Thèse de doctorat de l'Université de Cergy-Pontoise

Spécialité Traitement du Signal

Collaboration ONERA / UCP-ETIS

Detection in non-Gaussian Environment

Radar application

Supervisor	:	Patrick DUVAUT	ENSEA - ETIS
Co-supervisors	:	Jean-Philippe OVARLEZ	ONERA - DEMR/TSI
		David DECLERCQ	ENSEA - ETIS

Friday, June the 14^{th} of 2002

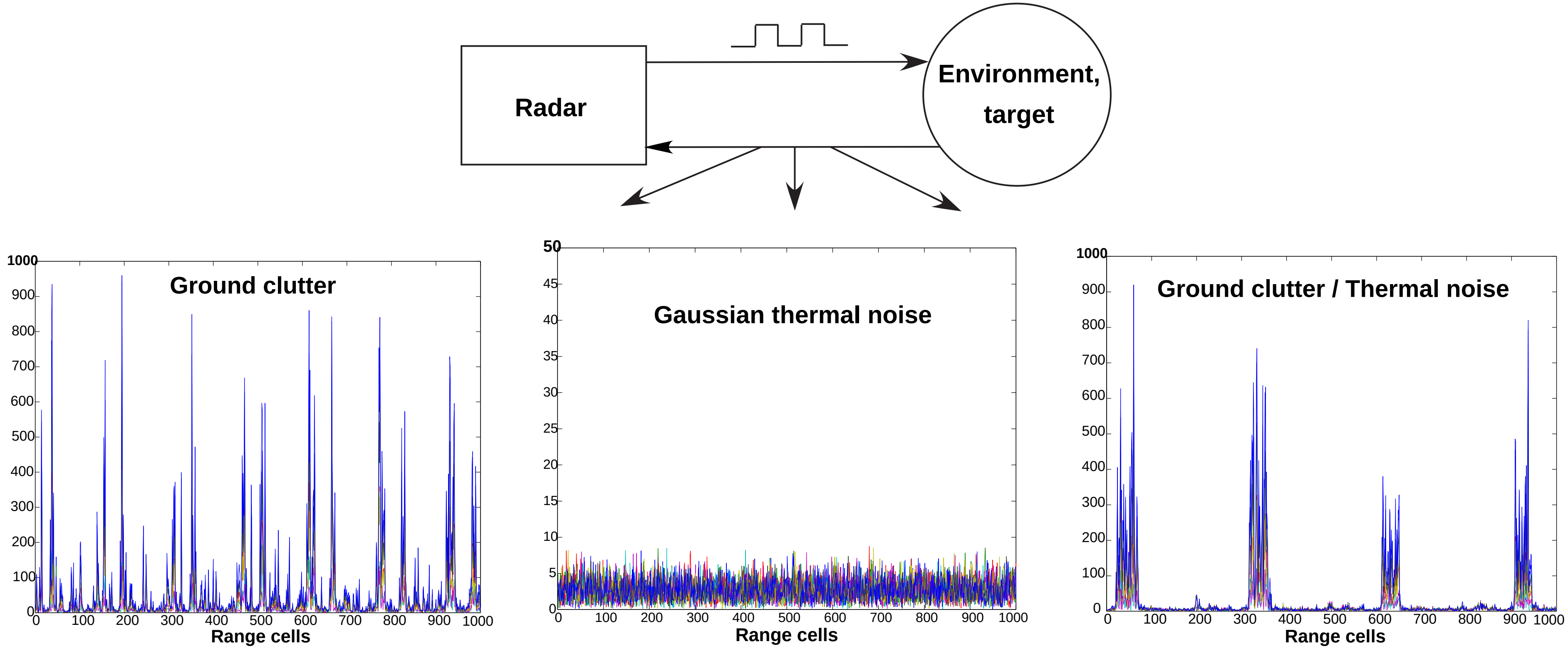
Emmanuelle Jay

Plan of the presentation

1 - Task of the subject
2 - Non-Gaussian environment characterization
3 - PEOD and BORD : two new optimal detection structures
4 - Performance analysis of BORD against experimental clutter data
5 - Conclusions et outlook

1 - Task of the subject

1.1 - Radar echoes



How to free itself from these echoes to detect the signal coming from the target ?

1 - Task of the subject

1.2 - Equation setting

◇ **Environment returns :** Random signals from the clutter

$H_0 : \mathbf{y} = \mathbf{b} \quad \Rightarrow \quad \mathbf{p(y/H_0) = p_b(y)} \quad \rightarrow$ **False Alarm (**P_{fa}**)** if H_1 au lieu de H_0

◇ **Target returns :** Deterministic or random signals

$H_1 : \mathbf{y} = \mathbf{b} + \mathbf{s} \quad \Rightarrow \quad \mathbf{p(y/H_1) = p_b(y - s)} \quad \rightarrow$ **Detection (**P_d**)** when H_1 is checked

◇ **Detection aim :** To build a detection test

$$\boxed{D(\mathbf{y}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta} \quad \text{such as} \quad \begin{cases} \textbf{for a fixed} \quad \mathbf{P_{fa}} = \mathbb{P}(D(\mathbf{y}) \overset{H_0}{>} \eta) \\ \mathbf{P_d} = \mathbb{P}(D(\mathbf{y}) \overset{H_1}{>} \eta) \quad \text{be } \textbf{optimal} \end{cases}$$

Identify the clutter statistics to build $D(\mathbf{y})$

1 - Task of the subject

1.3 - State of the art of the detection against non-Gaussian environment

◇ **Non-coherent detection** : CFAR Optimization of the matched filter ([Watt85, Gand88, Rifk94, Shni95])

⇒ **Non robust, non optimal**

◇ **Coherent detection** ([Fari85]) : Modelization with non-Gaussian complex processes (like SIRP)

[Yao73, Gold76, Cont87, Rang91, Rang93, Rang95, Barn96]

⇒ Adequacy to experimental ground and sea clutter ([Trun70, Cont87, Bill93, · · ·]),

⇒ Optimum detectors known in closed forms ([Cont87, Fari87, Cont91, Gini98, Sang99, · · ·]),

⇒ Detection performances improved if compared with those of the matched filter

⇒ **An** *A priori* **hypothesis is required to estimate the statistics parameters**

⇒ **Detectors optimality is checked only for the designed statistics**

Thesis Aim

To derive optimal coherent SIRP detectors, adaptive to the environment

2 - Non-Gaussian environment characterization

Many known non-Gaussian laws belong to the SIRV family

	SIRV Nature	**Optimum Detector**
	Gaussian	**OGD** - Optimum Gaussian Detector
	K-distributed	**OKD** - Optimum K Detector
	Cauchy, Student-t	**OCD, OStD** - Optimum Cauchy, Student-t Detector
	Weibull	**OWD** - Optimum Weibull Detector

2 - Non-Gaussian environment characterization

Many known non-Gaussian laws belong to the SIRV family

Texture **law**	**SIRV Nature**	**Optimum Detector**
Dirac in 1	Gaussian	**OGD** - Optimum Gaussian Detector
Gamma	K-distributed	**OKD** - Optimum K Detector
Inverse Gamma	Cauchy, Student-t	**OCD, OStD** - Optimum Cauchy, Student-t Detector
Weibull		
?	Weibull	**OWD** - Optimum Weibull Detector

How to characterize the environment statistics directly from the received data ?

