

Metrology = science of measurement

$$q = \{q\} [q]$$

{ } numerical value

[] unit

Towards a coherent system of units

•1789

Cahiers de doléances (Of grievances) :

« Ils demandent que les seigneurs à qui est dû des rentes en grains soient obligés de tenir au lieu principal du fief des mesures marquées et jaugées à l'ancienne et petite ou grande mesure, suivant que l'exigent les redevances. » (Cdd. Pas-de-Calais (Selles), II, P. 436)

« Qu'ils paient désormais leurs rentes à une seule et unique mesure royale » (Cdd. Quimper (Beuzac-Cap-Caval))

« Qu'il n'y eût plus qu'une mesure pour tout le Royaume, et que les grains de différentes espèces se mesurassent dans une même mesure.» (Cdd. Troyes (Chapvalonn), I, pp. 542/543)

•18 germinal an III (April 7, 1795)

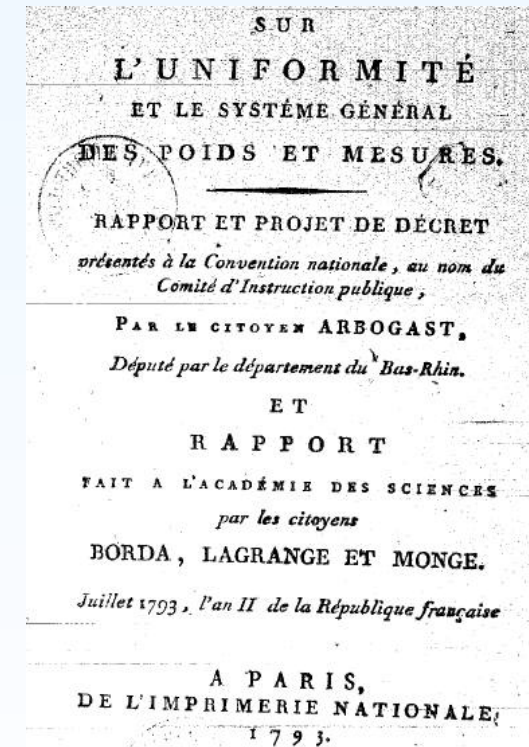
Décret relatif aux poids et aux mesures

The Convention decreed that the new "Republican Measures" were to be henceforth legal measures in France.

The metric (and decimal) system was adopted:

- the metre was defined as the length of 1/10 000 000 of the northern quadrant of the Paris meridian;

- the gram was defined as the absolute weight of a volume of pure water equal to a cube of one hundredth of a meter, and to the temperature of the melting ice.^[1]



M. J. de Caritat,
baron de Condorcet



G. Monge

Towards a coherent system of units

- 1832

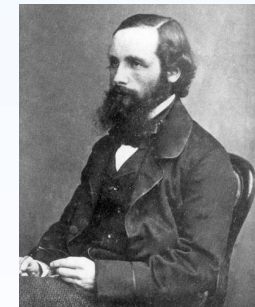
C.F. Gauss performed measurements of Earth's magnetic field, by using a decimal system based on three quantities: length, mass and time (mm, g, s).



C.F. Gauss

- 1860/1870

J.C. Maxwell proposed a coherent system of units: CGS (centimetre, gram, second)



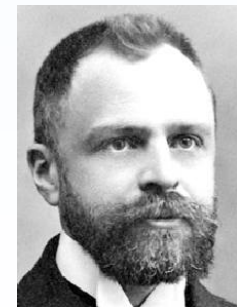
J.C. Maxwell

- 1875 : convention du mètre



- 1901

G. Giorgi completed the system MKS (metre, kilogram, second) by adding an electrical unit: the ampere



G. Giorgi

- 1948 -> 1960

Creation of the *Système international d'unités (SI)*



Outline

- The SI and the electrical units
- Single electron pump
- Experimental set-up
- Measurements
- The metrological triangle experiment

Outline

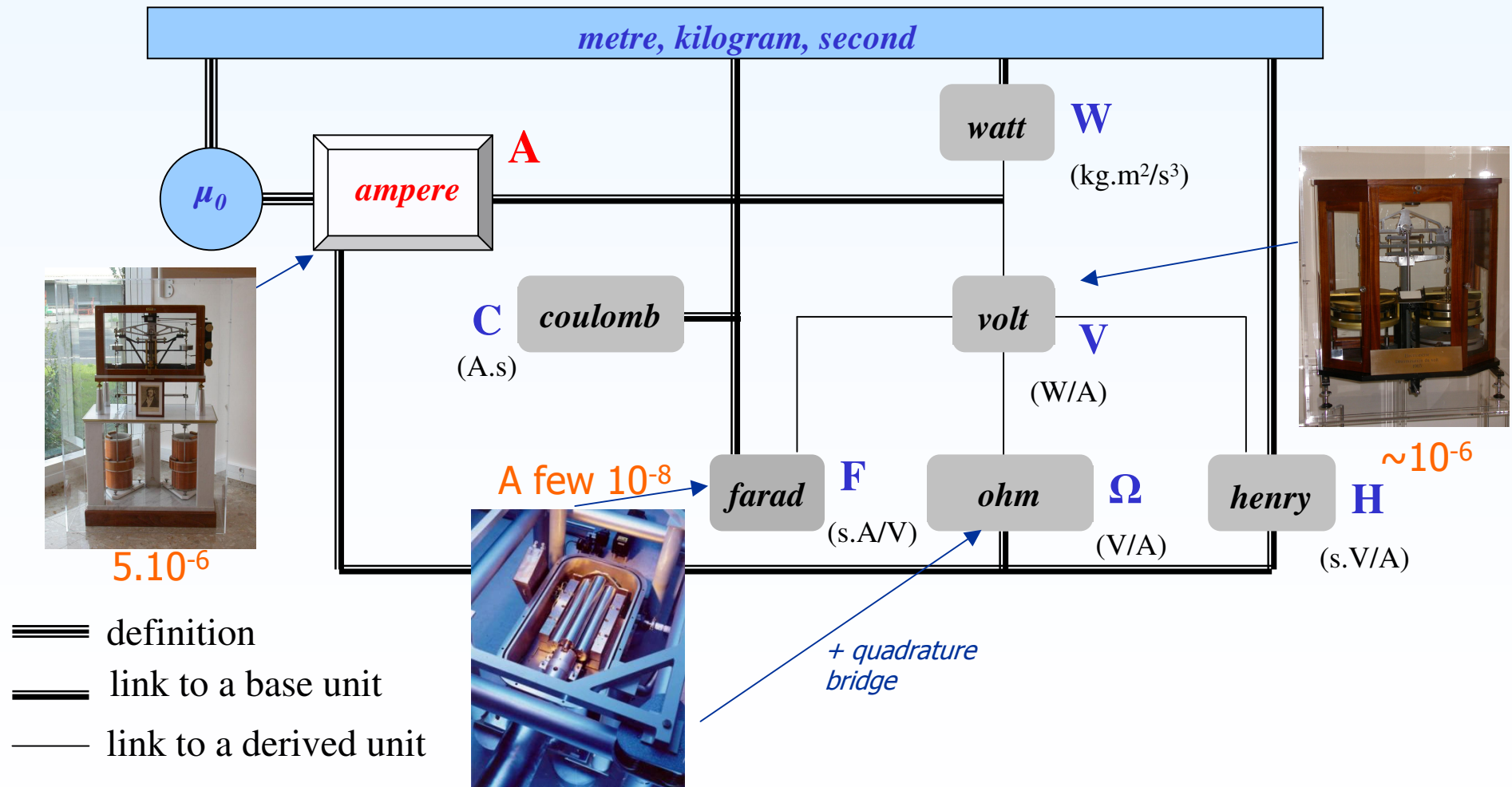
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Systeme international d'unités (SI)

Base quantity	Unit	Definition
mass	<i>kilogram (kg)</i>	The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram. (3 rd CGPM, 1901)
electric current	<i>ampere (A)</i>	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to $2 \cdot 10^{-7}$ newton per metre of length. (9 th CGPM, 1948)
thermodynamic temperature	<i>kelvin (K)</i>	The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (10 th CGPM, 1954)
time	<i>second (s)</i>	The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom. (13 th CGPM, 1967/68)
amount of substance	<i>mole (mol)</i>	1. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12. 2. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles. (14 th CGPM, 1971)
luminous intensity	<i>candela (cd)</i>	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. (16 th CGPM, 1979)
length	<i>metre (m)</i>	The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second. (17 th CGPM, 1983)

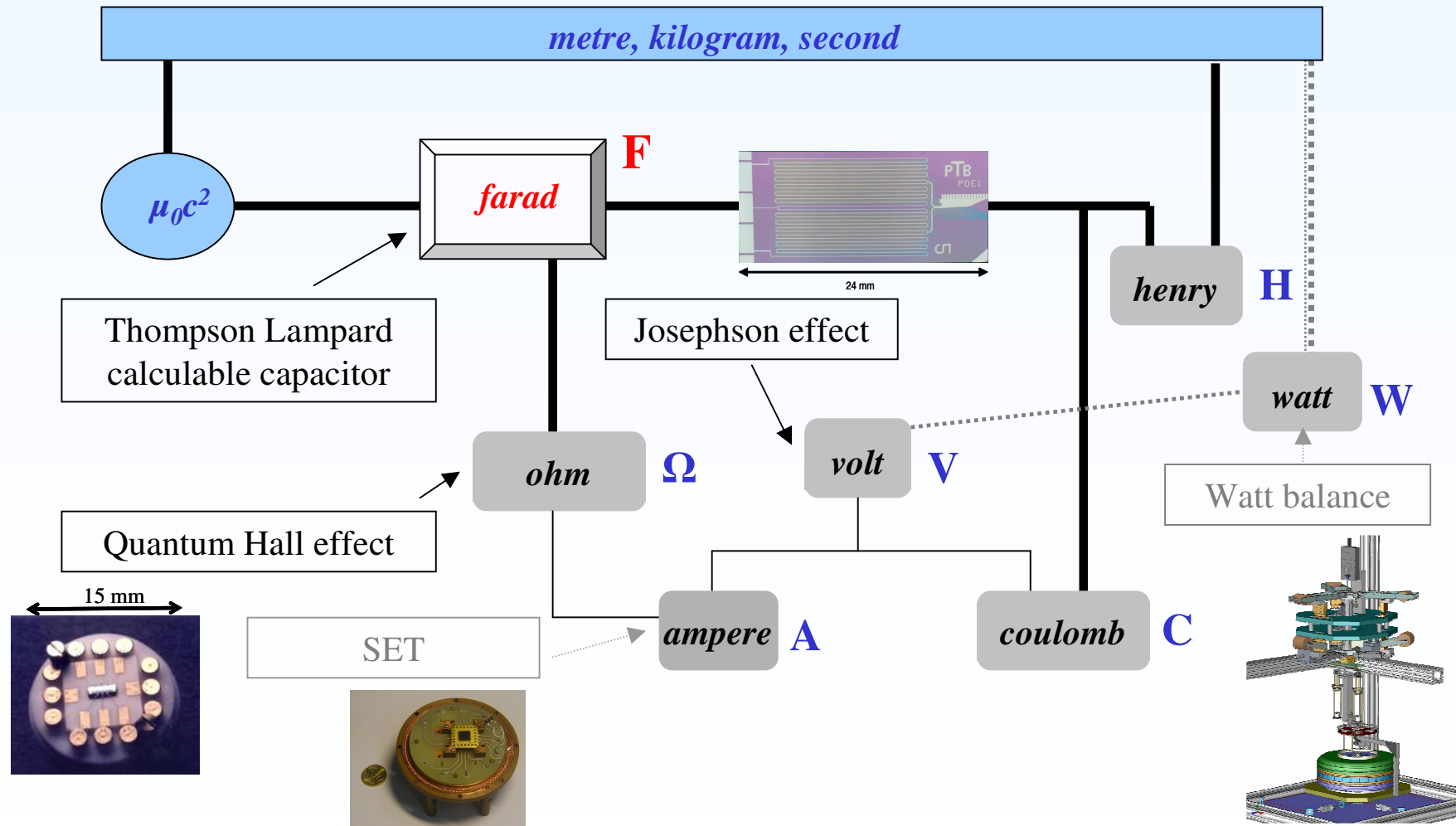
Electrical units : definition

The **realisation** of a unit is a physical experiment or artefact, based on well-established principles, that produces the unit in terms of **the SI definition**.



Electrical units : representation

The **representation** of a unit is an experiment or artefact which produces a quantity which can be routinely compared to other standards. The routine nature of a representation allows us to disseminate from one primary standard to a large number of secondary standards.



Quantum electrical metrology

- Josephson effect

$$V = \frac{n_j f_j}{K_J}$$

$$K_J \equiv \frac{2e}{h}$$

Volt reproducibility with a conventional value of K_J ($K_{J-90}=483\,597.9$ GHz/V):

1 part in 10^{10}

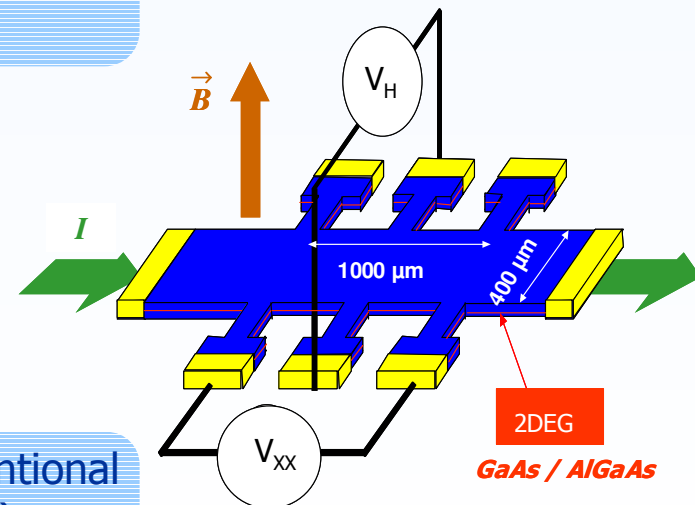
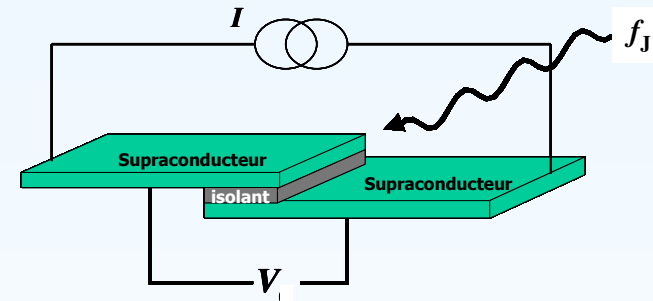
- Quantum Hall effect

$$R_H = \frac{R_K}{i}$$

$$R_K \equiv \frac{h}{e^2}$$

Ohm reproducibility with a conventional value of R_K ($R_{K-90}=25\,812.807$ Ω):

1 part in 10^9



Reproducibility of the ampere in the SI: 10^{-7}

The metrological triangle experiment (MTE)

- Checking the consistency of the three constants involved in electrical quantum metrology:

$$R_K, K_J, Q_X$$

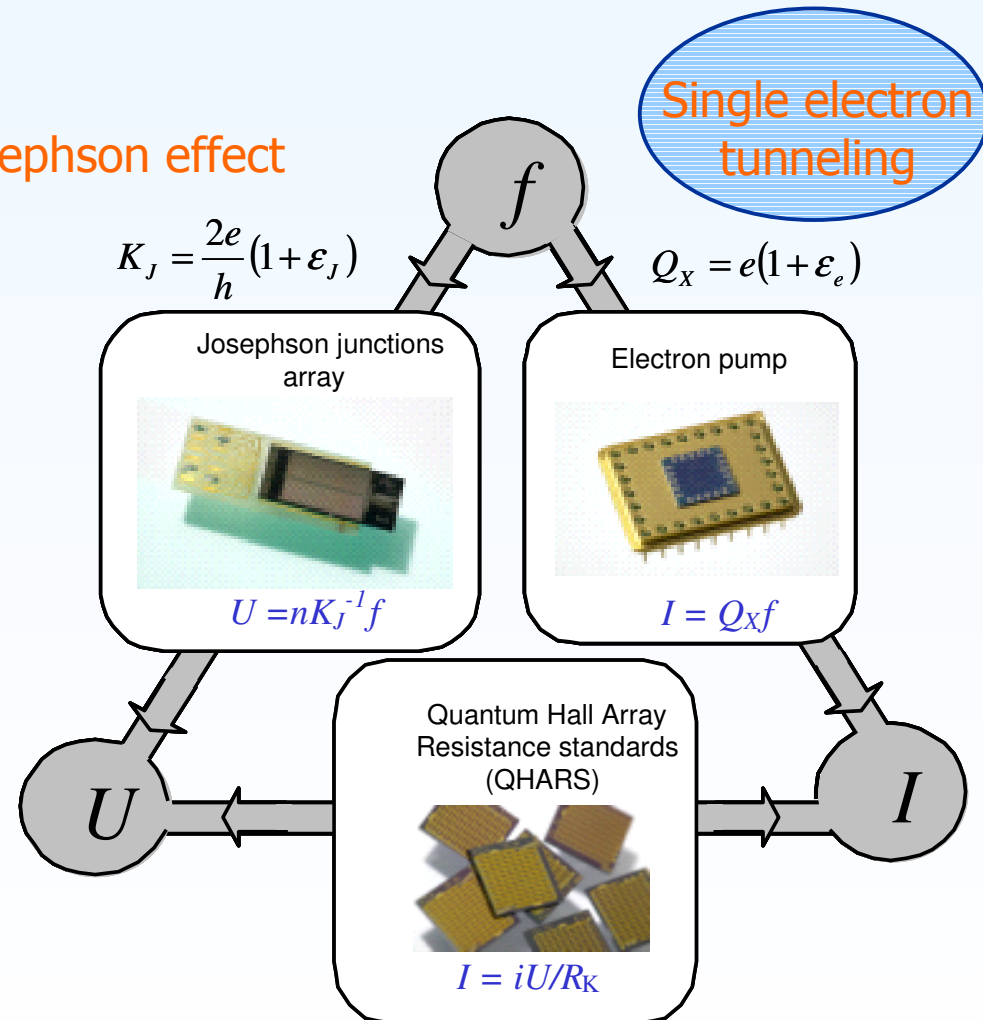
- Determination of the elementary charge Q_X involved in the pump

Josephson effect

$$K_J = \frac{2e}{h}(1 + \epsilon_J)$$

Single electron tunneling

$$Q_X = e(1 + \epsilon_e)$$



Quantum Hall effect

$$R_K = \frac{h}{e^2}(1 + \epsilon_K)$$

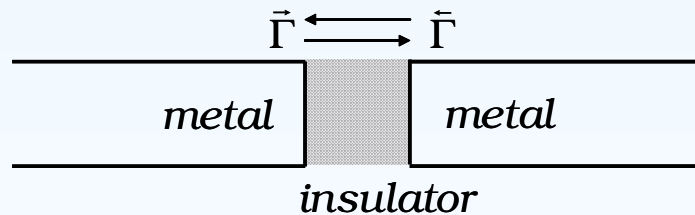
Likharev and Zorin, *J. Low. Temp. Phys.*, 59, p. 347, 1985

Outline

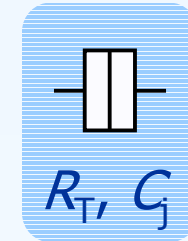
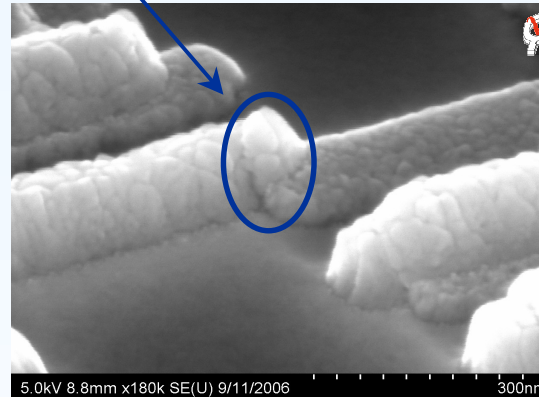
- The SI and the electrical units
- **Single electron pump**
- Experimental set-up
- Measurements
- The metrological triangle experiment

The Coulomb blockade

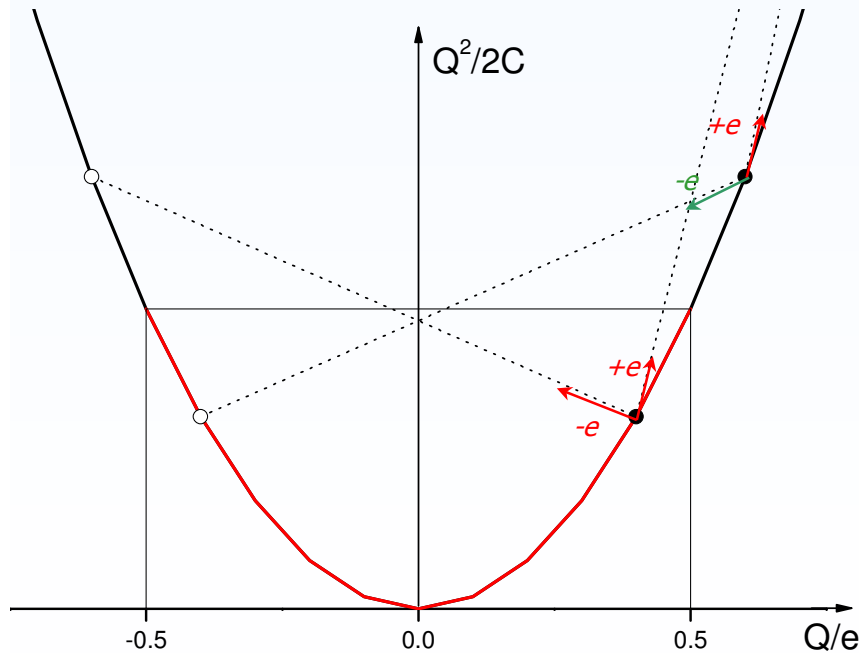
The tunnel junction



Tunnel junction



Typical values:
 $R_T \sim 100 \text{ k}\Omega$, $C_j \sim 100\text{-}200 \text{ aF}$
 ($C = 100 \text{ aF} \approx (50 \text{ nm})^2 \cdot 2 \text{ nm}$)



If thermal and quantum fluctuations are negligible compared to the charging energy $Q^2/2C_j$, i.e.:

