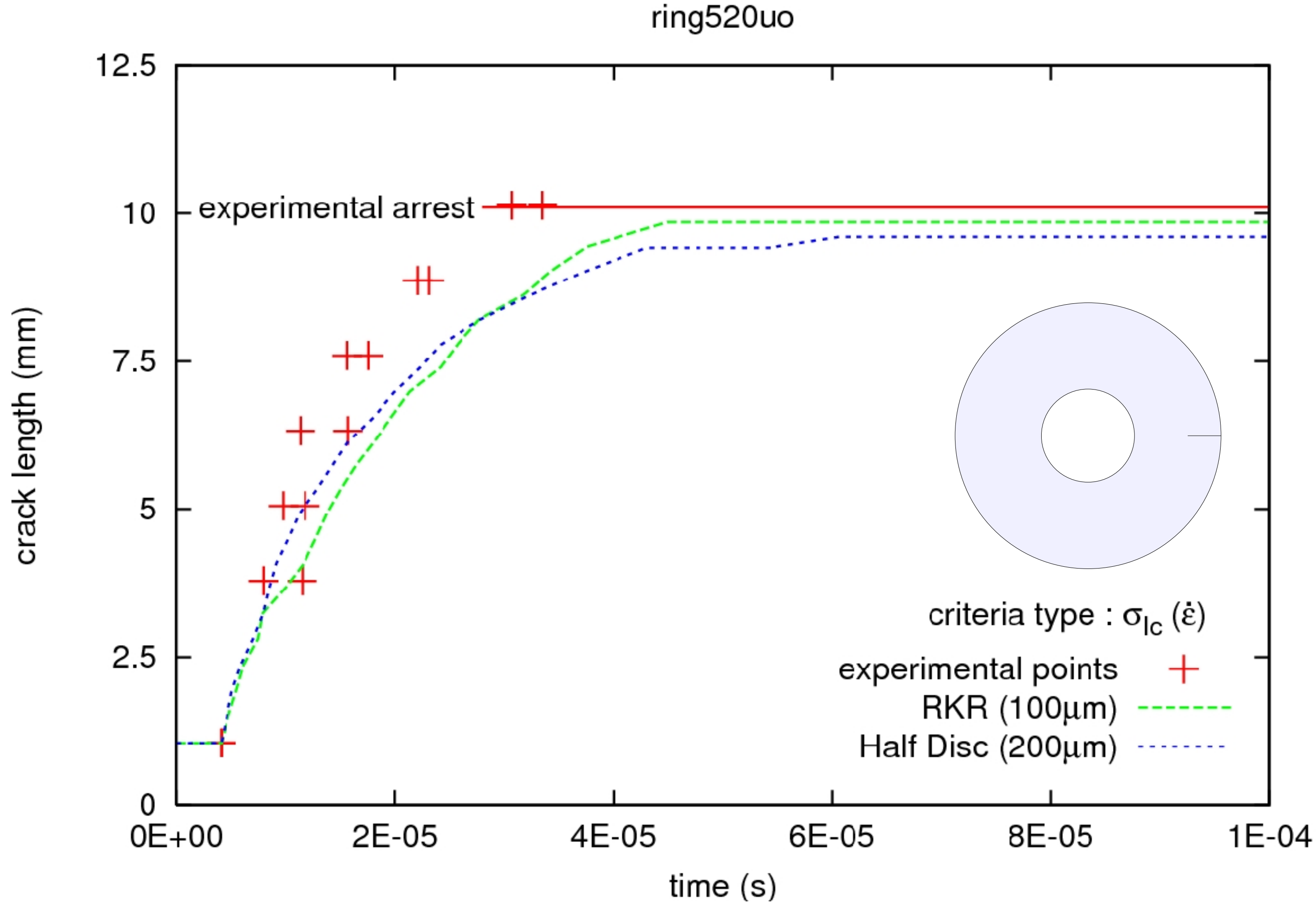

Model of Propagation – Step 2: Predictive Simulations

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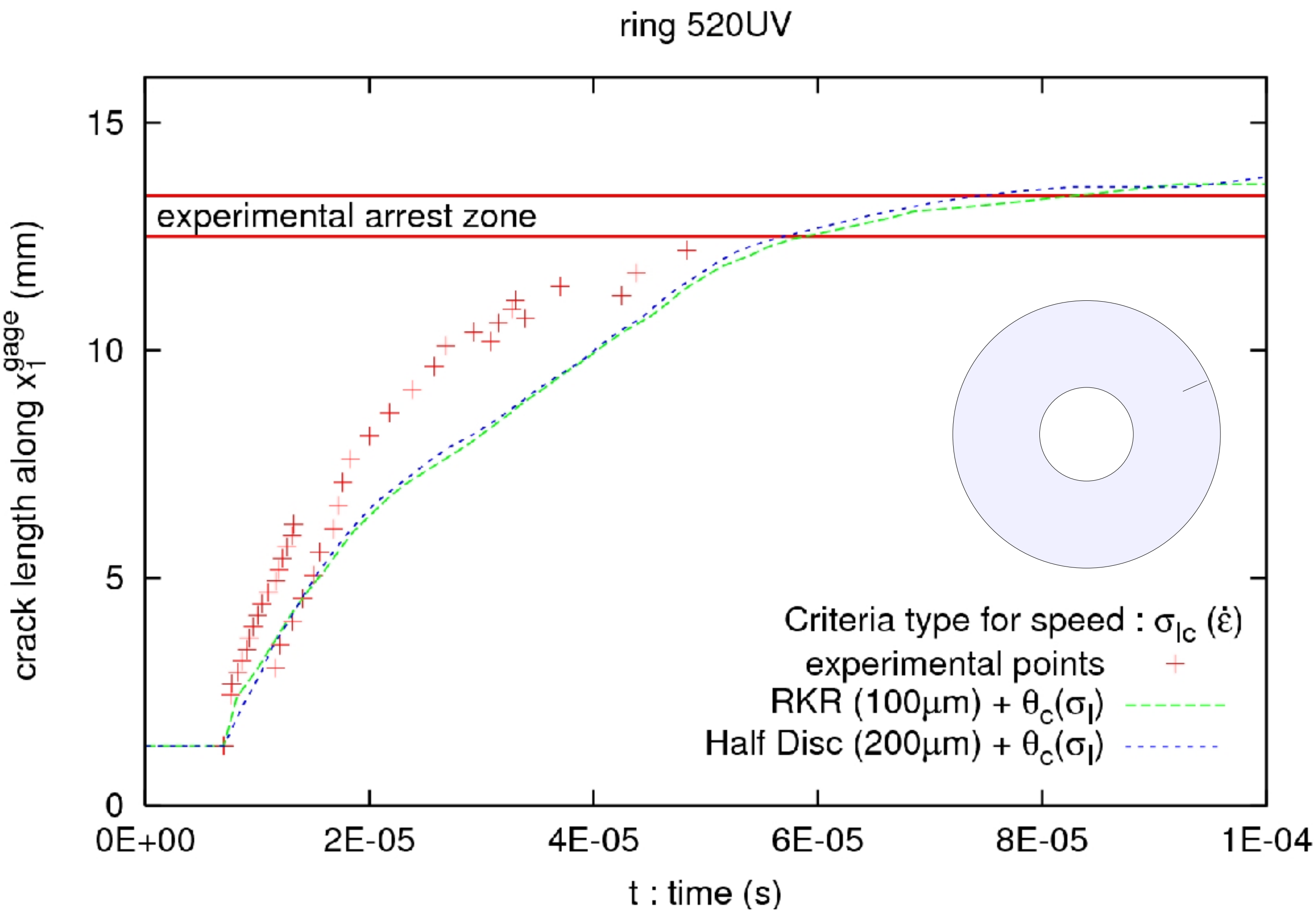
