Approximating Context-Free Grammars for Parsing and Verification

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LORIA, INRIA Nancy - Grand Est

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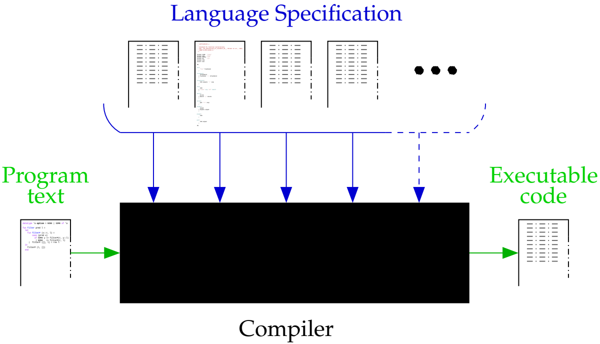
Standard ML

Milner et al. [1997]



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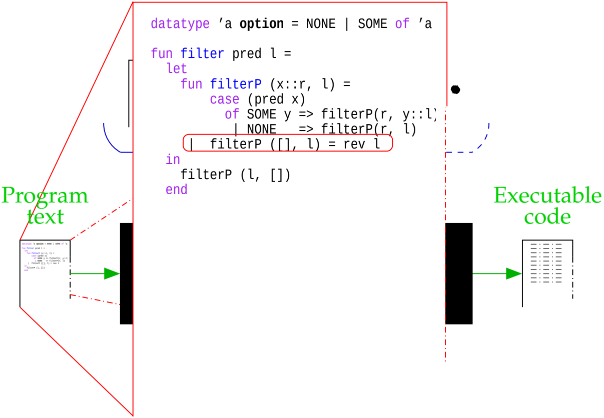

```
datatype 'a option = NONE | SOME of 'a

fun filter pred l =
  let
    fun filterP (x::r, l) =
        case (pred x)
          of SOME y => filterP(r, y::l)
           | NONE   => filterP(r, l)
    |  filterP ([], l) = rev l
  in
    filterP (l, [])
  end
```

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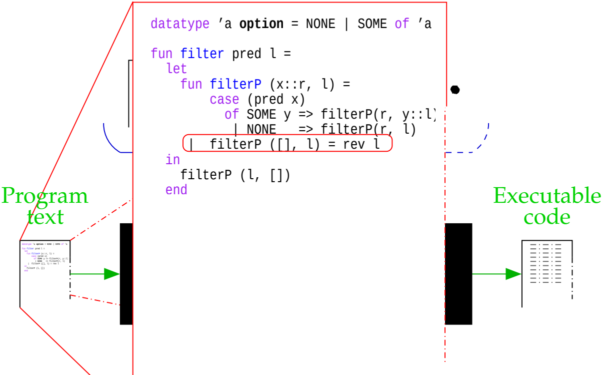

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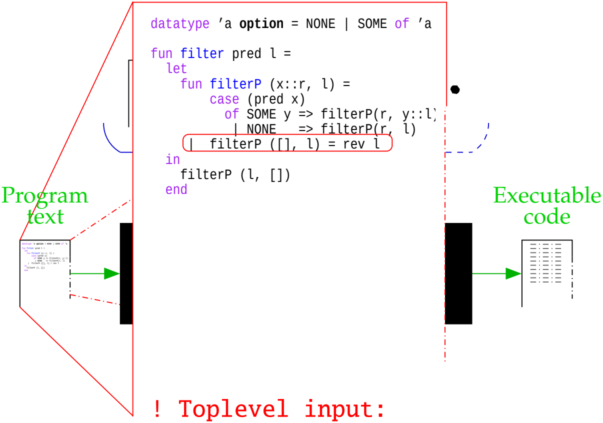
  end
```

```
Error: match.sml 9.25.
 Syntax error: replacing EQUALOP with DARROW
```

- MLton
- Moscow ML
- Poly/ML
- SML/NJ

Standard ML

Milner et al. [1997]



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```

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```
! Toplevel input:
!       |  filterP ([], l) = rev l
!                            ^
! Syntax error.
```

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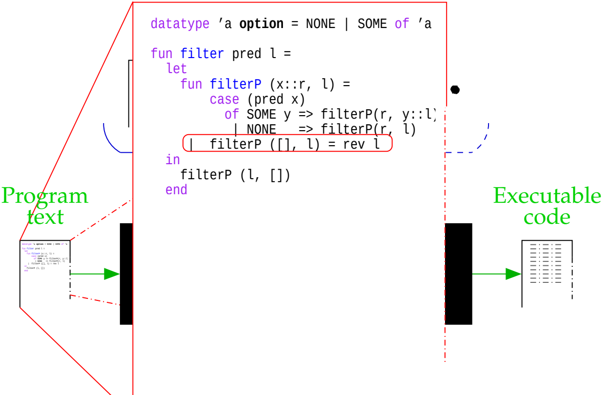

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Error: => expected but = was found

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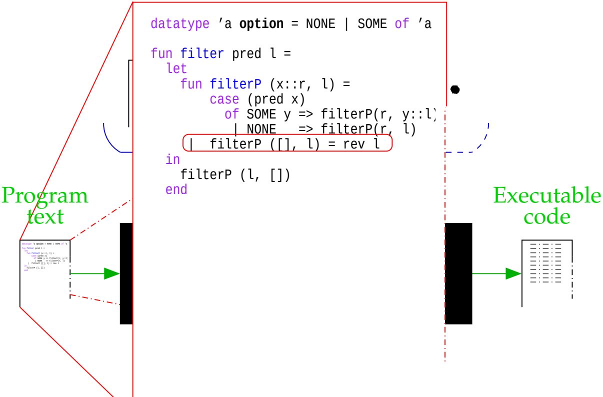

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  in
    filterP (l, [])
  end
```

```
stdIn:7.24-7.29 Error: syntax error:
  deleting  EQUALOP ID
```

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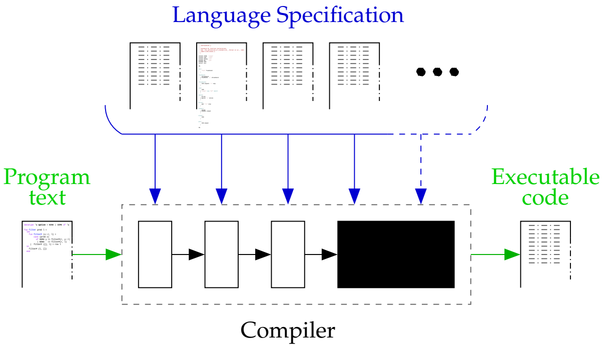
Parsers



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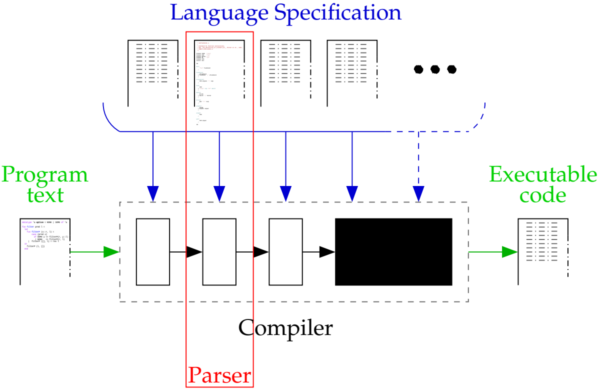

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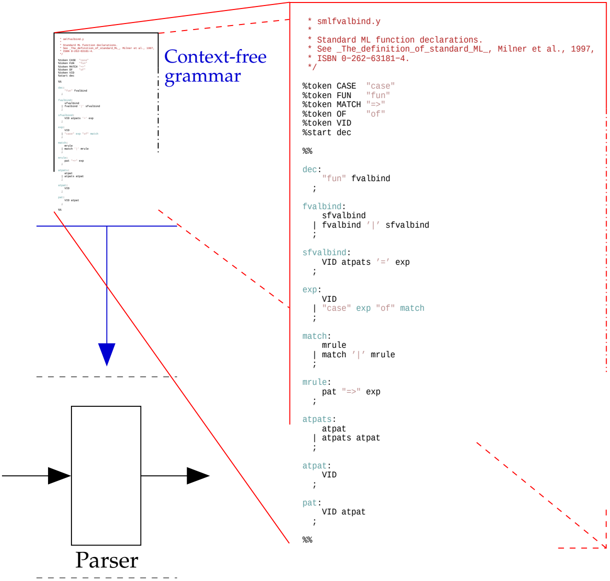


Context-free grammar

Parser

