Sur les solutions explicites des problèmes de diffraction par un diedre imparfaitement conducteur pour les équations de Maxwell

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SUR LES SOLUTIONS EXPLICITES DES PROBLEMES DE DIFFRACTION PAR UN DIEDRE IMPARFAITEMENT CONDUCTEUR POUR LES EQUATIONS DE MAXWELL

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soutenue le 25 Septembre 1995 devant le jury :

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à Patricia ,

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SUR LES SOLUTIONS EXPLICITES DES PROBLEMES DE DIFFRACTION PAR UN DIEDRE IMPARFAITEMENT CONDUCTEUR POUR LES EQUATIONS DE MAXWELL

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. sur le théorème d'inversion de Maliuzhinets :

G.D. Maliuzhinets , *'Inversion formula for the Sommerfeld integral'*, Sov. Phys. Dokl., 3 , pp. 52-56 , 1958.

CONCLUSION

INTRODUCTION

De nombreux ouvrages d'électromagnétisme classent les méthodes de résolution des problèmes de diffraction suivant le qualificatif d'analytique ou de numérique . Les premières donnent des formes explicites exactes ou asymptotiques des champs tandis que les secondes aboutissent à des expressions implicites en champ que l'on résout numériquement . Cette thèse se rapporte à certaines de nos publications relatives à la première catégorie. On y présente les solutions originales exactes ou asymptotiques , de problèmes de diffraction d'une onde électromagnétique par des corps élémentaires comportant une discontinuité de géométrie et/ou de matériau , en régime stationnaire ou instationnaire . Plusieurs de ces problèmes ainsi traités deviennent de nouveaux cas canoniques .

Le premier problème de diffraction par une discontinuité résolu explicitement fut celui du demi-plan parfaitement conducteur (conditions aux limites sur chacune des faces de Dirichlet et de Neumann , respectivement sur le champ électrique et magnétique tangentiel) étudié par Sommerfeld en 1896 [1]. Macdonald [2] et Carslaw [3] généralisèrent le résultat au cas du dièdre d'angle quelconque . Vint ensuite l'utilisation des techniques de Wiener-Hopf pour les cas avec rupture dans une structure rectiligne qui emploient la transformée de Fourier et usent directement d'une représentation du champ diffracté sous forme d'ondes planes sortantes satisfaisant l'équation des ondes [4]. De fait , si la solution de Sommerfeld pour le demi-plan a bien , elle aussi , la forme d'une somme pondérée d'ondes planes de directions complexes , son contour d'intégration diffère . Sur ce chemin , chaque onde plane ne satisfait pas, a priori, la condition d'onde sortante ou la convergence à l'infini , car Sommerfeld veut exprimer le champ total qui comporte une onde plane incidente provenant de l'infini . Ce sera donc grâce aux propriétés de la fonction analytique , retenue comme fonction de pondération dans son intégrale , que le champ proprement diffracté (champ total - champ incident) pourra satisfaire la condition de rayonnement d'onde sortante à l'infini . Le degré de liberté ainsi offert par ce type de représentation devait bientôt entraîner un nouveau développement : le théorème d'inversion de l'intégrale de Sommerfeld découvert par Maliuzhinets . Publié dans sa thèse en 1950 [5], puis démontré sous une forme aboutie en 1958 [6], il permet de traduire des conditions aux limites en champ sur la frontière d'un dièdre par des équations fonctionnelles sur les fonctions poids (dites aussi spectrales) associées à ce champ . Son auteur en déduit alors , de façon explicite , le rayonnement , en acoustique, d'un dièdre d'angle quelconque avec une distribution de vitesse donnée à sa surface , mais plus encore , celui du dièdre de surface non rigide, modélisée par des conditions d'impédances (dites aussi mixtes ou conditions de Léontovich) sur le champ en chaque face, et illuminé par une onde de pression plane . Le problème électromagnétique associé du dièdre avec des conditions différentielles de surface de Léontovich , illuminé par une onde plane de direction perpendiculaire à l'arête , s'en trouvait simultanément résolu . L'élégance des développements mathématiques de Maliuzhinets à partir de cette forme intégrale du champ nous a alors conduit à utiliser ce type de représentation pour aborder et résoudre de nouvelles classes de problèmes .

Nous classerons dans la thèse les articles correspondant à ces problèmes sous trois rubriques :

. Diffraction par un dièdre avec des conditions aux limites complexes , illuminé par une onde plane d'incidence normale par rapport à l'arête en régime stationnaire : problème scalaire en régime harmonique ,

. Diffraction par un dièdre avec des conditions aux limites complexes , illuminé par une onde plane d'incidence quelconque en régime stationnaire : problème vectoriel en régime harmonique ,

. Diffraction par un dièdre avec des conditions aux limites complexes , illuminé par une onde plane ou cylindrique quelconque en régime non stationnaire : problème vectoriel en régime temporel quelconque .

Avant cela , présentons une synthèse des articles concernés .

En 1970 , Maliuzhinets reprit le dernier problème cité , cette fois en considérant de nouvelles conditions aux limites , qui permettent de mieux modéliser les propriétés acoustiques de chacune des faces composant son dièdre . Aidé de son collègue Tuzhilin , il prit le cas de la plaque élastique modélisée à sa surface par une condition différentielle d'ordre de dérivées élevé (> 1) [7], travaux que Tuzhilin étendit , en 1973 , au cas du dièdre d'angle quelconque constitué de deux demi-plans élastiques d'épaisseur faible [8]. Morgan (avec des hypothèses très restrictives sur les matériaux) , puis nous-mêmes , traitions ce type de problème en électromagnétisme , en incidence normale par rapport à l'arête , et cela sans connaître alors les recherches de Tuzhilin . Morgan [9] considéra un système sans pertes . Il n'approfondit pas assez l'étude de l'équation fonctionnelle associée à son problème pour se rendre compte , en particulier , que sa fonction spectrale n'avait une forme réellement définie que s'il mettait des pertes , même infiniment petites , qui donnaient un signe à la partie réelle de certains de ces paramètres physiques . Quant aux résultats de mon étude publiée en 1987 [10], je le constatai après coup , ils recoupaient correctement les travaux de Tuzhilin . En outre , ils contenaient une démonstration complète pour le choix d'une condition aux limites de surface correcte au sens physique et mathématique , respectivement pour une bonne modélisation et un bon comportement de la fonction spectrale associée à la solution , un calcul original des fonctions spéciales nécessaires au calcul , ainsi que des applications numériques .

Le cas général de l'onde plane incidente de direction oblique par rapport à l'arête constituait une suite naturelle à ce travail . Mais le problème prend , en électromagnétisme , un degré de complexité supplémentaire, car en posant deux conditions aux limites à deux inconnues en champ pour chacune des faces du dièdre , on n'aboutit plus alors , de façon générale , à deux problèmes indépendants sur une composante fixe du champ . Pour la recherche d'une solution analytique complète , on retient alors comme inconnues les deux composantes du champ électromagnétique suivant l'arête. On note que le problème est vectoriel et que , de façon générale, la multiplication des opérateurs en jeu n'est alors plus commutative comme dans le cas précédemment étudié . Bucci et Franceschetti [11], puis Vaccaro

[12] et Rojas [13], avaient proposé une solution du cas où l'on pouvait écrire les deux conditions aux limites sur chacune des deux faces , à partir des composantes suivant la normale à une des faces . Ils avaient ainsi résolu le cas de la discontinuité plane et du demi-plan avec des impédances de faces quelconques , ainsi que celui du dièdre d'angle droit avec une face parfaitement conductrice . Voulant quant à nous , une méthode dans le cas d'un angle de dièdre quelconque et des conditions aux limites d'ordre de dérivées élevé sur chaque face , nous avons publié une démarche originale en 1989 [14] et 1990 [15-16]. En réduisant le problème de couplage à la résolution d'un nouveau type d'équation fonctionnelle scalaire , elle conduit , en particulier , à la détermination complète de la solution du problème concernant le dièdre d'impédance relative unité (impédance de surface couramment considérée pour modéliser un absorbant radioélectrique) d'angle quelconque . Notons que ce cas constitue , à ce jour , le seul cas canonique concernant un dièdre imparfaitement conducteur d'angle quelconque en incidence arbitraire .

Il fallait ensuite prendre en compte la courbure des faces d'un dièdre . Le dièdre à faces courbes d'angle quelconque avait été jusqu'alors étudié par Filippov [17] et Borovikov [18] dans le cas parfaitement conducteur , mais leurs expressions n'étaient pas uniformes au voisinage des plans tangents aux faces du dièdre . Michaeli [19] avait bien une solution uniforme, mais , déterminée heuristiquement à partir de la solution d'Idemen et Felsen [20] pour le demi-plan courbe (de courbure constante) parfaitement conducteur , elle ne pouvait prendre partout en compte l'influence de la courbure . Gérard et Lebeau [21] étudièrent le cas parfaitement conducteur , cette fois pour une courbure non constante . Buyukaskoy et Uzgören [22], prolongeant les travaux d'Idemen , avaient donné , quant à eux , une solution uniforme pour le demi-plan courbe à impédance . En 1991 [23] , nous avons donc publié une méthode pour le dièdre à faces courbes , d'angle et d'impédances de faces quelconques . On y considère une condition différentielle en puissance de la courbure , à imposer au champ sur chacun des plans tangents ; du type de celle dont Kaminetsky et Keller [24] se servent pour la discontinuité de rayon de courbure , elle nous permet d'arriver à une solution asymptotique explicite pour la fonction spectrale de l'intégrale de

Sommerfeld-Maliuzhinets attachée au champ . L'obtention d'un théorème d'inversion original nous permet de la raccorder , pour tout ordre du développement asymptotique , à une expression uniforme au passage des plans tangents , nous fournissant , par là-même , la transformation du champ diffracté d'arête en somme d'ondes rampantes.

Laissant le domaine harmonique , nous avons considéré , dans un article de 1991 [25] , la réponse d'un dièdre à une onde plane non stationnaire . En effet , s'il existait une solution analytique particulière au cas du dièdre à faces planes à impédance réelle , indépendante de la fréquence en régime harmonique [Papadopoulos [26], Sakharova et Filippov [27]] , il n'y avait en revanche aucune expression explicite générale quand l'impédance, arbitraire, peut varier avec la pulsation . En travaillant l'expression de la transformée de Laplace inverse de l'intégrale de Sommerfeld-Maliuzhinets , tout en utilisant la causalité et la réalité du champ dans le domaine temporel , on aboutit à certaines propriétés générales de la transformée de la fonction spectrale et ainsi , à une expression originale et compacte du champ . Dans cette expression , le contour d'intégration sur les angles complexes , infini en régime harmonique , est remplacé par un chemin fini , fonction du temps . Cette forme exacte permet , comme l'intégrale de Sommerfeld-Maliuzhinets dans le domaine fréquentiel , de distinguer aisément les termes d'optique géométrique du terme de champ proprement excité par l'arête (champ diffracté d'arête plus ondes de surface) . Elle permet d'effectuer facilement toutes les approximations nécessaires pour aboutir aux solutions de Papadopoulos , Sakharova , ou celle de Veruttipong [28] pour le cas parfaitement conducteur. Cette approche est , en outre , généralisable à une large classe de dièdres dont on connaît le champ diffracté , en régime harmonique , par une intégrale de Sommerfeld-Maliuzhinets , ainsi qu'au cas de l'illumination cylindrique , comme on l'expose dans [29] et [30] .

Revenant au domaine harmonique , nous avons abordé le problème du dièdre à impédances de faces fonction de la distance à l'arête [31]. Le seul résultat connu en 1992 , date de notre publication , concernait le cas d'une variation de l'impédance proportionnelle ou inversement proportionnelle à la distance à l'arête , découvert par Felsen [32]. Par une généralisation du théorème d'inversion de Maliuzhinets , nous avons obtenu l'équation fonctionnelle satisfaite par la fonction spectrale associée à des impédances exprimées comme une somme d'exponentielles . La résolvant alors par une technique de perturbation , nous avons trouvé une solution approchée explicite au problème .

La dernière publication décrite [33] concerne les liens entre la transformée de Fourier du champ sur un plan , appelée communément fonction de rayonnement grande distance , et la fonction spectrale de l'intégrale de Sommerfeld-Maliuzhinets attachée à ce champ. On connaissait l'expression de la première par la seconde , il manquait la réciproque , dont nous avons donné une forme explicite simple . Les conséquences en sont nombreuses : en premier lieu , la possibilité d'exprimer la fonction spectrale attachée au rayonnement d'une ligne-source de Dirac parallèle à l'arête , et par conséquent la fonction spectrale associée à une distribution arbitraire de sources , ou encore celle attachée à la réponse d'un objet exposé au rayonnement d'une ligne source , la réponse à une onde plane étant supposée connue ; puis , la traduction (et l'extension éventuelle) de toutes les expressions déjà obtenues par d'autres méthodes (ex. Théorie Géométrique de la Diffraction) , sous la forme d'une intégrale de Sommerfeld-Maliuzhinets ; enfin , deux résultats généraux complémentaires donnés dans [29-30] : pour l'un , l'obtention de l'expression analytique de la fonction spectrale attachée au rayonnement d'une face à partir de celle associée au rayonnement total , pour l'autre la détermination d'une équation fonctionnelle définie sur la variable d'angle d'incidence pour une illumination plane (non trivial car on ne peut interchanger les angles d'incidence et d'observation dans les fonctions spectrales de Sommerfeld-Maliuzhinets .)

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CHAPITRE I) Diffraction par un dièdre avec des conditions aux limites complexes , illuminé par une onde plane d'incidence normale par rapport à l'arête en régime stationnaire : problème scalaire en régime harmonique

I.1) J.M.L. Bernard ,*'Diffraction by a metallic wedge covered with a dielectric material'* , Wave Motion , 9 , pp. 543-561,1987.

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I.2.A) Errata et commentaires sur les démonstrations et formules

I.2.B) Compléments mathématiques

I.3) J.M.L. Bernard ,G. Pelosi ,*'Diffraction par un dièdre avec variation d'impédance dépendant de la distance à l'arête* ',Ann. Telecom. ,Vol.47 , 9-10,pp. 421-423,1992.

I.3.A) Errata et commentaires sur les démonstrations et formules

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NORTH-HOLLAND

DIFFRACTION BY A METALLIC WEDGE COVERED WITH A DIELECTRIC MATERIAL

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In the literature, the usual formulation which assumes a constant impedance boundary condition does not permit one to express both the value of the geometrical optics field and the actual excited surface wave, since the impedance associated with the material for the reflection and for the surface wave are generally different. To obtain these elements in the field expression, we replace the notion of constant impedance by a differential operator. The general solution presented here preserves enough degrees of freedom to satisfy the continuity of fields at the internal junction of the two materials which cover each face of the wedge. The numerical calculations are based upon a function, related to the Maliuzhinets function, in a form which is easily computable.

1. Introduction

We study here the problem of the diffraction by a metallic wedge with each face covered by a dielectric material, when it is illuminated by a plane wave with the magnetic field parallel to the edge. A study of this canonical wedge configuration allows one to better understand the phenomena of diffraction due to a discontinuity of material or (and) of tangent plane in a structure. Different approaches have been employed in by Senior [3], Tiberio et al. [4] and Bucci and Franceschetti [5]. These authors used the methods of Wiener-Hopf or Maliuzhinets [1, 2], and their analysis is based upon the notion of a constant impedance boundary. However, this notion is only an approximation. It does not allow one to express, in a straightforward manner, the actual geometrical optics field and the surface waves. In order to obtain a more complete expression for the field, we prefer to replace the constant impedance boundary condition by a differential condition of higher order. A condition of this type has already been used in [6, 7] for some specific cases. We express here in a more general manner its validity, and the solutions that it permits for the case of a wedge.

Therefore, we study first the development of the differential boundary condition valid on each face of the wedge. A new general solution for the field is then defined, where the remaining unknown coefficients can be obtained only by the conditions of field continuity, to be satisfied in the junction zone of the two dielectric layers. We will insist on the proper behaviour at infinity of the spectral function associated with this solution. The first term of the solution is a multimode term equivalent, in the case of constant passive impedance, to the one used in [4, 5]. We propose for it an original mathematical development, easily calculable for any wedge angle, from which we derive a new expression for the Maliuzhinets function. Different numerical results are presented; we validate the calculations and we compare the multimode results with the results given by a constant impedance. The time convention $\exp(i\omega t)$ is used in this paper.

2. Differential boundary condition

We study the boundary conditions satisfied by the field on the surface of the wedge shown in Fig. 1. It is a perfectly conducting wedge, covered on each face, + and −, by a different dielectric layer, of equal thickness d. The junction of these layers takes place at the junction zone (Fig. 1). The electromagnetic field is assumed to be of the H-type; i.e., the magnetic field is parallel to the edge of the wedge.

First, we consider one isolated face of the wedge where ε, μ are the permitivity and the permeability of its dielectric, and k is the associated wave number (Fig. 2). We define on the interface an (x, y, z) frame, y being directed outwards. The magnetic field H is parallel to the z-axis.

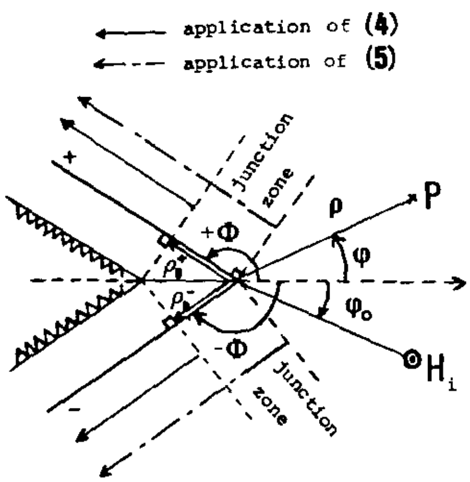

Fig. 1. Geometry of the wedge, and application of differential conditions.

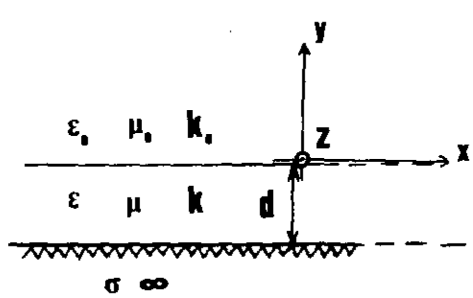

Fig. 2. Face of the wedge.

The knowledge of the electric field component E_x and its derivatives at $y = 0^-$ allows us to develop the field for $-d < y < 0$, by means of the Taylor expansion

$$E_x(y) = \sum_{n=0}^{\infty} \left(\frac{\partial^{2n} E_x}{\partial y^{2n}}\right)\Bigg|_{y=0^-} \frac{(y)^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \left(\frac{\partial^{2n+1} E_x}{\partial y^{2n+1}}\right)\Bigg|_{y=0^-} \frac{(y)^{2n+1}}{(2n+1)!}, \tag{1}$$

which, from the following properties of the electromagnetic field,

$$E_x = \frac{1}{\mathrm{i}\omega\varepsilon}\partial H_z/\partial y, \quad -d<y<0,$$

$$(\partial^2/\partial y^2)(H_z) = -(k^2+\partial^2/\partial x^2)(H_z), \quad -d<y<0$$

gives,

$$E_x(y) = \sum_{n=0}^{\infty}\left(\frac{\partial^{2n}E_x}{\partial y^{2n}}\right)\bigg|_{y=0^-}\frac{(y)^{2n}}{(2n)!} - \frac{\mathrm{i}}{\omega\varepsilon}\sum_{n=0}^{\infty}\left(\frac{\partial^{2n+2}H_z}{\partial y^{2n+2}}\right)\bigg|_{y=0^-}\frac{(y)^{2n+1}}{(2n+1)!}, \tag{2}$$

$$E_x(y) = \cos\left(y\sqrt{k^2+\frac{\partial^2}{\partial x^2}}\right)(E_x|_{y=0^-}) + \frac{\mathrm{i}}{\omega\varepsilon}\sqrt{k^2+\frac{\partial^2}{\partial x^2}}\sin\left(y\sqrt{k^2+\frac{\partial^2}{\partial x^2}}\right)(H_z|_{y=0^-}). \tag{3}$$

Now from the continuity of the tangential components of the field at the interface between the dielectric and the external medium, (3) can be used after replacing $y=0^-$ by $y=0^+$. Then we can write the cancellation of E_x on the prefectly conducting internal surface of the wedge and we can transform (3) into an equation satisfied by the field at the external surface outside the junction zone,

$$\left[\frac{\partial}{\partial y}\cos\left(d\sqrt{k^2+\frac{\partial^2}{\partial x^2}}\right) + \frac{1}{\varepsilon_r}\sqrt{k^2+\frac{\partial^2}{\partial x^2}}\sin\left(d\sqrt{k^2+\frac{\partial^2}{\partial x^2}}\right)\right](H_z)|_{y=0^+} = 0, \tag{4}$$

where $\varepsilon_r = \varepsilon/\varepsilon_0$, $\mu_r = \mu/\mu_0$ (ε_0, μ_0: free space permittivity and permeability). In this manner we obtain a local differential impedance boundary condition which depends only on the electric properties at a given x.

As seen in Fig. 1, equation (4) cannot be used in the junction zone. In order to have a valid boundary condition on the whole surface, H_z taken at x in (4) is replaced by its Taylor expansion, where the derivatives of H_z are taken around an abscissa shifted by a quantity equal to the width of the junction zone. This width is denoted by $\rho_0^{\pm}$. Following the + and – wedge faces, (4) becomes,

$$\left(\left[\cos\left(d\sqrt{(k^{\pm})^2+\frac{\partial^2}{\partial(x^{\pm})^2}}\right)\cdot\frac{\partial}{\partial y^{\pm}} + \frac{1}{\varepsilon_r^{\pm}}\sqrt{(k^{\pm})^2+\frac{\partial^2}{\partial(x^{\pm})^2}}\sin\left(d\sqrt{(k^{\pm})^2+\frac{\partial^2}{\partial(x^{\pm})^2}}\right)\right]\right.$$
$$\left.\cdot\sum_{n=0}^{\infty}\frac{(\rho_0^{\pm})^n}{n!}\cdot\frac{\partial^n}{\partial(\mp x^{\pm})^n}\right)(H_z)|_{\varphi=\pm\Phi} = 0, \tag{5}$$

where $(\cdot)^{\pm}$ relates to the face $\varphi = \pm\Phi$

$$x^{\pm} = \mp\rho\cos\Phi\mp\varphi, \qquad y^{\pm} = \rho\sin\Phi\mp\varphi,$$

where ρ, φ are cylindrical coordinates (Fig. 1)

$$\frac{\partial^n}{\partial(\mp x^{\pm})^n}(\cdot)\bigg|_{\varphi=\pm\Phi} = \frac{\partial^n}{\partial\rho^n}(\cdot)\bigg|_{\varphi=\pm\Phi}, \qquad \frac{\partial}{\partial y^{\pm}}(\cdot)\bigg|_{\varphi=\pm\Phi} = \mp\frac{\partial}{\rho\partial\varphi}(\cdot)\bigg|_{\varphi=\pm\Phi}.$$

In the next section, the solution of (5) is studied.

3. Derivation of the solution

We have to find the total magnetic field H_z for $|\varphi|\leqslant\Phi$, when the wedge is illuminated by a plane wave of unit amplitude, incident under the angle φ_0. The potential H_z is written in the form of a Sommerfeld integral,

$$H_z(\rho, \varphi) = \int_{\mathscr{C}} \frac{1}{4\pi \mathrm{i}} (f(\alpha+\varphi) - f(-\alpha+\varphi))\, \mathrm{e}^{\mathrm{i}k_0 r \cos\alpha}\, \mathrm{d}\alpha, \tag{6}$$

where $\mathscr{C}$ is defined in Fig. 3 and k_0 is the free space wave number.

The function $f(\alpha)$ has to satisfy the following properties cited by Maliuzhinets in [1],

(a) $f(\alpha) - 1/(\alpha - \varphi_0)$ is a regular function in the strip $|\mathrm{Re}\, \alpha| \leqslant \Phi$;

(b) $|f(\alpha) - f(\pm \mathrm{i}\infty)| < \exp(-a|\mathrm{Im}\, \alpha|)$, $0 < a < 1$ when $\mathrm{Im}\, \alpha \to \pm\infty$, $|\mathrm{Re}\, \alpha| \leqslant \Phi$;

(c) $f(-\mathrm{i}\infty) = -f(\mathrm{i}\infty)$; and

(d) $f(-\mathrm{i}\infty) = \frac{1}{2}\mathrm{i} H_z(0, \varphi)$, which is implied by (b) and (c),

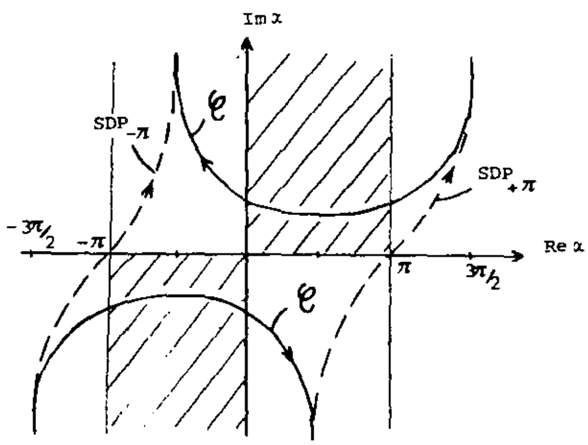

Fig. 3. Location of path $\mathscr{C}$ and steepest paths SDP$_{\pm\pi}$.

The direct application of (5) to (6) by differentiating the integrand cannot be directly done without knowing a priori a form of the solution. We, therefore, prefer to take a polynomial expansion of (5), of finite order $m+p$, which, with

$$-\partial^2/\partial(x^\pm)^2 \equiv k_0^2 + \partial^2/\partial(y^\pm)^2$$

gives us the following condition,

$$P_m^\pm\left(\frac{-\mathrm{i}}{k_0} \cdot \frac{\partial}{\partial y^\pm}\right) Q_p^\pm\left(\frac{-\mathrm{i}}{k_0} \cdot \frac{\partial}{\partial \rho}\right) (H_z)|_{\varphi = \pm\Phi} = 0, \tag{7}$$

where, $Q_p^\pm((-\mathrm{i}/k_0) \cdot \partial/\partial\rho)$ is a polynomial approximation of degree p of $\sum_{n=0}^{\infty} ((\rho_0^\pm)^n/n!) \cdot \partial^n/\partial\rho^n$, and $P_m^\pm$ is a polynomial of degree m (for parity of m see Section 5).

This type of condition has already been considered in [6] and is referred to as a "multimode condition", because each zero of $P_m^\pm$ corresponds to a constant surface impedance and a mode of propagation of a wave along the surface. We pursue the analysis in order to know the form of f allowed by (7), and with extensions allowed by (5). So we integrate (7) by parts, using the form (6) of the field, which gives us

$$\int_{\mathscr{C}} [(P_m^\pm(\mp \sin\alpha) f(\alpha \pm \Phi) - P_m^\pm(\pm \sin\alpha) f(-\alpha \pm \Phi)) Q_p^\pm(\cos\alpha)]\, \mathrm{e}^{\mathrm{i}k_0\rho\cos\alpha}\, \mathrm{d}\alpha = 0. \tag{8}$$

The asymptotic behaviour of the term within brackets in the integrand is written as $\mathrm{e}^{(m+p-a)|\mathrm{Im}\,\alpha|}$ where $0 < a < 1$.

We then apply a theorem due to Maliuzhinets [1] which allows us to write that $f(\alpha)$ satisfies,

$$(P_m^{\pm}(\mp \sin\alpha) f(\alpha \pm \Phi) - P_m^{\pm}(\pm \sin\alpha) f(-\alpha \pm \Phi)) Q_p^{\pm}(\cos\alpha)$$
$$= \sum_{n=1}^{m+p-1} [c_{n0} \pm c_{n1}] \cos^{n-1}\alpha \sin\alpha. \tag{9}$$

Each of the c_{nl} is independent of the sign of the wedge face, whereas $[c_{n0} \pm c_{n1}]$ refers to the $\pm$ face. Each coefficient c_{nl} corresponds to a degree of freedom of the solution. We notice that the right-hand side of (9) is zero for the constant impedance case ($m = 1$, $p = 0$).

The function f can then be expressed as the sum of two terms, which both satisfy (b) and (c) above as we will show in Section 5,

$$f(\alpha) = \frac{\Psi(\alpha)}{\Psi(\varphi_0)} \sigma(\alpha) + h(\alpha). \tag{10}$$

The first term is related to the equation with zero right-hand part. We choose $\sigma(\alpha)$ possessing the pole $\alpha = \varphi_0$ and allowing

$$P_m^{\pm}(\mp \sin\alpha) \Psi(\alpha \pm \Phi) - P_m^{\pm}(\pm \sin\alpha) \Psi(-\alpha \pm \Phi) = 0, \tag{11}$$

which leads us to write,

$$\sigma(\alpha \pm \Phi) = \sigma(-\alpha \pm \Phi),$$

and thus,

$$\sigma(\alpha) = \frac{\pi}{2\Phi} \cdot \frac{\cos\left(\frac{\pi}{2\Phi}\varphi_0\right)}{\sin\left(\frac{\pi}{2\Phi}\alpha\right) - \sin\left(\frac{\pi}{2\Phi}\varphi_0\right)}.$$

The second term $h(\alpha)$ is a particular solution of (9). It does not possess the pole $\alpha = \varphi_0$. The contrary would imply the use of $\sigma(\alpha)$ in $h(\alpha)$; this would lead to new incident and reflected fields, and the consistency with the first term would imply the loss of at least one degree of freedom of the term $h(\alpha)$ (another reason will be given at the end of Section 5). We let,

$$h(\alpha) = \sum_{l=0}^{1} \sum_{m=1}^{m+p-1} c_{nl} h_{nl}(\alpha), \tag{12}$$

where $h_{nl}(\alpha) = \chi_{nl}(\alpha) \Psi(\alpha)/\Psi(\varphi_0)$ satisfies,

$$(P_m^{\pm}(\mp \sin\alpha) h_{nl}(\alpha \pm \Phi) - P_m^{\pm}(\pm \sin\alpha) h_{nl}(-\alpha \pm \Phi)) Q_p^{\pm}(\cos\alpha)$$
$$= \begin{cases} +\cos^{n-1}\alpha \sin\alpha, & \text{if } l = 0, \\ \pm\cos^{n-1}\alpha \sin\alpha, & \text{if } l = 1. \end{cases} \tag{13}$$

Then we can look for an expression of the elementary functions Ψ and χ_{nl}. Equations (11) and (13) are respectively written as,

$$\frac{\Psi'}{\Psi}(\alpha \pm \Phi) + \frac{\Psi'}{\Psi}(-\alpha \pm \Phi) = \left(\mathrm{Log} \frac{P_m^{\pm}(\pm \sin\alpha)}{P_m^{\pm}(\mp \sin\alpha)}\right)', \tag{11'}$$

$$\chi_{nl}(\alpha \pm \Phi) - \chi_{nl}(-\alpha \pm \Phi) = \frac{\cos^{n-1}\alpha \cdot \Psi(\varphi_0)}{P_m^{\pm}(\mp \sin\alpha) \Psi(\alpha \pm \Phi) Q_p^{\pm}(\cos\alpha)} \cdot \begin{cases} +\sin\alpha, & \text{if } l = 0, \\ \pm\sin\alpha, & \text{if } l = 1, \end{cases} \tag{13'}$$

where $\partial(\cdot)/\partial\alpha$ is denoted by $(\cdot)'$.

By application of the theory of Fourier integrals we precisely know the solution of

$$s(\alpha \pm \Phi) + s(-\alpha \pm \Phi) = L^{\pm}(\alpha) \tag{14}$$

or

$$s(\alpha \pm \Phi) - s(-\alpha \pm \Phi) = K^{\pm}(\alpha), \tag{15}$$

where $L^{\pm}(\alpha)$ and $K^{\pm}(\alpha)$ are known functions. Particularly when $s(\alpha)$ is regular in the part $|\mathrm{Re}\,\alpha| \leq \Phi$, the function can be written in this region as

$$s(\alpha) = \frac{\mathrm{i}}{2\sqrt{2\pi}}\left(\int_{-\mathrm{i}\infty-\delta}^{\mathrm{i}\infty-\delta} + \int_{-\mathrm{i}\infty+\delta}^{\mathrm{i}\infty+\delta}\right)\left(\frac{R_+(\omega)\,\mathrm{e}^{-\mathrm{i}\omega\Phi}}{\mathrm{i}\sin(2\omega\Phi)} - \frac{R_-(\omega)\,\mathrm{e}^{\mathrm{i}\omega\Phi}}{\mathrm{i}\sin(2\omega\Phi)}\right)\cdot \mathrm{e}^{-\mathrm{i}\omega\alpha}\,\mathrm{d}\omega, \tag{16}$$

where

$$R_{\pm}(\omega) = \frac{-\mathrm{i}}{2\sqrt{2\pi}}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\begin{Bmatrix} L^{\pm}(\alpha) \\ K^{\pm}(\alpha)\end{Bmatrix}\mathrm{e}^{\mathrm{i}\omega\alpha}\,\mathrm{d}\alpha = \frac{-\mathrm{i}}{2\sqrt{2\pi}}r_{\pm}(\omega).$$

This expression after using (14) and (15) extends $s(\alpha)$ in the whole complex plane. Therefore, we have identified the general solution for $f(\alpha)$ in the case of an impedance condition of type (7). As expected, (7) (or (5)) is not enough to satisfy the uniqueness of the solution; the condition of continuity of tangential magnetic and electric field at the internal junction of the dielectric in the junction zone are then necessary to obtain the unknown coefficients c_{nl}. However, the first term (independent of c_{nl}) of the expansion (10) of $f(\alpha)$ is often sufficient for the case of small layer thickness or sufficiently high loss. Next, we derive an original expression for Ψ.

4. Development of Ψ

We have seen that Ψ is the solution of (11′) and (14), with $L^{\pm}(\alpha)$ given by

$$L^{\pm}(\alpha) = \left(\mathrm{Log}\left(\frac{P_m^{\pm}(\pm\sin\alpha)}{P_m^{\pm}(\mp\sin\alpha)}\right)\right)'. \tag{17}$$

Because $P_m^{\pm}$ is a polynomial, we decide to factorize it from the knowledge of its zeroes, which are taken as exact solutions of the characteristic equation derived from (5). Each zero is written as $\sin\theta_n^{\pm}$, which allows us to write,

$$L^{\pm}(\alpha) = \sum_{n=1}^{m}\left(\mathrm{Log}\left(\frac{-\sin\alpha \pm \sin\theta_n^{\pm}}{\sin\alpha \pm \sin\theta_n^{\pm}}\right)\right)' = \sum_{n=1}^{m} L_n^{\pm}(\alpha), \tag{18}$$

where

$$|\mathrm{Re}\sin\theta_n^{\pm}| \leq |\mathrm{Re}\sin\theta_{n+1}^{\pm}|.$$

By linearity in $L^{\pm}(\alpha)$ of the solution (16) applied to (11′), we obtain

$$\Psi(\alpha) = \prod_n \Psi_{0n}(\alpha), \tag{19}$$

where Ψ_{0n}'/Ψ_{0n} is a solution of (14) for $L^{\pm}(\alpha) = L_n^{\pm}(\alpha)$.

This amounts to a statement of the elementary problem "Ψ_{0n}, $L_n^\pm$" which is equivalent to the one studied in [2] for the case of constant passive impedance $Z^\pm = Z_0 \sin\theta_n^\pm$ (Z_0: free space impedance), but which needs a new development for an active impedance ($\mathrm{Re}\sin\theta_n^\pm < 0$). Moreover, as noted by Tiberio et al. [4], the calculation of the function Ψ_{0n} as written by Maliuzhinets [2] is not easy for an arbitrary wedge angle. Hence we intend, in our case, to develop (16) into a new form, which is easier to calculate and valid for any $\sin\theta_n^\pm$.

We determine, first, the analytical expression of $r_\pm(\omega)$, letting,

$$\frac{\sin\alpha - \sin\theta_n^\pm}{\sin\alpha + \sin\theta_n^\pm} = \frac{\tan((\alpha-\theta_n^\pm)/2)}{\tan((\alpha+\theta_n^\pm)/2)},$$

which is used in the integral expression of $r_\pm$. Then the calculation is performed using residues,

$$\begin{aligned} r_\pm(\omega) &= \frac{e^{i\omega\theta_m^\pm} + e^{i\omega(-\theta_n^\pm+\pi)}}{1+e^{i\omega\pi}} \cdot (-2\pi i), \qquad \text{if } 0 < \mathrm{Re}\,\theta_n^\pm \leqslant \tfrac{1}{2}\pi, \\ &= -\frac{e^{i\omega(\theta_n^\pm+\pi)} + e^{i\omega(-\theta_n^\pm)}}{1+e^{i\omega\pi}} \cdot (-2\pi i), \quad \text{if } 0 > \mathrm{Re}\,\theta_n^\pm \geqslant -\tfrac{1}{2}\pi, \end{aligned} \tag{20}$$

for $\mathrm{Im}\,\omega \geqslant 0$, and $r_\pm(\omega) = r_\pm(-\omega)$ for $\mathrm{Im}\,\omega \leqslant 0$.

The function Ψ_{0n} is separated into two parts,

$$\Psi_{0n}(\alpha) = \Psi_{0n}^+(\alpha)\Psi_{0n}^-(\alpha),$$

where $\Psi_{0n}^{\pm\prime}/\Psi_{0n}^\pm$ corresponding to $r_\pm$ in the expression of Ψ_{0n}'/Ψ_{0n}.

Therefore, when $\mathrm{Re}\,\theta_n^\pm > 0$ (the case $\mathrm{Re}\,\theta_n^\pm < 0$ is deduced by changing the sign of both $\theta_n^\pm$ and the expression) and $|\mathrm{Re}\,\alpha| \leqslant \Phi$, we can write

$$\frac{\Psi_{0n}^{\pm\prime}}{\Psi_{0n}^\pm}(\alpha) = -i\left[\int_0^{i\infty} \frac{e^{i\omega\theta_n^\pm} + e^{i\omega(-\theta_n^\pm+\pi)}}{(1+e^{i\omega\pi})\sin(2\omega\Phi)} \cdot \sin(\omega(\alpha\pm\Phi))\,d\omega\right]. \tag{21}$$

We expand $[1+e^{i\omega\pi}]^{-1}$ into series, and then the following equality, given in [10], is used:

$$\int_0^\infty e^{-\mu x}\frac{\sinh(\beta x)}{\sinh(bx)}\,dx = \frac{1}{2b}(\partial\,\mathrm{Log}\,\Gamma(\tfrac{1}{2}+\tfrac{1}{2}(\mu+\beta)/b) - \partial\,\mathrm{Log}\,\Gamma(\tfrac{1}{2}+\tfrac{1}{2}(\mu-\beta)/b)),$$

where $\partial\,\mathrm{Log}\,\Gamma$ is the logarithmic derivative of the gamma function Γ. This allows one to write the following original expansion of $\Psi_{0n}^\pm$ for $|\mathrm{Re}\,\alpha| \leqslant \Phi$,

$$\begin{aligned} \Psi_{0n}^\pm(\alpha) = \prod_{l=0}^{N} &\left(\frac{1}{\Gamma(\frac{1}{2}+\frac{1}{4}[\alpha\pm\Phi+(\theta_n^\pm+l\pi)]/\Phi)} \cdot \frac{1}{\Gamma(\frac{1}{2}-\frac{1}{4}[\alpha\pm\Phi-(\theta_n^\pm+l\pi)]/\Phi)}\right. \\ &\left.\cdot \frac{1}{\Gamma(\frac{1}{2}+\frac{1}{4}[\alpha\pm\Phi-(\theta_n^\pm-(l+1)\pi)]/\Phi)} \cdot \frac{1}{\Gamma(\frac{1}{2}-\frac{1}{4}[\alpha\pm\Phi+(\theta_n^\pm-(l+1)\pi)]/\Phi)}\right)^{(-1)^l} \\ &\cdot \exp\left(\int_0^\infty (-1)^{N+1} \cdot \frac{e^{-\nu\theta_n^\pm} + e^{\nu(\theta_n^\pm-\pi)}}{(1+e^{-\nu\pi})\sinh(2\nu\Phi)} \cdot e^{-\nu(N+1)\pi} \cdot \frac{1-\cosh(\nu(\alpha\pm\Phi))}{\nu}\,d\nu\right), \end{aligned} \tag{22a}$$

if $\mathrm{Re}\,\theta_n^\pm > 0$;

$$\Psi_{0n}^\pm(\alpha) = \frac{1}{(\text{previous expression with } \theta_n^\pm \text{ replaced by } -\theta_n^\pm)},$$

if $\mathrm{Re}\,\theta_n^\pm < 0$. (22b)

(Note: Expression (22) depends on N whereas $\Psi_{0n}^{\pm}(\alpha)/\Psi_{0n}^{\pm}(\varphi_0)$ is independent of N.) The remaining part of the section will discuss the original characteristics of the above expression.

The function $\Psi_{0n}^{\pm}$ has a different expression depending on the sign of Re $\theta_n^{\pm}$, which has not been noted elsewhere, as far as we know.

The remaining integral term is easy to evaluate since the integrand decreases rapidly. The evaluation of the Γ function is rather easy too. The expression (22) provides then an easy and fast way for obtaining Ψ_{0n} for an arbitrary wedge angle. Another result can be deduced from (22). Although it seems worthwhile to keep N finite for the calculation, (22) can also be considered when N tends towards infinity. We can then obtain a new expression for the Maliuzhinets function Ψ_Φ. As is done in [2] for constant passive impedance, we can write for Re $\theta_n^{+}>0$

$$\Psi_{0n}^{\pm}(\alpha)=K\Psi_\Phi(\alpha\pm\Phi+\tfrac{1}{2}\pi-\theta_n^{\pm})\,\Psi_\Phi(\alpha\pm\Phi-\tfrac{1}{2}\pi+\theta_n^{\pm})$$

(K: a constant), and then deduce

$$\Psi_\Phi(\alpha)=\prod_{l=0}^{\infty}\left[\frac{\Gamma(\frac{1}{2}+\frac{1}{4}(l+\frac{1}{2})\pi/\Phi)}{\Gamma(\frac{1}{2}+\frac{1}{4}[\alpha+(l+\frac{1}{2})\pi]/\Phi)}\cdot\frac{\Gamma(\frac{1}{2}+\frac{1}{4}(l+\frac{1}{2})\pi/\Phi)}{\Gamma(\frac{1}{2}+\frac{1}{4}[-\alpha+(l+\frac{1}{2})\pi]/\Phi)}\right]^{(-1)^l}. \tag{23}$$

This expression is rather interesting. The integral from which it is derived is valid only for $|\mathrm{Re}\,\alpha|\leqslant\Phi$, but we can prove that (23) is valid for any α (see Appendix). Moreover, the elementary properties of the function Γ,

$$\Gamma(1-z)\Gamma(z)=-z\Gamma(-z)\Gamma(z)=\pi/\sin(\pi z),$$

make it easy to find all the properties of Ψ_Φ given in [2].

5. Surface wave and behavior of $f(\alpha)$ at infinity

We study here the asymptotic behaviour of expression (10) for $f(\alpha)$, for $|\mathrm{Im}\,\alpha|\to\infty$, $|\mathrm{Re}\,\alpha|\leqslant\Phi$, and then investigate how conditions (b) and (c) are satisfied. This is very important because these conditions are necessary for a well-chosen spectral function. The first term of $f(\alpha)$ is the product of $\sigma(\alpha)$ with $\Psi(\alpha)/\Psi(\varphi_0)$. The asymptotic behaviour of $\sigma(\alpha)$ is ($K_{(\cdot)}$ being a constant):

$$\sigma(\alpha)\sim\mathrm{sgn}(\mathrm{Im}\,\alpha)\exp\left(-\frac{\pi}{2\Phi}\cdot|\mathrm{Im}\,\alpha|\right)K_\sigma+\mathrm{O}\left(\exp\left(-\frac{\pi}{\Phi}|\mathrm{Im}\,\alpha|\right)\right).$$

The behaviour of Ψ is determined from the behaviour of $\Psi_{0n}^{\pm}$, derived from [2] or (21):

$$\Psi_{0n}^{\pm}\sim\left[K_{0n}\exp\left(\frac{\pi}{4\Phi}\cdot|\mathrm{Im}\,\alpha|\right)+\mathrm{O}(1)\right]^{\mathrm{sgn\,Re\,sin}\,\theta_n^{\pm}},$$

where

$$\mathrm{sgn}\,x=\begin{cases}1, & \text{if } x>0,\\ -1, & \text{if } x<0,\end{cases}\qquad |\sin\theta_n^{\pm}|<\infty,$$

which gives us

$$\frac{\Psi(\alpha)}{\Psi(\varphi_0)}\sim K_\Psi\cdot\prod_{\pm}\prod_{n=1}^{m}\left[\exp\left(\frac{\pi}{4\Phi}\cdot|\mathrm{Im}\,\alpha|\right)+\mathrm{O}(1)\right]^{\mathrm{sgn\,Re\,sin}\,\theta_n^{\pm}}$$

$$\sim K_\Psi\cdot\prod_{\pm}\exp\left(\frac{\pi}{4\Phi}\cdot|\mathrm{Im}\,\alpha|\cdot\sum_{n=1}^{m}\mathrm{sgn\,Re\,sin}\,\theta_n^{\pm}\right). \tag{24}$$

So the calculation of $\sum_{n=1}^{m}$ sgn Re sin θ_n (where $\pm$ is omitted) is essential. In order to do that, we determine first the parity of m by discussing the number of modes that should be retained from the set of exact solutions of (5) in the lossless case (limit when losses tend to 0). The number of surface wave modes (Re sin $\theta_n = 0$) is finite and odd. On the other hand, the leaky wave modes (Re sin $\theta_n \neq 0$), corresponding to the zeroes sin θ_n and $-\sin^* \theta_n$, have to be kept together, so that the coefficient of reflection (see (30)),

$$R_{Hm} = \prod_{n=1}^{m} \frac{\sin\alpha - \sin\theta_n}{\sin\alpha + \sin\theta_n},$$

is of absolute value 1 for real α (physical in lossless case). Therefore, m is chosen as odd. We then note that, with lossy materials, the real parts of the zeroes corresponding to the surface waves have alternately plus or minus signs. We can finally write:

$$\sum_{n=1}^{m} \operatorname{sgn} \operatorname{Re} \sin\theta_n = +1. \tag{25}$$

We thus have

$$\frac{\Psi(\alpha)}{\Psi(\varphi_0)} \sim K_\Psi \cdot \exp\left(\frac{\pi}{2\Phi} \cdot |\operatorname{Im}\alpha|\right) + \mathrm{O}(1),$$

and, therefore,

$$\frac{\Psi(\alpha)}{\Psi(\varphi_0)} \sigma(\alpha) \sim K_\sigma K_\Psi \operatorname{sgn}(\operatorname{Im}\alpha) + \mathrm{O}\left(\exp\left(-\frac{\pi}{2\Phi} \cdot |\operatorname{Im}\alpha|\right)\right),$$

which satisfies conditions (b) and (c). From (12) the second member of $f(\alpha)$ is

$$h(\alpha) = \sum_{n,l} c_{nl} h_{nl}(\alpha).$$

We remark that from (16) we can write h_{nl} as,

$$h_{n0}(\alpha) = \frac{\Psi(\alpha)}{\Psi(\varphi_0)} \cdot \left(\int_0^{i\infty} \frac{-g_s(\omega)}{\cos(\omega\Phi)} \cdot \sin(\omega\alpha)\, d\omega + \int_0^{i\infty} \frac{g_a(\omega)}{\sin(\omega\Phi)} \cdot \cos(\omega\alpha)\, d\omega\right),$$

$$h_{n1}(\alpha) = \frac{\Psi(\alpha)}{\Psi(\varphi_0)} \cdot \left(\int_0^{i\infty} \frac{g_s(\omega)}{\sin(\omega\Phi)} \cdot \cos(\omega\alpha)\, d\omega - \int_0^{i\infty} \frac{g_a(\omega)}{\cos(\omega\Phi)} \cdot \sin(\omega\alpha)\, d\omega\right),$$

where $g_s(\omega)$ (=0 if $P_m^+ Q_p^+ = -P_m^- Q_p^-$) and $g_a(\omega)$ (=0 if $P_m^+ Q_p^+ = P_m^- Q_p^-$) are odd functions without singularities on the imaginary axis.

In order to obtain the asymptotic behaviour, we can approximate $g_{s(a)}(\omega)$ by $\sin(\omega\beta_{s(a)})$ and use [10, p. 344],

$$\int_0^{i\infty} \frac{\sin(\omega\beta_{s(a)})}{\sin(\omega\Phi)} \cdot \cos(\omega\alpha)\, d\omega \sim K_{s1(a0))} \exp\left(-\frac{\pi}{\Phi} \cdot |\operatorname{Im}\alpha|\right),$$

$$\int_0^{i\infty} \frac{\sin(\omega\beta_{s(a)})}{\cos(\omega\Phi)} \cdot \sin(\omega\alpha)\, d\omega \sim K_{s0(a1)} \operatorname{sgn}(\operatorname{Im}\alpha) \cdot \exp\left(-\frac{\pi}{2\Phi}|\operatorname{Im}\alpha|\right) + \mathrm{O}\left(\exp\left(-\frac{\pi}{\Phi} \cdot |\operatorname{Im}\alpha|\right)\right).$$

Then, with the behaviour of Ψ, one can write

$$h_{n0}(\alpha) \sim -K_\Psi \cdot K_{s0} \cdot \mathrm{sgn}(\mathrm{Im}\,\alpha) + K_\Psi \cdot K_{a0} \cdot \exp\left(-\frac{\pi}{2\Phi} \cdot |\mathrm{Im}\,\alpha|\right),$$
$$h_{n1}(\alpha) \sim K_\Psi \cdot K_{s1} \cdot \exp\left(-\frac{\pi}{2\Phi} \cdot |\mathrm{Im}\,\alpha|\right) - K_\Psi \cdot K_{a1} \cdot \mathrm{sgn}(\mathrm{Im}\,\alpha), \tag{26}$$

which satisfies conditions (b) and (c). These expressions also allow one to conclude with condition (d) that the coefficients c_{nl} contribute to the value of the field at the edge. This corresponds to the idea that the conditions at the junction of the dielectric material can influence the value of H_z at the edge, which seems physically correct. It could be shown that this is not true anymore if we had previously chosen $h(\alpha)$ with a pole in the strip $|\mathrm{Re}\,\alpha| \leqslant \Phi$ (cf. (13), (13′) and (16)).

6. Calculation of the field corresponding to the first term of the spectral function $f(\alpha)$

The calculation of the Sommerfeld integral (6) is performed by distorting the initial integration path into steepest descent paths (Fig. 3), and by taking into account the crossed poles corresponding to the incident field, the reflected field and the surface waves and leaky modes. Φ is taken greater than or equal to $\frac{1}{2}\pi$. We evaluate the remaining integral by an asymptotical method of the same type as the one used in [9]. The field resulting from the first term of the spectral function $f(\alpha)$ of (10) is then expressed as

$$\frac{1}{2\pi i}\int_{\mathscr{C}} \frac{\Psi(\alpha+\varphi)}{\Psi(\varphi_0)} \cdot \frac{\left(\frac{\pi}{2\Phi}\right)\cdot\cos\left(\frac{\pi}{2\Phi}\varphi_0\right)}{\sin\left(\frac{\pi}{2\Phi}(\alpha+\varphi)\right)-\sin\left(\frac{\pi}{2\Phi}\varphi_0\right)} \cdot e^{ik_0\rho\cos\alpha}\,d\alpha$$
$$= \left(\sum_p [H_{sp}^- + H_{sp}^+] + [H_i + H_r^+ + H_r^-]\right) + H_{SDP} + O((k_0\rho)^{-3/2}), \tag{27}$$

where the terms in brackets are residues; $H_{sp}^\pm$ are the surface and leaky wave terms:

$$H_{sp}^\pm = U(\pm\varphi - \varphi_p^\pm)\exp[ik\rho\cos(\theta_p^\pm + \pi + \Phi \mp \varphi)] \cdot \sigma(\pm(\pi + \theta_p^\pm + \Phi))$$
$$\cdot \mp\frac{2\sin\theta_p^\pm}{\cos\theta_p^\pm} \cdot \frac{\Psi_{0p}(\pm(-\pi - \theta_p^\pm \pm \Phi))}{\Psi(\varphi_0)} \cdot \prod_{q\neq p} \Psi_{0q}(\pm(\pi + \theta_p^\pm + \Phi)), \tag{28}$$
$$\varphi_p^\pm = \Phi + \mathrm{Re}\,\theta_p^\pm - 2\tan^{-1}(\tanh \mathrm{Im}(\tfrac{1}{2}\theta_p^\pm));$$

H_i is the incident field term:

$$H_i = U(\pi - |\varphi_0 - \varphi|)\,e^{ik_0\rho\cos(\varphi_0-\varphi)}, \quad \text{if } \varphi_0 \text{ real}; \tag{29}$$

$H_r^\pm$ is the reflected field term:

$$H_r^\pm = U(\pi - |\pm 2\Phi - (\varphi + \varphi_0)|)\,e^{ik_0\rho\cos(2\Phi-(\varphi+\varphi_0))} \cdot \frac{-\Psi(\pm 2\Phi - \varphi_0)}{\Psi(\varphi_0)}, \quad \text{if } \varphi_0 \text{ real}, \tag{30}$$

where

$$\frac{-\Psi(\pm 2\Phi - \varphi_0)}{\Psi(\varphi_0)} = -\frac{P_m^\pm(\sin(\Phi \mp \varphi_0))}{P_m^\pm(-\sin(\Phi \mp \varphi_0))} = R_{Hm}^\pm$$

(when $m \to \infty$, $R^{\pm}_{Hm}$ tends towards

$$R^{\pm}_{H} = \frac{\sin\alpha - z^{\pm}(\alpha)}{\sin\alpha + z^{\pm}(\alpha)}\bigg|_{\alpha=\Phi\mp\varphi_0},$$

where

$$z^{\pm}(\alpha) = \mathrm{i}\sqrt{\frac{\mu_r^{\pm}}{\varepsilon_r^{\pm}}}\cdot\sqrt{1-\frac{\cos^2\alpha}{\varepsilon_r^{\pm}\mu_r^{\pm}}}\cdot\tan(\sqrt{(k^{\pm})^2 - k_0^2\cos^2\alpha}\cdot d));$$

$U(\varphi)$ is the Heaviside function:

$$U(\varphi) = \begin{cases} 1, & \text{if } \varphi > 0, \\ 0, & \text{if } \varphi < 0; \end{cases}$$

and H_{SDP} is the asymptotically evaluated term:

$$\begin{aligned} H_{\mathrm{SDP}} \sim & -\frac{\pi}{4\Phi}\cdot\frac{\mathrm{e}^{-\mathrm{i}\pi/4}}{\sqrt{2\pi k_0}}\cdot\frac{\mathrm{e}^{-\mathrm{i}k_0\rho}}{\sqrt{\rho}} \\ & \cdot\left[\frac{\Psi(\pi+\varphi)}{\Psi(\varphi_0)}\left(\cot\left(\frac{\pi}{4\Phi}(\pi+\varphi-\varphi_0)\right)F(k_0\rho(1-\cos\beta_0^{+}))\right.\right. \\ & \qquad \left. + \tan\left(\frac{\pi}{4\Phi}(\pi+\varphi+\varphi_0)\right)F(k_0\rho(1-\cos\beta_1^{+}))\right) \\ & \quad -\frac{\Psi(-\pi+\varphi)}{\Psi(\varphi_0)}\left(\cot\left(\frac{\pi}{4\Phi}(-\pi+\varphi-\varphi_0)\right)F(k_0\rho(1-\cos\beta_0^{-}))\right. \\ & \qquad \left.\left. + \tan\left(\frac{\pi}{4\Phi}(-\pi+\varphi+\varphi_0)\right)F(k_0\rho(1-\cos\beta_1^{-}))\right)\right] \end{aligned} \tag{31}$$

also noted $D(\varphi, \varphi_0)\cdot\mathrm{e}^{-\mathrm{i}k_0\rho}/\sqrt{\rho}$ (D: diffraction coefficient) with

$$F(x^2) = 2\mathrm{i}x\,\mathrm{e}^{\mathrm{i}x^2}\int_x^{\infty}\mathrm{e}^{-\mathrm{i}\tau^2}\,\mathrm{d}\tau, \quad \text{if } \varphi_0 \text{ real}, \qquad F(x^2) = 1 \quad \text{if } \varphi_0 \text{ complex},$$

$$\beta_0^{\pm} = \pm\pi + \varphi - \varphi_0 - 4n_0^{\pm}\Phi,$$

$$\beta_1^{\pm} = 2\Phi - (\pm\pi + \varphi + \varphi_0) - 4n_1^{\pm}\Phi,$$

where $n_0^{\pm}$, $n_1^{\pm}$ are chosen so that $|\beta_0^{\pm}|$, $|\beta_1^{\pm}|$ are minima. The expression of H_{SDP} supresses the discontinuities of $H_{\mathrm{i}} + H_{\mathrm{r}}^{\pm}$; it could be improved by taking into account the discontinuity of $H_{\mathrm{sp}}^{\pm}$. One could point out, from (28), that some modes cannot be excited even though they are taken into account.