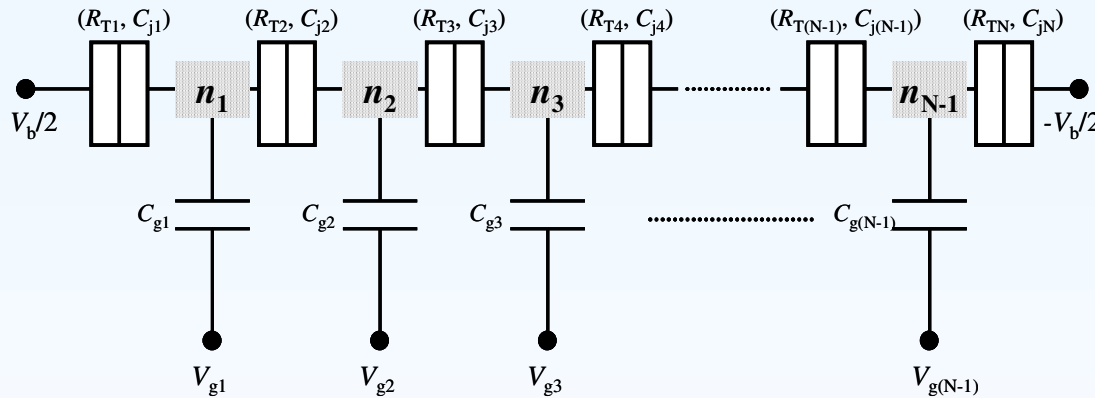
$$R_T \gg R_K (=h/e^2) \text{ and } C_j \ll 2/(e^2 k_B T)$$

then SET (*single electron tunneling*) phenomena occur.

$$|Q| < \frac{e}{2} \rightarrow \text{Transfer of electron is blocked}$$

Averin and Likharev J. Low. Temp. Phys., 62, p. 345, 1986

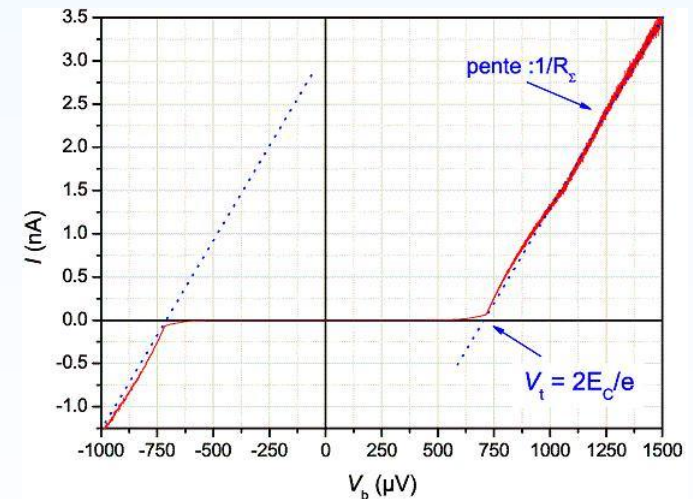
The single electron pump



N junctions (R_{Ti}, C_{ji}) ,
 $N-1$ islands controlled
 by $N-1$ gates C_{gi}, V_{gi}

$(n_1, n_2, \dots, n_{N-1})$ are the excess electrons on each island

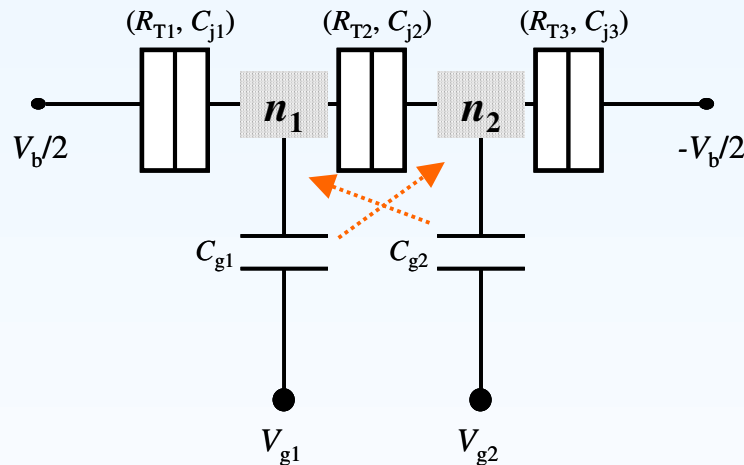
$I-V_b$ curve presents a plateau for $|V_b| < V_t$
 $(E_c = e^2/2C_\Sigma)$ when the junctions are in the
 blockade state.



Charging energy $E(n_1, n_2, \dots, n_{N-1})$ is a function of the capacitance parameters of the pump.

Each combination $(n_1, n_2, \dots, n_{N-1})$ has a stability domain in the space $(V_{g1}, V_{g2}, \dots, V_{gN-1})$

The single electron pump: stability diagram for 3 junctions

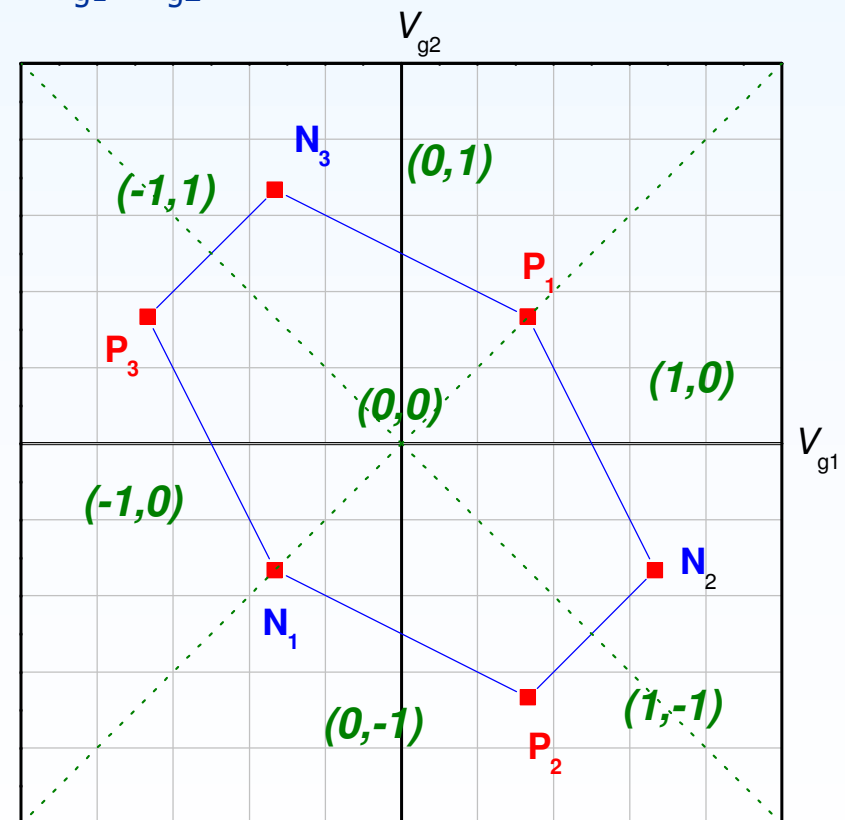


The stability domain of a couple (n_1, n_2) in the space (V_{g1}, V_{g2}) is an honeycomb lattice.

$$E(n_1, n_2) = \frac{e}{C_a C_b - C_{j2}^2} \left[\frac{e}{2} (C_b n_1^2 + C_a n_2^2) + e C_{j2} n_1 n_2 - V_{g1} C_{g1} (C_b n_1 + C_{j2} n_2) - V_{g2} C_{g2} (C_{j2} n_1 + C_a n_2) \right] + \text{termes indépendants de } (n_1, n_2)$$

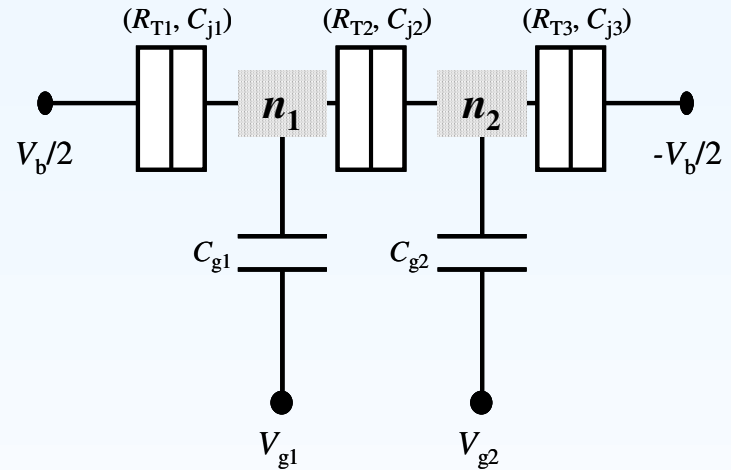
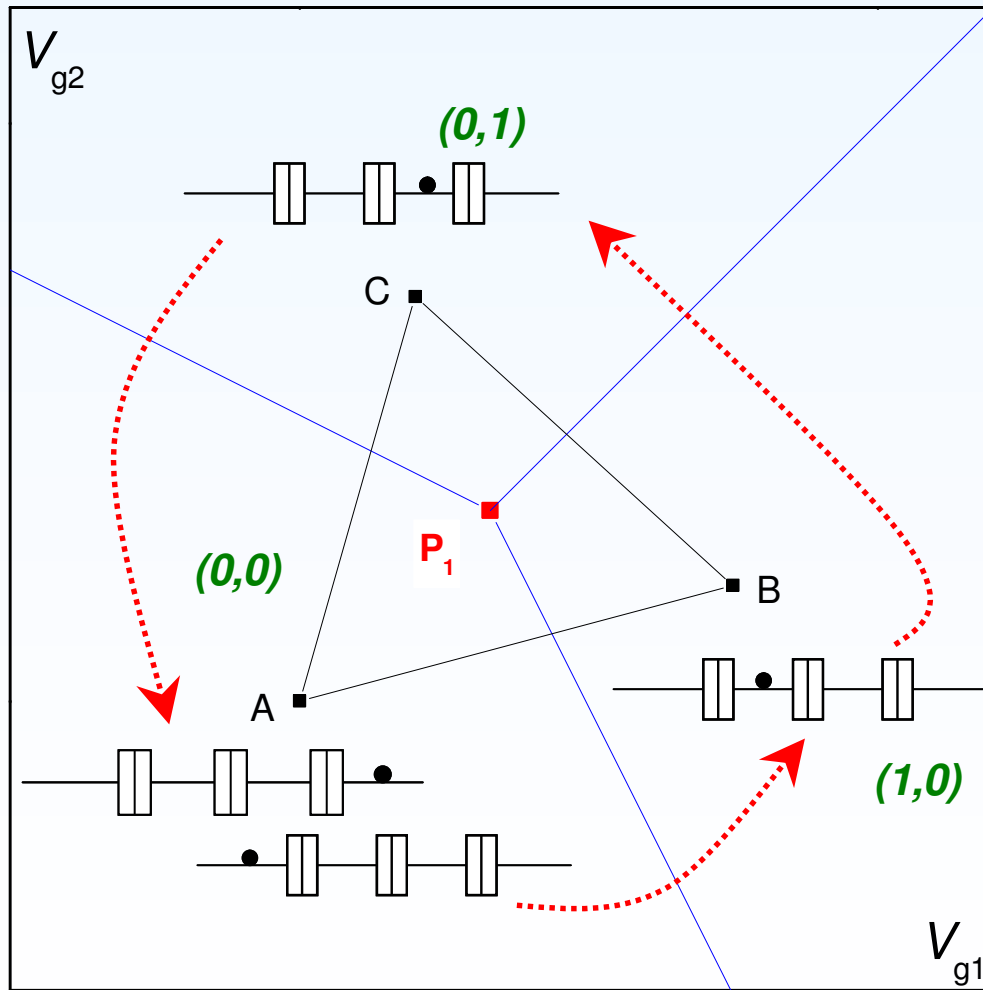
$$\begin{cases} C_a = C_{j1} + C_{j2} + C_{g1} \\ C_b = C_{j3} + C_{j2} + C_{g2} \end{cases}$$

In experiments, the honeycomb lattice is put out of shape by the dissymmetry of the junctions and by the cross talking terms.



Stability diagram for a 3 junctions pump with $R_{T1}=R_{T2}=R_{T3}$, $C_{j1}=C_{j2}=C_{j3}$ and $C_{g1}, C_{g2} \ll C_j$

Pumping operation



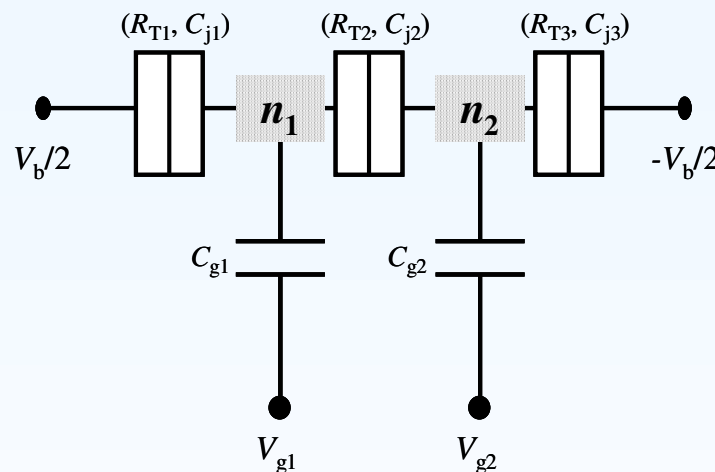
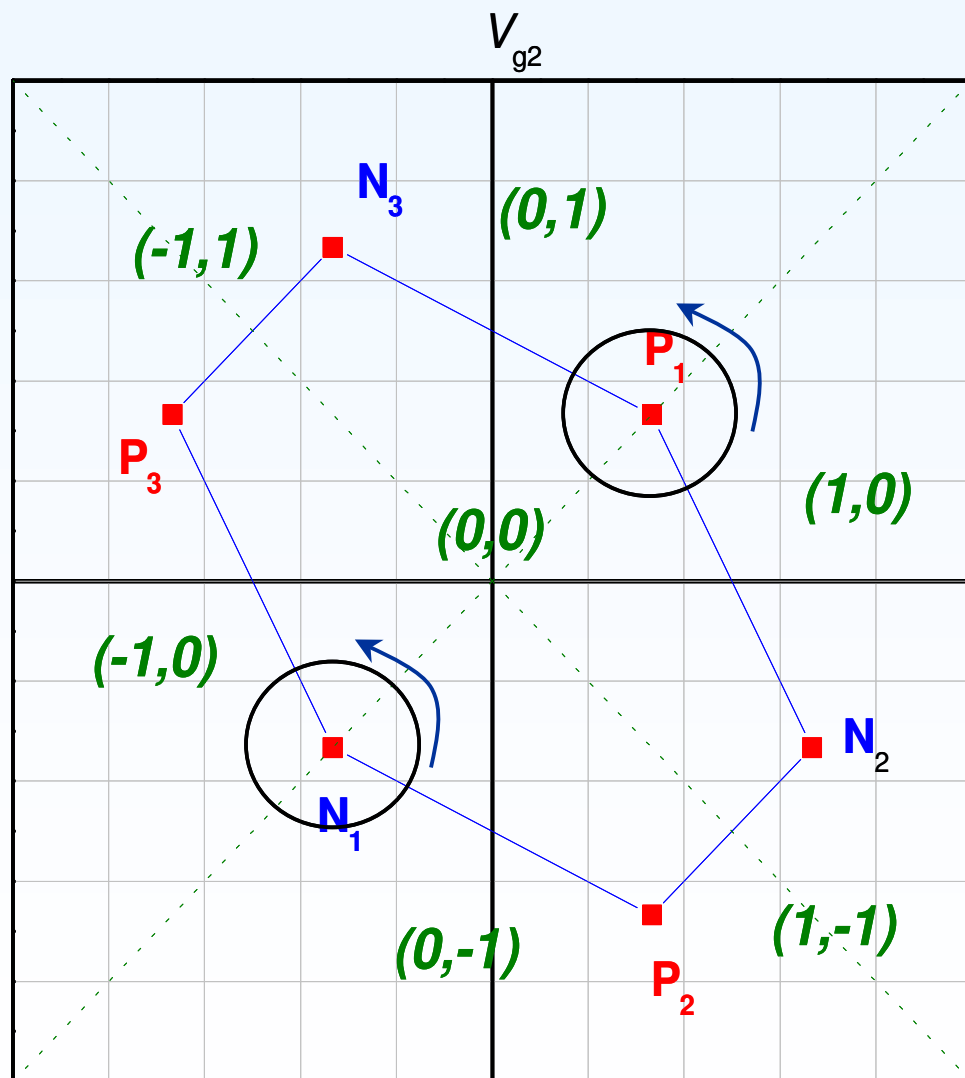
$$\begin{cases} V_{g1} = V_{g1DC} + A \cos(2\pi ft) \\ V_{g2} = V_{g2DC} + A \cos(2\pi ft + \varphi) \end{cases}$$

$$I = Q_X f$$

$$(Q_X \equiv e)$$

(i.e. $I \sim 0.16 \text{ pA/MHz}$ or $f = 10 \text{ MHz}$ implies $I \sim 1.6 \text{ pA}$ and $f = 60 \text{ MHz}$, $I \sim 10 \text{ pA}$)

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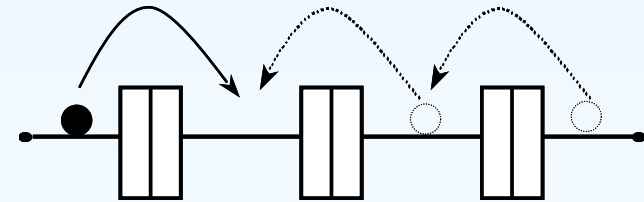
Error processes

- Cotunneling

Coherent transfer of n electrons through n junctions

$$q_0 = \sqrt{(\pi/2)(N-1)R_T C f}$$

$$\varepsilon_{ct} = b \left(\frac{R_K}{R_T} \right)^{N-2} |u - q_0|^{2N-2}$$



- Frequency effects

The larger the pumping frequency is, the greater the probability to miss an electron is. The intrinsic limit of the device is $R_T C$ (~ 10 GHz)

$$\varepsilon_f = \exp\left(-\frac{a}{R_T C f}\right)$$

- Thermal errors

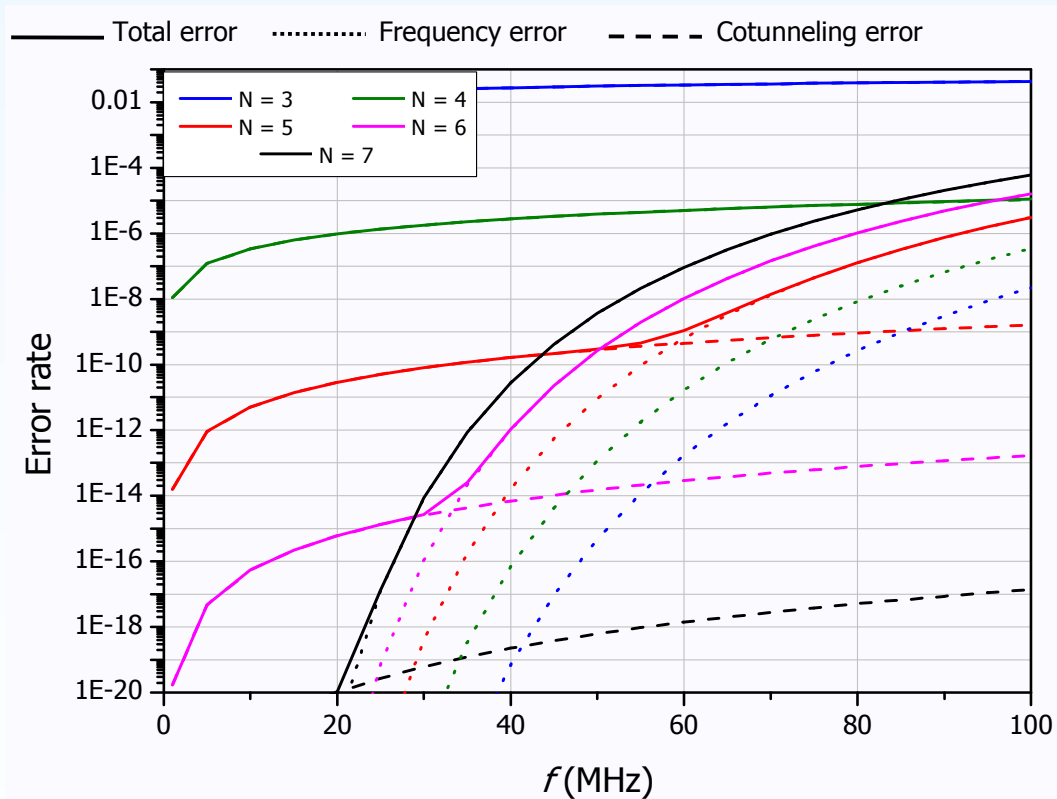
The electron temperature is not null in our measurement. This temperature is responsible for a modification of the rate transfer of the sequential transfer.

$$\varepsilon_{th} = c \exp\left(-d \frac{E_C}{k_B T}\right)$$

- Electromagnetic perturbations

Jensen and Martinis, Phys.Rev. B, 46, p. 13407, 1992

Metrological behaviour of a pump

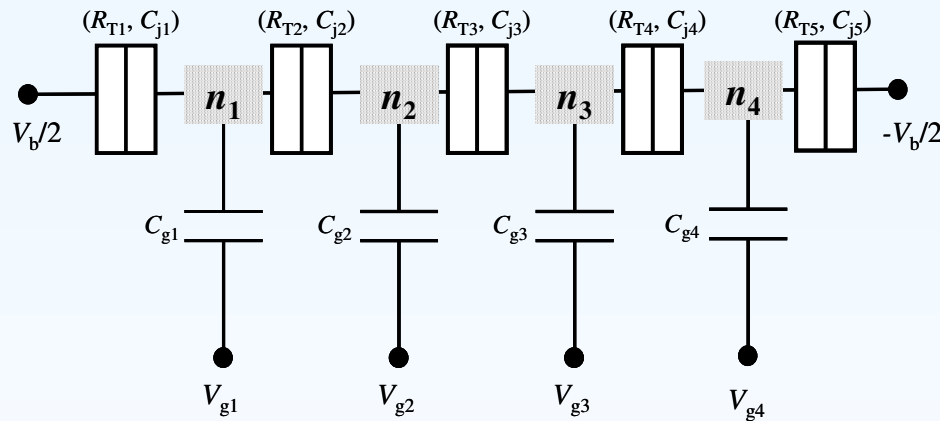


Total error for a N junctions electron pump with no bias voltage at 100 mK

N	$\epsilon_{ct}/\epsilon_f \sim 1$	Error $\sim 10^{-6}$	Error $\sim 10^{-8}$
3	$\gg 100$ MHz	Never	Never
4	> 100 MHz	~ 25 MHz	< 5 MHz
5	~ 60 MHz	~ 95 MHz	~ 70 MHz
6	~ 35 MHz	~ 80 MHz	~ 60 MHz
7	~ 20 MHz	~ 75 MHz	~ 55 MHz