```
 * smlfvalbind.y
 *
 * Standard ML function declarations.
 * See _The_definition_of_standard_ML_, Milner et al., 1997,
 * ISBN 0-262-63181-4.
 */

%token CASE  "case"
%token FUN   "fun"
%token MATCH "=>"
%token OF    "of"
%token VID
%start dec

%%

dec:
    "fun" fvalbind
  ;

fvalbind:
    sfvalbind
  | fvalbind '|' sfvalbind
  ;

sfvalbind:
    VID atpats '=' exp
  ;

exp:
    VID
  | "case" exp "of" match
  ;

match:
    mrule
  | match '|' mrule
  ;

mrule:
    pat "=>" exp
  ;

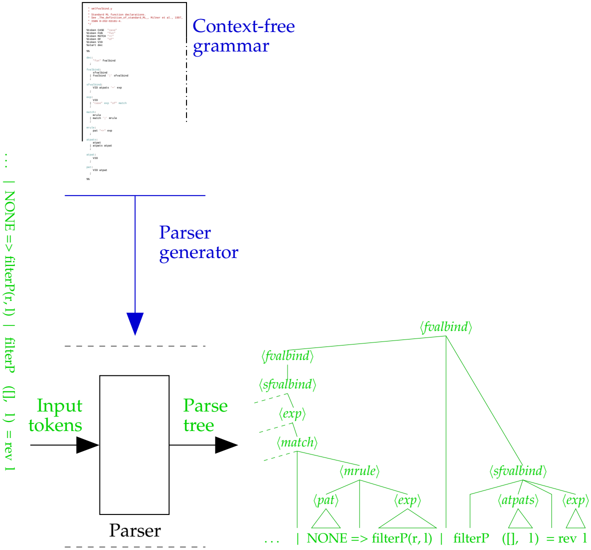
atpats:
    atpat
  | atpats atpat
  ;

atpat:
    VID
  ;

pat:
    VID atpat
  ;

%%
```

Parsers



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fvalbind:
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  | fvalbind '|' sfvalbind
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sfvalbind:
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  ;

exp:
    VID
  | "case" exp "of" match
  ;

match:
    mrule
  | match '|' mrule
  ;

mrule:
    pat "=>" exp
  ;

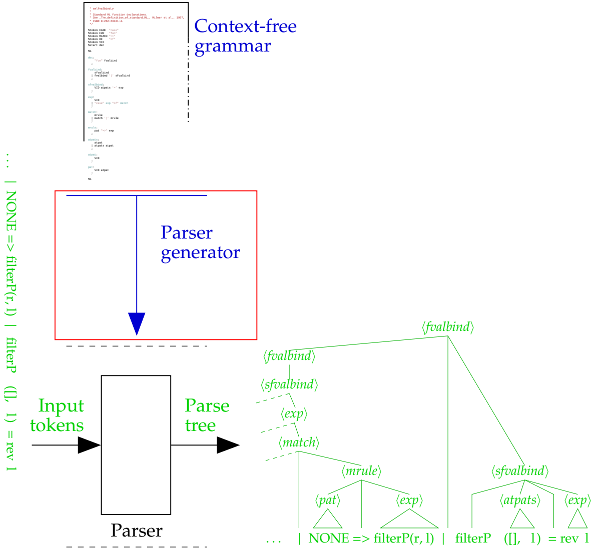
atpats:
    atpat
  | atpats atpat
  ;

atpat:
    VID
  ;

pat:
    VID atpat
  ;

%%
```

Parsers



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match:
    mrule
  | match '|' mrule
  ;

mrule:
    pat "=>" exp
  ;

atpats:
    atpat
  | atpats atpat
  ;

atpat:
    VID
  ;

pat:
    VID atpat
  ;