The zeroes $\sin\theta_p^{\pm}$ are calculated by approximating (5) (or (4)) with rational expressions of $\tan w$,

$$\tan w \approx \left(\frac{1}{w} + 2w\left(\sum_{k=1}^{M}\frac{1}{w^2 - k^2\pi^2} - \sum_{k=M+1}^{\infty}\frac{1}{k^2\pi^2}\right)\right)^{-1}.$$

We shall subsequently write the pth mode of a group of q modes, as $\sin\theta_{pq}$. We shall take q sufficiently large for convergence of the solution.

7. Numerical results

The numerical results given below deal with the field calculated using the first term of the spectral function (10). First, we verify the form (22) of $\Psi^{\pm}_{0n}$. We consider a case (Fig. 4), previously treated by Tiberio et al. with another expression of Ψ_{0n} [4], which is the diffraction of a wedge with exterior half angle $\Phi = 135°$, and relative face impedances $Z^{+}/Z_0 = \sin\theta^{+} = 4.0$, $Z^{-}/Z_0 = \sin\theta^{-} = 0.25$ (Z_0: free space impedance).

The agreement with the results of [4] is excellent. Next we consider the study of the basic case of Fig. 1, with $\Phi = 135°$, and to dielectric layers of the same kind; the relative permitivity, permeability and

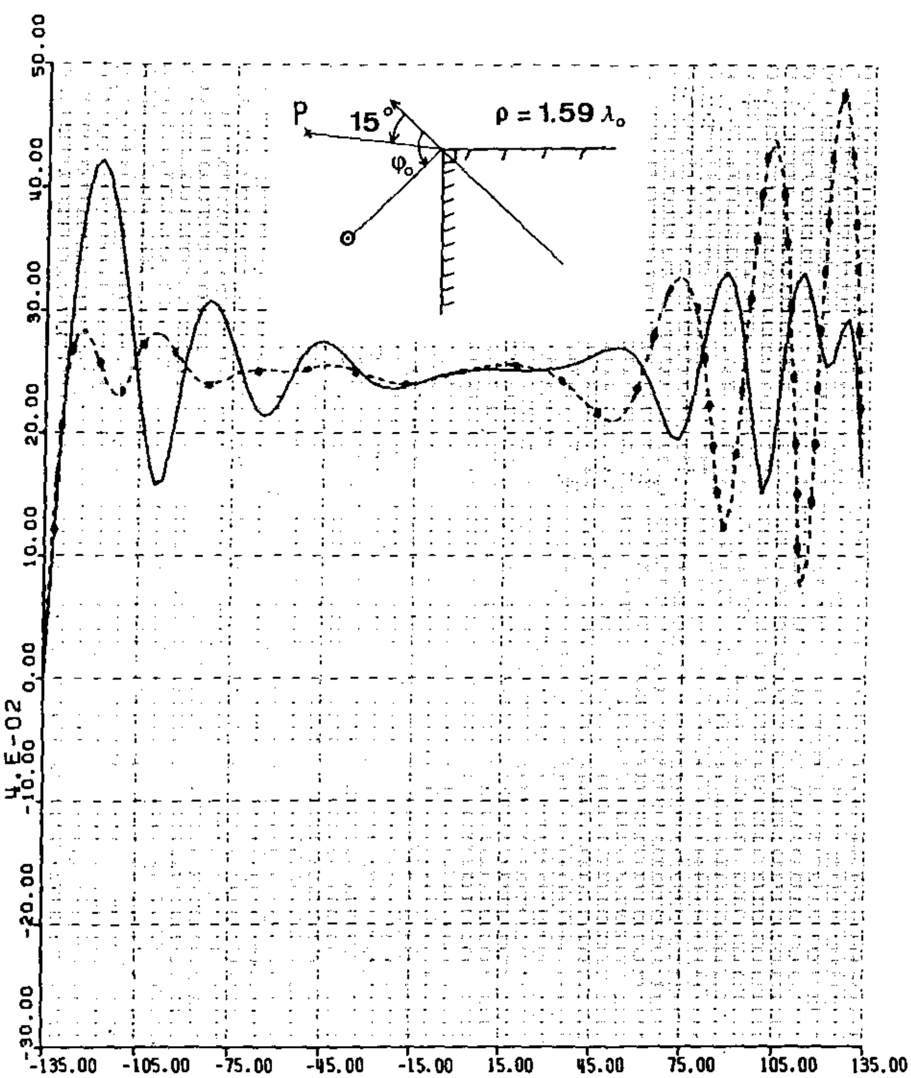

Fig. 4. Total field (a) $\sin\theta^{+} = 4.0$, $\sin\theta^{-} = 0.25$ with (22) and (27) (- - -), and results of [4] (· · · ·); (b) $\sin\theta^{+} = 0.25$, $\sin\theta^{-} = 4.0$ with (22) and (27) (——).

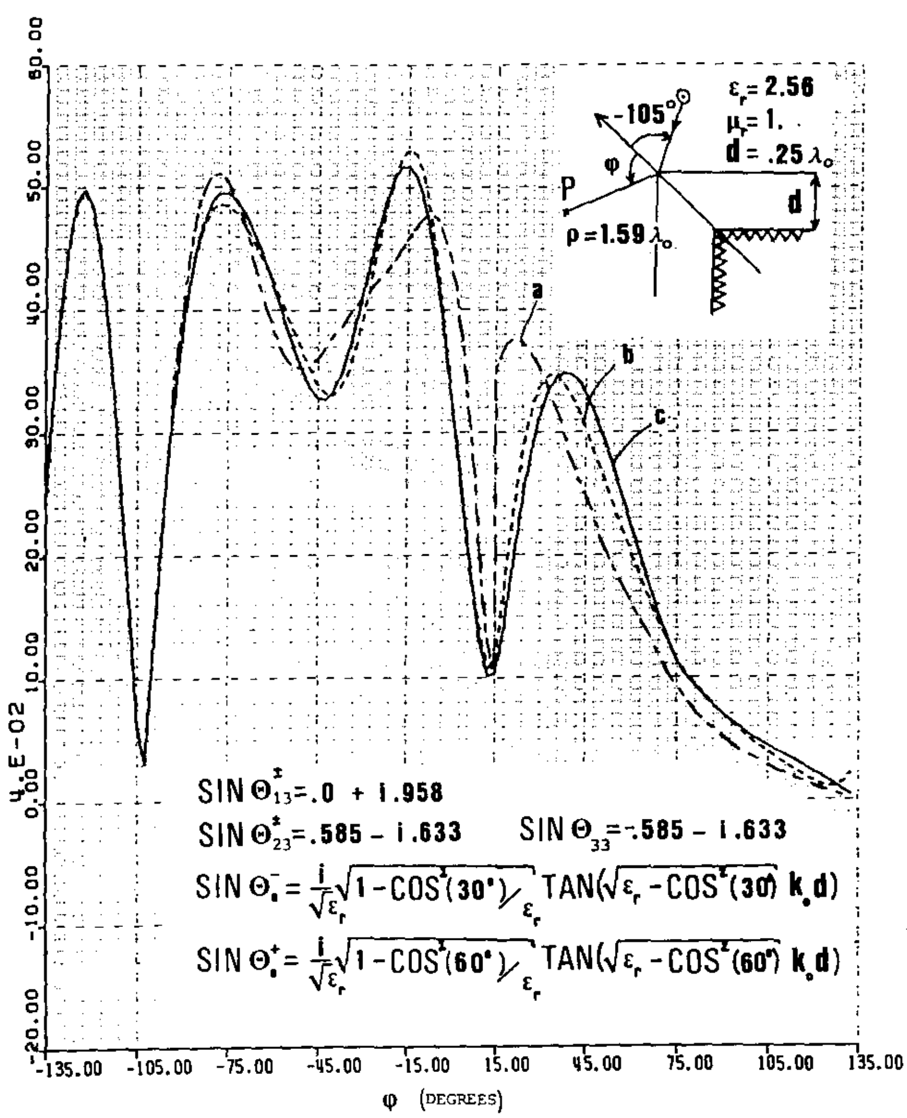

Fig. 5. H is the actual reflected field + respectively: (a) $(H-H_r)$ one mode $\{\sin\theta_{13}^{\pm}\}$; (b) $(H-H_r)$ three modes $\{\sin\theta_{i3}^{\pm},\ i=1,2,3\}$; (c) $(H-H_r)\sin\theta_0^{\pm}$.

thicknesses are, respectively, $\varepsilon_r=2.56-\mathrm{i}0^+$, $\mu_r=1.0$, $d=\frac{1}{4}\lambda_0$ (Figs. 5–7) and $\varepsilon_r=2.56-\mathrm{i}0.01$, $\mu_r=1.0$, $d=\frac{1}{2}\lambda_0$ (Figs. 8 and 9) (λ_0: free space wave length).

In Figs. 5, 6, 8 and 9 the computation of the field is done by adding the actual reflected and incident field to both the surface wave term and the asymptotically evaluated term (diffracted term H_{SDP}), which are calculated with a chosen set of modes. For Figs. 5 and 6 we successively retain the sets made of one mode $\{\sin\theta_{13}\}$, three modes $\{\sin\theta_{i3},\ i=1,2,3\}$, and again one "mode" corresponding to the constant relative impedance obtained in (30) and given by geometrical optics for the angle of incidence φ_0,

$$\left\{\sin\theta_0^{\pm}=z^{\pm}(\Phi\mp\varphi_0)=\mathrm{i}\sqrt{\frac{\mu_r^{\pm}}{\varepsilon_r^{\pm}}}\sqrt{1-\frac{\cos^2(\Phi\mp\varphi_0)}{\varepsilon_r^{\pm}\mu_r^{\pm}}}\tan\left(\sqrt{(k^{\pm})^2-k_0^2\cos^2(\Phi\mp\varphi_0)}\,d\right)\right\}.$$

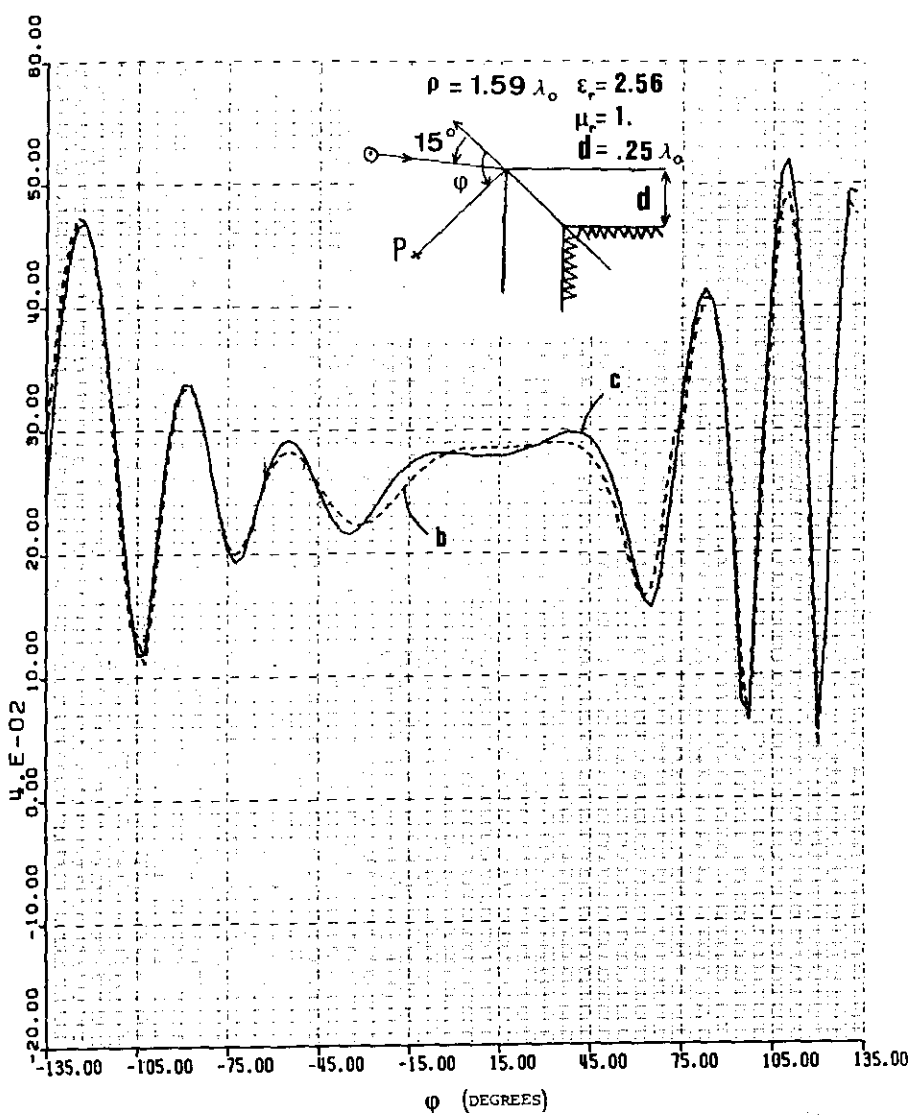

Fig. 6. H is the actual reflected field + respectively: (b) $(H - H_r)$ three modes $\{\sin\theta^{\pm}_{i3}, i = 1, 2, 3\}$; (c) $(H - H_r)\sin\theta^{\pm}_0$.

The agreement between the second and third choices, which allow the continuity of their respective curve, is very good. (This continuity is not respected by the first choice since it does not include the complex modes of orders 2 and 3 in its calculation.) It seems, therefore, that in this case a well-chosen constant impedance gives rather good results. We add to this remark by Fig. 7 which shows what we obtain with another constant impedance.

However, when the thickness of the dielectric increases, the constant impedance formulation is rather limited as compared to the multimode solution. This can be seen in Figs. 8 and 9, for $d = \frac{1}{2}\lambda_0$, where the difference between the two calculations becomes significant. This is also the case when we consider a surface wave guided by slab recessed in a metallic plane [8, 11], and the diffraction coefficient calculated with one mode (surface wave mode) is not as good as the one obtained with several modes (Fig. 10).

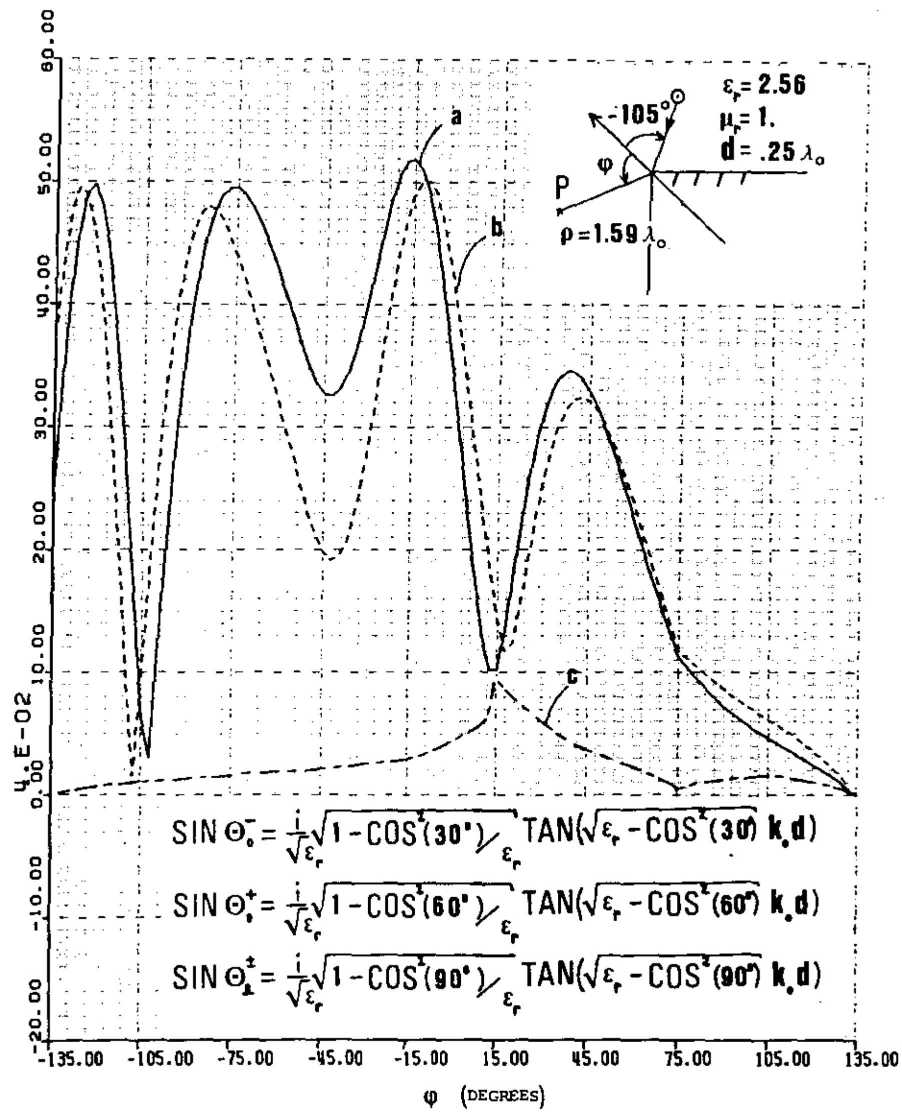

Fig. 7. Total field with relative constant face impedances (a) sin $\theta_0^{\pm}$; (b) sin $\theta_{\perp}^{\pm}$; (c) difference of diffraction terms (H_{SDP}) corresponding to (a) and (b).

8. Conclusion

We have analyzed a solution for the problem of scattering by a perfectly conducting wedge, each face being covered with dielectric layer. We have then defined a differential equation satisfied by the total field of H-type on the boundary of the wedge. In order to satisfy the radiation condition, we have written the field with a Sommerfeld integral and then derived the general expression for the spectral function which corresponds to the chosen boundary condition. Moreover we have verified the good choice of the solution by showing that the spectral function satisfies the conditions at infinity given by Maliuzhinets [1]. Concerning this, we notice the importance of the parity of the surface wave number (always odd) and of

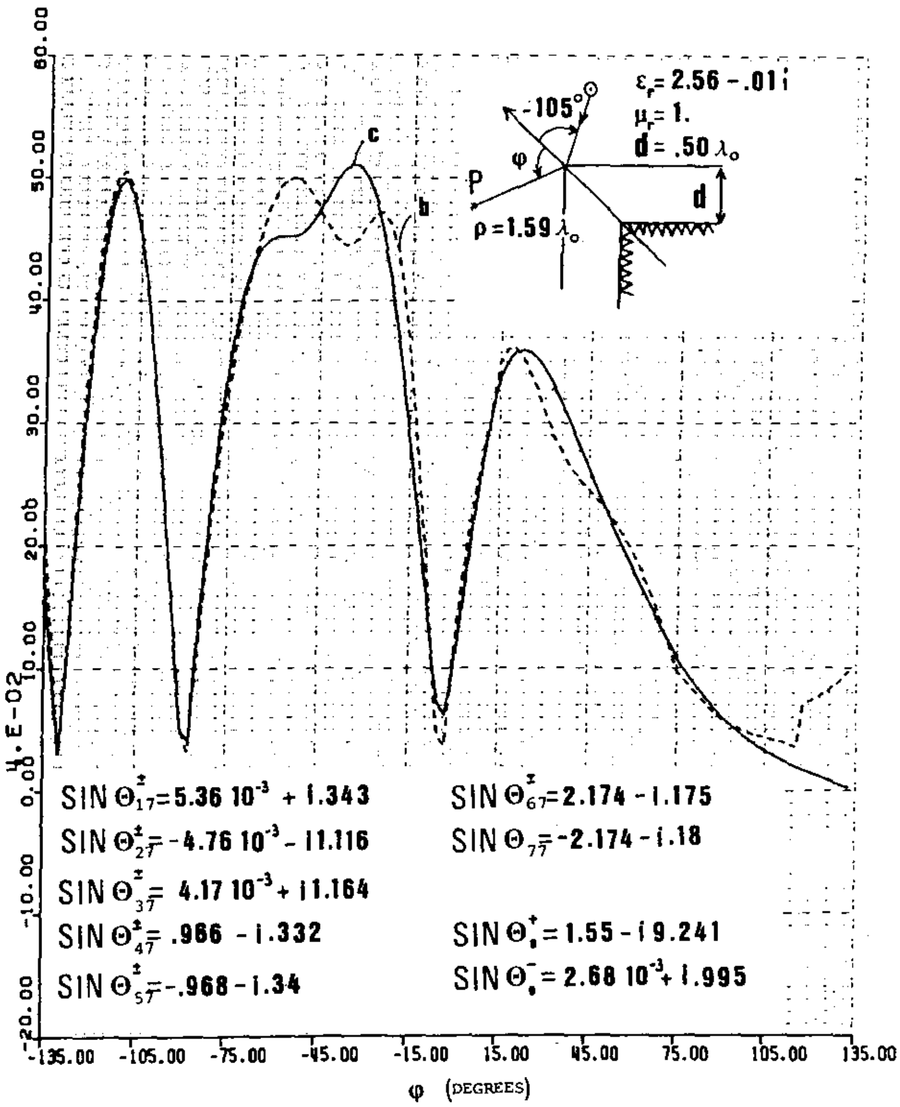

Fig. 8. H is the actual reflected field + respectively: (b) $(H - H_r)$ seven modes $\{\sin\theta_{i7}, i = 1, 7\}$; (c) $(H - H_r)\sin\theta_0^{\pm}$.

the alternance of the real part sign of the zeroes of the characteristic equation. Such a solution includes the necessary degrees of freedom which have to be used for complementary conditions at the internal junction of dielectric layers.

For layers with low thickness (junction thickness $\leqslant \lambda_0/\sqrt{|\varepsilon_r|}$) or relatively high loss, we have realized a complete and original calculation of the predominant term, needed in the spectral function, which shows the influence of several modes of propagation at the surface of the dielectric. Moreover, this derivation provides a new expression for the Maliuzhinets function. Let us notice that results derived in [2] can be used for a constant active impedance if only some changes are made as given in (22). Different numerical results are obtained. They particularly show that we can use a well chosen constant impedance for a real incident direction, up to a relatively high thickness, while keeping a good agreement with multimode

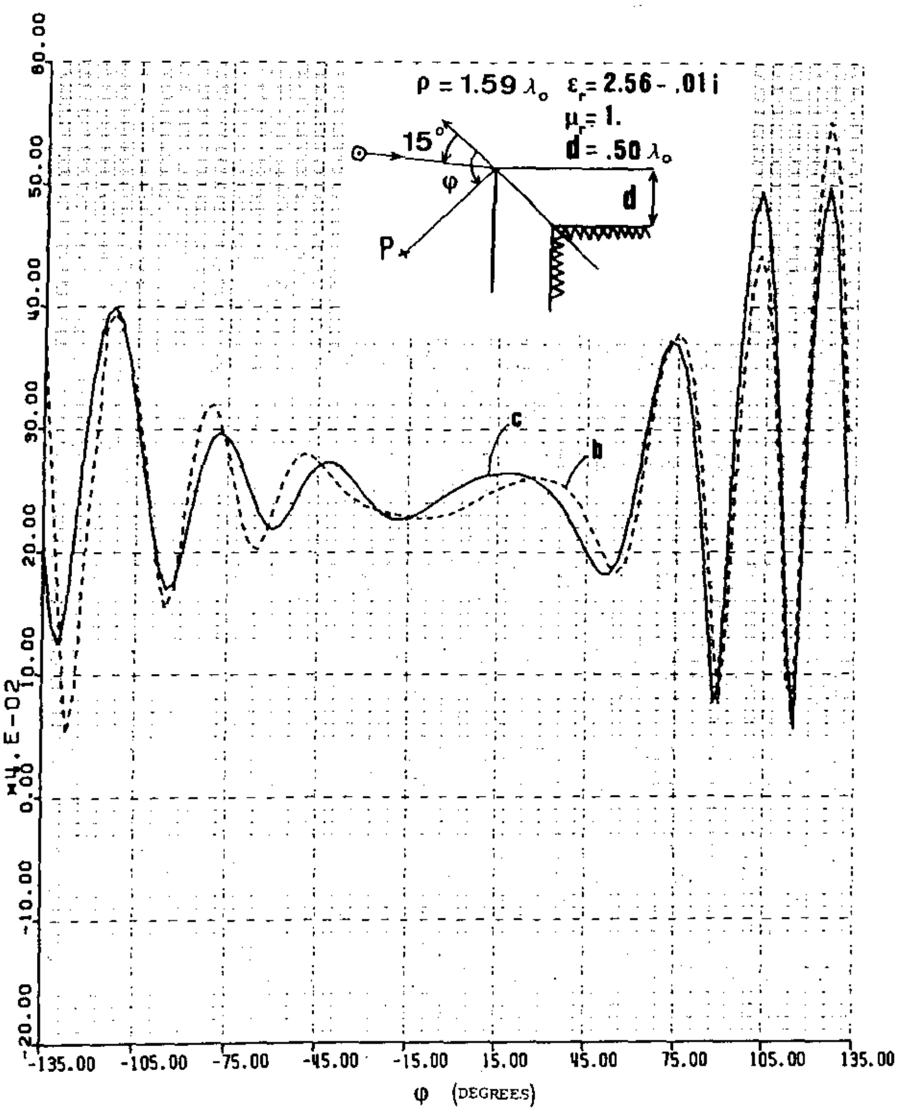

Fig. 9. H is the actual reflected field + respectively: (b) $(H - H_r)$ seven modes $\{\sin \theta_{i7}, i = 1, 7\}$; (c) $(H - H_r) \sin \theta_0^{\pm}$.

results. However this is not true anymore when the incident direction is complex, and induced surface waves become important.

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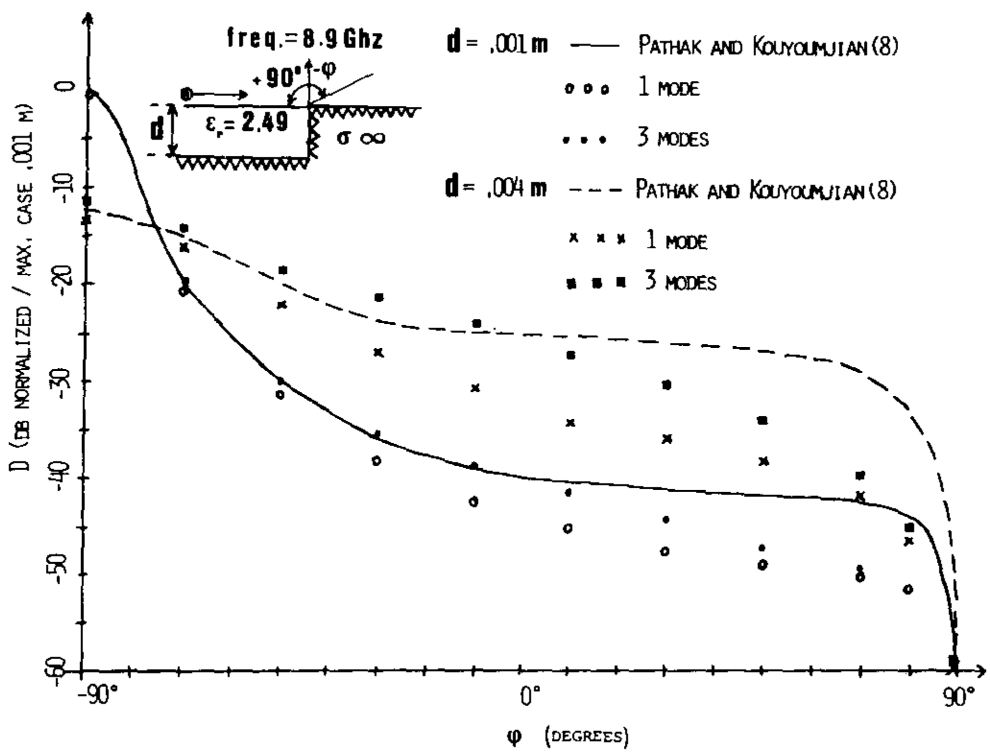

Fig. 10. Diffraction coefficient for an incident surface wave.

Appendix

As previously mentioned, the integral formulation used for the derivation of equation (23) converges only in a limited range of Φ, defined by $|\mathrm{Re}\,\alpha| \leq \Phi$. Moreover $\Psi_{\Phi[2]}$ (Ψ_Φ derived from [2]) and $\Psi_{\Phi(23)}$ (Ψ_Φ derived from (23)) are regular for $|\mathrm{Re}\,\alpha| \leq 2\Phi$. In order to show that $\Psi_{\Phi(23)} = \Psi_{\Phi[2]}$ for $|\mathrm{Re}\,\alpha| > 2\Phi$, it is then sufficient to show that $\Psi_{\Phi(23)}$ verifies the reflection condition which is satisfied by $\Psi_{\Phi[2]}$ [2]:

$$\frac{\Psi_{\Phi[2]}(\alpha+2\Phi)}{\Psi_{\Phi[2]}(\alpha-2\Phi)} = \cot(\tfrac{1}{2}(\alpha+\pi)).$$

After using $\Gamma(1+z) = z\Gamma(z)$ this condition is in fact satisfied by $\Psi_{\Phi(23)}$ since,

$$\frac{\Psi_{\Phi(23)}(\alpha+2\Phi)}{\Psi_{\Phi(23)}(\alpha-2\Phi)} = \prod_{l=0}^{\infty}\left[\frac{-\alpha+(l+\frac{1}{2})\pi}{\alpha+(l+\frac{1}{2})\pi}\right]^{(-1)^l} = \cot(\tfrac{1}{2}(\alpha+\tfrac{1}{2}\pi)).$$

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I.1.A) Errata et commentaires sur les démonstrations et formules

∗ p.545 : remplacer "$H_r = \mu/\mu_o$" par "$\mu_r = \mu/\mu_o$"

∗ p.545 et 546 : la prise en compte d'un décalage $\rho_o^{\pm}$ considéré dans l'article par l'application de l'opérateur $\sum \frac{(\rho_o^{\pm})^n}{n!} \cdot \frac{\partial^n}{\partial \rho^n}$ dans (5) , ainsi que par $\mathcal{Q}_p^{\pm}$ dans (7) et (9) , peut être modifiée suivant le commentaire ci-dessous tiré de notre exposé à la conférence GRECA /SEE de mars 1990 :

On définit un décalage de l'arête de $\rho_o^{\pm}$, pour l'application d'une condition différentielle ($\mathcal{P}$) sur le champ à la surface du dièdre , ceci pour prendre en compte l'inhomogénéité de matériaux au raccordement des deux faces du dièdre à cette même arête , en écrivant :

$$(\mathcal{P})^{\pm} \int_{\mathcal{C}} f(\alpha+\varphi)\, \mathrm{e}^{ik(\rho+\rho_o^{\pm})\cos\alpha} \mathrm{d}\alpha \,|_{\varphi = \pm\Phi} = 0 \text{ pour } \rho > 0 \ ,$$

ce qui se traduit , après une approximation polynomiale de ($\mathcal{P}$) de rang m et une intégration par parties , par :

$$\int_{\mathcal{C}} [P_m^{\pm}(\mp\sin\alpha))f(\alpha+\varphi)\mathrm{e}^{ik\rho_o^{\pm}\cos\alpha}]\, \mathrm{e}^{ik\rho\cos\alpha}\mathrm{d}\alpha \,|_{\varphi = \pm\Phi} = 0 \text{ pour } \rho > 0$$

qu'on ne peut (hélas) inverser pour obtenir une équation fonctionnelle sur f , à moins d'une approximation sur $Q(\cos\alpha) = \mathrm{e}^{ik\rho_o^{\pm}\cos\alpha}$ qui supprime son caractère trop croissant à l'infini (pour que le théorème d'inversion de Maliuzhinets soit applicable $|[..]|$ doit être $< \mathrm{e}^{a|Im\alpha|}$ quand $|\mathrm{Im}\alpha| \to \infty$, $\mathrm{Im}k\cos\alpha < 0$, $|\mathrm{Re}\alpha| < \pi$) . Morgan, Tuzhilin ou les auteurs plus récents (Rojas et Pathak par exemple) prennent implicitement $Q = 1$, en considérant que l'épaisseur de matériau sur lequel pénètre le champ reste faible . On peut pour autant donner à Q une approximation rationnelle

$$Q \simeq Q_p = \frac{R(\cos\alpha)}{S(\cos\alpha)} \ .$$

Dans notre article , nous prenons $S = 1$. Dans ce cas , on note *a posteriori* , que le terme ∞ dans (5) n'a que le sens de 'rang arbitraire de sommation' : il faut d'abord approximer Q pour définir un opérateur équivalent qui puisse s'appliquer sur l'expression intégrale de u ($\mathcal{C}$ étant un chemin d'intégration infini) , ce qui n'assure pas la convergence pour un rang infini et donne plutôt à l'opérateur différentiel de décalage dans (5) un sens de développement asymptotique pour ρ_o petit . Si l'on prend $S \neq 1$, on améliore bien sûr l'approximation mais le théorème d'inversion de Maliuzhinets doit être légérement modifié car , pour obtenir la meilleure approximation de Q par Q_p sur la plus grande partie de $\mathcal{C}$, on se retrouve nécessairement avec des pôles de [.] induits par S à l'intérieur des boucles de $\mathcal{C}$. On se ramène alors , après l'inversion de l'expression intégrale , à une équation (9) légèrement modifiée : certains c_n dépendent maintenant des singularités de Q_p et le terme '$Q_p^{\pm}$' de (9) est remplacé par R (numérateur de l'approximation rationnelle de Q) .

On notera que , la modification proposée pour mieux tenir compte de $\rho_o^{\pm}$ ne changeant pas la forme de l'équation fonctionnelle (9) , la suite de l'exposé donnée dans l'article reste valable .

* p.549 : . remplacer dans (20) " $r_{\pm}$ " par " $\pm r_{\pm}$ "
. changer dans (21) " $-i[\int_0^{i\infty}.....]$ " par " $+i[\int_0^{i\infty}.....]$ "

* p.550 : On indique au chapitre 5 de notre article que

$$\Psi_{on}^{\pm}(\alpha) \sim (\ K_{on}^{\pm}\, e^{(\frac{\pi}{4\Phi}|\,Im\alpha\,|)} + O(1)\ )^{\,signe\mathrm{Re}\theta_n^{\pm}} \quad \text{quand } |\mathrm{Im}\alpha| \to \infty\ ,$$

et on préférera écrire ,

$$\Psi_{on}^{\pm}(\alpha) \sim K\ (\ \cos(\tfrac{\pi}{4\Phi}(\alpha \pm \Phi)) + O(\alpha^{\nu} e^{(\frac{-\pi}{4\Phi}|\,Im\alpha\,|)})\ )^{\,signe\mathrm{Re}\theta_n^{\pm}} \quad \text{quand } |\mathrm{Im}\alpha| \to \infty\ ,$$

où K est indépendant de α , $\Phi \geq \pi/2$, et $\nu = 0$ si $\Phi \neq \pi/2$.
Pour poser cela , on aura utilisé tout d'abord que , d'après p.550 ,

$$\Psi_{on}^{\pm}(\alpha) = K_o\ (\Psi_{\Phi}(\alpha \pm \Phi + \pi/2 - \theta_n^{\pm}).\Psi_{\Phi}(\alpha \pm \Phi - \pi/2 + \theta_n^{\pm}))^{\,signe\mathrm{Re}\theta_n^{\pm}}\ ,$$

puis que , suivant M.P. Sakharova et A.F. Filippov , la fonction de Maliuzhinets Ψ_{Φ} a pour développement asymptotique , pour $\mathrm{Im}\alpha \to \pm\infty$,

$$\Psi_\Phi(\alpha) = \mathrm{A}\mathrm{e}^{\mp i\frac{\pi}{8\Phi}\alpha}\ \ (1 + C(\alpha)\ \mathrm{e}^{\pm i\frac{\pi}{2\Phi}\alpha} + D(\alpha)\ \mathrm{e}^{\pm i\alpha} + O(\alpha^{\nu'}\mathrm{e}^{\pm i\mu\alpha}))\ ,$$

où $C = O(\alpha^{\nu})$, $\nu = 0$ si $\Phi > \pi/2$, $\mu = min(\frac{\pi}{\Phi}\ ,\ \frac{\pi}{2\Phi}\ + 1)$ (pour les détails concernant C, D , voir l'article de Sakharova et Filippov référencé à la suite de l'Introduction).

* p.551 : . remplacer "$O(1)$" par "$O(\alpha^{\nu})$"
. remplacer " $O(\exp(\frac{-\pi}{2\Phi}\,|\,Im\alpha\,|\,))$" par "$O(\alpha^{\nu}\exp(\frac{-\pi}{2\Phi}\,|\,Im\alpha\,|\,))$"

* Reprenons pour les souligner (et en les généralisant) certaines considérations sur le choix des modes utilisés pour l'écriture de notre solution (p.551) : l'étude de l'ensemble complet des solutions de l'équation caractéristique d'un multicouche diélectrique plan (pour E ou H perpendicaire au plan d'incidence) dont la dernière couche est impénétrable (parfaitement conductrice par exemple) nous montre qu'il s'écrit, pour des pertes nulles ,

$$\begin{aligned}\{sin\theta_p\} = \{&\mathrm{Re}sin\theta_p = 0\ ,\ groupe\ A\ de\ cardinal\ fini\ impair\}\\ &+ \{sin\theta_p = sin\theta, sin\theta_{p+1} = -sin\theta^*\ ,\ groupe\ B\ infini\}\ ,\end{aligned}$$

dont on peut retenir le sous-ensemble $\{sin\theta_p\}_{p=1,m}$ qui doit contenir A ainsi qu'un nombre pair de modes de B ; ce sous-ensemble satisfait , pour des pertes non nulles (même infiniment petites) , à :

$$\sum_{n=1}^{m} \mathrm{sgn\ Re\ sin}\ \theta_n = 1 \text{ et } m \text{ impair .}$$

Ce choix permet directement que le coefficient de réflexion d'optique géométrique de chaque face soit de module 1 quand les pertes sont nulles et que $\lim_{|Im\alpha|\to\infty} f(\alpha)$ existe sans avoir à y employer aucun coefficient c_n ; ce choix est d'ailleurs celui retenu indépendamment par Tuzhilin et Maliuzhinets .

* p.552 : . remplacer dans (30) "$(2\Phi - (\varphi + \varphi_o))$" par "$(\pm 2\Phi - (\varphi + \varphi_o))$"

* p.560 : .remplacer "$\frac{\Psi_{\Phi[2]}(..)}{\Psi_{\Phi[2]}(..)} = \cot(\frac{1}{2}(\alpha + \pi))$" par "$\frac{\Psi_{\Phi[2]}(..)}{\Psi_{\Phi[2]}(..)} = \cot(\frac{1}{2}(\alpha + \frac{1}{2}\pi))$"

Diffraction par un dièdre à faces courbes non parfaitement conducteur (1)

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RÉSUMÉ. – *On étudie ici une expression du champ diffracté par un dièdre non parfaitement conducteur qui tient compte de l'angle à l'arête du dièdre et de la courbure de ses faces de façon globale. La solution est uniforme au voisinage des plans tangents à l'arête.*

ABSTRACT. – *The expression of the field diffracted by a non perfectly conducting wedge, taking into account both curvature and angle of the wedge, is studied here. The solution is uniform near the planes tangent at the edge.*

1. INTRODUCTION

La détermination du champ diffracté par un dièdre, en tenant compte de la courbure de ses faces a déjà été abordée pour différents cas précis de matériau ou de géométrie. Le cas « parfaitement conducteur » a été l'un des plus étudiés avec les travaux de Borovikov [1], de Michaeli [2], Idemen et Felsen [3], ainsi que ceux de Lee et Deschamp [4], Weston [5] et James [6]. Kaminetsky et Keller [7], Albertsen et Christiansen [8], ont considéré des matériaux non parfaitement conducteurs mais en ne tenant compte, pour leur coefficient de diffraction, que du premier ordre du développement de leur surface. Büyükaksoy *et al.* [9] ont analysé le cas du demi-plan courbe. On étudie ici des expressions du champ prenant en compte globalement l'angle à l'arête et la courbure des faces du dièdre.

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2. FORMULATION DU PROBLÈME ET SOLUTION

On considère un dièdre à faces courbes d'arête confondue avec l'axe z. Chaque face « ± » a pour surface un secteur circulaire de rayon $a^{\pm}$, qui a localement pour équation $\varphi = \pm(\Phi + \rho/2a^{\pm}) + O(\rho^2)$ dans les coordonnées polaires (ρ, φ). On prend ici $\Phi \geqq \pi/2$.

On désire déterminer le champ électrique ou magnétique rayonné, quand le dièdre est illuminé par une onde plane de direction perpendiculaire à l'arête et qu'on impose une condition d'impédance constante sur chacune des faces ±. Le problème se réduit alors à deux problèmes indépendants de même forme, respectivement pour chacune des composantes suivant l'arête du champ électrique et magnétique.

Ces deux composantes notées indifféremment u, on peut écrire le champ incident,

$$u^i(\rho, \varphi) = u_0\, e^{ik\rho \cos(\varphi - \varphi_0)} \tag{1}$$

et la condition d'impédance,

$$\left(\frac{\partial}{\partial n} - ik \sin \theta^{\pm}\right) u \Bigg|_{\text{face} \pm} = 0 \tag{2}$$

où $\partial/\partial n$ est la dérivée normale de surface.

On choisit alors d'écrire u sous la forme d'une intégrale de Sommerfeld-Maliuzhinets,

$$u(\rho, \varphi) = \frac{1}{2\pi i} \int_{\mathscr{C}} f(\alpha + \varphi)\, e^{ik\rho \cos \alpha}\, d\alpha \tag{3}$$

où $\mathscr{C}$ est constitué de deux branches : l'une allant de $(i\infty + 3\pi/2)$ à $(i\infty - \pi/2)$ au-dessus de toutes les singularités de l'intégrand et l'autre déduite par inversion par rapport à $\alpha = 0$. D'après Maliuzhinets [10], la fonction f satisfait à :

a) $(f(\alpha) - u_0/(\alpha - \varphi_0))$ régulière pour $|\mathrm{Re}\, \alpha| \leqq \Phi$;

b) $(f(\alpha) - f(\pm i\infty)) < \exp(-c\,|\mathrm{Im}\, \alpha|)$, $c > 0$, quand $\mathrm{Im}\, \alpha \to \pm\infty$.

Pour poursuivre, on modifie (2). On utilise un développement de Taylor de u et un développement polynomial de l'équation de la face ±, rapporté à la surface $\varphi = \pm\Phi$. Ceci donne, en laissant implicites les termes supérieurs au premier ordre,

$$(4)\quad \left[\mp\frac{\partial}{\rho\,\partial\varphi} - ik\sin\theta^{\pm} - \frac{1}{2a^{\pm}}\left(\left(\frac{\partial^2}{\partial\varphi^2} - \rho\frac{\partial}{\partial\rho}\right) \pm ik\sin\theta^{\pm}\,.\,\frac{\rho.\partial}{\partial\varphi}\right) + O\left(\frac{\rho}{a^{\pm 2}}\right)\right]u\Big|_{\varphi=\pm\Phi} = 0$$

On peut maintenant chercher la solution de (4) pour ρ quelconque. Ce type de condition aux limites déjà retenu par Kaminetzky et Keller [7] pour la discontinuité de rayon de courbure ($\Phi = \pi/2$) permet l'application du théorème d'inversion de Maliuzhinets [10].

Cela donne après application de (4) à (3),

$$(5)\quad \left(\sin\alpha \pm \sin\theta^{\pm} \pm \frac{1}{ik\,a^{\pm}}\mathrm{D}_{\alpha}(.) \pm O\left(\frac{1}{(ka)^2}\right)\right)f(\alpha\pm\Phi) - \left(-\sin\alpha \pm \sin\theta^{\pm} \pm \frac{1}{\mathrm{i}k\,a^{\pm}}\mathrm{D}_{-\alpha}(.) \pm O\left(\frac{1}{(ka)^2}\right)\right)f(-\alpha\pm\Phi) = 0$$

où

$$D_{\alpha}(.) = \frac{1}{2}\left[\left(\frac{\partial^2(.)}{\partial\alpha^2} - \frac{\partial}{\partial\alpha}\left(\operatorname{cotg}(\alpha)(.)\right)\right) \pm \sin\theta^{\pm}\frac{\partial}{\partial\alpha}\left(\frac{1}{\sin\alpha}\frac{\partial}{\partial\alpha}(.)\right)\right]$$

L'équation (5) suggère de poser

$$(6)\qquad f(\alpha) = \sum_{n\geq 0}\frac{1}{k^n}f_n(\alpha)$$

où les fonctions $f_n(\alpha)$ sont indépendantes de k. Appliquées à (5), les $f_n(\alpha)$ peuvent être déterminées par annulation des coefficients des puissances en k. On note que la convergence de la série (6) est essentiellement limitée par la convergence du développement adopté pour (4). Cette série peut en particulier fournir une bonne approximation du champ au voisinage de l'arête du dièdre. Hors du voisinage des pôles de $f(\alpha)$, cette série peut surtout servir pour le calcul du champ lointain rayonné au départ de l'arête qui est donné par

$$(7)\qquad u_d = -\frac{e^{-i\pi/4}}{\sqrt{2\pi k\rho}}\,.\,e^{-ik\rho}\left(f(\pi+\varphi) - f(-\pi+\varphi)\right)$$

appelé aussi champ diffracté d'arête. On poursuit donc maintenant le calcul des premiers termes du développement de f. Ils vont donner une approximation de l'influence conjuguée de l'angle d'arête Φ et des rayons de courbure $a^{\pm}$ des faces sur le champ u_d.

En appliquant l'expression (6) à (5), on obtient pour les termes f_0 et f_1

$$(8)\quad (\sin\alpha \pm \sin\theta^{\pm}) f_0(\alpha \pm \Phi) - (-\sin\alpha \pm \sin\theta^{\pm}) f_0(-\alpha \pm \Phi) = 0$$

$$(9)\quad (\sin\alpha \pm \sin\theta^{\pm}) f_1(\alpha \pm \Phi) - (-\sin\alpha \pm \sin\theta^{\pm}) f_1(-\alpha \pm \Phi) = \mp \frac{1}{i a^{\pm}} (D_{\alpha}(f_0(\alpha \pm \Phi)) - D_{-\alpha}(f_0(-\alpha \pm \Phi)))$$

Comme le suggérait l'écriture même de (6), f_0 satisfait à l'équation fonctionnelle (8) que l'on obtient pour le dièdre à faces planes; on en connaît la solution d'après [11] et [12]. La fonction f_0 connue, le second membre de (9) est déterminé et on peut utiliser la technique retenue dans [12] pour résoudre ce type d'équation. Du fait de la procédure récurrente infinie pour déterminer les f_n, on choisit que seule f_0 contribue au champ incident. Donc $f_{n \neq 0}$, en particulier f_1, est une fonction régulière pour $|\mathrm{Re}\,\alpha| \leqq \Phi$, assurant par là même l'unicité du choix de f_1. On pose ainsi

$$(10)\quad f_1(\alpha) = \Psi(\alpha) . \chi(\alpha)$$

où

- $\Psi(\alpha)$ est la solution, régulière pour $|\mathrm{Re}\,\alpha| \leqq \Phi$ de

$$\frac{\Psi(\alpha \pm \Phi)}{\Psi(-\alpha \pm \Phi)} = \frac{-\sin\alpha \pm \sin\theta^{\pm}}{\sin\alpha \pm \sin\theta^{\pm}}$$

déjà déterminée dans [11];

- $\chi(\alpha)$ est la solution de

$$\chi(\alpha \pm \Phi) - \chi(-\alpha \pm \Phi) = K^{\pm}(\alpha)$$

régulière pour $|\mathrm{Re}\,\alpha| \leqq \Phi$ qui est donnée pour $|\mathrm{Re}\,\alpha| \leqq \Phi$ par [13] et [14]

$$\chi(\alpha) = \frac{-i}{8\Phi} \int_{-i\infty}^{i\infty} d\alpha' \left(K^{+}(\alpha') \operatorname{tg}\left(\frac{\pi}{4\Phi}(\alpha + \Phi - \alpha') \right) - K^{-}(\alpha') \operatorname{tg}\left(\frac{\pi}{4\Phi}(\alpha - \Phi - \alpha') \right) \right)$$

ceci avec

$$K^{\pm}(\alpha) = \mp \frac{D_\alpha(f_0(\alpha \pm \Phi)) - D_{-\alpha}(f_0(-\alpha \pm \Phi))}{i a^{\pm} \Psi(\alpha \pm \Phi).(\sin\alpha \pm \sin\theta^{\pm})}$$

On a ainsi déterminé l'expression de f_0 et de f_1. L'expression de f ainsi obtenue $f_a = f_0 + f_1/k$ a l'originalité de rendre compte globalement de l'angle de dièdre Φ, des courbures des faces $\pm$ et de la nature des matériaux de face. On note que dans les cas limites, du dièdre à faces planes ($a^{\pm} \to \infty$) d'une part et de la discontinuité de rayon de courbure pour un matériau parfaitement conducteur ($\Phi = \pi/2$, $\sin\theta^{\pm} = 0$ ou ∞) d'autre part, on retrouve les expressions du champ diffracté par la discontinuité d'arête, déjà publiées respectivement par Maliuzhinets [11] et par Kaminetsky et Keller [7].

Ceci acquis, on peut maintenant s'intéresser à l'expression de u_d au voisinage du pôle de $f_a(\pm\pi + \varphi)$ pour $\varphi = \pm\Phi$. Ce voisinage correspond à la zone où des ondes rampantes peuvent être excitées et se propager.

On est alors amené à chercher une expression de f en ce voisinage, faisant passer continûment u_d d'une onde cylindrique à une somme d'ondes rampantes.

On pose pour $\varphi \sim \pm\Phi$

$$(11) \quad f(\pi + \varphi) - f(-\pi + \varphi) = e^{-i\xi_{\pm}^3/3} \int_{-\infty - i\varepsilon}^{+\infty - i\varepsilon} \frac{e^{-it\xi_{\pm}} L^{\pm}(t)}{w_2'(t) - q^{\pm} w_2(t)} dt$$

avec

$$\xi_{\pm} = (ka^{\pm}/2)^{1/3} \sin(\pm\varphi - \Phi)$$

$$w_2(t) = \sqrt{\pi}(Bi(t) - i\,Ai(t))$$

$$q^{\pm} = -i(ka^{\pm}/2)^{1/3} \sin\theta^{\pm}$$

Ce développement effectue le type de transition désiré pour le champ u_d de (7).

En effet, du fait que

$$ka^{\pm}(\pm\varphi - \Phi) \simeq ka^{\pm} \sin(\pm\varphi - \Phi) + \xi_{\pm}^3/3$$

on peut écrire pour $\xi_\pm \gg 0$, t_s étant les pôles de l'intégrand,

$$(12)\quad u_d = \frac{e^{-i\pi/4}}{\sqrt{2\pi k\rho}} . e^{-ik(\rho - a^\pm \sin(\pm\varphi - \Phi))} . e^{-ika^\pm(\pm\varphi-\Phi)} \times 2\pi i \sum_{\operatorname{Im} t_s < 0} e^{-it_s\xi_\pm} . \text{Résidu}\left(\frac{L^\pm(t)}{w_2' - q^\pm w_2}\right)\Bigg|_{t=t_s}$$

expression sous forme d'ondes rampantes du champ u_d

Par ailleurs, en supposant L faiblement oscillant, (11) fournit pour $\xi_\pm \ll 0$

$$(13)\quad u_d \simeq -\frac{e^{-i\pi/4}}{\sqrt{2\pi k\rho}} . e^{-ik\rho} . \frac{L^\pm(-\xi_\pm^2)}{\xi_\pm - iq^\pm} 2\sqrt{\pi}\,\xi_\pm$$

expression de u_d sous forme d'onde cylindrique. Dès lors, on cherche à déterminer $L(t)$. On peut alors remarquer que (13) s'applique dans une zone de φ où $f \simeq f_a$ est par ailleurs valable. Par comparaison on obtient une première approximation

$$(14)\quad L^\pm(-(ka^\pm/2)^{2/3}\sin^2\alpha) \simeq \frac{(\sin\alpha - \sin\theta^\pm)}{2\sqrt{\pi}\sin\alpha} \times (f_a(\pi \pm (\alpha+\Phi)) - f_a(-\pi \pm (\alpha+\Phi)))$$

où $-\pi/2 < \operatorname{Re}\alpha < 0$

Cette première expression de $L^\pm(t)$, déduite d'un développement par la phase stationnaire au premier ordre, possède une coupure « nuisible » à $\operatorname{Im} t = 0$, $t > 0$. On préfère donc une expression modifiée, dérivée d'une approche plus globale (Annexe). L'expression générale (A.10) tronquée au second ordre fournit en prenant $\sin\alpha$ pour $2\sin(\alpha/2)$ dans l'argument de $L^\pm$

$$(15)\quad L^\pm(-(ka^\pm/2)^{2/3}(\sin\alpha)^2) = 1/(2\sqrt{\pi}\sin\alpha) \times [(\sin\alpha - \sin\theta^\pm) . (f_a(\pi \pm (\alpha+\Phi)) - f_a(-\pi \pm (\alpha+\Phi))) - D^\pm_{\pm\alpha+\pi}(f_0(\pi \pm (\alpha+\Phi)) - f_0(-\pi \pm (\alpha+\Phi)))/(ika^\pm)]$$

où $|\operatorname{Re}\alpha| < \pi/2$. On note que l'expression (15) de $L^\pm(t)$ ne possède pas de coupure « nuisible » car le terme de droite dans (15) est pair en α.

On a ainsi obtenu un développement de u_d uniforme au voisinage des plans tangents $\varphi = \pm\Phi$. On notera que l'expression (11) avec (15) permet de retrouver, au premier ordre en $(ka^{\pm}/2)^{1/3}$, les résultats concernant un matériau parfaitement conducteur ($\sin\theta = 0$ ou ∞) publiés par Borovikov [1], Michaeli [15]. Cette expression tient compte globalement, comme f_a, de l'angle de dièdre, de la courbure et de la nature des faces.

3. CONCLUSION

On a considéré dans cette étude la diffraction par un dièdre à faces courbes non parfaitement conducteur, s'attachant à obtenir des expressions du champ diffracté d'arête tenant compte globalement de l'angle de dièdre à l'arête et de la courbure des faces. On a par ailleurs obtenu un développement uniforme du champ au voisinage des plans tangents à l'arête, rendant compte de la transformation du champ diffracté par l'arête, lorsqu'il passe d'une onde cylindrique à une somme d'ondes rampantes.

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ANNEXE

On considère que les champs diffusés au voisinage de chacune des faces $\varphi = +$ et $-\Phi$ correspondent aux rayonnements de courants « influencés » respectivement sur une portion de cylindre de rayon a^+ et a^-. En négligeant dans les expressions mathématiques les ondes faisant plus d'un tour de cylindre, la fonction de rayonnement de ces courants peut s'écrire :

$$(\mathrm{A.1}) \quad F^{\pm}(\varphi) = -\int_{\mathscr{E}} \frac{\mathscr{L}^{\pm}(\pm\nu)\, e^{-i\nu(\pm\varphi-\Phi)}}{H_\nu^{(2)'}(ka^{\pm}) + Q^{\pm} H_\nu^{(2)}(ka^{\pm})}\, d\nu\, e^{ika^{\pm}\sin(\pm\varphi-\Phi)}$$

où

$$Q^{\pm} = -i\sin\theta^{\pm}; \qquad H_\nu^{(1,2)} \text{ fonction de Hankel;}$$

$$\mathscr{E} \equiv]-\infty - i\varepsilon, +\infty - i\varepsilon[$$

On considère alors qu'on peut modéliser les sources « influençant » par les multipôles

$$\text{(A.2)}\quad \frac{\delta(\rho_c-a^\pm)}{\rho_c}\sum_n \mathscr{L}^\pm_{ne}(-1)^n\delta^{(n)}(\varphi_c),\qquad -\frac{\delta^{(1)}(\rho_c-a^\pm)}{k\rho_c}\sum_n \mathscr{L}^\pm_{nm}(-1)^n\delta^{(n)}(\varphi_c)$$

où (ρ_c, φ_c) sont les coordonnées polaires référencées au centre du cylindre de face $\pm$, $(\rho_c=a^\pm,\ \varphi_c=0)$ correspondant à l'arête.