Experimental difficulty

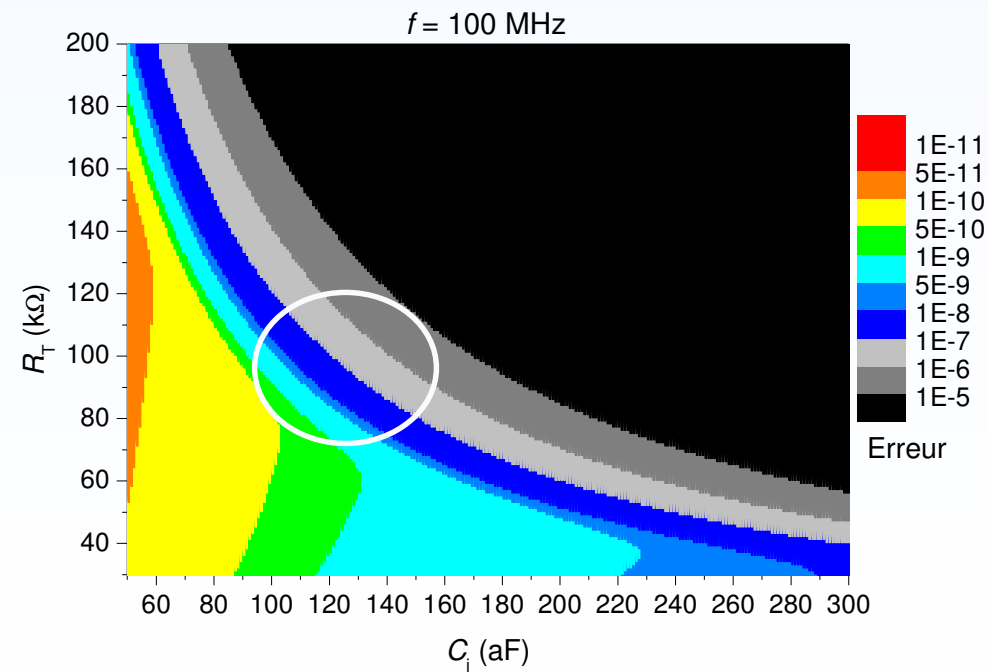
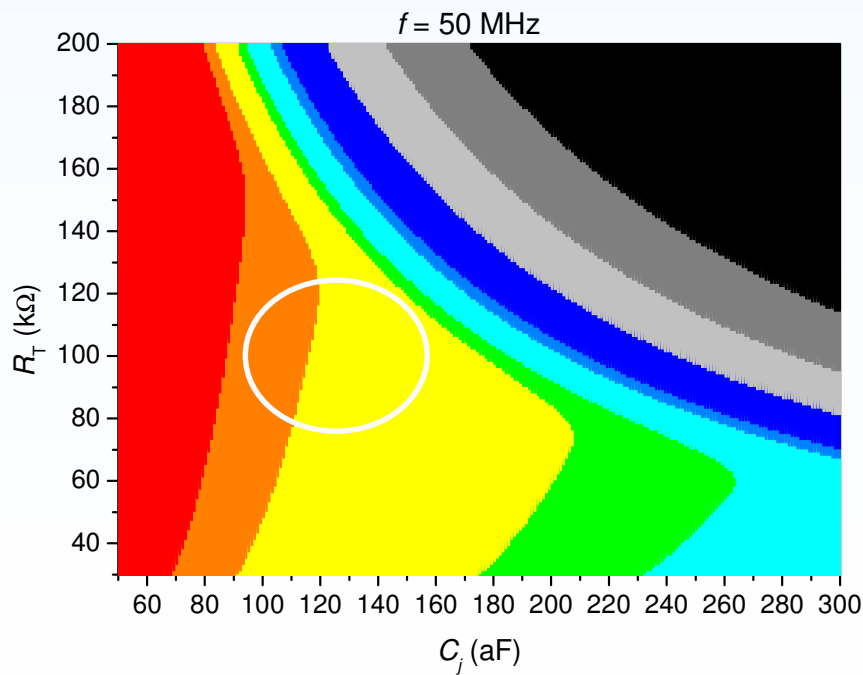
Metrological behaviour of a pump



Dependence on the parameters of the pump for a 5 junctions pump.

$$R_{T_i} = R_T \text{ and } C_{j_i} = C_j \text{ for } i=1..5$$

$$V_b=0 \text{ and } T= 100 \text{ mK}$$

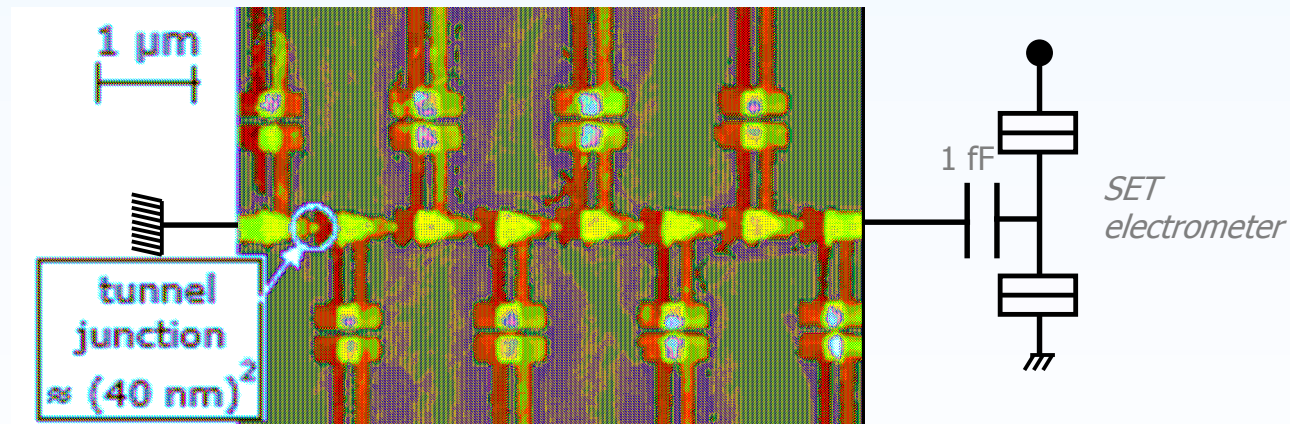


Devices expected to be metrological

Cotunneling is the main limitation for a metrological use of an electron pump

- 7 junctions electron pump

NIST, Keller et al., Appl. Phys. Lett., 69, p. 1804, 1996



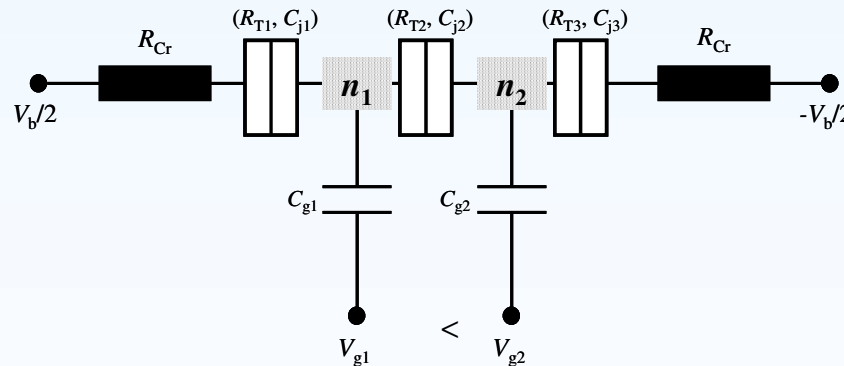
Transfer of electrons at 5 MHz,
error rate: 1.5 part in 10⁸

- The R-pump

Zorin et al., J. App. Phys., 88, p. 2665, 2000

The R-pump

Increasing the local impedance can reduce the cotunneling with preserving the transfer rate.



*Typical values:
R_{Cr} ~ 30-50 kΩ*

- Conditions on R_{Cr} :

$$R_K < 2R_{Cr} < R_T$$

Environment with large dissipation can absorb a part of the electron energy.

The charge equilibrium is established, then the pump behaves similarly to the case without resistor.

- The cotunneling current is roughly:

$$I_{ct} \propto V_b^\eta \quad \eta = 2 \left(N + \frac{2R_{Cr}}{R_K} \right) - 1$$

(i.e. that, in term of cotunneling, the N junctions pump is roughly equivalent to a $N + 2R_{Cr}/R_K$ junctions pump)

⇒ There is no complete theory of the error processes in a R-pump

Outline

- The SI and the electrical units
- Single electron pump
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Observational conditions

- $E_C = e^2/2C \gg k_B T$

if $C = 100$ aF (dimensions of the junction: $(50 \text{ nm})^2 \cdot 2 \text{ nm}$), then $T \ll 6.5 \text{ K}$.

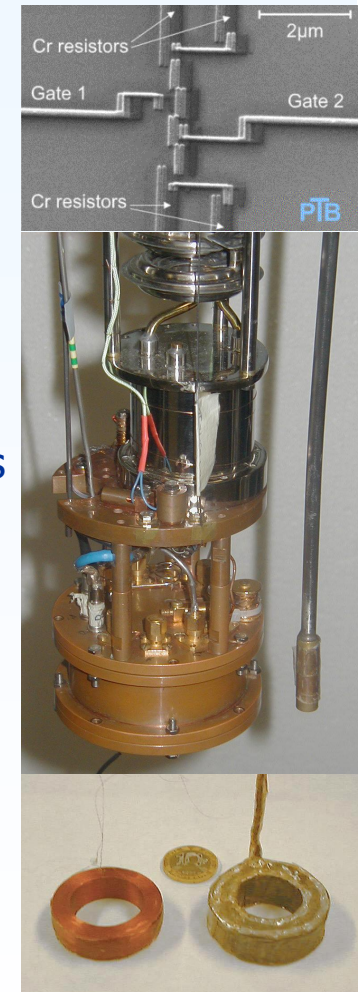
-> Fabrication of the samples by nano lithography to minimize the values of C (shadow evaporation technique)

-> Dilution unit to reach the temperature where the Coulomb blockade appears

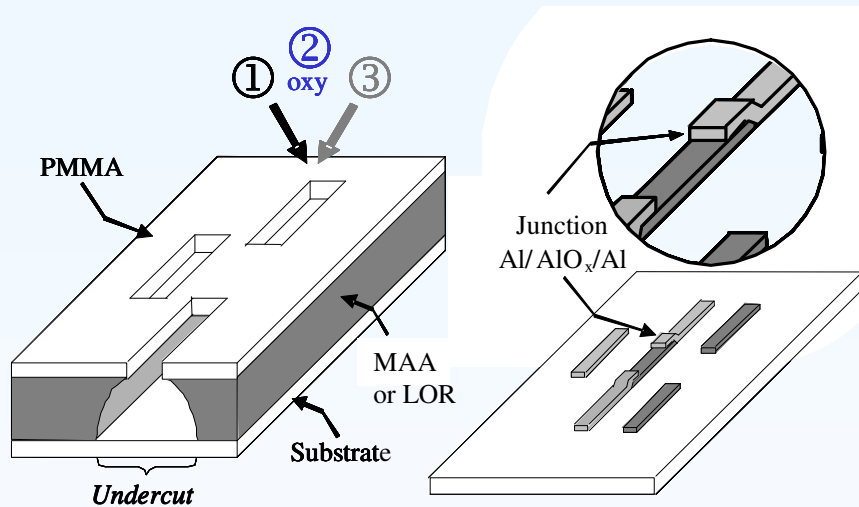
- $I \sim 1 - 10 \text{ pA}$

*a relative uncertainty of 1 part in 10^8 for a current of 10 pA (i.e. 0.1 aA)
=> we can miss 0.6 electron per second.*

-> Measurement of the intensity current by using a metrological current comparator



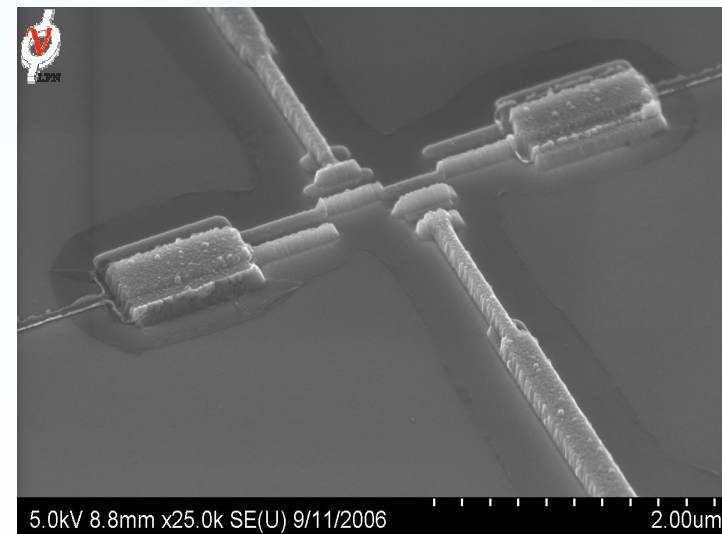
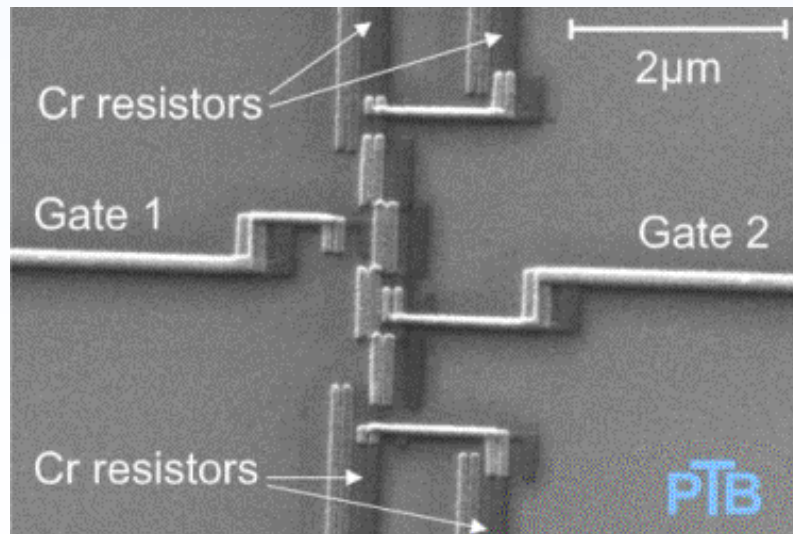
Samples

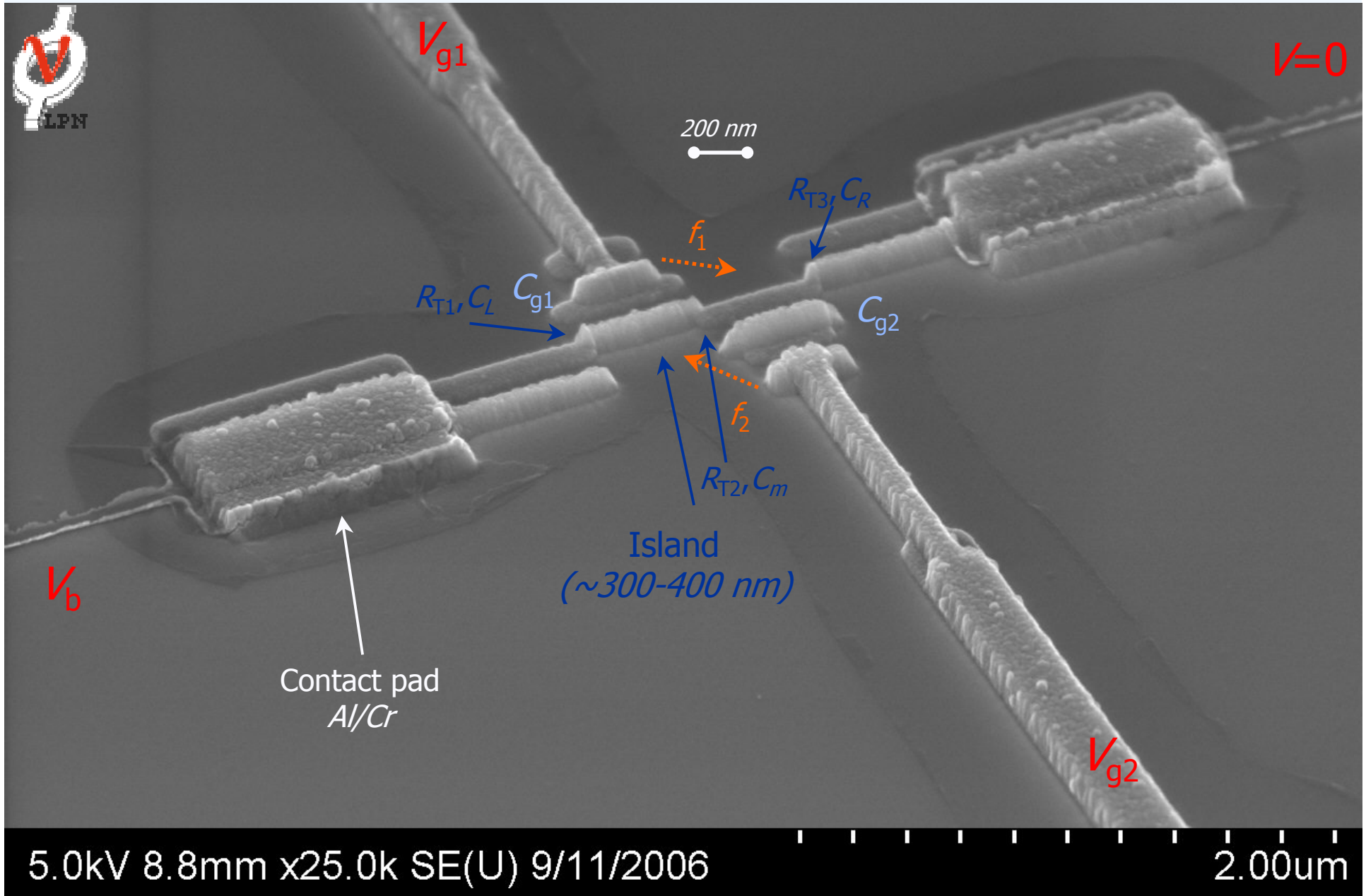


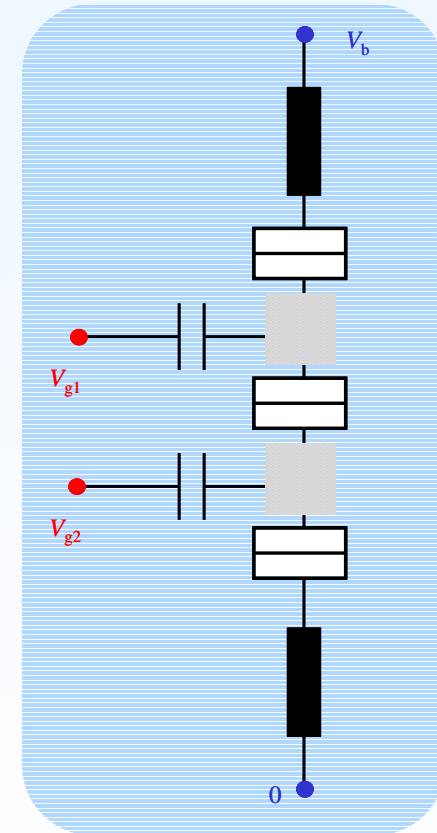
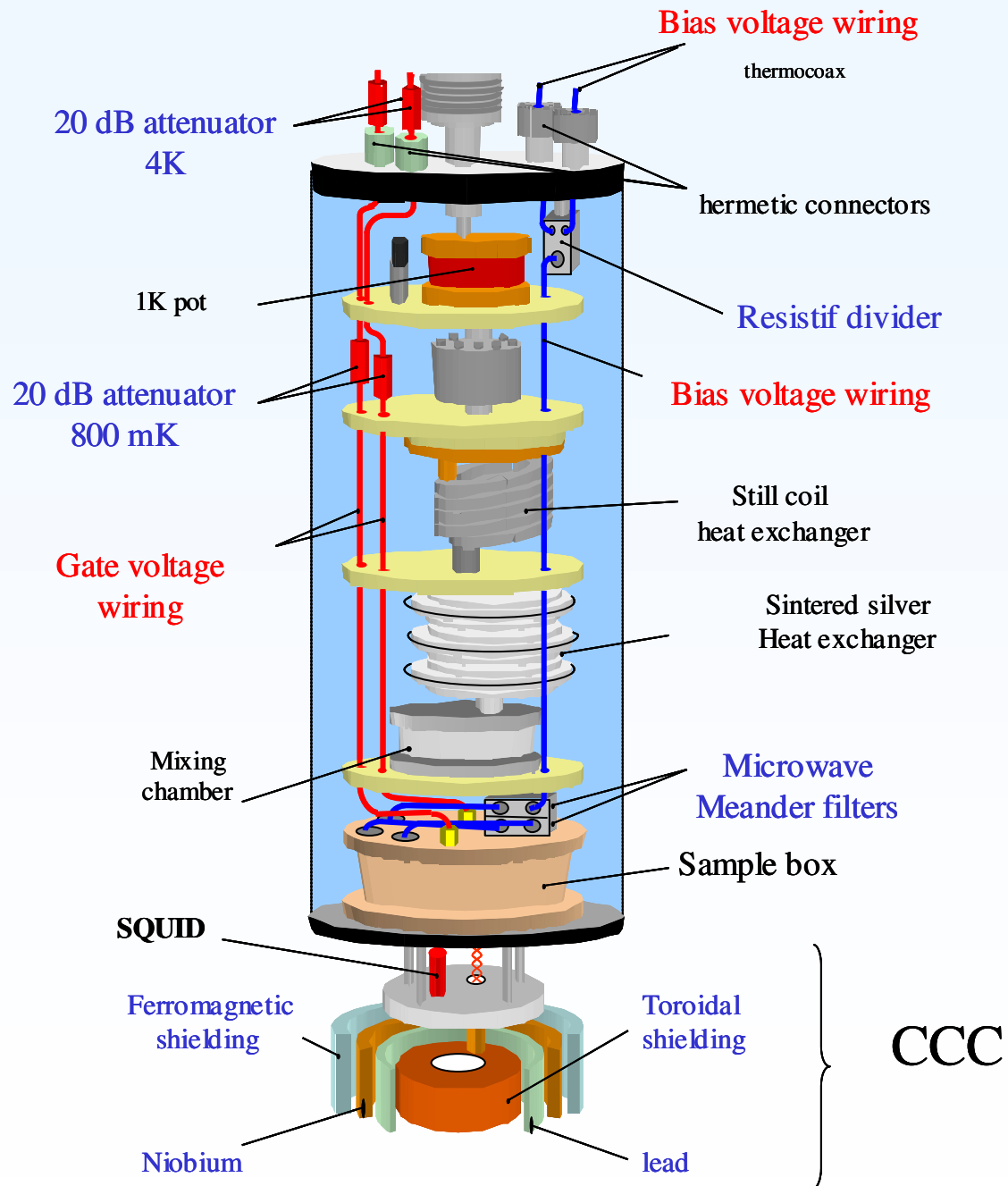
shadow evaporation technique