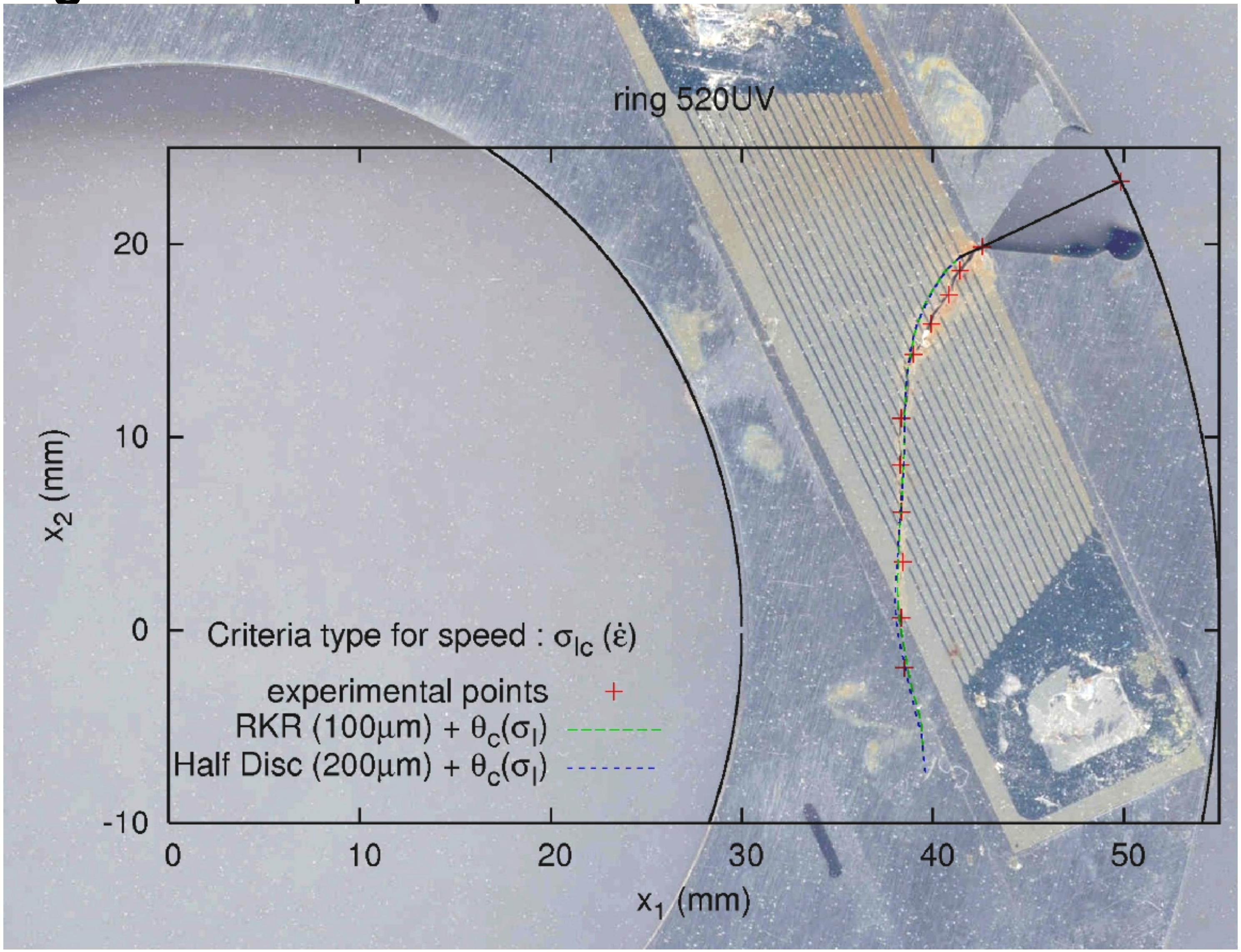
Application of the criteria in a **predictive** way to:
3. **Ring** under compression in **mixed mode**