On notera dès lors, que pour $|\varphi\mp\Phi\pm\pi/2|<\pi/2$, on peut prendre dans (A.1), $\mathscr{E}=$chemin de la méthode du col passant par le point de phase stationnaire avec

$$\text{(A.3)}\quad \mathscr{L}^\pm(\nu)=\sum_n(\mathscr{L}^\pm_{ne}(i\nu)^n\tilde{H}^{(2)}_\nu(ka^\pm)+\mathscr{L}^\pm_{nm}(i\nu)^n\tilde{H}^{(2)\prime}_\nu(ka^\pm))$$
$$\times(\tilde{H}^{(1)\prime}_\nu(ka^\pm)+Q^\pm\tilde{H}^{(1)}_\nu(ka^\pm))/2$$

où $\tilde{H}^{(1,2)}_\nu$ est l'expression asymptotique de Debye pour $\nu<ka^\pm$ de $H^{(1,2)}_\nu$.

Soit maintenant $\mathcal{O}^\pm$ la condition aux limites différentielle définie pour la face $\varphi=\pm\Phi$. D'après les définitions on peut écrire

$$\text{(A.4)}\quad \left[\mathcal{O}^\pm.\int_{\mathscr{S}}F^\pm(\alpha\mp(\pi/2-\Phi))\,e^{-ik\rho\cos(\alpha-\gamma)}\,d\alpha\Big|_{\gamma=\pm(\pi/2-\varepsilon)}\right]_{\varepsilon=0^+}$$
$$=\left[-\mathcal{O}^\pm.\left(\sum_n\mathscr{L}^\pm_{ne}\frac{\partial^n}{\partial\varphi_c^n}+\mathscr{L}^\pm_{nm}\frac{\partial^n}{\partial\varphi_c^n}\frac{\partial}{\partial k\,\rho_c}\right)\right.$$
$$\left.\times\int_{\mathscr{S}}e^{ik\rho\cos(\alpha+\gamma)}\,e^{-ik\rho'\cos(\alpha+\gamma')}\,d\alpha\Big|_{\gamma=\pm(\pi/2+\varepsilon)}\right]_{\varepsilon=0^+}$$

où (ρ', γ') sont les coordonnées polaires référencées à l'arête correspondant à (ρ_c, φ_c) et $\mathscr{S}=]-i\infty-\pi/2,\ -\pi/2]\cup]-\pi/2,\ \pi/2[\cup[\pi/2,\ \pi/2+i\infty[$

Par différentiation et intégration par parties, on peut alors écrire

$$\text{(A.5)}\quad \int_{\mathscr{S}}e^{\mp ik\rho\sin(\alpha\pm\varepsilon)}\,\mathcal{O}^\pm_{\alpha\pm\varepsilon}(F^\pm(\alpha\mp(\pi/2-\Phi)))\,d\alpha\Big|_{\varepsilon=0^+}$$
$$=-\int_{\mathscr{S}}e^{\mp ik\rho\sin(\alpha\pm\varepsilon)}\,\mathcal{O}_{\pi-(\alpha\pm\varepsilon)}(\sum_n\mathscr{L}^\pm_{ne}W_{ne}(a^\pm,\alpha)$$
$$+\mathscr{L}^\pm_{nm}W_{nm}(a^\pm,\alpha))\,d\alpha\Big|_{\varepsilon=0^+}$$

avec

$$W_{ne}(a,\ \alpha) = e^{ika\cos\alpha}\,\frac{\partial^n}{\partial\alpha^n}\,e^{-ika\cos\alpha}$$

$$W_{nm}(a,\ \alpha) = \left(\frac{\partial}{\partial ka} - i\cos\alpha\right) W_{ne}(a,\ \alpha)$$

La surface cylindrique de rayon $a^{\pm}$ considérée pour l'écriture de (A.1) et (A.3) n'est pas limitée par l'arête et (A.5) se prolonge à tout le plan tangent de face$\pm$ *i.e.* $\rho > 0$ et < 0, $\varepsilon = 0^{+}$ et 0^{-}.

En conséquence, on peut écrire

$$\text{(A.6)} \quad \mathcal{O}^{\pm}_{\alpha}\,(F^{\pm}\,(\alpha \mp (\pi/2 - \Phi))) = -\,\mathcal{O}^{\pm}_{\pi-\alpha}\big(\sum_{n} \mathscr{L}^{\pm}_{ne}\, W_{ne}(a,\ \alpha) + \mathscr{L}^{\pm}_{nm}\, W_{nm}(a,\ \alpha)\big)$$

On note alors que, en posant $\nu = ka^{\pm}\sin\alpha$

$$\text{(A.7)} \quad \begin{aligned} \mathcal{O}^{\pm}_{\pi-\alpha}(W_{ne}(a^{\pm},\ \alpha)) &= (i\nu)^{n}\,\mathcal{O}^{\pm}_{\pi-\alpha}(1) \\ \mathcal{O}^{\pm}_{\pi-\alpha}(W_{nm}(a^{\pm},\ \alpha)) &= (i\nu)^{n}\,\mathcal{O}^{\pm}_{\pi-\alpha}(-i\cos\alpha) \end{aligned}$$

ce qui, démontré pour plusieurs n, est admis pour n quelconque.

Par ailleurs, on constate que

$$\text{(A.8)} \quad \begin{aligned} \mathcal{O}^{\pm}_{\pi-\alpha}(1) &\simeq -\tilde{H}^{(2)}_{\nu}(ka^{\pm})\,(\tilde{H}^{(1)\prime}_{\nu}(ka^{\pm}) + Q^{\pm}\,\tilde{H}^{(1)}_{\nu}(ka^{\pm}))\cos\alpha \\ \mathcal{O}^{\pm}_{\pi-\alpha}(-i\cos\alpha) &\simeq -\tilde{H}^{(2)\prime}_{\nu}(ka^{\pm})\,(\tilde{H}^{(1)\prime}_{\nu}(ka^{\pm}) + Q^{\pm}\,\tilde{H}^{(1)}_{\nu}(ka^{\pm}))\cos\alpha \end{aligned}$$

La définition de $\mathscr{L}(\nu)$ prise dans (A.3) et le groupe d'égalités (A.6), (A.7), (A.8), permettent d'écrire

$$\text{(A.9)} \quad \mathscr{L}^{\pm}(\nu) = \mathcal{O}^{\pm}_{\alpha}\,(F^{\pm}\,(\alpha \mp (\pi/2 - \Phi)))/(2\cos\alpha)$$

ou encore, directement pour (A.1)

$$\text{(A.10)} \quad \mathscr{L}^{\pm}(\pm ka^{\pm}\cos\alpha) = -\,\mathcal{O}^{\pm}_{\pm(\alpha+\pi/2)}(F^{\pm}\,(\pm(\alpha+\Phi)))/(2\sin\alpha)$$

où $|\mathrm{Re}\,\alpha| < \pi$. Une fois $F^{\pm}$ identifiée au voisinage de la zone de transition, l'expression (A.1) permet alors un passage uniforme dans la zone d'excitation des ondes rampantes.

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I.2.A) Errata et commentaires sur les démonstrations et formules

* p.326 : l'expression (15) de $L^{\pm}$ provient de l'expression générale (A.1) de la fonction de rayonnement (dont la forme est déduite rigoureusement d'un développement original utilisant les harmoniques cylindriques) . A ce titre , l'expression (15) , dérivée d'un développement du noyau $\mathcal{L}$ de (A.1) en posant $\nu = ka^{\pm} + (ka^{\pm}/2)^{2/3}\tau$, supprime , certes , la coupure constatée dans (14), mais doit être , si l'on veut être complet , introduite dans un développement lui aussi au second ordre de (A.1) (l'expression (11) n'en est en effet que le développement au premier ordre) .

* p.328 : remplacer dans A.3 "$Q^{\pm} H_{\nu}^{(1)'}$..." par "$Q^{\pm} H_{\nu}^{(1)}$..."

* Signalons ici que l'on peut éliminer de notre solution le terme d'optique géométrique séculaire (croissance polynomiale en ρ) lié intrinsèquement au développement d'une condition aux limites sous une forme asymptotique : ce terme d'optique géométrique de réflexion , séculaire dès le deuxième ordre en courbure , peut être identifié et corrigé sur la surface du dièdre par référence à la solution exacte pour le cylindre circulaire , ce qui nous fournit alors , après rayonnement des sources fictives de surface (théorème de Helmotz) associées au champ ainsi modifié , une expression du champ rayonné sans terme séculaire . (A ce sujet , notons que la fonction spectrale associée au rayonnement de toute répartition de sources données peut être obtenue grâce à la fonction spectrale attachée à la fonction de Hankel dont nous avons obtenu l'expression dans un de nos articles (référencé dans l'introduction en [33] , et dont la copie est au chapitre IV)) .

I.2.B) Compléments mathématiques

I.2.B.1) Une nouvelle démonstration de la formule A.9 de $\mathcal{L}^{\pm}$ pour (A.1)

L'expression (A.1) permet le prolongement du développement asymptotique initial de la fonction de rayonnement pour des directions , au départ de l'arête , en deçà de la tangente à la surface courbe du dièdre (zone d'excitation des rayons dits rampants guidés par la courbure) . On en donne ici une nouvelle démonstration .

Soit u un champ composé d'ondes sortantes , satisfaisant l'équation des ondes en deux dimensions, pour un régime harmonique en temps $\mathrm{e}^{i\omega t}$,

$$(1) \qquad \Delta\, \mathrm{u} \;+ k^2\, \mathrm{u} = 0\ ,$$

dans le domaine Ω extérieur à un cylindre circulaire de rayon a , avec k nombre d'onde complexe , $\mathrm{Im}k < 0$ (le cas $\mathrm{Im}k = 0$ pourra être considéré au sens de la limite) . On définit des coordonnées cartésiennes x et y telles que la surface $\partial\Omega = \mathcal{C}$ a pour équation $x^2 + (y+a)^2 = a^2$ .L'écriture de la dépendence en temps t est dès à présent omise . On définit la quantité S en écrivant

$$(2) \qquad \left(\frac{\partial \mathrm{u}}{\partial n} - ik\, \sin\theta\ \mathrm{u} \right) \Big|_{\mathcal{C}} = S\ ,$$

où n est l'abscisse suivant la normale sortante au cylindre et θ une constante. Choisissons de développer u en harmoniques cylindriques , $\mathrm{e}^{-im\varphi_c} H_m^{(2)}(k\rho_c)$, où (ρ_c,φ_c) sont les coordonnées polaires , définies de façon que les origines en ρ_c et φ_c correspondent respectivement au centre du cylindre et à l'axe des y , $H_m^{(2)}$ étant la fonction de Hankel de deuxième espèce d'ordre m . On obtient , d'après (2) ,

$$(3) \qquad \mathrm{u}|_{\mathcal{C}} = \frac{1}{2\pi k} \sum_m \mathrm{e}^{-im\varphi_c} \frac{\displaystyle\int_{\mathcal{C}} \mathrm{e}^{im\varphi_c}\, S(\varphi_c)\, \mathrm{d}\varphi_c}{H_m^{(2)\prime}(ka) + Q\ H_m^{(2)}(ka)}\ H_m^{(2)}(ka)\ ,$$

où $Q = -i\, \sin\theta$.

Définissons alors F , la fonction de rayonnement , en écrivant :

$$(4) \qquad u(x,y) = \frac{-1}{2\pi i} \int_{\mathcal{I}} F(\alpha)\, e^{-ik\rho\cos(\alpha-\varphi'')} d\alpha \;,$$

avec $x = -\rho\sin\varphi''$, $y = \rho\cos\varphi''$, et $\mathcal{I}$ un chemin de $-\arg(ik) - i\infty + \delta'$ à $\arg(ik) + i\infty + \delta'$, ($\mathrm{Im}k < 0$) . Cette expression est valide dans le demi-espace sans source avec $y\cos\delta' - x\sin\delta' > 0$. On notera que u à grande distance s'exprime suivant :

$$(5) \qquad u = \frac{-e^{-i\pi/4}}{\sqrt{2\pi k\rho}}\, e^{-ik\rho} F(\varphi'') + O((k\rho)^{-3/2}) \;.$$

On suppose alors que l'on possède un développement asymptotique de F pour a grand , à un ordre donné arbitraire. Reporté dans (4) , il nous fournit une expression de u , valide quand le point d'observation ou le lieu des sources se situe dans le voisinage de l'origine.On en a un exemple avec le cas de la diffraction par un dièdre à faces courbes , pour lequel on peut obtenir , grâce à la première partie de notre article , le développement de la fonction F pour une illumination plane ou cylindrique (en effet , on peut dériver $F_{ill.plane}$ de (5) ou , par réciprocité , écrire $F_{ill.cylind.} \equiv u_{ill.plane,\rho\ fini}$) . On détermine dans la suite une transformation intégrale particulière de F, permettant d'obtenir directement un développement valide pour des directions φ telles que $|\varphi''|$ soit voisin de ou supérieur à $\pi/2$.

En comparant (5) au développement asymptotique de u donné par la représentation de Helmhotz [1] pour $\rho\rightarrow\infty$, on obtient :

$$(6) \quad F(\varphi'_c) = \frac{e^{-ika\cos\varphi'_c}}{2} \int_{\mathcal{C}} \Big(u \frac{\partial e^{ik\rho_c\cos(\varphi_c-\varphi'_c)}}{\partial\rho_c} - \frac{\partial u}{\partial\rho_c}\, e^{ik\rho_c\cos(\varphi_c-\varphi'_c)}\Big) a\, d\varphi_c,$$

ce qui nous donne , d'après (2) ,

$$(7) \qquad F(\varphi'_c) = \frac{-e^{-ika\cos\varphi'_c}}{2} \int_{\mathcal{C}} \big(iu(-k\cos(\varphi_c-\varphi'_c) + k\sin\theta) + $$

$$+ S\big)\, e^{ika\cos(\varphi_c-\varphi'_c)} a\, d\varphi_c \;.$$

On utilise alors (3) et l'égalité $e^{ika\cos(\varphi_c-\varphi'_c)} = \sum_m i^m J_m(ka)\, e^{-im(\varphi'_c-\varphi_c)}$, qui nous permettent d'écrire :

$$(8) \qquad \mathrm{F}(\varphi_c') = \frac{-\mathrm{e}^{-ika\cos\varphi_c'}}{2} \Big(\sum_m i^m \, \mathrm{e}^{-im\varphi_c'} \int_{\mathcal{C}} S(\varphi_c) \, \mathrm{e}^{im\varphi_c} \, a \, \mathrm{d}\varphi_c \times$$
$$\times \Big[J_m(ka) - \frac{J_m'(ka) + Q\, J_m(ka)}{H_m^{(2)\prime}(ka) + Q\, H_m^{(2)}(ka)} H_m^{(2)}(ka)\Big]\Big) \, ,$$

nous donnant , d'après $\mathrm{W}(J_m, H_m^{(2)}) = -2i/\pi ka$,

$$(9a) \qquad \mathrm{F}(\varphi_c') = \frac{-\mathrm{e}^{-ika\cos\varphi_c'}}{i\pi ka} \sum_m \mathrm{e}^{-im\varphi_c'} \frac{i^m \mathcal{M}(m)}{H_m^{(2)\prime}(ka) + Q\, H_m^{(2)}(ka)} \, ,$$

ou encore sous une autre forme ,

$$(9b) \qquad \mathrm{F}(\varphi_c') = \sum_m \mathrm{e}^{\mp im\varphi_c'} d_m \mathcal{M}(\pm m) \, ,$$

où $\mathcal{M}(\nu) = \int_{\mathcal{C}} S(\varphi_c) \, \mathrm{e}^{i\nu\varphi_c} a \mathrm{d}\varphi_c$ pour ν entier, et $d_\nu = d_{-\nu} = \dfrac{\dfrac{-\mathrm{e}^{-ika\cos\varphi_c'}}{i\pi ka} i^\nu}{H_\nu^{(2)\prime}(ka) + Q\, H_\nu^{(2)}(ka)}$.

En assimilant la somme précédente à celle de résidus d'une expression intégrale ,comme G.N. Watson [2] le suggère classiquement , on peut écrire :

$$(10) \qquad \sum_m \mathrm{e}^{\mp im\varphi_c'} d_m \mathcal{M}(\pm m) = \Big(\int_{-\infty - i\epsilon}^{+\infty - i\epsilon} + \int_{+\infty + i\epsilon}^{-\infty + i\epsilon}\Big) d_\nu \mathcal{M}(\pm\nu) \, \frac{\mathrm{e}^{-i\nu(\pm\varphi_c' - \pi)}}{2i\sin\nu\pi} \, \mathrm{d}\nu,$$

avec $\epsilon > 0$. La fonction $(\frac{\mathrm{e}^{i\nu\pi}}{2i\sin\nu\pi})$ est alors développée suivant $(\sum_{p \geq 0} \mathrm{e}^{-i\nu 2p\pi})$ si $\mathrm{Im}\nu < 0$, et $(-\sum_{p<0} \mathrm{e}^{-i\nu 2p\pi})$ si $\mathrm{Im}\nu > 0$. On choisit alors de ne retenir que le terme en $p = 0$ en considérant $|\pm\varphi_c' - \frac{\pi}{2}| < \pi/2$, omettant donc les ondes correspondant à $\varphi_c' \equiv \varphi_c' \pm 2p\pi$ avec $p \neq 0$, qui font virtuellement plus d'un tour en φ_c' . On obtient ainsi , pour $|\pm\varphi_c' - \frac{\pi}{2}| < \pi/2$,

$$(11) \qquad \mathrm{F}(\varphi_c') \equiv \frac{-\mathrm{e}^{-ika\cos\varphi_c'}}{i\pi ka} \int_{-\infty - i\epsilon}^{+\infty - i\epsilon} \mathrm{e}^{-i\nu(\pm\varphi_c' - \pi/2)} \frac{\mathcal{M}(\pm\nu)}{H_\nu^{(2)\prime}(ka) + Q\, H_\nu^{(2)}(ka)} \, \mathrm{d}\nu,$$

avec $\epsilon > 0$. On s'intéresse maintenant au développement de $\mathcal{M}(\pm\nu)$ pour ν grand , obtenu quand on développe l'expression de $S(\varphi_c)$ au voisinage de $\varphi_c = 0$. Considèrons alors , pour la surface $\mathcal{C}$, un développement de y au voisinage de l'origine $\rho = 0$, asymptotique pour a grand , donné par

$$(12) \qquad y \sim a \sum_{p \geq 0} c_p (x/a)^{2p+2} \,,$$

définissant 'asymptotiquement' une nouvelle surface $\mathcal{C}_a$, pour laquelle l'abscisse curviligne (croissant en même temps que x) est noté $\tilde{s}$, avec $\varphi_c \sim \tilde{s}\,/a$. Par la suite , un développement asymptotique de (2) en puissance inverse de a nous conduit à une condition aux limites sur u en $y = 0^+$, au voisinage de l'origine , ce qui nous permet d'écrire :

$$(13) \qquad S \sim \mathcal{B}\, \mathrm{u}\,|_{y = 0^+, x = x_1(\tilde{s}\,)} = \frac{-1}{2\pi i} \int_{\mathcal{I}} B_\alpha(\mathrm{F}(\alpha))\, \mathrm{e}^{ikx_1(\tilde{s}\,)\sin\alpha} \mathrm{d}\alpha \,,$$

où l'on notera que la fonction x_1 n'est pas précisée , et donc que $\mathcal{B}$ et B_α ne sont pas complètement définis . Par ailleurs , on considèrera que $B_\alpha(\mathrm{F}(\alpha)) = O(e^{d\,|\,\mathrm{Im}\alpha\,|\, - ik\delta_1 \cos\alpha})$ quand $|\,\mathrm{Im}\alpha\,| \to \infty$ sur $\mathcal{I}$, où $\delta_1 > 0$ (aucune source dans le demi-espace $y \geq -\delta_1$) , le cas $\delta_1 = 0$ (présence de sources de rayonnement en $y = 0$) pouvant être considéré au sens de la limite .
Considérant , à ce stade , le développement asymptotique $\tilde{S}$ de S fourni par (13) , on peut écrire :

$$(14) \qquad \mathcal{M}(\nu) \sim \tilde{\mathcal{M}}_0(\nu) = \int_{-\infty}^{+\infty} \tilde{S}\ \mathrm{e}^{i\nu(-\tilde{s}/a)}\, \mathrm{d}\tilde{s} \,.$$

On assume que $\tilde{S} < \mathrm{e}^{-c\tilde{s}}$ quand $\tilde{s} \to +\infty$ et $\tilde{S} < \mathrm{e}^{d\tilde{s}}$ quand $\tilde{s} \to -\infty$, $-d < c$, que $-d < \mathrm{Im}(\frac{\nu}{a}) < c$, et que x_1 est une fonction analytique sur $\mathbb{R}$ telle que $x_1(\tilde{s}\,) = \tilde{s} + O(1)$ quand $|\,\tilde{s}\,| \to \infty$. On pourra dès lors écrire , pour $-d < \mathrm{Im}(\frac{\nu}{a}) < c$,

$$(15) \qquad \tilde{\mathcal{M}}_0(\nu) = \frac{-1}{2\pi i} \int_{\mathcal{I}} B_\alpha(\mathrm{F}(\alpha)) \int_{-\infty}^{+\infty} \mathrm{e}^{-i(\nu(\tilde{s}/a) - kx_1(\tilde{s}\,)\sin\alpha)}\, \mathrm{d}\tilde{s}\, \mathrm{d}\alpha \,,$$

où on aura soin que $\mathcal{I}$, chemin de $-\pi/2 - \arg(ik)$ à $\pi/2 + \arg(ik)$, soit alors choisi de façon que $\mathrm{Im}(k\sin\alpha) = \mathrm{Im}(\frac{\nu}{a})$. Par ailleurs , on considère que , pour (15) , aucune singularité de $B_\alpha(\mathrm{F}(\alpha))$, si il en est , ne puisse intercepter $\mathcal{I}$.
On pose $\nu = ka\sin\Lambda$, puis on assume , maintenant , qu'on a pour x_1 , un développement au voisinage de $\varphi_c = 0$, asymptotique pour a grand , donné par :

$$(16) \qquad x_1 \sim \tilde{s} + a \sum_{p \geq 0} g_p\, (\,\tilde{s}\,/a\,)^{p+3} \,.$$

On obtient alors le développement asymptotique de $\widetilde{\mathcal{M}}_o$, donc de $\mathcal{M}$, pour a grand :

(17) $\widetilde{\mathcal{M}}(\nu) \equiv \frac{-1}{2\pi i} \int_{\mathcal{S}} B_\alpha(\mathrm{F}(\alpha)).(1 + \sum_{p \geq 0} \mathcal{P}_p \; / \; a^{p+2}).\; \frac{2\pi}{k} \; (k\delta(k\sin\alpha - k\sin\Lambda)) \; d\alpha$,

ou encore

$$(18) \qquad \widetilde{\mathcal{M}}(\nu) = \frac{i\pi ka}{2\cos\alpha} . \mathcal{O}_\alpha(\mathrm{F}(\alpha)) \,|_{\,\alpha = \Lambda} \;,$$

avec $\mathcal{O}_\alpha(.) = \frac{2\cos\alpha}{\pi k^2 a} \; [(\; 1 + \sum_{p \geq 0} \frac{\mathcal{P}_p^a}{a^{p+2}} \;) \; \frac{B_\alpha(.)}{\cos\alpha}]$, où $\mathcal{P}_p^a$ et $\mathcal{P}_p$ sont des opérateurs linéaires différentiels par rapport à la variable $\sin\alpha$ dont les coefficients sont polynomiaux . Cette expression étant analytique , l'hypothèse sur Imν prise pour écrire (14) n'est plus nécessaire : (18) peut être utilisée directement dès que ν est grand . On note de plus que $\widetilde{\mathcal{M}}$ (donc $\mathcal{O}_\alpha$) ne dépend pas du choix de la fonction x_1 tandis que $\mathcal{P}_p^a$ et B_α en dépendent .

Une fois l'expression (18) de $\widetilde{\mathcal{M}}$ reportée dans (11) , on obtient une expression intégrale de F valide en particulier pour $\pm\varphi_c' \sim$ et $> \pi/2$ (zone voisinant la surface physique du cylindre) , qui ne nécessite pour son noyau qu'une expression asymptotique de F permettant d'exprimer u au voisinage de l'origine pour $\pm\varphi_c' \sim \pi/2$.

Pour le cas du dièdre à faces courbes , le développement précédent se rapporte à chacun des arcs composant chaque face $\pm$ de la surface , que l'on prolonge . A chacune des deux faces $+ ou -$ satisfaisant (2) sans second membre , on fait correspondre $- ou + \widetilde{s} > 0$; on remarquera alors que $\widetilde{S} \;|_{\,face\,\pm} = 0$ et $\widetilde{S} < e^{b|\widetilde{s}|}$ entraînent que l'expression ,

$$\widetilde{\mathcal{M}}_o(\pm\nu) = \int_{-\infty}^{+\infty} \widetilde{S} \;\; \mathrm{e}^{\pm i\nu(-\widetilde{s}/a)} \, d\widetilde{s} \;\; ,$$

est une fonction régulière pour $\mathrm{Im}(\frac{\nu}{a}) < -b$. Par ailleurs , on note que l'on pourra changer l'origine des angles pour la fonction de rayonnement et modifier alors le couple (11)-(18) suivant :

. dans (11) : $\varphi_c' \to \varphi \pm (\pi/2 - \Phi)$, $\widetilde{\mathcal{M}}(\pm\nu) \to i\pi k a^{\pm} \mathcal{L}^{\pm}(\pm\nu)$,
$\mathrm{F}(\varphi_c')$ (pour $|\pm\varphi_c' - \frac{\pi}{2}| < \pi/2$) $\to F^{\pm}(\varphi)$ (pour $|\pm\varphi - \Phi| < \pi/2$),

. dans (11) et (18) : $\mathcal{O}_\alpha$, a , $\sin\theta \to \mathcal{O}_\alpha^{\pm}$, $a^{\pm}$, $\sin\theta^{\pm}$,

. dans (18) : $\mathrm{F}(\alpha) \;\rightarrow\; F^{\pm}(\alpha \mp (\pi/2 - \Phi))$,

ce qui nous permet de retrouver exactement l'expression (A.9) de $\mathcal{L}^{\pm}$ pour (A.1) , ce que nous cherchions .

Notes :

* On note que pour un problème de diffraction , l'expression (A.1) pour $F^{\pm}(\varphi)$ peut être considéré pour des sources d'illumination extérieures au cylindre de rayon $a^{\pm}$ en faisant varier continuement ces sources de l'intérieur vers l'extérieur .

* Par ailleurs , pour obtenir une expression de $F(\varphi)$ pour une illumination plane , on peut procéder de deux façons :
  . considérer ce cas comme déduit à la limite d'un cas d'illumination cylindrique créée par une ligne source à distance ρ' de l'origine , en écrivant :

$$\text{(19a)}\quad F_{ill.plane}(\varphi) \equiv \sqrt{k\rho'}\, e^{ik\rho'}\Big[\underset{\substack{(\textit{déduit de } (11)\\ \textit{avec } \mathcal{M}\ (\textit{asympt.}))}}{F_{ill.\ cylindrique}(\varphi)}\Big] \quad \text{quand } \rho' \to \infty$$

ou , mieux encore ,

$$\text{(19b)}\quad F_{ill.plane}(\varphi) \equiv \sqrt{k\rho'}\, e^{ik\rho'}\Big[\Big(\underset{\substack{(\textit{déduit de } (11)\\ \textit{avec } \mathcal{M}\ (\textit{asympt.}))}}{F_{ill.cylindrique}(\varphi)} - \underset{\substack{(\textit{déduit de } (11)\\ \textit{avec } \widetilde{\mathcal{M}}_{1,\rho'}\ (\textit{asympt.}))}}{F_{o}(\varphi)}\Big)$$

$$+ \Big(\underset{\substack{(\textit{déduit de } (11)\\ \textit{avec } \mathcal{M}_{1,\rho'}\ (\textit{exact}))}}{F_{o}(\varphi)}\Big)\Big] \quad \text{quand } \rho' \to \infty$$

au sens fonctionnel où chacune de ces expressions , reportée dans la formule (4) du champ u en fonction de F , nous fournit le champ pour une illumination plane, en faisant pour l'évaluation de (4) $\rho' \to \infty$ et $\rho/\rho' \to 0$. La fonction $\mathcal{M}_{1,\rho'}$ devra correspondre à S obtenue pour une configuration telle que $[\widetilde{\mathcal{M}} - \widetilde{\mathcal{M}}_{1,\rho'}]$ reste faiblement oscillant quand $\rho' \to \infty$. L'expression obtenue d'après (19b) sera alors valable,quel que soit le point d'observation.

. ou encore considérer que l'on peut écrire :

(20a) $F_{ill.plane}\ (\varphi) \equiv \underset{\substack{(\text{déduit de } (11) \\ \text{avec } \mathcal{M} \ (asympt.))}}{F_{ill.\ plane}\ (\varphi)}$

ou , mieux encore ,

(20b) $F_{ill.plane}\ (\varphi) \equiv [\ (\ \underset{\substack{(\text{déduit de } (11) \\ \text{avec } \mathcal{M} \ (asympt.))}}{F_{ill.plane}\ (\varphi)} \ - \ \underset{\substack{(\text{déduit de } (11) \\ \text{avec } \widetilde{\mathcal{M}}_1 \ (asympt.))}}{F_o\ (\varphi)}\ )$

$+ \ (\ \underset{\substack{(\text{déduit de } (11) \\ \text{avec } \mathcal{M}_1 \ (exact))}}{F_o\ (\varphi)}\ )\ ]$,

au sens fonctionnel où chacune de ces expressions , reportée dans la formule (4) du champ u en fonction de F , nous fournit le champ pour une illumination plane,en considérant, pour l'évaluation de l'intégrale (4), ka grand et ρ/a petit . La fonction de rayonnement asymptotique utilisée pour le calcul de $\widetilde{\mathcal{M}}$ possède alors des pôles en ν de champs incident et réfléchis . Pour (20a) on devra situer correctement ces pôles par rapport au chemin d'intégration en ν , en considérant que ceux-ci sont complexes (Im$k < 0$) . Tandis que pour (20b) , $\mathcal{M}_1$ sera choisi de façon que $[\widetilde{\mathcal{M}} - \widetilde{\mathcal{M}}_1]$ ne comportera plus toutes les singularités initiales de champs réfléchis .L'expression obtenue d'après (20b) sera alors valable , quel que soit le point d'observation.

* pour clarifier le sens de ' ≡ ' dans les expressions (19a-b) , on note , comme exemple , que , quand $\rho' \to \infty$,

$$\frac{-1}{2\pi i} \int_{\mathcal{S}} \left(\sqrt{k\rho'}\ e^{ik\rho'} e^{ik\rho' \cos(\alpha - \varphi')}\right) e^{-ik\rho\cos(\alpha - \varphi)}\ d\alpha \qquad \to \ \frac{-e^{i(k\rho\cos(\varphi' - \varphi) - \pi/4)}}{\sqrt{2\pi}} \ .$$

* l'expression du champ u obtenue dans notre article sur le dièdre à faces courbes pour une illumination plane , peut être considéré

par réciprocité comme celle d'une fonction de rayonnement F pour une illumination cylindrique , pouvant être rigoureusement utilisée pour la transformation intégrale décrite précédemment .

(Remerciements : je tiens à remercier O. Lafitte pour d'intéressantes discussions au cours de l'année 1994 , qui m'ont conduit à poser (2) comme point de départ d'une nouvelle démonstration de (A.9))

REFERENCES :

[1] D.S. Jones , *The theory of electromagnetism* , Pergamon press (1964) .
[2] G.N. Watson , *The transmission of electric waves around the earth* , Proc. Roy. Soc. Ser. A , 95 , p. 546-563 (1918).

I.2.B.2) Considérations sur la régularité du noyau $\mathcal{L}^{\pm}$ donné par (A.9) et ses conséquences

On remarquera que le calcul à partir de l'expression intégrale (A.1) de $F^{\pm}$ est fortement influencé par l'uniformité de $\mathcal{L}^{\pm}$ dans une certaine région du plan complexe en ν . Ainsi pour $\pm\varphi \sim \Phi$, la transformation de tout ou certaines parties de $F^{\pm}$ sous forme d'une somme de résidus aux zéros de $H_{\nu}^{(2)'}(ka^{\pm}) + Q^{\pm} H_{\nu}^{(2)}(ka^{\pm})$ (correspondant aux ondes pouvant être guidées par la surface courbe) est conditionnée par l'uniformité de $\mathcal{L}^{\pm}(\pm\nu)$ au voisinage de $\nu = ka^{\pm}$. Pour le cas du dièdre étudié dans notre article , avec conditions aux limites mixtes , on a , quelle que soit la source d'illumination située dans la domaine $\pm\varphi < \Phi$, directement d'après [1] ,

$$\mho^{\pm}_{\alpha \pm \pi/2}(F^{\pm}(\alpha \pm \Phi)) + \mho^{\pm}_{-\alpha \pm \pi/2}(F^{\pm}(-\alpha \pm \Phi)) = 0 \quad .$$

Cela nous donne immédiatement la parité en α de l'expression de $\mathcal{L}^{\pm}(\pm ka^{\pm}\cos\alpha)$ donnée dans (A.10) par :

$$\mathcal{L}^{\pm}(\pm ka^{\pm}\cos\alpha) = -\mho^{\pm}_{\pm(\alpha + \pi/2)}(F^{\pm}(\pm(\alpha + \Phi)))/(2\sin\alpha) \quad ,$$

et donc l'absence de coupure de $\mathcal{L}^{\pm}(\pm\nu)$ pour $\nu = ka^{\pm}$. Dans ce cas , le terme $\mathcal{L}^{\pm}$, une fois introduit dans l'expression intégrale (A.1) , nous permet bien d'obtenir directement une transformation uniforme en somme de résidus pour tout ou partie de $F^{\pm}$ pour $\pm\varphi > \Phi$, (en particulier la partie faiblement oscillante correspondant au champ rayonné par l'arête qui peut s'exprimer dans cette zone en somme de rayons 'rampants' , comme le prévoit la physique du problème) , ce que ne pouvait nous donner le développement asymptotique initial de $F^{\pm}$ (en puissances inverses de $a^{\pm}$) défini pour $\pm\varphi < \Phi$ et singulier pour $\pm\varphi = \Phi$.

On constate ainsi que le problème délicat du passage entre développement asymptotique de $F^{\pm}$ en puissances entières et celui en puissances fractionnaires, rencontré pour les directions qui passent la frontière $\pm\varphi = \Phi$, se trouve alors résolu de façon globale et directe .

Montrons par ailleurs , maintenant , que les deux premiers ordres en courbures de $F^{\pm}$ ne nous donnent aucun pôle en $\alpha = 0$ pour $\mathcal{L}^{\pm}$. En effet , toute opération linéaire en α sur $F^{\pm}(\pm(\alpha+\Phi))$ se ramenant à cette même opération sur $f(\alpha', \pm(\alpha+\Phi))$ $(f(\alpha',\alpha'')$ est la fonction spectrale où l'on a précisé α'' , la direction de l'onde plane incidente) , on peut étudier les deux premiers ordres de l'expression suivante :

$$-\mho^{\pm}_{\pm(\alpha+\pi/2)}(f(\alpha', \pm(\alpha+\Phi)))/(2\sin\alpha)$$

$$= \frac{-2i}{ka^{\pm}\sqrt{\pi}}\left(\frac{1}{2\sqrt{\pi}sin\alpha}\left[(sin\alpha - sin\theta^{\pm}).(f_0(\alpha', \pm(\alpha+\Phi))\right.\right.$$

$$\left. f_1(\alpha', \pm(\alpha+\Phi))/k) - D^{\pm}_{\pm\alpha+\pi}(f_0(\alpha', \pm(\alpha+\Phi)))/ika^{\pm}\right]) + O(1/(ka^{\pm})^2),$$

où

$$D^{\pm}_{\alpha}(.) = \frac{1}{2}\left[\frac{\partial^2(.)}{\partial\alpha^2} - \frac{\partial}{\partial\alpha}(\mathrm{cotg}\alpha(.)) \pm \sin\theta^{\pm}\frac{\partial}{\partial\alpha}\left(\frac{1}{\sin\alpha}\frac{\partial}{\partial\alpha}(.)\right)\right] .$$

Or , d'après [1] , on a :

$$\mho^{\pm}_{\alpha\pm\pi/2}(f(\alpha',\alpha\pm\Phi)) + \mho^{\pm}_{-\alpha\pm\pi/2}(f(\alpha', -\alpha\pm\Phi)) = 0 .$$

Cette équation , au premier ordre (i.e. en $1/k^0$) , nous indique que $f_0(\alpha', \pm\Phi) = 0$, pour $\sin\theta \neq 0$. Au deuxième ordre (i.e. en $1/k^1$) , elle nous montre , en posant $f_0(\alpha', \pm(\alpha+\Phi)) = \sum_{p\geq 1} a_p\alpha^p$, que du fait de l'imparité de l'expression étudiée , les pôles de $D^{\pm}_{\pm\alpha+\pi}(f_0(\alpha', \pm(\alpha+\Phi)))$ et f_1 en $\alpha = 0$, d'ordre 2 , s'annulent l'un l'autre dans l'expression étudiée . L'expression est donc non-singulière en $\alpha = 0$.

REFERENCES :

[1] J.M.L. Bernard , *Progresses on the diffraction by a wedge : transient solution for line source illumination , single face contribution to scattered field , and new consequence of reciprocity on the spectral function* , Rev. Techn. Thom. , vol.25 , 4, p. 1209-1220 ,(1993).

I.2.B.3) Considérations sur l'expression (10) , p.324-325 .

On remarque que l'on peut sans difficulté , en intégrant par parties , réécrire l'expression intégrale donnée pour la formule (10) p.324-325 de notre article, suivant l'égalité :

$$\int_{-i\infty}^{i\infty} K^{\pm}(\alpha')\ t(\alpha',\alpha)\ d\alpha'$$
$$= \frac{\pm i}{a \pm \Psi(\varphi_o)} [\partial_{\varphi_o} \int_{-i\infty}^{i\infty} A.t(\alpha',\alpha)\ d\alpha' + \partial_{\varphi_o}\partial_\alpha \int_{-i\infty}^{i\infty} B.t(\alpha',\alpha)\ d\alpha'$$
$$+ \int_{-i\infty}^{i\infty} C.t(\alpha',\alpha)\ d\alpha' + \partial_\alpha \int_{-i\infty}^{i\infty} D.t(\alpha',\alpha)\ d\alpha'] ,$$

où A,B,C,D et t sont donnés par :

$A = s_{-}(\alpha',\varphi_o)\partial_{\alpha'}\mathcal{L}og((-\sin\alpha' \pm \sin\theta^{\pm})\Psi(-\alpha' \pm \Phi))$,

$B = s_{-}(\alpha',\varphi_o)$,

$$C = s_+(\alpha',\varphi_o)\partial_{\alpha'}\mathcal{L}og\left(\frac{\Psi(-\alpha'\pm\Phi)}{(sin\alpha'\pm sin\theta^{\pm})}\right)\partial_{\alpha'}\mathcal{L}og((-sin\alpha'\pm sin\theta^{\pm})\Psi(-\alpha'\pm\Phi)) ,$$

$$D = s_+(\alpha',\varphi_o)\partial_{\alpha'}\mathcal{L}og\left(\frac{\Psi(-\alpha'\pm\Phi)}{(sin\alpha'\pm sin\theta^{\pm})}\right) ,$$

$$t(\alpha',\alpha) = tg(\tfrac{\pi}{4\Phi}(\alpha\pm\Phi-\alpha')) ,$$

avec $s_\epsilon(\alpha',\varphi_o) = \frac{\pi}{4\Phi\ \sin\alpha'}\,(tg(\frac{\pi}{4\Phi}(\alpha'\pm\Phi+\varphi_o)) + \epsilon\ tg(\frac{\pi}{4\Phi}(-\alpha'\pm\Phi+\varphi_o)))$.

On notera que l'expression du type précédent , définie pour $|\mathrm{Re}\alpha| < \Phi$, peut être prolongée analytiquement pour $|\mathrm{Re}\alpha| \geq \Phi$ (avec $G.t < e^{-a|Im\alpha'|}$, $|\mathrm{Im}\alpha| \to \infty$, $a > 0$, pour $G \equiv A,B,C,D$) par translation du contour d'intégration de chacun des termes , en prenant soin , pendant l'opération , de prendre en compte les singularités captées (A,B,C,D et t n'ont que des pôles simples en α') . Remarquons que le prolongement est particulièrement nécessaire si l'on veut obtenir une expression de la fonction de rayonnement $f(\pi+\varphi) - f(-\pi+\varphi)$.

I.2.B.4) Equation fonctionnelle asymptotique . Développement de la démonstration générale permettant en particulier le passage des équations (4) à (5) de notre article .

Nous allons préciser ici certains détails concernant le passage , pour chacune des faces du dièdre , d'une condition aux limites en champ asymptotique en puissance de la courbure à une équation fonctionnelle asymptotique sur la fonction spectrale associée au champ .

Ecrivons une condition aux limites asymptotique en puissances entières de la courbure $1/a^{\pm}$. On a :

(1) $$\sigma_a(u) = 0 ,$$

où

(2) $$\sigma_a(u) = [\mp\frac{\partial}{\rho\partial\varphi} - ik sin\theta^{\pm} - \sum_{q>0}\frac{1}{(a^{\pm})^q}Q_q]\ u\,|_{\varphi=\pm\Phi} ,$$

avec , par homogénéité :

$$(3) \qquad Q_q = \sum_{\substack{n,r,p \geq 0 \\ avec\ n-r=q-1}} (b^q_{nrp} + d^q_{nrp}\ k sin\theta^{\pm} \rho)\ \rho^n \frac{\partial^r}{\partial\rho^r} \frac{\partial^p}{\partial\varphi^p} \ ,$$

ceci quel que soit ρ fini , y compris l'origine (c'est le cas pour notre problème mais signalons qu'on devrait toujours pouvoir s'y ramener en prenant $\rho^p \sigma_a$ au lieu de σ_a) .
Montrons que l'on peut écrire :

$$(4) \qquad (sin\alpha \pm sin\theta^{\pm} \mp i \sum_{q>0} \frac{1}{(ka^{\pm})^q} D^q_{\alpha}(\ .\ ))f(\alpha \pm \Phi)$$

$$-(-sin\alpha \pm sin\theta^{\pm} \mp i \sum_{q>0} \frac{1}{(ka^{\pm})^q} D^q_{-\alpha}(\ .\ ))f(-\alpha \pm \Phi) = 0 \ ,$$

équation fonctionnelle asymptotique en puissances de la courbure . L'équation (4) déduite de la condition aux limites asymptotique a un sens , si le théorème d'inversion de Maliuzhinets , appliqué terme à terme à la condition en champ u , nous donne bien une équation fonctionnelle asymptotique . Ceci implique que la transformée de Laplace de la condition asymptotique

$$(5) \qquad \int_0^{\infty} \sigma_a(\mathrm{u})\ e^{-(c'+ic'')\rho} d\rho \ ,$$

pour $c' > b$, b positif , définie comme la somme des transformées de chacun des termes , est un développement asymptotique , ce que nous allons démontrer. On négligera dans la suite l'indice supérieure de face $\pm$. (On note par ailleurs que le second membre de l'équation fonctionnelle (4) est nul car on aura considéré que f , définie à une constante additive près, est telle que $f(i\infty) = -f(-i\infty)$, et que $Q_q(\mathrm{u})$ reste fini à l'origine (si l'on écrivait $\rho^p \sigma_a$ au lieu de σ_a , ce ne serait plus le cas et on aurait $\sum_{n=1}^{p} c_n \sin\alpha(\cos\alpha)^{n-1}$ en second membre).)

Soit donc σ la somme des $n+1$ premiers termes de σ_a. On suppose , ce qui est loisible , qu'il existe b indépendant de n et d tels que ,

$$(6) \qquad |\, a^n \sigma(u) \,| < e^{b\rho} \rho^d \ , \ d > 0 \ , \text{ quand } \rho > X \text{ et } a > A \ .$$

Montrons que , soit ϵ donné , on peut choisir A tel que

$$(7) \qquad |\, a^n \int_0^\infty \sigma(u)\, e^{-(c'+ic'')\rho} d\rho \,| < 2\epsilon \ ,$$

pour c', c'' réels , $c' > b$.

L'inégalité (6) entraîne qu'il existe une constante K indépendante de r et de a telle que , si $r > X$ et $a > A$,

$$(8) \qquad |\, a^n \int_r^\infty \sigma(u)\, e^{-(c'+ic'')\rho} d\rho \,| \ < \ K\, e^{-cr}\, r^d = T \ ,$$

avec $c > 0$. Par ailleurs , utilisant la définition de σ_a ainsi que (6) , on peut supposer que , pour $0 < \rho < r$, A est tel que

$$(9) \qquad |a^n \sigma(u) e^{-(c'+ic'')\rho}| < K'/a \leq \epsilon/r \ ,$$

où K' est une constante indépendante de r et de a . Ceci posé , on montre que r peut aussi être choisi tel que $T < \epsilon$. Cela équivaut , en effet , à écrire :

$$(10) \qquad \epsilon a \ \geq K'r > \ K'/c \ \times \operatorname{Log}(Kr^d/\epsilon) \ ,$$

ce qui est toujours possible en prenant $\epsilon a = K'r$ et en choisissant A convenablement .

On aura alors :

$$(11) \quad |\, a^n \int_0^\infty \sigma(u)\, e^{-(c'+ic'')\rho} d\rho \,| < |a^n \int_0^r \sigma(u)\, e^{-(c'+ic'')\rho} d\rho| \ + \epsilon < 2\epsilon \ ,$$

ce qui achève la démonstration .

pp. 421-423

Diffraction par un dièdre avec variation d'impédance dépendant de la distance à l'arête

Jean-Michel L. BERNARD *
Giuseppe PELOSI **

Résumé

On détermine par une méthode perturbative la solution pour la diffraction par un dièdre dont l'impédance de chacune des faces varie en fonction de la distance à l'arête. Cette variation de l'impédance ou de son inverse, l'admittance, est considérée comme une perturbation de faible amplitude mais de forme quelconque.

Mots clés : Diffraction onde, Dièdre, Impédance surface, Méthode perturbation.

DIFFRACTION BY A WEDGE WITH VARIATION OF IMPEDANCE DEPENDING OF THE DISTANCE FROM THE EDGE

Abstract

The solution for the diffraction by a wedge with its faces impedances depending of the distance from the edge is determined. The impedance or admittance variations are considered here as weak amplitude perturbations of arbitrary form.

Key words : Wave diffraction, Dihedron, Surface impedance, Perturbation method.

Sommaire

I. INTRODUCTION

Une variation des caractéristiques électriques de part et d'autre d'une discontinuité du type dièdre peut être rencontrée dans un problème pratique et on désire calculer son effet sur la diffraction d'une onde plane. Jusqu'à maintenant, ce cas a été considéré pour des variations proportionnelles ou inversement proportionnelles à la distance à l'arête, en particulier par Felsen [1]. Cette étude s'étend au cas plus général d'une variation de forme quelconque dans la limite de faible amplitude. A cette fin, on considère une onde plane incidente sur un dièdre à impédance dont les conditions aux limites sur chacune des faces sont perturbées, relativement au cas où les impédances des faces sont constantes. La méthode présentée permet d'obtenir une expression explicite du terme de perturbation.

II. FORMULATION DU PROBLÈME ET SOLUTION

On considère un dièdre d'arête confondue avec l'axe z, et de secteur angulaire externe défini par $|\varphi| < n\pi/2$, dans les coordonnées polaires (ρ, φ). Ses caractéristiques électriques ne varient que dans le plan polaire. Ce dièdre est illuminé par une onde plane, de direction perpendiculaire à l'arête, de sorte qu'on se ramène à deux problèmes scalaires indépendants pour les composantes suivant z du champ électrique E_z (cas TM ou E, $H_z = 0$) et magnétique H_z (cas TE ou H, $E_z = 0$), noté indifféremment u (Fig. 1).

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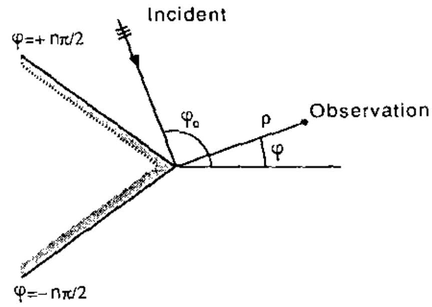

FIG. 1. — Géométrie du dièdre.
Geometry of the wedge.

Le champ incident s'écrit :

(1) $$u^i(\rho, \varphi) = e^{jk\rho\cos(\varphi - \varphi_0)},$$

où k est le nombre d'onde du vide, et la dépendance en temps $\exp(j\omega t)$ est prise en compte implicitement. On impose que les faces $\varphi = \pm n\pi/2$ soient caractérisées par une impédance (resp. admittance) relative $Z^{\pm}(\rho)/Z_0$ (resp. $Y^{\pm}(\rho)/Y_0$), fonction de la distance à l'arête telle que :

(2) $$Z^{\pm}(\rho)/Z_0 \text{ (resp. } Y^{\pm}(\rho)/Y_0) = \sin\theta^{\pm} + \delta S^{\pm}(\rho),$$

où $\sin\theta^{\pm}$ est complexe, δ, un paramètre d'amplitude, $Z_0 = 1/Y_0$, l'impédance du vide, et

(3) $$S^{\pm}(\rho) = \int_{\mathcal{I}^{\pm}} s^{\pm}(a)\, e^{-ka\rho}\, da \equiv \mathbf{W}^{\pm} \circ (e^{-ak\rho}),$$

une superposition d'exponentielles. $\mathcal{I}^{\pm}$ est un contour dans le plan complexe avec $\mathrm{Re}(ka) \geq 0, \mathbf{W}^{\pm}$ représentant un opérateur intégral sur a. Dans le cas d'une variation exponentielle de $S(\rho), \mathbf{W}^{\pm}$ est un scalaire et $\circ$ une simple multiplication.

On traite simultanément des perturbations de l'impédance et de l'admittance grâce au terme « respectivement ». On préfèrera l'un ou l'autre choix de façon à minimiser le terme $\delta S^{\pm}(\rho)$ de (2).

La condition aux limites sera du type, noté H_o (resp. E_o) pour $E_z = 0$ (resp. $H_z = 0$), défini par :

(4) $$\left[\frac{1}{\rho}\frac{\partial}{\partial\varphi} \pm jk\left(\sin\theta^{\pm} + \delta S^{\pm}(\rho)\right)\right] u(\rho, \varphi) = 0, \quad \varphi = \pm\frac{n\pi}{2},$$

ou du type, noté E_1 (resp. H_1) pour $H_z = 0$ (resp. $E_z = 0$) :

(5) $$\left[\left(\sin\theta^{\pm} + \delta S^{\pm}(\rho)\right)\frac{1}{\rho}\frac{\partial}{\partial\varphi} \pm jk\right] u(\rho, \varphi) = 0, \quad \varphi = \pm\frac{n\pi}{2}.$$

Dans la suite, on considère que (4) ou (5) s'applique aux deux faces, mais la méthode pourrait s'appliquer au cas mixte.

On choisit maintenant d'écrire le champ total $u(\rho, \varphi)$ sous la forme d'une intégrale de Sommerfeld-Maliuzhinets :

(6) $$u(\rho, \varphi) = \frac{1}{2\pi j}\int_{\gamma} f(\alpha + \varphi)\, e^{jk\rho\cos\alpha}\, d\alpha,$$

où le contour d'intégration γ est constitué de deux branches : l'une, γ^+, allant de $(+j\infty + \pi + \varepsilon)$ à $(+j\infty - \varepsilon)$, $\varepsilon > 0$, au-dessus de toutes les singularités de l'intégrant, et l'autre, γ^-, déduite par symétrie par rapport à $\alpha = 0$ (Fig. 2). Par ailleurs, la fonction spectrale f satisfait à $|f(\alpha) - f(\pm j\infty)| < \exp(-c|\mathrm{Im}(\alpha)|)$, $c > 0$, quand $\mathrm{Im}(\alpha) \to \pm\infty$.

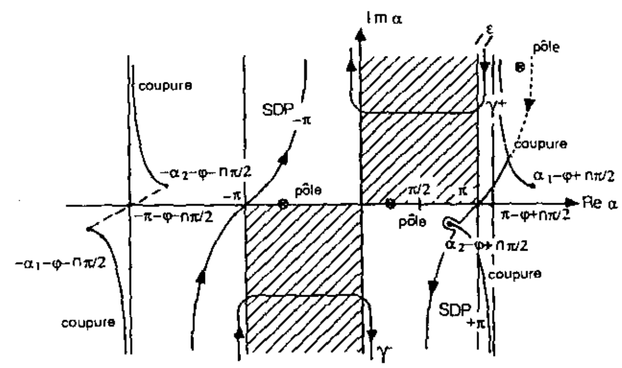

FIG. 2. — Détails des contours $\gamma^{\pm}$, SDP$\pm\pi$. et des pôles et coupures de $f(\alpha + \varphi)$.
Details of paths $\gamma^{\pm}$, SDP$\pm\pi$. and of poles and branch cuts of $f(\alpha + \varphi)$.

On suit maintenant une démarche comparable à celle de Maliuzhinets [2, 3], ce qui consiste à prendre la transformée de Laplace suivant ρ de (4)-(5) et à utiliser l'analycité de f. On obtient alors pour (4) (cas $H_o(E_o)$),

(7) $$(\sin\alpha \pm \sin\theta^{\pm})f(\alpha \pm n\pi/2) - (-\sin\alpha \pm \sin\theta^{\pm})f(-\alpha \pm n\pi/2) = \mp\delta\sin\alpha\mathbf{W}^{\pm}\circ \left[\frac{f(\alpha' \pm n\pi/2) - f(-\alpha' \pm n\pi/2)}{\sin\alpha'}\right]\Bigg|_{\cos\alpha' = \cos\alpha - ja},$$

et pour (5) (cas $E_1(H_1)$),

(8) $$(\sin\theta^{\pm}\sin\alpha \pm 1)f(\alpha \pm n\pi/2) - (-\sin\theta^{\pm}\sin\alpha \pm 1)f(-\alpha \pm n\pi/2) = -\delta\sin\alpha\mathbf{W}^{\pm}\circ\big((f(\alpha' \pm n\pi/2) + f(-\alpha' \pm n\pi/2)\big)_{\cos\alpha' = \cos\alpha - ja}.$$

De façon à utiliser une méthode perturbative, le paramètre δ est maintenant supposé petit, ce qui suggère de poser :

(9) $$f \simeq f_0 + \delta f_1.$$

Dans cette expression, f_0 est la solution connue de (7) ou (8) pour $\delta = 0$, c'est-à-dire celle correspondant au problème non perturbé avec impédances constantes des faces du dièdre [3]. Pour f_1, l'application des équations inhomogènes (7) ou (8) nous fournit, pour (7) (cas $H_o(E_o)$) :

(10) $$(\sin\alpha \pm \sin\theta^{\pm})f_1(\alpha \pm n\pi/2) - (-\sin\alpha \pm \sin\theta^{\pm})f_1(-\alpha \pm n\pi/2) = \mathbf{W}^{\pm}\circ L^{\pm}(\alpha),$$

où $L^{\pm}$ dépendant de f_0 s'écrit :

(11a) $$L^{\pm}(\alpha) = \mp\Big(\frac{\sin\alpha}{\sin\alpha'}[f_0(\alpha' \pm n\pi/2) - f_0(-\alpha' \pm n\pi/2)]\Big)\Big|_{\cos\alpha'=\cos\alpha - \mathrm{j}a},$$

ou

(11b) $$L^{\pm}(\alpha) = \Big(\frac{\sin\alpha}{\sin\theta^{\pm}}[f_0(\alpha' \pm n\pi/2) + f_0(-\alpha' \pm n\pi/2)]\Big)\Big|_{\cos\alpha'=\cos\alpha - \mathrm{j}a}.$$

On note qu'on choisira plutôt (11a) pour $\sin\theta^{\pm}$ tendant vers 0, (11b) quand $\sin\theta^{\pm}$ tend vers ∞. Pour (8) (cas $E_1(H_1)$), $L^{\pm}$ défini suivant (11b), ($\neq$ (11a) pour (8)),

(12) $$(\sin\alpha \pm 1/\sin\theta^{\pm})f_1(\alpha \pm n\pi/2) - (-\sin\alpha \pm 1/\sin\theta^{\pm})f_1(-\alpha \pm n\pi/2) = -\mathbf{W}^{\pm}\mathrm{o}L^{\pm}(\alpha).$$

Le second membre de (10) ou (12) connu, on peut appliquer la procédure déjà utilisée dans [4, 5, 6]. On pose :

(13) $$f_1(\alpha) = \Psi(\alpha)\chi(\alpha),$$

où pour (10) et (12), Ψ est la solution de l'équation sans second membre, régulière pour $|\mathrm{Re}(\alpha)| \leq n\pi/2$, calculée d'après [3]. La fonction $\chi(\alpha)$ satisfait, pour (10) ou (12) :

(14) $$\chi(\alpha \pm n\pi/2) - \chi(-\alpha \pm n\pi/2) = K^{\pm}(\alpha),$$

avec, pour (10) (cas $H_o(E_o)$) :

(15) $$K^{\pm}(\alpha) = \frac{\mathbf{W}^{\pm}\mathrm{o}L^{\pm}(\alpha)}{(\sin\alpha \pm \sin\theta^{\pm})\Psi(\alpha \pm n\pi/2)},$$

et pour (12) (cas $E_1(H_1)$) :

(16) $$K^{\pm}(\alpha) = \frac{-\mathbf{W}^{\pm}\mathrm{o}L^{\pm}(\alpha)}{(\sin\alpha \pm 1/\sin\theta^{\pm})\Psi(\alpha \pm n\pi/2)}.$$

On connaît la solution de (14)-(15) et (14)-(16) par [4] et [5], régulière pour $|\mathrm{Re}(\alpha)| \leq n\pi/2$, suivant :

(17) $$\chi(\alpha) = -\frac{\mathrm{j}}{4n\pi}\int_{-\mathrm{j}\infty}^{+\mathrm{j}\infty}\left[K^{+}(\alpha')\tan\left(\frac{\alpha + n\pi/2 - \alpha'}{2n}\right) - K^{-}(\alpha')\tan\left(\frac{\alpha - n\pi/2 - \alpha'}{2n}\right)\right]\mathrm{d}\alpha'.$$

On note que $f_1(\alpha)$ possède d'après (10) et (12) les singularités dues, d'une part, aux pôles de f_0, et, d'autre part, celles dues à certaines coupures de la fonction $\cos^{-1}(\cos\alpha - \mathrm{j}a)$. En particulier pour $|\mathrm{Re}(\alpha)| < n\pi$, $f_1(\alpha \pm n\pi/2)$ est régulier pour $\pm\mathrm{Re}(\alpha) < 0$ et ses coupures, pour $\pm\mathrm{Re}(\alpha) > 0, n \geq 1$, sont, en négligeant l'opération de $\mathbf{W}^{\pm}$, celles de $L^{\pm}$, d'équation :

(18) $$\mathrm{Im}(\cos\alpha - \mathrm{j}a) = 0, \quad \mathrm{Re}(\cos\alpha - \mathrm{j}a) < -1,$$

d'origines $\alpha = \pm\alpha_i$, $\cos\alpha_i - \mathrm{j}a = -1$, $i = 1, 2$. On note que $L^{\pm}$ ne possède pas de coupure pour $a = 0$ (impédance constante), et $1/n$ entier, $\sin\theta^{\pm} = 0$ (en particulier cas non perturbé du plan avec impédance ou admittance nulle).