Cr deposition, 7 nm, angle null
(20 nm*70 μm)

- Al deposition: 25 nm, angle 10°
- Al/Oxidation: a few nm (2-3)
- Al deposition: 60 nm, angle -10°





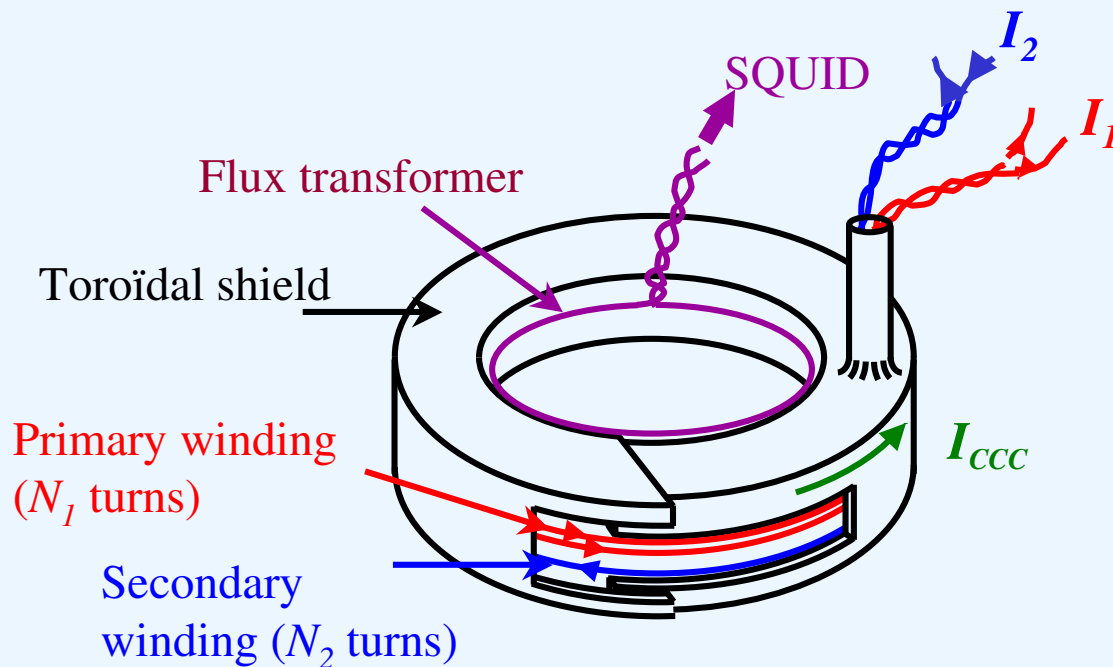
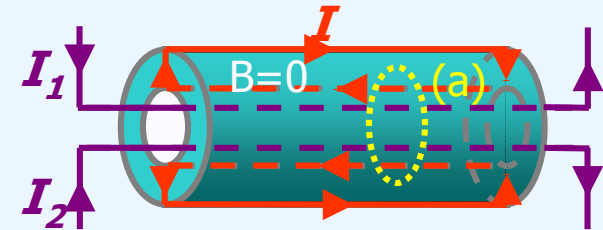


Cryogenic Current Comparator (CCC)

Theoretical principles :

Ampere's law : $\oint_{(a)} B \cdot dl = \mu_0 \cdot (I_1 + I_2 - I)$

Meissner effect : $\oint_{(a)} B \cdot dl = 0$



$$I_{CCC} = N_1 I_1 - N_2 I_2$$

I_2 is adjusted in order to have $I_{CCC} = 0$,
then: $N_1 I_1 = N_2 I_2$

$$I_2 = \frac{N_1}{N_2} I_1$$

Cryogenic Current Comparator (CCC)

- Current resolution δI (=the measurable minimum current circulating in the toroidal shield)

- 3 components :

Johnson noise of the input resistance + SQUID resolution + external magnetic flux noise

Typical value: for a perfect coupling between the DC SQUID and the toroidal shield, and $N_1=20000$ turns, $\delta I \sim 0.7 \text{ fA/Hz}^{1/2}$

- Exactness of the CCC current ratio

- Winding ratio (*a leakage flux is generated by the currents at the end of the shield*)

-> The toroidal shield has to overlap itself over a sufficient length. In practice with 2 overlaps, the uncertainty on N_1/N_2 is below 1 part in 10^{10}

- Frequency effect (*resonance at f_R and capacitive leakage due to parasitic capacitances*)

-> f_R decreases when N_1 increases (=2 kHz for $N_1=20\,000$). The current ratio error is proportional to $(f/f_R)^2$

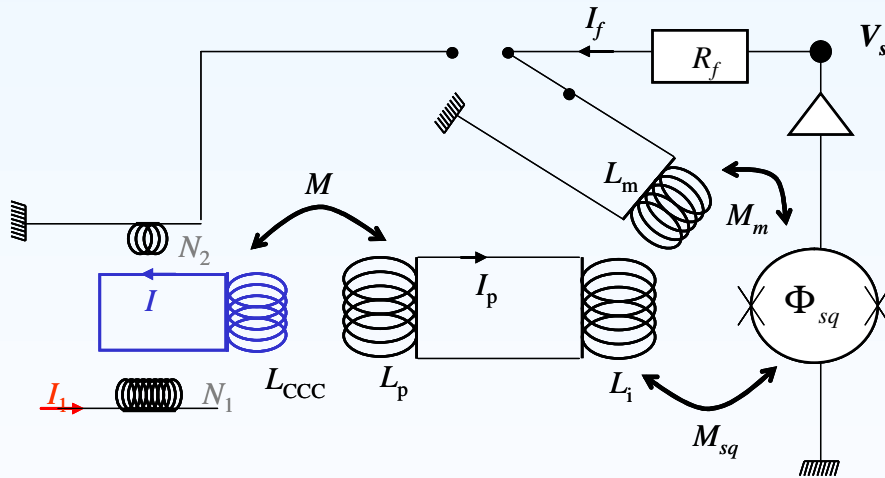
- Finite open loop feedback gain

-> An adjustment of the feedback current at 10^{-4} is necessary to have a current ratio error below 1 part in 10^9

=> With optimized parameters and shielding of the CCC and a pre-adjusting feedback current source **uncertainty on the amplification of the current is below one part in 10^8**

Operating modes of the CCC

Internal feedback mode



$$G_{IFB} = \frac{V_S}{I_1} = \frac{M_{sq}}{M_m} \frac{k}{2} \sqrt{\frac{L_{CCC}}{L_i}} N_1 R_f$$

Typical values:

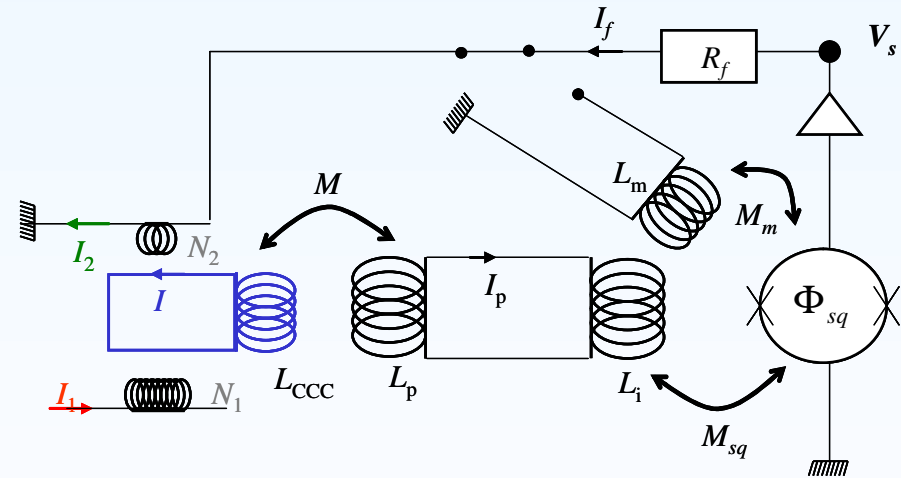
$$N_1 = 20\,000 \quad (N_2 = 1)$$

$$M_{sq} = 10\text{ nH}, \quad M_m = 1\text{ nH}, \quad L_i = 1.8\ \mu\text{H}, \quad L_{CCC} = 15\text{ nH}$$

and $k = 0.8$

$$\rightarrow G_{IFB}/R_f \sim 7 \cdot 10^3$$

External feedback mode



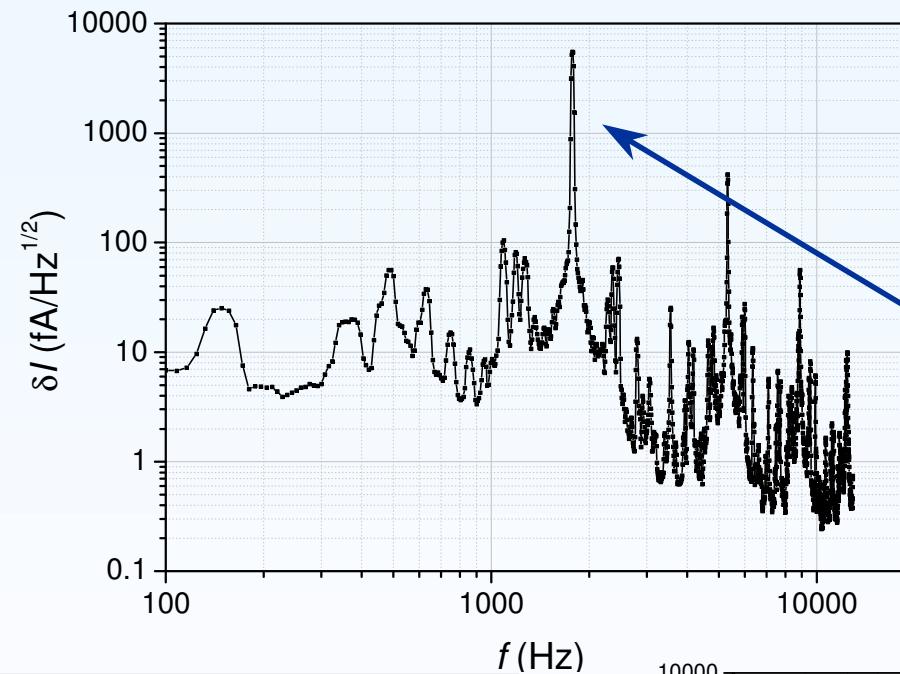
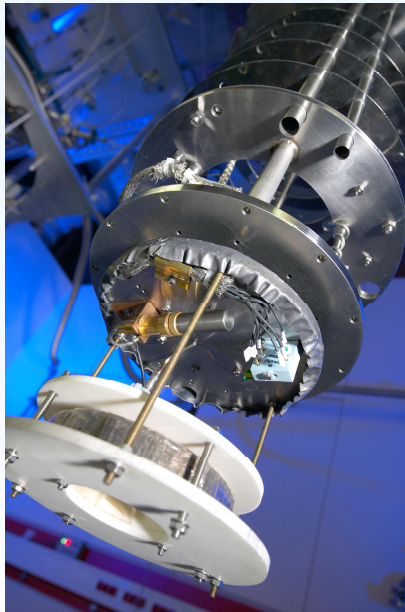
$$G_{EFB} = \frac{V_S}{I_1} = \frac{N_1}{N_2} R_f$$

Typical values:

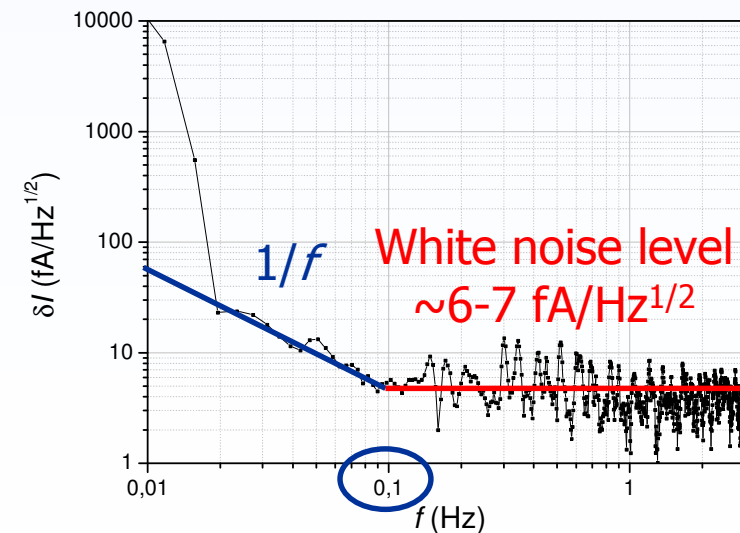
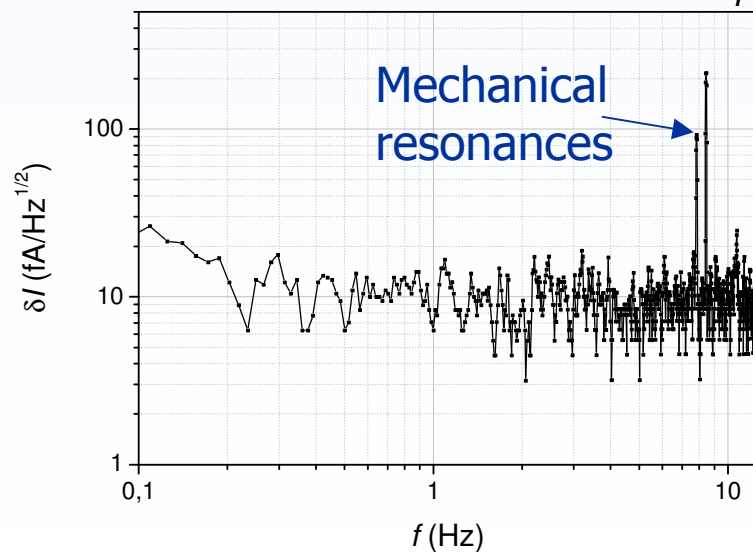
$$N_1 = 20\,000, \quad N_2 = 1$$

$$\rightarrow G_{EFB}/R_f = 2 \cdot 10^4$$

Experimental characterization of the set-up



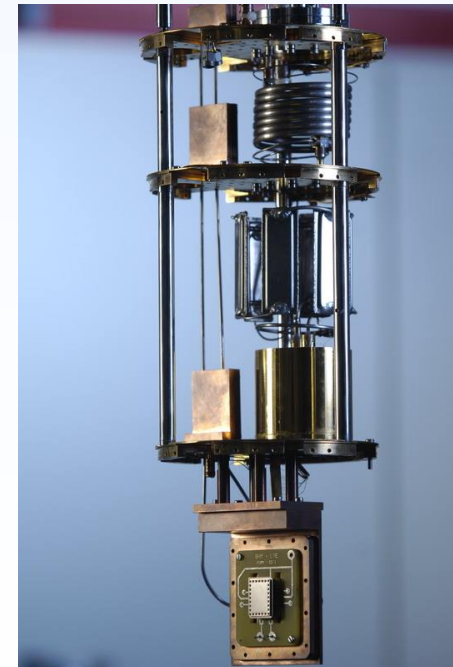
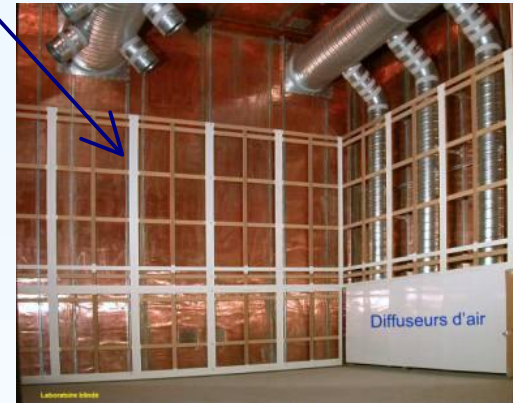
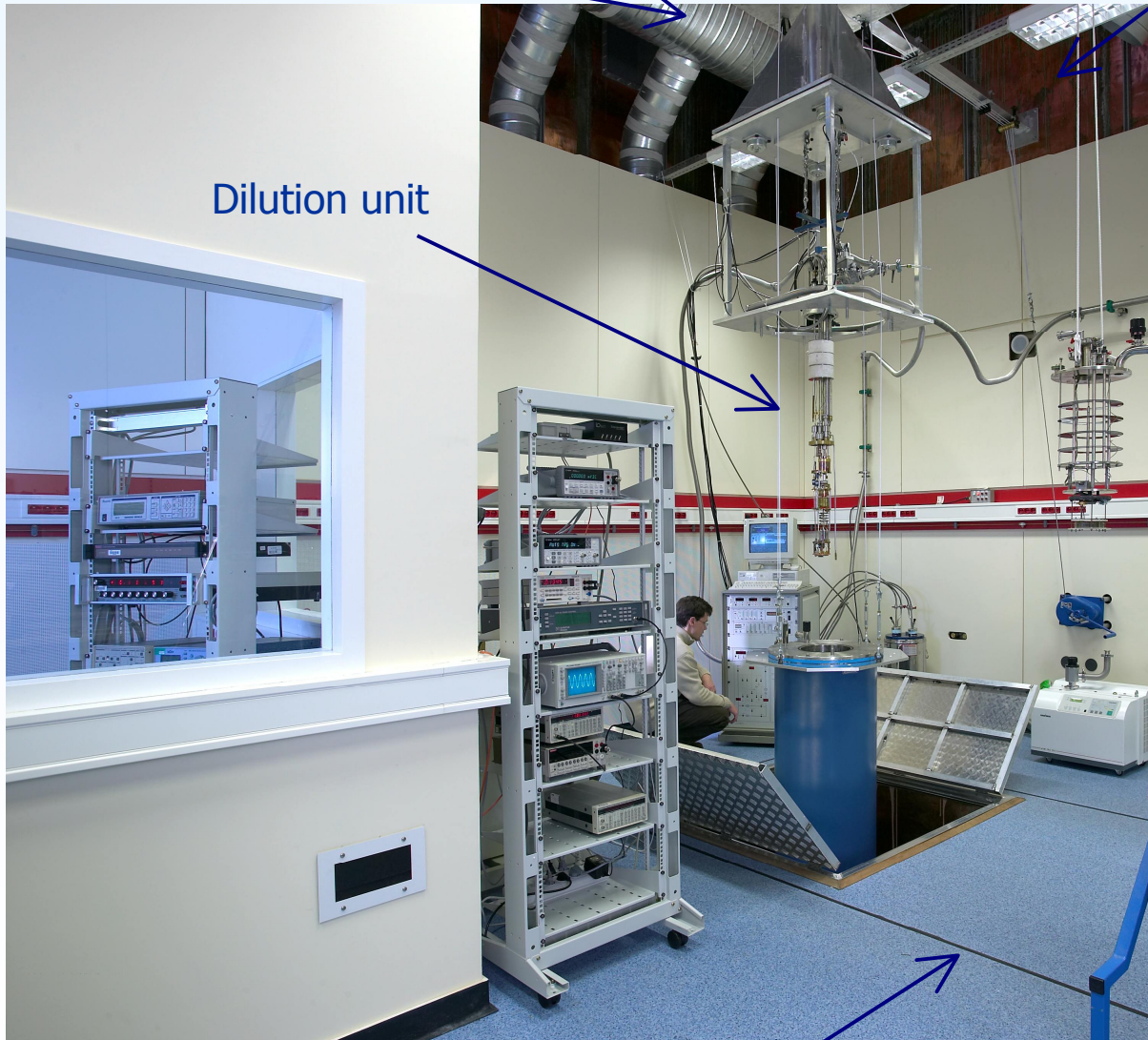
N_1	20 000
N_2	1
L (H)	8
f_R (kHz)	1.8
C (nF)	1



Thermal regulation

Copper shielding

Dilution unit



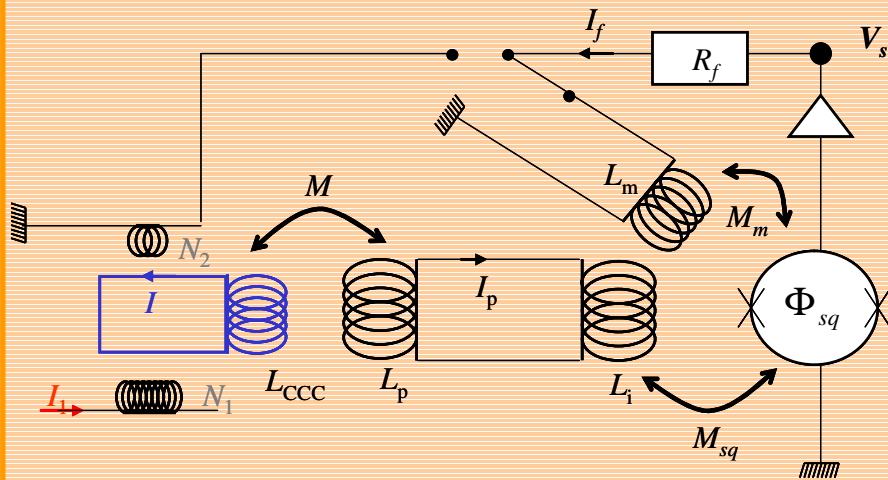
Mechanical insulation

Outline

- The SI and the electrical units
- Single electron pump
- Experimental set-up
- **Measurements**
- The metrological triangle experiment