2 - Non-Gaussian environment characterization

2.1 - SIRV : Spherically Invariant Random Vector

◇ **The K. Yao's representation theorem, [Yao73]** :

$$\mathbf{y} = \mathbf{x}\sqrt{\tau} \begin{cases} \mathbf{y} & \text{SIRV or } m\text{-complex compound Gaussian Vector} \\ \tau & \textit{Texture} \text{ of the SIRV, positive r.v. with pdf } p(\tau) \\ \mathbf{x} & \textit{Speckle} \text{ of the SIRV, stat. indepdt of the texture } : \mathbf{x} \sim \mathcal{CN}(\mathbf{0}_m, 2\,\mathbf{M}) \end{cases}$$

$$\Longrightarrow \boxed{p(\mathbf{y}) = \int_0^{+\infty} \underbrace{p(\mathbf{x}/\tau)}_{\text{Known}}\; p(\tau)\, d\tau}$$

***Texture* pdf characterize the SIRV statistics**

2 - Non-Gaussian environment characterization

2.2 - SIRV : Properties

◇ **Radial coherent characteristic function : Spherically invariant :**

$$\text{SIRV} : \mathbf{y} = \mathbf{y}_I + j\,\mathbf{y}_Q \begin{cases} \mathbf{y}_I, \mathbf{y}_Q & : \quad \text{Statistically independant} \\ \mathbf{y}_I, \mathbf{y}_Q/\tau & \sim \quad \mathcal{N}(\mathbf{0}_m, \tau\,\mathbf{M}) \\ \arg(\mathbf{y}_I, \mathbf{y}_Q) & : \quad \textbf{Uniformely distributed on } [-\pi, +\pi] \\ p(\mathbf{y}) & = \quad h(\mathbf{y}^\dagger\,\mathbf{M}^{-1}\,\mathbf{y}) \end{cases}$$

$$\Downarrow$$

$$\boxed{F_y(\mathbf{u}) = \int_{\mathbb{R}^{2m}} p(\mathbf{y})\, e^{j\,\mathbf{u}^\dagger\,\mathbf{y}}\, d\mathbf{y} = g(\mathbf{u}^\dagger\,\mathbf{M}\,\mathbf{u})}$$

◇ **Invariance under linear transformation : $\mathbf{A}\,\mathbf{y} + \mathbf{b}$ is also a SIRV**, with the same texture pdf than $\mathbf{y}$

2 - Non-Gaussian environment characterization

2.2 - SIRV : Properties

◇ **Radial coherent characteristic function : Spherically invariant :**

$$\text{SIRV} : \mathbf{y} = \mathbf{y}_I + j\,\mathbf{y}_Q \begin{cases} \mathbf{y}_I, \mathbf{y}_Q & : \quad \text{Statistically independant} \\ \mathbf{y}_I, \mathbf{y}_Q/\tau & \sim \quad \mathcal{N}(\mathbf{0}_m, \tau\,\mathbf{M}) \\ \arg(\mathbf{y}_I, \mathbf{y}_Q) & : \quad \textbf{Uniformely distributed on } [-\pi, +\pi] \\ p(\mathbf{y}) & = \quad h(\mathbf{y}^\dagger\,\mathbf{M}^{-1}\,\mathbf{y}) \end{cases}$$

$$\Downarrow$$

$$F_y(\mathbf{u}) = \int_{\mathbb{R}^{2m}} p(\mathbf{y})\, e^{j\,\mathbf{u}^\dagger\,\mathbf{y}}\, d\mathbf{y} = g(\mathbf{u}^\dagger\,\mathbf{M}\,\mathbf{u})$$

◇ **Invariance under linear transformation : $\mathbf{A}\,\mathbf{y} + \mathbf{b}$ is also a SIRV**, with the same texture pdf than $\mathbf{y}$

Proposed solution : Bayesian Approach

2 - Non-Gaussian environment characterization

2.3 - *A posteriori* pdf of the *texture*

◇ **Bayes' rule :**

$$p(\tau/\mathbf{y}) = \frac{\overbrace{p(\mathbf{y}/\tau)}^{known}\ \overbrace{p(\tau)}^{unknown}}{\underbrace{p(\mathbf{y})}_{unknown}} = \frac{p(\mathbf{y}/\tau)\, p(\tau)}{\int_0^{+\infty} p(\mathbf{y}/\tau)\, p(\tau)\, d\tau}$$

◇ **Learning with N reference SIRV$[\mathbf{r}_1, \cdots, \mathbf{r}_N]^t$: Jeffrey's Non-informative *Prior* choosen for τ**

$$\mathbf{g}(\tau) = \frac{1}{\tau}$$

$$\Longrightarrow \quad p(\tau/\mathbf{r}_i) = \frac{\overbrace{p(\mathbf{r}_i/\tau)}^{known}\ \overbrace{g(\tau)}^{known}}{\underbrace{p(\mathbf{r}_i)}_{known}} = \frac{p(\mathbf{r}_i/\tau)\, g(\tau)}{\int_0^{+\infty} p(\mathbf{r}_i/\tau)\, g(\tau)\, d\tau} = \mathcal{IG}\left(m, \frac{2}{\mathbf{r}_i^\dagger\, \mathbf{M}^{-1}\, \mathbf{r}_i}\right)$$

Need to estimate the *speckle* correlation matrix

2 - Non-Gaussian environment characterization

2.4 - Normalized Structured Covariance Matrix (NSCM) estimation of the *speckle*

◇ **From the N reference vectors of size m, $[\mathbf{r}_1, \cdots, \mathbf{r}_N]^t$:**

$$\mathbf{M}_r = \mathbb{E}(\mathbf{r}\,\mathbf{r}^\dagger) = \mathbb{E}(\tau\,\mathbf{x}\,\mathbf{x}^\dagger) = \mathbb{E}(\tau)\,\mathbf{M}_x = 2\,\mathbb{E}(\tau)\,\mathbf{M}$$

$$\Longrightarrow \boxed{\widehat{\mathbf{M}}_x = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{r}_i\,\mathbf{r}_i^\dagger}{\mathbf{r}_i^\dagger\,\mathbf{r}_i}} \quad \Longleftrightarrow \quad \boxed{\widehat{\mathbf{M}}_x = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{x}_i\,\mathbf{x}_i^\dagger}{\mathbf{x}_i^\dagger\,\mathbf{x}_i}}$$

Estimation : statistically independent of the *texture* pdf

[Gini99, Gini00b, Gini00c]

2 - Non-Gaussian environment characterization

2.5 -*Texture* pdf estimation

◇ **With a Padé approximation :**

$$\widehat{p}(\tau) = \sum_{k=1}^{M} \lambda_k \, e^{-\alpha_k \, \tau}$$

i) *Texture* r.v. resampling according to its instantaneous *a posteriori* pdf : $\tilde{\tau}_{i=1}^{N} \sim p(\tau/\mathbf{r}_i)$

ii) Moments estimation : $\widehat{\mu}_n = N^{-1} \sum_{i=1}^{N} \tilde{\tau}_i^n$

iii) Computation of the M Padé coefficients $\{\alpha_k\}$ et $\{\lambda_k\}$ from the estimated moments

◇ **With a Monte-Carlo Bayesian estimator :**

$$p(\tau) = \int_{\mathbb{R}^m} p(\tau/\mathbf{r}) \, p(\mathbf{r}) \, d\mathbf{r} \quad \overset{\textbf{MC}}{\Longrightarrow} \quad \widehat{p}_N(\tau) = \frac{1}{N} \sum_{i=1}^{N} p(\tau/\mathbf{r}_i)$$