Sans nuire à la généralité, on peut discuter de la solution en champ u obtenue pour le cas $S^{\pm}(\rho) = \mathrm{e}^{-ka\rho}$, i.e. $W \equiv$ constante. L'expression déduite de (6) se simplifie en déformant γ vers les deux chemins de descente rapide $\mathrm{SDP}_{\pm\pi}$, $\mathrm{Im}(\mathrm{j}k\cos\alpha) = -k$, $\mathrm{Re}(\mathrm{j}k\cos\alpha) \leq 0$, centrés sur $+\pi$ et $-\pi$. La solution comprend alors les termes de résidus aux pôles de f_0 de champ incident, de $(f_0 + \delta f_1)$ des champs réfléchis et ondes de surface (ou à pertes), la contribution des chemins $\mathrm{SDP}_{\pm\pi}$ appelée champ diffracté d'arête mais aussi, dues à f_1 quand $L^{\pm}$ a des coupures, les intégrales suivant les branches définies en (18) qui sont interceptées par la déformation de γ ainsi que ceci est représenté en figure 2. Ces derniers éléments de champ peuvent s'interpréter comme un continuum d'ondes à pertes, excitées par l'arête et guidées par la surface.

On a ainsi déterminé f_1, terme de la solution prenant en compte l'angle du dièdre, l'impédance (admittance) et ses variations.

III. CONCLUSION

On a considéré la diffraction d'une onde plane par un dièdre d'angle quelconque dont l'impédance ou l'admittance de surface a des variations qui sont fonctions de la distance à l'arête, de faible amplitude mais de forme quelconque, ceci relativement au cas non perturbé d'impédance ou admittance constante. Le terme de perturbation dû à cette variation a été calculé explicitement.

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BIOGRAPHIE

Jean-Michel L. BERNARD est né le 8 février 1958; Ingénieur de l'Ecole supérieure d'électricité, il travaille actuellement au Centre Thomson d'applications radars (LCTAR) au service Electromagnétisme et antennes.
Giuseppe PELOSI est né à Pise, le 25 décembre 1952. Il est diplômé de l'Université de Florence et professeur d'électromagnétisme. Ses recherches concernent l'étude de la diffraction, par des techniques asymptotiques ou numériques.

I.3.A) Errata et commentaires sur les démonstrations et formules

∗ p.422 : juste avant (10) , remplacer ".. nous fournit , pour (7)..."
par " .. nous fournit deux types d'équations . Pour (7)..."

CHAPITRE II) Diffraction par un dièdre avec des conditions aux limites complexes , illuminé par une onde plane d'incidence quelconque en régime stationnaire : problème vectoriel en régime harmonique

II.1) J.M.L. Bernard ,*'On the diffraction of an electromagnetic skew incident wave by a non perfectly conducting wedge'*,Ann. Telecom. , vol.45, 1-2 , pp. 30-39 , 1990 (Errata n°9-10,p.577) .

II.1.A) Errata et commentaires sur les démonstrations et formules

II.1.B) Compléments mathématiques

II.2) J.M.L. Bernard ,*'Exact analytical solution for the diffraction at skew incidence by a class of wedge with absorbing material'*,Rev. Techn. Thomson-Csf , Vol. 20-21 , pp. 523-527 , 1989.

II.2.A) Errata et commentaires sur les démonstrations et formules

II.3) J.M.L. Bernard ,*'Properties of the solution and reciprocity theorem for a class of wedge problem at skew incidence '*, conf. IEEE-AP de Dallas 1990 .

pp. 30–39

On the diffraction of an electromagnetic skew incident wave by a non perfectly conducting wedge

Jean-Michel BERNARD *

Abstract

We study the diffraction by a wedge of an electromagnetic plane wave with skew incidence on the edge, when boundary conditions give us two equations by face with combined electric and magnetic fields. The problem is reduced principally to a non linear scalar functional equation with one unknown. As an example of application, the solution for a wedge with arbitrary angle and relative impedance unity (the most usual model for absorbing material) is given.

Key words : Wave diffraction, Electromagnetic wave, Oblique incidence, Wedge, Plane wave, Non ideal conductor.

SUR LA DIFFRACTION D'UNE ONDE PLANE EN INCIDENCE OBLIQUE PAR UN DIÈDRE NON PARFAITEMENT CONDUCTEUR

Résumé

L'auteur étudie la diffraction par un dièdre d'une onde plane avec incidence oblique par rapport à l'arête quand les conditions aux limites donnent deux équations par face couplant les composantes suivant l'arête du champ électrique et du champ magnétique. Le problème est ici principalement réduit à une équation fonctionnelle non linéaire à une inconnue. Comme exemple particulier d'application, la solution pour un dièdre d'impédance relative unité (le modèle le plus courant d'absorbant) est développée.

Mots clés : Diffraction onde, Onde électromagnétique, Incidence oblique, Coin, Onde plane, Conducteur non idéal.

Contents

I. INTRODUCTION

We study here the diffraction by a non perfectly conducting wedge, when it is illuminated by an electromagnetic plane wave of skew incidence on the edge. As in [2] where normal incidence was studied, boundary conditions with high order derivatives are considered. Now in the case of skew incidence, boundary conditions lead to two equations (instead of one for normal incidence [2]) involving both electric and magnetic field for each face of the wedge.

In case of a half plane or full plane, one can use the Wiener-Hopf method and its developments [3], as the method used by Rojas [4], or the generalised Wiener-Hopf methods given by Hurd [5], Daniele [6]. In another way, with some modifications from Maliuzhinets method [1], solutions have been obtained by Bucci and Franceschetti [7] for the half plane, Vaccaro [8], Senior [9] for the right angled wedge.

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In this paper, one tries to define a generalisation of the Maliuzhinets method, which could be applied to the case of coupled equations with no restriction about the wedge angle or the order of the boundary conditions. This method allows in particular to reduce the problem of this coupling to a non linear scalar functional equation of order two with one unknown. As an example, the solution of the diffraction by a wedge of arbitrary angle, with relative impedance unity is particularly developed. Numerical results are given. Moreover a section is devoted to practical mathematical equalities which are satisfied by the derivatives of the field solution at the edge. The time dependence $\exp(i\omega t)$ is used in this paper.

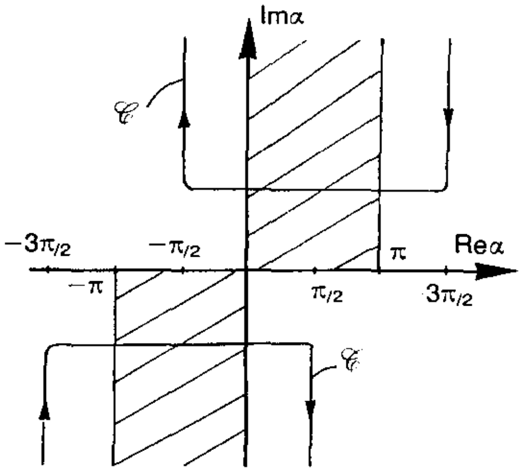

FIG. 2. — Location of path C.

Description du chemin C.

II. HOW TO MODIFY THE PROBLEM

The wedge considered here is of external angle 2Φ, with the edge parallel to the z-axis of origin O. The incidence of the incoming plane wave is defined by the angles β and φ_0 given in Figure 1 ; it is characterized by the z-components of electric and magnetic fields respectively,

$$(1)\quad \overline{E}_i\hat{z} = D_1 \exp[ik_0(\rho\sin\beta\cos(\varphi-\varphi_0) - z\cos\beta)],$$
$$\overline{H}_i\hat{z} = \frac{D_2}{Z_0}\exp[ik_0(\rho\sin\beta\cos(\varphi-\varphi_0) - z\cos\beta)],$$

with k_0 being, the free space wave number and Z_0 the free space impedance; (ρ,φ,z) the coordinates of the observation point P are indicated on Figure 1.

The total field depends only on the knowledge of the z-components of electric and magnetic fields respectively E_z and H_z (appendix 1). As in [2], one uses the Sommerfeld integral and writes,

$$(2)\quad E_z(\rho,\varphi) = \frac{e^{-ik_0 z\cos\beta}}{2\pi i}\int_C f_1(\alpha+\varphi)e^{ik_0\rho\sin\beta\cos\alpha}\,d\alpha,$$
$$H_z(\rho,\varphi) = \frac{e^{-ik_0 z\cos\beta}}{2\pi i Z_0}\int_C f_2(\alpha+\varphi)e^{ik_0\rho\sin\beta\cos\alpha}\,d\alpha,$$

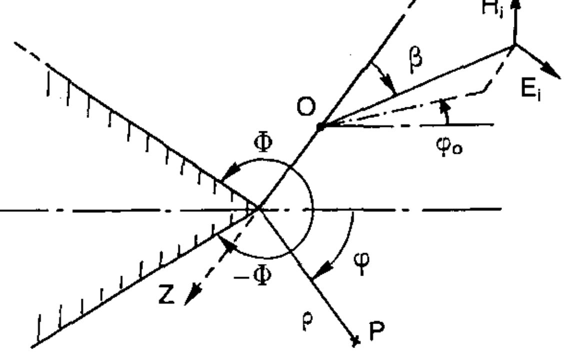

FIG. 1. — Geometry of wedge.

Géométrie du dièdre.

where the two branches of the contour C given in Figure 2 are above all the singularities of the integrand [1].

The functions $f_j (j = 1,2)$ have to satisfy to regularity conditions defined by Maliuzhinets [1],

(a) $f_j(\alpha) - D_j/(\alpha-\varphi_0)$ is a regular function in the strip $|\mathrm{Re}\alpha| \leq \Phi$,

(b) $|f_j(\alpha) - f_j(\pm i\infty)| < \exp(-a|\mathrm{Im}\alpha|), a > 0$, when $\mathrm{Im}\alpha \to \pm\infty, |\mathrm{Re}\alpha| \leq \Phi$.

Then one can write the boundary conditions on the wedge faces. Derived with the same principles as in [2], they are obtained in appendix 1 and are written as,

$$(3)\quad \int_C \left[A^{\pm}_{\alpha}\left|\begin{matrix} f_1(\alpha\pm\Phi)\\ f_2(\alpha\pm\Phi)\end{matrix}\right. - A^{\pm}_{-\alpha}\left|\begin{matrix} f_1(-\alpha\pm\Phi)\\ f_2(-\alpha\pm\Phi)\end{matrix}\right.\right] e^{ik_0\rho\sin\beta\cos\alpha}\,d\alpha = 0,$$

where $A^{\pm}_{\alpha}$ is a 2×2 matrix function of α composed with degree q polynomials of $\sin\alpha$ and $\cos\alpha$.

To go further, one needs the asymptotic behaviour of the term within the brackets, which is from the condition (b) on f_j,

$$O(A^{\pm}_{\alpha}) = O(e^{q|\mathrm{Im}\alpha|}),$$

as $|\mathrm{Im}\alpha| \to \infty$.

One then applies a theorem due to Maliuzhinets [1] which allows us to write that f_1, f_2 satisfy,

$$(4)\quad A^{\pm}_{\alpha}\left|\begin{matrix} f_1(\alpha\pm\Phi)\\ f_2(\alpha\pm\Phi)\end{matrix}\right. - A^{\pm}_{-\alpha}\left|\begin{matrix} f_1(-\alpha\pm\Phi)\\ f_2(-\alpha\pm\Phi)\end{matrix}\right.$$
$$= \sum_{n=1}^{q}\left|\begin{matrix} c^{\pm}_{1n}\\ c^{\pm}_{2n}\end{matrix}\right. \sin\alpha\cos^{n-1}\alpha \quad \left(\text{that we note}\left|\begin{matrix} c^{\pm}_{1}(\alpha)\\ c^{\pm}_{2}(\alpha)\end{matrix}\right.\right),$$

with constant numbers $c^{\pm}_{1n}, c^{\pm}_{2n}$.

As in [2], $c^{\pm}_{1n}, c^{\pm}_{2n}$ lead to degrees of freedom necessary for the application of supplementary boundary conditions :

(c) the field is finite at infinity :

$f_j(\alpha) - D_{j/(\alpha-\varphi_0)}$ has no poles α_p as,

$$-\pi-\Phi < \mathrm{Re}\alpha_p < \Phi, \quad \mathrm{Im}\alpha_p < 0,$$
$$-\Phi < \mathrm{Re}\alpha_p < \pi+\Phi, \quad \mathrm{Im}\alpha_p > 0;$$

(d) edge conditions given by internal conditions at the junction of the materials of each face.

Now, the heart of the matter is to modify (4) in order to be solved. It is decided to search for an even or odd linear operator function of $\alpha, B_\alpha^\pm$, such that when applied to (4) one obtains,

$$(5)\quad C_{\alpha\pm\Phi}\left|\begin{matrix} f_1(\alpha\pm\Phi) \\ f_2(\alpha\pm\Phi)\end{matrix}\right. - \varepsilon^\pm C_{-\alpha\pm\Phi}\left|\begin{matrix} f_1(-\alpha\pm\Phi) \\ f_2(-\alpha\pm\Phi)\end{matrix}\right. = B_\alpha^\pm\left|\begin{matrix} C_1^\pm(\alpha) \\ C_2^\pm(\alpha)\end{matrix}\right.,$$

with,

$$C_{\alpha\pm\Phi} = (B_\alpha^\pm)(A_\alpha^\pm),$$
$$C_\alpha = \begin{pmatrix} a(\alpha) & c(\alpha) \\ d(\alpha) & b(\alpha)\end{pmatrix}\frac{1}{(ab-cd)},$$

a, b, c, d being functions of α,

$\varepsilon^\pm = 1$ if $B_\alpha^\pm$ is even, $\varepsilon^\pm = -1$ if $B_\alpha^\pm$ is odd, which leads to two independant equations of the type,

$$t(\alpha\pm\Phi) - \varepsilon^\pm t(-\alpha\pm\Phi) = \text{ known function},$$

that can be solved from [2] (see section IV). The expression of C_α and the parity property of $B_\alpha^\pm$ then imply the following necessary and sufficient condition on C_α,

$$(6)\quad (A_\alpha^\pm)(C_{\alpha\pm\Phi})^{-1} - \varepsilon^\pm(A_{-\alpha}^\pm)(C_{-\alpha\pm\Phi})^{-1} = 0,$$

that one has now to reduce.

III. REDUCTION OF C_α EQUATION

The equation (6) is homogeneous but difficult to reduce specially because of the non commutativity of the matrix product. By multiplication with $(A_\alpha^\pm)^{-1}$ and $C_{\alpha\pm\Phi}$, the equation (6) is written as,

$$(7)\quad (C_{\alpha\pm\Phi})[(A_\alpha^\pm)^{-1}(A_{-\alpha}^\pm)](C_{-\alpha\pm\Phi})^{-1} = \begin{bmatrix}\varepsilon^\pm & 0 \\ 0 & \varepsilon^\pm\end{bmatrix},$$

where the expression in brackets can be written as,

$$(A_\alpha^\pm)^{-1}(A_{-\alpha}^\pm) = \begin{pmatrix} l^\pm(\alpha) & n^\pm(\alpha) \\ p^\pm(\alpha) & l^\pm(-\alpha)\end{pmatrix}\frac{1}{\det A_\alpha^\pm},$$

$n^\pm(\alpha), p^\pm(\alpha)$ being odd functions. For the sake of simplicity, from now on, superscript indices of face $\pm$ are omitted, and :

$$(8)\quad a_+ = a(\alpha\pm\Phi), a_- = a(-\alpha\pm\Phi),$$

and so on for b, c, d;

$$(9)\quad \begin{bmatrix} l_+ & n \\ p & l_-\end{bmatrix} = \begin{bmatrix} l(\alpha) & n(\alpha) \\ p(\alpha) & l(-\alpha)\end{bmatrix},$$
$$A_\pm = A_{\pm\alpha}.$$

One supposes subsequently the non trivial case $n, p \neq 0$ and so $c, d \neq 0$.

Equation (7) can be expanded in the following form,

$$(10)\quad \frac{1}{\det(A_+)}\begin{bmatrix} -c_+b_-g\left(\frac{a_+}{c_+},\frac{d_-}{b_-}\right) & c_+c_-g\left(\frac{a_+}{c_+},\frac{a_-}{c_-}\right) \\ b_+b_-g\left(\frac{d_+}{b_+},\frac{d_-}{b_-}\right) & c_-b_+g\left(\frac{d_+}{b_+},\frac{a_-}{c_-}\right)\end{bmatrix} = \begin{bmatrix}\varepsilon & 0 \\ 0 & \varepsilon\end{bmatrix}(a_+b_+ - c_+d_+),$$

with,

$$g(r,s) = nrs - (l_+r - l_-s) - p.$$

For the non diagonal terms of (10) one obtains,

$$(11a)\quad g\left(\frac{a_+}{c_+},\frac{a_-}{c_-}\right) = 0,$$

$$(11b)\quad g\left(\frac{d_+}{b_+},\frac{d_-}{b_-}\right) = 0,$$

with (since $\det(C_\alpha)^{-1} \neq 0$),

$$(11c)\quad \frac{a}{c} \neq \frac{d}{b}.$$

For the diagonal term of (10) one has,

$$(12a)\quad \frac{1}{\det(A_+)}g\left(\frac{a_+}{c_+},\frac{d_-}{b_-}\right) = -\frac{\varepsilon}{c_+b_-}(a_+b_+ - c_+d_+),$$

$$(12b)\quad \frac{1}{\det(A_+)}g\left(\frac{d_+}{b_+},\frac{a_-}{c_-}\right) = \frac{\varepsilon}{c_-b_+}(a_+b_+ - c_+d_+),$$

which is equivalent (see appendix 2) to,

$$(13a)\quad \left(\frac{b_+}{c_+}\right)\left(\frac{b_-}{c_-}\right)^{-1} = \frac{-g(a_+/c_+, d_-/b_-)}{g(d_+/b_+, a_-/c_-)},$$

$$(13b)\quad \frac{(b_+a_+)}{(b_-a_-)} = \frac{\det(A_-)}{\det(A_+)}\frac{1-(c_-/a_-)(d_-/b_-)}{1-(c_+/a_+)(d_+/b_+)}.$$

The group of the equations (11a), (11b), (11c) and (13a), (13b), identified as equivalent to (7), can now be reduced.

Then, consider first a/c and d/b given by (11a), (11b) and (11c) as known quantities. One takes the logarithmic derivatives of (b/c) and (ba) as unknown functions, and then (13a), (13b) lead to equations of the following type,

$$(14)\quad s(\alpha\pm\Phi) + (\text{or}-)s(-\alpha\pm\Phi) = \mathrm{H}^\pm(\alpha),$$

where $\mathrm{H}^\pm(\alpha)$ is known.

The solution of (14) is given by an application of the theory of Fourier integrals, initiated by Maliuzhinets in [11], [1]. Particularly when $s(\alpha)$ is regular in the part $|\mathrm{Re}\alpha| \le \Phi$, and $\mathrm{H}(\alpha) = \mathrm{O}(\mathrm{e}^{-a|\mathrm{Im}\alpha|}), a > 0$, for $|\mathrm{Im}\alpha| \to \infty$, the function can be written in the band $|\mathrm{Re}\alpha| \le \varphi$ as,

$$(15)\quad s(\alpha) = \frac{\mathrm{i}}{2\sqrt{2\pi}}\left(\int_{-\mathrm{i}\infty-\delta}^{\mathrm{i}\infty-\delta} + \int_{-\mathrm{i}\infty+\delta}^{\mathrm{i}\infty+\delta}\right) \left(\frac{R_+(\omega)\mathrm{e}^{-\mathrm{i}\omega\Phi}}{\mathrm{i}\sin(2\omega\Phi)} - \frac{R_-(\omega)\mathrm{e}^{\mathrm{i}\omega\Phi}}{\mathrm{i}\sin(2\omega\Phi)}\right)\mathrm{e}^{-\mathrm{i}\omega\alpha}\,\mathrm{d}\alpha,$$

with,

$$R_\pm(\omega) = \frac{-\mathrm{i}}{2\sqrt{2\pi}}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} H^\pm(\alpha)\mathrm{e}^{\mathrm{i}\omega\alpha}\,\mathrm{d}\alpha = \frac{-\mathrm{i}}{2\sqrt{2\pi}}\,r_\pm(\omega),$$

which expression after using (14) gives $s(\alpha)$ in the whole complex plane. If $R_\pm$ is not simple, we can modify (15) by changing the order of integration. This allows us to write,

$$s(\alpha) = \frac{-\mathrm{i}}{8\Phi}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\mathrm{d}\alpha'\left[H^+(\alpha')\tan\left(\frac{\pi}{4\Phi}(\alpha+\Phi-\alpha')\right) - H^-(\alpha')\tan\left(\frac{\pi}{4\Phi}(\alpha-\Phi-\alpha')\right)\right],$$

integral which has been studied extensively by Tuzhilin in [13].

Therefore, the problem is then reduced to the solutions a/c and d/b of (11a), (11b) and (11c) that are studied now.

Suppose one has a solution a/c of (11a). One can let,

$$(16)\qquad \frac{d}{b} = \frac{a}{c} + \frac{1}{w},$$

that can be used in (11b). This gives us for w the following equation (superscript indice $\pm$ for face $\pm\varphi$),

$$(17)\quad \left(l_- + n\frac{a_+}{c_+}\right)^\pm w(\alpha\pm\Phi) - \left(l_+ - n\frac{a_-}{c_-}\right)^\pm w(-\alpha\pm\Phi) = (-n)^\pm,$$

of which we know a solution, since as in [2], one can write,

$$(18)\quad w(\alpha) = \psi(\alpha)\chi(\alpha) + K\psi(\alpha),\ K \text{ a constant},$$

with

$$\left(l_- + n\frac{a_+}{c_+}\right)^\pm \psi(\alpha\pm\Phi) - \left(l_+ - n\frac{a_-}{c_-}\right)^\pm \psi(-\alpha\pm\Phi) = 0,$$

$$\chi'(\alpha\pm\Phi) + \chi'(-\alpha\pm\Phi) = \left(\frac{-n^\pm}{\psi(\alpha\pm\Phi)\left(l_- + n\frac{a_-}{c_-}\right)^\pm}\right)',$$

which leads to equation (14) for ψ'/ψ and χ'.

So the condition (11c) can be satisfied if one has a solution for (11a).

Thus, it would be useful to show the existence of this solution. So, let us use (4). It is known that for every incident plane wave corresponds a solution for the field and thus for f_1 and f_2, which gives us the c_{1n}, c_{2n}, $n = 1, -, q$. By linear combination (with coefficients not all zero) of solutions for different incidences, one can find g_1, g_2 (linearity of (4)) satisfying :

$$(19)\qquad A_\alpha^\pm\begin{vmatrix} g_1(\alpha\pm\Phi) \\ g_2(\alpha\pm\Phi)\end{vmatrix} - A_{-\alpha}^\pm\begin{vmatrix} g_1(-\alpha\pm\Phi) \\ g_2(-\alpha\pm\Phi)\end{vmatrix} = 0,$$

which corresponds to (4) without second member. The functions g_1 and g_2 are not zero because the corresponding incident field is not zero. Then by comparing (19) with (6), one can notice rather easily (see appendix 3) that one can take,

$$(20)\qquad \frac{a}{c} = -\frac{g_2}{g_1},$$

which proves the existence of a solution for (11a).

Therefore, the problem of coupled equation given by (4) has been essentially reduced to the non linear functional equation with one unknown (11a). As far as we know, there is no general explicit expansion of its solution. Therefore some particular case, from which one solution has been isolated, is analysed thereafter.

Notes :

- (11a), (11b), (11c) and (13a), (13b) are independant of $\varepsilon^\pm$. a, b, c, d defined, $\varepsilon^\pm$ are determined by (6) or (7). For example, if $A_\alpha^\pm C_{\alpha\pm\Phi}^{-1}$ is definite non zero for $\alpha = 0$, then $\varepsilon^\pm = 1$.
- From (6), one obtains that,

$$(21)\quad (C_{\alpha\pm\Phi})^{-1} = \varepsilon^\pm(A_\alpha^\pm)^{-1}(A_{-\alpha}^\pm)(C_{-\alpha\pm\Phi})^{-1}.$$

So if $(C_\alpha)^{-1}$ is analytic in the strip $|\mathrm{Re}\alpha| \le \Phi$ the poles of $(C_\alpha)^{-1}$ for $|\mathrm{Re}\alpha| > \Phi$ can be deduced from zeroes of $\det(A_\alpha^\pm)$.

- The function $d/b(a/c)$ can always be changed to be « analytic in the strip $|\mathrm{Re}\alpha| \le \Phi$ » by letting (as in (16)),

$$\left[\frac{d}{b}\left(\frac{a}{c}\right)\right]_{\text{non analytic}} = \left[\frac{d}{b}\left(\frac{a}{c}\right)\right]_{\text{analytic}} + \frac{1}{w},$$

and using (18) with,

$\psi(\alpha) = \psi(\alpha)$ (deduced from (15))×

(rational function of $\sin\left(\frac{\pi}{2\Phi}\alpha\right)$

which has the poles of $\left[\frac{d}{b}\left(\frac{a}{c}\right)\right]_{\text{non analytic}}$ as zeroes).

- The chapter VI allows us to say that (19) doesn't imply $g_j(\mathrm{i}\infty) - g_j(-\mathrm{i}\infty) = 0 (j = 1, 2)$. Seeing that the g_j satisfy to (b), one can conjecture from (20) that there exists a/c so that,

$$\left|\frac{a}{c}(\alpha) - \frac{a}{c}(\pm\mathrm{i}\infty)\right| < \mathrm{e}^{-a|\mathrm{Im}\alpha|}, a > 0,\ \ \mathrm{Im}\alpha \to \pm\infty,$$

which is confirmed by the case studied in the chapter IV.

IV. APPLICATION TO SOME CLASSES OF WEDGES

As a specific example of application, one considers here the case where (A-ll) (appendix 1) can be applied i.e. when the penetration of the field into the wedge is small. It is the case of the constant impedance boundary conditions. This allows to write,

$$(22)\quad A_\alpha^\pm = \begin{bmatrix} \cos\beta\cos\alpha & \sin\alpha \pm \eta^\pm \sin\beta \\ \sin\alpha \pm \sin\beta / \eta^\pm & -\cos\beta\cos\alpha \end{bmatrix},$$

$$\det(A_\alpha^\pm) = -(\sin\alpha\sin\beta \pm \eta^\pm)(\sin\alpha\sin\beta \pm 1/\eta^\pm),$$

and thus,

$$(23)\quad l(\alpha) = \cos^2\beta\sin^2\alpha - \cos^2\alpha \pm \sin\alpha\sin\beta\,(\eta^\pm - 1/\eta^\pm),$$
$$n(\alpha) = -p(\alpha) = \cos\beta\sin 2\alpha,$$

where $\eta^\pm$ is usually named relative impedance.

With this expression of $A_\alpha^\pm$, (4) has to be written with $q = 1$ and $c_{11}^\pm = c_{11}, c_{21}^\pm = c_{21}$. One can notice that both expressions of $\det(A_\alpha^\pm)$ and the second note at the end of chapter III allow to write that : if $\mathrm{Re}\eta^\pm > 0$ and $(C_\alpha)^{-1}$ is analytic in the strip $|\mathrm{Re}\alpha| \leq \varphi$ then $(C_\alpha)^{-1}$ has no singularities of the type listed in condition (c).

IV.1. Wedge with $\eta^\pm = 1/\eta^\pm = 1$.

The equation $g(a_+/c_+, a_-/c_-) = 0$ can be written :

$$(24)\quad \frac{(l_+ + l_-)}{2}\,\frac{[(a_+/c_+) - (a_-/c_-)]}{[(a_+/c_+)(a_-/c_-) + 1]} + \frac{(l_+ - l_-)}{2}\,\frac{[(a_+/c_+) + (a_-/c_-)]}{[(a_+/c_+)(a_-/c_-) + 1]} = n.$$

This formulation is specially well suited to derive the solution for the case $\eta^\pm = 1$ where $l_+ = l_- = l$. It is this case, known as the most usual model for absorbing material, that we study now. So letting,

$$\frac{a}{c}(\alpha) = \tan(v(\alpha)),$$

one obtains,

$$(25)\quad v'(\alpha \pm \Phi) + v'(-\alpha \pm \Phi) = \left(\arctan\left(\frac{n(\alpha)}{l(\alpha)}\right)\right)'.$$

The derivation of v' is done from (15). By using the following expression,

$$\arctan\left(\frac{n(\alpha)}{l(\alpha)}\right) = \mathrm{i}\log\left(\frac{-\tan(\alpha)\cos\beta + \mathrm{i}}{\tan(\alpha)\cos\beta + \mathrm{i}}\right),$$

and the technique of residues, one determines that,

$$(26)\quad r_\pm(\omega) = (2\pi)\left(\frac{\mathrm{e}^{\mathrm{i}\omega\left(\frac{\pi}{2} - \mathrm{i}\theta\right)} - \mathrm{e}^{\mathrm{i}\omega\left(\frac{\pi}{2} + \mathrm{i}\theta\right)}}{1 - \mathrm{e}^{\mathrm{i}\omega\pi}}\right),$$

for $\mathrm{Im}\omega > 0$, and $r_\pm(\omega) = r_\pm(-\omega)$ for $\mathrm{Im}\omega < 0$, with,

$$\frac{\pi}{2} - \mathrm{i}\theta = \arctan\left(\frac{\mathrm{i}}{\cos\beta}\right).$$

This gives for $|\mathrm{Re}\alpha| \leq \Phi$,

$$(27)\quad v(\alpha) = \mathrm{i}\int_0^\infty \frac{\mathrm{e}^{-\nu\left(\frac{\pi}{2} - \mathrm{i}\theta\right)} - \mathrm{e}^{-\nu\left(\frac{\pi}{2} + \mathrm{i}\theta\right)}}{(1 - \mathrm{e}^{-\nu\pi})}\left(\int \frac{\sinh(\nu(\alpha - \Phi)) - \sinh(\nu(\alpha + \Phi))}{\sinh(2\nu\Phi)}\,\mathrm{d}\alpha\right)\mathrm{d}\nu + v_0,$$

where v_0 is a constant.

When $|\mathrm{Im}\alpha| \to \infty$, the principal contribution comes from $\nu \sim 0$; now from [10],

$$\int_0^\infty \frac{\sinh(ax)}{\sinh(bx)}\,\mathrm{d}x = \frac{\pi}{2b}\tan\left(\frac{\pi a}{2b}\right),$$

which allows to write,

$$(28)\qquad v(\alpha) = O(1) \quad \text{when } |\mathrm{Im}\alpha| \to \infty.$$

Since the solution given by (27) for $|\mathrm{Re}\alpha| \leq \Phi$ has no pole, (28) indicates that v is finite in the strip $|\mathrm{Re}\alpha| \leq \Phi$. Consequently, v_0 can be chosen so that $a/c = \tan(v)$ has no pole or zero for $|\mathrm{Re}\alpha| \leq \varphi$.

For d/b satisfying to (11b) one can use the expression of a/c with a constant term added to v. This allows to choose the following expression,

$$\frac{d}{b} = \tan\left(v + \frac{\pi}{2}\right)\left(= \left(\frac{-a}{c}\right)^{-1}\right).$$

(13a) and (13b) can then be written as :

$$(29a)\qquad \left(\frac{b_+}{c_+}\right)\left(\frac{b_-}{c_-}\right)^{-1} = \left(\frac{a_+}{c_+}\right)\left(\frac{a_-}{c_-}\right)^{-1},$$

$$(29b)\qquad \frac{(b_+a_+)}{(b_-a_-)} = \left[\frac{-\sin\alpha\sin\beta \pm 1}{\sin\alpha\sin\beta \pm 1}\right]^2 \times \frac{1 + [\cot(v(-\alpha \pm \Phi))]^2}{1 + [\cot(v(\alpha \pm \Phi))]^2}.$$

(29a) is directly equivalent to :

$$\frac{a}{c} = \frac{b}{c} \quad \text{so} \quad a = b.$$

Then from (29b) one can let,

$$(30)\qquad a(\alpha) = \psi(\alpha)\sin(v(\alpha)),$$

where, $\quad \dfrac{\psi(\alpha \pm \Phi)}{\psi(-\alpha \pm \Phi)} = \dfrac{-\sin\alpha\sin\beta \pm 1}{\sin\alpha\sin\beta \pm 1}.$

$\psi(\alpha)$ is taken from expressions given in [2].

Thus, one has :

$$(31)\qquad \begin{aligned} a(\alpha) &= \psi(\alpha)\sin(v(\alpha)), \\ b(\alpha) &= a(\alpha), \\ c(\alpha) &= \psi(\alpha)\cos(v(\alpha)), \\ d(\alpha) &= -c(\alpha), \end{aligned}$$

which are analytic functions for $|\mathrm{Re}\alpha| \leq \Phi$, and of asymptotic behaviour $O\left(\mathrm{e}^{\frac{\pi}{2\Phi}|\mathrm{Im}\alpha|}\right)$ when $|\mathrm{Im}\alpha| \to \infty$.

As noticed in the first note at the end of section III, since $(C_{\pm\Phi})^{-1}$ are definite non zero, the quantities $\varepsilon^{\pm}$ of (5) are equal to 1. (5) is then written as :

$$(32)\quad \left|\begin{matrix} t_1(\alpha\pm\Phi)\\ t_2(\alpha\pm\Phi)\end{matrix}\right. - \left|\begin{matrix} t_1(-\alpha\pm\Phi)\\ t_2(-\alpha\pm\Phi)\end{matrix}\right. = B^{\pm}_{\alpha}\left|\begin{matrix} C_{11}\\ C_{21}\end{matrix}\right. \sin\alpha,$$

where,

$(T_\alpha)\left|\begin{matrix} C_{11}\\ C_{21}\end{matrix}\right.$ is a particular solution of (32) which is regular for $|\mathrm{Re}\alpha| \leq \Phi$. This is taken directly from (15); $\left|\begin{matrix} t_{1s}\\ t_{2s}\end{matrix}\right.$ satisfies (32) with a null second member and possesses the pole $\alpha = \varphi_0$.

Thus one get :

$$(34)\quad \left|\begin{matrix} f_1(\alpha)\\ f_2(\alpha)\end{matrix}\right. = (C_\alpha)^{-1} \left[(C_{\varphi_0})\left|\begin{matrix} D_1\\ D_2\end{matrix}\right.\sigma(\alpha) + (T_\alpha)\left|\begin{matrix} C_{11}\\ C_{21}\end{matrix}\right.\right],$$

where,

$$\left|\begin{matrix} t_1(\alpha)\\ t_2(\alpha)\end{matrix}\right. = (C_\alpha)\left|\begin{matrix} f_1(\alpha)\\ f_2(\alpha)\end{matrix}\right.,$$
$$B^{\pm}_{\alpha} = (C_{\alpha\pm\Phi})(A^{\pm}_{\alpha})^{-1}.$$

One can let,

(errata, 9-10, p. 544)

$$(33)\quad \left|\begin{matrix} t_1(\alpha)\\ t_2(\alpha)\end{matrix}\right. = \left|\begin{matrix} t_{1s}(\alpha)\\ t_{2s}(\alpha)\end{matrix}\right. + (T_\alpha)\left|\begin{matrix} C_{11}\\ C_{21}\end{matrix}\right.,$$

with,

$$\sigma(\alpha) = \frac{\pi}{2\Phi}\,\frac{\cos\left(\frac{\pi}{2\Phi}\varphi_0\right)}{\sin\left(\frac{\pi}{2\Phi}\alpha\right) - \sin\left(\frac{\pi}{2\Phi}\varphi_0\right)}.$$

Since $(C_\alpha)^{-1}$ is analytic for $|\mathrm{Re}\alpha| \leq \Phi$, one can use the remark made at the beginning of chapter IV on the singularities of $(C_\alpha)^{-1}$. This allows expression (34) with $c_{11} = c_{21} = 0$ to satisfy condition (c). Then the following expression is obtained,

$$(35)\quad \left|\begin{matrix} f_1(\alpha)\\ f_2(\alpha)\end{matrix}\right. = \frac{\psi(\alpha)}{\psi(\varphi_0)} \begin{bmatrix} \cos(v(\alpha)-v(\varphi_0)) & \sin(v(\alpha)-v(\varphi_0))\\ -\sin(v(\alpha)-v(\varphi_0)) & \cos(v(\alpha)-v(\varphi_0))\end{bmatrix} \left|\begin{matrix} D_1\\ D_2\end{matrix}\right.\sigma(\alpha),$$

where finally the constant term of v has no use. It can be noticed that taking $c_{11} = c_{21} = 0$ is not implied by (c) because $[(C_\alpha)^{-1}(T_\alpha)]$ has no poles listed in (c). Then the unicity of the field solution leads to,

$$(C_\alpha)^{-1}(T_\alpha) = \text{ constant matrix.}$$

IV.2. Half and full plane, $\eta^{\pm}$ arbitrary.

From the expressions of n and $l_{\pm}$, one can notice the following equalities,

$$g\left(\frac{-\cos\beta}{\cot\alpha}, \frac{\cos\beta}{\cot\alpha}\right) = 0,$$

and

$$g\left(\frac{\cot\alpha}{\cos\beta}, \frac{-\cot\alpha}{\cos\beta}\right) = 0.$$

When $\Phi = \pi/2$ or π, this allows straight to take,

$$\frac{a_+}{c_+} = \frac{-\cos\beta}{\cot\alpha},$$
$$\frac{d_+}{b_+} = \frac{\cot\alpha}{\cos\beta},$$

which gives from (13a) and (13b),

$$(36)\quad \left(\frac{b_+}{c_+}\right)\left(\frac{b_-}{c_-}\right)^{-1} = -\frac{R(\pm\eta^{\pm},\alpha)}{R(\pm 1/\eta^{\pm},\alpha)},$$
$$\frac{(b_+a_+)}{(b_-a_-)} = R(\pm\eta^{\pm},\alpha)R(\pm 1/\eta^{\pm},\alpha),$$

where :

$$R(\eta,\alpha) = \frac{-\sin\alpha\sin\beta+\eta}{\sin\alpha\sin\beta+\eta}.$$

This choice allows to find the solution given in [7, 9], which can serve as reference for the expressions of f_1, f_2.

V. CONSEQUENCE ON THE CHOICE OF $a/c(d/b)$.

The compactness of the solution defined in chapter IV.1., comes from the choice of a/c and d/b which are finite in the strip $|\mathrm{Re}\alpha| \leq \Phi$. If one takes this choice (possible in all case if we refer to chapter III notes) the procedure can be rather systematic.

On the other hand, when one has a/c or d/b with poles for $|\mathrm{Re}\alpha| \leq \Phi$ as seen in § IV.2, the form of the solution is rather less general since adapted to one particular case but is valuable if the expression of regular a/c and d/b are too complicated.

VI. NOTE ON THE RELATION BETWEEN $C^{\pm}_{n}$ AND FIELD DERIVATIVES ON THE EDGE

For the sake of simplicity if one writes f_1, f_2 as f then it can be noticed that the boundary conditions on each wedge face lead to equalities of the following type,

$$(37)\quad \left[\sin\alpha\left(\sum_{n=0}^{N} a^{\pm}_{n}\cos^{2n}\alpha\right)(f(\alpha\pm\Phi)+f(-\alpha\pm\Phi)) + \left(\sum_{n=0}^{2N+1} b^{\pm}_{m}\cos^{m}\alpha\right)(f(\alpha\pm\Phi)-f(-\alpha\pm\Phi))\right]$$
$$\left(\sum_{p=0}^{P} d^{\pm}_{p}\cos^{p}\alpha\right) = \sum_{n=1}^{2N+P+1} c^{\pm}_{n}\sin\alpha\cos^{n-1}\alpha.$$

Then one can rewrite the above equation by using the following developments,

$$(38)\quad \frac{\sum_{n=0}^{2N+1} b_m^{\pm} \cos^m \alpha}{\sum_{n=0}^{N} a_n^{\pm} \cos^{2n} \alpha} = \sum_{n=-1}^{M} \frac{e_n^{\pm}}{\cos^n \alpha} + E(\alpha),$$

$$\frac{\sum_{n=1}^{2N+P+1} c_n^{\pm} \cos^{n-1} \alpha}{\sum_{p=0}^{P} d_p^{\pm} \cos^p \alpha \sum_{n=0}^{N} a_n \cos^{2n} \alpha} = \sum_{n=0}^{M} \frac{l_n^{\pm}}{\cos^n \alpha} + L(\alpha),$$

where $E(\alpha)$ and $L(\alpha)$ are $\mathrm{O}(1 / \cos^{M+1} \alpha)$ when $|\mathrm{Im}\alpha| \to \infty, l_n^{\pm}$ being linear combination of $c_n^{\pm}$.

This allows to formulate (37) as,

$$(39)\quad (f(\alpha \pm \Phi) + f(-\alpha \pm \Phi)) + \sum_{n=-1}^{M} \frac{e_n^{\pm}}{\cos^n \alpha} \frac{1}{\sin \alpha} (f(\alpha \pm \Phi) - f(-\alpha \pm \Phi)) = \sum_{n=0}^{M} \frac{l_n^{\pm}}{\cos^n \alpha} + \left[\frac{-E(\alpha)}{\sin \alpha} (f(\alpha \pm \Phi) - f(-\alpha \pm \Phi)) + L(\alpha) \right].$$

Then one notices two types of equalities that can be used : firstly, for,

$$\mathrm{S}(\rho, \varphi) = \frac{1}{4\pi \mathrm{i}} \int_C (s(\alpha + \varphi) - s(-\alpha + \varphi)) \mathrm{e}^{\mathrm{i}k\rho \cos \alpha} \, \mathrm{d}\alpha,$$

one has the following equalities,

$$\lim_{\rho \to 0} S(\rho, \varphi) = \mathrm{i}(s(\mathrm{i}\infty) - s(-\mathrm{i}\infty)) \text{ if } |s(\pm \mathrm{i}\infty)| < \infty,$$

$$\frac{\partial^n}{\partial(\mathrm{i}k\rho)^n} \left(\frac{\partial}{\mathrm{i}k\rho \partial\varphi} \right) S(\rho, \varphi) = \frac{1}{4\pi \mathrm{i}} \int_C \sin \alpha \cos^n \alpha \, (s(\alpha + \varphi) + s(-\alpha + \varphi)) \, \mathrm{e}^{\mathrm{i}k\rho \cos \alpha} \, \mathrm{d}\alpha;$$

and secondly,

$$\lim_{\rho \to 0} \int_C \sin \alpha \cos^n \alpha \, \mathrm{e}^{\mathrm{i}k\rho \cos \alpha} \, \mathrm{d}\alpha = 0.$$

Thus, if one takes the field expression related to f,

$$\mathrm{u}(\rho, \varphi) = \frac{1}{4\pi \mathrm{i}} \int_C (f(\alpha + \varphi) - f(-\alpha + \varphi)) \, \mathrm{e}^{\mathrm{i}k\rho \cos \alpha} \, \mathrm{d}\alpha,$$

(39) and the condition (b) on f allows to write for $m \geq 0$:

$$(40)\quad \lim_{\rho \to 0} \left(\frac{\partial^m}{\partial(\mathrm{i}k\rho)^m} \frac{\partial}{\mathrm{i}k\rho \partial\varphi} + \sum_{n=-1}^{m} \mathrm{e}_n^{\pm} \frac{\partial^{m-n}}{\partial(\mathrm{i}k\rho)^{m-n}} \right) u(\rho, \varphi)|_{\varphi = \pm\Phi} = -l_{m+1}^{\pm}.$$

As a matter of fact, a recent reading of the papers of Tuzhilin [12, 13] (which by the way are not very much quoted) has been very rewarding.

VII. NUMERICAL RESULTS

The expression of the spectral functions f_j given in (34) for the field scattered by a wedge with the relative impedance of an usual absorbing material $\eta^{\pm} = 1$, involves two special functions $\psi(\alpha)$ and $v(\alpha)$. It is worth being tested numerically. Now the reciprocity principle is known to be satisfied without the condition (d) when the boundary conditions are of constant impedance type, so that it can be a test. The field apart from residues terms corresponding to poles of f_1 are considered. For ρ tending to infinity, it is written [2],

$$E_{\mathrm{d}} = (E_+ - E_-) \frac{\mathrm{e}^{-\mathrm{i}k_0(\rho \sin \beta + z \cos \beta)}}{\sqrt{k_0 \rho \sin \beta}},$$

with,

$$E_{\pm} = -\frac{\mathrm{e}^{\;-\mathrm{i}\pi/4}}{\sqrt{2\pi}} f_1(\pm\pi + \varphi).$$

The Figures 3 and 4 give the absolute values of $E_+, E_-, E_+ - E_-$ respectively for $\varphi_0 = 57.5°, \beta = 30°, \varphi$ varying and $\varphi = 57.5°, \beta = 150°, \varphi_0$ varying. The comparaison shows us the expected symmetry $\varphi \leftrightarrow \varphi_0, \beta \leftrightarrow \pi - \beta$ for the term $E_+ - E_-$.

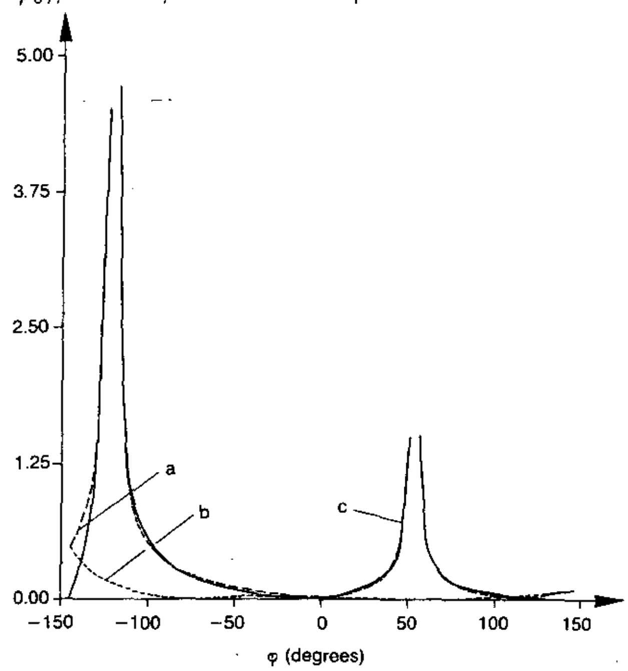

FIG. 3. — Asymptotic terms (a) E_+ (b) E_- (c) $E_+ - E_-$, for $\varphi_0 = 57.5°, \beta = 30°, \Phi = 145°, |E_z^i| = .5, H_z^i = 0$.

Termes asymptotiques (a) E_+ (b) E_- (c) $E_+ - E_-$, pour $\varphi_0 = 57,5°, \beta = 30°, \Phi = 145°, |E_z^i| = 0,5, H_z^i = 0$.

VIII. CONCLUSION

The method presented here gives a way to reduce the difficult problem of the diffraction of a skew incident plane wave by a wedge of arbitrary angle for boun-

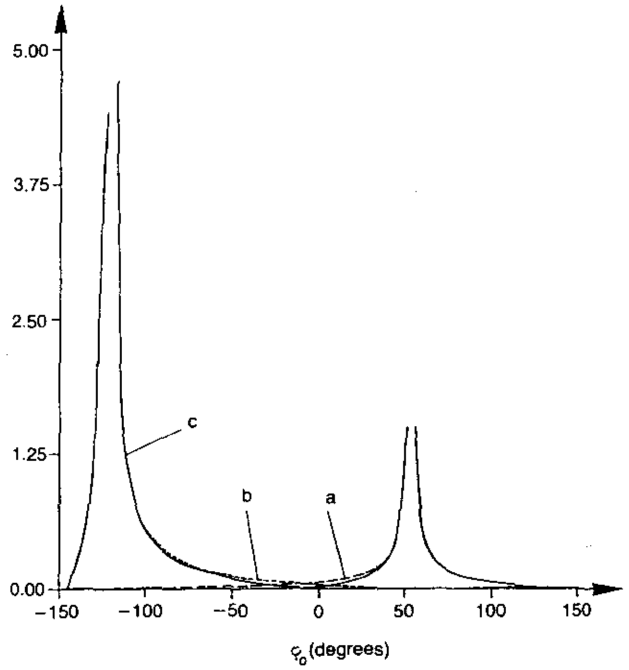

FIG. 4. — Asymptotic terms (a) E_+ (b) E_- (c) $E_+ - E_-$, for $\varphi = 57.5°, \beta = 150°, \Phi = 145°, |E^i_z| = .5, H^i_z = 0$.

Termes asymptotiques (a) E_+ (b) E_- (c) $E_+ - E_-$, pour $\varphi = 57,5°, \beta = 150°, \Phi = 145°, |E^i_z| = 0,5, H^i_z = 0$.

APPENDIX 1

We consider a perfectly conducting wedge, covered on each face, + and −, by a different dielectric layer, of equal thickness d. To determine boundary conditions, we first isolate one face. ε_1, μ_1 are the permittivity and the permeability of its dielectric, and k_1 is the associated wave number. $\varepsilon_0, \mu_0, k_0$ are the permittivity, the permeability, and the wave number of the free space. We take the incident field as a plane wave of direction angle β with the edge. In the coordinate system defined in Figure 3, the different components of the field can be expressed in terms of the z-components of electric and magnetic fields :

$$\text{(A.1)} \quad E_x = \frac{-\gamma}{\gamma^2 + k_j^2}\frac{\partial E_z}{\partial x} - \frac{\mathrm{i}\omega\mu_j}{\gamma^2 + k_j^2}\frac{\partial H_z}{\partial y},$$

$$\text{(A.2)} \quad E_y = \frac{-\gamma}{\gamma^2 + k_j^2}\frac{\partial E_z}{\partial y} + \frac{\mathrm{i}\omega\mu_j}{\gamma^2 + k_j^2}\frac{\partial H_z}{\partial x},$$

$$\text{(A.3)} \quad H_x = \frac{-\gamma}{\gamma^2 + k_j^2}\frac{\partial H_z}{\partial x} + \frac{\mathrm{i}\omega\varepsilon_j}{\gamma^2 + k_j^2}\frac{\partial E_z}{\partial y},$$

$$\text{(A.4)} \quad H_y = \frac{-\gamma}{\gamma^2 + k_j^2}\frac{\partial H_z}{\partial y} - \frac{\mathrm{i}\omega\varepsilon_j}{\gamma^2 + k_j^2}\frac{\partial E_z}{\partial x},$$

$$\gamma = \mathrm{i}k_0\cos\beta, j = 0, 1.$$

E_x, E_z electric field components have to be zero on the conductor. This is done by using Taylor expansions :

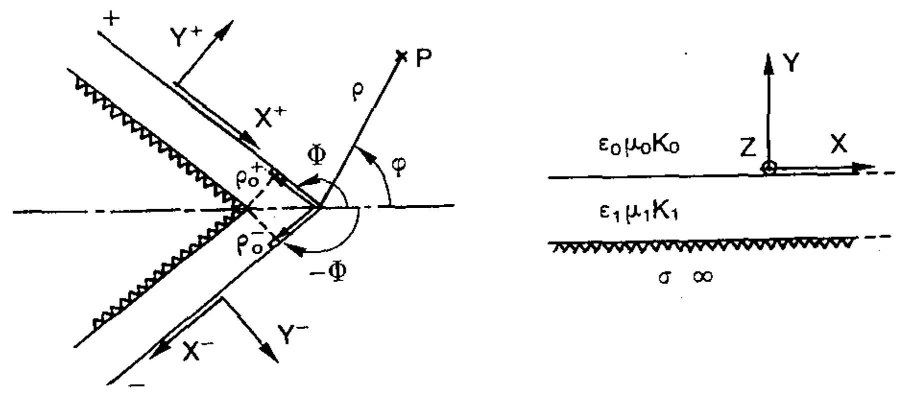

FIG. 5. — Geometry of wedge and face of wedge.

Géométrie du dièdre et d'une face.

dary conditions with high order derivatives. Each face of this wedge gives us two coupled equations for the two spectral functions corresponding to E and H field. An operator is then searched in order to obtain uncoupled equations that can be solved. Its existence is demonstrated in the same time that its determination is reduced to the knowledge of the solution of an original non linear functional equation with one unknown. The examples of application show the ability to easily recover known results as for the half and full plane and to explicit new solutions as for the interesting case of wedge covered with absorbing material of relative impedance unity.

$$\text{(A.5)} \quad \left[\sum_{n=0}^{\infty} \left.\frac{\partial^{2n} E_x}{\partial y^{2n}}\right|_{y=0^-}\frac{y^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \left.\frac{\partial^{2n+1} E_x}{\partial y^{2n+1}}\right|_{y=0^-}\frac{y^{2n+1}}{(2n+1)!}\right]_{y=-d} = 0,$$

$$\text{(A.6)} \quad \left[\sum_{n=0}^{\infty} \left.\frac{\partial^{2n} E_z}{\partial y^{2n}}\right|_{y=0^-}\frac{y^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \left.\frac{\partial^{2n+1} E_z}{\partial y^{2n+1}}\right|_{y=0^-}\frac{y^{2n+1}}{(2n+1)!}\right]_{y=-d} = 0,$$

After computing (A.5) $+\frac{\gamma}{\gamma^2+k_1^2}\frac{\partial}{\partial x}$ (A.6) and using (A.1), one get :

$$\text{(A.7)}\quad \left[\cos\left(d\sqrt{\frac{-\partial^2}{\partial y^2}}\right)\left(E_x+\frac{\gamma}{\gamma^2+k_1^2}\cdot\frac{\partial}{\partial x}E_z\right)-\frac{\mathrm{i}\omega\mu_1}{\gamma^2+k_1^2}\sqrt{\frac{-\partial^2}{\partial y^2}}\sin\left(d\sqrt{\frac{-\partial^2}{\partial y^2}}\right)(H_z)\right]\Bigg|_{y=0^-}=0,$$

with,

$$\frac{-\partial^2}{\partial y^2}\equiv k_1^2+\gamma^2+\frac{\partial^2}{\partial x^2},$$

and by using (A.3) in (A.6),

$$\text{(A.8)}\quad \left[\frac{\mathrm{i}\omega\varepsilon_1}{\gamma^2+k_1^2}\cos\left(d\sqrt{\frac{-\partial^2}{\partial y^2}}\right)(E_z)-\frac{1}{\sqrt{-\partial^2/\partial y^2}}\sin\left(d\sqrt{\frac{-\partial^2}{\partial y^2}}\right)\left(H_x+\frac{\gamma}{\gamma^2+k_1^2}\frac{\partial}{\partial x}H_z\right)\right]\Bigg|_{y=0^-}=0.$$

Now from the continuity of the tangential components of the field at the interface between the dielectric and external medium, $y=0^-$ can be replaced by $y=0^+$.