%%
```

LALR(1) Parser Generator

- GNU Bison

```
state 20
    6 exp: "case" exp "of" match .
    8 match: match . '|' mrule

    '|'  shift, and go to state 24
    '|'       [reduce using rule 6 (exp)]
```

- Restricted grammar class

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state 20
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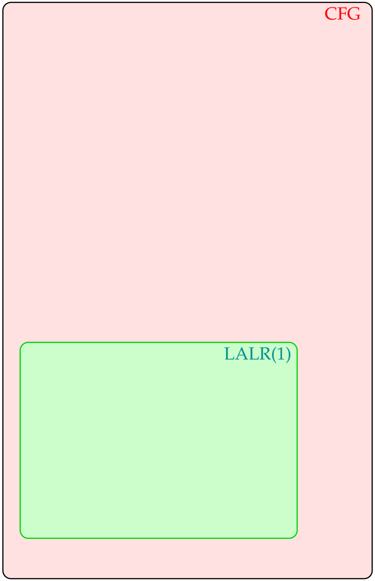
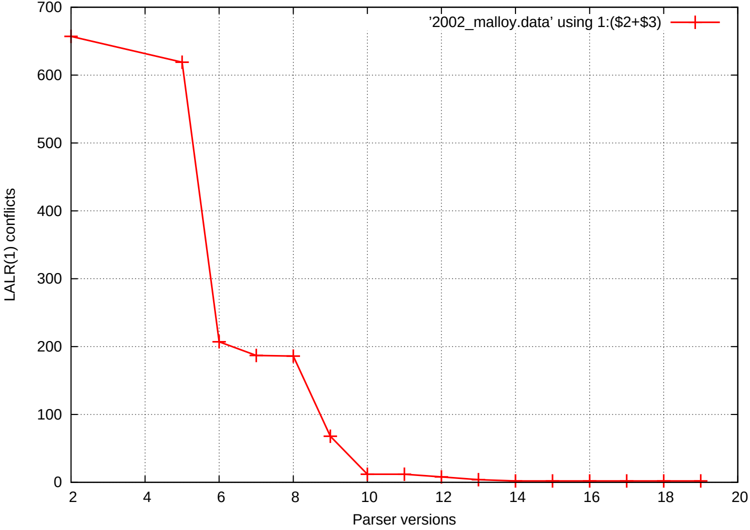


Dealing with Conflicts

An Objective Measure [Malloy et al., 2002] on a C# Grammar



Dealing with Conflicts

A Subjective Measure

Courtesy of `http://www.phdcomics.com`.

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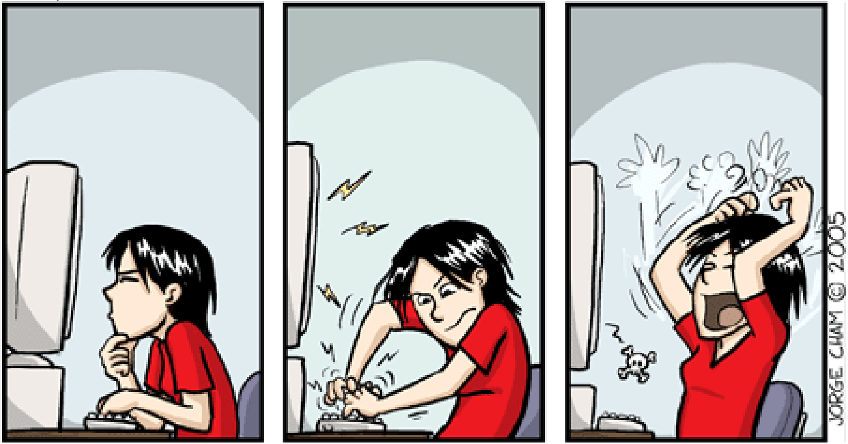

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State of the Art

- LR(k) [Knuth, 1965]
- LR-Regular [Čulik and Cohen, 1973]
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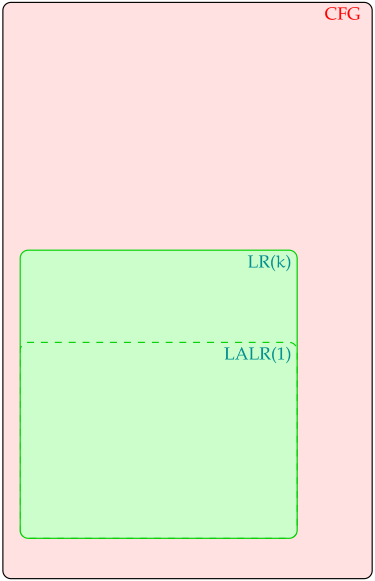

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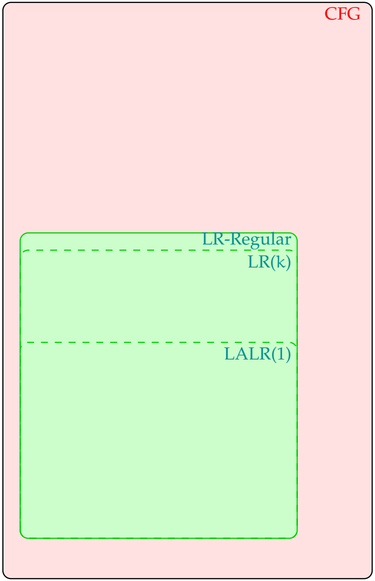

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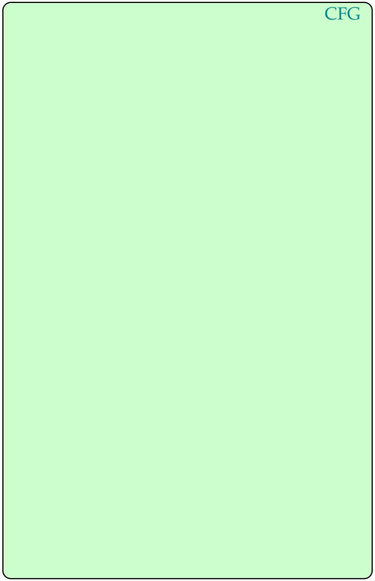
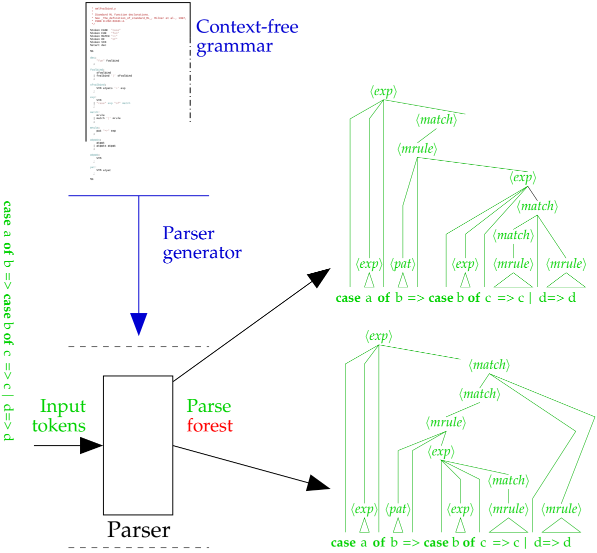


Ambiguity



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    VID
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pat:
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  ;

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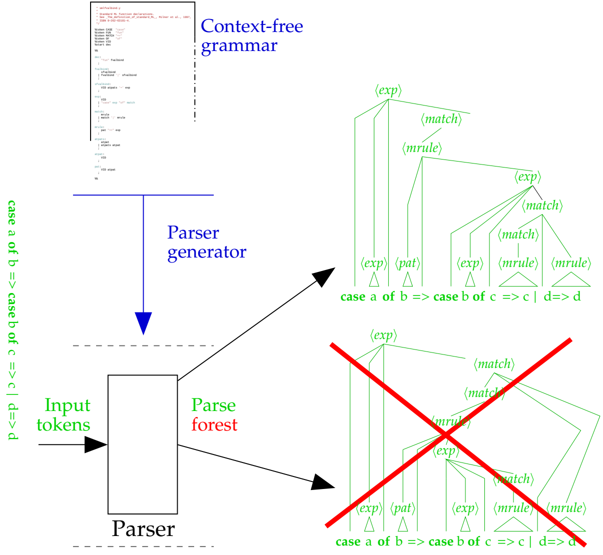
```

Ambiguity



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    VID
  ;

pat:
    VID atpat
  ;