2 - Non-Gaussian environment characterization

2.5 -*Texture* pdf estimation

◇ **With a Padé approximation :**

$$\widehat{p}(\tau) = \sum_{k=1}^{M} \lambda_k \, e^{-\alpha_k \, \tau}$$

i) *Texture* r.v. resampling according to its instantaneous *a posteriori* pdf : $\tilde{\tau}_{i=1}^{N} \sim p(\tau/\mathbf{r}_i)$

ii) Moments estimation : $\widehat{\mu}_n = N^{-1} \sum_{i=1}^{N} \tilde{\tau}_i^n$

iii) Computation of the M Padé coefficients $\{\alpha_k\}$ et $\{\lambda_k\}$ from the estimated moments

◇ **With a Monte-Carlo Bayesian estimator :**

$$p(\tau) = \int_{\mathbb{R}^m} p(\tau/\mathbf{r}) \, p(\mathbf{r}) \, d\mathbf{r} \quad \overset{\text{MC}}{\Longrightarrow} \quad \widehat{p}_N(\tau) = \frac{1}{N} \sum_{i=1}^{N} p(\tau/\mathbf{r}_i)$$

Two new detection strategies come from these propositions

2 - Non-Gaussian environment characterization

2.5 -*Texture* pdf estimation

◇ **With a Padé approximation :**

$$\widehat{p}(\tau) = \sum_{k=1}^{M} \lambda_k \, e^{-\alpha_k \, \tau}$$

PEOD

i) *Texture* r.v. resampling according to its instantaneous *a posteriori* pdf : $\tilde{\tau}_{i=1}^{N} \sim p(\tau/\mathbf{r}_i)$

ii) Moments estimation : $\widehat{\mu}_n = N^{-1} \sum_{i=1}^{N} \tilde{\tau}_i^n$

iii) Computation of the M Padé coefficients $\{\alpha_k\}$ et $\{\lambda_k\}$ from the estimated moments

◇ **With a Monte-Carlo Bayesian estimator :**

$$p(\tau) = \int_{\mathbb{R}^m} p(\tau/\mathbf{r}) \, p(\mathbf{r}) \, d\mathbf{r} \quad \overset{\text{MC}}{\Longrightarrow} \quad \widehat{p}_N(\tau) = \frac{1}{N} \sum_{i=1}^{N} p(\tau/\mathbf{r}_i)$$

Two new detection strategies come from these propositions

ETIS ONERA

2 - Non-Gaussian environment characterization

2.5 -*Texture* pdf estimation

◇ **With a Padé approximation :**

$$\widehat{p}(\tau) = \sum_{k=1}^{M} \lambda_k \, e^{-\alpha_k \, \tau}$$

PEOD

i) *Texture* r.v. resampling according to its instantaneous *a posteriori* pdf : $\tilde{\tau}_{i=1}^{N} \sim p(\tau/\mathbf{r}_i)$

ii) Moments estimation : $\widehat{\mu}_n = N^{-1} \sum_{i=1}^{N} \tilde{\tau}_i^n$

iii) Computation of the M Padé coefficients $\{\alpha_k\}$ et $\{\lambda_k\}$ from the estimated moments

◇ **With a Monte-Carlo Bayesian estimator :**

BORD

$$p(\tau) = \int_{\mathbb{R}^m} p(\tau/\mathbf{r}) \, p(\mathbf{r}) \, d\mathbf{r} \quad \overset{\text{MC}}{\Longrightarrow} \quad \widehat{p}_N(\tau) = \frac{1}{N} \sum_{i=1}^{N} p(\tau/\mathbf{r}_i)$$

Two new detection strategies come from these propositions

3 - Optimum detection strategies derivation

3.1 - Coherent detection theory applied to SIRV

◇ **Each observation $\mathbf{y}_{obs}$ is a SIRV of size m :**

$$\mathbf{y}_{obs} = \zeta\,\mathbf{s} + \mathbf{b} = \zeta\,\mathbf{s} + \mathbf{x}\sqrt{\tau}\begin{cases} \mathbf{b} & : \quad \text{SIRV} \;:\; p_b(\mathbf{b}) = p_{\mathsf{b}}(\sqrt{\tau}\,x_1, \cdots, \sqrt{\tau}\,x_m) \\ \mathbf{s} & = \quad \mathbf{s}(A, \underline{\theta}) \;:\; \text{Target Signal with amplitude } A \text{ and where } \underline{\theta} = (f_d, \tau, \cdots) \end{cases}$$

◇ **Hypothesis** $\begin{cases} H_0 & : \quad \zeta = 0 \quad \Rightarrow \quad p(\mathbf{y}_{obs} \,/\, H_0) = p_{\mathsf{b}}(\mathbf{y}_{obs}) \\ H_1 & : \quad \zeta = 1 \quad \Rightarrow \quad p(\mathbf{y}_{obs} \,/\, H_1) = p_{\mathsf{b}}(\mathbf{y}_{obs} - \mathbf{s}) \end{cases}$

◇ **Possible errors** $\begin{cases} \text{To Choose } H_0 \text{ if } H_1 & : \quad \textbf{Non-detection :}\ 1 - P_d \\ \text{To Choose } H_1 \text{ si } H_0 & : \quad \textbf{False Alarm :}\ P_{fa} \end{cases}$

3 - Optimum detection strategies derivation

Decision Criterion

◇ **Neymann-Pearson criterion : To fix P_{fa} and to optimize P_d**

◇ **Optimal decision :** The likelihood ratio test : $\Lambda(\mathbf{y}_{obs}) = \dfrac{p_{\mathbf{b}}(\mathbf{y}_{obs} - \mathbf{s} \,/\, H_0)}{p_{\mathbf{b}}(\mathbf{y}_{obs} \,/\, H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$

◇ **Case where the target amplitude is unknown :** The Generalized likelihood ratio

$$\mathbf{s}(A,\theta) = A\,\mathbf{p} \begin{cases} \widehat{A}_{mv} = \dfrac{|\mathbf{p}^\dagger\,\mathbf{M}^{-1}\,\mathbf{y}_{obs}|^2}{\mathbf{p}^\dagger\,\mathbf{M}^{-1}\,\mathbf{p}} \\ \Lambda(\mathbf{y}_{obs}, \widehat{A}_{mv}) = \dfrac{\displaystyle\int_0^{+\infty} \tau^{-m} \exp\left(-\frac{q_1(\mathbf{y}_{obs})}{2\,\tau}\right) p(\tau)\, d\tau}{\displaystyle\int_0^{+\infty} \tau^{-m} \exp\left(-\frac{q_0(\mathbf{y}_{obs})}{2\,\tau}\right) p(\tau)\, d\tau} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \end{cases}$$

All depend on the *texture* pdf of the SIRV

3 - Optimum detection strategies derivation

3.2 - PEOD - Padé Estimated Optimum Detector

◇ *Texture* **pdf estimated with Padé (from N references) :**

$$p(\tau) \longleftrightarrow \widehat{p}(\tau) = \sum_{k=1}^{M} \lambda_k \, e^{-\alpha_k \, \tau}$$

◇ **NSCM Estimation (from N references) :**

$$\widehat{\mathbf{M}} = \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{r}_i \, \mathbf{r}_i^\dagger}{\mathbf{r}_i^\dagger \, \mathbf{r}_i}$$

PEOD ([Jay01]) :

$$\left(\frac{q_1(\mathbf{y}_{obs})}{q_0(\mathbf{y}_{obs})} \right)^{\frac{1-m}{2}} \frac{\displaystyle\sum_{k=1}^{M} \lambda_k \, (\alpha_k)^{\frac{m-1}{2}} \, K_{1-m}\left(\sqrt{2\, \alpha_k \, q_1(\mathbf{y}_{obs})} \right)}{\displaystyle\sum_{k=1}^{M} \lambda_k \, (\alpha_k)^{\frac{m-1}{2}} \, K_{1-m}\left(\sqrt{2\, \alpha_k \, q_0(\mathbf{y}_{obs})} \right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