Then, one uses (A.1) with (A.7) which gives,

$$\text{(A.9)}\quad \left[\cos\left(d\sqrt{k_1^2+\gamma^2+\frac{\partial^2}{\partial x^2}}\right)\left(\left(\frac{-\gamma}{\gamma^2+k_0^2}+\frac{\gamma}{\gamma^2+k_1^2}\right)\frac{\partial E_z}{\partial x}-\frac{\mathrm{i}\omega\mu_0}{\gamma^2+k_0^2}\frac{\partial H_z}{\partial y}\right)-\frac{\mathrm{i}\omega\mu_1}{\gamma^2+k_1^2}\sqrt{k_1^2+\gamma^2+\frac{\partial^2}{\partial x^2}}\sin\left(d\sqrt{k_1^2+\gamma^2+\frac{\partial^2}{\partial x^2}}\right)(H_z)\right]\Bigg|_{y=0^+}=0,$$

and then (A.3) with (A.8),

$$\text{(A.10)}\quad \left[\frac{\mathrm{i}\omega\varepsilon_1}{\gamma^2+k_1^2}\cos\left(d\sqrt{k_1^2+\gamma^2+\frac{\partial^2}{\partial x^2}}\right)(E_z)-\frac{1}{\sqrt{k_1^2+\gamma^2+\frac{\partial^2}{\partial x^2}}}\sin\left(d\sqrt{k_1^2+\gamma^2+\frac{\partial^2}{\partial x^2}}\right)\left(\left(\frac{-\gamma}{\gamma^2+k_0^2}+\frac{\gamma}{\gamma^2+k_1^2}\right)\frac{\partial H_z}{\partial x}+\frac{\mathrm{i}\omega\varepsilon_0}{\gamma^2+k_0^2}\frac{\partial E_z}{\partial y}\right)\right]\Bigg|_{y=0^+}=0.$$

When one uses (A.9) and (A.10) for the wedge, one has to shift (as in [2]) their zone of application by $\rho_0^\mp$ (see Fig. 3). One then approximates the differential equation of infinite order, by polynomial differential equation of finite rank q. Particularly when $k_1 \gg k_0$ one can write,

$$\text{(A.11)}\quad \frac{\gamma}{\gamma^2+k_1^2}\sim 0,\ \sqrt{k_1^2+\gamma^2+\frac{\partial^2}{\partial x^2}}\equiv\sqrt{\ }\sim k_1,$$

$$\left(-\omega\mu_1\frac{\sqrt{\ }}{k_0}\frac{\gamma^2+k_0^2}{\gamma^2+k_1^2}\tan(d\sqrt{\ })\right)\Bigg|_{\varphi=\pm\Phi}\sim \mathrm{i}k_0\eta^{\pm}\sin^2\beta\sqrt{\frac{\mu_0}{\varepsilon_0}},$$

$$\left(\omega\varepsilon_1\frac{\sqrt{\ }}{k_0}\frac{\gamma^2+k_0^2}{\gamma^2+k_1^2}\cot(d\sqrt{\ })\right)\Bigg|_{\varphi=\pm\Phi}\sim \mathrm{i}k_0\frac{\sin^2\beta}{\eta^{\pm}}\sqrt{\frac{\varepsilon_0}{\mu_0}},$$

$$\rho_0^{\pm}\sim 0.$$

If one uses the expressions given in (2) for the field at the surface of the wedge, then one obtains by integration by parts,

$$\int_C\begin{pmatrix}\mathrm{P}_{11}^{\pm}(\alpha) & \mathrm{P}_{12}^{\pm}(\alpha)\\ \mathrm{P}_{21}^{\pm}(\alpha) & \mathrm{P}_{22}^{\pm}(\alpha)\end{pmatrix}\left|\begin{matrix}f_1(\alpha\pm\Phi)\\ f_2(\alpha\pm\Phi)\end{matrix}\right.\ \mathrm{e}^{\mathrm{i}k_0\rho\sin\beta\cos\alpha}\,d\alpha=0,$$

where $P_{lj}^{\pm}, l=1,2, j=1,2$ are degree q polynomials of $\sin\alpha$ and $\cos\alpha$. In the text one will let $A_\alpha^{\pm}=(P_{lj}^{\pm})_{\substack{l=1,2\\ j=1,2}}$.

APPENDIX 2

It is shown here that (12a), (12b) with (11a), (11b) and (11c) are equivalent to (13a), (13b) with (11a), (11b), (11c). If one divides (12a) by (12b), one obtains (13a),

$$\frac{-g\left(\frac{a_+}{c_+},\frac{d_-}{b_-}\right)}{g\left(\frac{d_+}{b_+},\frac{a_-}{c_-}\right)}=\left(\frac{b_+}{c_+}\right)\left(\frac{b_-}{c_-}\right)^{-1}.$$

By use of the fact that $g(d_+/b_+, a_-/c_-)$ changes into $-g(a_+/c_+, d_-/b_-)$ when α changes into $-\alpha$ (12b) gives us,

$$\frac{\det A_-}{\det A_+}\frac{1-(c_-/a_-)(d_-/b_-)}{1-(c_+/a_+)(d_+/b_+)}=\frac{(b_+a_+)}{(b_-a_-)}.$$

Inversely, (13a) and (13b) allows to write,

$$\frac{1}{\det A_+}\,g\left(\frac{a_+}{c_+},\frac{d_-}{b_-}\right)=-\frac{k(\alpha)}{c_+b_-}(a_+b_+-c_+d_+),$$

$$\frac{1}{\det A_+}\,g\left(\frac{d_+}{b_+},\frac{a_-}{c_-}\right)=\frac{k(\alpha)}{c_-b_+}(a_+b_+-c_+d_+),$$

which looks like (12a) and (12b) with ε replaced by a even function $k(\alpha)$.

Therefore, by (11a), (11b), (11c) and the notations of (8), one obtains :

$$(C_{\alpha\pm\Phi})[(A_\alpha^{\pm})^{-1}(A_{-\alpha}^{\pm})](C_{-\alpha\pm\Phi})^{-1}=\begin{pmatrix}k^{\pm}(\alpha) & 0\\ 0 & k^{\pm}(\alpha)\end{pmatrix},$$

($k(\alpha)\equiv k^{\pm}(\alpha)$ for each face).

Now if one changes the sign of α in the last equation, one obtains the inverse of the left side and thus,

$$k(-\alpha) = 1/k(\alpha).$$

Since $k(\alpha)$ is even, this leads to,

$$k(\alpha) = +1 \text{ or } -1,$$

which completes the demonstration.

One notices that if $(C_{\alpha\pm\Phi})(A_{\alpha}^{\pm})^{-1}|_{\alpha=0}$ and their inverse are definite, then one has $k(\alpha) = 1$.

APPENDIX 3

One note $g_{j+} = g_j(\alpha \pm \Phi), g_{j-} = g_j(-\alpha \pm \Phi)$ and « · » being an arbitrary quantity.

(19) allows to write that,

$$(A_{\alpha}^{\pm})\begin{pmatrix} \cdot & g_{1+} \\ \cdot & g_{2+} \end{pmatrix} - (A_{-\alpha}^{\pm})\begin{pmatrix} \cdot & g_{1-} \\ \cdot & g_{2-} \end{pmatrix} = \begin{pmatrix} \cdot & 0 \\ \cdot & 0 \end{pmatrix}.$$

It is always possible to choose $\begin{pmatrix} \cdot & g_1 \\ \cdot & g_2 \end{pmatrix}$ so that it is invertible. This leads to,

$$\begin{pmatrix} \cdot & g_{1+} \\ \cdot & g_{2+} \end{pmatrix}^{-1} [(A_{\alpha}^{\pm})^{-1}(A_{-\alpha}^{\pm})] \begin{pmatrix} \cdot & g_{1-} \\ \cdot & g_{2-} \end{pmatrix} = \begin{pmatrix} \cdot & 0 \\ \cdot & \cdot \end{pmatrix},$$

and then from (7), (8) and (10) to,

$$g(-g_{2+}/g_{1+}, -g_{2-}/g_{1-}) = 0.$$

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BIOGRAPHY

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II.1.A) Errata et commentaires sur les démonstrations et formules

* p.32 : remplacer "$|\mathrm{Re}\alpha| \leq \varphi$" par "$|\mathrm{Re}\alpha| \leq \Phi$"
* p.33 : remplacer dans (15) "$s(\alpha) = \ldots\, d\alpha$" par "$s(\alpha) = \ldots\, d\omega$"
* p.34 : remplacer "$|\mathrm{Re}\alpha| \leq \varphi$" par "$|\mathrm{Re}\alpha| \leq \Phi$"
* p.35 (erratum vol.45 , 9-10 ,p.577) : un pavé de texte a été déplacé dans la première colonne . La partie de texte

" where ,

$$\left| \begin{matrix} t_1(\alpha) \\ t_2(\alpha) \end{matrix} \right. = (C_\alpha) \left| \begin{matrix} f_1(\alpha) \\ f_2(\alpha) \end{matrix} \right.$$

....

$$(33) \qquad \ldots\ldots\ldots\ldots\ldots + (T_\alpha) \left| \begin{matrix} c_{11} \\ c_{21} \end{matrix} \right. \quad "$$

doit être transférée dans cette même colonne juste après la formule (32) .

* p.36 chap. VII : remplacer "f_j given in (34)" par "f_j given in (35)"
* On note que v développée (suivant [16] (réf. introduction)) nous donne

$$v(\alpha) = i \sum_{\pm} \Big(\sum_{l=0}^{N} \sum_{\epsilon' = -1 et +1} \mathcal{L}og\, \big[\Gamma(\tfrac{1}{2} + \tfrac{1}{4\Phi}(\epsilon'(\alpha \pm \Phi) + \tfrac{\pi}{2} - i\theta + l\pi))^{\mp 1} \times$$

$$\times \Gamma(\tfrac{1}{2} + \tfrac{1}{4\Phi}(\epsilon'(\alpha \pm \Phi) + \pi - \tfrac{\pi}{2} + i\theta + l\pi))^{\pm 1}\big] +$$

$$+ \int_0^{\infty} \frac{e^{-\nu(\frac{\pi}{2} - i\theta)} - e^{-\nu(\frac{\pi}{2} + i\theta)}}{(1 - e^{-\nu\pi})} \times e^{-\nu(N+1)\pi} \times$$

$$\times \frac{\mp(\mathrm{ch}(\nu(\alpha \pm \Phi)) - 1)}{\nu \mathrm{sh} 2\nu\Phi}\, d\nu \Big) ,$$

et que , en faisant $N \to \infty$, pour $\Phi = \pi$ (cas du demi plan) ,

$$v_{|\Phi = \pi}(\alpha) = i \sum_{\pm} \mathcal{L}og\, \big[\Gamma(\tfrac{1}{2} + \tfrac{1}{4\pi}(\mp\alpha - \tfrac{\pi}{2} - i\theta))^{\mp 1} \Gamma(\tfrac{1}{2} + \tfrac{1}{4\pi}(\mp\alpha + \tfrac{\pi}{2} - i\theta))^{\mp 1} \times$$

$$\times \Gamma(\tfrac{1}{2} + \tfrac{1}{4\pi}(\mp\alpha - \tfrac{\pi}{2} + i\theta))^{\pm 1} \Gamma(\tfrac{1}{2} + \tfrac{1}{4\pi}(\mp\alpha + \tfrac{\pi}{2} + i\theta))^{\pm 1}\big]$$

$$= i\, \mathcal{L}og\Big(\frac{\cos(\frac{1}{4}(-\alpha - \frac{\pi}{2} - i\theta))}{\cos(\frac{1}{4}(+\alpha - \frac{\pi}{2} - i\theta))} \, \frac{\cos(\frac{1}{4}(-\alpha + \frac{\pi}{2} - i\theta))}{\cos(\frac{1}{4}(+\alpha + \frac{\pi}{2} - i\theta))} \Big) ,$$

ou encore , la forme

$$v_{\,|\,\Phi=\pi}(\alpha) = \pm i\ \mathcal{L}og\ \left(\sin(\pm\alpha-(\pi/2-i\theta))\right)\ \mp$$

$$\mp\, i\mathcal{L}og[8\ \sin(\tfrac{1}{4}(\mp\alpha-\tfrac{\pi}{2}-i\theta))\cos(\tfrac{1}{4}(\pm\alpha-\tfrac{\pi}{2}-i\theta))\times$$

$$\times\sin(\tfrac{1}{4}(\mp\alpha+\tfrac{\pi}{2}-i\theta))\cos(\tfrac{1}{4}(\pm\alpha+\tfrac{\pi}{2}-i\theta))]\ ,$$

(où l'on peut choisir indifféremment dans l'expression le signe supérieur ou inférieur) qui nous permet , dans ce cas particulier , de retrouver sans difficulté la solution pour le demi plan de Bucci et Franceschetti (réf. [11] introd.) , car on peut alors mettre en forme $\cos v$ et $\sin v$ de façon à mettre en facteur le terme $(\prod_{\pm}\sin(\pm\alpha-(\pi/2-i\theta)))^{-1} = ((1-(\sin\alpha)^2(\sin\beta)^2)/(\sin\beta)^2)^{-1}$.

II.1.B) Compléments mathématiques : sur la recherche d'une solution pour des cas complexes de matrice d'impédance $A_\alpha^\pm$, suivant la méthode développée dans notre article 'On the diffraction ...' (on se référera aux formules et notations de notre article).

II.1.B.1) Considérations sur $\det A_\alpha^\pm$ pour une condition aux limites exacte tirée de (A.9) et (A.10) de l'annexe 1.

Dans le cas de conditions aux limites du type (A.9) et (A.10) considérées dans l'annexe 1 , pour un plan parfaitement conducteur recouvert d'une couche de diélectrique d'épaisseur d , on a :

$$A_\alpha^\pm = \begin{bmatrix} \dfrac{k_1^2-k_o^2}{k_1^2}\cos\beta\cos\alpha & \dfrac{\gamma^2+k_1^2}{k_1^2}\sin\alpha \pm \eta_h^\pm\sin\beta \\ \dfrac{\gamma^2+k_1^2}{k_1^2}\sin\alpha \pm \sin\beta/\eta_e^\pm & -\ \dfrac{k_1^2-k_o^2}{k_1^2}\cos\beta\cos\alpha \end{bmatrix},$$

où , en notant $\epsilon_1 = \epsilon_o \epsilon_r$ et $\mu_1 = \mu_o \mu_r$,

$$\eta_h^{\pm} = i\sqrt{\frac{\mu_r}{\epsilon_r}}\,\frac{1}{k_1}\,\sqrt{k_1^2 + \gamma^2 - (k_o \cos\alpha \sin\beta)^2}\ tg(d\sqrt{k_1^2 + \gamma^2 - (k_o \cos\alpha \sin\beta)^2})\ ,$$

$$1/\eta_e^{\pm} = -i\sqrt{\frac{\epsilon_r}{\mu_r}}\,\frac{1}{k_1}\,\sqrt{k_1^2 + \gamma^2 - (k_o \cos\alpha \sin\beta)^2}\ cotg(d\sqrt{k_1^2 + \gamma^2 - (k_o \cos\alpha \sin\beta)^2})\ .$$

Après quelques calculs élémentaires que nous ne détaillons pas ici , on obtient :

$$\det A_\alpha^{\pm} = -\frac{\gamma^2 + k_1^2}{k_1^2}\,(\sin\alpha \sin\beta \pm \eta_h^{\pm})(\sin\alpha \sin\beta \pm 1/\eta_e^{\pm})\ .$$

On notera qu'en choisissant $k_1 \gg k_o$, on retrouve l'expression (22) de notre article , associée au cas d'une impédance constante .

II.1B.2) Considérations sur la solution exacte pour une certaine classe de matrices d'impédance $A_\alpha^{\pm}$ attachées à une surface anisotrope .

On considère maintenant le cas où l'on a

$$A_\alpha^{\pm} = \begin{bmatrix} \cos\beta \cos\alpha & \sin\alpha \pm \eta_h^{\pm} \sin\beta \\ \sin\alpha \pm \sin\beta/\eta_e^{\pm} & -\cos\beta \cos\alpha \end{bmatrix},$$

où $\eta_h^{\pm}$ et $\eta_e^{\pm}$ sont des constantes avec $\eta_h^{\pm}\eta_e^{\pm} = 1$, ce qui définit une certaine classe de conditions aux limites anisotropes de surface (conditions isotropes dans le cas particulier où $\eta_h^{\pm} = \eta_e^{\pm} = 1$). On pose $\eta_h^{\pm} = \sin\theta_1^{\pm}$, $0 < \mathrm{Re}\theta_1^{\pm} \leq \pi/2$. On a alors

$$l(\alpha) = \cos^2\beta\ \sin^2\alpha\ - \cos^2\alpha - (\eta_h^{\pm}/\eta_e^{\pm} - 1)\sin^2\beta\ ,$$

$$n(\alpha) = -p(\alpha) = \cos\beta\ \sin(2\alpha)\ ,$$

$$\det A_\alpha^{\pm} = -(\sin\alpha\,\sin\beta\ \pm(\ \eta_h^{\pm} + \cos\beta\,\sqrt{(\eta_h^{\pm})^2 - 1}\ ))\ \times$$
$$\times(\sin\alpha\,\sin\beta\ \pm(\ \eta_h^{\pm} - \cos\beta\,\sqrt{(\eta_h^{\pm})^2 - 1}\ ))\ ,$$

où l'on note que $(\ \eta_h^{\pm} + \epsilon\ \cos\beta\ \sqrt{(\eta_h^{\pm})^2 - 1}\ ) = \sin(\beta)\times\sin(\theta_1^{\pm} - \epsilon i\delta)$ avec $\delta = \mathcal{L}og(\mathrm{tg}(\beta/2))$, $\epsilon = +ou - 1$. On remarque alors que l est une fonction paire et que l'on peut donc suivre une démarche comparable à celle du chapitre IV.1 de l'article . On peut ainsi écrire dans un premier stade , pour résoudre (11a) et (11b) ,

$$\tfrac{a}{c}(\alpha) = \tan(v_{an}(\alpha)) \quad , \quad \tfrac{d}{b}(\alpha) = \tan(v_{an}(\alpha) + \pi/2)\ ,$$

$$v'_{an}(\alpha \pm \Phi) + v'_{an}(-\alpha \pm \Phi) = (\arctan\ (\ \frac{n(\alpha)}{l(\alpha)}))'\ ,$$

où l'expression de v_{an} se détermine de façon standart d'après (14) et (15) . On peut la développer , d'une façon comparable à celle utilisée pour $\eta_h^{\pm} = 1$ (proceedings conf. IEEE /Dallas 1990) , suivant :

$$v_{an}(\alpha) = i \sum_{\pm}\ \sum_{\epsilon = +1 et -1} (\ \sum_{l=0}^{N} \mathcal{L}og\ (\ [\Gamma(\tfrac{1}{2} + \tfrac{1}{4\Phi}(-(\alpha\pm\Phi) + \theta_1^{\pm} - \epsilon i\delta + l\pi))^{\mp\frac{1}{2}} \times$$
$$\times\Gamma(\tfrac{1}{2} + \tfrac{1}{4\Phi}((\alpha\pm\Phi) + \theta_1^{\pm} - \epsilon i\delta + l\pi))^{\mp\frac{1}{2}} \times \Gamma(\tfrac{1}{2} + \tfrac{1}{4\Phi}((\alpha\pm\Phi) + \pi - \theta_1^{\pm} + \epsilon i\delta + l\pi))^{\pm\frac{1}{2}} \times$$
$$\times\Gamma(\tfrac{1}{2} + \tfrac{1}{4\Phi}(-(\alpha\pm\Phi) + \pi - \theta_1^{\pm} + \epsilon i\delta + l\pi))^{\pm\frac{1}{2}}\,]^{\epsilon}\ )\ +$$
$$+\,\epsilon\int_0^{\infty} \frac{\mathrm{e}^{-\nu(\theta_1^{\pm} - \epsilon i\delta)} - \mathrm{e}^{-\nu(\pi - \theta_1^{\pm} + \epsilon i\delta)}}{2\times(1 - \mathrm{e}^{-\nu\pi})} \times \mathrm{e}^{-\nu(N+1)\pi}$$
$$\times\frac{\mp(\mathrm{ch}(\nu(\alpha\pm\Phi)) - 1)}{\nu\,\mathrm{sh}2\nu\Phi}\,\mathrm{d}\nu\ )\ .$$

Dans une seconde étape , pour (13a) et (13b) , on obtient :

$$(\ \frac{b_+}{c_+}\ )\ (\ \frac{b_-}{c_-}\ )^{-1} = (\ \frac{a_+}{c_+}\ )\ (\ \frac{a_-}{c_-}\ )^{-1}\ ,$$

$$(\ \frac{a_+ b_+}{a_- b_-}\ ) = \frac{\det A_{-\alpha}^{\pm}}{\det A_\alpha^{\pm}}\ \frac{1 + (\cot(v(-\alpha\pm\Phi)))^2}{1 + (\cot(v(\alpha\pm\Phi)))^2}\ ,$$

ce qui nous permet d'écrire l'équivalent de (31) :

$$
\begin{aligned}
&a(\alpha) = \Psi_{an}(\alpha)\ \sin(v_{an}(\alpha))\ ,\\
&b(\alpha) = a(\alpha)\ \ ,\\
&c(\alpha) = \Psi_{an}(\alpha)\ \cos(v_{an}(\alpha))\ \ ,\\
&d(\alpha) = -c(\alpha)\ \ ,
\end{aligned}
$$

où , $\Psi_{an}(\alpha)$ défini par

$$
\left(\frac{\Psi_{an}(\alpha \pm \Phi)}{\Psi_{an}(-\alpha \pm \Phi)}\right)^2 = \frac{\det A^{\pm}_{-\alpha}}{\det A^{\pm}_{\alpha}}\ ,
$$

est calculé de façon standart en prenant la dérivé logarithmique de l'égalité , ce qui permet d'obtenir une équation fonctionnelle du type de (14) que l'on résout avec (15) , expression que l'on peut développer (d'une façon comparable à celle utilisée dans notre article de *Wave Motion*(1987)) suivant

$$
\begin{aligned}
\Psi_{an}(\alpha) = \prod_{\epsilon = -1 et +1} \prod_{\pm}\Big(\prod_{l=0}^{N} \Big[\ &\Gamma\big(\tfrac{1}{2}+\tfrac{1}{4\Phi}((\alpha\pm\Phi)+\theta_1^{\pm}-\epsilon i\delta+l\pi)\big)^{-\frac{1}{2}} \times\\
\times\,\Gamma\big(\tfrac{1}{2}+\tfrac{1}{4\Phi}(-(\alpha\pm\Phi)+\theta_1^{\pm}-\epsilon i\delta+l\pi)\big)^{-\frac{1}{2}} &\times \Gamma\big(\tfrac{1}{2}+\tfrac{1}{4\Phi}((\alpha\pm\Phi)+\pi-\theta_1^{\pm}+\epsilon i\delta+l\pi)\big)^{-\frac{1}{2}} \times\\
\times\,\Gamma\big(\tfrac{1}{2}+\tfrac{1}{4\Phi}(-(\alpha\pm\Phi)+\pi-\theta_1^{\pm}+\epsilon i\delta+l\pi)\big)^{-\frac{1}{2}}\Big]^{(-1)^l} &\times\\
\times\, exp\Big(\int_0^{\infty}(-1)^{N+1}\times\frac{e^{-\nu(\theta_1^{\pm}-\epsilon i\delta)}+e^{-\nu(\pi-\theta_1^{\pm}+\epsilon i\delta)}}{2\times(1+e^{-\nu\pi})}&\times e^{-\nu(N+1)\pi}\\
&\times\frac{1-\mathrm{ch}(\nu(\alpha\pm\Phi))}{\nu\,\mathrm{sh}2\nu\Phi}\,d\nu\ \Big)\Big)\,.
\end{aligned}
$$

Continuant alors de suivre la même démarche que dans IV.1 , on obtient de façon explicite les fonctions spectrales attachées à $(E_z, Z_o H_z)$:

$$
\begin{bmatrix} f_1(\alpha) \\ f_2(\alpha) \end{bmatrix} = \frac{\Psi_{an}(\alpha)}{\Psi_{an}(\varphi_o)} \begin{bmatrix} \cos(v_{an}(\alpha)-v_{an}(\varphi_o)) & \sin(v_{an}(\alpha)-v_{an}(\varphi_o)) \\ -\sin(v_{an}(\alpha)-v_{an}(\varphi_o)) & \cos(v_{an}(\alpha)-v_{an}(\varphi_o)) \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \sigma(\alpha)\ ,
$$

ce qui résout le problème .

Exact analytical solution for the diffraction at skew incidence by a class of wedge with absorbing boundary ([1])

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ABSTRACT. – *By an original approach of the problem of the diffraction of a skew incident wave on a wedge, an analytical solution for the diffraction of a plane wave with any incidence by a wedge of arbitrary angle covered with an absorbing material of relative impedance unity, is obtained here.*

RÉSUMÉ. – *Par une approche originale du problème de la diffraction d'une onde plane par un dièdre, une solution analytique de la diffraction d'une onde plane d'incidence arbitraire, par un dièdre d'angle quelconque recouvert d'un matériau absorbant d'impédance relative unité est développée ici.*

1. INTRODUCTION

The problem of the diffraction of a plane wave with skew incidence on a wedge with impedance boundary condition is delicate, because instead of being coupled with one equation as in normal incidence, the electric and magnetic fields must satisfy two equations by face [1]. The important case with the wedge of arbitrary angle, and relative impedance of face unity (the most usual model for absorbing material), had not yet been solved analytically. The solution of this problem by an original approach is given here. The time convention $\exp(i\omega t)$ is used.

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2. FORMULATION AND SOLUTION OF THE PROBLEM

The wedge considered here is with the edge parallel to the z-axis, and an external sector defined by $|\varphi|<\Phi$ in polar coordinates (ρ, φ). An incoming plane wave is characterized by the z-components of the electric (magnetic) field,

$$E_z^i\,(H_z^i)=D_1\,(D_2/Z_0)\,\mathrm{e}^{ik\,[\rho\sin\beta\cos(\varphi-\varphi_0)-z\cos\beta]} \tag{1}$$

with k, Z_0 being respectively the free space wave number and impedance, β the angle of the incident direction with the edge of the wedge. As in [2] for normal incidence, the Sommerfeld integral is used for the expression of the z-components of the electric (magnetic) field,

$$E_z\,(H_z)=\frac{\mathrm{e}^{-ikz\cos\beta}}{2\pi i\,(Z_0)}\int_{\mathscr{C}}\mathrm{d}\alpha\,f_1\,(f_2)\,(\alpha+\varphi)\,\mathrm{e}^{ik\rho\sin\beta\cos\alpha} \tag{2}$$

where the $\mathscr{C}$ consists of two branches: one going from $(i\infty+3\pi/2)$ to $(i\infty-\pi/2)$ above all singularities of the integrand, and the other obtained by its inversion with respect to $\alpha=0$. From Maliuzhinets [3], the functions $f_j\,(j=1, 2)$ are assumed to satisfy (*a*) $[f_j(\alpha)-D_j/(\alpha-\varphi_0)]$ regular for $|\mathrm{Re}\,\alpha|\leqq\Phi$, (*b*) $|f_j(\alpha)-f_j(\pm i\infty)|<\exp(-a|\mathrm{Im}\,\alpha|)$, $a>0$, when $\mathrm{Im}\,\alpha\to\pm\infty$.

The impedance boundary condition can then be written by differentiation of (2), and integration by parts, for each face $\varphi=\pm\Phi$,

$$\int_{\mathscr{C}}\mathrm{d}\alpha\left[A_\alpha^{\pm}\left|\begin{matrix}f_1\,(\alpha\pm\Phi)\\ f_2\,(\alpha\pm\Phi)\end{matrix}\right.-A_{-\alpha}^{\pm}\left|\begin{matrix}f_1\,(-\alpha\pm\Phi)\\ f_2\,(-\alpha\pm\Phi)\end{matrix}\right.\right]\mathrm{e}^{ik\rho\sin\beta\cos\alpha}=0 \tag{3}$$

with $A_\alpha^{\pm}$ a matrix function of α given for a constant relative impedance $\eta^{\pm}$ by,

$$A_\alpha^{\pm}=\begin{pmatrix}\cos\beta\cos\alpha & \sin\alpha\pm\eta^{\pm}\sin\beta\\ \sin\alpha\pm\sin\beta/\eta^{\pm} & -\cos\beta\cos\alpha\end{pmatrix} \tag{4}$$

As in [2] for the scalar case, we use a theorem due to Maliuzhinets [3] which allows us to write,

$$[\text{term within brackets in (3)}]=\left|\begin{matrix}c_1\\ c_2\end{matrix}\right..\sin\alpha \tag{5}$$

with constant numbers c_1, c_2 to be used here if the developped solution has no physical behaviour at infinity. Now the heart of the matter is to modify (5). An even or odd linear operator $B_\alpha^\pm$ is to be found so that $B_\alpha^\pm A_\alpha^\pm = C_{\alpha\pm\Phi}$ and $B_\alpha^\pm = \varepsilon^\pm B_{-\alpha}^\pm$, which gives us when applied to (5),

$$\begin{vmatrix} t_1(\alpha\pm\Phi) \\ t_2(\alpha\pm\Phi) \end{vmatrix} - \varepsilon^\pm \begin{vmatrix} t_1(-\alpha\pm\Phi) \\ t_2(-\alpha\pm\Phi) \end{vmatrix} = B_\alpha^\pm \begin{vmatrix} c_1 \\ c_2 \end{vmatrix} . \sin\alpha \tag{6}$$

with $\begin{vmatrix} t_1(\alpha) \\ t_2(\alpha) \end{vmatrix} = C_\alpha \begin{vmatrix} f_1(\alpha) \\ f_2(\alpha) \end{vmatrix}$

The equation (6) is equivalent to two independant equations of Maliuzhinets type which can be solved [3]. The problem is now to find C_α as previously defined, which is equivalent to write,

$$(A_\alpha^\pm)(C_{\alpha\pm\Phi})^{-1} - \varepsilon^\pm (A_{-\alpha}^\pm)(C_{-\alpha\pm\Phi})^{-1} = 0 \tag{7}$$

Then we let,

$$C_\alpha^{-1} = \begin{pmatrix} b(\alpha) & -c(\alpha) \\ -d(\alpha) & a(\alpha) \end{pmatrix} \tag{8}$$

and note for the sake of simplicity, from now on, $a_+ = a(\alpha\pm\Phi)$, $a_- = a(-\alpha\pm\Phi)$ and so on for b, c, d and $A_\pm = A_{\pm\alpha}$ with no superscript indice of face $\varphi = \pm\Phi$.

After some rather tedious manipulations, one arrives to the necessary and sufficient conditions,

$$\left(\frac{b_+}{c_+}\right)\left(\frac{b_-}{c_-}\right)^{-1} = \frac{-g(a_+/c_+,\ d_-/b_-)}{g(d_+/b_+,\ a_-/c_-)} \tag{9a}$$

$$\frac{(b_+ a_+)}{(b_- a_-)} = \frac{\det A_-}{\det A_+} . \frac{1-(c_-/a_-)(d_-/b_-)}{1-(c_+/a_+)(d_+/b_+)} \tag{9b}$$

$$g(a_+/c_+,\ a_-/c_-) = g(d_+/b_+,\ d_-/b_-) = 0 \tag{9c}$$

where $a/c \neq d/b$ with,

$$g(r, s) = n . r . s - (l_+ . r - l_- . s) + n \tag{10}$$

where $n = \sin 2\alpha \cos\beta$ and from $\eta^\pm = 1$, $l_\pm = l = \cos^2\beta \sin^2\alpha - \cos^2\alpha$.

So letting $a(\alpha)/c(\alpha)=\tan(v(\alpha))$, one obtains from (9 c) and (10),

$$v'(\alpha\pm\Phi)+v'(-\alpha\pm\Phi)=[\operatorname{atan}(n/l)]' \tag{11}$$

where $(.)'$ is the derivative of the function $(.)$.

From use of the properties of Fourier transformations as in [2], [3], a regular solution of (11) for $|\operatorname{Re}\alpha|\leq\Phi$ is obtained,

$$v(\alpha)=i\int_0^{\infty}\frac{e^{-\nu(\pi/2-i\theta)}-e^{-\nu(\pi/2+i\theta)}}{[1-\exp(-\nu\pi)]}\cdot\left[\int\frac{\sinh[\nu(\alpha-\Phi)]-\sinh[\nu(\alpha+\Phi)]}{\sinh(2\nu\Phi)}d\alpha\right]d\nu \tag{12}$$

with $\pi/2-i\theta=\operatorname{atan}(i/\cos\beta)$. This expression can be completed for $|\operatorname{Re}\alpha|>\Phi$ with (11). Then from (9 c) and in order to simplify (9 a) we choose $d/b=\tan(v+\pi/2)$.

The expression of a/c and d/b can then be used in (9 a), (9 b). This allows to obtain $a=b$, $d=-c$ and,

$$a(\alpha)=\Psi(\alpha)\sin[v(\alpha)],\qquad c(\alpha)=\Psi(\alpha)\cos[v(\alpha)] \tag{13}$$

with,

$$\Psi(\alpha\pm\Phi)/\Psi(-\alpha\pm\Phi)=(-\sin\alpha\sin\beta\pm1)/(\sin\alpha\sin\beta\pm1) \tag{14}$$

where (14) is the usual equation that is encountered and solved in [4] for a wedge with the impedance face $\eta=1/\sin\beta$.

The principal problem of the determination of C_α satisfying (7) is then solved. Because $(C_{\pm\Phi})^{-1}$ is definite, one obtains from (7), $\varepsilon^{\pm}=1$. Now the equation (6) has to be solved which is rather easy with [2]-[4]. As in [1]-[4], the function $\sigma(\alpha)=\nu\cos\nu\varphi_0(\sin\nu\alpha-\sin\nu\varphi_0)^{-1}$, $\nu=\pi/2\Phi$, is used, which contains the pole corresponding to the incident wave [condition (a)] and satisfies $\sigma(\alpha\pm\Phi)-\sigma(-\alpha\pm\Phi)=0$. From behaviour at infinity of the function Ψ, v and σ, this allows to write the solution satisfying (a), (b) and boundary conditions in the form,

$$\begin{vmatrix} f_1(\alpha)\\ f_2(\alpha)\end{vmatrix}=(C_\alpha)^{-1}(C_{\varphi 0})\begin{vmatrix} D_1\\ D_2\end{vmatrix}.\sigma(\alpha)+(C_\alpha)^{-1}(T_\alpha)\begin{vmatrix} c_1\\ c_2\end{vmatrix} \tag{15}$$

where the second term of the right part is a particular solution of (6) which can be derived from [2]-[3]. Then we note that the first term of the right part of (15) has no pole that could lead to infinity behaviour of the associated field. Therefore we have no use of c_1, c_2 that can be set to zero and the analytical solution of the field defined in (2) can finally be obtained with,

$$\begin{vmatrix} f_1(\alpha) \\ f_2(\alpha) \end{vmatrix} = \frac{\Psi(\alpha)}{\Psi(\varphi_0)} \cdot \begin{pmatrix} \cos\Delta(\alpha) & \sin\Delta(\alpha) \\ -\sin\Delta(\alpha) & \cos\Delta(\alpha) \end{pmatrix} \begin{vmatrix} D_1 \\ D_2 \end{vmatrix} \cdot \sigma(\alpha) \tag{16}$$

where $\Delta(\alpha) = v(\alpha) - v(\varphi_0)$.

It has to be noticed that when $\beta = \pi/2$, $v(\alpha)$ is constant and the expression (16) gives us the known solution found for normal incidence [4].

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II.2.A) Errata et commentaires sur les démonstrations et formules

* p.523 : remplacer ".. being coupled with one equation as in normal incidence , the electric and magnetic fields must satisfy two equations by face [1]" par "..being uncoupled as in normal incidence , the components of the electric and magnetic fields parallel to the edge are coupled by two equations by face [1]"

* p.525 : remplacer ".. developped.." par ".. developed.."

PROPERTIES OF THE SOLUTION AND RECIPROCITY THEOREM FOR A CLASS OF WEDGE PROBLEM AT SKEW INCIDENCE.

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The problem of the diffraction of a plane wave with skew incidence on a wedge with impedance boundary condition is delicate ,because instead of involving ,as for TE or TM cases, one scalar equation by face [1] [2] ,two scalar equations are needed . The problem is then vectorial as pointed by Vaccaro in [3]. New results concerning an example of application of the method developed in [4] ,or more specifically in [5], for the skew incidence are indicated here.

The electric and magnetic components of the fields along the edge , E_z and H_z ,resulting from the scattering by a wedge of arbitrary angle and relative impedance of face unity (the most usual model for absorbing material), are given by ,

$$\left| \begin{matrix} E_z \\ Z_0 H_z \end{matrix} \right. = \frac{e^{-ikz\cos\beta}}{2\pi i} \cdot \int_{\mathcal{C}} \left| \begin{matrix} f_1(\alpha+\varphi) \\ f_2(\alpha+\varphi) \end{matrix} \right. \cdot e^{ik\rho\sin\beta\cos\alpha} \, d\alpha \, , \quad (1)$$

where ,

$|\psi| < \phi$ defines the external sector of the wedge in polar coordinates (fig. 1),

k , Z_0 being the free space wave number and impedance,

$$\begin{vmatrix} f_1(\alpha) \\ f_2(\alpha) \end{vmatrix} = \frac{\psi(\alpha)}{\psi(\varphi_0)} \cdot \begin{pmatrix} \cos\Delta(\alpha) & \sin\Delta(\alpha) \\ -\sin\Delta(\alpha) & \cos\Delta(\alpha) \end{pmatrix} \cdot \begin{vmatrix} D_1 \\ D_2 \end{vmatrix} \cdot \sigma(\alpha) \quad , \qquad (2)$$

with ,

ψ satisfying $\psi(\alpha\pm\phi)/\psi(-\alpha\pm\phi) = (-\sin\alpha\sin\beta\pm1)/(\sin\alpha\sin\beta\pm1)$

and σ , referenced in [1],

φ_0 , β , D_1, D_2 derived from the incident field components , $\begin{vmatrix} E_z^i \\ Z_0 H_z^i \end{vmatrix} = \begin{vmatrix} D_1 \\ D_2 \end{vmatrix} \cdot e^{ik(\rho\sin\beta\cos(\psi-\varphi_0) - z\cos\beta)}$,

$\Delta(\alpha) = v(\alpha) - v(\varphi_0)$.

The special function $v(\alpha)$ is developed here from [4,5] and presented in the following original form ,which is deduced from the power series of $(1-e)^{-1}$,

$$\begin{aligned} v(\alpha) = i \Big(\sum_{l=0}^{N} \sum_{\pm} \operatorname{Log} \Big[& \Gamma\Big(\tfrac{1}{2} + \tfrac{1}{4\phi}\big(\pm(\alpha-\phi) + \tfrac{\pi}{2} - i\theta + l\pi\big)\Big) \\ & \cdot \Gamma\Big(\tfrac{1}{2} + \tfrac{1}{4\phi}\big(\pm(\alpha+\phi) + \tfrac{\pi}{2} - i\theta + l\pi\big)\Big)^{-1} \cdot \Gamma\Big(\tfrac{1}{2} + \tfrac{1}{4\phi}\big(\pm(\alpha+\phi) \\ & + \tfrac{\pi}{2} + i\theta + l\pi\big)\Big) \cdot \Gamma\Big(\tfrac{1}{2} + \tfrac{1}{4\phi}\big(\pm(\alpha-\phi) + \tfrac{\pi}{2} + i\theta + l\pi\big)\Big)^{-1} \Big] \\ & + \int_0^{\infty} \frac{e^{-\nu(\pi/2 - i\theta)} - e^{-\nu(\pi/2 + i\theta)}}{(1 - e^{-\nu\pi})} \cdot e^{-\nu(N+1)\pi} \cdot \frac{\operatorname{ch}\nu(\alpha-\phi) - \operatorname{ch}\nu(\alpha+\phi)}{\nu \operatorname{sh} 2\nu\phi} d\nu \Big) , \end{aligned} \qquad (3)$$

with , $\pi/2 - i\theta = \arctan(i/\cos\beta)$,

for $|\operatorname{Re}\alpha| \leq \phi$, $N \geq -1$ (N=-1 in [4,5]) .

The integral term decreases rapidly as N increases ,and all the properties of v can be deduced ,as $N \to \infty$, from the properties of the gamma function Γ ,

$$\Gamma(z)\,\Gamma(1-z) = -z\,\Gamma(z)\,\Gamma(-z) = \pi / \sin(\pi z) \quad . \qquad (4)$$

An example of the use of the properties of v comes with the reciprocity theorem . This theorem is satisfied without any need for conditions at the edge internal junction when the boundary condition are of constant impedance type , so that the asymptotic term $E_{+}-E_{-}$, with ,

$$E_{\pm} = f_1(\pm\pi+\varphi)\, e^{-i\pi/4} / \sqrt{2\pi} \quad ,$$

has the symmetry $\varphi \leftrightarrow \varphi_0 \quad , \quad \beta \leftrightarrow \pi-\beta$.

Figures 3 and 4 illustrate this symmetry ,which comes from a specific property of v ,

$$v(\pm\pi+\varphi) - v(\varphi) = \pm i \log\left(\frac{\cos\left(\frac{\pi}{4\Phi}\left(\varphi-\Phi\pm\frac{\pi}{2}-i\theta\right)\right)\cos\left(\frac{\pi}{4\Phi}\left(\varphi+\Phi\pm\frac{\pi}{2}+i\theta\right)\right)}{\cos\left(\frac{\pi}{4\Phi}\left(\varphi-\Phi\pm\frac{\pi}{2}+i\theta\right)\right)\cos\left(\frac{\pi}{4\Phi}\left(\varphi+\Phi\pm\frac{\pi}{2}-i\theta\right)\right)} \right) \quad . (5)$$

Acknowledgments: this study has been supported by the DRET under contract n° 87414 .

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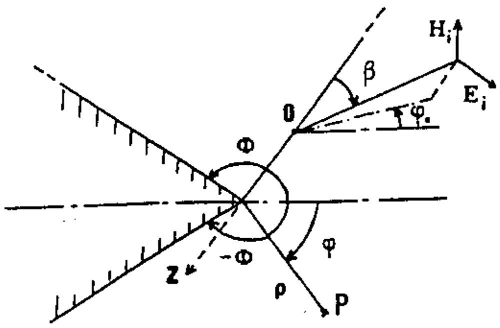

Figure 1 : Geometry of wedge

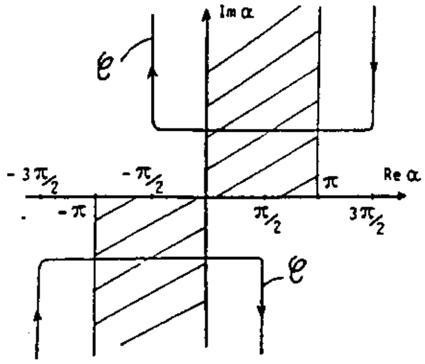

Figure 2 : Location of path $\mathscr{C}$

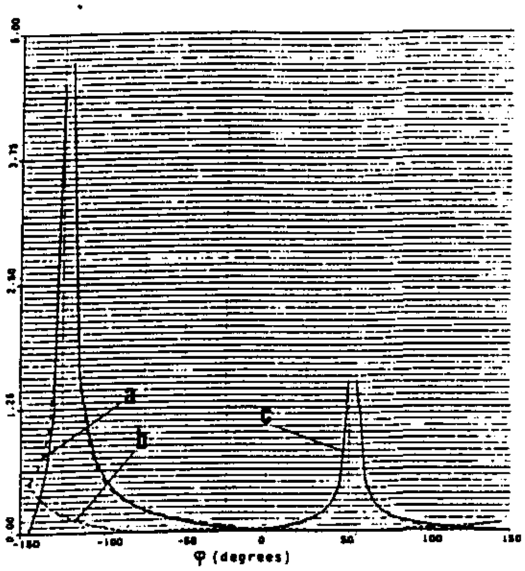

Figure 3 : Asymptotic terms (a) E_+ (b) E_- (c) $E_+ - E_-$ for $\varphi_0 = 57.5°$, $\beta = 30°$, $\phi = 145°$, $|E^i_z| = .5$, $H^i_z = 0.$.

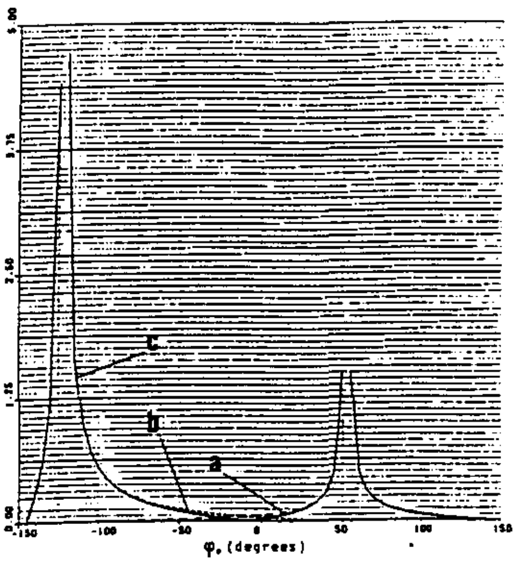

Figure 4 : Asymptotic terms (a) E_+ (b) E_- (c) $E_+ - E_-$ for $\varphi = 57.5°$, $\beta = 150°$, $\phi = 145°$, $|E^i_z| = .5$, $H^i_z = 0.$.

CHAPITRE III) Diffraction par un dièdre avec des conditions aux limites complexes , illuminé par une onde plane ou cylindrique quelconque en régime non stationnaire : problème vectoriel en régime temporel quelconque

III.1) J.M.L. Bernard *'On the time-domain scattering by a passive classical frequency dependent wedge-shaped region in a lossy dispersive medium'* , Ann. Telecom. , vol. 49, n° 11-12 , pp.673-683 , 1994 .

III.1.A) Errata et commentaires sur les démonstrations et formules

III.1.B) Compléments mathématiques

III.2) J.M.L. Bernard ,G. Pelosi,G. Manara,A. Freni ,*'Time domain scattering by an impedance wedge for skew incidence'*,Proceedings conf. ICEAA / Turin , pp.11-14 , 1991.

III.3) J.M.L. Bernard *'Progresses on the diffraction by a wedge : transient solution for line source illumination, single face contribution to scattered field , and new consequence of reciprocity on the spectral function'*, Rev. Techn. Thomson-Csf , Vol.25 , n°4 , pp.1209-1220 , déc. 1993 .

III.3.A) Errata et commentaires sur les démonstrations et formules

pp. 673-683

On the time-domain scattering by a passive classical frequency dependent wedge-shaped region in a lossy dispersive medium

Jean-Michel L. BERNARD *

Abstract

The time-scattering of an electromagnetic wave by a wedge-shaped region in a lossy medium, both with frequency dependent electric characteristics, is analyzed for plane wave and line source illuminations. New exact analytical expressions, satisfying explicitely the causality and allowing useful physical decompositions of the field, are obtained in these cases, using the particularities of the harmonic response in Sommerfeld-Maliuzhinets integral, especially original properties of the spectral function attached to it.

Key words : Wave diffraction, Dispersive medium, Wedge, Lossy medium, Non sinusoidal wave, Frequency characteristic, Electromagnetic wave, Plane wave, Cylindrical wave, Analytical wave.

SUR LA DIFFRACTION EN RÉGIME NON STATIONNAIRE DANS UN MILIEU DISPERSIF À PERTES PAR UN SECTEUR DIÈDRE CLASSIQUE PASSIF DONT LES CARACTÉRISTIQUES DÉPENDENT DE LA FRÉQUENCE

Résumé

La diffraction en régime non stationnaire d'une onde électromagnétique par un secteur dièdre dans un milieu à pertes, tous deux de caractéristiques électriques dispersives, est analysée pour des ondes incidentes plane et cylindrique (ligne source). Dans ces cas, on obtient des expressions analytiques exactes, satisfaisant explicitement la causalité et permettant d'intéressantes décompositions du champ, en utilisant certaines particularités de la représentation du champ en intégrale de Sommerfeld-Maliuzhinets en régime harmonique, particulièrement des propriétés originales de la fonction spectrale qui y est attachée.

Mots clés : Diffraction onde, Milieu dispersif, Coin, Milieu dispersif, Onde non sinusoïdale, Caractéristique fréquentielle, Onde électromagnétique, Onde plane, Onde cylindrique, Méthode analytique.

Contents

I. INTRODUCTION

The explicit transient solutions of diffraction problems are of particular mathematical interest concerning classical objects with edges. Several time-domain analytical solutions have already been derived for the canonical problem of electromagnetic wave scattering for perfectly [1-6] and non-perfectly conducting wedges [7-11]. The scattering of a time-dependent plane wave by a wedge defined in harmonic state with two different faces impedances has been analyzed in [8-10] by a method valid only when the surface impedances are frequency independent, whereas it is in [11] that, for the first time, this restriction has been removed.

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In this paper, we develop the theory to obtain time-domain analytical expressions for the diffraction, in a linear isotropic homogeneous lossy medium, by a wedge-shaped region, both frequency dependent. The Sommerfeld-Maliuzhinets integral has proven to be particularly important to express the field in frequency domain for wedge type problems [12-26]. Obtaining new properties of the spectral function attached to this form, we determine a new exact expression of the time response, satisfying explicitely causality and allowing useful physical decompositions of the field. We particularize the treatment following the behaviour of the spectral function f in the α complex plane for real frequency, distinguishing the classical case wherein $f(\alpha) = 0(e^{(1-a)|\mathrm{Im}\alpha|})$, $a \geq 0$, for $|\mathrm{Im}\alpha| \to \infty$, $|\mathrm{Re}\alpha| \leq \pi + \Phi$ (2Φ the external wedge angle), as this occurs when a plane wave illuminates an imperfectly conducting wedge of the type considered in [14-26]; this behaviour corresponds, for the far field radiation function F, when $\Phi \geq \pi/2$, to the case in which $F(\alpha)/\cos\alpha$ is integrable at infinity within the whole band $|\mathrm{Re}\alpha| \leq \Phi$. The method is developed, at once for a plane wave incident field, in chapter III, and extended to the case of a line source illumination, in chapter IV.

II. GENERAL CONSIDERATIONS ABOUT THE FIELD DERIVED FROM SOMMERFELD-MALIUZHINETS INTEGRAL

Let us consider an electromagnetic field with a direction of propagation of constant skew angle β with z axis. The cylindrical coordinates (ρ, φ, z) and the related cartesian axis are used (Fig. 1). Let us take the case wherein the sources of radiation are harmonic, in the wedge sector $|\varphi| \geq \Phi$; each component of the field is $0(e^{w\rho})$ as $\rho \to \infty$, w real, for $|\varphi| < \Phi$, summable with

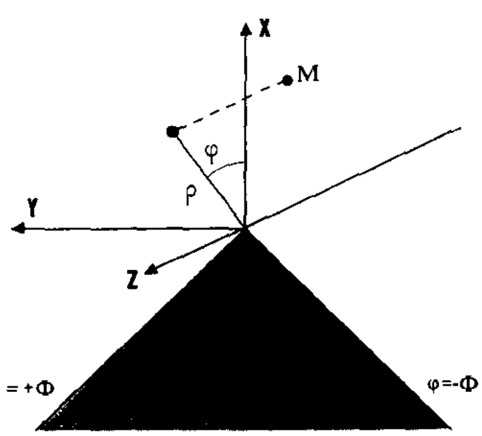

FIG. 1. — Geometry for the three-dimensional wedge.

Géométrie pour le dièdre en 3 dimensions.

respect to ρ in the vicinity of $\rho = 0$ (i.e. $\int_{\rho_1}^{\rho_2} |()|d\rho \to 0$ if $\rho_j \to 0$). They derive from the z-components of the electric and magnetic fields (see Appendix I), which tend to constants independent of φ at $\rho = 0$. Then, each cartesian component $\tilde{u}$ of the electric and magnetic field E and H can be written in the form of a Sommerfeld-Maliuzhinets integral [12-13] (Appendix I) :

$$(1) \quad \tilde{u}(\rho, \varphi, z, \omega) = \frac{e^{-ikz\cos\beta}}{2\pi i} \int_\gamma f(\alpha + \varphi, \omega)\, e^{ik\rho\sin\beta\cos\alpha}\, d\alpha,$$

for $|\varphi| < \Phi$, $f(\alpha, \omega)$ being an analytic function in α regular when $|\mathrm{Re}\ \alpha| < \Phi$ (except possibly at infinity), the dependence in time $e^{i\omega t}$ being suppressed, k being the wave number of the isotropic homogeneous lossy medium of propagation with $\mathrm{Im}\ k < 0$. The path γ is depicted in Figure 2a. It is composed of two symmetric loops with no singularity of the analytical function $f(\alpha + \varphi)$ on and within them, except possibly

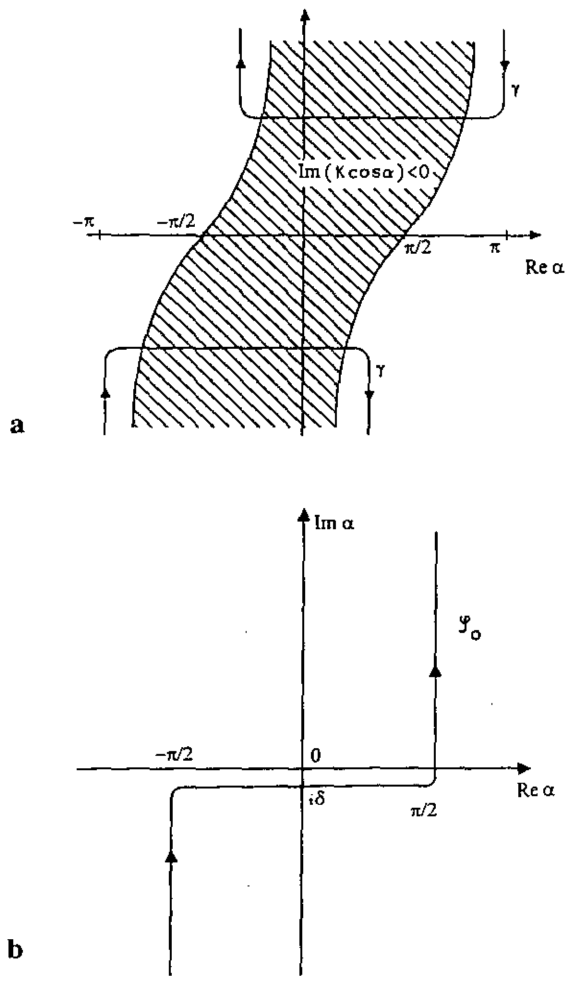

FIG. 2. — Contours of integration :
a) integration path γ for complex k;
b) integration path S_0 for real positive k.