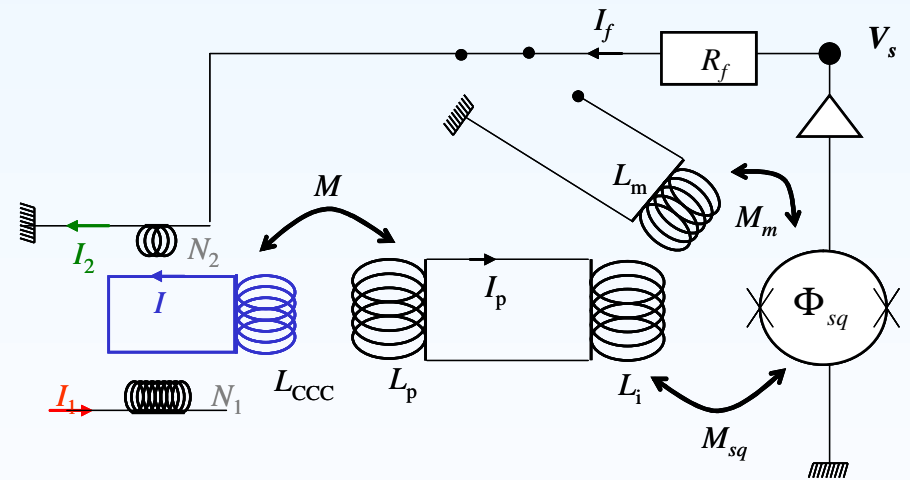
Operating modes of the CCC

Internal feedback mode



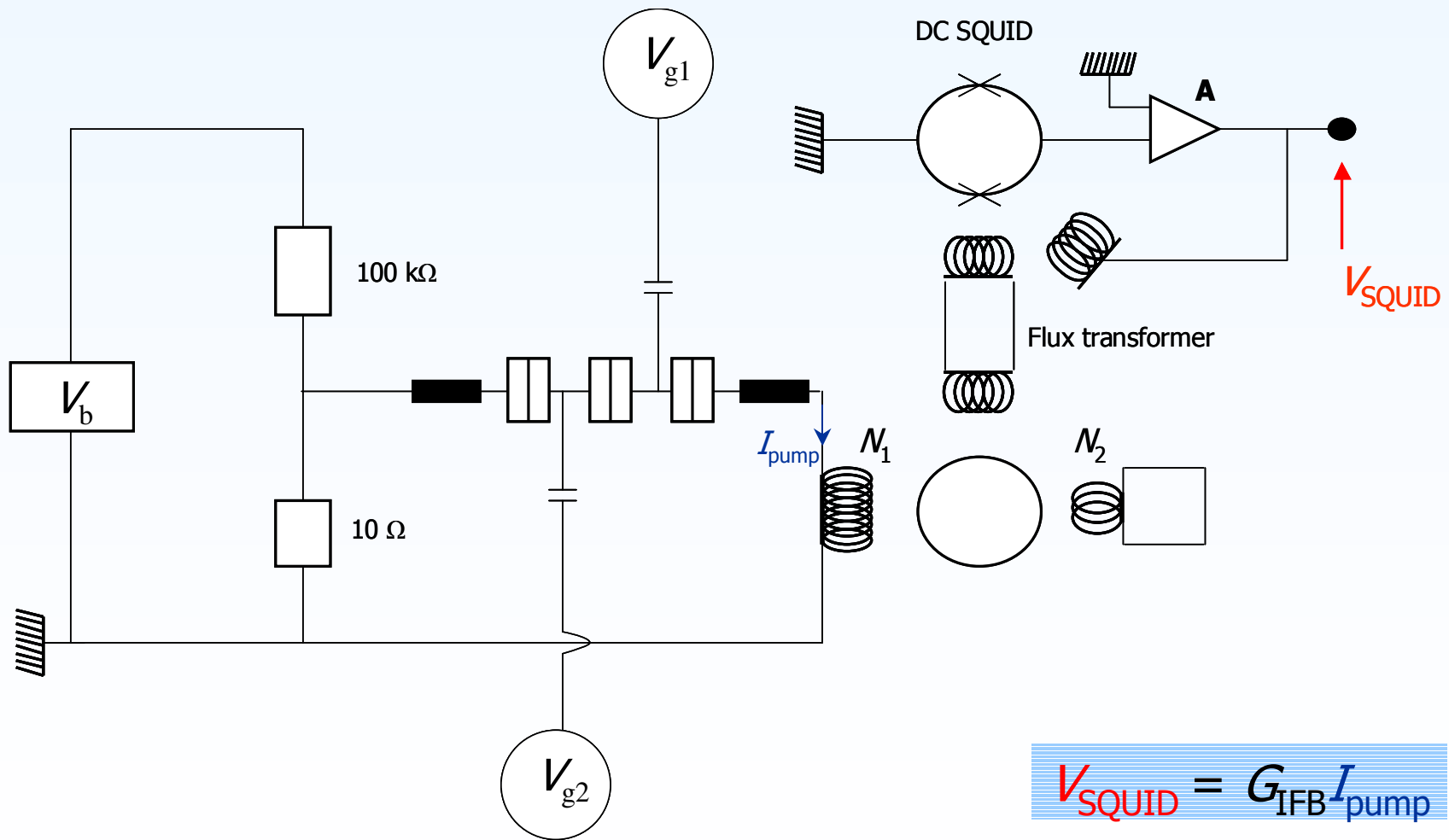
$$G_{IFB} = \frac{V_S}{I_1} = \frac{M_{sq}}{M_m} \frac{k}{2} \sqrt{\frac{L_{ccc}}{L_i}} N_1 R_f$$

External feedback mode



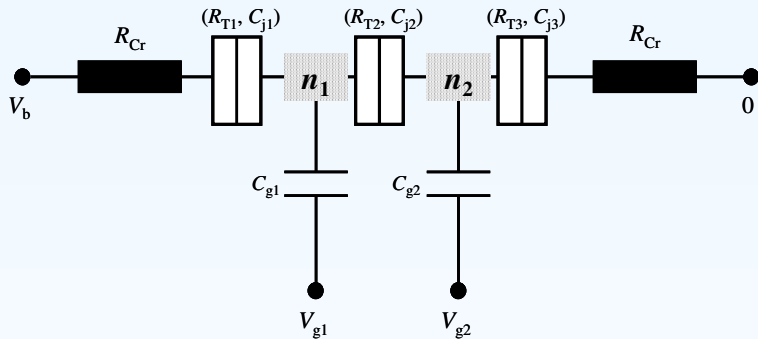
$$G_{EFB} = \frac{V_S}{I_1} = \frac{N_1}{N_2} R_f$$

The internal feedback mode



$$V_{\text{SQUID}} = G_{\text{IFB}} I_{\text{pump}}$$

Characterization of a pump: static measurements



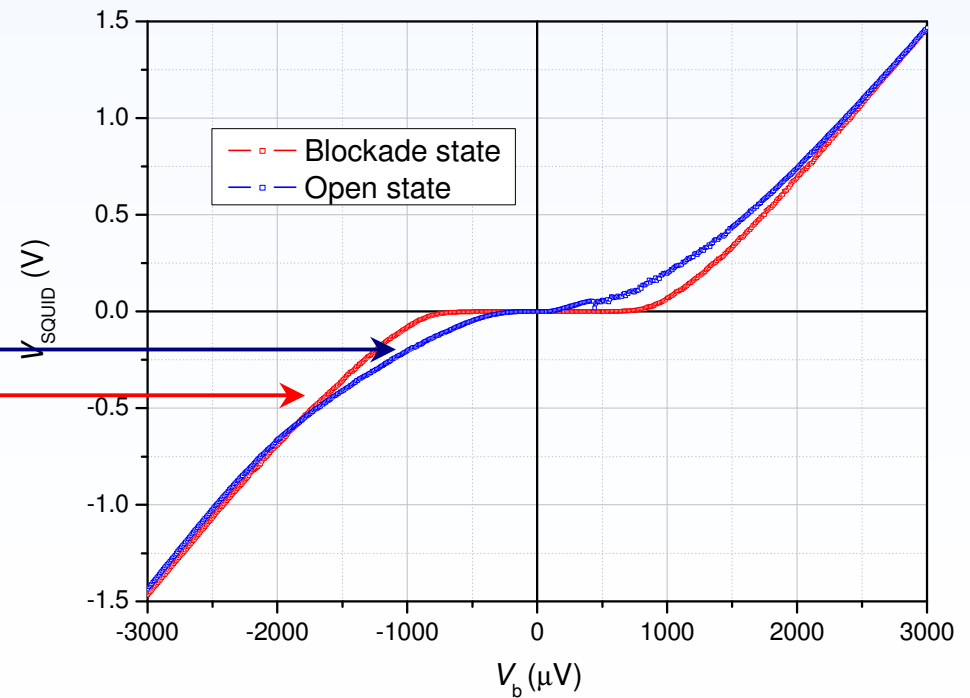
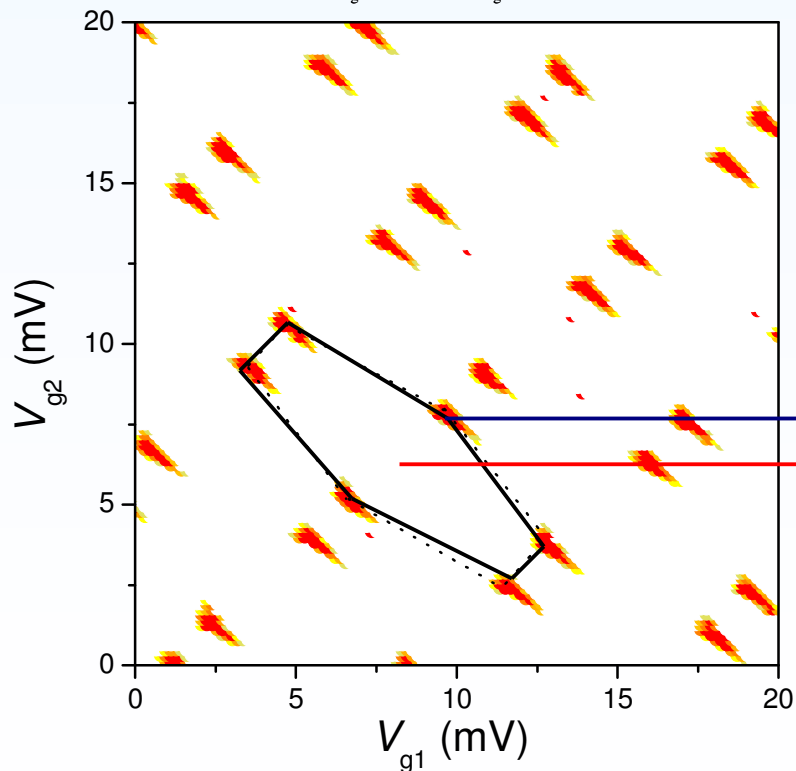
V_{g1} and V_{g2} are biased with DC voltages

-> Stability diagram :

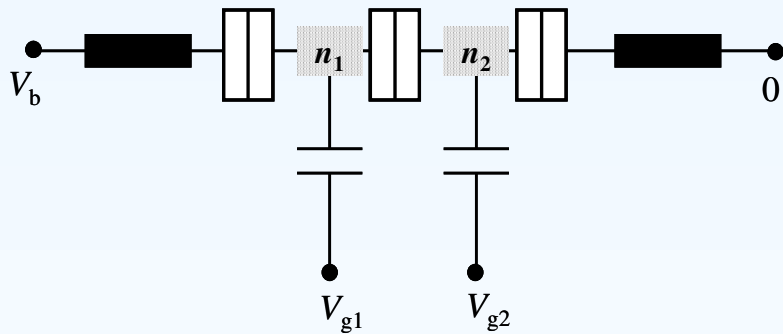
V_b is fixed and V_{g1} and V_{g2} change

-> I - V_b curve :

A couple (V_{g1}, V_{g2}) is fixed and V_b varies

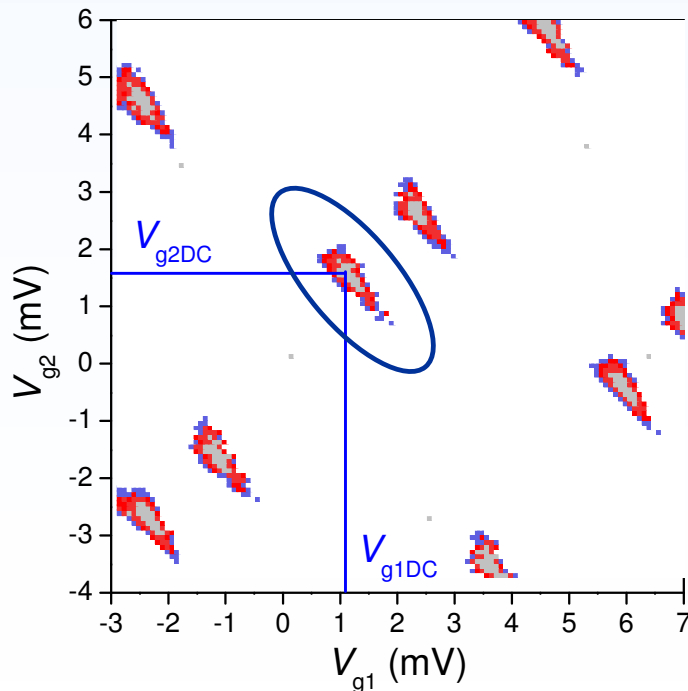


Characterization of a pump: pumping parameters

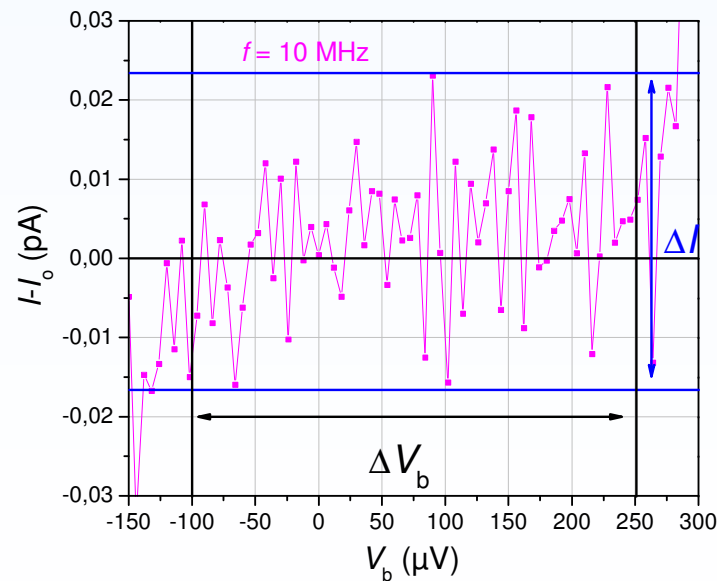


$$\begin{cases} V_{g1} = V_{g1DC} + A \cos(2\pi ft) \\ V_{g2} = V_{g2DC} + A \cos(2\pi ft + \varphi) \end{cases}$$

- V_{g1DC} and V_{g2DC} are coordinates of a triple point
- Adjusting parameters: A and φ



In dynamic mode I - V_b curve presents a current step at $I=ef$.

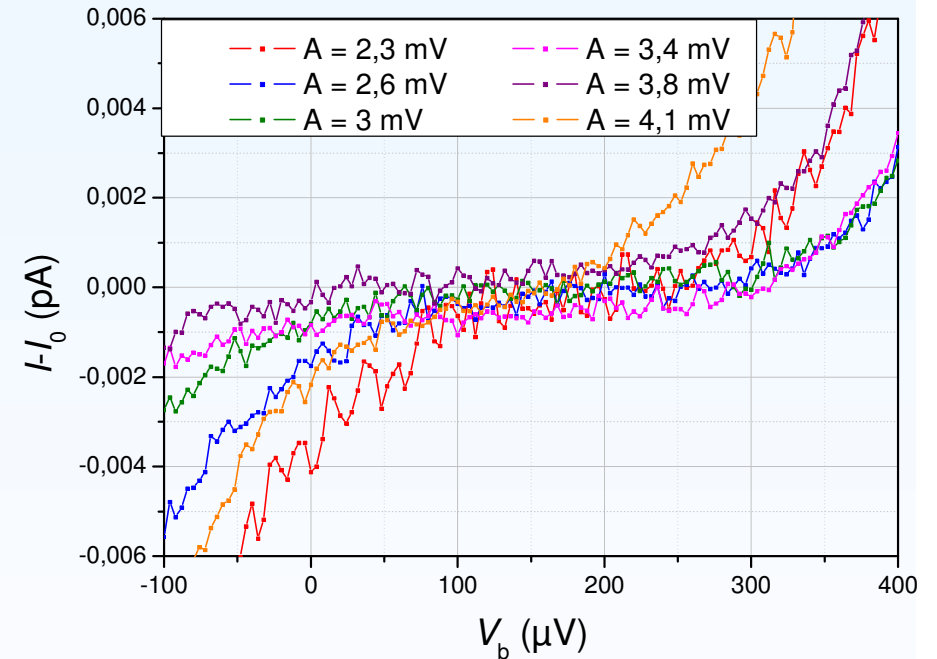
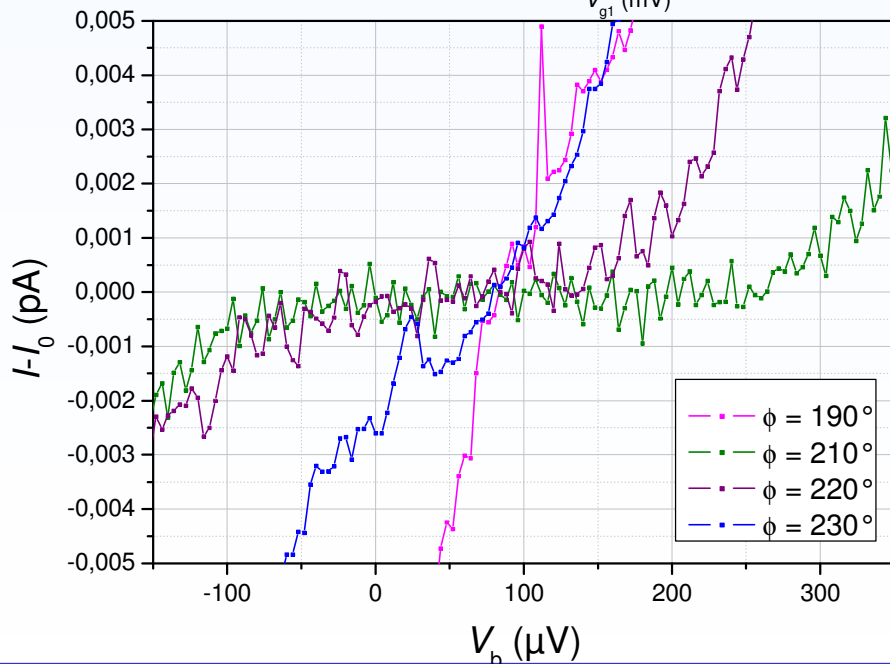
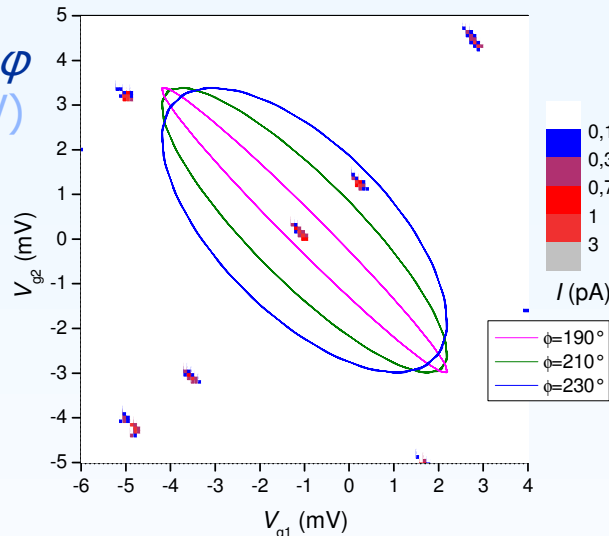


Criterion:
for ΔI corresponding to observable fluctuations, A and φ maximize ΔV_b