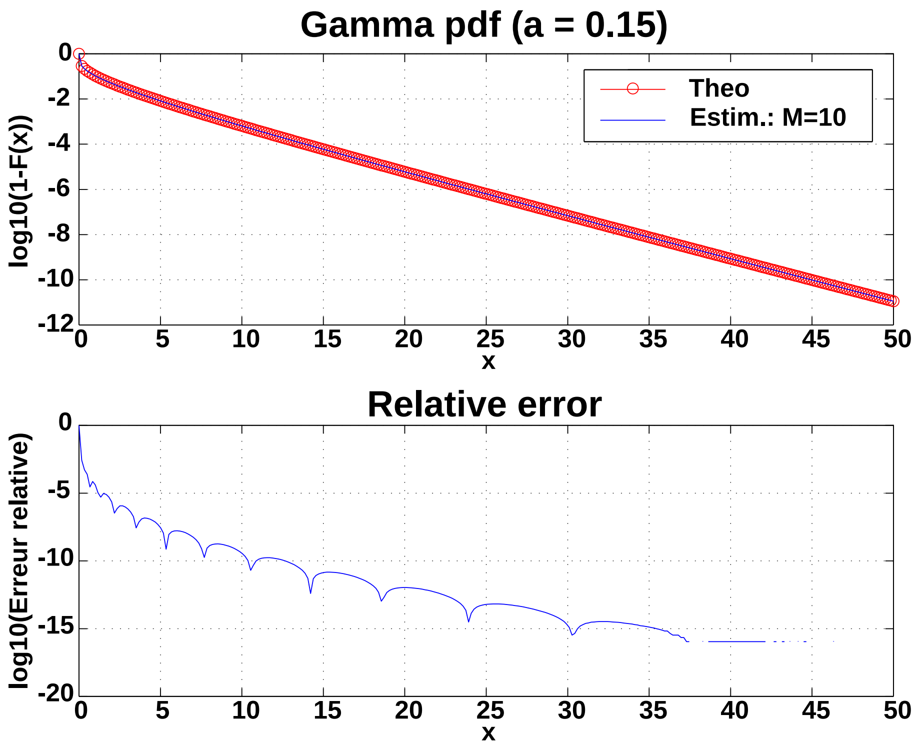
$q_0(\mathbf{y}_{obs}) = \mathbf{y}_{obs}^\dagger \, \widehat{\mathbf{M}}^{-1} \, \mathbf{y}_{obs}$	$q_1(\mathbf{y}_{obs}) = \mathbf{y}_{obs}^\dagger \, \widehat{\mathbf{M}}^{-1} \, \mathbf{y}_{obs} - \dfrac{\lvert \mathbf{p}^\dagger \, \widehat{\mathbf{M}}^{-1} \, \mathbf{y}_{obs} \rvert^2}{\mathbf{p}^\dagger \, \widehat{\mathbf{M}}^{-1} \, \mathbf{p}}$	F.A. $\equiv q_0(\mathbf{y}) - q_1(\mathbf{y})$