Contours d'intégration :
a) chemin γ pour k complex ;
b) chemin S_0 pour k réel positif.

at infinity; one of them consists of the two half lines $\mathrm{Re}\ \alpha = \arg(\mathrm{i}k) \pm (\varepsilon + \pi/2)$, $\varepsilon > 0$, $\mathrm{Im}\ \alpha \geq d$ and a segment at $\mathrm{Im}\ \alpha = d$. The spectral function f can be expressed from [12], or more precisely from (A.14) given in Appendix II, for $\Phi > \pi/2$, according to :

$$\text{(2a)}\quad f(\alpha,\omega) = \frac{1}{4\pi\mathrm{i}} \int_{\mathcal{S}_0} F(\alpha',\omega) \tan\left(\frac{1}{2}(\alpha-\alpha')\right) \mathrm{d}\alpha' - \frac{1}{4\pi\mathrm{i}} \int_{l_s} f(\mp\pi+\alpha',\omega) \tan\left(\frac{1}{2}(\alpha-\alpha')\right) \mathrm{d}\alpha',$$

for $\tilde{u}$ finite at $\rho = 0$, or more generally,

$$\text{(2b)}\quad f(\alpha,\omega) = \frac{1}{4\pi\mathrm{i}} \int_{\mathcal{S}_0} F(\alpha',\omega)\left(\tan\left(\frac{1}{2}(\alpha-\alpha')\right) + \tan\left(\frac{1}{2}\alpha'\right)\right) \mathrm{d}\alpha' - \frac{1}{4\pi\mathrm{i}} \int_{l_s} f(\mp\pi+\alpha',\omega) \tan\left(\frac{1}{2}(\alpha-\alpha')\right) \mathrm{d}\alpha',$$

for $\tilde{u}$ summable in the vicinity of $\rho = 0$. The term F is the far field radiation function, so called because the field, if no sources at infinity, is equivalent, as $\rho \to \infty$, to $\tilde{u}_\infty$ with :

$$\text{(3a)}\quad F(\varphi,\omega) = -\sqrt{2\pi k\rho \sin\beta}\ \mathrm{e}^{\mathrm{i}(k\rho\sin\beta + kz\cos\beta + \pi/4)}\ \tilde{u}_\infty .$$

The expressions (2a) and (2b), valid only for α between $\mathcal{S}_0 - \pi$ and $\mathcal{S}_0 + \pi$, can be analytically continued in the whole complex plane from the equality :

$$\text{(3b)}\qquad F(\alpha,\omega) = f(\pi+\alpha,\omega) - f(-\pi+\alpha,\omega).$$

Let us notice that F is unique, but that a constant or a linear combination of $\sin\alpha$ and $\cos\alpha$ can be added to f without changing $\tilde{u}$. For the derivation of (2a)-(2b), the field is considered, at first, $0(\rho^N)$ as $\rho \to \infty$, N integer, in the region $|\varphi| \leq \pi/2$. Then, the contour $\mathcal{S}_0$ corresponds to $\mathrm{Im}\ (k \sin\alpha) = 0$ shifted with $\mathrm{i}\,\mathrm{sign}(\mathrm{Re}\ k)\delta$, $\pm\ \delta > 0$, given in Figure 2b when, in limit, k is real positive; the contour l_s is a closed contour around the singularities of $f(\mp\pi + \alpha, \omega)$ with $\mathrm{Im}\ k \sin\alpha = 0$, $|\mathrm{Im}\ \alpha| < d$. We must, however, notice that we can consider displacements of the singularities of f, which deform $\mathcal{S}_0$ and l_s, and allow the field to have an exponentially divergent behaviour at infinity. In some cases, the contour $\mathcal{S}_0$ can be selected to have the contribution of l_s in (2a)-(2b) equal to 0 : in particular, if the singularities of f, corresponding to plane waves not attenuated at infinity along $|\varphi| = \pi/2$, are radiated by only one face of the wedge $\varphi = \pm\ \Phi$ (as for a reflected plane wave, for instance), $f(\mp\pi + \alpha)$ has no singularity within l_s, which gives us zero.

Then, let us consider any wedge sector $|\varphi| \geq \Phi$ with electrical properties independent of z, illuminated with a z-dependence $\mathrm{e}^{-\mathrm{i}kz\cos\beta}$. The scattered field, since it is equal to the radiation of equivalent sources on the scatterer surface, can be expressed, with the previous conditions on the field, in the form (1) of a Sommerfeld-Maliuzhinets integral. Moreover, the representation (1) being valid for a plane wave, without any restriction on the directions of observation or incidence (Appendix III), the total field in presence of a wedge illuminated by a plane wave has the form (1).

Now, let us consider the transform to apply to $\tilde{u}$, $0(\omega^{-v})$ as $|\mathrm{Re}\ \omega| \to \infty$ for any finite $\mathrm{Im}\ \omega \leq 0$, $v > 1$, regular and analytic for any finite ω with $\mathrm{Im}\ \omega \leq 0$, in order to pass onto the time response $u(t)$, real and causal.

A first form is given by :

$$\text{(4)}\qquad u(t) = \frac{1}{\pi}\ \mathrm{Re} \int_0^{+\infty} \tilde{u}(\rho,\varphi,z,\omega)\ \mathrm{e}^{\mathrm{i}\omega t}\ \mathrm{d}\omega.$$

When (1) is used for $\tilde{u}$, (4) has the advantage of needing no change of γ with ω in the lossless case. Nevertheless, except when an exact suitable form of the integral is known, the term *real part* needs to be developed in the integral before searching an expression which exhibits all the properties of u.

So, let us introduce an other form. An important case is when (a) $f(\alpha + \varphi, \omega) = 0(\mathrm{e}^{(1-a)|\mathrm{Im}\alpha|})$, $a \geq 0$, for $|\mathrm{Im}\ \alpha| \to \infty$, $|\mathrm{Re}\ \alpha| \leq \pi$, $|\varphi| \leq \Phi$, which is equivalent for $\Phi > \pi/2$, provided the previous defined behaviour of $\tilde{u}$ at $\rho = 0$ is satisfied, to the condition on the far field function (see Appendix I) (b) $(F(\alpha,\omega)/\cos\alpha)$ is integrable at infinity (i.e. $\int_{\alpha_1}^{\alpha_2}()\ \mathrm{d}\alpha \to 0$ if $\mathrm{Im}\ \alpha_j \to +\infty$ (or $-\infty$)) for $|\mathrm{Re}\ \alpha| < \Phi$. Then, if it is possible to show that the asymptotic behaviour (a) remains, the contour γ can be deformed into the fixed paths $\mathrm{Re}\ \alpha = \pm\ \pi$ (summing the contribution of singularities if any intercepted during the deformation) for any ω with $\mathrm{Im}\ \omega \leq 0$. In this case, the expression of $u(t)$ as an inverse Laplace transform must be prefered to (4) according to :

$$\text{(5)}\qquad u(t) = \frac{1}{2\pi} \int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma} \tilde{u}(\rho,\varphi,z,\omega)\ \mathrm{e}^{\mathrm{i}\omega t}\ \mathrm{d}\omega,$$

with $\sigma \geq 0$. The term *real part* of (4) has disappeared and it is then possible to use at best the analytical properties of the spectral function f to translate the reality and the causality of u.

The latter form, with the condition (a) satisfied (assumed for ω real and extended for $\mathrm{Im}\ \omega \leq 0$), is used in chapter III for the problem of the diffraction by a wedge with a plane wave illumination. The method developed is then extended in chapter IV when a line source field is incident and a double Sommerfeld-Maliuzhinets integral needed.

III. PLANE WAVE ILLUMINATION

III.1. Causal time expression of the total field.

The diffraction of a plane wave by a passive wedge-shaped sector $|\varphi| \geq \Phi$ in a lossy medium is now considered. The electric properties of the sector are supposed to be independent of z, and the behaviour of the field, as described at the beginning of the previous chapter. The wedge is considered to belong to the class of scatterers, which, for ω real, have a spectral function f satisfying the asymptotic behaviour (a) equivalent, for $\Phi \geq \pi/2$, to the condition (b) on the far field radiation function. It is also assumed that it exists Φ' so that $f(\alpha+\varphi,\omega)$ has no singularities in ω for ω real, $|\mathrm{Re}\,(\alpha)| < \pi$ and $|\varphi| < \Phi'$. These conditions are not very restrictive since they can be satisfied, for instance, when differential boundary conditions are considered on each face of the wedge as in [14-26]. Besides, let us note that Φ', depending on the problem, is equal to Φ in the case of passive impedance boundary conditions [14-18]. The time-dependent plane wave impinges on the edge from a direction which is determined by the two angles β and φ'. The angle β is a measure of the incident direction skew with respect to the edge of the wedge and $\beta = \pi/2$ corresponds to normal incidence.

Each cartesian component of the total field in time domain, noted indifferently u, is expressed, as prefered in the previous chapter, by :

$$(6)\quad u(\rho,\varphi,z,t) = \frac{1}{2\pi}\int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma} \tilde{u}(\rho,\varphi,z,\omega)\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}\omega,$$

with $\sigma \geq 0$. In (6), $\tilde{u}$ is the response to an incident time-harmonic plane wave field deriving from the z-components vector $\tilde{\boldsymbol{u}}_i = (E_{\mathrm{i}z}, Z_0 H_{\mathrm{i}z})$ (Appendix I) of the form :

$$(7)\quad \tilde{\boldsymbol{u}}_i(\rho,\varphi,z,\varphi',\beta,\omega) = U^i(\omega)\mathrm{e}^{\mathrm{i}k\rho\sin\beta\cos(\varphi-\varphi')}\,\mathrm{e}^{-\mathrm{i}kz\cos\beta}\,\boldsymbol{e},$$

with $U^i(\omega) = 0(\omega^{-b})$ for $|\omega| \to \infty$, $b > 1$ assumed so $\tilde{u} = 0(\omega^{-v})$ with $v > 1$ as $|\mathrm{Re}\,\omega| \to \infty$ for any finite $\mathrm{Im}\,\omega \leq 0$, regular and analytic for any finite ω with $\mathrm{Im}\,\omega \leq 0$, Z_0 the impedance of the medium of propagation, E_{iz} and H_{iz} the z-components of the electric and magnetic incident field, $\boldsymbol{e} = (e_i, Z_0 h_i)$ a constant vector. The wave number k is equal to $\zeta(\omega)\omega/c$, c being the speed of the light, ζ depending on the characteristic of the lossy medium with $\zeta \to 1$ when $|\omega| \to \infty$, $\mathrm{Im}\,k < 0$ (except possibly for ω real infinite) and ζ regular for $\mathrm{Im}\,\omega \leq 0$. As in the previous chapter, $\tilde{u}$ is taken in the form of a Sommerfeld-Maliuzhinets integral :

$$(8)\quad \tilde{u}(\rho,\varphi,z,\omega) = \frac{\mathrm{e}^{-\mathrm{i}kz\cos\beta}}{2\pi\mathrm{i}} \int_\gamma f(\alpha+\varphi,\omega)\mathrm{e}^{\mathrm{i}k\rho\sin\beta\cos\alpha}\,\mathrm{d}\alpha,$$

where the contour γ is the integration path depicted in Figure 2a. In order to simplify the notation, the dependence of the fields from the space coordinates (observation point and direction of incidence), if not strictly necessary, is suppressed and understood subsequently.

Now, some useful properties of the spectrum function $f(\alpha+\varphi,\omega)$ and its inverse Fourier transform are derived, principally as consequences of the reality and of the causality of the wedge response. They will be used to obtain a new expression of u.

Let us consider the Maliuzhinets' inversion formula [13] (Appendix I) :

$$(9)\quad f(\alpha+\varphi,\omega) - f(-\alpha+\varphi,\omega) = -\mathrm{i}k\sin\beta\sin\alpha\mathrm{e}^{\mathrm{i}kz\cos\beta} \int_0^{+\infty} \tilde{u}(\rho,\varphi,z,\omega)\mathrm{e}^{-\mathrm{i}k\rho\sin\beta\cos\alpha}\,\mathrm{d}\rho,$$

for $\mathrm{Im}(k\sin\beta\cos\alpha) < \mathrm{Im}\,k\sin\beta$ in our case, $|\mathrm{Re}\,\alpha| < \pi$; let us note that the angles $\pm\alpha+\varphi$, loci of the singularities of f corresponding to the incident, reflected and transmitted (if the wedge is not opaque) plane waves, are then excluded. From the reality of the field, directly follows that $\tilde{u}(-\omega) = [\tilde{u}(\omega)]^*$ for ω real. This can be used in (9) to obtain (see Appendix IV) :

$$(10)\qquad f(\alpha+\varphi,-\omega) - [f(\alpha^*+\varphi,\omega)]^* = \mathrm{i}\kappa_1,$$

where κ_1 is a constant with respect to α which, without loss of generality, we can choose equal to zero. Let us consider now $\mathrm{Re}\,(\alpha) = 0$. The spectral function $f(\alpha+\varphi,\omega)$ is then regular in ω, for ω real : the scattered field is composed of outgoing waves and the only singularity of $f(\alpha,\omega)$ in the band $|\mathrm{Re}\,(\alpha)| < \Phi$, except possibly at infinity, is the pole in α associated with the incident field at $\alpha = \varphi'$, independent of ω. Considering that the electric characteristics of the wedge tend to a constant as $\pm\omega \to \infty$, $f(\alpha,\omega) \sim U^i(\omega)f_0(\alpha)$ at infinity. Then, an inverse Fourier transform of $f(\alpha+\varphi,\omega)$, denoted by $\mathcal{F}(\alpha+\varphi,\tau)$, can be performed. From (9) and the regularity of $\tilde{u}$ for $\mathrm{Im}\,\omega \leq 0$, the following equality is obtained :

$$(11)\quad \mathcal{F}(\alpha+\varphi,\tau) - \mathcal{F}(-\alpha+\varphi,\tau) = -\frac{\sin\alpha\sin\beta}{2\pi c}\lim_{\sigma\to 0}\frac{\partial}{\partial\tau}\int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma}\zeta(\omega)\,\mathrm{e}^{\mathrm{i}\omega\tau+\mathrm{i}kz\cos\beta} \int_0^{+\infty}\tilde{u}(\omega)\mathrm{e}^{-\mathrm{i}k\rho\sin\beta\cos\alpha}\,\mathrm{d}\rho\mathrm{d}\omega.$$

By interchanging the order of the integration, the expression (11) can be written as :

$$(12)\quad \mathcal{F}(\alpha+\varphi,\tau) - \mathcal{F}(-\alpha+\varphi,\tau) = -\frac{\sin\alpha\sin\beta}{2\pi c}\lim_{\sigma\to 0}\frac{\partial}{\partial\tau} \int_0^\infty\left(\int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma}\zeta(\omega)\tilde{u}(\omega)\,\mathrm{e}^{\mathrm{i}\omega t'}\,\mathrm{d}\omega\right)\mathrm{d}\rho,$$

where $ct' = c\tau + \zeta(\omega)\ (z\cos\beta - \rho\sin\beta\ \cos\alpha)$. The quantity ζ is complex but we can use the fact that it tends to 1 when $|\omega| \to \infty$. $u(t)$ is causal with respect to the front of the incident plane wave, so that, in (12), the contour of integration in ω can be closed at infinity for $\mathrm{Im}\,(\omega) < 0$ when $ct'|_{|\omega|\to\infty} < (z\cos\beta - \rho\sin\beta)$; in this case, as $\zeta(\omega)$ and $\tilde{u}(\omega)$ are non singular for any finite ω with $\mathrm{Im}\,\omega < 0$, the integral is null. Accordingly, since $\cos\alpha = \mathrm{ch}\mu > 1$, we obtain, when $\tau < 0$:

$$(13)\qquad \mathcal{F}(\mathrm{i}\mu+\varphi,\tau) - \mathcal{F}(-\mathrm{i}\mu+\varphi,\tau) = 0.$$

Moreover, by taking into account relationship (10), it is found that :

(14) $\mathcal{F}(-\mathrm{i}\mu+\varphi,\tau)-(\mathcal{F}(\mathrm{i}\mu+\varphi,\tau))^*=0.$

Adding (13) and (14), it is easily seen that, for $\tau<0$,

(15) $\mathcal{F}(\mathrm{i}\mu+\varphi,\tau)=(\mathcal{F}(\mathrm{i}\mu+\varphi,\tau))^*.$

The inverse Fourier transform $\mathcal{F}(\alpha',\tau)$ of f used in (15) is an analytical function of $\alpha'=\mathrm{i}\mu+\varphi$. Thus, when $\tau<0$, we obtain :

(16) $\mathcal{F}(\alpha+\varphi,\tau)=\kappa_2,$

where κ_2 is a real constant with respect to α which, without loss of generality, can be chosen equal to 0. The property (16) can be extended to the whole region $|\mathrm{Re}\ (\alpha)|<\pi$, $|\varphi|<\Phi'$: $f(\alpha+\varphi,\omega)$ having no singularities in ω for ω real, when $|\mathrm{Re}\ (\alpha)|<\pi$ and $|\varphi|<\Phi'$, and being $0(U^i(\omega))$ as $\pm\ \omega\to\infty$, $\mathcal{F}(\alpha+\varphi,\tau)$ is a function which presents a unique analytical expression when α and φ lie in the bands just defined.

By complement, we note that, when $\omega\to\pm\infty$, $\tilde{u}$ tends to be finite, $\zeta\to 1$, and thus, from (9), $f(\alpha+\varphi,\omega)$ tends to be regular for $\pm$ Im $(\cos\alpha)<0$; accordingly, all the singularities of $f(\alpha+\varphi,\omega)$ independent of ω, for $|\mathrm{Re}\ (\alpha)|<\pi$, are located on the real axis; they are assumed isolated, not essential ones.

Consequently, for $|\mathrm{Re}\ (\alpha)|<\pi$, $|\varphi|<\Phi'$, the Fourier transform of $\mathcal{F}$ in τ becomes a Laplace transform which analytically continues $f(\alpha+\varphi,\omega)$ as a regular analytic function in ω for Im $(\omega)\le 0$ with $f(\alpha+\varphi,\omega)\sim U^i(\omega)f_0(\alpha+\varphi)$ as $|\omega|\to\infty$, and in α, except possibly for Im $\alpha\ \neq\ 0$ and at infinity, with the behaviour $0(\mathrm{e}^{(1-a)|\mathrm{Im}\ \alpha|})$ when $|\mathrm{Im}\ \alpha|\to\infty$, $a\ge 0$, initially supposed for ω real at the beginning of the chapter.

Note on the extension for φ' complex :

- If we want to consider φ' complex, we have to assume that it exists Φ'' so that $f(\alpha+\varphi,\omega)$ has no singularity in ω for ω real, when $|\mathrm{Re}\ (\alpha)|<\pi$, $|\varphi|<\Phi'$ and $|\mathrm{Re}\ \varphi'|<\Phi''$. The response $\tilde{u}$ with φ' complex independent of ω does not satisfy $\tilde{u}(-\omega)=(\tilde{u}(\omega))^*$ and its inverse Fourier transform does not exist, but (16) can be continued analytically in φ'. Then, by this argument, last sentence can be adapted following : « Consequently, for $|\mathrm{Re}\ (\alpha)|<\pi$, $|\varphi|<\Phi'$ and $|\mathrm{Re}\ \varphi'|<\Phi''$, the Fourier transform of $\mathcal{F}$ in τ becomes a Laplace transform which analytically continues $f(\alpha+\varphi,\omega)$ as a regular analytical function in ω for Im $(\omega)\le 0$ with $f(\alpha+\varphi,\omega)\sim U^i(\omega)f_0(\alpha+\varphi)$ as $|\omega|\to\infty$, and in α, except possibly at singular points independent of ω and at infinity, with the behaviour $0(\mathrm{e}^{(1-a)|\mathrm{Im}\ \alpha|})$ when $|\mathrm{Im}\ \alpha|\to\infty$, $a\ge 0$, as initially supposed for ω real at the beginning of the chapter ». Let us notice that, for the passive impedance case, $\Phi''=\Phi'=\Phi$.

At this step, we can begin to manipulate the expression of the field. According to the behaviour of f, the Sommerfeld integration path can be deformed into the $\mathcal{D}$ contour depicted in Figure 3a, with $\varepsilon'=\mathrm{i}0^+$. Therefore, from expressions (6)-(8), we obtain, for $|\varphi|<\Phi'$:

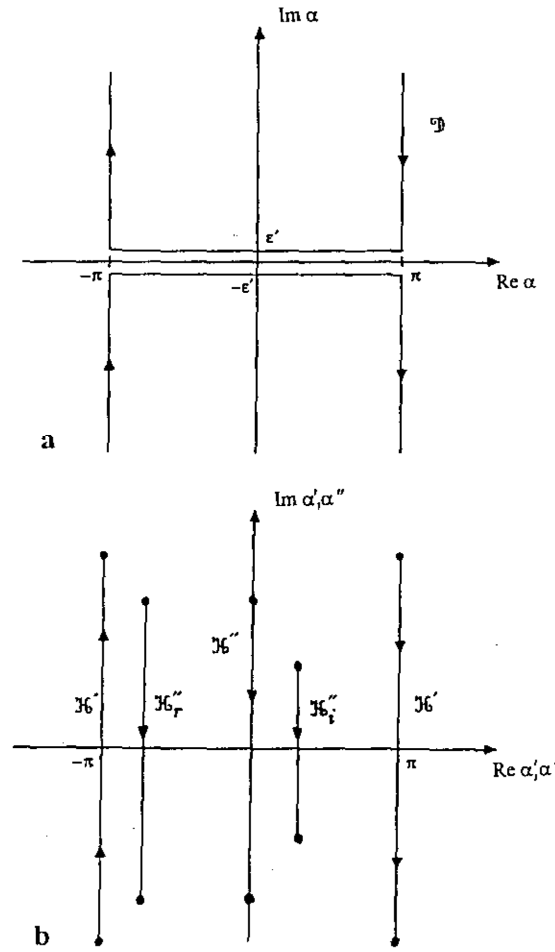

fig 3b must be exchanged for fig. 4

FIG. 3. — Contours of integration for plane wave illumination :
a) integration path $\mathcal{D}$;
b) integration path $\mathcal{H}$ (α_τ is the value of α for which $\tau(\alpha)=0^-$).

Contours d'intégration pour une illumination plane :
a) chemin $\mathcal{D}$;
b) chemin $\mathcal{H}$ (α_τ est la valeur de α pour laquelle $\tau(\alpha)=0^-$).

(17)
$$u(t)=\frac{1}{2\pi}\int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma}\frac{1}{2\pi\mathrm{i}}\int_{\mathcal{D}}f(\alpha+\varphi,\omega)\mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\alpha)}\,\mathrm{d}\alpha\mathrm{d}\omega,$$

with $\sigma>0$, $\tau_\zeta(\alpha)=\zeta(\omega)(\rho\sin\beta\cos\alpha-z\cos\beta)/c+t$. Then, let us define $\mathcal{F}_\zeta$, given by :

(18)
$$\mathcal{F}_\zeta(\alpha+\varphi,\tau(\alpha),t)=\frac{1}{2\pi}\int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma}f(\alpha+\varphi,\omega)\mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\alpha)}\,\mathrm{d}\omega,$$

for $\tau(\alpha)=\tau_\zeta(\alpha)|_{|\omega|\to\infty}=(\rho\sin\beta\cos\alpha-z\cos\beta)/c+t$ real. The functions $f(\alpha,\omega)$ and $\zeta(\omega)$ having no singularities in ω for Im $\omega\le 0$, $|\mathrm{Re}\ \alpha|<\pi+\Phi'$, $\mathcal{F}_\zeta(\alpha+\varphi,\tau(\alpha),t)$ is equal to zero for $\tau(\alpha)$ real negative, and consequently, we obtain, for $|\varphi|<\Phi'$ (Φ' being equal to Φ in the case of passive impedance conditions on the faces of the wedge) :

(19)
$$u(t)=\frac{1}{2\pi}\int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma}\frac{1}{2\pi\mathrm{i}}\int_{\mathcal{H}(t,\rho,z)}f(\alpha+\varphi,\omega)\ \mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\alpha)}\,\mathrm{d}\alpha\mathrm{d}\omega,$$

where $\mathcal{H}(t,\rho,z)$ is the finite integration path depicted in Figure 3b. Furthermore, if we can suppose it exists ε_1 so the electric characteristics of the wedge continue to tend to constants and $f(\alpha+\varphi,\omega) \sim U^i(\omega) f_0(\alpha+\varphi)$ as $|\omega| \to \infty$ with $|\arg(\omega^2)| < 2\varepsilon_1$, $\mathcal{F}_\zeta$ can be analytically continued when $|\arg \tau(\alpha)| < \varepsilon_1$. Then, it can be written :

$$(20) \quad u(t) = \frac{1}{2\pi \mathrm{i}} \int_{\mathcal{H}(t,\rho,z)} \mathcal{F}_\zeta(\alpha+\varphi, \tau(\alpha), t)\, \mathrm{d}\alpha.$$

Let us notice that (20) can be extended to the region $\Phi' < |\varphi| < \Phi$, $\mathcal{H}$ being deformed continuously by any singularity which could move in the band $|\mathrm{Re}\ \alpha| < \pi$. Now, let us show the interest of the expression (19) with the impedance wedge case.

III.2. Discussion of the impedance wedge case.

In the passive impedance case, the electric field E and magnetic field H are related on each face of the wedge $\varphi = \pm\,\Phi$ by $E - n^{\pm}(n^{\pm}E) = Z^{\pm} n^{\pm} \wedge H$, for harmonic state, where $n^{\pm}$ is the unit vector along the outward-pointing normal to the face, and $Z^{\pm}$ the impedance with Re $Z^{\pm} > 0$. Then, the spectral function f has been completely derived only for β equal to $\pi/2$ [14], for the half plane [15], for the plane discontinuity, for the right angled wedge [16-17] and for an absorbing wedge with unit relative impedance [18], nevertheless, the behaviour and the physical meaning of all singularities of f are well identified [18]. The spectral function f satisfies the condition (a) and (19) applies with $\Phi' = \Phi$. So, (19) can be used and discussed in this case.

As shown in Figure 3b, $\mathcal{H}(t,\rho,z)$ is composed by a closed contour $\mathcal{H}_g$ corresponding to $\max[0; t(\rho,z)] > \tau(\alpha) > \max[0; t(-\rho,z)]$, and by two limited branches $\mathcal{H}_e$ at Re $\alpha = \pm\pi$, corresponding to $t(-\rho,z) > \tau(\alpha) > 0$, with $t(\rho,z) = (\rho\sin\beta - z\cos\beta)/c + t$. The contour $\mathcal{H}_g$ includes all the singularities of $f(\alpha+\varphi,\omega)$, which are displaced along the real axis with $|\mathrm{Re}\ \alpha| < \pi$. For the impedance wedge case, these singularities are first order poles which correspond to the incident and reflected fields, that constitute the geometrical optics part u_g of the total field. The open contour $\mathcal{H}_e$ is directly related to an edge field contribution u_e, which exists only for $t(-\rho,z) > 0$.

Thus, the expression (19) of u can be developed according to :

$$(21) \qquad u(t) = u_g(t) + u_e(t),$$

for $(\rho\sin\beta + z\cos\beta) < ct$, and :

$$(22) \qquad u(t) = u_g(t),$$

for $(\rho\sin\beta + z\cos\beta) > ct$. The term u_g and u_e are expressed as :

$$(23) \quad u_g(t) = \frac{1}{2\pi} \int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma} \sum_{\alpha_s} \mathrm{Residue}[f(\alpha+\varphi,\omega)\mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\alpha)}]|_{\alpha=\alpha_s}\, \mathrm{d}\omega,$$

where α_s are the poles of $f(\alpha+\varphi,\omega)$ along the real axis, with $\max[0, t(\rho,z)] > \tau(\alpha_s) > \max[0, t(-\rho,z)]$, and :

$$(24) \quad u_e(t) = \frac{1}{2\pi} \int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma} \mathrm{d}\omega \frac{1}{2\pi\mathrm{i}} \int_{\mathcal{H}_e} f(\alpha+\varphi,\omega)\, \mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\alpha)}\, \mathrm{d}\alpha.$$

The two regions being in evidence are identified by the following inequalities $(\rho\sin\beta + z\cos\beta) > ct$ and $(\rho\sin\beta + z\cos\beta) < ct$. In the first region only the incident and reflected fields are present, while in the second, a term u_e, related to the presence of the edge of the wedge, arises.

These results can be compared with the one obtained in [8]. In [8], the field expressions, related to the two-dimensional case ($\beta = \pi/2$), are obtained when an independence of the surface impedances with ω is assumed. Our approach, though considering frequency dependent media, continues to provide equivalent physical insights in the solution, specifying $\rho\sin\beta + z\cos\beta < ct$ and $\rho\sin\beta + z\cos\beta > ct$ regions, and satisfying explicitly the causality principle.

Finally, it is important to note that the UTD (or GTD) time-domain expressions of the edge field, developed in [6] for the perfectly conducting wedge, can be recovered from (24) by simply developing $f(\alpha+\varphi)/\sin(\alpha/2)$ as a rational function (or Taylor series) in terms of powers of $\cos(\alpha/2)$, considering (or not) the proximity of the poles.

Note. — In the impedance case, for $\beta = \pi/2$, the spectral function f related to the z components of the field has the form :

$$f(\alpha,\omega) = U^i(\omega) \frac{\Psi(\alpha,\omega)}{\Psi(\varphi',\omega)}\, \sigma(\alpha),$$

where σ containing the poles of incident and reflected fields doesn't depend of ω; the remaining part possesses all the properties in ω of f derived in § III.1.

IV. LINE SOURCE ILLUMINATION

IV.1. The form of the frequency response.

Now, we consider that the previous wedge is illuminated by the field radiated by a harmonic line source, parallel to the edge of the wedge. All the components of the incident field derive from the z-components vector $\tilde{\boldsymbol{u}}^i = (E_{iz}, Z_0 H_{iz})$ (Z_0 the impedance of the medium of propagation) :

$$(25) \quad \tilde{\boldsymbol{u}}_i = \frac{-\mathrm{i}}{4}\, \mathrm{e}^{-\mathrm{i}kz\cos\beta}\, U^i(\omega) H_0^{(2)}(k\sin\beta R)\, \boldsymbol{e},$$

with $R = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\varphi-\varphi')}$, (ρ', φ') being the coordinates of the line source, $\boldsymbol{e} = (e_i, Z_0 h_i)$ a constant vector. From [12], the z-components vector

$(E_{sz}, Z_0 H_{sz})$ of the scattered field in the frequency domain is given by a double Sommerfeld-Maliuzhinets integral according, for $\Phi \geq \pi/2$, to :

$$(26)\quad \tilde{u}_s(\rho,\varphi) = \frac{\mathrm{e}^{-\mathrm{i}kz\cos\beta}}{2\pi\mathrm{i}} \int_\gamma f_s(\alpha+\varphi,\omega)\, \mathrm{e}^{\mathrm{i}k\rho\sin\beta\cos\alpha}\, \mathrm{d}\alpha,$$

where :

$$(27)\quad f_s(\alpha+\varphi,\omega) = \frac{\mathrm{i}}{4\pi} \int_\gamma f_{sd}(\alpha+\varphi,\alpha'+\varphi',\omega)\, \mathrm{e}^{\mathrm{i}k\rho'\sin\beta\cos\alpha'}\, \mathrm{d}\alpha',$$

with $f_{sd} = \overline{\overline{f}}_{sd} e$ given by :

$$(28)\quad \overline{\overline{f}}_{sd}(\alpha,\alpha',\omega) = \frac{1}{4\pi\mathrm{i}} \int_{S_0} ({}^a\overline{\overline{f}}(\alpha',\alpha'',\omega) - \overline{\overline{f}}_i(\alpha',\alpha'',\omega)) \tan\left(\frac{1}{2}(\alpha-\alpha'')\right) \mathrm{d}\alpha'',$$

and its analytic continuation. The term ${}^a\overline{\overline{f}}$ is defined, so that $\overline{\overline{f}} e$ is the vector spectral function attached to the z-components vector of the total field $(E_z, Z_0 H_z)$ obtained for a plane wave illumination as in chapter III, but determined by reciprocity for a skew angle $\pi - \beta$, with $({}^a B)_{ii} = (B)_{ii}$, $({}^a B)_{ij} = -(B)_{ji}$ $i \neq j$, with unchanged electrical characteristics when the wedge-shaped region is considered isotropic, or, more generally with transposed electrical characteristics if the wedge is anisotropic; $\overline{\overline{f}}_i$ is defined so that $\overline{\overline{f}}_i e = \frac{1}{2} U^i(\omega) \mathrm{cotan}\left(\frac{1}{2}(\alpha'-\alpha'')\right) e$ is attached to the z-components vector of the incident field for plane wave illumination (Appendix III). Let us note that this expression is well distinct from a Fourier transform used in [27] and that its form doesn't need to change according to the source position and the observation point.

IV.2. Causal time expression in impedance case.

Only a part of the frequency far field radiation function related to the problem depicted in chapter IV.1. satisfies the condition (b) at infinity, and this part is no more analytic whereas it was such in chapter III. It corresponds to the part of f_s with γ deformed into the branches at Re $\alpha' = \pm\,\pi$, then deprived of all contributions of singularities which would go through the path. For the impedance wedge case considered now, this part is associated to the edge field.

Neglecting the contribution of poles of f, the paths γ, considered for $\tilde{u}_s$ in (26)-(28), are deformed to the branches at Re $\alpha, \alpha' = \pm\,\pi$, and the term $\tilde{u}_{se}$, that we name edge field, is obtained. Then, applying an inverse Laplace transform to it, and using the properties of f in ω complex plane determined in chapter III which have permitted to write (19), we can derive the time domain expression u_{se} :

$$(29)\quad u_{se}(t) = \frac{1}{2\pi} \int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma} \mathrm{d}\omega \left(\frac{1}{8\pi} \int_{\mathcal{H}''} \frac{1}{\pi} \int_{\mathcal{H}'} \mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\alpha',\alpha'')} f(\alpha'+\varphi', \alpha''+\varphi,\omega)\, \mathrm{d}\alpha' \mathrm{d}\alpha'' \right),$$

where, $f = {}^a\overline{\overline{f}} e$, $\tau_\zeta(\alpha',\alpha'') = \zeta(\omega)((\rho'\cos\alpha' - \rho\cos\alpha'')\sin\beta - z\cos\beta)/c + t$, $\mathcal{H}''$ and $\mathcal{H}'$ being two finite paths, respectively on the branch Re $\alpha'' = 0$ and the branches Re $\alpha' = \pm\pi$ (Fig. 4), so that :

$$(30)\quad \tau(\alpha',\alpha'') = \tau_\zeta(\alpha',\alpha'')|_{|\omega|\to\infty} = ((\rho'\cos\alpha' - \rho\cos\alpha'')\sin\beta - z\cos\beta)/c + t > 0.$$

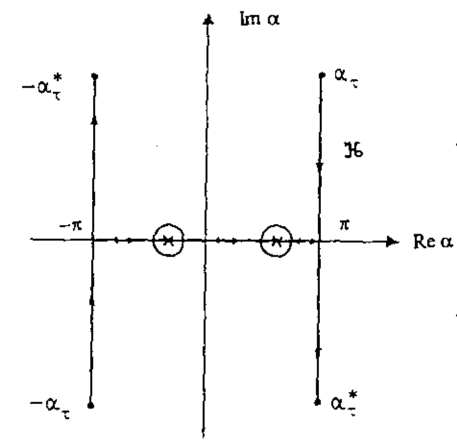

fig. 4 must be exchanged for fig. 3b

FIG. 4. — Contours of integration for line source illumination.
Contours d'intégration pour illumination cylindrique (ligne source).

Let us note that it is possible to change (29)-(30) by reciprocity, according to :

$$(31)\quad f = {}^a\overline{\overline{f}} e \text{ for skew angle } \pi-\beta \to f = \overline{\overline{f}} e \text{ for skew angle } \beta,\ \varphi \to \varphi', \varphi' \to \varphi, \rho \to \rho', \rho' \to \rho,$$

and then to obtain :

$$(32)\quad u_{se}(t) = \frac{1}{2\pi} \int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma} \mathrm{d}\omega \left(\frac{1}{8\pi} \int_{\mathcal{H}''} \frac{1}{\pi} \int_{\mathcal{H}'} \mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\alpha',\alpha'')} f(\alpha'+\varphi, \alpha''+\varphi',\omega)\, \mathrm{d}\alpha' \mathrm{d}\alpha'' \right),$$

with :

$$\tau_\zeta(\alpha',\alpha'') = \zeta(\omega)((\rho\cos\alpha' - \rho'\cos\alpha'')\sin\beta - z\cos\beta)/c + t,$$

$$(33)\quad \tau(\alpha',\alpha'') = \tau_\zeta(\alpha',\alpha'')|_{|\omega|\to\infty} = ((\rho\cos\alpha' - \rho'\cos\alpha'')\sin\beta - z\cos\beta)/c + t.$$

In the latter form, (32) can be used directly for any Cartesian component of the field, with f, instead of f, the spectral function associated to it for a plane wave illumination, as defined in chapter III.

Now, let us recover the expression of the total field from the edge field. When $|\varphi'| > \pi - \Phi$, $|\varphi - \varphi'| > \pi$, it is a shadowed zone and we easily verify that the total field $\tilde{u}$ is equal to $\tilde{u}_{se}$. Then, from an analytical continuation of $\tilde{u}_{se}$ in φ and φ', the Heaviside function being denoted U, the expression of the total field $u(t)$ for $\Phi \geq \pi/2$ follows :

$$(34) \quad u(t) = u_{se}(t) + U(\pi - |\varphi - \varphi'|)u_i(t) + \sum_{\pm} U(\pi - |\pm 2\Phi - (\varphi + \varphi')|)u_r^{\pm}(t);$$

the term u_i corresponds to the pole of incident field in f and is written :

$$(35) \quad u_i(t) = \frac{1}{2\pi}\int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma} \mathrm{d}\omega \frac{\mathrm{i}}{4\pi} \int_{\mathcal{H}_i''} \mathrm{d}\alpha'' \mathrm{Residue}[\mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\alpha',\alpha'')} f(\alpha' + \varphi, \alpha'' + \varphi', \omega)]|_{\alpha'=\alpha''+\varphi'-\varphi},$$

$\mathcal{H}_i''$ being a finite path on Re $\alpha'' = \Omega_i$ branch ($|\Omega_i| < \pi$), so that $(\rho\cos(\alpha'' + \varphi' - \varphi) - \rho'\cos\alpha'') = -R_i\cos(\alpha'' - \Omega_i)$ is a real negative quantity, and $\tau(\alpha'' + \varphi' - \varphi, \alpha'')$ is positive (Fig. 4); the terms $u_r^{\pm}$ correspond to the poles of reflected fields in f and are written as :

$$(36) \quad u_r^{\pm}(t) = \frac{1}{2\pi}\int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma} \mathrm{d}\omega \frac{\mathrm{i}}{4\pi} \int_{\mathcal{H}_r''} \mathrm{d}\alpha'' \mathrm{Residue}[\mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\alpha',\alpha'')} f(\alpha' + \varphi, \alpha'' + \varphi', \omega)]|_{\alpha'=\pm 2\Phi-(\alpha''+\varphi'+\varphi)},$$

$\mathcal{H}_r''$ being a finite path on Re $\alpha'' = \Omega_r$ line ($|\Omega_r| < \pi$), so that $(\rho\cos(\pm 2\Phi - (\alpha'' + \varphi' + \varphi)) - \rho'\cos\alpha'') = -R_r^{\pm}\cos(\alpha'' - \Omega_r)$ is a real negative quantity and $\tau(\pm 2\Phi - (\alpha'' + \varphi' + \varphi), \alpha'')$ is positive (Fig. 4).

Then, let us notice some properties of the terms u_{se}, u_i, u_r which, in particular, allows $u(t)$ to satisfy the causality, i.e., to be null for $ct - z\cos\beta < 0$. From (32), u_{se} exists only if $ct - z\cos\beta > (\rho + \rho')\sin\beta$, which corresponds properly to the necessary path source-edge-observation point of the edge field. The term u_i is null for $ct - z\cos\beta < R_i\sin\beta$, R_i being the straight path between the line source and the observation point; as we expect, it doesn't depend on the electric characteristics of the wedge. The term $u_r^{\pm}$, directly related to the reflected field, is null for $ct - z\cos\beta < R_r^{\pm}\sin\beta$, $R_r^{\pm}$ being the straight path between the observation point and the image line source corresponding to the face $\pm\,\Phi$; its expression contains the residue of f with the reflection coefficients of the faces.

Notes.

• The residue term related to the z-components of the field in (35) is $U^i(\omega)$ if e neglected in this case,

$$u_i(t) = \frac{1}{2\pi}\int_{-\infty-\mathrm{i}\sigma}^{+\infty-\mathrm{i}\sigma} \mathrm{d}\omega \left(\frac{\mathrm{i}}{4\pi}\int_{\mathcal{H}_i''} \mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\alpha''+\varphi'-\varphi,\alpha'')} U^i(\omega)\mathrm{d}\alpha''\right).$$

For $\zeta = 1$ (free space), $U^i = 1$, the term u_i is then given by :

$$u_i(t) = U(t - z\cos\beta/c - R_i\sin\beta/c)\frac{1}{2\pi\sqrt{(t - z\cos\beta/c)^2 - (R_i\sin\beta/c)^2}}.$$

• For $\beta = \pi/2$, e neglected, the residue term related to the z-component of the field is in (36) :

$$\mathrm{Residue}[\;] = U^i(\omega)\mathrm{e}^{\mathrm{i}\omega\tau_\zeta(\pm 2\Phi-(\alpha''+\varphi'+\varphi),\alpha'')} \frac{\sin(\Phi \mp (\alpha'' + \varphi')) - \sin\theta^{\pm}}{\sin(\Phi \mp (\alpha'' + \varphi')) + \sin\theta^{\pm}},$$

where the reflection coefficient in $\sin\theta^{\pm} = Z^{\pm}/Z_0$ or $Z_0/Z^{\pm}$ can be recognized.

• A Fourier transform applied to the limit form (34) with $t \to \infty$, gives us a new form of the field in frequency domain corresponding to (26) developed in the impedance case. Let us notice that, for (32), paths of integration can then be deformed to the steepest descent paths, summing the contributions of the poles which could be captured during the deformation.

V. CONCLUSION

The time response of a wedge-shaped sector surrounded by a lossy medium, both frequency dependent, has been originally developed by using the particular form of the field in Sommerfeld-Maliuzhinets integral in frequency domain. A particular causality principle, derived on the inverse Fourier transform of the spectral function associated to the integral for plane wave illumination, explicites the properties of this function in ω complex plane.

These properties are used to obtain original analytical causal expressions for plane wave and line source illuminations with finite paths of integration on complex angle, when the whole field or a part of it (a part which permits by analytic continuation to recover everywhere all the field) has a far field radiation function $F(\alpha)$ with $F(\alpha)/\cos\alpha$ integrable at infinity within a certain band of α for ω real, as it is the case, for instance, for an impedance wedge. The solution gives a useful physical insight, allowing to decompose the field in several contributions, in particular the part due to the edge.

APPENDIX I

Let us consider a harmonic electromagnetic field with a z dependence $\mathrm{e}^{-\gamma z}$, $\gamma = \mathrm{i}k\cos\beta$. Let ε, μ, k, be the permittivity, the permeability and the associated wave number. The application of Maxwell equations leads us to the expressions of the Cartesian components of the electric and magnetic field E and H according to :

$$\text{(A.1)}\quad E_x = \frac{-\gamma}{\gamma^2+k^2}\frac{\partial E_z}{\partial x} - \frac{\mathrm{i}\omega\mu}{\gamma^2+k^2}\frac{\partial H_z}{\partial y},$$

$$\text{(A.2)}\quad E_y = \frac{-\gamma}{\gamma^2+k^2}\frac{\partial E_z}{\partial y} + \frac{\mathrm{i}\omega\mu}{\gamma^2+k^2}\frac{\partial H_z}{\partial x},$$

$$\text{(A.3)}\quad H_x = \frac{-\gamma}{\gamma^2+k^2}\frac{\partial H_z}{\partial x} + \frac{\mathrm{i}\omega\varepsilon}{\gamma^2+k^2}\frac{\partial E_z}{\partial y},$$

$$\text{(A.4)}\quad H_y = \frac{-\gamma}{\gamma^2+k^2}\frac{\partial H_z}{\partial y} - \frac{\mathrm{i}\omega\varepsilon}{\gamma^2+k^2}\frac{\partial E_z}{\partial x},$$

Let us take, at first, the case described in chapter II, the field being radiated by sources in the region $|\varphi| \geq \Phi$. The expression :

$$\text{(A.5)}\quad \tilde{u}(\rho,\varphi,z) = \frac{\mathrm{e}^{-ikz\cos\beta}}{2\pi\mathrm{i}}\int_\gamma f(\alpha+\varphi)\mathrm{e}^{ik\rho\sin\beta\cos\alpha}\,\mathrm{d}\alpha,$$

and the inversion formula of Maliuzhinets [13] :

$$\text{(A.6)}\quad f(\alpha+\varphi)-f(-\alpha+\varphi) = -\mathrm{i}k\sin\beta\sin\alpha\mathrm{e}^{ikz\cos\beta}\int_0^{+\infty}\tilde{u}(\rho,\varphi,z)\mathrm{e}^{-ik\rho\sin\beta\cos\alpha}\mathrm{d}\rho,$$

with $f(\alpha')$ an analytical function regular for any finite α' when $|\mathrm{Re}\ \alpha'| < \Phi$ (no ingoing plane wave incident on the wedge), $\mathrm{Im}(k\sin\beta\cos\alpha) < -w$, $|\mathrm{Re}\ \alpha| < \pi$, $|\varphi| < \Phi$, are verified, from [12-13], for the z-components of the field. In particular, the z-components of the field radiated by any line source in the region $|\varphi| \geq \Phi$, absorbed at infinity ($\mathrm{Im}\ k < 0$) and derived from Hankel function $H_0^{(2)}$, has a known spectral function f_H given in [12]; $f_H(\alpha)$ is naturally regular when $|\mathrm{Re}\ \alpha| < \Phi$ (the regularity need to be imposed when we search f the solution of the functional equations deduced from (A.6) with given values of $\tilde{u}$ for $\varphi = \pm\ \Phi$, as in [13]); in any case, the regularity condition must be absolutely specified if we mean to consider $\tilde{u} = 0(\mathrm{e}^{w\rho})$ with $w \geq -\ \mathrm{Im}\ k\sin\beta$. The formula (A.5) and (A.6), then verified for the z-components, are satisfied for the x and y components, with the spectral function associated to them and obtained by replacing, in (A.1 - A.4), $\frac{\partial\tilde{u}}{\partial x}$ by $f_1(\alpha) = \mathrm{i}k\sin\beta\cos\alpha(f(\alpha)+a_1)+b_1\sin\alpha$ and $\frac{\partial\tilde{u}}{\partial y}$ by $f_2(\alpha) = \mathrm{i}k\sin\beta\sin\alpha(f(\alpha)+a_2)+b_2\cos\alpha$ (a_j and b_j constants being chosen so that $f_j(\alpha)/\sin\alpha = o(1)$ as $|\mathrm{Im}\ \alpha| \to \infty$) with f the spectral function related to $\tilde{u}$. We notice that, in the case of an additional plane wave illuminating the wedge as in chapter III, (A.5 - A.6) apply, this time, to $f(\alpha)$ with a pole when $|\mathrm{Re}\ \alpha| < \Phi$.

Now let us consider the condition given in the chapter II $(a) f(\alpha+\varphi) = 0(\mathrm{e}^{(1-a)|\mathrm{Im}\alpha|})$, $a \geq 0$, for $|\mathrm{Im}\ \alpha| \to \infty$, $|\mathrm{Re}\ \alpha| \leq \pi$, $|\varphi| < \Phi$. The z-components tending to constants, independent of φ, at $\rho = 0$, and being $0(\mathrm{e}^{w\rho})$ as $\rho \to \infty$ for $|\varphi| < \Phi$, we obtain, from (A.6), that the spectral functions $f(\alpha+\varphi)$ related to them tend to constants, as $\mathrm{Im}\ \alpha \to \pm\ \infty$ and $\mathrm{Im}\ k\sin\beta\cos\alpha < -w$ (with $\tilde{u}(\rho=0) = \mathrm{i}(f(\mathrm{i}\infty)-f(-\mathrm{i}\infty))$), so that $a = 1$ in (a). The regularity of f at infinity then obtained, allows us to write that the far field radiation function given by :

$$F(\alpha) = f(\pi+\alpha) - f(-\pi+\alpha),$$

is integrable for $|\mathrm{Im}\ \alpha| \to \infty$ (i.e. $\int_{\alpha_1}^{\alpha_2}()\mathrm{d}\alpha \to 0$ if $\mathrm{Im}\ \alpha_j \to +\infty$ (or $-\infty$)), $|\mathrm{Re}\ \alpha| < \Phi$. More generally, with (A.1 - A.4), we deduce that F divided by $\cos\alpha$ is integrable when f are related to any component. Reciprocally, if the far field radiation function divided by $\cos\alpha$ is integrable at infinity for $|\mathrm{Re}\ \alpha| < \Phi$, the spectral function is given by (2b) for $\Phi > \pi/2$, and then satisfies (a) with $a = 0$.

APPENDIX II

Determination of the spectral function corresponding to the radiation of equivalent currents on each face $\varphi = \pm\ \Phi$ of a wedge-shaped region.

Let $\tilde{u}$ be a Cartesian component of the field associated to the sources being in the wedge-shaped region $|\varphi| \geq \Phi > \pi/2$. From the Green theorem on equivalent sources, the field $\tilde{u}$ for $|\varphi| < \pi/2$ can be written as the sum of $\tilde{u}_+$ and $\tilde{u}_-$, the radiation of equivalent surface currents carried respectively by the faces $\varphi = +\ \pi/2$ and $-\ \pi/2$, according to the expression :

$$\text{(A.7)}\quad \tilde{u}_\pm(\rho,\varphi) = -\frac{\mathrm{i}}{4}\mathrm{e}^{-ikz\cos\beta}\int_{\varphi=\pm\pi/2,z=0}\tilde{u}\frac{\partial H_0^{(2)}(kR\sin\beta)}{\partial n} - \frac{\partial\tilde{u}}{\partial n}H_0^{(2)}(kR\sin\beta)\mathrm{d}\rho_i,$$

with $R = \sqrt{\rho^2+\rho_i^2 \mp 2\rho\rho_i\sin\varphi}$, $\mathrm{Im}\ k < 0$, supposing, at first, that $\tilde{u}$ and $\frac{\partial\tilde{u}}{\partial n}$ are $0(\rho^N)$ as $\rho \to \infty$, $|\varphi| \leq \pi/2$, and respectively finite and summable at $\rho = 0$. One can let, for $|\varphi| < \Phi$, the total field $\tilde{u}$ in the form of a Sommerfeld-Maliuzhinets integral :

$$\text{(A.8)}\quad \tilde{u}(\rho,\varphi) = \frac{\mathrm{e}^{-ikz\cos\beta}}{2\pi\mathrm{i}}\int_\gamma f(\alpha+\varphi)\mathrm{e}^{ik\rho\sin\beta\cos\alpha}\,\mathrm{d}\alpha,$$

with $f(\alpha+\varphi)$ tending to constants $f(\pm\ \mathrm{i}\infty)$ as $\mathrm{Im}\ \alpha \to \pm\ \infty$ within γ, and $f(\mathrm{i}\infty)+f(-\ \mathrm{i}\infty) = 0$. On the other hand, from [12] :

$$\text{(A.9)}\quad H_0^{(2)}(kR\sin\beta) = \frac{1}{2\pi\mathrm{i}}\int_\gamma\frac{-1}{2\pi}\int_{S_0}\mathrm{e}^{ik\rho_i\sin\beta\cos(\alpha'\mp\pi/2)}\left(\tan\left(\frac{1}{2}(\alpha+\varphi-\alpha')\right)-g_0(\alpha')\right)\mathrm{d}\alpha'\mathrm{e}^{ik\rho\sin\beta\cos\alpha}\,\mathrm{d}\alpha,$$

S_0 being a path from $-\ \mathrm{i}\infty - \arg(ik)$ to $\mathrm{i}\infty + \arg(ik)$, and g_0, normally unnecessary because of the oddness of

γ, chosen as $g_0(\alpha) = -\tan(\alpha/2)$ in order to simplify the demonstration. Because $\tilde{u}$ is $0(\rho^N)$ as $\rho \to \infty$, $|\varphi| \leq \pi/2$, $f(\alpha+\varphi)$ has no singularity in the domain defined by Im $k\cos\alpha < 0$; then, we can deform γ into γ_0, composed by two infinite branches with Im $k\cos\alpha > 0$ separated by 2π as $|\text{Im}\ \alpha| \to \infty$ and admitting no singularity within them. Then, shifting $\mathcal{S}_0$ with isign(Re $k)\delta$ with respect to the path Im $k\sin\alpha = 0$, $\pm\ \delta > 0$, and considering that 2π is the period of the exponential term in (A.8), we can inverse the paths of integration and integrate in ρ_i. Because $f(\mathrm{i}\infty) + f(-\ \mathrm{i}\infty) = 0$, we can close γ_0 at infinity and evaluate the integration on γ_0 by the method of residues. Then, the following expression is obtained :

$$\text{(A.10)}\quad \tilde{u}_\pm(\rho,\varphi) = \frac{\mathrm{e}^{-\mathrm{i}kz\cos\beta}}{2\pi\mathrm{i}} \int_\gamma f_\pm(\alpha+\varphi)\mathrm{e}^{\mathrm{i}k\rho\sin\beta\cos\alpha}\,\mathrm{d}\alpha,$$

with :

$$\text{(A.11)}\quad f_\pm(\alpha) = \frac{1}{4\pi\mathrm{i}} \int_{\mathcal{S}_0} \pm f(\pm\pi+\alpha')\left(\tan\left(\frac{1}{2}(\alpha-\alpha')\right) - g(\alpha')\right)\mathrm{d}\alpha',$$

for α between $\mathcal{S}_0 - \pi$ and $\mathcal{S}_0 + \pi$, $\mathcal{S}_0$ given in Figure 2b, when k is, in limit, real positive, with $\pm\ \delta > 0$, and g arbitrarily chosen to make the integral converge. One will let :

$$\text{(A.12)}\quad f_\pm(\alpha) = \frac{1}{4\pi\mathrm{i}}\ Pg \int_{\mathcal{S}_0} \pm f(\pm\pi+\alpha')\left(\tan\left(\frac{1}{2}(\alpha-\alpha')\right)\right)\ \mathrm{d}\alpha'.$$

It is worth noticing that it is possible to choose the condition $f_\pm(\mathrm{i}\infty) = -f_\pm(-\ \mathrm{i}\infty)$, usually used to fix the expression of spectral functions, only if g can be equal to zero. This expression of $f_\pm$ can be continued analytically by the equality :

$$\text{(A.13)}\quad f_\pm(\pi+\alpha) - f_\pm(-\pi+\alpha) = \pm f(\pm\pi+\alpha).$$

Let us add that the previous formula (A.8)-(A.13) can be generalized to $f(\alpha)$ with the behaviour $f(\alpha) = 0(\mathrm{e}^{(1-a)|\mathrm{Im}\alpha|})$, $a > 0$, corresponding to any component of the field, summable in vicinity of $\rho = 0$. We will take the precaution to define the function f so $\tilde{u}_\pm$ comprises no term $d_1 H_0^{(2)}(k\rho\sin\beta)$, with d_1 a constant. This choice corresponds to the possibility we have to add a constant to f without changing the total field $\tilde{u}$.

As a direct consequence of (A.10)-(A.13), we can obtain the expression (2) of $f = f_+ + f_-$. To this end, the paths $\mathcal{S}_0$ will be deformed into a same path of integration, with particular attentions to the contribution of possible singularities of $f(\mp\pi+\alpha)$ with Im $(k\sin\alpha) = 0$, corresponding to plane waves, each one radiating with no attenuation along $|\varphi| = \pi/2$. Thus, we get :

$$\text{(A.14)}\quad f(\alpha) = \frac{1}{4\pi\mathrm{i}}\ Pg \int_{\mathcal{S}_0} F(\alpha')\tan\left(\frac{1}{2}(\alpha-\alpha')\right)\mathrm{d}\alpha' - \frac{1}{4\pi\mathrm{i}}\int_{l_s} f(\mp\pi+\alpha')\tan\left(\frac{1}{2}(\alpha-\alpha')\right)\mathrm{d}\alpha',$$

where $F(\alpha) = f(\pi+\alpha) - f(-\pi+\alpha)$, $\mathcal{S}_0$ corresponding to Im $k\sin\alpha = 0$ shifted with isign(Re $k)\delta$, $\pm\ \delta > 0$, and l_s a closed contour around the singularities of $f(\mp\pi+\alpha)$ with Im $k\sin\alpha = 0$, $|\text{Im}\ \alpha| < d$. However, let us notice that it is possible to consider displacements of the singularities which can deform $\mathcal{S}_0$ and l_s, and allow the field to have an exponentially divergent behaviour at infinity. Besides, (A.14) easily allows to recover one result given in [12], considering a lossy passive wedge illuminated by a source disposed between one face and the plane $|\varphi| = \pi/2$: this result consists to use (A.14) for f related to the total field, and then, select δ so the term with l_s in (A.14) is zero even if the sources radiate discrete plane waves. Let us note that another form of this development can be found in [28].

APPENDIX III

The z-components vectors $\tilde{\boldsymbol{u}}_i = (E_{iz}, Z_0 H_{iz})$ of the field radiated by a harmonic plane wave of direction (φ', β) can be expressed as :

$$\text{(A.15)}\quad \tilde{\boldsymbol{u}}_i(\rho,\varphi,z,\omega) = \frac{\mathrm{e}^{-\mathrm{i}kz\cos\beta}}{2\pi\mathrm{i}} \int_\gamma \boldsymbol{f}_i(\alpha+\varphi,\varphi',\omega)\mathrm{e}^{\mathrm{i}k\rho\sin\beta\cos\alpha}\,\mathrm{d}\alpha,$$

with $\boldsymbol{f}_i(\alpha,\alpha') \equiv \dfrac{U^i(\omega)}{2}\mathrm{cotan}\left(\dfrac{1}{2}(\alpha-\alpha')\right)\boldsymbol{e}$, $\boldsymbol{e} = (e_i, Z_0 h_i)$ a constant vector. The spectral function of each Cartesian component of the field can be then derived from the general expressions given in Appendix I.