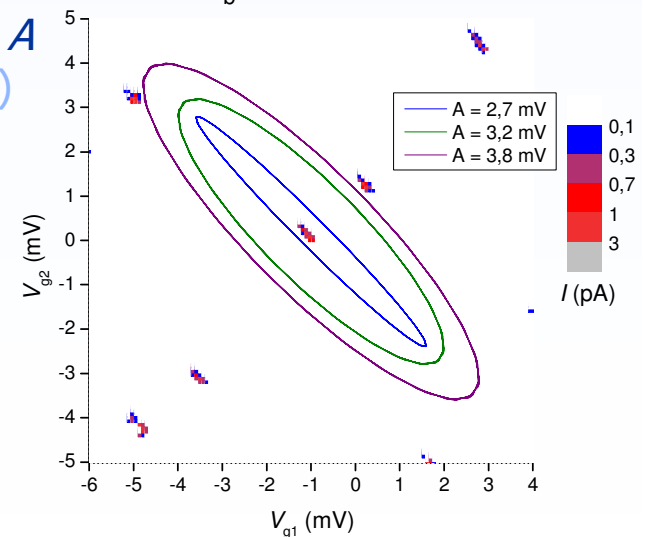
Current step measured at 10 MHz

Characterization of a pump: pumping parameters

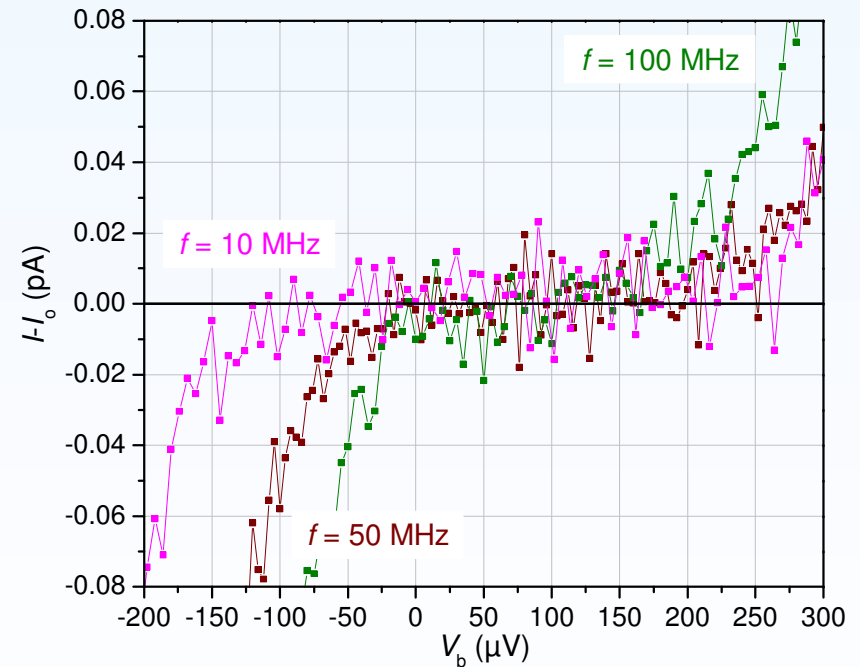
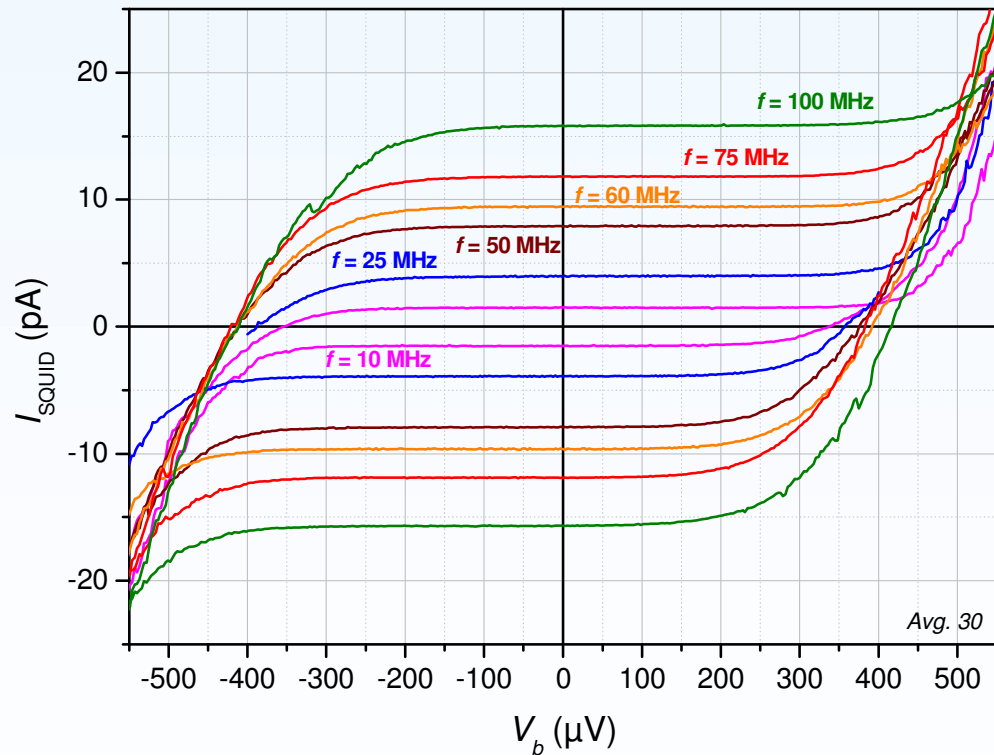
Dependence on ϕ
(with $A=3.2$ mV)



Dependence on A
(with $\phi=210^\circ$)



Characterization of a pump: current steps



Current steps obtained with a 3 junctions R-pump in the internal feedback mode.
The current was reversed by modifying φ .

f (MHz)	10	50	100
ΔV_b (μV)	350	300	150

($\Delta I = 40$ fA)

Type A uncertainty

GUM gives some rules to express the result of a measurement.

Uncertainty is divided in two parts:

- type A: uncertainty linked to statistical processes
- type B: uncertainty linked to other processes

- Objective: to reduce the type A uncertainty
(*by accumulating data*)
- If measurement are performed in white noise regime, then the type A uncertainty is the *experimental standard deviation of the mean*:

$$s^2(\bar{I}) = \frac{1}{n(n-1)} \sum_{k=1}^n \left(I_k - \frac{1}{n} \sum_{i=1}^n I_i \right)^2$$

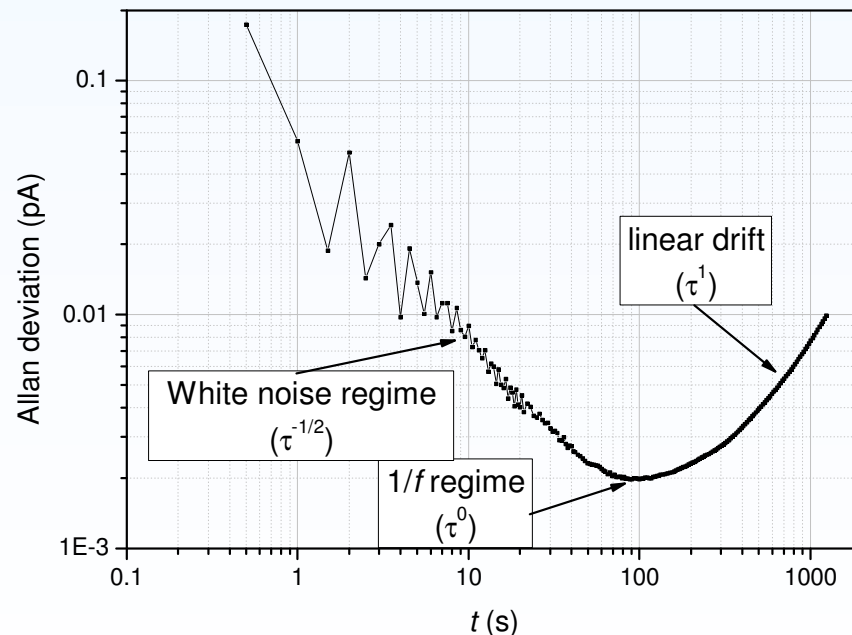
(*n*: number of measurements of the current intensity *I*)

Hypothesis of white noise must be validate

Allan variance

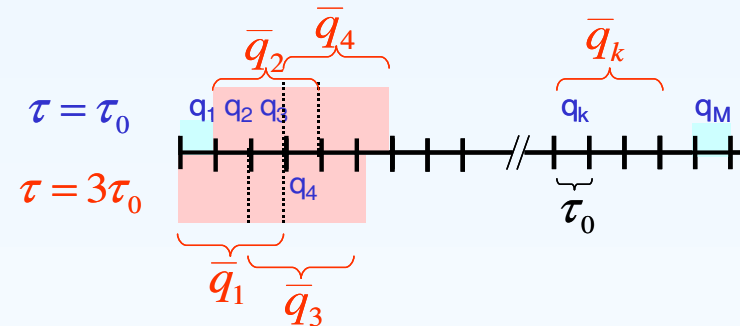
Hypothesis to use the Allan variance:

- Normal distribution
- PSD has a power law dependence on frequency
- Observations are equally spaced in time



Allan deviation calculus made on an accumulation of data of a R-pump (with $f = 10$ MHz)

The overlapping Allan variance:



$$\sigma_q^2(m\tau_0) = \frac{1}{2m^2(M-2m+1)} \sum_{j=1}^{M-2m+1} \left(\sum_{i=1}^{j+m-1} [q_{i+m} - q_i] \right)^2$$

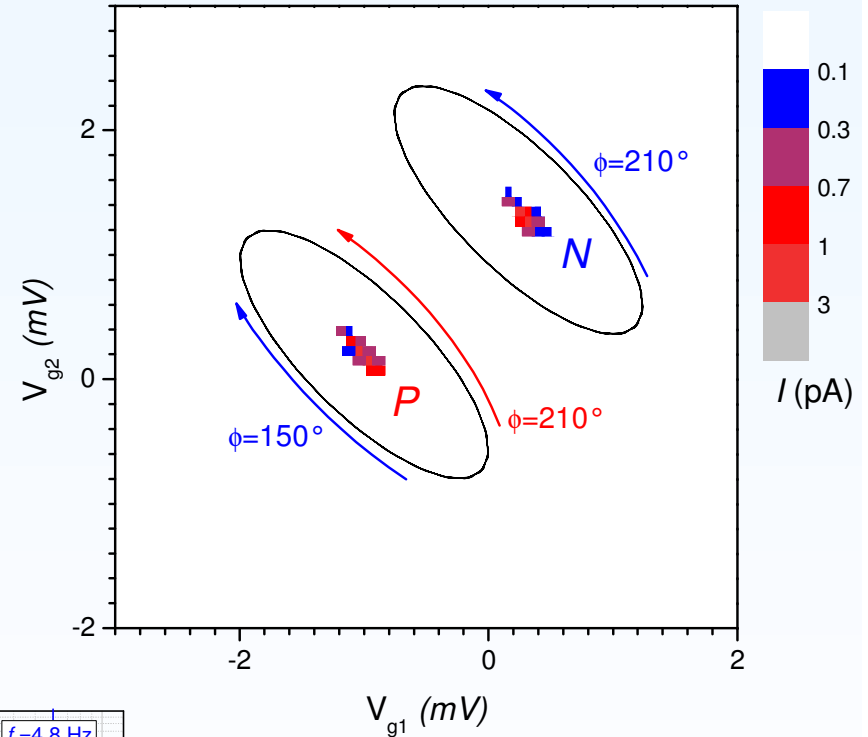
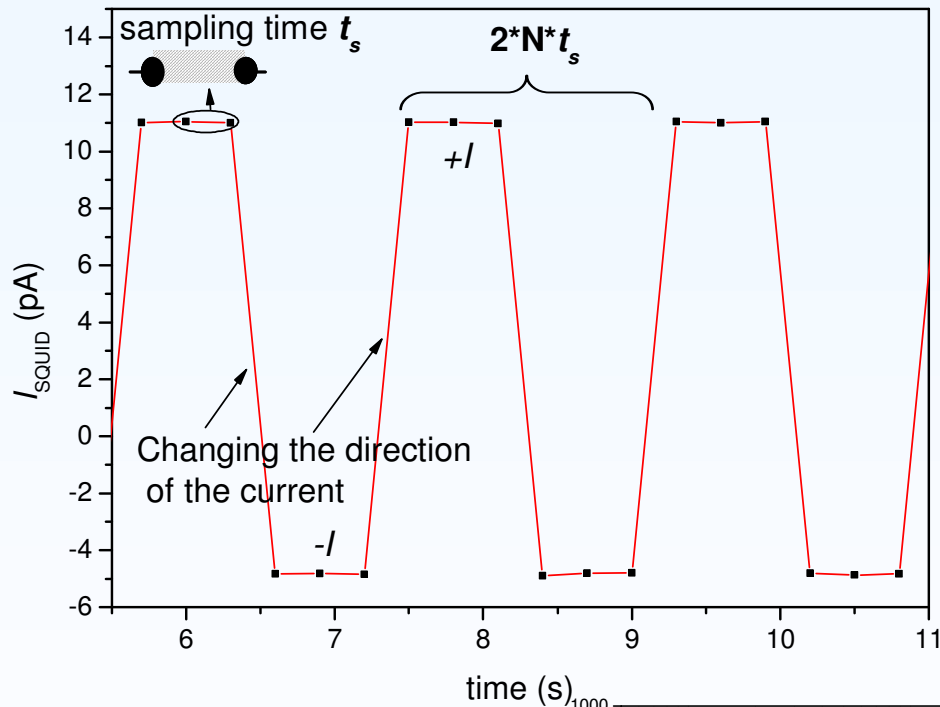
In case of white noise regime, Allan variance is an unbiased estimator of the true variance

$$\sigma_q(m\tau_0) = \sqrt{\frac{h_0}{2m\tau_0}}$$

$h_0^{1/2}$ is the white noise level (in $A/Hz^{1/2}$)

Allan, IEEE Trans. Instrum. Meas., 36, p. 646, 1987 ; Witt, IEEE Trans. Instrum. Meas., 50, p. 445, 2001

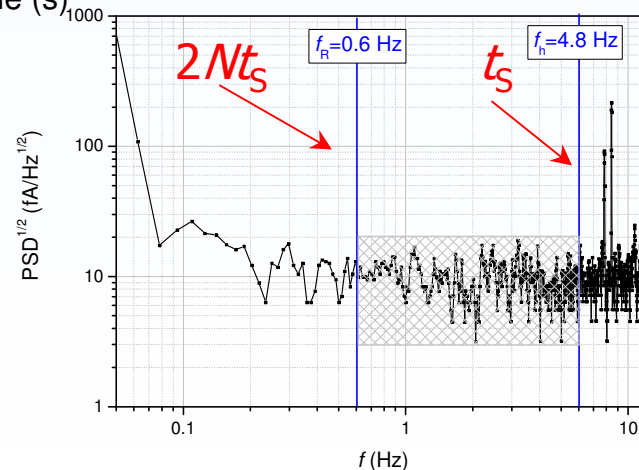
Long term measurements: principle



Determination of N and t_s :

-> Measurement of the PSD

-> Determination of an interval $[f_R, f_H]$ where the noise is white.

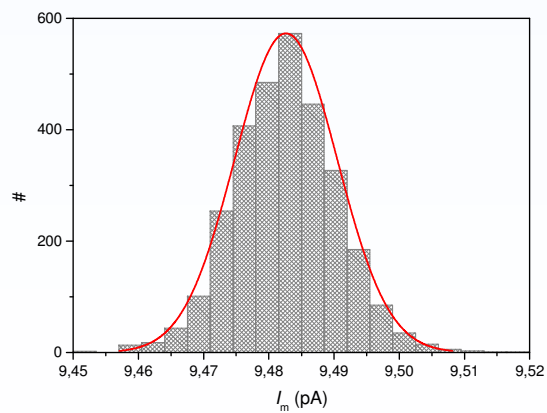
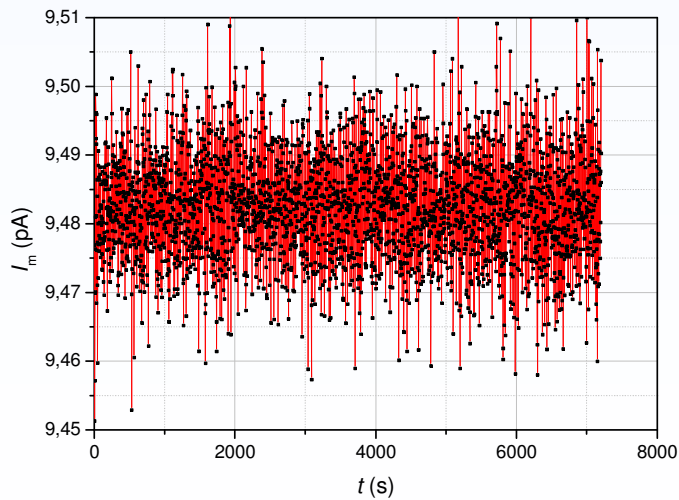
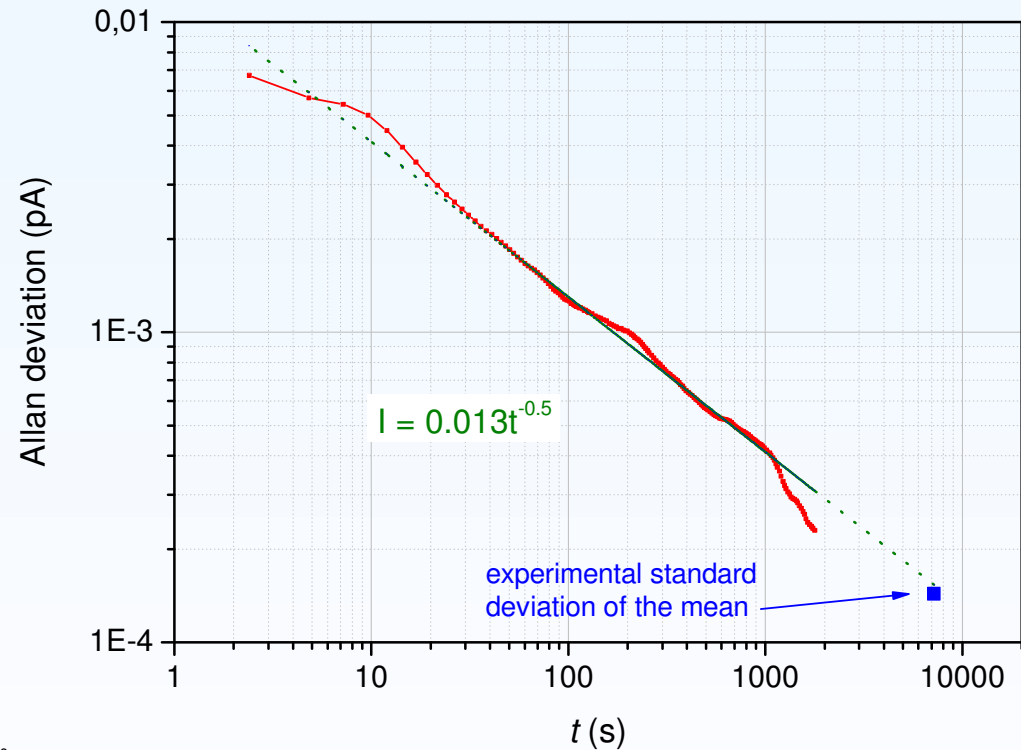
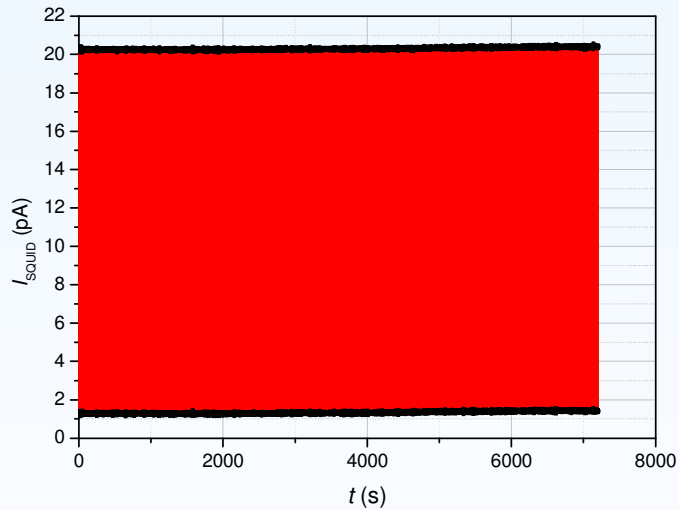


Changing the direction of the current:

-> Around a triple point: modification of the phase

-> With a given phase: modification of the triple point

A long term measurement at $f = 60$ MHz



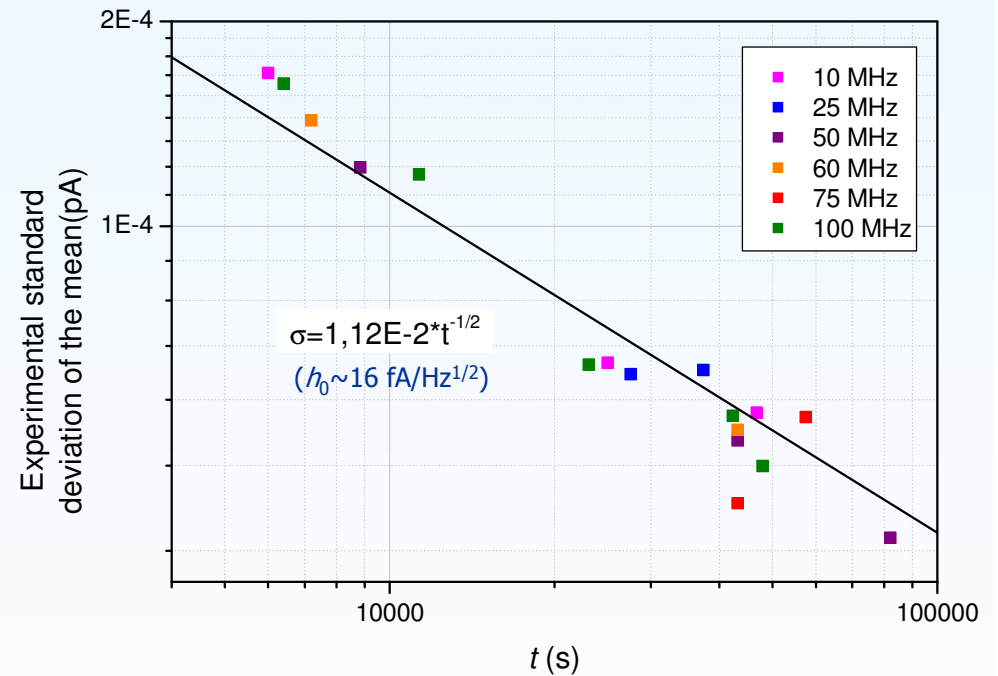
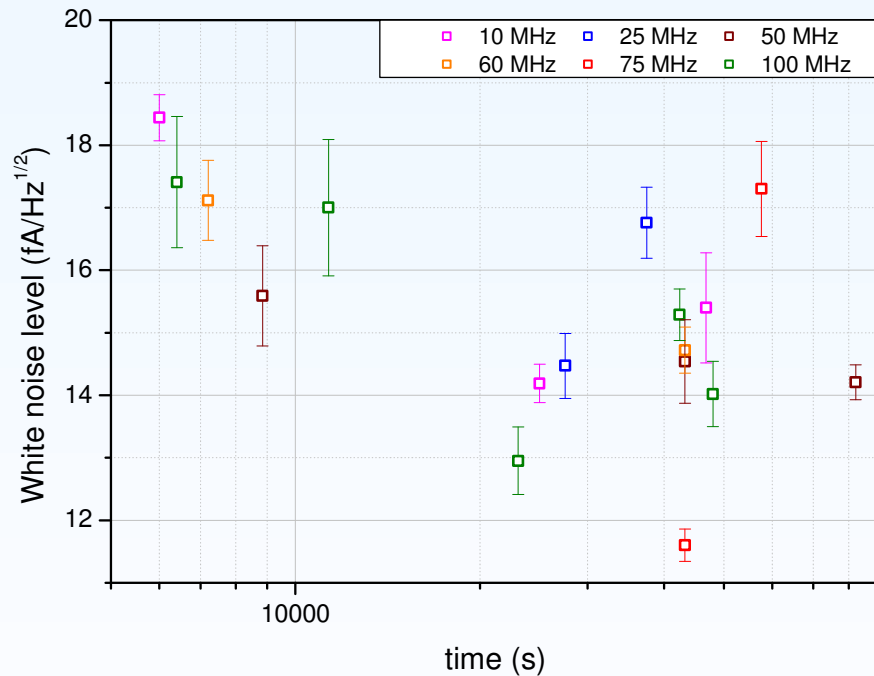
Results

Measuring time
2 h

White noise level:
 $18 \text{ fA/Hz}^{1/2}$

Relative uncertainty:
 $1.5 \text{ parts in } 10^5$

Long term measurements: results



f (MHz)	10	25	50	60	75	100
T (h)	13	7.7	12	12	12	6.5
White noise level ($\text{fA}/\text{Hz}^{1/2}$)	14	16	14	15	12	12
Relative type A uncertainty (ppm)	33.6	15.3	6.1	5.3	3.3	3.9

Results and limitations of this mode of measurement

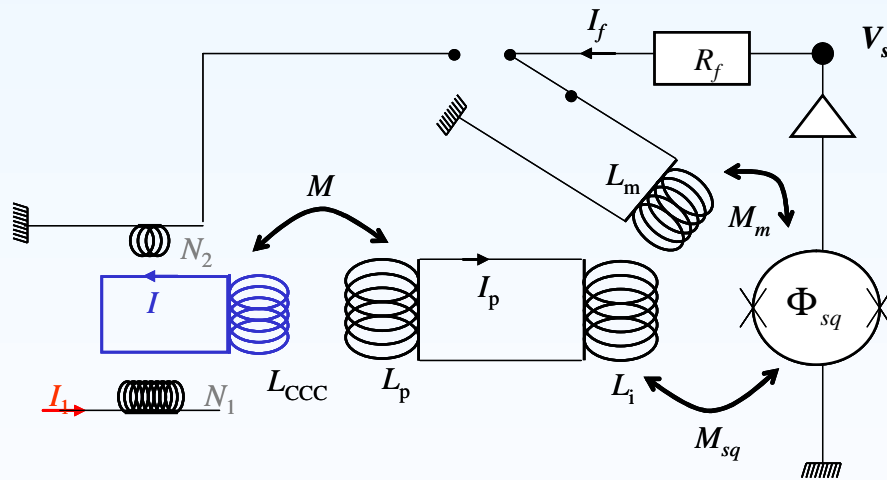
- Operating frequency up to 100 MHz. No effect of this frequency on the stability of the current at the level of accuracy measured.
- Stability of the current generated by the pump on long term (up to 12 h) with a relative uncertainty of a few parts in 10^6 .