3.2 - PEOD - Padé Estimated Optimum Detector

Relevance of the Padé approximation

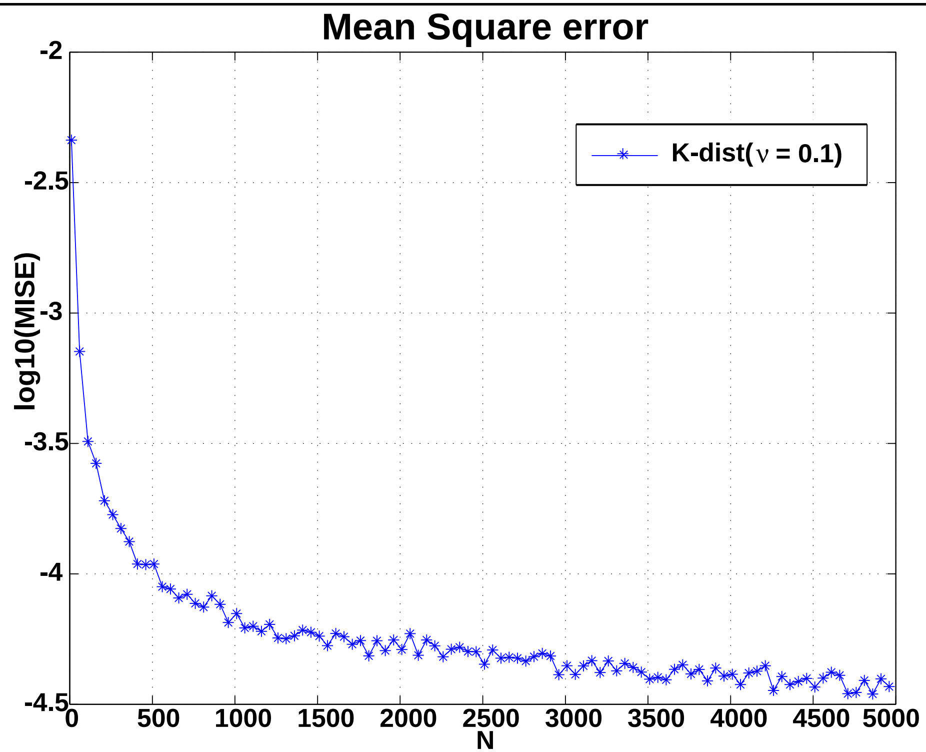
From the theoritical moments

Gamma pdf - $\log_{10}(1 - F(x))$



From the estimated moments

K-dist ($\nu = 0.1$) - MSE as a function of N

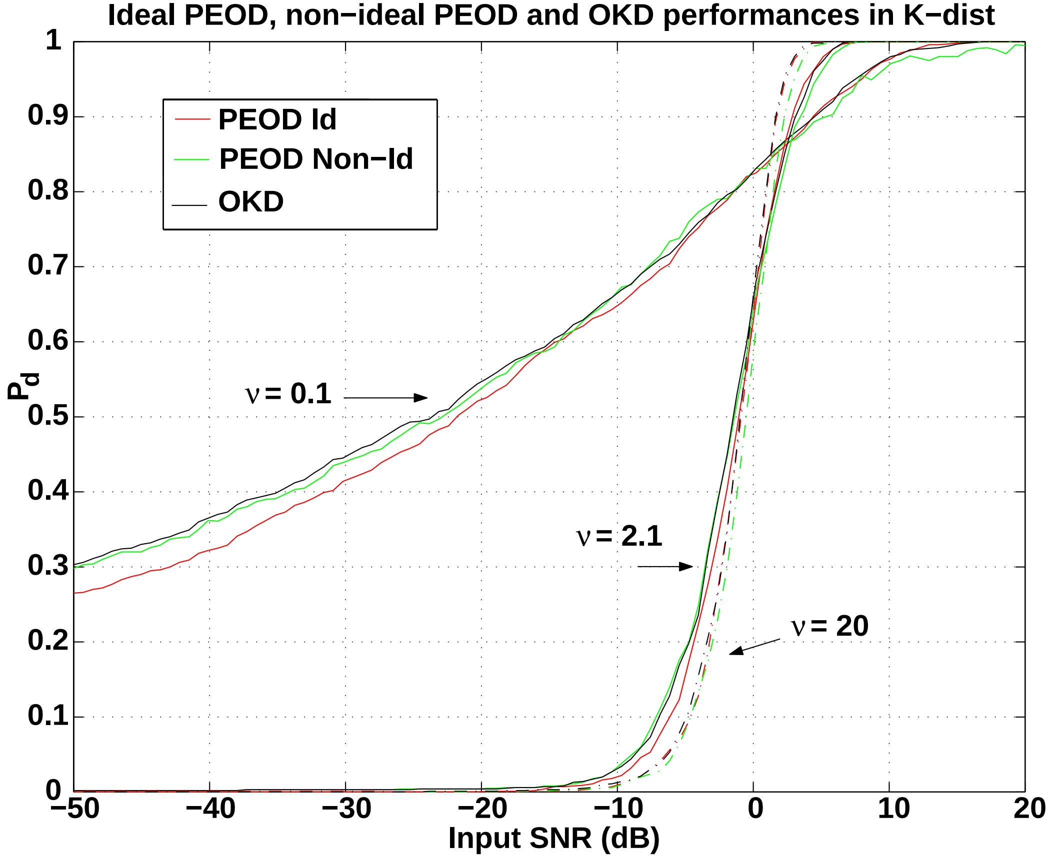


3.2 - Le PEOD - Padé Estimated Optimum Detector

Detection performances - $m = 10$

Comparison "ideal" PEOD / "non-ideal" PEOD - $P_{fa} = 10^{-3}$

K-dist. $M_{\nu=0.1} = 9, M_{\nu=2.1} = 6, M_{\nu=20} = 3$



3 - Optimum detection strategies derivation

3.3 - BORD - Bayesian Optimum Radar Detector

◇ **MC-bayesian estimate of** $p(\tau)$ **:**

$$\widehat{p}_N(\tau) = \frac{\tau^{-m-1}}{2^m\,\Gamma(m)\,N} \sum_{i=1}^{N} \left(\mathbf{r}_i^\dagger\,\widehat{\mathbf{M}}^{-1}\,\mathbf{r}_i\right)^m \exp\left(-\frac{\mathbf{r}_i^\dagger\,\widehat{\mathbf{M}}^{-1}\,\mathbf{r}_i}{2\,\tau}\right)$$

◇ **BORD depends directly on the received data (Reference and Observations)**

BORD ([Jay02a,b]) :

$$\frac{\displaystyle\sum_{i=1}^{N}\left[\frac{\mathbf{r}_i^\dagger\,\widehat{\mathbf{M}}^{-1}\,\mathbf{r}_i}{(q_1(\mathbf{y}_{obs})+\mathbf{r}_i^\dagger\,\widehat{\mathbf{M}}^{-1}\,\mathbf{r}_i)^2}\right]^m}{\displaystyle\sum_{i=1}^{N}\left[\frac{\mathbf{r}_i^\dagger\,\widehat{\mathbf{M}}^{-1}\,\mathbf{r}_i}{(q_0(\mathbf{y}_{obs})+\mathbf{r}_i^\dagger\,\widehat{\mathbf{M}}^{-1}\,\mathbf{r}_i)^2}\right]^m} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

$$q_0(\mathbf{y}_{obs}) = \mathbf{y}_{obs}^\dagger\,\widehat{\mathbf{M}}^{-1}\,\mathbf{y}_{obs} \qquad \widehat{\mathbf{M}} = \frac{m}{N}\sum_{i=1}^{N}\frac{\mathbf{r}_i\,\mathbf{r}_i^\dagger}{\mathbf{r}_i^\dagger\,\mathbf{r}_i} \qquad q_1(\mathbf{y}_{obs}) = \mathbf{y}_{obs}^\dagger\,\widehat{\mathbf{M}}^{-1}\,\mathbf{y}_{obs} - \frac{|\mathbf{p}^\dagger\,\widehat{\mathbf{M}}^{-1}\,\mathbf{y}_{obs}|^2}{\mathbf{p}^\dagger\,\widehat{\mathbf{M}}^{-1}\,\mathbf{p}}$$

3 - Optimum detection strategies derivation

Expected characteristics for a detector

◇ **Detection threshold computation**

$$P_{fa} = \mathbb{P}\left(\int D(\mathbf{y}/\tau) p(\tau)\, d\tau \overset{H_0}{>} \eta \right)$$

◇ **If independance with respect to the texture pdf :**

CFAR Property (Constant False Alarm Rate) with respect to the texture pdf

$\Rightarrow$ **Independent of the environment statistics**

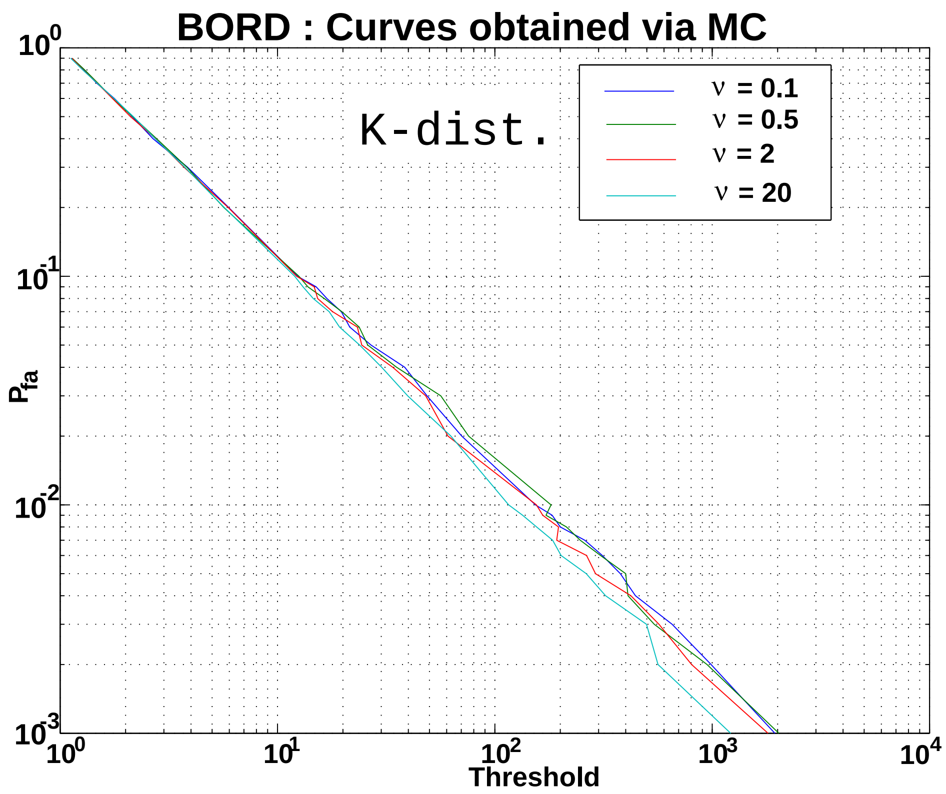
◇ **Adaptability to the environment statistics, Optimality**