APPENDIX IV

For ω real, the use of $\tilde{u}(\omega) = (\tilde{u}(-\omega))^*$ in (9) allows to write :

$$\text{(A.16)}\quad f(\alpha+\varphi,-\omega) - f(-\alpha+\varphi,-\omega) = [f(\alpha^*+\varphi,\omega) - f(-\alpha^*+\varphi,\omega)]^*,$$

for any φ real with $|\varphi| < \Phi$. From (A.16) follows $\partial_\varphi f(\varphi,-\omega) - [\partial_\varphi f(\varphi,\omega)]^* = 0$ which, by the analycity of f, gives us :

$$\text{(A.17)}\quad f(\alpha+\varphi,-\omega) - [f(\alpha^*+\varphi,\omega)]^* = \mathrm{i}\kappa_1,$$

where κ_1 is a constant with respect to α and φ.

Let us notice about (9), for ω real, Im $k < 0$, that the growth of the field at infinity is implied by the behaviour of the incident plane wave of real incidence angle : consequently, $\tilde{u}$ is $0(\mathrm{e}^{-\mathrm{Im}k\rho\sin\beta})$ as $\rho \to \infty$, and then (9) can be used, in particular, for any α with Re $\alpha = 0$, except at the origin.

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BIOGRAPHY

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III.1.A) Errata et commentaires sur les démonstrations et formules

* p. 677 et 679 : les contenus des figures **3.b** et **4** doivent être échangés
* p. 677 : lire $\mathrm{Im}\,\alpha = 0$ au lieu de $\mathrm{Im}\,\alpha \neq 0$
* pour l'appendice II , p.681 et 682 , certaines précisions peuvent être apportées :

. les formules (A.7)-(A.9) peuvent être réécrites , pour une lecture plus aisée , ainsi :

$$\text{(A7)} \quad \tilde{u}_{\pm}(\rho,\varphi) = -\frac{i}{4}\,\mathrm{e}^{-ikz\cos\beta} \int_{\varphi_i = \pm\pi/2\,,z=0} \tilde{u}(\rho_i,\varphi_i)\,\frac{\partial H_o^{(2)}(kR\sin\beta)}{\partial n} - \frac{\partial\tilde{u}(\rho_i,\varphi_i)}{\partial n}\,H_o^{(2)}(kR\sin\beta)\;\mathrm{d}\rho_i\;,$$

with $R = \sqrt{\rho^2 + \rho_i^2 - 2\rho\rho_i\cos(\varphi - \varphi_i)}$, ...

$$\text{(A.9)}\quad H_o^{(2)}(kR\sin\beta) =$$

$$= \frac{1}{2\pi i}\int_{\gamma} \frac{-1}{2\pi}\int_{\mathcal{S}_o} \mathrm{e}^{ik\rho_i\sin\beta\cos(\alpha'-\varphi_i)}\left(\mathrm{tg}\left(\tfrac{1}{2}\,(\alpha+\varphi-\alpha')\right) - g_o(\alpha')\right)d\alpha'\,\mathrm{e}^{ik\rho\sin\beta\cos\alpha}\,d\alpha \;,\ldots$$

. au cours de la démonstration aboutissant à (A.11) , on intègre en ρ_i. Soit γ_o le contour γ déformé, défini dans l'appendice , comportant deux branches infinies identiques séparées de 2π ne comportant aucune singularité de $f(\alpha+\varphi)$ entre elles , on obtient une expression comportant le terme

$$\int_{\gamma_o} \left[\, \frac{\epsilon(\sin(\alpha' - \epsilon\frac{\pi}{2}) - \sin\alpha)}{\cos(\alpha' - \epsilon\frac{\pi}{2}) + \cos\alpha}\, f(\alpha+\varphi)\right]\,\mathrm{d}\alpha$$

où $\epsilon = +\,ou -1$. Montrons que cette intégrale a bien un sens . En effet , pour α' donné , l'expression [.] sous le signe intégrale est non singulière pour $|\mathrm{Im}\alpha| > A$. En coupant γ_o de deux segments de droites définis par $\mathrm{Im}\alpha = \mathrm{a}_1$ et a_2 , tels que $|\mathrm{a}_i| > A$, on constitue alors un contour fermé sur lequel l'intégrale est nulle . De ce fait , et $\lim_{|Im\alpha|\to\infty}$ [.] existant , la partie de l'intégrale suivant γ_o , avec $\mathrm{Im}\alpha$ compris entre a_1 et a_2 , tend vers 0 quand $|\mathrm{a}_i| \to \infty$, ce qui montre que l'intégrale précédente est bien définie .

. on peut ajouter , comme remarque sur les expressions (A11) et (A12) , que $f_{\pm}$ donnée par

$$\text{(A11)} \qquad f_{\pm}(\alpha) = \frac{1}{4\pi i} \int_{\mathcal{S}_0} \pm f(\pm\pi + \alpha')\left(\operatorname{tg}(\tfrac{1}{2}(\alpha - \alpha')) - g(\alpha')\right) \mathrm{d}\alpha' ,$$

peut être infinie quand $|\mathrm{Im}\alpha| \to \infty$, si $\pm f(\epsilon\pi + \alpha') \to constante \neq \mathbf{0}$ (avec $\epsilon = +ou-1$) quand $\pm\mathrm{Im}\alpha' \to \infty$ sur $\mathcal{S}_0$ (dans ce cas g est indispensable à la convergence de l'intégrale) . Pour autant , notons que cela n'entraîne pas un comportement divergent de $u_{\pm}$ à l'origine car

$$f_{\pm}(\pi + \alpha) - f_{\pm}(-\pi + \alpha) = \pm f(\pm\pi + \alpha)$$

nous permet d'écrire :

$$\tilde{u}_{\pm}(\rho,\varphi) = \frac{-\mathrm{e}^{-ikz\cos\beta}}{2\pi i} \int_{\mathcal{S}_0} \pm f(\pm\pi + \alpha + \varphi)\, \mathrm{e}^{-ik\rho\sin\beta\cos\alpha} \mathrm{d}\alpha \; +$$

$$+ \text{ termes de résidus ,}$$

expression qui , en général , reste finie lorsque $\rho \to 0$, puisque dans le cas le plus courant $|f(\epsilon\pi + \varphi + \alpha) + f(\epsilon\pi + \varphi - \alpha)|$ est intégrable suivant le contour $\mathcal{S}_0$.

* On note que , pour la détermination de $\mathcal{H}''$ et $\mathcal{H}'$, la condition

$$\tau(\alpha',\alpha'') = \tau_{\zeta}(\alpha',\alpha'')\,\big|_{|\omega| \to \infty} = ((\rho\cos\alpha' - \rho'\cos\alpha'')\sin\beta - z\cos\beta)/c + t \;> 0 ,$$

pour (32) , signifie qu'on détermine d'abord $\mathcal{H}''$ (ou resp. $\mathcal{H}'$) par la condition $\tau(\pm\pi,\alpha'') > 0$ (ou resp. $\tau(\alpha',0) > 0$) , puis $\mathcal{H}'$ (ou resp. $\mathcal{H}''$) par $\tau(\alpha',\alpha'') > 0$. Remarquons que $\mathcal{H}'$ (resp. $\mathcal{H}''$) tend vers un chemin infini quand ρ (resp. ρ') tend vers 0 , autorisant ainsi certaine composante du champ à être singulière à l'origine .

III.1.B) Complément mathématique : condition de Meixner et finitude des composantes du champ parallèle à l'arête .

Considérons qu'en régime harmonique , chaque composante E_z ou H_z , notée ũ , du champ parallèle à l'arête , en présence du dièdre , domaine passif avec pertes illuminé par une onde plane , satisfait aux deux conditions suivantes:

(a) ũ , $\frac{\rho\partial \tilde{u}}{\partial\rho}$, et $\frac{\partial\tilde{u}}{\partial\varphi}$ ont des parties réelles et imaginaires fonction de ρ , analytiques (excepté à l'origine), et monotones en un voisinage de l'origine $\rho < \epsilon$;

(b) la fonction spectrale $f(\alpha)$, associée à ũ , est une fonction analytique telle qu'il existe un polynôme $f_p(\alpha)$ de façon que $|f(\alpha) - f_p(\alpha)| \to 0$ et $|f'(\alpha) - f'_p(\alpha)| \to 0$ quand $|\mathrm{Im}\alpha| \to \infty$.

La condition de Meixner de sommabilité du carré du module du champ au voisinage d'une singularité nous permet d'écrire :

$$\int_{\rho_1}^{\rho_2} \left|\frac{\rho\partial\tilde{u}}{\partial\rho}\right|^2 \frac{1}{\rho}\, \mathrm{d}\rho \mathrm{d}\varphi \quad \text{et} \quad \int_{\rho_1}^{\rho_2} \left|\frac{\partial\tilde{u}}{\partial\varphi}\right|^2 \frac{1}{\rho}\, \mathrm{d}\rho \mathrm{d}\varphi \to 0 \text{ quand } \rho_i \to 0 \ ,$$

ce qui entraîne , d'après (a) ,

$$\frac{\rho\partial\tilde{u}}{\partial\rho} \text{ et } \frac{\partial\tilde{u}}{\partial\varphi} \to 0 \text{ quand } \rho \to 0 \ .$$

La fonction ũ est alors au voisinage de l'origine $o(\rho^{-a})$, quel que soit $a > 0$. On peut l'écrire , pour $|\varphi| < \Phi$, sous la forme :

$$\tilde{u}(\rho,\varphi) = \frac{\mathrm{e}^{-ikz\cos\beta}}{2\pi i} \int_\gamma f(\alpha + \varphi)\, \mathrm{e}^{ik\sin\beta\rho\cos\alpha}\, \mathrm{d}\alpha \ ,$$

avec , d'après le théorème d'inversion de Maliuzhinets ,

$$f(\alpha+\varphi) - f(-\alpha+\varphi) =$$

$$= -ik\sin\beta \ \mathrm{tg}\alpha \ e^{ikz\cos\beta} \int_0^{+\infty\cos\alpha} \tilde{u}(x/\cos\alpha,\varphi,z) e^{-ik\sin\beta x} \ \mathrm{d}x \ ,$$

pour $\mathrm{Im}(k\sin\beta\cos\alpha) < \mathrm{Im}(k\sin\beta) < 0$, $|\mathrm{Re}\alpha| < \pi$. D'après les conditions précédentes de convergence déduites du théorème de Meixner , il s'en suit que $f'(\alpha+\varphi)+f'(-\alpha+\varphi)$ et $f'(\alpha+\varphi)-f'(-\alpha+\varphi) \to 0$ quand $|\mathrm{Im}\alpha| \to \infty$, et que donc $f'(\alpha+\varphi) \to 0$ quand $|\mathrm{Im}\alpha| \to \infty$. On ajoute alors la condition sur f de comportement à l'infini donnée par (b) , ce qui entraîne que $f \to$constante quand $|\mathrm{Im}\alpha| \to \infty$ et donc que $\tilde{\mathrm{u}}(\rho,\varphi) \to$constante quand $\rho \to 0$.

<u>Notes</u> :

- si $f(\alpha+\varphi)$ ne possède aucune singularité pour $|\mathrm{Im}\alpha| > d$ (excepté peut-être à l'infini) et $a_1 < \mathrm{Re}\alpha < a_2$, la condition $f'(\alpha+\varphi) \to 0$ quand $|\mathrm{Im}\alpha| \to \infty$ aura pour conséquence que $|f(a_1+\alpha+\varphi)-f(a_2+\alpha+\varphi)| \to 0$ quand $|\mathrm{Im}\alpha| \to \infty$.

- développons l'expression précédente de $f(\alpha+\varphi)-f(-\alpha+\varphi)$ en fonction de $\tilde{\mathrm{u}}$. Posons $x=\rho\cos\alpha$, $\tilde{\mathrm{u}}(\rho)=g(Log\rho)$ et considérons que $\mathrm{Re}\alpha=0$. Soit ϵ , il existe ϵ' et δ positifs avec $\epsilon'<\delta$ tels que

$$| \; (\int_0^{+\infty} (.)\mathrm{d}x \; - \; \int_{\epsilon'}^{\delta} (.)\mathrm{d}x) \; | < \epsilon \; ,$$

où δ dépend de $\mathrm{Im}k$. On écrit alors g suivant

$$g(Log\rho) = g(-Log\cos\alpha) + Log(\rho\cos\alpha) \times (\mathrm{Re}g'(Log\rho_1) + i\mathrm{Im}g'(Log\rho_2)) \; ,$$

où les ρ_i appartiennent à l'intervalle $[\rho, 1/\cos\alpha]$, et $g'(Log\rho) = \frac{\rho\partial\tilde{\mathrm{u}}}{\partial\rho}$. On note alors que :

$$\int_{\epsilon'}^{\delta} (\mathrm{Re}g'(Log\rho_1) + i\mathrm{Im}g'(Log\rho_2)) \; Log(x) \; e^{-ikx\sin\beta} \, \mathrm{d}x =$$

$$= O(\frac{\rho\partial\tilde{\mathrm{u}}}{\partial\rho}|_{\rho=a/\cos\alpha}) \; , \; a<\delta \; , \text{ quand } \cos\alpha \to \infty \; ,$$

et donc , en fin de compte ,

$$f(i\mathrm{Log}(\rho)+\varphi) - f(-i\mathrm{Log}(\rho)+\varphi) \sim i \; \tilde{\mathrm{u}}(\rho,\varphi) + O(\frac{\rho\partial\tilde{\mathrm{u}}}{\partial\rho}|_{\rho=a\rho}) \; \text{ quand } \rho \to 0 \; .$$

TIME-DOMAIN SCATTERING BY AN IMPEDANCE WEDGE FOR SKEW INCIDENCE

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ABSTRACT - The scattering of an electromagnetic time-dependent plane wave by the edge of an impedance wedge is analyzed. The time-domain behavior of the fields in the wedge-shaped region is obtained by applying an inverse Laplace transform to the steady-state response, also considering the total field to be real and causal. This procedure can be used whenever a time-harmonic representation of the field is known in the form of a Sommerfeld integral, both in two and three-dimensional cases.

1. INTRODUCTION

Time-domain field solutions are of relevant importance for a large variety of practical applications. For instance, they can be extremely useful in electromagnetic pulse (EMP) studies, when the effects of the exposition of a complex structure to a lightning flash or to a nuclear blast have to be evaluated. In particular, several time-domain analytical solutions have been derived for the canonical problem of electromagnetic wave scattering from perfectly [1]-[6] and non-perfectly conducting wedges [7]-[10]. Nevertheless, extensive numerical investigations on these solutions are not available in the literature, particularly in the non-perfectly conducting case.

The two-dimensional scattering of a time-dependent plane wave by a wedge with two different, isotropic impedance faces has been analyzed in [10]. There, the analysis was based on the method presented in [8] and [9], that is valid only whenever the surface impedance of the wedge is frequency independent.

In this paper, a suitable, general time-domain solution for plane wave scattering by an impedance wedge is determined from the knowledge of the response of the wedge in the frequency domain. The time-domain expressions are derived by performing an inverse Laplace transform [4]. The integral formulation for the field is then manipulated by simply considering the causality principle and the reality of the total field in the time-domain. The final expressions for the total field allow a useful physical interpretation of the solution and are suitable for efficient numerical evaluations.

It is worth pointing out that, the solution which is presented in this paper contains as a particular case the one in [8]. In the analysis, the hypothesis of frequency independence on the impedances of the faces has been removed, allowing to consider a wider class of practical problems. To this end, it is noted that, if a physical model as impedance boundary conditions is used, it is usually valid within a limited frequency band. Consequently, in practical simulations it is necessary to refer to an incident signal consisting of a pulse with an arbitrary time dependence, modulating an high-frequency carrier.

An important feature of the proposed method is that it can be applied to treat both two and three-dimensional cases for which the solution in the frequency domain is available in the form of a Sommerfeld integral [11]-[16]. Moreover, it is noted that this procedure is also valid when the boundary conditions are expressed in a more general form with respect to the Leontovich's one.

In the next section, the response of an impedance wedge to an arbitrary time-dependent incident plane wave is obtained from the solution for the time-harmonic problem. Finally, the expressions for the field are discussed in Section III.

~~For the sake of simplicity, it is supposed that the two faces of the wedge can not support surface wave propagation.~~(*)

2. THE THREE DIMENSIONAL SOLUTION

The geometry for the three-dimensional scattering by an impedance wedge is depicted in Fig. 1. Two different uniform isotropic, arbitrary impedance boundary conditions are imposed on the two faces. A time-dependent plane wave impinges on the edge from a direction which is determined

(*) cette phrase peut être supprimée car elle ne concerne que les applications numériques présentées à la conférence.

by the two angles β and ϕ'. The angle β is a measure of the incident direction skewness with respect to the edge of the wedge and $\beta=\pi/2$ corresponds to normal incidence. The observation point is P and the exterior angle of the wedge is $n\pi$. The electric properties of the wedge are supposed to be independent of z.

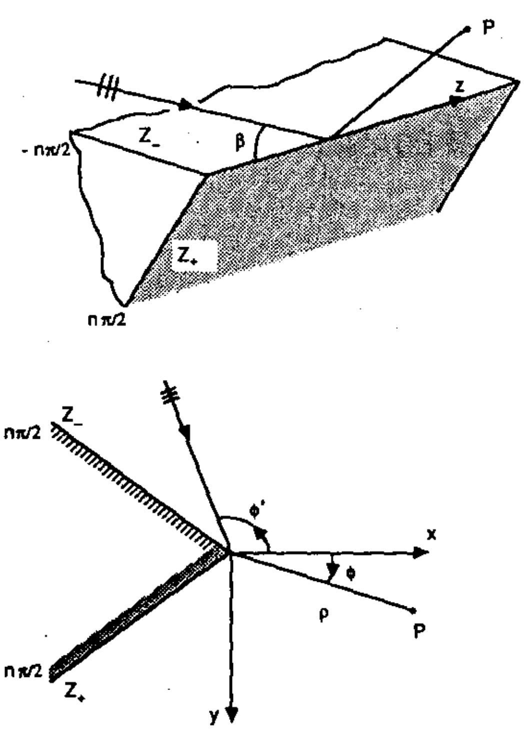

Fig. 1 Geometry for the three-dimensional scattering at an impedance wedge

The surface impedances of the $\phi=-n\pi/2$ and $\phi=n\pi/2$ faces, at the considered frequency ω, are complex constants; they will be denoted by Z_- and Z_+, respectively. It is worth pointing out that in practical applications an impedance boundary condition can usually be defined within a limited frequency band, with Z_- and Z_+ depending on frequency. So, it will be possible to determine the response to signals with a band in which suitable impedance boundary conditions can be used to approximate the actual situation. ~~For this reason we will examine in detail the response of the wedge to an incident wave packet with a limited frequency band, also supposing that no surface wave can be excited in this frequency band.~~(*)

The time-domain response of the wedge can be obtained from the time-harmonic one, by directly applying an inverse Laplace transform. In particular, by denoting with $u^i(t)$ and $u(t)$ the z components of the incident and total magnetic (electric) fields respectively, we have

$$u(t) = \frac{1}{2\pi} \int_{-\infty-j\sigma}^{+\infty-j\sigma} U^i(\omega) G_o(\omega) e^{j\omega t} d\omega \qquad (1)$$

with $\sigma \geq 0$. In (1), $U^i(\omega)$ is the spectrum of the incident signal (it is supposed that $U^i(\omega) \sim \omega^{-a}$, $a>1$) and $G_o(\omega)$, that can be identified with the z component of the electric or the magnetic field in the exterior wedge region, is the response of the wedge to an incident unit time-harmonic plane wave

$$G^i(\rho,\phi,z;\phi',\beta;\omega) = e^{jk\rho\sin\beta\cos(\phi-\phi')} e^{-jkz\cos\beta} \qquad (2)$$

with $k=\omega/c$. In particular

$$G_o(\rho,\phi,z;\omega) = \frac{e^{-jkz\cos\beta}}{2\pi j} \int_\gamma F_o(\alpha+\phi;\omega) e^{jk\rho\sin\beta\cos\alpha} d\alpha \qquad (3)$$

where the contour γ is the Sommerfeld integration path depicted in Fig. 2a and $F_o(\alpha+\phi;\omega)$ is a pertinent spectrum function, subject to appropriate boundary conditions on the two faces of the wedge. It has been derived by Maliuzhinets in the case $\beta=\pi/2$ [11]. With some modifications from Maliuzhinets method, solutions have been also obtained in some three-dimensional cases for the half plane [12], for the right angled wedge [13],[14] and for an absorbing wedge with unit relative impedance [16]. It is noted that $F_o(\alpha+\phi;\omega)$ has no pole singularities in the region $\Im(\omega\cos\alpha)<0$, $\Im(\alpha)\neq 0$, when $|\Re(\alpha)|\leq\pi$. In order to simplify the notation, the dependence of the fields from the space coordinates (observation point and direction of incidence), if not strictly necessary, is suppressed and understood in the following.

It is now convenient to introduce a new spectral function

$$F(\alpha+\phi;\omega)=U^i(\omega)F_o(\alpha+\phi;\omega) \; . \qquad (4)$$

Consequently, a different expression for the total field $G(\rho,\phi,z;\omega)$ is obtained from expression (3). Some useful properties of the inverse Fourier transform of the spectrum function $F(\alpha+\phi;\omega)$ are derived in the following, which directly follow from the reality and causality of the response of the wedge.

Let us consider the Maliuzhinets' inversion formula [17]

$$\begin{aligned} &F(\alpha+\phi;\omega)-F(-\alpha+\phi;\omega) = \\ &= -jk\sin\beta\sin\alpha e^{jkz\cos\beta} \int_0^{+\infty} G(\rho,\phi,z;\omega) e^{-jk\rho\sin\beta\cos\alpha} d\rho \end{aligned} \qquad (5)$$

From the reality of the field, it follows that $G(-\omega)=[G(\omega)]^*$ for ω real. This can be used in (5) to obtain

$$F(\alpha+\phi;-\omega)-[F(\alpha^*+\phi;\omega)]^* = j\kappa_1 \, , \qquad (6)$$

where κ_1 is a constant with respect to α that, without loosing generality, can be chosen equal to zero. By performing an inverse Fourier transform of (5) for $\Re(\alpha)=0$, where $F(\alpha+\phi;\omega)$ is regular in ω, the following expression is obtained

$$\begin{aligned} &\mathfrak{F}(\alpha+\phi;\tau)-\mathfrak{F}(-\alpha+\phi;\tau)= \\ &= -\frac{\sin\alpha\sin\beta}{2\pi c} \int_{-\infty}^{+\infty} j\omega e^{jk(c\tau+z\cos\beta)} \int_0^{+\infty} G(\omega) e^{-jk\rho\sin\beta\cos\alpha} d\rho d\omega \end{aligned} \qquad (7)$$

where the inverse Fourier transform of $F(\alpha+\phi;\omega)$ has been denoted by $\mathfrak{F}(\alpha+\phi;\tau)$. By interchanging the order of integration, expression (7) can be rewritten as

$$\begin{aligned} &\mathfrak{F}(\alpha+\phi;\tau)-\mathfrak{F}(-\alpha+\phi;\tau) = \\ &= -\frac{\sin\alpha\sin\beta}{2\pi c} \int_0^{\infty} \frac{\partial}{\partial t'} \left(\int_{-\infty}^{+\infty} G(\omega) e^{j\omega t'} d\omega \right) d\rho \end{aligned} \qquad (8)$$

(*) même note que page précédente.

where $ct'=c\tau+z\cos\beta-\rho\cos\alpha\sin\beta$. It can be noted that the inverse Fourier transform of $G(\omega)$ is causal, so that it is zero for $ct'<(z\cos\beta-\rho\sin\beta)$. As a consequence, for $\cos\alpha=\mathrm{ch}\mu>1$ and $\tau<0$ we obtain from (8)

$$\mathcal{F}(j\mu+\phi;\tau)-\mathcal{F}(-j\mu+\phi;\tau)=0 \qquad (9)$$

Moreover, by taking into account relationship (6), it is found that

$$\mathcal{F}(-j\mu+\phi;\tau)-(\mathcal{F}(j\mu+\phi;\tau))^{*}=0. \qquad (10)$$

Adding (9) and (10), it is easily seen that, for $\tau<0$,

$$\mathcal{F}(j\mu+\phi;\tau)=(\mathcal{F}(j\mu+\phi;\tau))^{*}. \qquad (11)$$

The inverse Fourier transform $\mathcal{F}$ of F used in (11) is an analytical function of α. Thus, for $\tau<0$, we obtain

$$\mathcal{F}(\alpha+\phi;\tau)=\kappa_2 \quad , \qquad (12)$$

where κ_2 is a real constant with respect to α that, without loss of generality, can be chosen equal to 0. In the case of a passive impedance, it can be easily shown that $\mathcal{F}(\alpha+\phi;\tau)$ is a function which presents a unique analytical expression for $|\Re(\alpha)|<\pi$, as $F(\alpha+\phi,\omega)$ has no poles in ω when $|\Re(\alpha)|<\pi$, $\Im(\alpha)\neq 0$. Then, by taking into account that $\mathcal{F}(\alpha+\phi;\tau)$ is analytic with respect to α, property (12) can be extended to the whole region $|\Re(\alpha)|<\pi$. Consequently, for $|\Re(\alpha)|<\pi$, the Fourier transform of $\mathcal{F}$ in τ becomes a Laplace transform which analytically continues F as a regular function in ω for $\Im(\omega)<0$, so that

$$\mathcal{F}(\alpha+\phi,\tau)=\frac{1}{2\pi}\int_{-\infty-j\sigma}^{+\infty-j\sigma} F(\alpha+\phi,\omega)e^{j\omega\tau}d\omega \quad , \qquad (13)$$

with $\sigma>0$.

From (5), we note that when ω is real $F(\alpha+\phi,\omega)$ is regular for $\Im(\omega\cos\alpha)<0$ and thus, all the singularities of $F(\alpha+\phi;\omega)$ which do not depend on ω, for $|\Re(\alpha)|<\pi$, are located on the real axis. This, with the regularity in ω for $\Im(\omega)<0$, allows us to write that, when $\Im(\omega)<0$, $F(\alpha+\phi,\omega)$ is regular for $\Im(\alpha)\neq 0$ and $|\Re(\alpha)|<\pi$. So, the Sommerfeld integration path can be deformed onto the $\mathfrak{J}$ contour depicted in Fig. 2b. Consequently, from expressions (1)-(3), we obtain

$$u(t)=\frac{1}{2\pi}\int_{-\infty-j\sigma}^{+\infty-j\sigma} U^{i}(\omega)\left(\frac{1}{2\pi j}\int_{\mathfrak{J}} F_0(\alpha+\phi,\omega)e^{j\omega\tau(\alpha)}\,d\alpha\right)d\omega \qquad (14)$$

with $\sigma>0$ and $\tau(\alpha)=(\rho\cos\alpha\sin\beta-z\cos\beta)/c+t$. Moreover, it is noted that $\tau(\alpha)$ can be considered real when α varies on $\mathfrak{J}$, so that, by taking also into account the asymptotic behavior of $U^{i}(\omega)$, the order of integration in (14) can be interchanged. Furthermore, by applying (12) and (13),

$$\int_{-\infty-j\sigma}^{+\infty-j\sigma} U^{i}(\omega)F_0(\alpha+\phi,\omega)e^{j\omega\tau(\alpha)}d\omega=0 \qquad (15)$$

for $\tau(\alpha)<0$, so that we can write

$$u(t)=\frac{1}{2\pi j}\int_{\mathcal{H}(t,\rho,z)}\left(\frac{1}{2\pi}\int_{-\infty-j\sigma}^{+\infty-j\sigma} U^{i}(\omega)F_0(\alpha+\phi,\omega)e^{j\omega\tau(\alpha)}d\omega\right)d\alpha \qquad (16)$$

where $\mathcal{H}(t,\rho,z)$ is the integration path depicted in Fig. 2c.

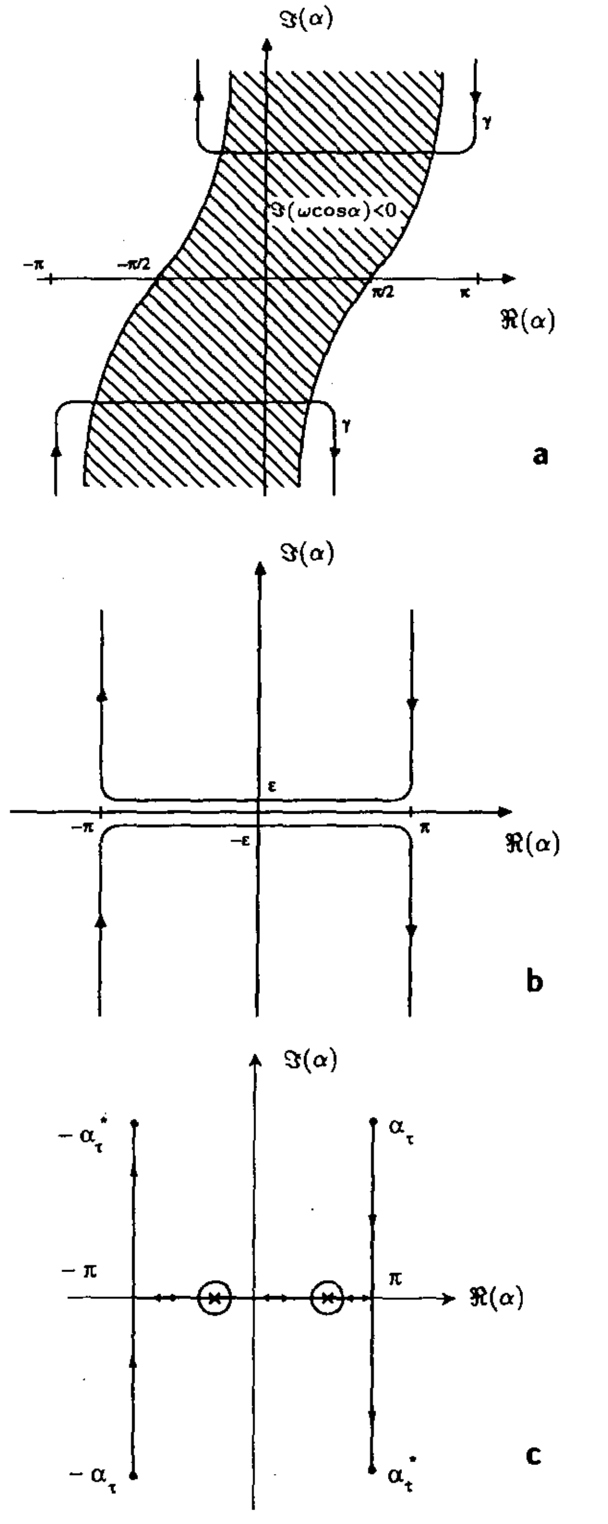

Fig. 2 Countours of integration: a) Sommerfeld integration path γ; b) integration path $\mathfrak{J}$; c) integration path $\mathcal{H}$ (α_τ is the value of α for which $\tau(\alpha)=0$).

3. DISCUSSION

As shown in the same Fig. 2c, $\mathcal{H}(t,\rho,z)$ is composed by a closed contour $\mathcal{H}_g$, corresponding to $\max[0;t_0(\rho,z)]>\tau(\alpha)>\max[0;t_0(-\rho,z)]$, and by two limited branches $\mathcal{H}_e$, parallel to the imaginary axis for $\Re(\alpha)=\pm\pi$, corresponding to $t_0(-\rho,z)>\tau(\alpha)>0$, with

$t_o(\rho,z)=(\rho\sin\beta-z\cos\beta)/c+t$. It is easily seen that the closed contour $\mathcal{K}_g$ includes all the pole singularities of $F_o(\alpha+\phi;\omega)$, which are displaced along the real axis with $|\Re(\alpha)|<\pi$. These singularities correspond to the incident and reflected fields, that constitute the Geometrical Optics part u_g of the total field. The open contour $\mathcal{K}_e$ is directly related to an edge field contributions u_e, which exists only for $t_o(-\rho,z)>0$.

The following expressions are obtained

$$u(t)=u_g(t)+u_e(t) \tag{17}$$

for $(\rho\sin\beta+z\cos\beta)<ct$, and

$$u(t)=u_g(t) \tag{18}$$

for $(\rho\sin\beta+z\cos\beta)>ct$. In particular, in (17) and (18) u_g can be expressed as

$$u_g(t)=\frac{1}{2\pi}\int_{-\infty-j\sigma}^{+\infty-j\sigma} U^i(\omega)\sum_{\alpha_s}\mathrm{Res}\left\{F_o(\alpha+\phi,\omega)\,e^{j\omega\tau(\alpha)}\right\}_{\alpha=\alpha_s}d\omega \tag{19}$$

where α_s are the poles of $F_o(\alpha+\phi,\omega)$ along the real axis, with $\max[0;t_o(\rho,z)]>\tau(\alpha_s)>\max[0;t_o(-\rho,z)]$. Moreover, $u_e(t)$ in (17) can be interpreted as an edge field contribution

$$u_e(t)=\frac{1}{2\pi}\int_{-\infty-j\sigma}^{+\infty-j\sigma} d\omega\,\frac{U^i(\omega)}{2\pi j}\int_{\mathcal{K}_e} F_o(\alpha+\phi,\omega)\,e^{j\omega\tau(\alpha)}\,d\alpha \tag{20}$$

with $\tau(\alpha)=(\rho\cos\alpha\sin\beta-z\cos\beta)/c+t$.

It is important to note that in general, for a wedge type problem, the terms poles and residues in (19) can be changed to take into account any kind of singularity along the real axis. The expressions which have been derived can then be applied whenever a frequency-domain integral representation of the total field in terms of a Sommerfeld integral is known, with a spectral function F whose inverse Fourier transform $\mathcal{F}(\alpha+\phi;\tau)$ has its domain of analyticity in α connected, for $|\Re(\alpha)|\leq\pi$.

The solution is valid for any incident, causal signal with arbitrary time dependence. In particular it gives evidence to two regions, which are identified by the following inequalities $(\rho\sin\beta+z\cos\beta)>ct$ and $(\rho\sin\beta+z\cos\beta)<ct$. In the impedance case, in the first region only the incident and reflected fields are present, while in the second a term u^s, related to the presence of the edge of the wedge, arises (diffracted field).

It can be shown that the results in [8], which are related to the two-dimensional case $(\beta=\pi/2)$, are obtained when an independence of F_o (or of the surface impedances) on ω is assumed and we consider an incident unit step function, i.e. $j\omega U^i(\omega)=1$. This confirms that the present approach, which is based on the Laplace transform, is more general than that in [8], providing equivalent physical insights in the solution ($\rho\sin\beta+z\cos\beta<ct$ or $\rho\sin\beta+z\cos\beta>ct$ regions) when also the causality principle and the reality of the signal are considered. Moreover, it allows the analysis of some three-dimensional cases, for which the spectral function is known.

Finally, it is important to note that in the case of a perfectly conducting wedge $(Z_-=Z_+=0)$, the UTD (or GTD) time-domain expressions [6] of the edge field can be recovered from (20) by simply developing $F_o/\sin(\alpha/2)$ as a rational function (or Taylor series) in terms of powers of $\cos(\alpha/2)$, considering (or not) the presence of the poles.

Several numerical examples will be shown at the oral presentation to investigate into the influence of the surface impedance and of the exterior angle $n\pi$ on the response of the wedge.

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Progresses on the diffraction by a wedge: transient solution for line source illumination, single face contribution to scattered field, and new consequence of reciprocity on the spectral function ([1])

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ABSTRACT. – *This paper is devoted to three original results on diffraction by an imperfectly conducting wedge. Firstly, the transient solution for the diffraction of an electromagnetic wave by an impedance wedge in a lossy medium, both with frequency dependent electric characteristics, is analyzed for line source illumination, and new expressions, satisfying explicitely the causality and allowing useful physical decompositions of the field, are obtained. Secondly, the exact expression of the spectral function, attached to the scattered field radiated by each face of a wedge-shaped region, is given. Thirdly, the theorem of reciprocity finds an interesting application on a recently published expression of the spectral function, by giving a new functional equation that it satisfies.*

RÉSUMÉ. – *Cet article décrit trois résultats originaux concernant la diffraction par un dièdre imparfaitement conducteur. En premier lieu, la diffraction en régime non stationnaire d'une onde électromagnétique par un dièdre à impédance dans un milieu à pertes, tous deux de caractéristiques électriques dispersives en régime harmonique, est analysée pour une onde incidente cylindrique (ligne source), et des expressions analytiques exactes, satisfaisant explicitement la causalité et permettant d'intéressantes décompositions du champ, sont obtenues. Deuxièmement, on détermine l'expression analytique de la fonction spectrale attachée au champ diffusé par chacune des faces d'un secteur dièdre. Enfin troisièmement, on trouve une application originale du théorème de réciprocité à une expression récente de la fonction spectrale, qui permet de démontrer une nouvelle équation à laquelle doit satisfaire cette fonction.*

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1. INTRODUCTION

This paper is here devoted to three new results on the analytical solution of the diffraction by a imperfectly conducting wedge for transient and harmonic response.

Firstly, our development about the time-domain diffraction is presented in chapter 2. The scattering of a time-dependent plane wave by a wedge with two different impedance faces has been analyzed in [1] by a method only valid when the surface impedances are frequency independent, whereas in [2], for the first time, this restriction has been removed. In this paper, the theory to obtain time-domain analytical expressions for the diffraction by an impedance wedge in a lossy medium, both with frequency dependent electric characteristics, is developed. Obtaining new properties of the spectral function attached to the Sommerfeld-Maliuzhinets integral, we determine a new exact expression of the time response for line source illumination, satisfying explicitely causality and allowing useful physical decomposition of the field.

Then, we give the original results concerning the harmonic response in chapters 3 and 4. It is known that Michaeli had recognized the contribution of each face in the particular solution of the diffraction by an perfectly conducting wedge [3]. Here, we show that a suitable analytical expression of the contribution of each face is possible in the general case of an imperfectly conducting wedge-shaped region. Furthermore, a new important consequence of the theorem of reciprocity is derived. So, we determine that the property of reciprocity, applied to a new expression of the spectral function found in [4], allows the function to satisfy a new functional equation.

2. TRANSIENT SOLUTION FOR LINE SOURCE ILLUMINATION

2.1. Initial forms of the field

Let us consider an electromagnetic field with a propagation of constant skew with z axis characterized by the angle β, resulting from the diffraction by a wedge, defined by $|\varphi| \geq \Phi$, with impedance boundary conditions on each face. The cylindrical coordinates $(\rho,\, \varphi,\, z)$ and the related cartesian axis are used. Each component of the field in frequency domain derives from the z-components of the electric and magnetic field. We denote $\tilde{u}$

one of these. At once, let us take a harmonic line source with pulsation ω, parallel to the edge of the wedge, which illuminates the wedge with the incident field

$$\tilde{u}_i = \frac{-i}{4} e^{-ikz \cos \beta} U^i(\omega) H_0^{(2)}(k \sin \beta R) \tag{1}$$

with $R = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\varphi - \varphi')}$, (ρ', φ') being the coordinates of the line source, $k = \zeta(\omega)\omega/c$ the wave number of the lossy medium of propagation and $U^i(\omega)$ assumed $0(\omega^{-b})$, $b>1$, for $|\omega| \to \infty$. Then, from [4], the total field $\tilde{u} = \tilde{u}_s + \tilde{u}_i$ in frequency domain is given with $\tilde{u}_s$ a double Sommerfeld-Maliuzhinets integral, following, for $\Phi \geq \pi/2$,

$$\tilde{u}_s(\rho, \varphi) = \frac{e^{-ikz \cos \beta}}{2\pi i} \int_\gamma f_s(\alpha + \varphi; \omega) e^{ik\rho \sin \beta \cos \alpha} \mathrm{d}\alpha \tag{2}$$

where

$$f_s(\alpha + \varphi; \omega) = \frac{i}{4\pi} \int_\gamma e^{ik\rho' \sin \beta \cos \alpha'} f_{sd}(\alpha + \varphi, \alpha' + \varphi'; \omega) \mathrm{d}\alpha' \tag{3}$$

with f_{sd} given by

$$f_{sd}(\alpha, \alpha'; \omega) = \frac{1}{4\pi i} \int_{S_0} \left(f(\alpha', \alpha''; \omega) - \frac{U^i(\omega)}{2} \operatorname{cotg}\left(\frac{1}{2}(\alpha' - \alpha'')\right)\right) \operatorname{tg}\left(\frac{1}{2}(\alpha - \alpha'')\right) \mathrm{d}\alpha'' \tag{4}$$

and its analytic continuation, f being the spectral function corresponding to the expression of the field $\tilde{u}_p$ for a plane wave illumination with a polar angle α'' and a skew angle $\pi - \beta$. The path γ is composed of two symmetric loops with no singularity within them, one of them consisting of the two half lines $\operatorname{Re} \alpha = \arg(ik) \pm (\varepsilon + \pi/2)$, $\varepsilon > 0$, $\operatorname{Im} \alpha \geq d$ and a segment at $\operatorname{Im} \alpha = d$. The contour S_0 is a path from $-i\infty - \arg(ik)$ to $i\infty + \arg(ik)$, following $\operatorname{Im} k \sin \alpha = 0$. Let us notice that this expression is different from the one used in [5] and doesn't need to change according to the source position and the observation point.

The expression of the transient field $u(t)$ as an inverse Laplace transform can then be let

$$u(t) = \frac{1}{2\pi} \int_{-\infty - i\sigma}^{+\infty - i\sigma} \tilde{u}(\rho, \varphi, z; \omega) e^{i\omega t} \mathrm{d}\omega \tag{5}$$

with $\sigma \geq 0$. Our aim is to use at best the analytical properties of the spectral function f to translate the reality and the causality of u. In the impedance case, the behaviour and the physical meaning of all singularities of the spectral function f are well identified in the general case, for ω real [6]. The next chapter is devoted to derive the particular properties of f in ω-complex plane in order to be able to modify (5).

2.2. The spectral function f in ω-complex plane

Here, some useful properties of the spectrum function $f(\alpha+\varphi;\omega)$ and its inverse Fourier transform are derived with a real incidence angle, principally as consequences of the reality and the causality of the response of the wedge. They will be used to obtain a new expression of $u(t)$.

Let us consider the Maliuzhinet's inversion formula [7]

$$(6) \quad f(\alpha+\varphi;\omega) - f(-\alpha+\varphi;\omega) = -ik\sin\beta\sin\alpha\, e^{ik\rho z\cos\beta} \times \int_0^{+\infty} \tilde{u}_p(\rho,\varphi,z;\omega)\, e^{-ik\rho\sin\beta\cos\alpha}\, \mathrm{d}\rho$$

for $\operatorname{Im} k\cos\alpha < 0$, $|\operatorname{Re}\alpha| < \pi$, excluding the angles $\pm\alpha+\varphi$ loci of the poles of f corresponding to the incident, and reflected plane waves. From the reality of the field, directly follows that $\tilde{u}_p(-\omega) = [\tilde{u}_p(\omega)]^*$ for ω real. This can be used in (6) to obtain

$$(7) \qquad f(\alpha+\varphi;-\omega) - [f(\alpha^*+\varphi;\omega)]^* = i\,\kappa_1$$

where κ_1 is a real constant with respect to α which, without loss of generality, we can choose equal to zero. Let us consider now $\operatorname{Re}(\alpha)=0$. The spectral function $f(\alpha+\varphi;\omega)$ is then regular in ω: the scattered field is composed of outgoing waves and the only pole of $f(\alpha;\omega)$ in the band $|\operatorname{Re}(\alpha)| < \Phi$ is the one associated with the incident field at $\alpha=\varphi'$, independent of ω. Considering that the electric characteristics of the wedge tend to a limit when $|\omega| \to \infty$, $f(\alpha+\varphi;\omega)$ is $0(\omega^{-b})$, $b>1$, as $U^i(\omega)$. Then, an inverse Fourier transform of f, denoted by $\mathcal{F}(\alpha+\varphi;\tau)$, can be performed. From (6), the following equality is obtained

$$(8)\ \mathcal{F}(\alpha+\varphi;\tau) - \mathcal{F}(-\alpha+\varphi;\tau) = -\frac{\sin\alpha\sin\beta}{2\pi c}\frac{\partial}{\partial\tau} \times \int_{-\infty}^{+\infty} \zeta(\omega)\, e^{i\omega\tau+ikz\cos\beta} \int_0^{+\infty} \tilde{u}_p(\omega)\, e^{-ik\rho\sin\beta\cos\alpha}\, \mathrm{d}\rho\, \mathrm{d}\omega$$

By interchanging the order of the integration, the expression (8) can be written as

$$(9)\quad \mathcal{F}(\alpha+\varphi\,;\,\tau)-\mathcal{F}(-\alpha+\varphi\,;\,\tau) \\ = -\frac{\sin\alpha\,\sin\beta}{2\,\pi\,c}\,\frac{\partial}{\partial\tau}\int_0^{\infty}\left(\int_{-\infty}^{+\infty}\zeta(\omega)\,\tilde{u}_p(\omega)\,e^{i\,\omega\,t'}\,\mathrm{d}\omega\right)\mathrm{d}\rho$$

where $ct' = c\tau + \zeta(\omega)\,(z\cos\beta - \rho\cos\alpha\sin\beta)$. The quantity ζ is complex but we can use the fact that it tends to 1 when $|\omega| \to \infty$. $u_p(t)$ is causal with respect to the front of the incident plane wave, so that, in (9), the contour of integration in ω can be closed at infinity for $\mathrm{Im}(\omega) < 0$ when $ct'|_{|\omega|\to\infty} < (z\cos\beta - \rho\sin\beta)$; in this case, as $\zeta(\omega)$ and $\tilde{u}_p(\omega)$ are non singular for any finite ω with $\mathrm{Im}\,\omega < 0$, the integral is null. Accordingly, since $\cos\alpha = \mathrm{ch}\,\mu > 1$, we obtain when $\tau < 0$

$$(10)\qquad \mathcal{F}(i\,\mu+\varphi\,;\,\tau)-\mathcal{F}(-i\,\mu+\varphi\,;\,\tau)=0$$

Moreover, by taking into account relationship (7), it is found that

$$(11)\qquad \mathcal{F}(-i\,\mu+\varphi\,;\,\tau)-(\mathcal{F}(i\,\mu+\varphi\,;\,\tau))^{*}=0$$

Adding (10) and (11), it is easily seen that, for $\tau < 0$

$$(12)\qquad \mathcal{F}(i\,\mu+\varphi\,;\,\tau)=(\mathcal{F}(i\,\mu+\varphi\,;\,\tau))^{*}$$

The inverse Fourier transform $\mathcal{F}(\alpha'\,;\,\tau)$ of f used in (12) is an analytical function of $\alpha' = i\,\mu + \varphi$. Thus, when $\tau < 0$, we obtain

$$(13)\qquad \mathcal{F}(\alpha+\varphi\,;\,\tau)=\kappa_2$$

where κ_2 is a real constant with respect to α which, without loss of generality, can be chosen equal to 0. The property (13) can be extended to the whole region $|\mathrm{Re}(\alpha)| < \pi$: $f(\alpha+\varphi, \omega)$ having no singularities in ω for ω real, as $|\mathrm{Re}(\alpha)| < \pi$ and $|\varphi| < \Phi$ for an impedance wedge, and being $0(U^i(\omega))$ when $|\omega| \to \infty$, $\mathcal{F}(\alpha+\varphi\,;\,\tau)$ is a function which presents a unique analytical expression when α and φ lie in the bands just defined.

By complement, we note that, when $\omega \to \pm\infty$, $\tilde{u}_p$ tends to be finite, $\zeta \to 1$, and thus, from (6), $f(\alpha+\varphi, \omega)$ tends to be regular for $\pm\text{Im}(\cos\alpha) < 0$; accordingly, all the singularities of $f(\alpha+\varphi;\omega)$ independent of ω, for $|\text{Re}(\alpha)| < \pi$, are located on the real axis.

Consequently, for $|\text{Re}(\alpha)| < \pi$, $|\varphi| < \Phi$, the Fourier transform of $\mathcal{F}$ in τ becomes a Laplace transform which analytically continues $f(\alpha+\varphi;\omega)$ as a regular analytic function in α, for $\text{Im}\,\alpha \neq 0$, and in ω for $\text{Im}(\omega) \leq 0$, with the behaviours $0(\omega^{-b})$ for $|\omega| \to \infty$, and $0(1)$ for $|\text{Im}\,\alpha| \to \infty$ (as initially showed for ω real in [6]).

2.3. Causal time expression in impedance case

Let us name the edge field the term $\tilde{u}_{se}$ obtained when, neglecting the contribution of poles, the paths γ are deformed to the branches at $\text{Re}\,\alpha$, $\alpha' = \pm\pi$ in the expression of $\tilde{u} - \tilde{u}_i$. Then , applying an inverse Laplace transform to it and using the properties of f given in previous chapter, we derive the time domain expression u_{se}

$$(14)\quad u_{se}(t) = \frac{1}{2\pi}\int_{-\infty-i\sigma}^{+\infty-i\sigma} \mathrm{d}\omega \times\left(\frac{1}{8\pi}\int_{\mathcal{H}''}\frac{1}{\pi}\int_{\mathcal{H}'} e^{i\omega\tau_\zeta(\alpha',\alpha'')} f(\alpha'+\varphi',\alpha''+\varphi;\omega)\,\mathrm{d}\alpha'\,\mathrm{d}\alpha''\right)$$

where $\tau_\zeta(\alpha',\alpha'') = \zeta(\omega)((\rho'\cos\alpha' - \rho\cos\alpha'')\sin\beta - z\cos\beta)/c + t$, $\mathcal{H}''$ and $\mathcal{H}'$ being two finite paths on the branch $\text{Re}\,\alpha'' = 0$ and the branches $\text{Re}\,\alpha' = \pm\pi$ respectively (Fig. 1), so

$$(15)\qquad \tau(\alpha',\alpha'') = \tau_\zeta(\alpha',\alpha'')|_{|\omega|\to\infty} = ((\rho'\cos\alpha' - \rho\cos\alpha'')\sin\beta - z\cos\beta)/c + t > 0$$

Now, let us recover the expression of the total field. When $|\varphi'| > \pi - \Phi$, $|\varphi - \varphi'| > \pi$, it is a shadowed zone and we verify easily that $\tilde{u} = \tilde{u}_{se}$. Then, from an analytical continuation of $\tilde{u}_{se}$ in φ and φ', the Heaviside function being denoted U, the complete expression of $u(t)$ for $\Phi \geq \pi/2$ follows

$$(16)\qquad u(t) = u_{se}(t) + U(\pi - |\varphi-\varphi'|)\,u_i(t) + \sum_{\pm} U(\pi - |\pm 2\Phi - (\varphi+\varphi')|)\,u_r^{\pm}(t)$$

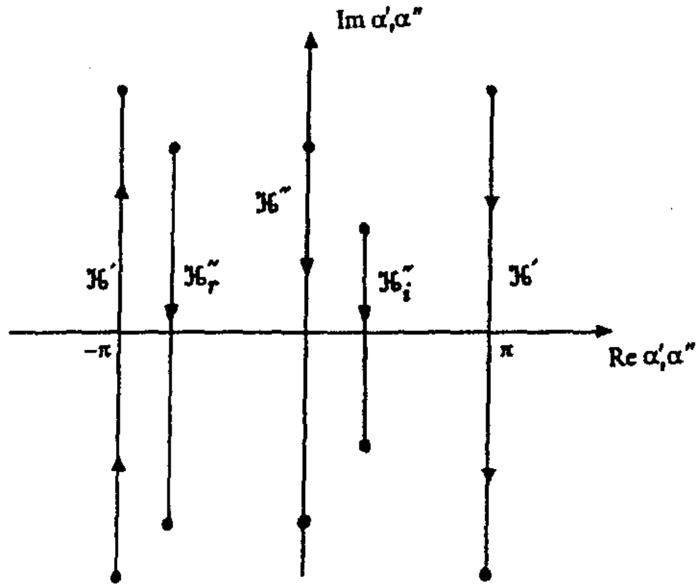

Fig. 1. – Contour of integration for line source illumination

the term u_i corresponds to the pole of incident field in f and is written

$$(17) \qquad u_i(t) = \frac{1}{2\pi} \int_{-\infty - i\sigma}^{+\infty - i\sigma} \mathrm{d}\omega \times \left(\frac{i}{4\pi} \int_{\mathcal{H}_i''} e^{i\omega\tau_\zeta(\alpha'' + \varphi - \varphi', \alpha'')} U^i(\omega)\, \mathrm{d}\alpha'' \right)$$

$\mathcal{H}_i''$ being a finite path on $\mathrm{Re}\,\alpha'' = \Omega_i$ branch ($|\Omega_i| < \pi$), so that $(\rho' \cos(\alpha'' + \varphi - \varphi') - \rho \cos \alpha'') = -\mathrm{R}_i \cos(\alpha'' - \Omega_i)$ is a real negative quantity, and $\tau(\alpha'' + \varphi - \varphi', \alpha'')$ is positive (Fig. 1); the term u_r corresponds to the poles of reflected fields in f and is written as

$$(18) \quad u_r^{\pm}(t) = \frac{1}{2\pi} \int_{-\infty - i\sigma}^{+\infty - i\sigma} \mathrm{d}\omega\, \frac{i}{4\pi} \int_{\mathcal{H}_r''} \mathrm{d}\alpha''\, \mathrm{Residue} \times \left[e^{i\omega\tau_\zeta(\alpha', \alpha'')} f(\alpha' + \varphi', \alpha'' + \varphi;\, \omega) \right] \Big|_{\alpha' = \pm 2\Phi - (\alpha'' + \varphi' + \varphi)}$$

$\mathcal{H}''_r$ being a finite path on $\mathrm{Re}\,\alpha'' = \Omega_r$ line ($|\Omega_r| < \pi$), so that $(\rho' \cos(\pm 2\,\Phi - (\alpha'' + \varphi' + \varphi)) - \rho \cos \alpha'') = -R_r \cos(\alpha'' - \Omega_r)$ is a real negative quantity and $\tau\,(\pm 2\,\Phi - (\alpha'' + \varphi' + \varphi),\, \alpha'')$ is positive (*Fig.* 1).

Then, let us notice some properties of the terms u_{se}, u_i, u_r which, in particular, allows $u\,(t)$ to satisfy the causality, *i. e.*, to be null for $ct - z \cos\beta < 0$. From (14), u_{se} exists only if $ct - z \cos\beta > (\rho + \rho') \sin\beta$, which corresponds properly to the necessary path source-edge-observation point of the edge field. The term u_i is null for $ct - z \cos\beta < R_i \sin\beta$, R_i being the straight path between the line source and the observation point; as we expect, it doesn't depend on the electric characteristics of the wedge. The term $u_r^{\pm}$, directly related to the reflected field, is null for $ct - z \cos\beta < R_r^{\pm} \sin\beta$, $R_r^{\pm}$ being the straight path between the observation point and the image line source corresponding to the face $\pm\Phi$; its expression contains the residue of f with the reflection coefficients of the faces.

3. DETERMINATION OF THE SPECTRAL FUNCTION ATTACHED TO THE FIELD SCATTERED BY EACH FACE OF A WEDGE-SHAPED REGION

In this chapter, let us consider $\tilde{u}$, a cartesian component of a harmonic field with a skew angle β of propagation. It is the sum of $\tilde{u}_s$ and $\tilde{u}_i$, radiated by harmonic sources, respectively located in a wedge-shaped region $|\varphi| \geq \Phi$ and at the angle φ', with $|\varphi'| < \Phi$. From the Green theorem on equivalent sources, the field $\tilde{u}_s$, for $|\varphi| < \Phi$, can be written as the sum of $\tilde{u}_+$ and $\tilde{u}_-$, respectively the radiations of equivalent surface currents carried by the faces $\varphi = +\Phi$ and $\varphi = -\Phi$, following the expression

$$(19)\quad \tilde{u}_{\pm}(\rho, \varphi) = -\frac{i}{4} e^{-ikz \cos\beta} \times \int_{\varphi = \pm\Phi,\, z=0} \tilde{u} \frac{\partial H_0^{(2)}(k \sin\beta\, R)}{\partial n} - \frac{\partial \tilde{u}}{\partial n} H_0^{(2)}(k \sin\beta\, R)\, \mathrm{d}\rho_i$$

with $R = \sqrt{\rho^2 + \rho_i^2 - 2\,\rho\rho_i \cos(\varphi \mp \Phi)}$, or more generally, with $\varphi_i \leq \Phi$

$$(20)\quad \tilde{u}_{\pm}(\rho, \varphi) = -U(\pm\varphi' - \varphi_i)\, \tilde{u}_i - \frac{i}{4} e^{-ikz \cos\beta} \int_{\varphi = \pm\varphi_i,\, z=0} \tilde{u} \frac{\partial H_0^{(2)}(k \sin\beta\, R)}{\partial n} - \frac{\partial \tilde{u}}{\partial n} H_0^{(2)}(k \sin\beta\, R)\, \mathrm{d}\rho_i$$

with $R = \sqrt{\rho^2 + \rho_i^2 - 2\,\rho\rho_i \cos(\varphi \mp \varphi_i)}$, supposing, at once, that $\tilde{u}$ is finite and $\dfrac{\partial \tilde{u}}{\partial n}$ integrable.