%%
```

Context-free grammar
Parser generator
case a of b => case b of c => c | d=> d
Input tokens
Parser
Parse forest
⟨exp⟩
⟨match⟩
⟨mrule⟩
⟨exp⟩
⟨match⟩
⟨match⟩
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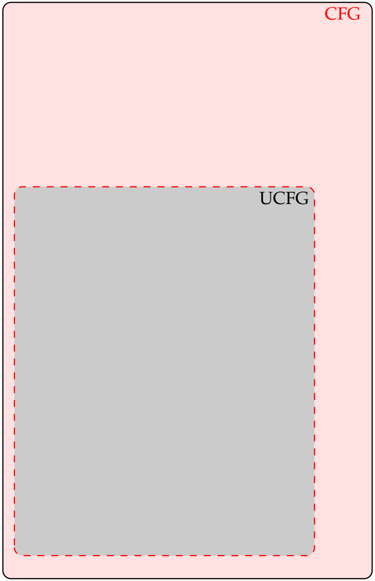

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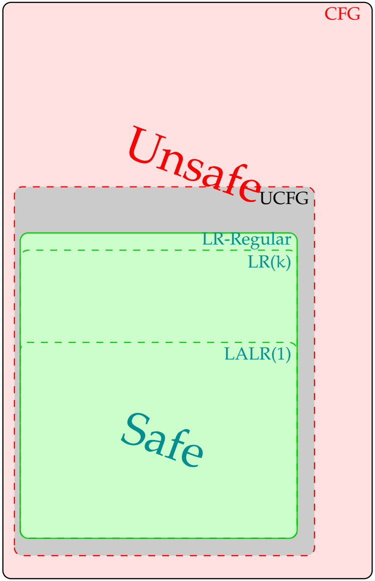

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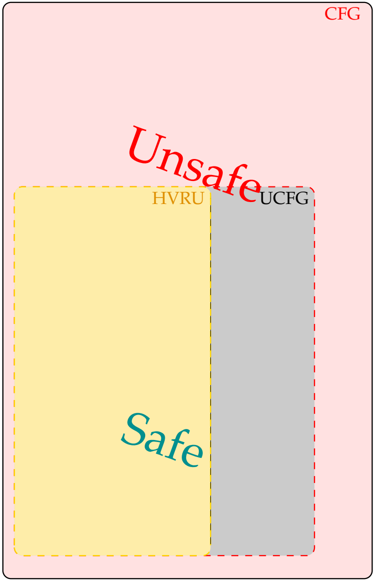

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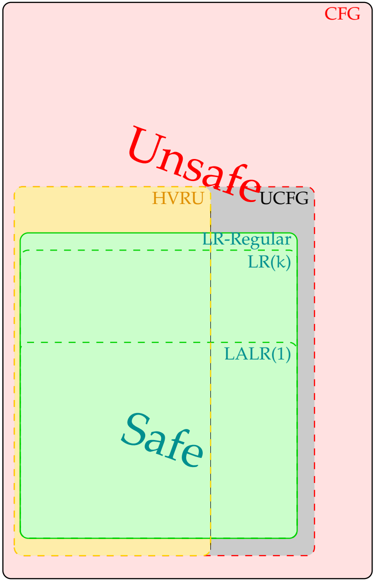

Contributions

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  - Noncanonical LALR(1)
  - Shift-Resolve
- Noncanonical unambiguity test
- Framework for grammar approximations

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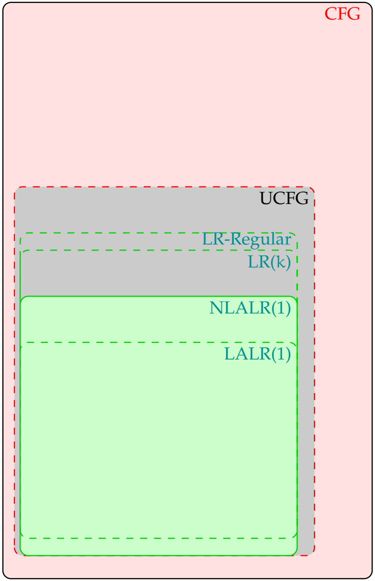

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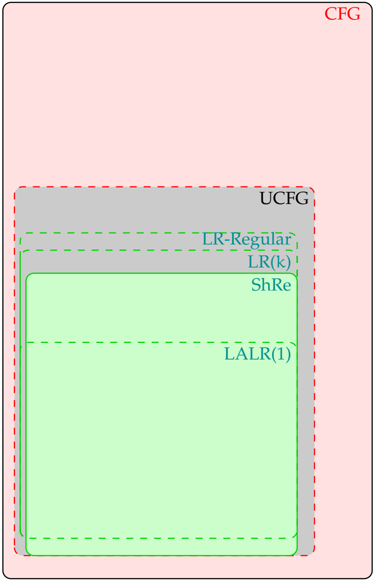

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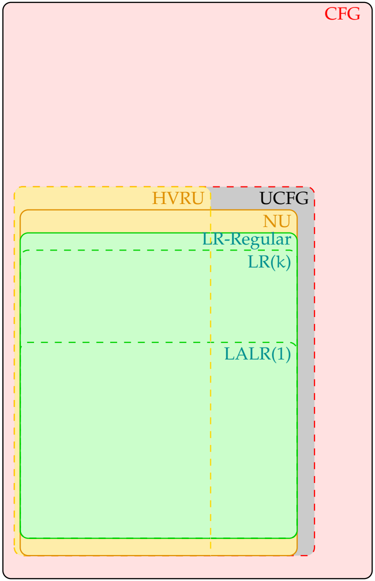

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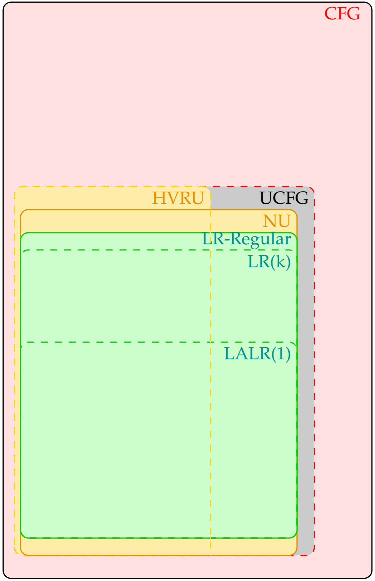


Bracketed Grammars

$$\mathcal{G} = \langle N, T, P, S \rangle,\ V = N \cup T$$

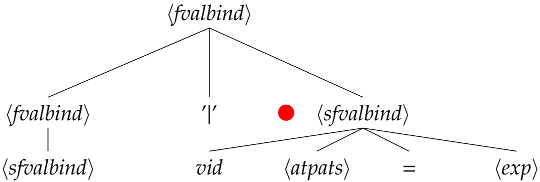
$$\begin{array}{rcl}
\langle \mathit{dec} \rangle & \xrightarrow{1} & \mathbf{fun}\ \langle \mathit{fvalbind} \rangle \\
\langle \mathit{fvalbind} \rangle & \xrightarrow{2} & \langle \mathit{sfvalbind} \rangle \\
\langle \mathit{fvalbind} \rangle & \xrightarrow{3} & \langle \mathit{fvalbind} \rangle\ '|'\ \langle \mathit{sfvalbind} \rangle \\
\langle \mathit{sfvalbind} \rangle & \xrightarrow{4} & \mathit{vid}\ \langle \mathit{atpats} \rangle\ =\ \langle \mathit{exp} \rangle \\
\langle \mathit{exp} \rangle & \xrightarrow{5} & \mathbf{case}\ \langle \mathit{exp} \rangle\ \mathbf{of}\ \langle \mathit{match} \rangle \\
\langle \mathit{match} \rangle & \xrightarrow{6} & \langle \mathit{mrule} \rangle \\
\langle \mathit{match} \rangle & \xrightarrow{7} & \langle \mathit{match} \rangle\ '|'\ \langle \mathit{mrule} \rangle \\
\langle \mathit{mrule} \rangle & \xrightarrow{8} & \langle \mathit{pat} \rangle \ \mathtt{=>}\ \langle \mathit{exp} \rangle \\
\langle \mathit{atpats} \rangle & \xrightarrow{9} & \langle \mathit{atpat} \rangle \\
\langle \mathit{atpats} \rangle & \xrightarrow{10} & \langle \mathit{atpats} \rangle\ \langle \mathit{atpat} \rangle \\
\langle \mathit{pat} \rangle & \xrightarrow{11} & \mathit{vid}\ \langle \mathit{atpat} \rangle \\
\langle \mathit{atpat} \rangle & \xrightarrow{12} & \mathit{vid}
\end{array}$$

Bracketed Grammars

$$\mathcal{G}_b = \langle N, T_b, P_b, S\rangle,\ V_b = N \cup T_b$$

$$\begin{array}{rcl}
\langle \mathit{dec}\rangle & \xrightarrow{1} & d_1\ \mathbf{fun}\ \langle \mathit{fvalbind}\rangle\ r_1 \\
\langle \mathit{fvalbind}\rangle & \xrightarrow{2} & d_2\ \langle \mathit{sfvalbind}\rangle\ r_2 \\
\langle \mathit{fvalbind}\rangle & \xrightarrow{3} & d_3\ \langle \mathit{fvalbind}\rangle\ '|'\ \langle \mathit{sfvalbind}\rangle\ r_3 \\
\langle \mathit{sfvalbind}\rangle & \xrightarrow{4} & d_4\ \mathit{vid}\ \langle \mathit{atpats}\rangle\ =\ \langle \mathit{exp}\rangle\ r_4 \\
\langle \mathit{exp}\rangle & \xrightarrow{5} & d_5\ \mathbf{case}\ \langle \mathit{exp}\rangle\ \mathbf{of}\ \langle \mathit{match}\rangle\ r_5 \\
\langle \mathit{match}\rangle & \xrightarrow{6} & d_6\ \langle \mathit{mrule}\rangle\ r_6 \\
\langle \mathit{match}\rangle & \xrightarrow{7} & d_7\ \langle \mathit{match}\rangle\ '|'\ \langle \mathit{mrule}\rangle\ r_7 \\
\langle \mathit{mrule}\rangle & \xrightarrow{8} & d_8\ \langle \mathit{pat}\rangle\ \text{=>}\ \langle \mathit{exp}\rangle\ r_8 \\
\langle \mathit{atpats}\rangle & \xrightarrow{9} & d_9\ \langle \mathit{atpat}\rangle\ r_9 \\
\langle \mathit{atpats}\rangle & \xrightarrow{10} & d_{10}\ \langle \mathit{atpats}\rangle\ \langle \mathit{atpat}\rangle\ r_{10} \\
\langle \mathit{pat}\rangle & \xrightarrow{11} & d_{11}\ \mathit{vid}\ \langle \mathit{atpat}\rangle\ r_{11} \\
\langle \mathit{atpat}\rangle & \xrightarrow{12} & d_{12}\ \mathit{vid}\ r_{12}
\end{array}$$

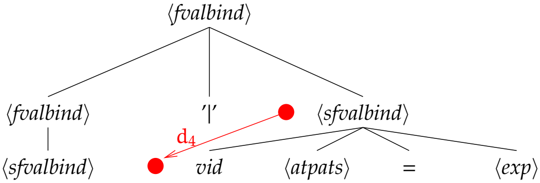
Positions



$$\mathrm{d}_3\ \mathrm{d}_2\ \langle \mathit{sfvalbind} \rangle\ \mathrm{r}_2\ '|'\ {\color{red}\bullet}\, \mathrm{d}_4\ \mathit{vid}\ \langle \mathit{atpats} \rangle\ =\ \langle \mathit{exp} \rangle\ \mathrm{r}_4\ \mathrm{r}_3$$

Position Graph Γ

Left-to-right Walks in Trees



$$\mathrm{d}_3\ \mathrm{d}_2\ \langle sfvalbind\rangle\ \mathrm{r}_2\ \text{'|'}\ \mathrm{d}_4 \bullet\ vid\ \langle atpats\rangle\ =\ \langle exp\rangle\ \mathrm{r}_4\ \mathrm{r}_3$$

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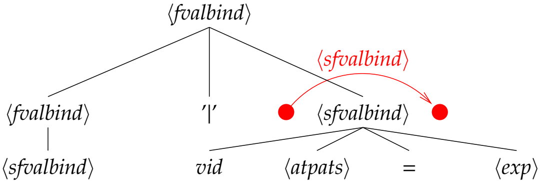

$$d_3\ d_2\ \langle \mathit{sfvalbind} \rangle\ r_2\ '|'\ d_4\ \mathit{vid}\ \langle \mathit{atpats} \rangle\ =\ \langle \mathit{exp} \rangle\ r_4 \bullet\ r_3$$

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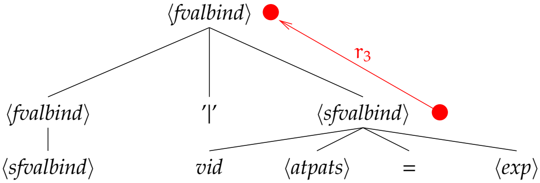

$$\mathrm{d}_3\ \mathrm{d}_2\ \langle \mathit{sfvalbind} \rangle\ \mathrm{r}_2\ '|'\ \mathrm{d}_4\ \mathit{vid}\ \langle \mathit{atpats} \rangle\ =\ \langle \mathit{exp} \rangle\ \mathrm{r}_4\ \mathrm{r}_3 \bullet$$

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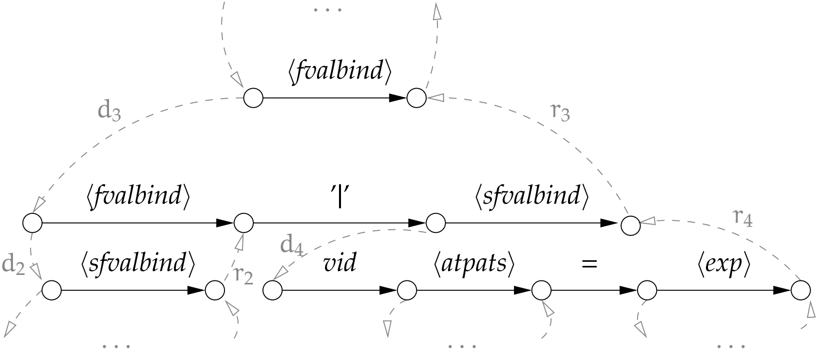


Position Automaton $\Gamma/{\equiv}$

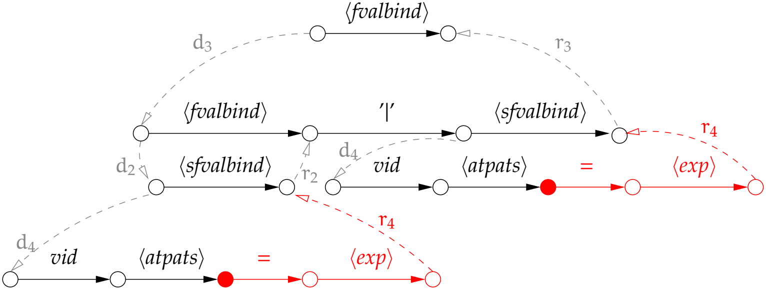
Definition

$\Gamma/{\equiv}$ is the quotient of Γ by an equivalence relation $\equiv$ between positions.

Theorem (Language over-approximation)

$$\mathcal{L}(\mathcal{G}_b) \subseteq \mathcal{L}(\Gamma/{\equiv}) \cap T_b^*$$

Example: item_0 Equivalence



- equivalence class

  $[\langle \mathit{sfvalbind}\rangle \xrightarrow{4} \mathit{vid}\ \langle \mathit{atpats}\rangle \bullet\ =\ \langle \mathit{exp}\rangle]$
- LR(0) items
- Γ/item_0: nondeterministic LR(0) automaton

Example: item$_0$ Equivalence

$$\begin{tikzcd}