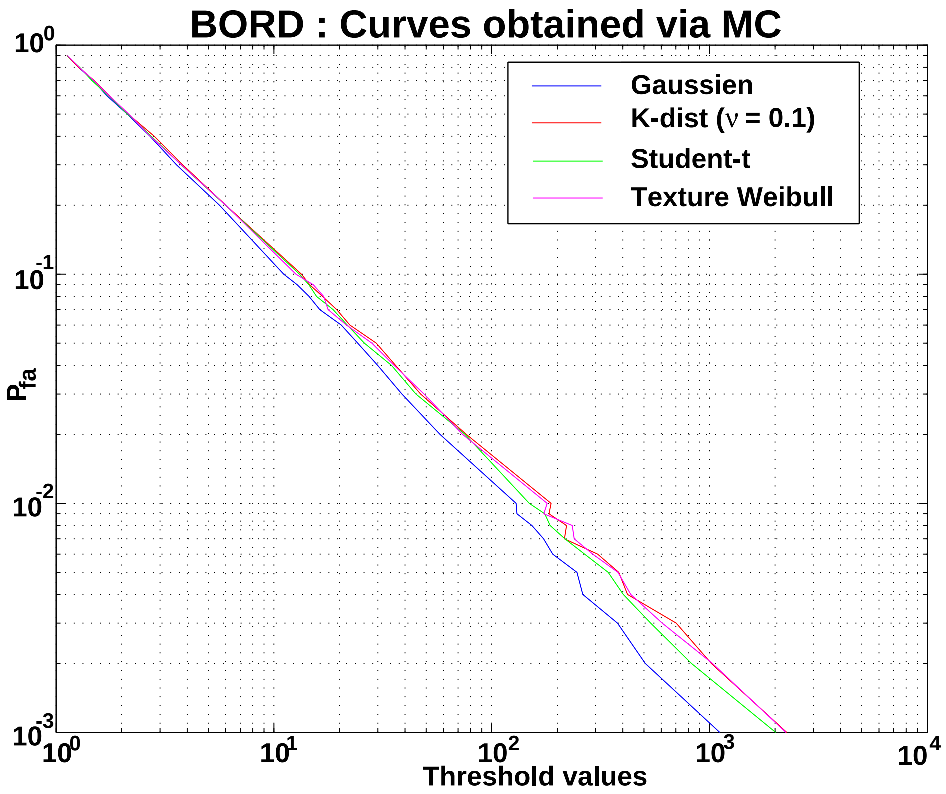
◇ **Computation of the detection test pdf**

3.3 - BORD - Bayesian Optimum Radar Detector

Operational characteristics

- **Adaptive detector**
- **Keep the CFAR property with respect to the *texture* pdf :**





3.3 - BORD - Bayesian Optimum Radar Detector

Asymptotical behaviour

◇ **Asymptotical means : the number N of reference data goes to ∞**

$$\widehat{\mathbf{M}} \xrightarrow{p.s.} \mathbf{M} \qquad \textbf{ET} \qquad \textbf{BORD} \xrightarrow{Loi} \left(\frac{q_0(\mathbf{y}_{obs})}{q_1(\mathbf{y}_{obs})}\right)^m$$

◇ **The "Asymptotic BORD" coincide with ALQ Detector** ([Gini97a])

◇ **The "Asymptotic BORD" is CFAR with respect to the *texture* pdf :**

$$\textbf{"Asymptotic BORD"} \text{ equivalent to } \left(\frac{\overbrace{q_0(\mathbf{x}_{obs})}^{\text{F.Q.Gauss}}}{q_1\underbrace{(\mathbf{x}_{obs})}_{\text{Gauss}}}\right)^m$$

The "Asymptotic BORD"

Pdf of "Asymptotic BORD"

◇ The "Asymptotic BORD" **depends only on the *speckle*** of the observations (complex Gaussian vectors)

"Asymptotic BORD" pdf

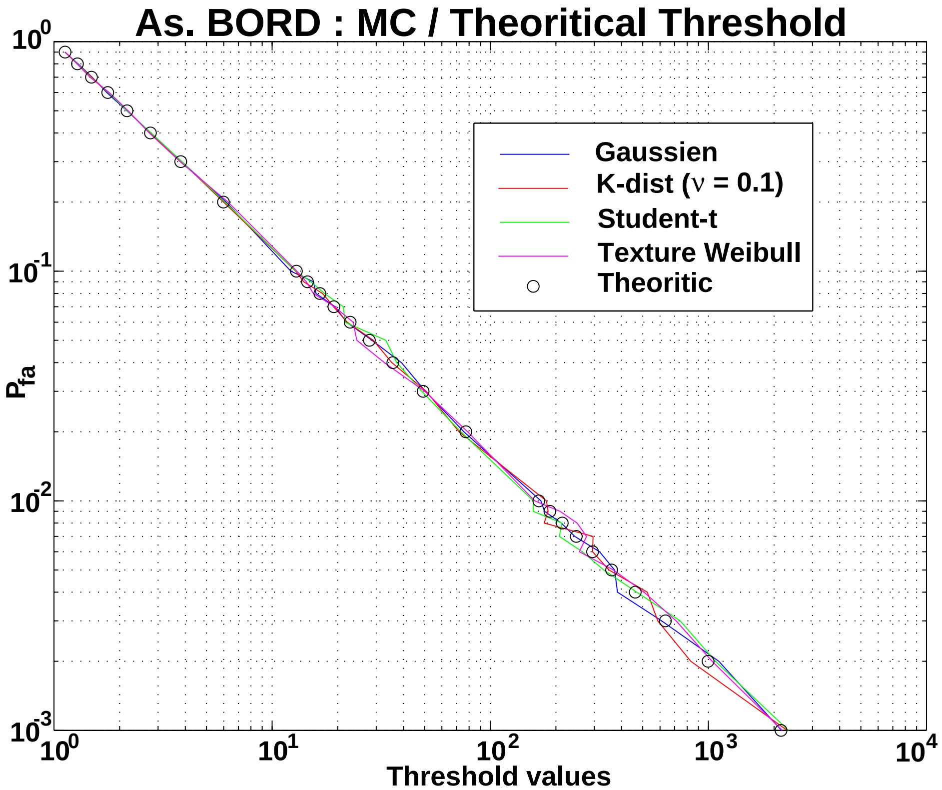
- **The "Asymptotic BORD" test is equivalent to :**

$$\frac{\left|\mathbf{p}^{\dagger}\,\mathbf{M}^{-1}\,\mathbf{x}_{obs}\right|^{2}}{(\mathbf{p}^{\dagger}\,\mathbf{M}^{-1}\,\mathbf{p})\,(\mathbf{x}_{obs}^{\dagger}\,\mathbf{M}^{-1}\,\mathbf{x}_{obs})}\ \underset{H_0}{\overset{H_1}{\gtrless}}\ \frac{\sqrt[m]{\eta}-1}{\sqrt[m]{\eta}}$$