BUT

- No information on the quantization level and on the flatness of the current steps.

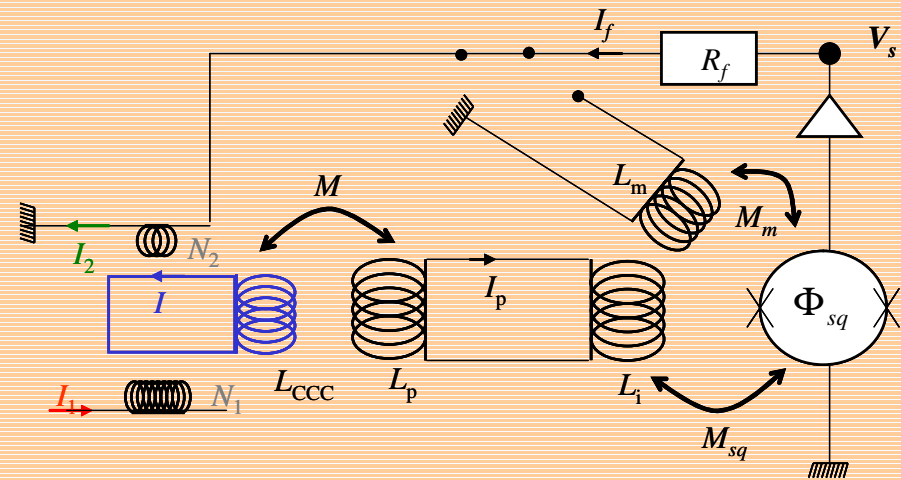
Operating modes of the CCC

Internal feedback mode



$$G_{IFB} = \frac{V_S}{I_1} = \frac{M_{sq}}{M_m} \frac{k}{2} \sqrt{\frac{L_{ccc}}{L_i}} N_1 R_f$$

External feedback mode

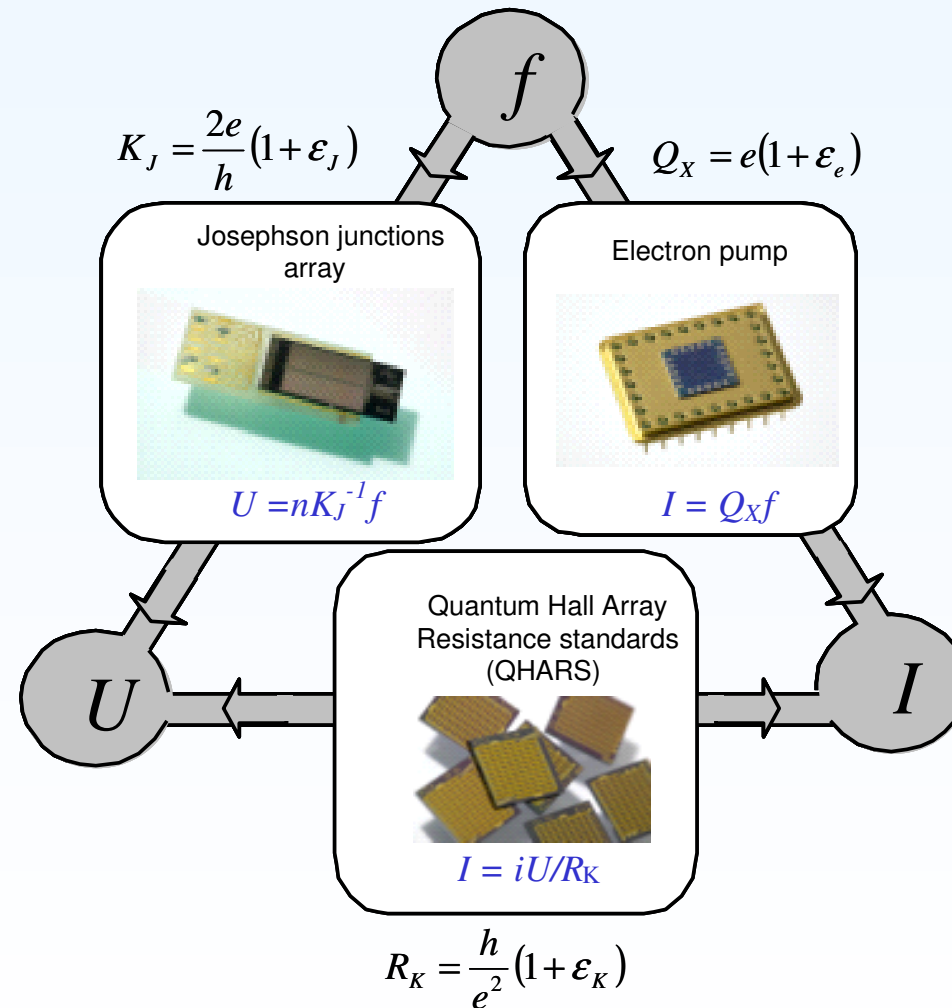


$$G_{EFB} = \frac{V_S}{I_1} = \frac{N_1}{N_2} R_f$$

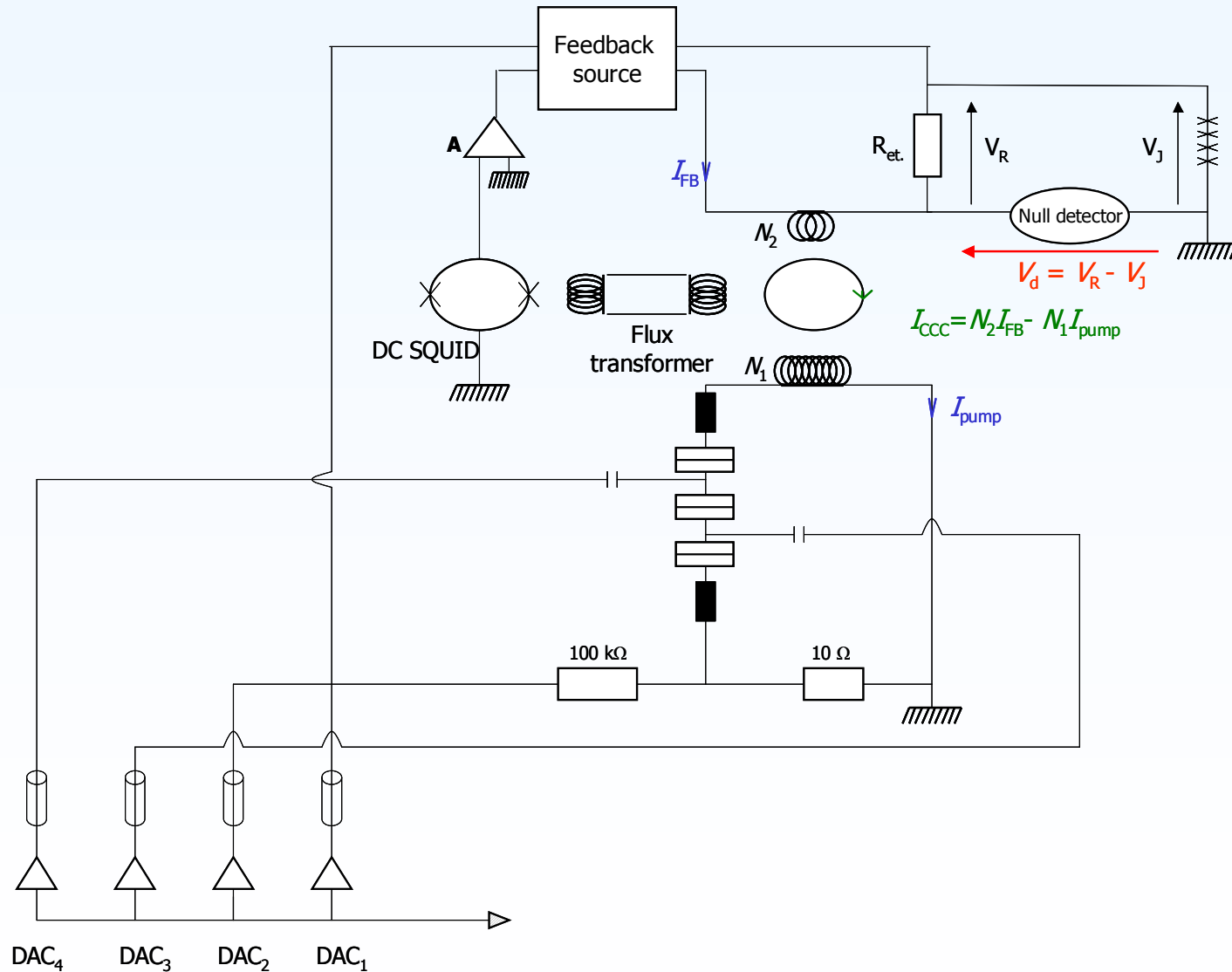
Outline

- Electrical units
- Single electron pump
- Experimental set-up
- Measurements
- **The metrological triangle experiment**

The metrological triangle experiment (MTE)



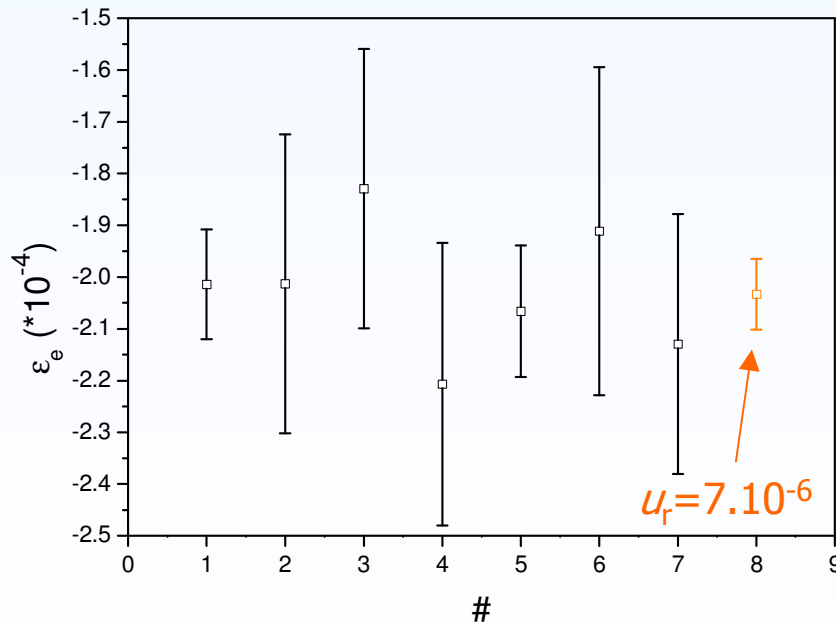
Experimental realisation at LNE



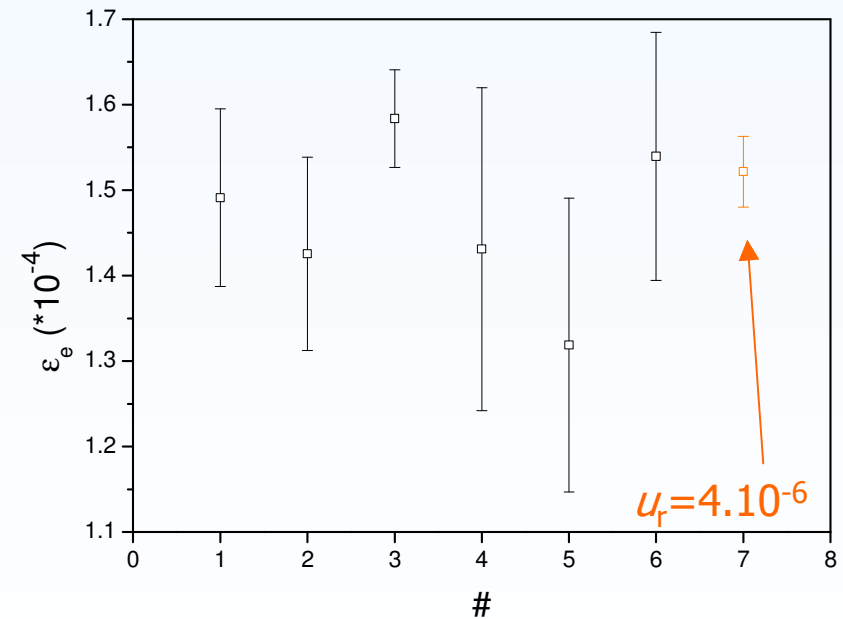
First results

Measurements were performed on long time by reversing voltage and current (modification of current was made by changing the triple point)

$$Q_X = e(1 + \varepsilon_e) = \frac{N_2}{N_1} \frac{1}{f_{SET} R_{et}} \left(V_d + \frac{n_j f_j}{K_J} \right)$$



$n_j=5, f_j=73 \text{ GHz}, f_{SET}=23.55 \text{ MHz}$



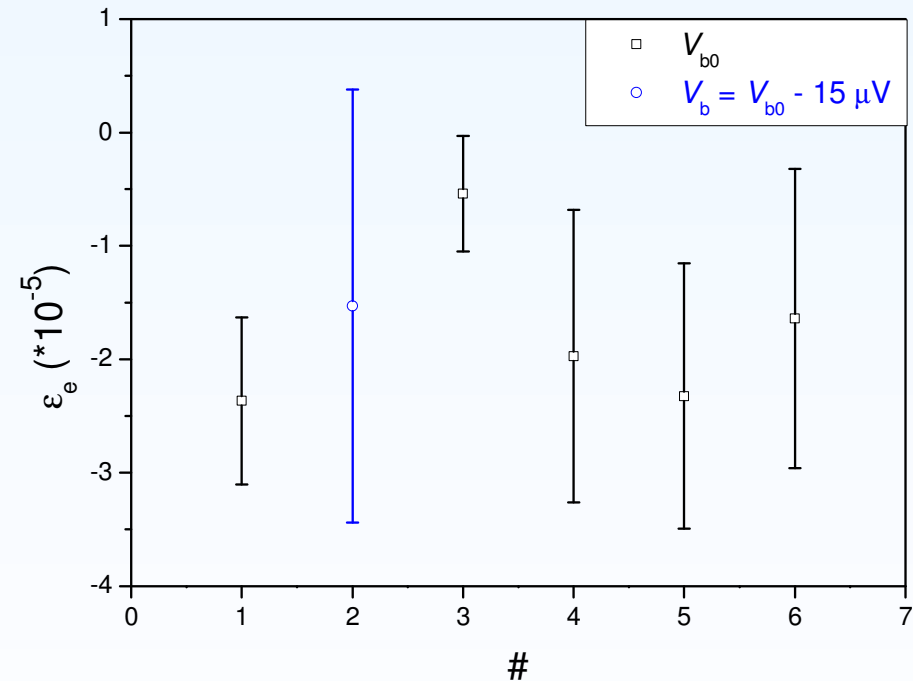
$n_j=8, f_j=73 \text{ GHz}, f_{SET}=37.69 \text{ MHz}$

⇒ there is, at the moment, an irreproducibility **BUT** feasibility of the measurement with a relative uncertainty of a few parts in 10^6 is demonstrated.

Properties of the R-pump

This set-up permits to determine the metrological properties of the R-pump

- Long term measurements made with different bias voltages give the flatness of the current steps.



$n_j=8, f_j=73 \text{ GHz}, f_{\text{SET}}=37.69 \text{ MHz}$

- Determination of the quantization level.
- Then this set-up can be used to make comparison between samples and to characterize their metrological properties.

Interests of the MTE

- MTE permits to calculate $R_K K_J Q_X$ then to measure:

$$\varepsilon_K + \varepsilon_J + \varepsilon_e = \varepsilon_m \pm u_r$$

ε_m : measured value

u_r : uncertainty of the measurement

- ⇒ If $0 \in [\varepsilon_m - u_r, \varepsilon_m + u_r]$, improvement of our confidence in the theoretical relations:

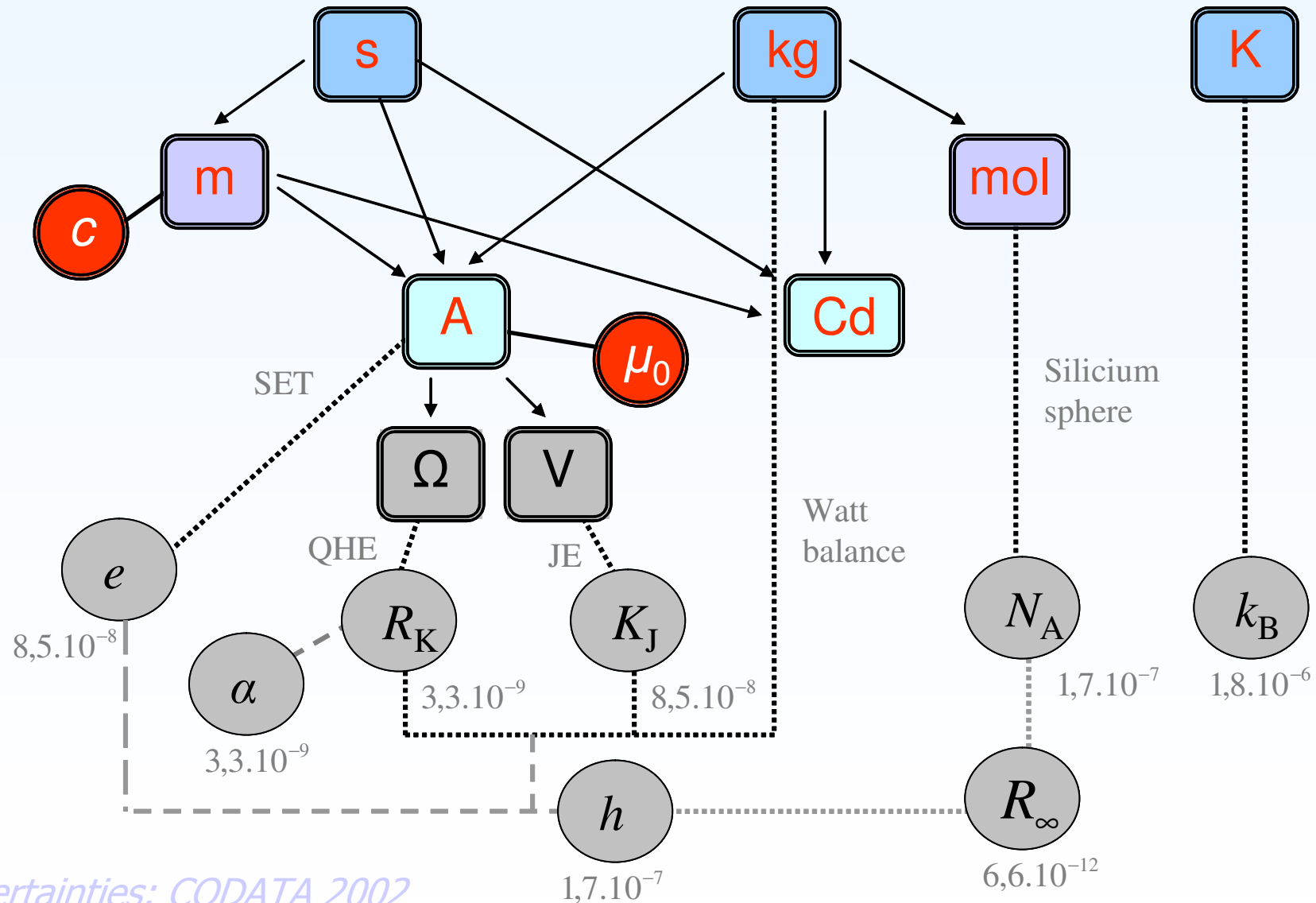
$$R_K = h/e^2, K_J = 2e/h, Q_X = e$$

- Thompson Lampard capacitor gives a determination of R_K , then the watt balance gives a determination of K_J
- ⇒ MTE becomes a determination of Q_X

Systeme international d'unités (SI)