Now, let, for $|\varphi| < \pi/2, \Phi \geq \pi/2$, $|\varphi'| > \pi/2$, the total field $\tilde{u}$ in the form of a Sommerfeld-Maliuzhinets integral

$$\tilde{u}(\rho, \varphi) = \frac{e^{-ikz \cos \beta}}{2\,\pi\, i} \int_\gamma f(\alpha + \varphi)\, e^{ik \sin \beta \rho \cos \alpha}\, \mathrm{d}\alpha \tag{21}$$

with $f(\alpha) = 0(1)$ on γ. On the other hand, from [4], let us write, for $\varphi_i \geq \pi/2$

$$\begin{aligned} H_0^{(2)}(k \sin \beta\, R) &\\ = \frac{1}{2\,\pi\, i} \int_\gamma \frac{-1}{2\,\pi} \int_{\mathcal{S}_0} & e^{ik \sin \beta \rho_i \cos(\alpha' \mp \varphi_i)} \\ \times \operatorname{tg} & \left(\frac{1}{2} (\alpha + \varphi - \alpha') \right) \mathrm{d}\alpha'\, e^{ik \sin \beta \rho \cos \alpha}\, \mathrm{d}\alpha \end{aligned} \tag{22}$$

$\mathcal{S}_0$ being a path from $-i\,\infty - \arg(ik)$ to $i\,\infty + \arg(ik)$. Then, letting φ_i equal to $\pi/2$, adapting and inversing the paths of integration, a new expression of $u_\pm$ is obtained

$$\begin{aligned} \tilde{u}_\pm(\rho, \varphi) = -U(\pm\varphi' - \pi/2)\tilde{u}_i + & \frac{e^{-ikz \cos \beta}}{2\pi\, i} \\ \times \int_\gamma & f_\pm(\alpha + \varphi)\, e^{ik \sin \beta \rho \cos \alpha}\, \mathrm{d}\alpha \end{aligned} \tag{23}$$

with

$$f_\pm(\alpha) = \frac{1}{4\,\pi\, i} \int_{\mathcal{S}_0} \pm f(\pm\pi + \alpha') \left(\operatorname{tg} \left(\frac{1}{2} (\alpha - \alpha') \right) - g(\alpha') \right) \mathrm{d}\alpha' \tag{24}$$

for α between $\mathcal{S}_0 - \pi$ and $\mathcal{S}_0 + \pi$, $\mathcal{S}_0$ defined, when k real positive, by $]-\pi/2 - i\,\infty, -\pi/2 + i\,\delta] \cup [-\pi/2 + i\,\delta, \pi/2 + i\,\delta] \cup [\pi/2 + i\,\delta, \pi/2 + i\,\infty[$ with $\pm\delta > 0$, and g arbitrarily chosen to make the integral converge.

It is worth noticing that we can choose the condition $f_\pm(i\,\infty) = -f_\pm(-i\,\infty)$, usually used to fix the expression of f, only if g can be equal to zero. The expression of $f_\pm$ can be continued analytically by the equality

$$f_\pm(\pi + \alpha) - f_\pm(-\pi + \alpha) = \pm f(\pm\pi + \alpha) \tag{25}$$

Let us add that the previous formula (23)-(25) can be generalized to $f(\alpha)$ with the behaviour $f(\alpha) = 0\,(e^{(1-c)|\mathrm{Im}\,\alpha|})$, $c > 0$, as $|\mathrm{Im}\,\alpha| \to \infty$ on γ, corresponding to the case $\tilde{u}$ integrable at $\rho = 0$.

Now, let us consider an example of application: the problem of an impedance wedge illuminated by a plane wave, with $f(\alpha) = 0\,(e^{(1-c)|\mathrm{Im}\,\alpha|})$, $c > 0$, as $|\mathrm{Im}\,\alpha| \to \infty\ \forall \mathrm{Re}\,\alpha$. The term $\tilde{u}_i$ normalized to unity, (23) is written

$$(26)\ \tilde{u}_\pm(\rho, \varphi) = \frac{e^{-ikz\cos\beta}}{2\pi i}\int_\gamma \Bigg(f_\pm(\alpha+\varphi) - \frac{U(\pm\varphi' - \pi/2)}{2}\,\mathrm{cotg}\left(\frac{1}{2}(\alpha+\varphi-\varphi')\right)\Bigg)\, e^{ik\sin\beta\rho\cos\alpha}\,\mathrm{d}\alpha$$

Then, the field $\tilde{u}_{\pm e}$ due to the edge on the face $\varphi = \pm\Phi$, can be obtained by deforming γ to the branches $\mathrm{Re}\,\alpha = +\pi$ et $-\pi$, while neglecting the poles of geometrical optics, following

$$(27)\quad \tilde{u}_{\pm e}(\rho, \varphi) = \mp\frac{e^{-ikz\cos\beta}}{2\pi i}\int_{-i\infty}^{i\infty} f(\pm\pi+\alpha+\varphi)\, e^{-ik\sin\beta\rho\cos\alpha}\,\mathrm{d}\alpha$$

This form is easy to check with the result obtained by Michaeli [3] on the particular case of the field diffracted by a perfectly conducting wedge.

4. A NEW FUNCTIONAL EQUATION FOR THE SPECTRAL FUNCTION

Now, let us consider an imperfectly conducting wedge-shaped region $|\varphi| \geq \Phi \geq \pi/2$ illuminated by a harmonic plane wave with z-components amplitude vector $\bar{c} = (e_i, Z_0 h_i)$ (Z_0: free space impedance), coming from the direction defined by a polar angle α' and a skew angle β. The expression of $\bar{f}$, a vector composed of the two spectral functions attached to the z-components of the field $(E_z, Z_0 H_z)$, is given, from [4] by

$$(28)\qquad \bar{f}(\alpha, \alpha', \beta) = \frac{1}{4\pi i}\int_{\mathcal{S}_0} \bar{F}(\alpha'', \alpha', \beta)\,\mathrm{tg}\left(\frac{1}{2}(\alpha - \alpha'')\right)\mathrm{d}\alpha''$$

where $\bar{F}(\alpha, \alpha', \beta) = \bar{f}(\pi+\alpha, \alpha', \beta) - \bar{f}(-\pi+\alpha, \alpha', \beta)$, α lying in the domain limited by $\mathcal{S}_0 + \pi$ and $\mathcal{S}_0 - \pi$, with, for $|\mathrm{Re}\ \alpha'| \geq \pi/2$, $\mathcal{S}_0$ defined as in chapter 3 with, this time, δ so that $\delta\,\mathrm{Re}\,\alpha' > 0$ and $|\delta| > |\mathrm{Im}\,\alpha'|$. From the reciprocity theorem, it is known that, with $\bar{F} = \bar{\bar{F}}\,\bar{c}$,

$$(29)\qquad \bar{\bar{F}}(\alpha, \alpha', \beta) = {}^a\bar{\bar{F}}(\alpha', \alpha, \pi - \beta)$$

where $({}^a\bar{\bar{F}})_{ii} = (\bar{\bar{F}})_{ii}$, $({}^a\bar{\bar{F}})_{ij} = -(\bar{\bar{F}})_{ji}$ with $i \neq j$. Then, let us assume that boundary conditions, on each face $\varphi = \pm\Phi$, allow to write, as in [6]

$$(30)\qquad \begin{aligned} &\mathcal{A}^{\pm}_{\alpha,\beta}\,\bar{f}\,(\alpha \pm \Phi,\, \alpha',\, \beta) - \mathcal{A}^{\pm}_{-\alpha,\beta}\,\bar{f}\,(-\alpha \pm \Phi,\, \alpha',\, \beta) \\ &= \sum c_n \sin\alpha \cos^n \alpha \end{aligned}$$

$\mathcal{A}^{\pm}_{\alpha,\beta}$ being a linear operator with $\mathcal{A}^{\pm}_{\alpha+2\pi,\beta} = \mathcal{A}^{\pm}_{\alpha,\beta}$. We derive easily the functional equational on F

$$(31)\qquad \mathcal{A}^{\pm}_{\pi+\alpha,\beta}\,\bar{F}\,(\alpha \pm \Phi,\, \alpha',\, \beta) + \mathcal{A}^{\pm}_{\pi-\alpha,\beta}\,\bar{F}\,(-\alpha \pm \Phi,\, \alpha',\, \beta) = 0$$

which is equivalent, from (29), to

$$(32)\qquad \mathcal{A}^{\pm}_{\pi+\alpha,\pi-\beta}\,{}^a\bar{\bar{F}}\,(\alpha',\, \alpha \pm \Phi,\, \beta) + \mathcal{A}^{\pm}_{\pi-\alpha,\pi-\beta}\,{}^a\bar{\bar{F}}\,(\alpha',\, -\alpha \pm \Phi,\, \beta) = 0$$

At this step, the expression (28) and the linearity of $\mathcal{A}$ is used, and F can be replaced by f in (32):

$$(33)\qquad \mathcal{A}^{\pm}_{\pi+\alpha,\pi-\beta}\,{}^a\bar{\bar{f}}\,(\alpha',\, \alpha \pm \Phi,\, \beta) + \mathcal{A}^{\pm}_{\pi-\alpha,\pi-\beta}\,{}^a\bar{\bar{f}}\,(\alpha',\, -\alpha \pm \Phi,\, \beta) = 0$$

This new functional equation on the second variable of f is a new tool to study the behaviour of the spectral function on the angle complex plane.

5. CONCLUSION

Three new results concerning the analytical expression of the transient and the harmonic response of an imperfectly conducting wedge have been presented.

Firstly the time response of an impedance wedge in a lossy medium, both frequency dependent, has been originally developed by using the particular form of the field in Sommerfeld-Maliuzhinets integral in frequency domain. A particular causality principle, derived on the inverse Fourier transform of the spectral function associated to the integral for plane wave illumination, explicites the properties of this function in ω complex plane. These properties are used to obtain original analytical causal expressions for line source illuminations, with finite paths of integration on complex angles. The solution gives a useful physical insight, allowing to decompose the field in several contributions, in particular the part due to the edge.

Secondly a suitable analytical expression of the spectral function corresponding to the field radiated by a single face of an arbitrary wedge-shaped region, illuminated by an external source, has been extracted from the knowledge of the spectral function attached to the total field.

The boundary conditions applied to the faces of a wedge are equivalent to functional equations on the spectral function attached to the field. From the initial functional equation, we show, and this is the third point, that we can derive, for a plane wave illumination, a new equation that the spectral function has to satisfy in function, this time, of the angle of arrival of the incident field.

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III.3.A) Errata et commentaires sur les démonstrations et formules

* on note que, dans le chapitre 2.1, on ne considère, pour simplifier l'exposé, que la composante du champ total parallèle à celle du champ incident . On pourra se référer , pour le calcul des influences entre composantes , au chap. IV de l'article des *Annales des télécom.* présenté en III.1 .

* p.1212 : . remplacer " $e^{ik\rho z\cos\beta}$ " par " $e^{ikz\cos\beta}$ "
. remplacer " k_1 is a real constant " par " k_1 is a constant "

* p. 1216 : les formules (19)-(22) peuvent être réécrites de façon plus explicite sous la forme :

$$(19) \quad \tilde{u}_\pm(\rho,\varphi) = -\frac{i}{4}\, e^{-ikz\cos\beta} \int_{\varphi_i' = \pm\Phi ,z=0} \tilde{u}(\rho_i,\varphi_i') \frac{\partial H_0^{(2)}(kR\sin\beta)}{\partial n} - \frac{\partial \tilde{u}(\rho_i,\varphi_i')}{\partial n} H_0^{(2)}(kR\sin\beta)\, d\rho_i ,$$

with $R = \sqrt{\rho^2 + \rho_i^2 - 2\rho\rho_i\cos(\varphi - \varphi_i')}$, ...

$$(20) \quad \tilde{u}_\pm(\rho,\varphi) = -U(\pm\varphi' - \varphi_i)\, \tilde{u}_i - \frac{i}{4}\, e^{-ikz\cos\beta} \int_{\varphi_i' = \pm\varphi_i\, ,\, z=0} \tilde{u}(\rho_i,\varphi_i') \frac{\partial H_0^{(2)}(kR\sin\beta)}{\partial n} - \frac{\partial \tilde{u}(\rho_i,\varphi_i')}{\partial n} H_0^{(2)}(kR\sin\beta) d\rho_i ,$$

with $R = \sqrt{\rho^2 + \rho_i^2 - 2\rho\rho_i cos(\varphi - \varphi_i')}$, ...

$$(22) \quad H_0^{(2)}(kR\sin\beta) = \frac{1}{2\pi i}\int_\gamma \frac{-1}{2\pi}\int_{\mathcal{I}_0} e^{ik\rho_i\sin\beta\cos(\alpha' - \varphi_i')}\mathrm{tg}(\tfrac{1}{2}\,(\alpha + \varphi - \alpha'))d\alpha' e^{ik\rho\sin\beta\cos\alpha} d\alpha ,$$

* p.1218-1219 : pour un dièdre de caractéristiques électriques anisotropes, prendre pour $a(\bar{\bar{\cdot}})$ des caractéristiques transposées.

CHAPITRE IV) Annexe : Compléments de publications

. sur le problème de l'illumination cylindrique :

J.M.L. Bernard ,G. Pelosi ,*'Nouvelle formule d'inversion pour la fonction spectrale de Sommerfeld-Maliuzhinets et applications à la diffraction '*, Rev. Techn. Thomson-Csf , Vol.25 , n°4 , pp.1189-1200 , déc. 1993 .

. sur le théorème d'inversion de Maliuzhinets :

G.D. Maliuzhinets ,*'Inversion formula for the Sommerfeld integral'*, Sov. Phys. Dokl. , 3 , pp. 52-56 , 1958.

Nouvelle formule d'inversion pour la fonction spectrale de l'intégrale de Sommerfeld-Maliuzhinets et applications à la diffraction ([1])

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RÉSUMÉ. – *On étudie ici une formule d'inversion permettant d'obtenir la fonction spectrale de l'intégrale de Sommerfeld-Maliuzhinets à partir de l'expression de la fonction de rayonnement. On l'applique à différents problèmes de rayonnement concernant en particulier la diffraction par un dièdre.*

ABSTRACT. – *An inversion formula to obtain the spectral function of the Sommerfeld-Maliuzhinets integral from the expression of the far-field radiation function is studied here. It is applied to different problems of radiation concerning in particular the diffraction by a wedge.*

1. INTRODUCTION

Le champ électromagnétique peut être représenté sous différentes formes satisfaisant par avance l'équation de propagation des ondes et la condition de rayonnement à l'infini. Parmi ces expressions, celle du champ total avec une intégrale de Sommerfeld-Maliuzhinets a le grand avantage d'avoir une forme généralement indépendante de la position de la source et du point d'observation, ce qui, par exemple, n'est pas le cas de l'expression du

([1]) Manuscrit reçu le 8 février 1993.

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champ sous la forme commune de la transformée de Fourier d'une fonction de rayonnement.

On considère ici une formule d'inversion originale permettant de calculer la fonction spectrale f utilisée pour l'expression du champ en intégrale de Sommerfeld-Maliuzhinets ([1], [2]), à partir de l'expression de la fonction de rayonnement. Cette méthode va permettre, en particulier, d'obtenir les expressions avec une intégrale de Sommerfeld-Maliuzhinets

– du potentiel de rayonnement d'une ligne source,

– du champ électromagnétique total en présence d'un dièdre quelconque illuminé par une ligne source à distance finie, dont on connaît la fonction spectrale quand il est illuminé par une onde plane,

– du champ en présence d'une discontinuité de rayon de courbure dans un corps imparfaitement conducteur, à partir du coefficient de diffraction de Kaminetzky et Keller [3].

2. DÉTERMINATION DE LA FORMULE D'INVERSION

Formule d'inversion

On considère un dièdre d'angle externe $n\pi$, d'arête confondue avec l'axe z, de propriétés électriques invariantes suivant z. On se place dans le cas où la dépendance en temps et en z est donnée par le terme exponentiel $\exp(i\omega t - ik\cos\beta z)$, ω représentant la pulsation de l'onde, k, le nombre d'onde du milieu de propagation et β, l'angle de la direction de l'onde incidente avec l'arête. On laissera implicite ce terme dans la suite et on notera $k_t = k\sin\beta$. La composante suivant z du champ électrique ou magnétique en présence du dièdre, dans un secteur angulaire ne contenant aucune source, sauf à l'infini, peut s'exprimer sous la forme d'une intégrale de Sommerfeld-Maliuzhinets [1]

$$u(\rho, \varphi) = \frac{1}{2\pi i} \int_\gamma f(\alpha + \varphi)\, e^{ik_t \rho \cos\alpha}\, d\alpha \tag{1}$$

où (ρ, φ) sont les coordonnées polaires définies dans le plan perpendiculaire à z, et γ donné en figure 1 est constitué d'une branche, allant de $(i\infty + \pi + h)$ à $(i\infty - h)$, et de son symétrique, avec $|Im\,\alpha| > d$, h, d réels positifs, et f, définis de telle façon que $f(\alpha + \varphi)$ est régulière partout dans le domaine interne à chaque branche, u supposé fini en $\rho = 0$. Par ailleurs on sait, par

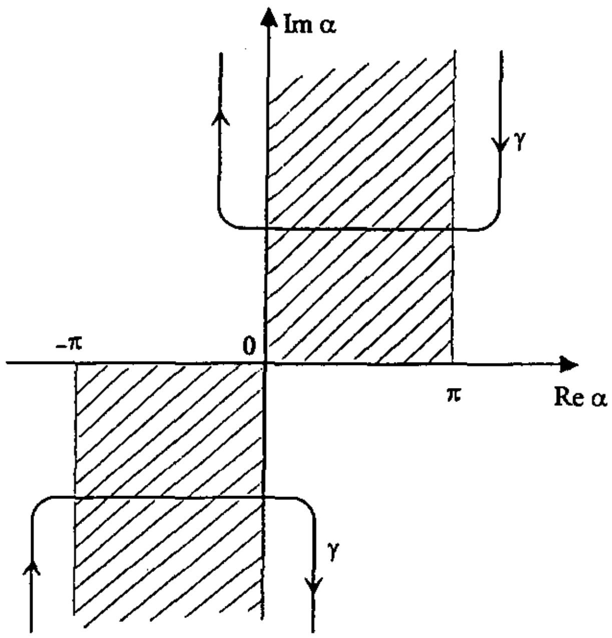

Fig. 1. – Chemin d'intégration γ.

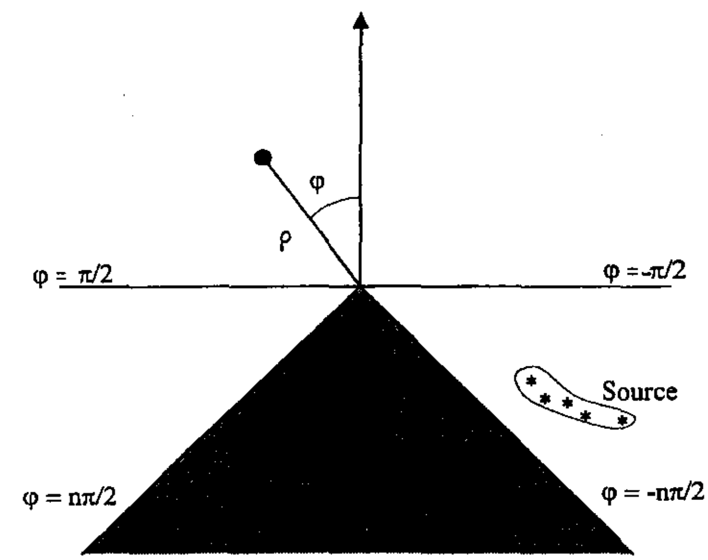

Fig. 2. – Géométrie du problème.

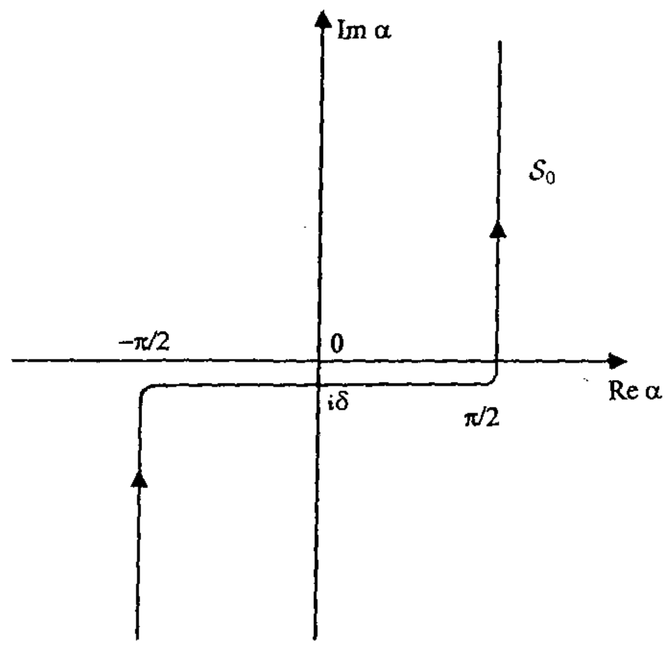

Fig. 3. – Chemin d'intégration $\mathcal{S}_0$.

construction [2], que dans le cas où la source illuminant le dièdre se situe dans le demi-espace $|\varphi| > \pi/2$ (*fig.* 2), on a, pour $|\varphi| < \pi/2$,

$$(2) \quad u(\rho, \varphi) = -\frac{1}{2\pi i} \int_{\mathcal{S}_0} (f(\pi + \alpha) - f(-\pi + \alpha))\, \mathrm{e}^{-ik_t \rho \cos(\alpha - \varphi)}\, \mathrm{d}\alpha$$

où $\mathcal{S}_0$ est défini en figure 3. Si le champ incident est une onde plane de direction $\varphi = \varphi'$, on veillera à ce que $\delta \, . \, \varphi' > 0$. Pour une source passée dans le second demi-espace, la solution s'obtient par prolongement analytique.

On désire maintenant exprimer $f(\alpha)$ à partir de la fonction de rayonnement $[f(\pi+\alpha) - f(-\pi+\alpha)]$. Pour cela, on multiple (2) par $-ik_t \sin\alpha' \, \mathrm{e}^{-ik_t\rho \cos \alpha'}$, ($Im\, k_t \cos\alpha' < 0$), et on intègre de 0 à ∞ en ρ. Pour le terme de gauche, on utilise le théorème de Maliuzhinets [1], tandis que pour celui de droite, il est permis d'inverser l'ordre d'intégration. Cela donne

$$(3) \quad f(\alpha' + \varphi) - f(-\alpha' + \varphi) = \frac{\sin \alpha'}{2\pi i} \int_{\mathcal{S}_0} \frac{f(\pi + \alpha) - f(-\pi + \alpha)}{\cos \alpha' + \cos(\alpha - \varphi)}\, \mathrm{d}\alpha$$

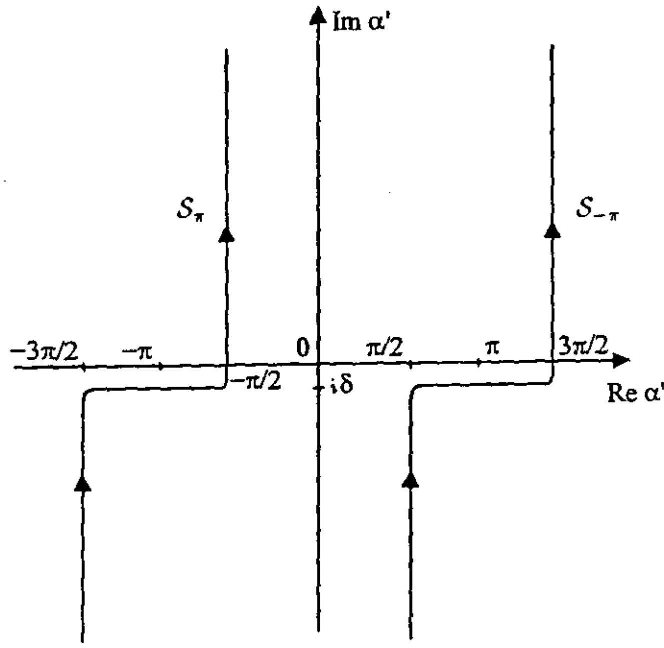

Fig. 4. – Contours dans le plan complexe S_π et $S_{-\pi}$.

Comme

$$\frac{\sin \alpha'}{\cos \alpha' + \cos(\alpha - \varphi)} = \frac{1}{2}\left(\operatorname{tg}\left(\frac{1}{2}(\alpha' + \varphi - \alpha)\right) - \operatorname{tg}\left(\frac{1}{2}(-\alpha' + \varphi - \alpha)\right)\right)$$

on dérive de (3) la formule d'inversion recherchée,

$$(4) \quad f(\alpha') = \frac{1}{4\pi i} \int_{S_0} (f(\pi + \alpha) - f(-\pi + \alpha)) \operatorname{tg}\left(\frac{1}{2}(\alpha' - \alpha)\right) d\alpha$$

valide pour α' compris entre $S_{-\pi}$ et S_π définis en figure 4. Cette expression peut être étendue à tout le plan complexe, soit par prolongement analytique, soit par l'utilisation directe de l'expression de $[f(\pi+\alpha)-f(-\pi+\alpha)]$.

Comme pour (2), (4) est donnée pour une source disposée en $|\varphi|>\pi/2$, mais comme (2), elle est prolongeable analytiquement en dehors de ce cas (particulièrement utile pour $n \leq 1$).

On note que si f nous permet de déformer, pour (1), le contour γ vers les chemins de descente rapide concernant le terme exponentiel, on n'a plus besoin du calcul proprement dit de l'expression intégrale de f donnée dans (4), mais plutôt de celui de $[f(\pi+\alpha)-f(-\pi+\alpha)]$ d'une part, et d'autre part, des contributions aux singularités de f interceptées au cours de la transformation de γ. Ce calcul aux singularités de la fonction f est obtenu assez simplement, quelle que soit la position de la source et du point d'observation, par prolongement analytique de l'expression donnée en (4).

Mais l'utilisation directe de cette démarche n'est qu'un des choix de développement du champ que permet (1) associée à (4).

On considère maintenant différents exemples précis d'application de la formule d'inversion.

3. APPLICATIONS

3.1. Expression de f pour le potentiel d'une ligne source $H_0^{(2)}(k_t R)$

Le potentiel de rayonnement d'une ligne source parallèle à z, en (ρ', φ'), est donné avec l'expression bien connue de la fonction de Hankel, suivant

$$(5) \qquad H_0^{(2)}(k_t R) = \frac{1}{\pi} \int_{S_0} e^{ik_t \rho' \cos(\alpha-\varphi')} e^{-ik_t \rho \cos(\alpha-\varphi)} d\alpha$$

pour

$$R = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\varphi - \varphi')}, \qquad |\varphi| < \pi/2, \quad |\varphi'| > \pi/2$$

Comparant (2) à (5), et utilisant (4), on obtient directement l'expression originale de f pour $H_0^{(2)}(k_t R)$,

$$(6) \qquad f(\alpha') = -\frac{1}{2\pi} \int_{S_0} e^{ik_t \rho' \cos(\alpha-\varphi')} \operatorname{tg}\left(\frac{1}{2}(\alpha' - \alpha)\right) d\alpha$$

Le prolongement analytique de f permet d'écrire directement (1) pour φ tel que signe $(\varphi').(\varphi' - \varphi) > 0$; mais, cela n'est pas valable pour les autres cas, à cause du comportement exponentiel de f. On relèvera qu'une simple sommation pondérée de la fonction élémentaire donnée en (6) permet de calculer la fonction spectrale associée à une distribution quelconque de sources. Ces remarques serviront pour l'expression du champ, pour φ et φ' quelconques, avec une source à distance finie rayonnant devant un dièdre.

Parmi les propriétés de (6), on notera que, pour $-\pi/2 \leqq \pm Re\,\alpha' \leqq 3\,\pi/2$

$$(7) \qquad f(\alpha')|_{Im\ \alpha' \to \pm i\infty} = \mp \frac{i}{2} H_0^{(2)}(k_t R)|_{\rho=0} = \mp \frac{i}{2} u(0, \varphi)$$

ce qui est identique à une caractéristique générale de f énoncée dans [1].

On remarquera, de plus, en faisant tendre ρ' vers ∞ dans (6), qu'on obtient sans difficulté la fonction spectrale associée à une onde plane de direction φ'. Celle-ci est $(1/2)\operatorname{cotg}((1/2)(\alpha'-\varphi'))$, de période 2π. La fonction de rayonnement associée, au sens des fonctions analytiques utilisé pour (2), est nulle. On en déduit donc qu'une pure onde plane admet (1) mais pas (2) comme représentation, au sens des fonctions analytiques.

3.2. **Expression du champ pour une ligne source et un point d'observation à distances finies d'un dièdre quelconque**

On suppose connaître la réponse à une onde plane du dièdre sous la forme (1). On sait alors écrire, par application du théorème de réciprocité, la fonction de rayonnement pour une ligne source en (ρ', φ'), $|\varphi'| > \pi/2$, rayonnant en espace libre $u^i = H_0^{(2)}(k_t R)$, sous la forme suivante :

$$(8) \quad f(\pi + \alpha'') - f(-\pi + \alpha'') = -\frac{1}{\pi} \int_{\gamma_{\varepsilon'}} e^{ik_t \rho' \cos\alpha'} f_0(\alpha' + \varphi', \alpha'')\, d\alpha'$$

où $\gamma_{\varepsilon'}$ donné en figure 5 n'intercepte aucune singularité de $f_0(\alpha' + \varphi', \alpha'')$ quand α'' appartient à S_0, f correspond à un angle d'obliquité de β, f_0 de $\pi - \beta$. Les expressions (4) et (1) permettent alors d'écrire

$$(9) \quad u(\rho, \varphi) = \frac{1}{4\pi i} \int_{\gamma_\varepsilon} \frac{e^{ik_t \rho \cos\alpha}}{2\pi i} \left(\int_{S_0} \frac{\sin\alpha}{\cos\alpha + \cos(\alpha'' - \varphi)} \times \left(-\frac{1}{\pi} \int_{\gamma_{\varepsilon'}} e^{ik_t \rho' \cos\alpha'} f_0(\alpha' + \varphi', \alpha'')\, d\alpha' \right) d\alpha'' \right) d\alpha$$

pour $|\varphi| < \pi/2$, $|\varphi'| > \pi/2$. Avec le chemin $\gamma_{\varepsilon'}$ et le comportement exponentiel décroissant de l'intégrand en α'', on peut inverser l'ordre d'intégration en α'' et α'. Cela donne, en utilisant la parité de γ_ϵ

$$(10) \quad u(\rho, \varphi) = \frac{1}{2\pi i} \int_{\gamma_\varepsilon} e^{ik_t \rho \cos\alpha} \times \left(-\frac{1}{\pi} \int_{\gamma_{\varepsilon'}} e^{ik_t \rho' \cos\alpha'} f_s(\alpha + \varphi, \alpha' + \varphi')\, d\alpha' \right) d\alpha$$

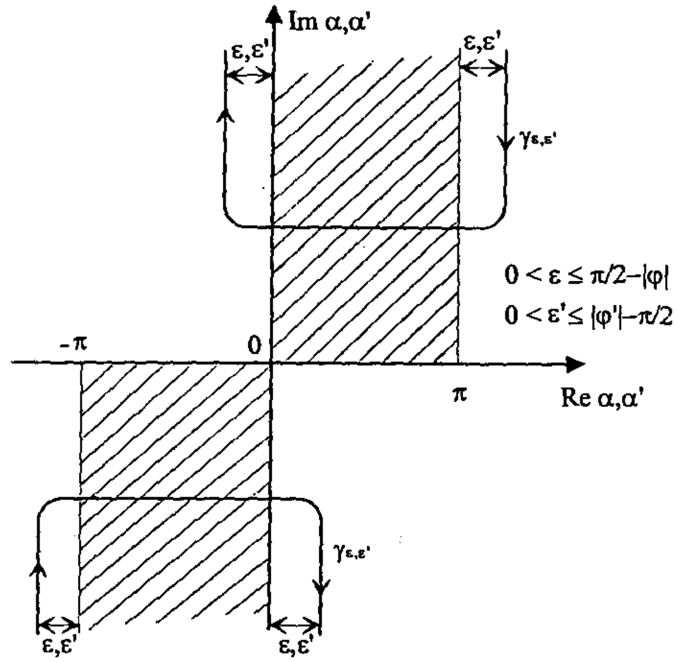

Fig. 5. – Chemins d'intégration γ_ε et $\gamma_{\varepsilon'}$.

avec f_s écrit suivant

$$f_s(\alpha, \alpha') = \frac{1}{4\pi i} \int_{S_0} f_0(\alpha', \alpha'') \operatorname{tg}\left(\frac{1}{2}(\alpha - \alpha'')\right) \mathrm{d}\alpha''$$

Pour obtenir une expression de u valide pour φ et φ' quelconques, on doit suivre une remarque du paragraphe précédent. On sépare la fonction spectrale attachée au champ incident, qui ne permet pas d'écrire (1) quel que soit φ, de celle du champ proprement diffracté qui, rayonné par la distribution équivalente de courants portés par la surface du dièdre $|\varphi| = n\pi/2$, permet, d'après le paragraphe 3.1., de poser (1) quel que soit $|\varphi| < n\pi/2$. On écrit donc

$$(11) \quad u(\rho, \varphi) = \frac{1}{2\pi i} \int_{\gamma_\varepsilon} e^{ik_t \rho \cos\alpha} \times \left(-\frac{1}{\pi} \int_{\gamma_{\varepsilon'}} e^{ik_t \rho' \cos\alpha'} f_{sd}(\alpha + \varphi, \alpha' + \varphi')\, \mathrm{d}\alpha'\right) \mathrm{d}\alpha + H_0^{(2)}(k_t R)$$

avec $R = \sqrt{\rho^2 + \rho'^2 - 2\,\rho\rho' \cos(\varphi - \varphi')}$, et f_{sd} écrit suivant

$$(12)\quad f_{sd}(\alpha, \alpha') = \frac{1}{4\pi i} \int_{S_0} \left(f_0(\alpha', \alpha'') - \frac{1}{2}\operatorname{cotg}\left(\frac{1}{2}(\alpha' - \alpha'')\right)\right) \times \operatorname{tg}\left(\frac{1}{2}(\alpha - \alpha'')\right) \mathrm{d}\alpha''$$

L'expression est alors prolongeable pour φ et φ' quelconques en prenant f_{sd} pour son prolongement analytique, ainsi que γ_ε et $\gamma_{\varepsilon'}$ pour γ. Elle constitue une détermination originale du champ, indépendante des positions de la ligne source et du point d'observation, avec une double intégrale de Sommerfeld-Maliuzhinets. Cette expression est bien distincte de la double transformée de Fourier utilisée dans [4], dont la forme doit varier avec la position de la source et du point d'observation.

On montre maintenant, sur un exemple, la validité du développement précédent, en retrouvant, dans le cas parfaitement conducteur, l'expression donnée par Maliuzhinets [5]. Pour cela, on remarque, dans le cas particulier parfaitement conducteur, pour l'intégration suivant α', que

$$(13)\quad \text{partie impaire } f_0(\alpha' + \varphi', \alpha'') = \text{partie impaire } f_0(\alpha' + \alpha'', \varphi')$$

On peut alors modifier (10) en posant $\Omega = \alpha' + \alpha''$ comme nouvelle variable, et en dériver l'expression suivante, pour γ avec $h = \pi/2$:

$$(14)\quad u(\rho, \varphi) = \frac{1}{2\pi i} \int_\gamma f_0(\Omega, \varphi') \times \left(\frac{1}{\pi} \int_{S_0} \mathrm{e}^{ik_t\rho' \cos(\Omega - \alpha'')}\, \mathrm{e}^{-ik_t\rho\cos(\varphi - \alpha'')}\, \mathrm{d}\alpha'' \right) \mathrm{d}\Omega$$

valide quand ρ est suffisamment grand pour la convergence de l'intégrale. Reconnaissant dans l'intégrale en α'' une fonction de Hankel, on obtient, en fin de compte, dans ce cas particulier, la forme suivante :

$$(15)\quad u(\rho, \varphi) = \frac{1}{2\pi i} \int_\gamma f_0(\Omega + \varphi, \varphi')\, H_0^{(2)}(k_t R)\, \mathrm{d}\Omega$$

où $R = \sqrt{\rho^2 + \rho'^2 - 2\,\rho\rho' \cos\Omega}$, qui est bien l'expression obtenue dans [5].

3.3. **Détermination de f pour le cas de la discontinuité de rayon de courbure à partir du coefficient de diffraction de Kaminetzky et Keller [3]**

Dans le cas d'une discontinuité de rayon de courbure illuminée par une onde plane de direction φ', $n\pi/2$ est égal à $\pi/2$ et, puisque $|\varphi'|<n\pi/2$, on a besoin de raisonner par prolongement analytique. Pour cela, on doit modifier (4). On considère le problème du dièdre de faces courbes de rayon de courbure $a^{\pm}$ pour $n \geqq 1$, $\beta=\pi/2$ [6]. On sépare pour (4) l'expression de la fonction de rayonnement en trois termes, suivant $f=f_0+f_+/ka^+ + f_-/ka^-$. On note que, d'après la position des singularités de f [6], quand $|\varphi'|>\pi/2$, on peut écrire pour (4)

$$(16)\quad \begin{aligned} f(\alpha') =&[f_0(\alpha')] \\ &+\left[\frac{1}{4\pi ika^+}\int_{-i\infty-(\pi/2)}^{+i\infty-(\pi/2)}(f_+(\pi+\alpha)-f_+(-\pi+\alpha))\right. \\ &\qquad\left.\times \mathrm{tg}\left(\frac{1}{2}(\alpha'-\alpha)\right)\mathrm{d}\alpha\right] \\ &+\left[\frac{1}{4\pi ika^-}\int_{-i\infty+(\pi/2)}^{+i\infty+(\pi/2)}(f_-(\pi+\alpha)-f_-(-\pi+\alpha))\right. \\ &\qquad\left.\times \mathrm{tg}\left(\frac{1}{2}(\alpha'-\alpha)\right)\mathrm{d}\alpha\right] \end{aligned}$$

Cette nouvelle expression peut, elle, être appliquée à $|\varphi'|<\pi/2$, donc, au cas particulier de la discontinuité de rayon de courbure $n=1$, avec des impédances de faces égales, cas traité par Kaminetzky et Keller [3] qui obtiennent, par leur méthode de la couche limite, le développement asymptotique de la fonction de rayonnement. On note que l'on récupère avec (16) des informations sur la réflexion, qu'avait perdues le coefficient $[f(\pi+\varphi)-f(-\pi+\varphi)]$, puisque celui-ci est en particulier nul pour $a^+=a^-$. On a, pour le cas particulier $n=1$, suivant la notation de [6],

$$(17)\quad \begin{aligned} &(f_{\pm}(\pi+\alpha)-f_{\pm}(-\pi+\alpha)) \\ &\quad=\pm\frac{i}{(\cos\alpha+\sin\theta)} \\ &\quad\times(D_{((\pi/2)+\alpha)}(f_0(\pi+\alpha))-D_{-((\pi/2)+\alpha)}(f_0(-\alpha))) \end{aligned}$$

où,

$$D_\alpha(.) = \frac{1}{2}\left[\frac{\partial^2(.)}{\partial\alpha^2} - \frac{\partial}{\partial\alpha}(\operatorname{cotg}\alpha(.)) + \sin\theta\,\frac{\partial}{\partial\alpha}\left(\frac{1}{\sin\alpha}\,\frac{\partial}{\partial\alpha}(.)\right)\right]$$

$$f_0(\alpha) = \frac{\cos\alpha + \sin\theta}{\cos\varphi' + \sin\theta}\,\frac{\cos\varphi'}{\sin\alpha - \sin\varphi'}$$

où $\sin\theta$ est l'impédance relative de surface.

Suivant alors quelques calculs élémentaires, on retrouve sans difficulté, dans cette application, la même expression de f que dans [6]. On notera donc la possibilité d'associer (4) à un coefficient de diffraction calculé par la GTD (Théorie Géométrique de la Diffraction), pour obtenir l'expression du champ total sous la forme d'une intégrale de Sommerfeld-Maliuzhinets.

4. CONCLUSION

On a déterminé l'expression de la fonction spectrale de Sommerfeld-Maliuzhinets à partir de l'expression de la fonction de rayonnement pour le problème de la diffraction par une structure cylindrique. On peut en déduire alors des expressions originales du champ total, utilisables quelle que soit la position de la source et du point d'observation. L'application est faite à l'expression de la fonction de Hankel, au rayonnement d'une ligne source face à un dièdre quelconque, et à la diffraction par une discontinuité de rayon de courbure.

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Mathematical Physics

INVERSION FORMULA FOR THE SOMMERFELD INTEGRAL

G. D. Maliuzhinets

(Presented by Academician M.A. Leontovich, September 20, 1957)

It is well known that a solution of the two-dimensional wave equation

$$\Delta S - m^2 S = 0, \quad \left(-\frac{\pi}{2} \leqslant \arg m \leqslant \frac{\pi}{2}\right) \tag{1}$$

in the form

$$S(r, \varphi) = \frac{1}{2\pi i} \int\limits_{\gamma} e^{mr\cos\alpha} s(\alpha + \varphi)\, d\alpha \tag{2}$$

was employed by Sommerfeld [1] for a rigorous consideration of the problem of the diffraction of a plane wave by a wedge with boundary conditions $S = 0$ or $\partial S/\partial n = 0$. A systematic method of finding the function $s(\alpha)$ was proposed not long ago [2] which permits the solution of this diffraction problem for boundary conditions

$$\partial S / \partial n + hS = 0.$$

Theorem 1 of this paper enables one to obtain the solution of certain new boundary-value problems in wedge-like regions in the form of the Sommerfeld integral (2).

Theorem 1. Let M, a, b, c, d be positive numbers; let ϵ, m be numbers satisfying the conditions: $0 < \epsilon < \pi$, $|\arg m| \leq \pi/2$. Given the integral equation

$$F(r) = \frac{1}{2\pi i} \int\limits_{\gamma} e^{mr\cos\alpha} f(\alpha)\, d\alpha. \tag{3}$$

The given function F(r) satisfies the inequality $|F(r)| < M|r|^{-1+a} e^{b|r|}$ for positive values of r and also in the entire region $c < |r| < \infty$, $|\arg r| < \epsilon_1$, where this function is analytic and regular.

The contour of integration γ (Figure 1) is made up of two loops. The loop γ_1 consists of the two half lines $\mathrm{Re}\,\alpha = \arg m \pm (\epsilon + \pi/2)$, $\mathrm{Im}\,\alpha \geq d$ and the line segment $\mathrm{Im}\,\alpha = d$. The loop γ_2 is symmetric to γ_1 with respect to inversion in the origin $\alpha = 0$.

Then, among those analytic functions $f(\alpha)$, which are regular on the contour γ and within both loops except possibly at infinitely distant points, and which satisfy in these regions the inequality $|f(\alpha)| < M_1 \exp[(1 - a_1)|\mathrm{Im}\,\alpha|]$, there exists one and only one odd function which is a solution of the integral Equation (3). For $\mathrm{Re}(m\cos\alpha) > b$ this function is represented by the integral

$$f(\alpha) = -\frac{m\sin\alpha}{2} \int\limits_0^\infty F(r)\, e^{-mr\cos\alpha}\, dr. \tag{4}$$

For this function $a_1 = a$.

Proof. For an odd function $f(\alpha)$, in view of the symmetry of the contour γ, we can use a single loop γ_1 and rewrite (3) as

$$F(r) = \frac{1}{\pi i}\int\limits_{\gamma_1} e^{mr\cos\alpha} f(\alpha)\,d\alpha. \tag{5}$$

If we introduce the function

$$g(w) = -2f(\alpha)\exp(-i\arg m)/\sin\alpha,$$

where $w = \exp(i\arg m)\cos\alpha$, the Integral (5) becomes

$$F(r) = \frac{1}{2\pi i}\int\limits_{\Gamma} e^{|m|rw} g(w)\,dw, \tag{6}$$

where the contour Γ, into which γ_1 is transformed (Figure 2), intersects the real axis between the origin and $\cosh d$ and coincides at infinity with the rays $\arg w = \pm(\epsilon + \pi/2)$.

The function $g(w)$ is regular in the region to the right of the contour Γ since this region is the transform of the interior of the loop γ_1, where the function $f(\alpha)/\sin\alpha$ is regular. From the limitations imposed on the order of increase of the allowed functions $f(\alpha)$, we obtain $|g(w)| < 4M_1|w|^{-a_1}$ for $|w| \to \infty$, $|\arg w| < \epsilon + \pi/2$.

In order to find $g(w)$ from Equation (6), we multiply both sides of the equation by $\exp(-|m|rw)$, where $\operatorname{Re} w > \cosh d$ and integrate with respect to r, obtaining

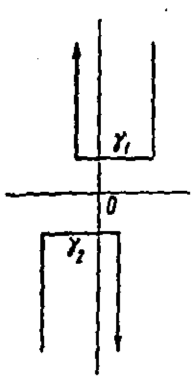

Fig. 1

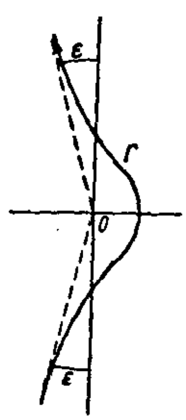

Fig. 2

$$\int\limits_0^\infty F(r)\,e^{-|m|rw}\,dr =$$

$$= \frac{1}{2\pi i}\int\limits_{\to 0}^{\to\infty} dr \int\limits_{\Gamma} e^{|m|r(w_1-w)} g(w_1)\,dw_1.$$

Changing the order of integration on the right-hand side and then taking the limit, we obtain

$$\int\limits_0^\infty F(r)\,e^{-|m|rw}\,dr = -\frac{1}{2\pi i|m|}\int\limits_{\Gamma}\frac{g(w_1)}{w_1 - w}\,dw_1 = \frac{g(w)}{|m|}, \tag{7}$$

where, thanks to the condition $a_1 > 0$, the integration over the contour Γ has reduced to a residue calculation. From this, transforming back to the function $f(\alpha)$ we obtain the desired odd solution (4) of the Integral Equation (3).

In order to convince ourselves that our solution $f(\alpha)$ is really regular, as we have assumed, we note that in the Integral (7) the portion of the path of integration $r > c$ may be displaced to an arbitrary half-line $|\arg r| = \epsilon_1 > \epsilon$, thanks to the analyticity of $F(r)$, after which we may convince ourselves of the regularity of the function $g(w)$ for large $|w|$ for all intervals $|\arg w| < \epsilon + \pi/2$, and consequently of the regularity of $f(\alpha)/\sin\alpha$ for large $\operatorname{Im}\alpha \geq d$ in the strip $|\operatorname{Re}\alpha - \arg m| < \epsilon + \pi/2$, which corresponds to the interior of the loop γ_1.

Finally, using the condition $|F(r)| < M|r|^{-1+\alpha}e^{b|r|}$, we obtain from (7)

$$|g(w)| < \int_n^{\infty} |F(r)|\, e^{-r|wm|\cos(\arg w)}\, dr < M\,|mw|^{-a} \int_0^{\infty} e^{-x\,[\cos(\arg w) - b/|mw|]}\, x^{1-a}\, dx.$$

Since the integral on the right is bounded in magnitude for large values of $|w|$, we have the estimate $|g(w)| < M_2|w|^{-a}$. It follows that for the solution $f(\alpha)$ we have also $a_1 = a$.

We have thus proven Theorem 1. Formulas (3) and (4) are inversion formulas for the Sommerfeld integral.

The uniqueness of the solution, which is a consequence of the fact that, by virtue of (4), an odd solution $f(\alpha)$ of the homogeneous integral equation

$$\frac{1}{2\pi i} \int_{\gamma} e^{mr\cos\alpha} f(\alpha)\, d\alpha = 0 \tag{8}$$

must vanish identically, is closely connected with the condition $f(\alpha) = O\{\exp[(1-a)|\operatorname{Im}\alpha|]\}(a > 0)$. This is evident from the following theorem.

<u>Theorem 2.</u> Let $f(\alpha)$ be an analytic function, regular on the contour γ and in the interior of the loops γ_1 and γ_2 of Theorem 1 everywhere except for infinitely distant points. For $|\operatorname{Im}\alpha| \to \infty$, let $f(\alpha) = O\{\exp[(n + 1 - a)|\operatorname{Im}\alpha|]\}$ in these regions, where $0 < a < 1$ and <u>n</u> is a positive integer or zero.

Then, in order for (8) to hold for $r > 0$, it is necessary and sufficient that the function $f(\alpha)$ have the form

$$f(\alpha) = f_1(\alpha) + \sin\alpha \sum_{\nu=0}^{n} c_{\nu}\nu \cos^{\nu-1}\alpha, \tag{9}$$

where $f_1(\alpha)$ is an arbitrary even function and the coefficients c_ν are arbitrary constants; or, as follows from (9), that the function $f(\alpha)$ satisfy the functional equation

$$f(\alpha) - f(-\alpha) = 2\sin\alpha \sum_{\nu=0}^{n} c_{\nu}\nu \cos^{\nu-1}\alpha. \tag{10}$$

<u>Proof.</u> When an even function $f_1(\alpha)$ is substituted in (8), this integral vanishes by virtue of the symmetry of the contour γ. The remaining odd part of the function $f(\alpha)$, equal to $\frac{1}{2}[f(\alpha) - f(-\alpha)]$, we shall designate by $f_2(\alpha)$. By means of an n-fold integration by parts, we may transform the integral Equation (8) into the form $\frac{1}{2\pi i}\int_{\gamma} e^{mr\cos\alpha} D^n[f_2(\alpha)]\, d\alpha = 0$, where D is an operator given by

$$D[f(\alpha)] = \frac{1}{m}\frac{d}{d\alpha}\left[\frac{f(\alpha)}{\sin\alpha}\right].$$

Since the odd function $D^n[f_2(\alpha)]$ for $|\operatorname{Im}\alpha| \to \infty$ is of order $\exp[(1-a)|\operatorname{Im}\alpha|]$, the conditions of Theorem 1 now obtain, and hence, in view of (4), we have $D^n[f_2(\alpha)] = 0$. From this, by an n-fold integration, we obtain an expression containing <u>n</u> arbitrary constants which we may write in the form

$$f_2(\alpha) = \sin\alpha \sum_{\nu=0}^{n} c_{\nu}\nu \cos^{\nu-1}\alpha.$$

Thus, the necessity and sufficiency of either (9) or (10) is clear and Theorem 2 is proven.

We note that the formula

$$D^n[f(\alpha)] = -\frac{m\sin\alpha}{2}\int_0^\infty e^{-mr\cos\alpha}F(r)r^n dr \tag{11}$$

may serve as an inverse to (3) in the case when, for $r \to 0$, the function $F(r)$ is of the order $|r|^{a-(n+1)}$.

Theorem 2 enables one to solve boundary value problems in wedge-like regions with derivatives of arbitrary order at the boundary, in particular the problem of the diffraction of sound waves by a half-infinite elastic plate. In this case, the arbitrary constants assume a simple meaning.

In all the formulas given above, we have, following V.A. Fok, introduced the complex quantity m in place of the wave number k. For the problem of wave propagation in a nonabsorptive medium possessing a positive phase velocity [3], taking the time dependence to be a factor $\exp(-i\omega t)$, we may assume that $m = -ik$, where k is a positive number. Then, the Sommerfeld Integral (2) and the inversion Formulas (3) and (4) become

$$S(r,\varphi) = \frac{1}{2\pi i}\int_\gamma e^{-ikr\cos\alpha}s(\alpha+\varphi)\,d\alpha; \tag{12}$$

$$F(r) = \frac{1}{2\pi i}\int_\gamma e^{-ikr\cos\alpha}f(\alpha)\,d\alpha; \tag{13}$$

$$f(\alpha) = \frac{ik\sin\alpha}{2}\int_0^\infty e^{ikr\cos\alpha}F(r)\,dr \tag{14}$$

and the path of integration over the two loops of the contour γ runs between the limits: $\gamma_1(i\infty+\epsilon,\ i\infty-\pi-\epsilon)$ and $\gamma_2(-i\infty-\epsilon,\ -i\infty+\pi+\epsilon)$.

As the simplest example of the application of the inversion formulas (14) in conjuction with the method developed earlier [2, 4], we derive the function of forced vibration [5] $S(r,\varphi)$, satisfying the equation $\Delta S + k^2S = 0$ in the wedge-like region $-\Phi < \varphi < \Phi$, excited by the common action of the wave $S_0\exp[-ikr\cos(\varphi - \varphi_0)](|\mathrm{Re}\,\varphi_0| < \Phi - \epsilon)$, incident from infinity and sources distributed over the boundary of the wedge determined by a given function $F_\pm(r)$ at the boundary: $-\partial S/\partial\varphi + ikrF_\pm(r) = 0\ (\varphi = \pm\Phi)$, where $|F_\pm(r)| < \infty$ for $r < \infty$.

Then, the unique continuous and single-valued (including points on the boundary) solution may be represented in the form of a Sommerfeld integral if we look for the function $s(\alpha)$ among those functions satisfying the following conditions: 1) the function $s(\alpha) - S_0/(\alpha - \varphi_0)$ is regular in the strip $|\mathrm{Re}\,\alpha| < \Phi$; 2) in this strip $|s(\alpha) - s(\pm i\infty)| < M\exp(-a|\mathrm{Im}\,\alpha|)$ for $\mathrm{Im}\,\alpha \to \pm\infty$; 3) $s(-i\infty) = -s(i\infty) = iS(0,\varphi)/2$ ($S(0,\varphi)$ does not depend on φ).

We represent both members of the left-hand side of the boundary condition as Sommerfeld integrals:

$$-\frac{1}{ikr}\frac{\partial S}{\partial\varphi}\bigg|_{\varphi=\pm\Phi} = \frac{1}{2\pi i}\int_\gamma e^{-ikr\cos\alpha}\frac{\sin\alpha}{2}[s(\pm\Phi+\alpha)+s(\pm\Phi-\alpha)]\,d\alpha,$$

$$F_\pm(r) = \frac{1}{2\pi i}\int_\gamma e^{-ikr\cos\alpha}f_\pm(\alpha)\,d\alpha,$$

where the functions $f_\pm(\alpha)$ are expressed in terms of the given functions $F_\pm(r)$ by the inversion Formula (14). Then, by virtue of Theorem 2 and Conditions 2) and 3), we obtain the two functional equations $s(\alpha \pm \Phi) + s(-\alpha \pm \Phi) = -2f_\pm(\alpha)/\sin\alpha$. The solution of these equations, which has a single pole with principal part $S_0/(\alpha - \varphi_0)$ in the strip $|\mathrm{Re}\,\alpha| < \Phi + \epsilon$, may easily be obtained with the help of a Fourier integral:

$$S(\alpha) = S_0\frac{\pi}{2\Phi}\cos\frac{\pi\alpha}{2\Phi}\Big/\left[\sin\frac{\pi\alpha}{2\Phi} - \sin\frac{\pi\varphi_0}{2\Phi}\right] +$$
$$+\frac{i}{2\sqrt{2\pi}}\left(\int_{-i\infty-\delta}^{i\infty-\delta} + \int_{-i\infty+\delta}^{i\infty+\delta}\right)\frac{R_+(w)e^{-iw\Phi} - R_-(w)e^{iw\Phi}}{i\sin 2w\Phi}e^{-iw\alpha}dw, \tag{15}$$

where

$$R_{\pm}(w) = \frac{i}{\sqrt{2\pi}} \int_{-i\infty}^{i\infty} \frac{f_{\pm}(\alpha)}{\sin\alpha} e^{i\omega x} d\alpha.$$

Formulas (12), (15) provide a solution to the problem. For the particular case $F_{\pm}(r) \equiv 0$ when only the first member of (15) is present, we obtain the well-known solution of Sommerfeld for the problem of the diffraction of a plane wave by an ideal rigid wedge. The other extreme case $S_0 = 0$ is the solution of the two-dimensional equation $\Delta S + k^2 S = 0$ for a boundary value problem of the Neumann type.

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CONCLUSION

Si la diffraction d'une onde électromagnétique par des discontinuités pose des problèmes mathématiques fort complexes , on montre , dans cette synthèse de nos publications sur le sujet , que la recherche de solutions analytiques rigoureuses pour des classes très générales de problèmes , exigeant dès le départ une démarche originale , peut se révéler fort fructueuse . On obtient ainsi des expressions explicites des champs diffractés par un dièdre pour des régimes stationnaires ou non , des illuminations planes ou cylindriques quelconques , des géométries avec faces planes ou courbes , tout en considérant des conditions aux limites imparfaitement conductrices . Les expressions obtenues apportent des progrès significatifs dans la description du comportement des champs dans des situations de plus en plus réalistes , tout en restant , du fait qu'elles sont analytiques , d'un coût calcul raisonnable .