- **Cochran's theorem (if M non singular) :**

$$\eta = \mathbf{P}_{\mathbf{fa}}^{\frac{\mathbf{m}}{\mathbf{1-m}}}$$

Validation of η from simulations

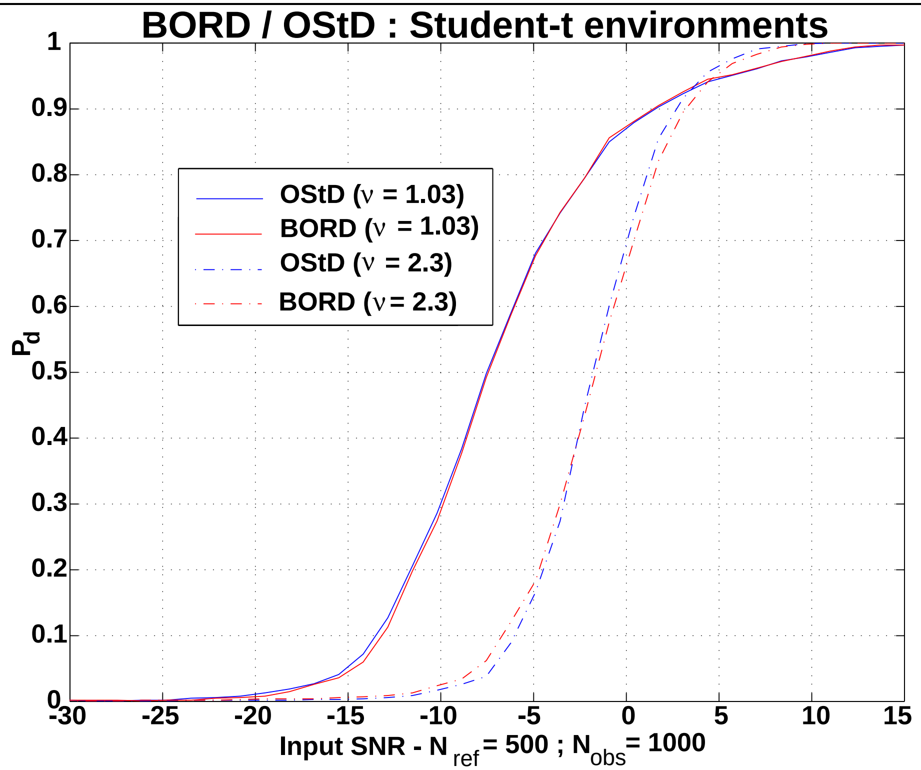


3.3 - BORD - Bayesian Optimum Radar Detector

Detection performances - $m = 10$ - $\rho_1 \approx 0.1$

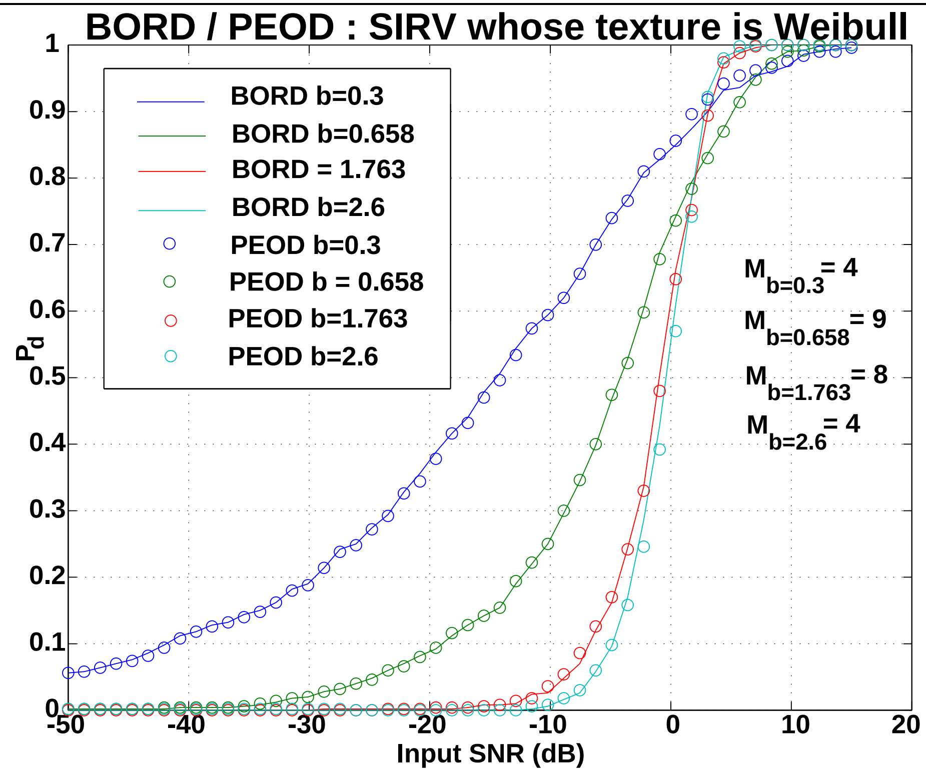
Comparison with OStD - $P_{fa} = 10^{-3}$

Student-t $\nu = 1.03$ et $\nu = 2.3$ - $N_{ref} = 500$



Comparison "ideal" PEOD - $P_{fa} = 10^{-3}$

Env. whose *texture* is Weibull ($b = 0.3, 0.658, 1.763, 2.6$)