{[\langle \mathit{fvalbind}\rangle \xrightarrow{2} \bullet \langle \mathit{sfvalbind}\rangle]} \arrow[dr, dashed, "d_4"'] & & {[\langle \mathit{fvalbind}\rangle \xrightarrow{3} \langle \mathit{fvalbind}\rangle\ '|'\ \bullet \langle \mathit{sfvalbind}\rangle]} \arrow[dl, dashed, "d_4"] \\
& {[\langle \mathit{sfvalbind}\rangle \xrightarrow{4} \bullet \mathit{vid}\ \langle \mathit{atpats}\rangle\ =\ \langle \mathit{exp}\rangle]} \arrow[d, "\mathit{vid}"] & \\
& {[\langle \mathit{sfvalbind}\rangle \xrightarrow{4} \mathit{vid} \bullet \langle \mathit{atpats}\rangle\ =\ \langle \mathit{exp}\rangle]} \arrow[d, "\langle \mathit{atpats}\rangle"] & \\
& {[\langle \mathit{sfvalbind}\rangle \xrightarrow{4} \mathit{vid}\ \langle \mathit{atpats}\rangle \bullet\ =\ \langle \mathit{exp}\rangle]} \arrow[d, "="] & \\
& {[\langle \mathit{sfvalbind}\rangle \xrightarrow{4} \mathit{vid}\ \langle \mathit{atpats}\rangle\ =\ \bullet \langle \mathit{exp}\rangle]} \arrow[d, "\langle \mathit{exp}\rangle"] & \\
& {[\langle \mathit{sfvalbind}\rangle \xrightarrow{4} \mathit{vid}\ \langle \mathit{atpats}\rangle\ =\ \langle \mathit{exp}\rangle \bullet]} \arrow[dl, dashed, "r_4"] \arrow[dr, dashed, "r_4"'] & \\
{[\langle \mathit{fvalbind}\rangle \xrightarrow{2} \langle \mathit{sfvalbind}\rangle \bullet]} & & {[\langle \mathit{fvalbind}\rangle \xrightarrow{3} \langle \mathit{fvalbind}\rangle\ '|'\ \langle \mathit{sfvalbind}\rangle \bullet]}
\end{tikzcd}$$

Summary

- general framework for approximations
- applications:
  - parser construction
  - ambiguity detection
  - XML validation [Segoufin and Vianu, 2002]?
  - symbolic supertagging [Boullier, 2003]?

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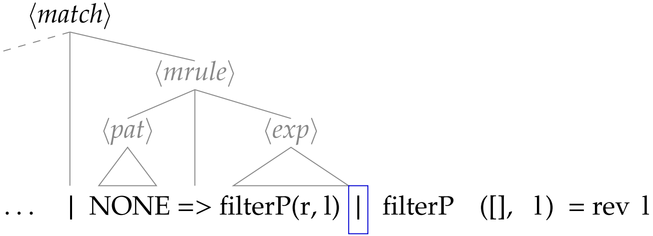
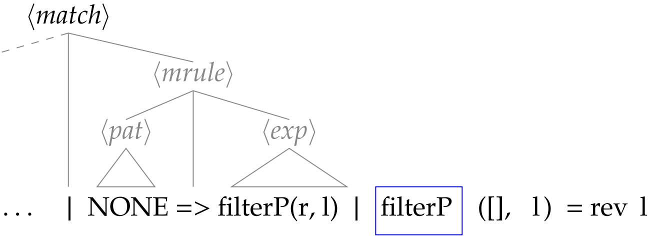
. . . | NONE => filterP(r, l) | filterP   ([],   l) = rev l

Shift-Resolve Parse



Shift-Resolve Parse



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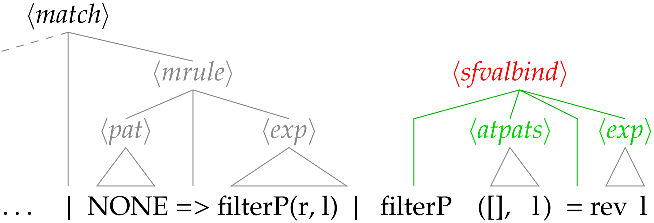

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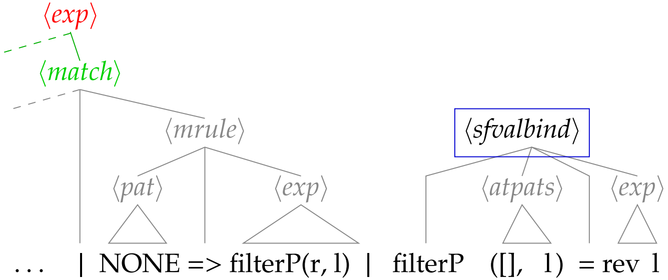

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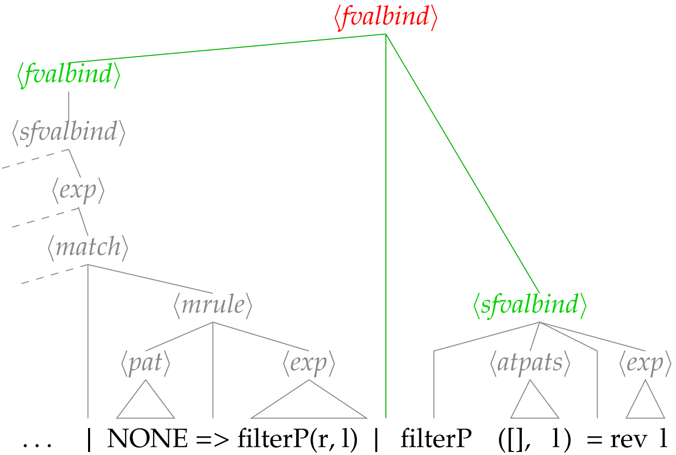

Generating the Parser

1. position automaton
2. determinization by subset construction

Subset Construction

Principle

- d_i transitions denote traditional item closures
- r_i transitions denote a phrase that should be reduced
- other transitions denote shifts
- items in the construction hold
  1. a state of the position automaton
  2. a parsing action
  3. a pushback length

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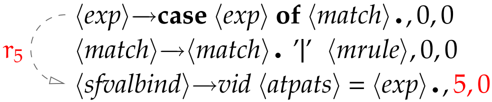
Subset Construction

Example

$$\langle exp\rangle \rightarrow \textbf{case}\ \langle exp\rangle\ \textbf{of}\ \langle match\rangle \bullet, 0, 0$$
$$\langle match\rangle \rightarrow \langle match\rangle \bullet\ '|'\ \ \langle mrule\rangle, 0, 0$$

Subset Construction

Example



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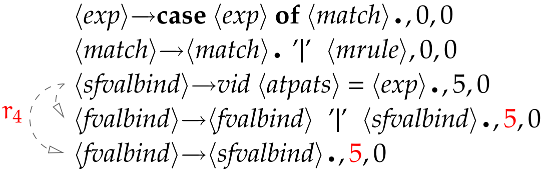
$\langle sfvalbind\rangle \rightarrow vid\ \langle atpats\rangle = \langle exp\rangle \bullet, 5, 0$

Subset Construction

Example



$\langle exp\rangle \to \textbf{case}\ \langle exp\rangle\ \textbf{of}\ \langle match\rangle \bullet, 0, 0$

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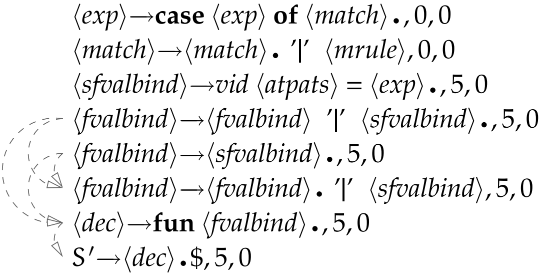

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$\langle fvalbind\rangle \rightarrow \langle fvalbind\rangle \bullet\ \text{'|'}\ \langle sfvalbind\rangle, 5, 0$

$\langle dec\rangle \rightarrow \textbf{fun}\ \langle fvalbind\rangle \bullet, 5, 0$

$S' \rightarrow \langle dec\rangle \bullet \$, 5, 0$

Subset Construction

Example

$\langle exp\rangle \rightarrow \textbf{case}\ \langle exp\rangle\ \textbf{of}\ \langle match\rangle \bullet, 0, 0$

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Subset Construction

Example

$$\langle \mathit{exp} \rangle \rightarrow \textbf{case}\ \langle \mathit{exp} \rangle\ \textbf{of}\ \langle \mathit{match} \rangle \bullet, 0, 0$$
$$\langle \mathit{match} \rangle \rightarrow \langle \mathit{match} \rangle \bullet\ \text{'|'}\ \langle \mathit{mrule} \rangle, 0, 0$$
$$\langle \mathit{sfvalbind} \rangle \rightarrow \mathit{vid}\ \langle \mathit{atpats} \rangle = \langle \mathit{exp} \rangle \bullet, 5, 0$$
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$$\Big\downarrow \text{'|'}$$

$$\langle \mathit{fvalbind} \rangle \rightarrow \langle \mathit{fvalbind} \rangle\ \text{'|'} \bullet \langle \mathit{sfvalbind} \rangle, 5, 1$$
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Subset Construction

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$\langle exp\rangle\rightarrow\textbf{case}\ \langle exp\rangle\ \textbf{of}\ \langle match\rangle\,\bullet,0,0$

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$\langle sfvalbind\rangle\rightarrow vid\ \langle atpats\rangle = \langle exp\rangle\,\bullet,5,0$

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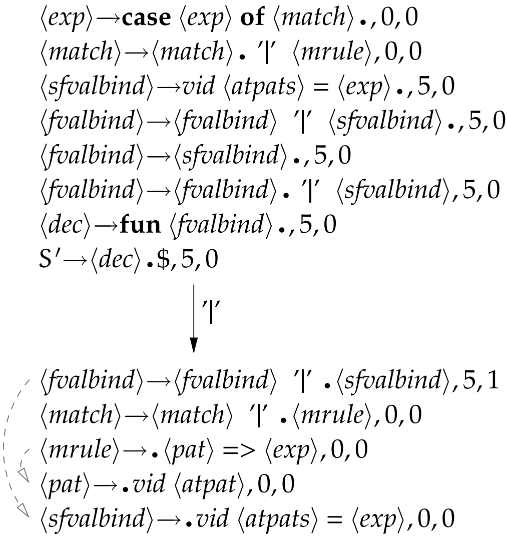
d_8 $\langle match\rangle\rightarrow\langle match\rangle\ \text{'|'}\ \bullet\langle mrule\rangle,0,0$

$\langle mrule\rangle\rightarrow\bullet\langle pat\rangle \Rightarrow \langle exp\rangle,0,0$

Subset Construction

Example



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$S' \to \langle dec\rangle \bullet \$, 5, 0$

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$\langle match\rangle \to \langle match\rangle\ \text{'|'}\ \bullet \langle mrule\rangle, 0, 0$

$\langle mrule\rangle \to \bullet \langle pat\rangle \Rightarrow \langle exp\rangle, 0, 0$

$\langle pat\rangle \to \bullet vid\ \langle atpat\rangle, 0, 0$

$\langle sfvalbind\rangle \to \bullet vid\ \langle atpats\rangle = \langle exp\rangle, 0, 0$

Construction Failure

$$\langle exp \rangle \rightarrow \textbf{case}\ \langle exp \rangle\ \textbf{of}\ \langle match \rangle \bullet, 0, 0$$