Base quantity	Unit	Definition
mass	<i>kilogram (kg)</i>	The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram. (3 rd CGPM, 1901)
electric current	<i>ampere (A)</i>	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to $2 \cdot 10^{-7}$ newton per metre of length. (9 th CGPM, 1948)
thermodynamic temperature	<i>kelvin (K)</i>	The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (10 th CGPM, 1954)
time	<i>second (s)</i>	The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom. (13 th CGPM, 1967/68)
amount of substance	<i>mole (mol)</i>	1. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12. 2. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles. (14 th CGPM, 1971)
luminous intensity	<i>candela (cd)</i>	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. (16 th CGPM, 1979)
length	<i>metre (m)</i>	The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second. (17 th CGPM, 1983)

Units and fundamental constants



Uncertainties: CODATA 2002

Towards a new SI

- kilogram

- Microscopic kilogram: *silicium sphere*
 - N_A is fixed
 - Electric kilogram: *watt balance*
 - $K_J^2 R_K$ is fixed
- > Fixation of h if

$$\begin{cases} R_K = \frac{h}{e^2} \\ K_J = \frac{2e}{h} \end{cases}$$

- kelvin

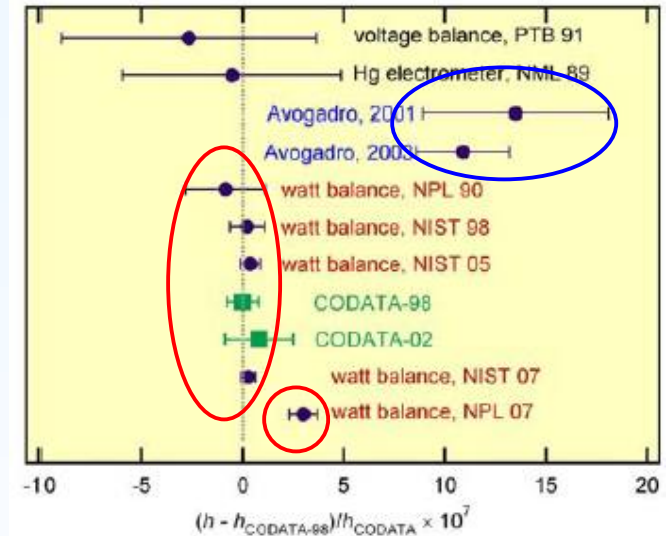
- k_B is fixed

- ampere

- $Z_0 (= \mu_0 c)$ is fixed
- e is fixed

=> *Thompson Lampard*

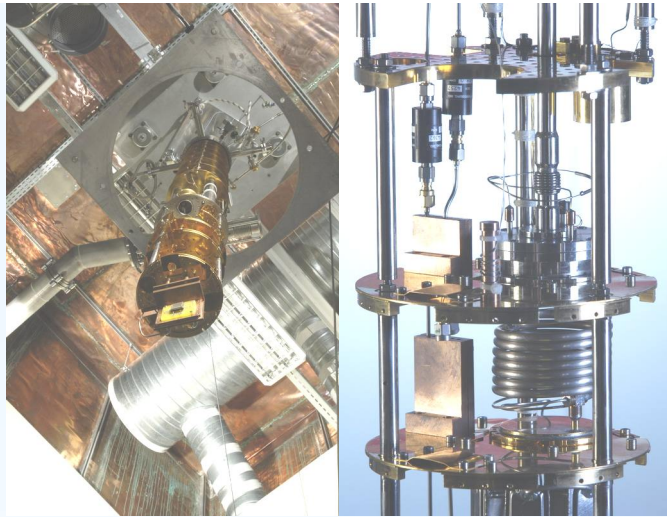
=> *Metrological triangle*



Source: A. Eichenberger, Houches 2007

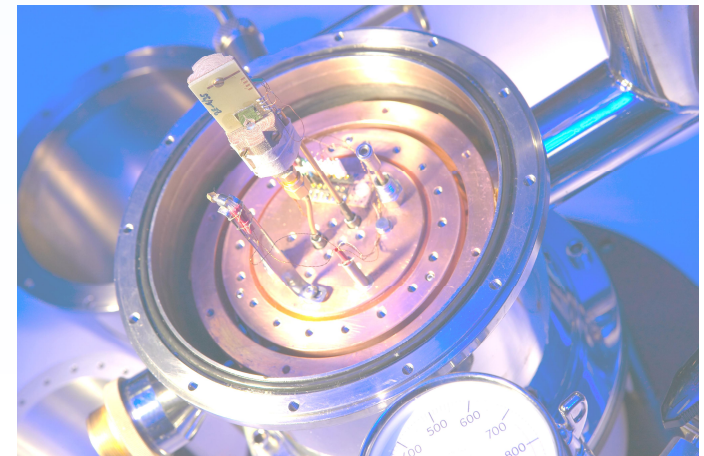
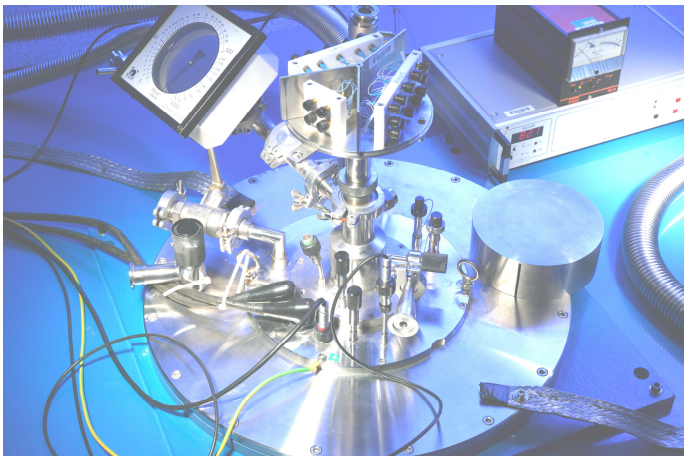
Outlook

- Obtaining of an uncertainty of 1 part in 10^6 is a first step important in the framework on a new definition of units and in characterization of R-pump
- Such a step permits to validate the measurement principle
- This set-up could then be used to test other devices which generate more current



Thanks for your attention

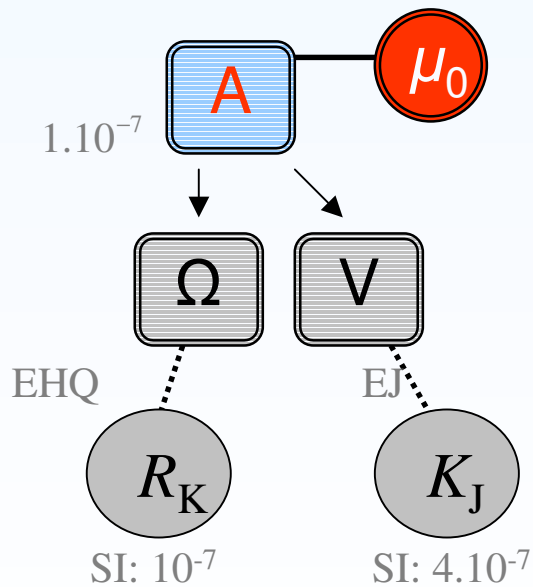
*and special thanks to the quantum
metrology group at LNE*



Electrical units : the "90" system

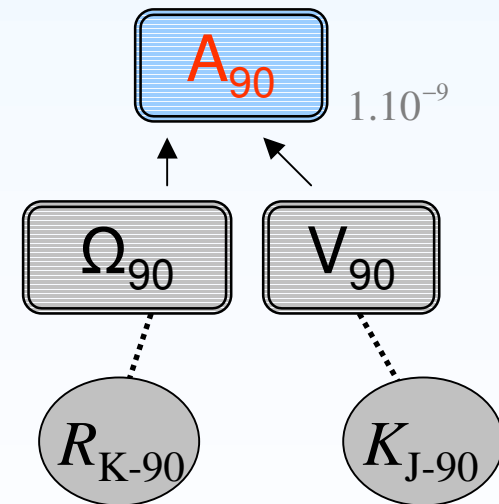
- CIPM recommendations (1988)

set exact values for the Josephson and von Klitzing constants, and called for laboratories to base their standards on these values from 1 January 1990. These values are noted K_{J-90} and R_{K-90} .



$$\left\{ \begin{array}{l} V_{90} = \frac{K_{J-90}}{K_J} V_{SI} \\ \Omega_{90} = \frac{R_K}{R_{K-90}} \Omega_{SI} \end{array} \right.$$

Reproducibility:
 -> QHE ohm: 10^{-10}
 -> JE volt: 10^{-9}



$$K_{J-90} = 483\,597.9 \text{ GHz/V}$$

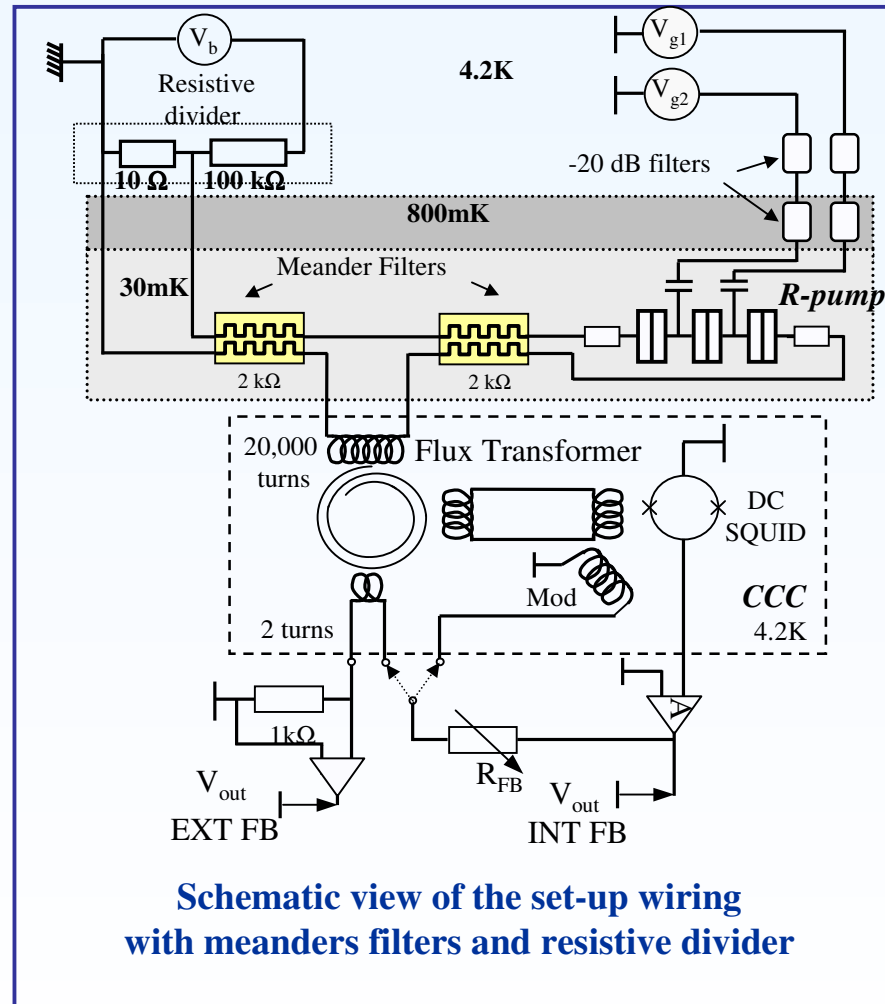
$$R_{K-90} = 25\,812.807 \, \Omega$$

CODATA 2002:

$$K_J = K_{J-90}(1 - 4.3 \cdot 10^{-8} \pm 8.5 \cdot 10^{-8})$$

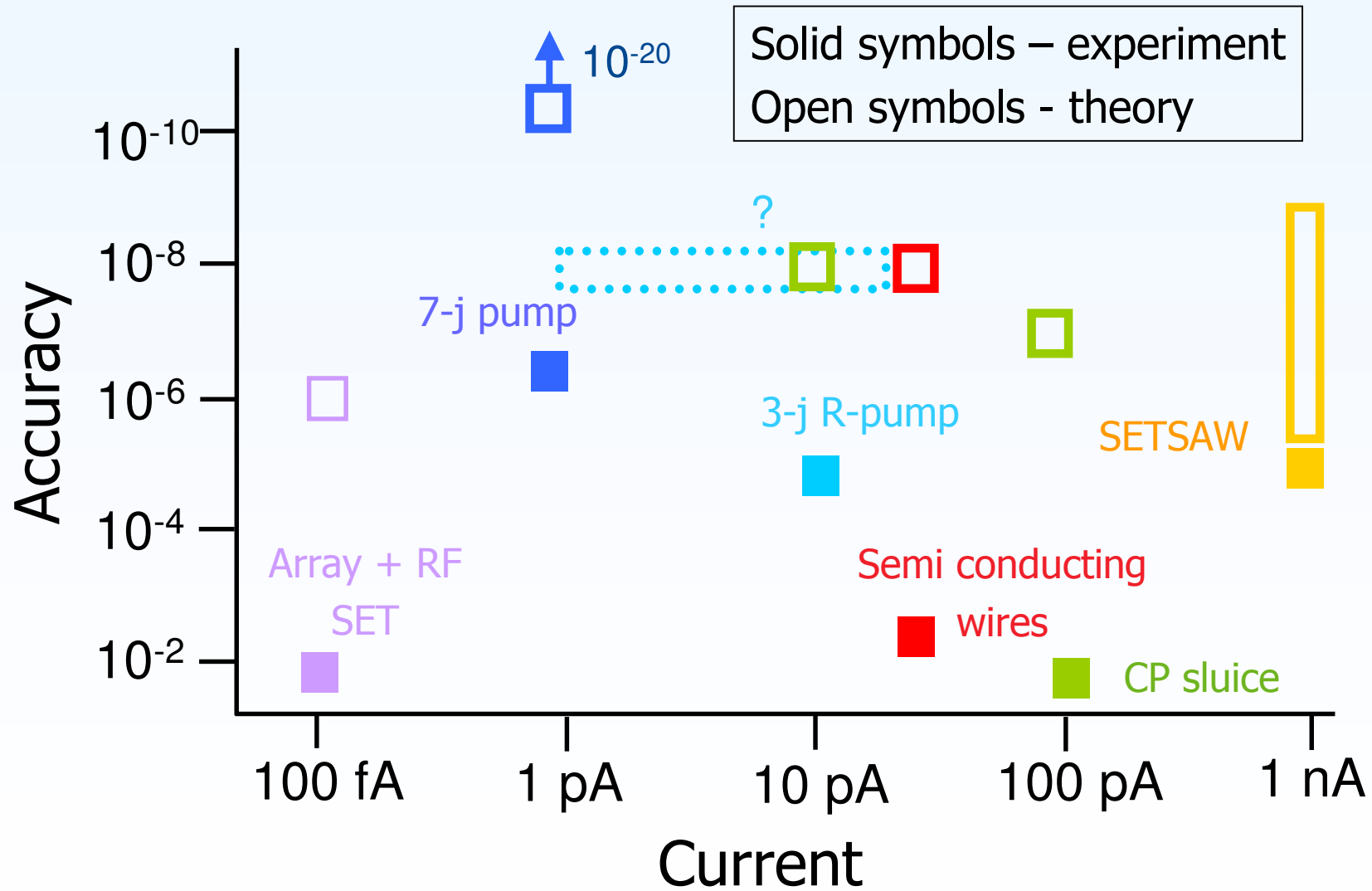
$$R_K = R_{K-90}(1 + 1.74 \cdot 10^{-8} \pm 0.33 \cdot 10^{-8})$$

Electrical schematic view of the set-up

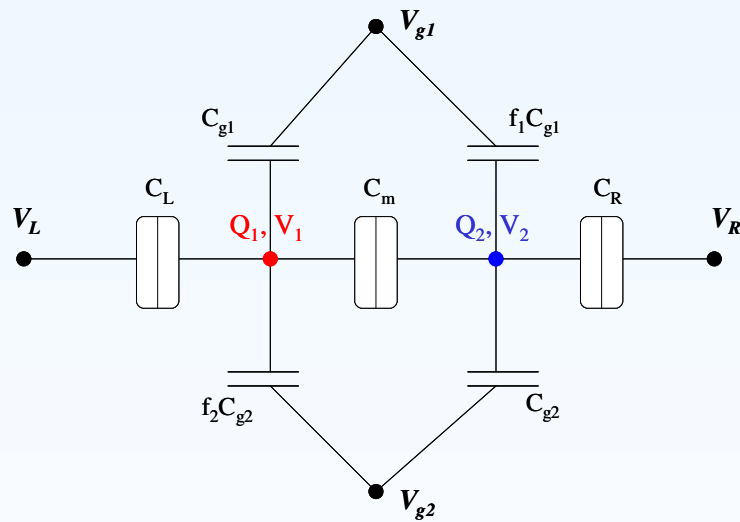


Meander filters supplied by CEA (Saclay)

Others devices



Characterization of a pump: determination of the parameters



$$E(n_1, n_2) = \frac{e}{C_1 C_2 - C_m^2} \left[\frac{e}{2} (C_2 n_1^2 + C_1 n_2^2) + e C_m n_1 n_2 - V_{g1} C_{g1} ((C_2 + f_1 C_m) n_1 + (C_m + f_1 C_1) n_2) - V_{g2} C_{g2} ((C_m + f_2 C_2) n_1 + (C_1 + f_2 C_m) n_2) \right] + \text{termes indépendants de } (n_1, n_2)$$

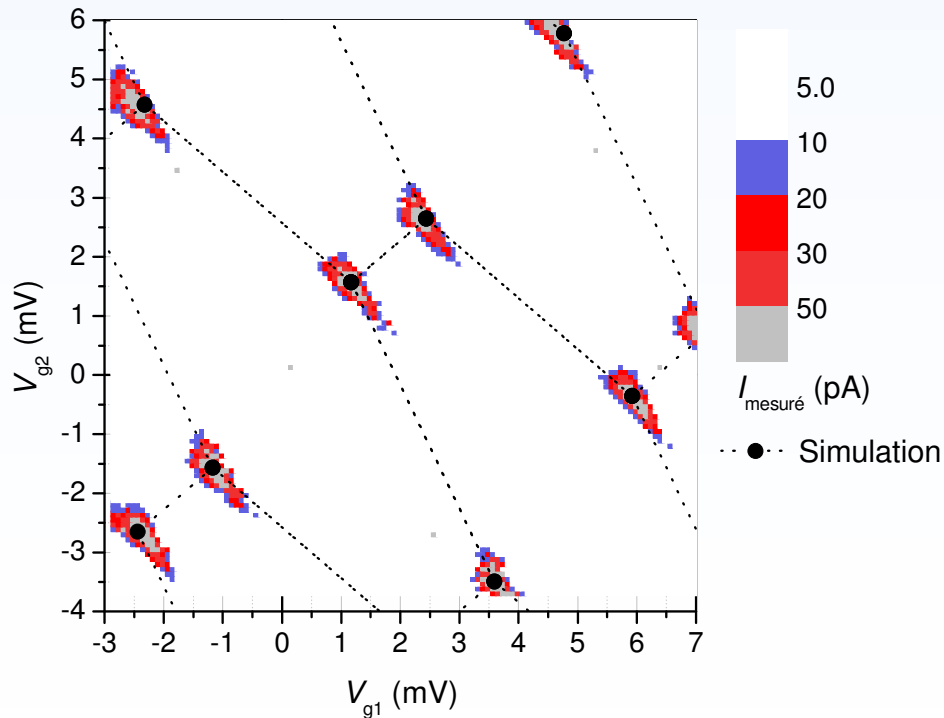
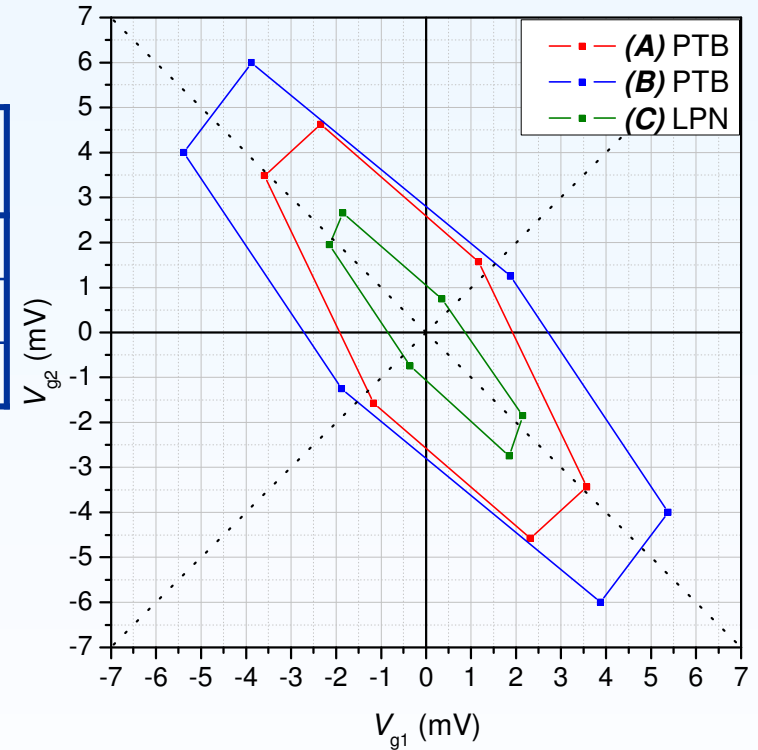
$$\begin{cases} C_1 = C_L + C_m + C_{g1} + f_2 C_{g2} \\ C_2 = C_R + C_m + f_1 C_{g1} + C_{g2} \end{cases}$$

- Determination of the boundaries of the domain where $(0,0)$ is stable
-> 6 segments delimit this domain, the positions of the 6 triple points are function of the capacitor parameters of the pump.
- Measurements of the stability diagram **and** of the threshold voltage permit to determine the 7 parameters unknown:

$$C_L, C_m, C_R, C_{g1}, C_{g2}, f_1, f_2$$

Characterization of a pump: determination of the parameters

Pump	C_{g1} (aF)	C_{g2} (aF)	f_1	f_2	C_L (aF)	C_m (aF)	C_R (aF)	T_C (K)
(A)	35	30	0.30	0.25	75	85	60	7
(B)	25	25	0.4	0.3	55	120	75	5.5
(C)	80	60	0.35	0.60	95	215	135	3



Pump	C_L/C_m (%)	C_R/C_m (%)	C_L/C_R (%)
(A)	90	70	125
(B)	45	60	75
(C)	45	60	70