Model of Propagation – Step 2: Predictive Simulations

Application of the criteria in a **predictive** way to:

3. **Ring** under compression in **mixed mode**



Model of Propagation – Step 2: Predictive Simulations

Application of the criteria in a **predictive** way to:
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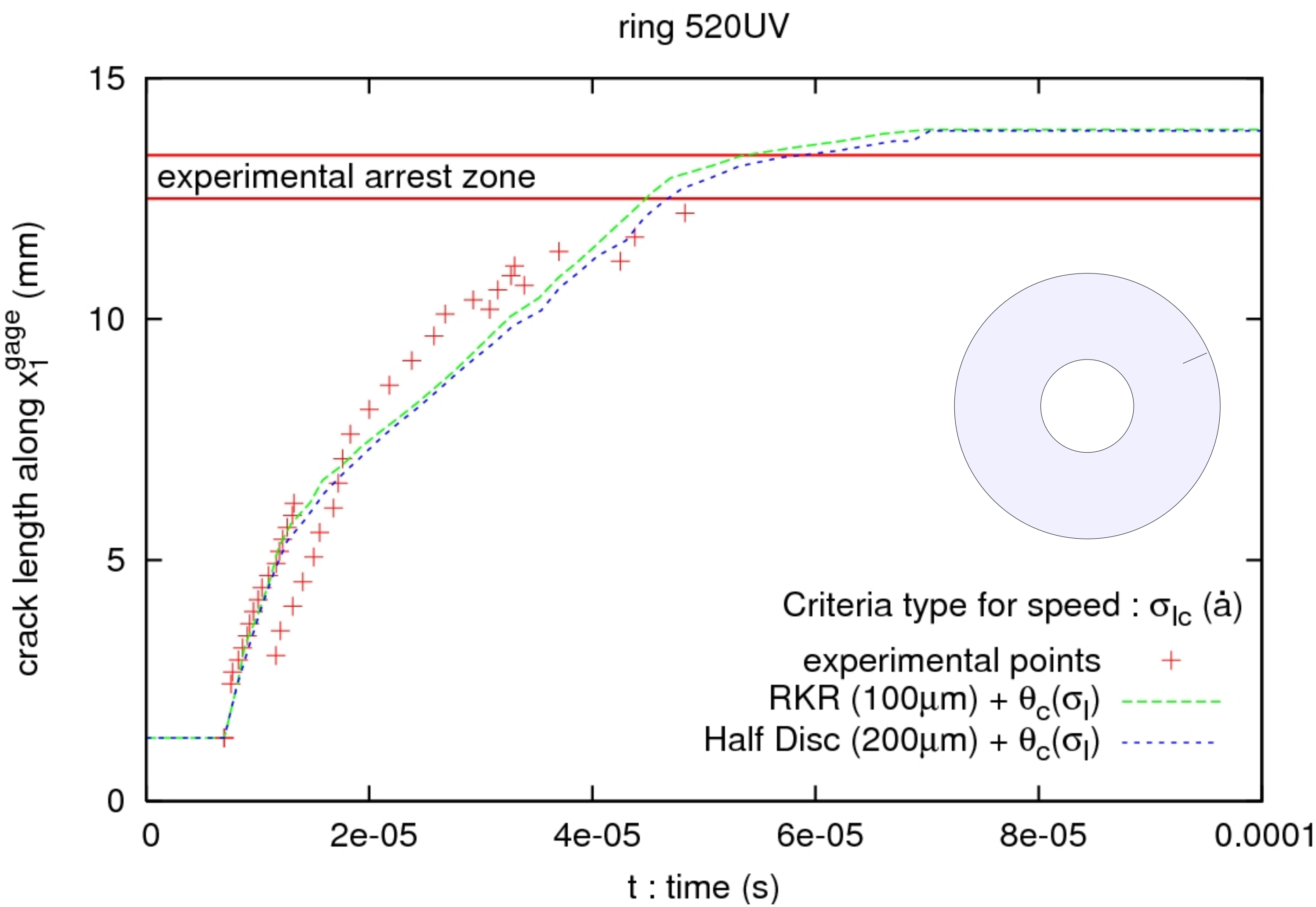

Model of Propagation – Step 2: Predictive Simulations

Application of the criteria in a **predictive** way to:
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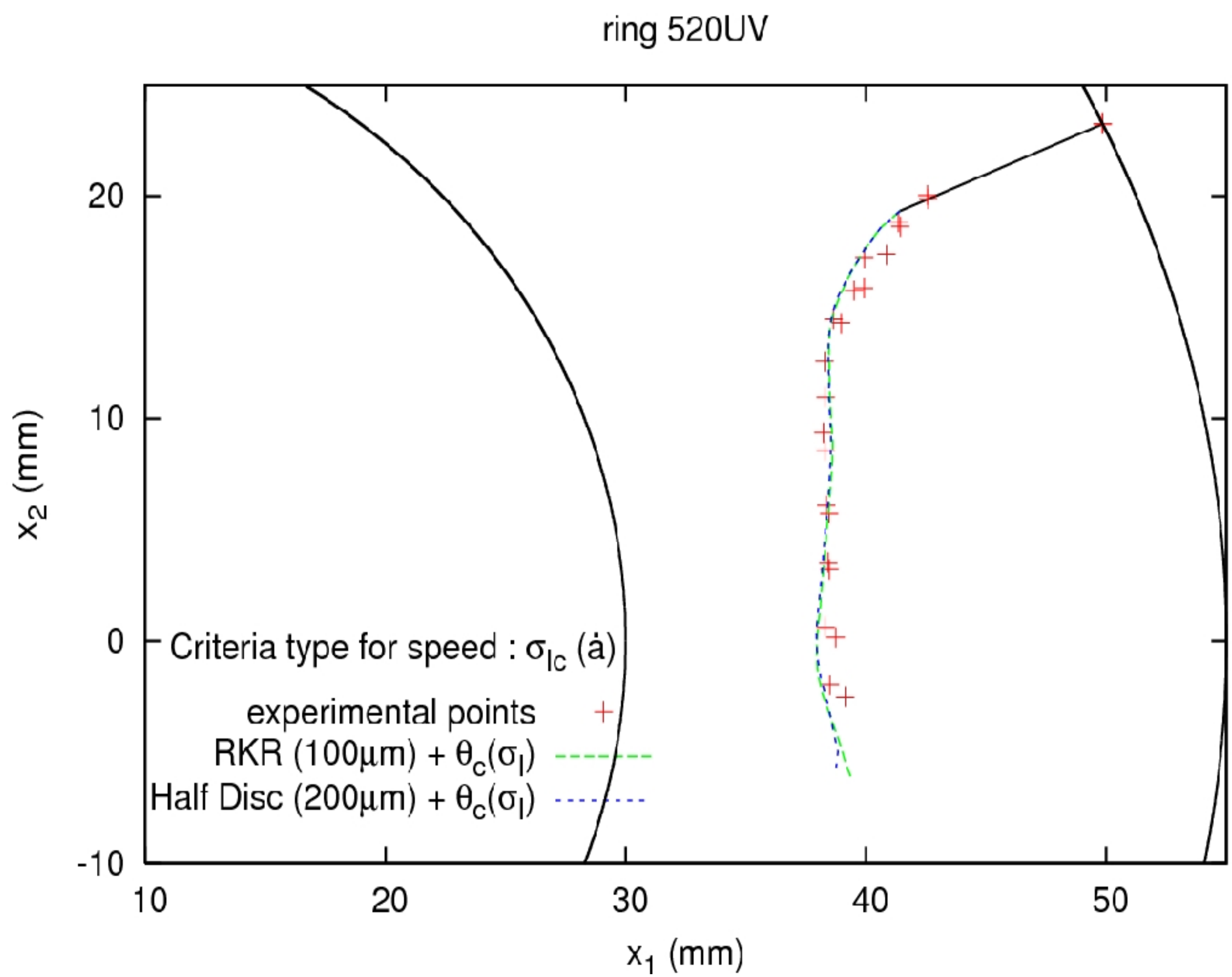