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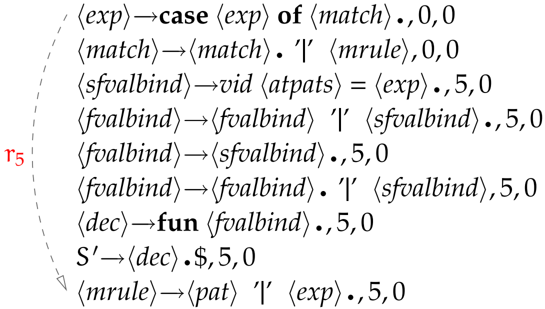

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r_5

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$S' \rightarrow \langle \mathit{dec}\rangle \bullet \$, 5, 0$

$\langle \mathit{mrule}\rangle \rightarrow \langle \mathit{pat}\rangle\ \text{'|'}\ \langle \mathit{exp}\rangle \bullet, 5, 0$

$\langle \mathit{match}\rangle \rightarrow \langle \mathit{mrule}\rangle \bullet, 5, 0$

$\langle \mathit{match}\rangle \rightarrow \langle \mathit{match}\rangle \bullet\ \text{'|'}\ \langle \mathit{mrule}\rangle, 5, 0$

Complexity

- $|\Gamma/{\equiv}|$: size of the position automaton
- $|\mathcal{A}|$: size of the parser: $\mathcal{O}(2^{|\Gamma/{\equiv}|\,|P|})$
- parsing time complexity for input w: $\mathcal{O}(|w|)$

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  $|\Gamma/\mathsf{item}_0| = \mathcal{O}(|\mathcal{G}|)$
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Summary

- Shift Resolve parsers
  1. Large class of grammars accepted
  2. Unambiguity
  3. Linear time parsing
- 2-steps construction
  1. Simple
  2. Flexible

Principles

- a bracketed sentence = a derivation tree
- ambiguity = more than one tree with the same yield

$d_6 d_8 d_{13}$ *vid* r_{13} => d_5 **case** d_{14} *vid* r_{14} **of** $d_7 d_6 d_8 d_{13}$ *vid* r_{13} => d_{14} *vid* $r_{14} r_8 r_6$ ′|′ $d_8 d_{13}$ *vid* r_{13} => d_{14} *vid* $r_{14} r_8 r_7 r_5 r_8 r_6$

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- construct a FSA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{G}_b) \subseteq \mathcal{L}(\mathcal{A})$, and look for bracketed sentences with the same yield

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$d_7 d_6 d_8 d_{13}$ *vid* r_{13} => d_5 **case** d_{14} *vid* r_{14} **of** $d_7 d_8 d_{13}$ *vid* r_{13} => d_{14} *vid* $r_{14} r_8 r_7 r_5 r_8 r_6$ '|' $d_8 d_{13}$ *vid* r_{13} => d_{14} *vid* $r_{14} r_8 r_7$

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$d_7 d_6 d_8 d_{13}$ *vid* r_{13} => d_5 **case** d_{14} *vid* r_{14} **of** $d_7 d_8 d_{13}$ *vid* r_{13} => d_{14} *vid* $r_{14} r_8 r_7 r_5 r_8 r_6$ '|' $d_8 d_{13}$ *vid* r_{13} => d_{14} *vid* $r_{14} r_8 r_7$

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RU($\equiv$)

- $\mathcal{G}$ is regular unambiguous for $\equiv$ of finite index, if there does not exist $w_b \neq w'_b$ in $\mathcal{L}(\Gamma/\equiv) \cap T^*_b$ with $h(w_b) = h(w'_b)$
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Nonterminal Transitions

- $\mathcal{SF}(\mathcal{G}_b) \subseteq \mathcal{L}(\Gamma/{\equiv})$
- look for two different bracketed sentential forms in $\mathcal{L}(\Gamma/{\equiv})$

$$\mathrm{d}_6\mathrm{d}_8\ \langle \mathit{pat}\rangle \texttt{ => } \mathrm{d}_5\ \textbf{case}\ \langle \mathit{exp}\rangle\ \textbf{of}\ \mathrm{d}_7\ \langle \mathit{match}\rangle\ '|'\ \langle \mathit{mrules}\rangle \mathrm{r}_7\mathrm{r}_5\mathrm{r}_8\mathrm{r}_6$$
$$\mathrm{d}_7\mathrm{d}_6\mathrm{d}_8\ \langle \mathit{pat}\rangle \texttt{ => } \mathrm{d}_5\ \textbf{case}\ \langle \mathit{exp}\rangle\ \textbf{of}\ \langle \mathit{match}\rangle \mathrm{r}_5\mathrm{r}_8\mathrm{r}_6\ '|'\ \langle \mathit{mrules}\rangle\ \mathrm{r}_7$$

- a nonterminal transition represents exactly its derived context-free language

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- a nonterminal transition represents exactly its derived context-free language

Nonterminal Transitions

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- a nonterminal transition represents exactly its derived context-free language

Mutual Accessibility Relations

- between pairs of states of $\Gamma/\!\equiv$, (q_1, q_2)
- synchronized left-to-right walks from an initial pair (q_s, q_s)

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epsilon: mae

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nothing!

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conflict: mac

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shift: mas

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reduce: mar

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conflict: mac

NU($\equiv$)

- $\mathsf{ma}{=}\mathsf{mas} \cup \mathsf{mae} \cup \mathsf{mac} \cup \mathsf{mar}$
- $\mathcal{G}$ is noncanonically unambiguous if there does not exist a relation $(q_s, q_s)\ \mathsf{ma}^*\ (q_f, q_f)$ that uses mac at some step
- Computation in $\mathcal{O}(|\Gamma/{\equiv}|^2)$ in space

Comparisons

- Regular Unambiguity RU($\equiv$)
- Bounded-length detection schemes
- LR(k) and LR-Regular (LR(Π))
- Horizontal and vertical ambiguity (HVRU($\equiv$))

Bounded-length detection

[Gorn, 1963, Cheung and Uzgalis, 1995, Schröer, 2001, Jampana, 2005]

- generate sentences
- not conservative
- prefix_m prevents from false positives in sentences of length $< m$
- need to generate a^{2^n+1} to find $\mathcal{G}_4^n$ ambiguous, but $\mathcal{G}_4^n \notin \mathrm{NU}(\mathsf{item}_0)$

$$S \rightarrow A \mid B_n a,\ A \rightarrow Aaa \mid a,\ B_1 \rightarrow aa,\ B_2 \rightarrow B_1 B_1,\ \ldots,\ B_n \rightarrow B_{n-1} B_{n-1} \qquad (\mathcal{G}_4^n)$$

LR(k) and LR-Regular

[Knuth, 1965, Hunt III et al., 1975, Čulik and Cohen, 1973, Heilbrunner, 1983]

- conservative tests
- define item_Π s.t. $\mathrm{LR}(\Pi) \subset \mathrm{NU}(\mathsf{item}_\Pi)$
- need a LR(2^n) test to prove $\mathcal{G}_3^n$ unambiguous, but $\mathcal{G}_3^n \in \mathrm{NU}(\mathsf{item}_0)$

$$S \rightarrow A \,|\, B_n,\ A \rightarrow Aaa \,|\, a,\ B_1 \rightarrow aa,\ B_2 \rightarrow B_1 B_1,\ \ldots,\ B_n \rightarrow B_{n-1} B_{n-1} \qquad (\mathcal{G}_3^n)$$

Implementation

- For the whole SML grammar:
  - conflicts in the LALR(1) parser

    ```
    sml.y: conflicts: 223 shift/reduce, 35 reduce/reduce
    ```
  - Our tool:

    ```
    89 potential ambiguities with LR(1) precision detected
    ```

- For the SML grammar fragment:

  ```
  2 potential ambiguities with LR(0) precision detected:
      (match -> mrule . , match -> match . '|' mrule )
      (match -> match . '|' mrule , match -> match '|' mrule . )
  ```

- NU(item$_1$) correctly identifies 87% of our unambiguous grammars—73% of the non-LALR(1) ones

Summary

- conservative ambiguity detection
- provably better than several other techniques
- also experimentally better

Conclusion

- Main issues in parser development:
  - nondeterminism
  - ambiguity in particular
- Deterministic parsers for larger classes of grammars
- Ambiguity detection algorithm

Directions for Future Work

- Linear time parsing for NU($\equiv$) grammars?
- Improved implementation
- Noncanonical languages
- Regular approximations

Thanks!

Our Issue

Shift/Reduce Conflict

GNU Bison

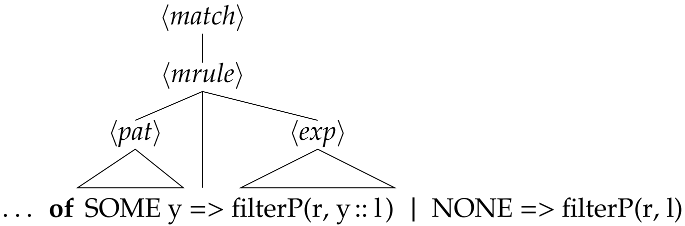
```
state 20
    6 exp: "case" exp "of" match .
    8 match: match . ’|’ mrule