Model of Propagation – Step 2: Predictive Simulations

Application of the criteria in a **predictive** way to:
3. **Ring** under compression in **mixed mode**

Title:crackpath_anomix_s1c_v_3.eps
Creator:gnuplot 4.0 patchlevel 0
CreationDate:Thu Jun 21 09:54:29 2007

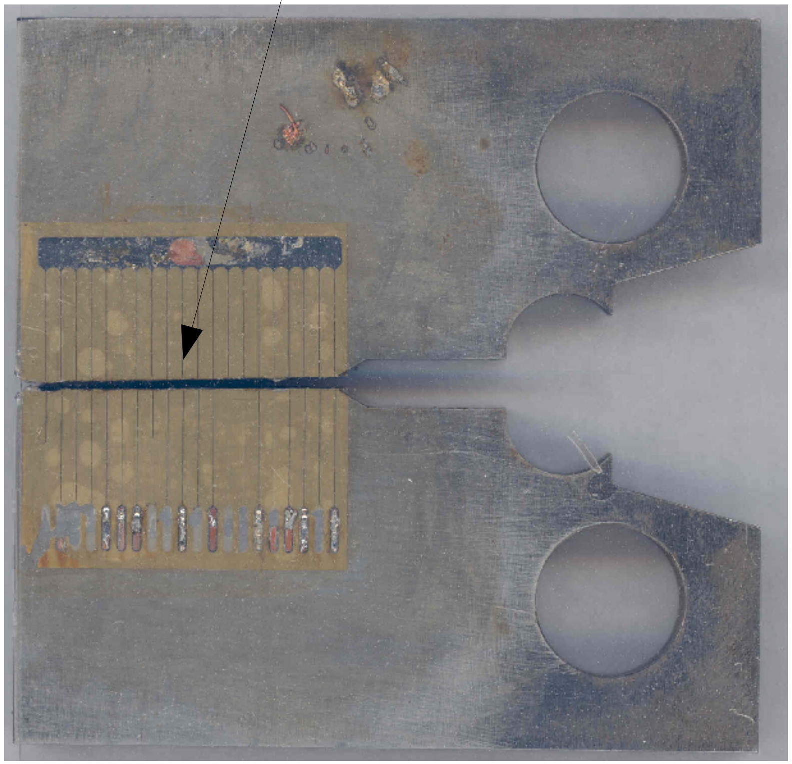
Conclusion and prospect

Prospect:

- Explain differences between crack paths observed for thin specimen:

for fracture at low load

→ **straight** crack path



high fracture load initiation

→ **crack branching**