    ’|’  shift, and go to state 24
    ’|’       [reduce using rule 6 (exp)]
```

Our Issue

Shift/Reduce Conflict

Which action to choose?

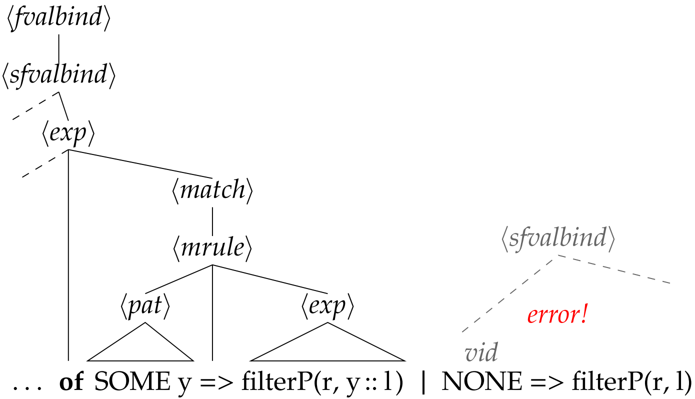


. . . **of** SOME y => filterP(r, y :: l) **|** NONE => filterP(r, l)

Our Issue

Shift/Reduce Conflict

Which action to choose? Reduce?

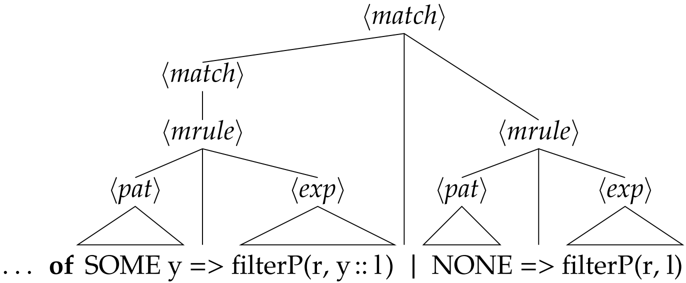


. . . **of** SOME y => filterP(r, y :: l) | NONE => filterP(r, l)

Our Issue

Shift/Reduce Conflict

Which action to choose? Shift?



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Shift/Reduce Conflict

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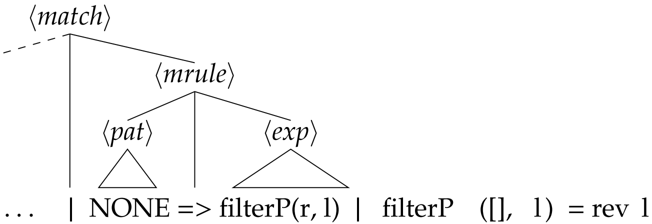

Our Issue

Shift/Reduce Conflict

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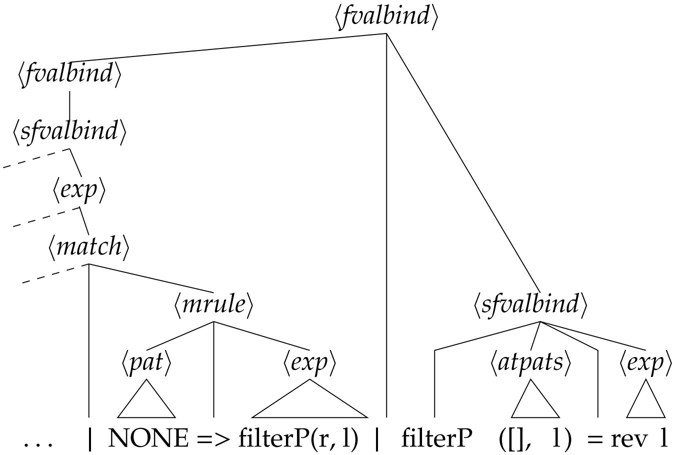

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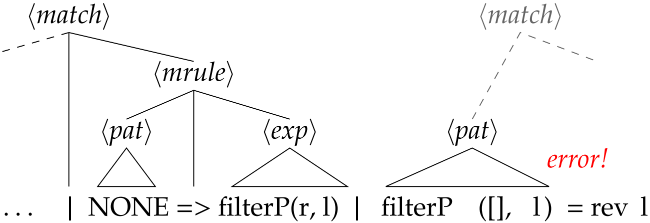
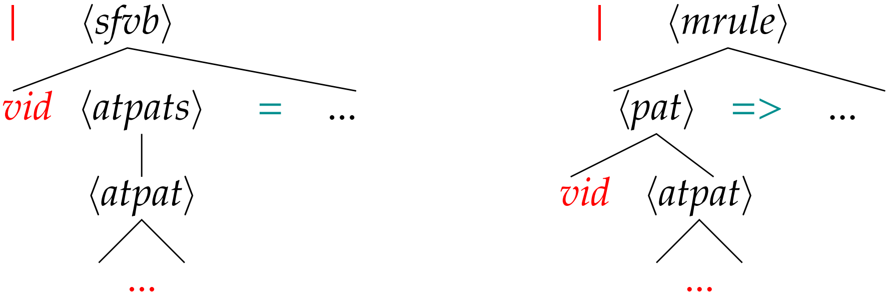


Unbounded Lookahead



| $\langle sfvb \rangle$

vid $\langle atpats \rangle$ = ...

$\langle atpat \rangle$

...

| $\langle mrule \rangle$

$\langle pat \rangle$ => ...

vid $\langle atpat \rangle$

...

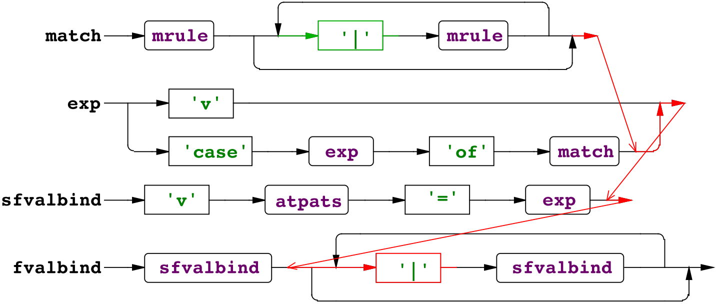
Limitations

Ambiguity Report

- grambiguity [Brabrand et al., 2007]

```
*** horizontal ambiguity at E[plus]: Exp <--> '+' Exp
    ambiguous string: "x+x+x"
```

- ANTLRWorks [Parr, 2007]



Other Limitations

- memory requirements: a solution could be a NLALR test
- dynamic disambiguation: inverse problem, some means to deciding equivalence needed

H. J. S. Basten. Ambiguity detection methods for context-free grammars. Master's thesis, Centrum voor Wiskunde en Informatica, Universiteit van Amsterdam, Aug. 2007.

P. Boullier. Supertagging: A non-statistical parsing-based approach. In IWPT'03, pages 55–65, 2003. URL `ftp://ftp.inria.fr/INRIA/Projects/Atoll/Pierre.Boullier/supertaggeur_final.pdf`.

C. Brabrand, R. Giegerich, and A. Møller. Analyzing ambiguity of context-free grammars. In J. Holub and J. Žďárek, editors, CIAA'07, 2007. URL `http://www.brics.dk/~brabrand/grambiguity/`. To appear in Lecture Notes in Computer Science.

D. G. Cantor. On the ambiguity problem of Backus