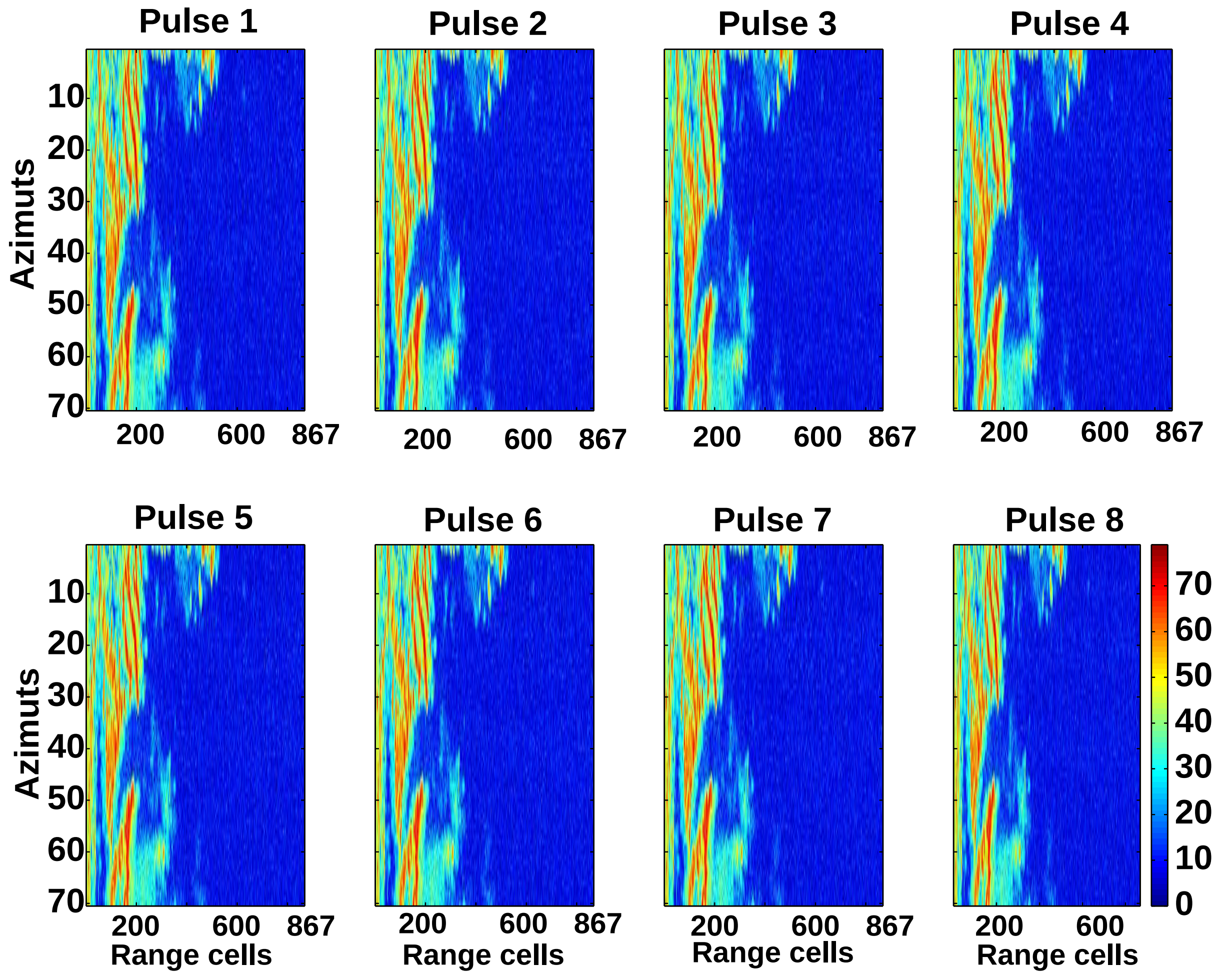


4 - The BORD, applied to experimental data

Data description

Data given by Thalès TAD

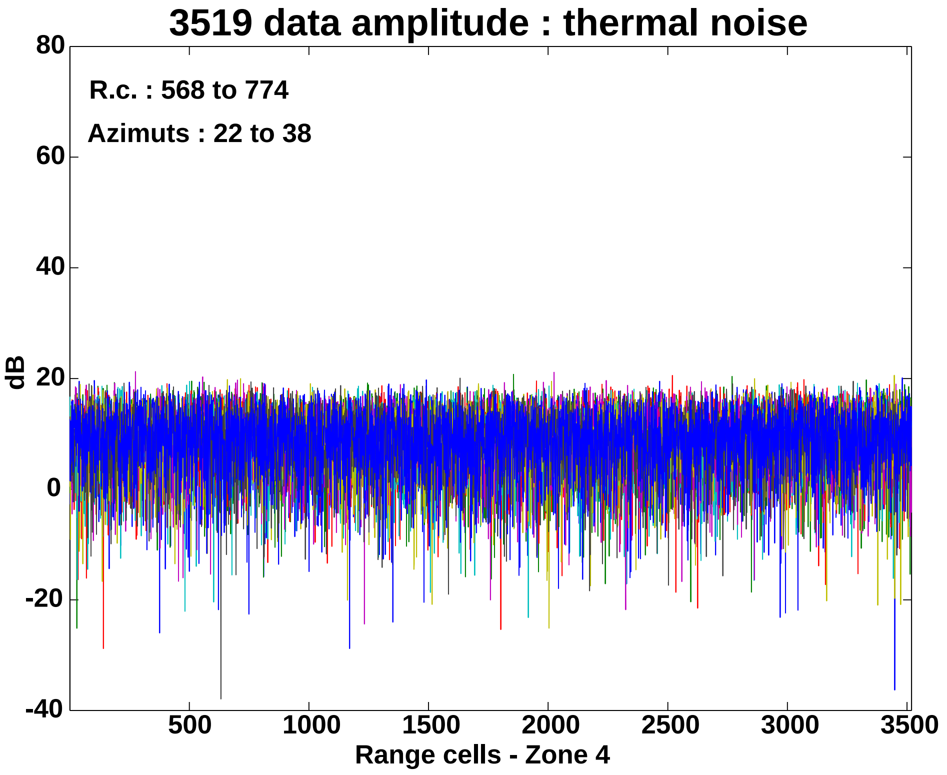
- Ground clutter (low grazing angle)
- 70 azimut
- 868 range cells of 60 m.
- 8 pulses transmitted
- Radioelectric horizon to ∼ 15 kms (∼ rc 250)

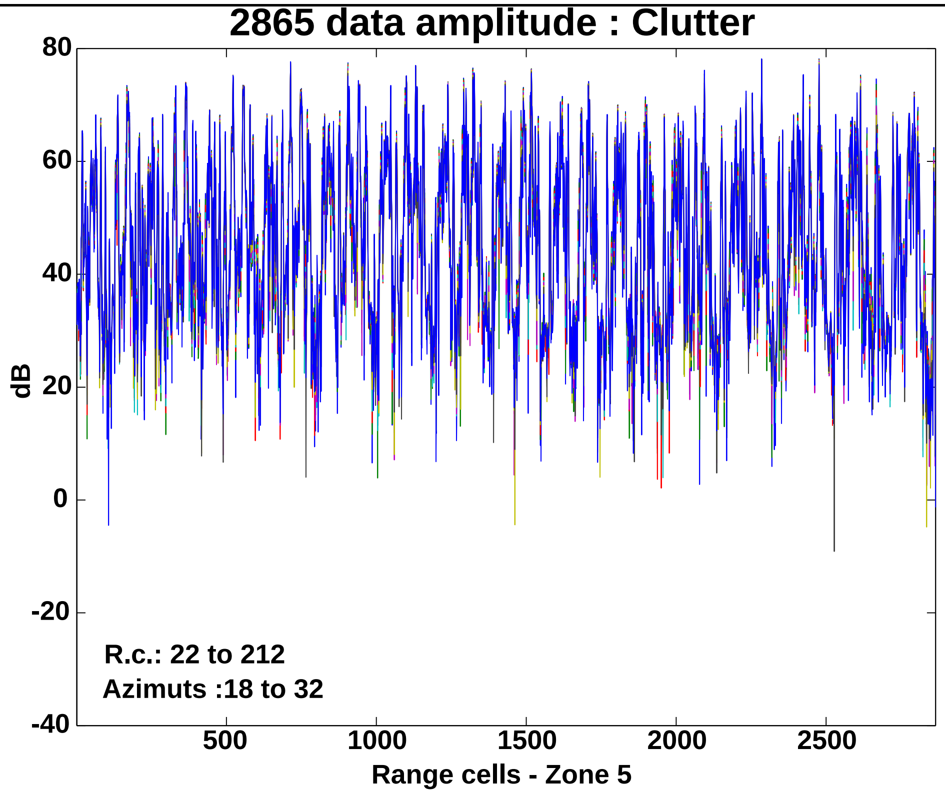


4 - The BORD, applied to experimental data

Data Amplitudes

Thermal noise	Clutter

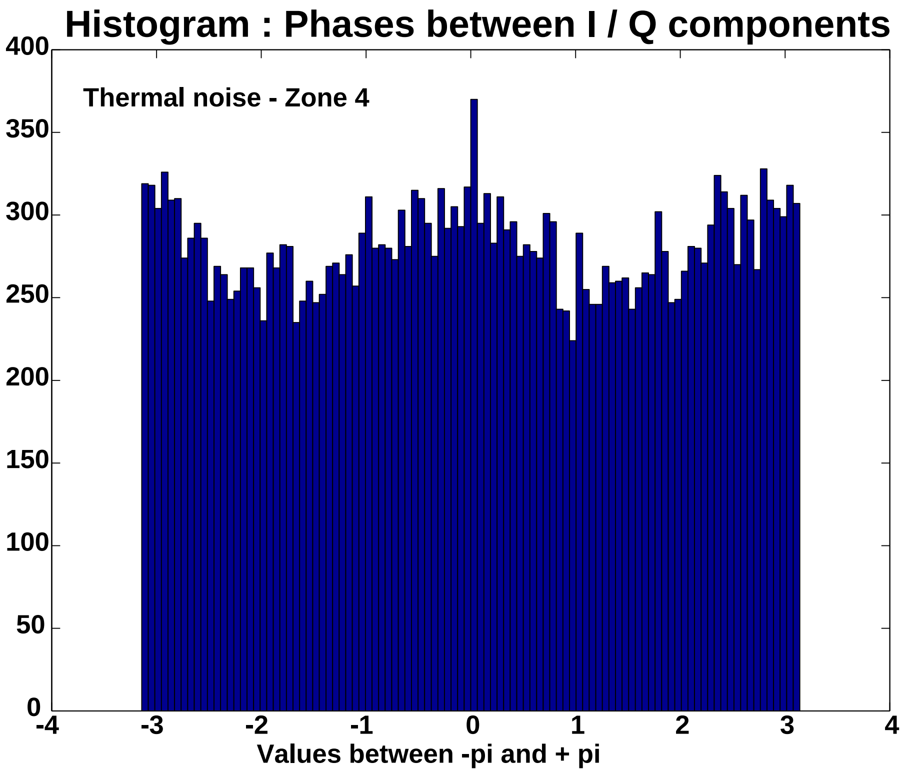


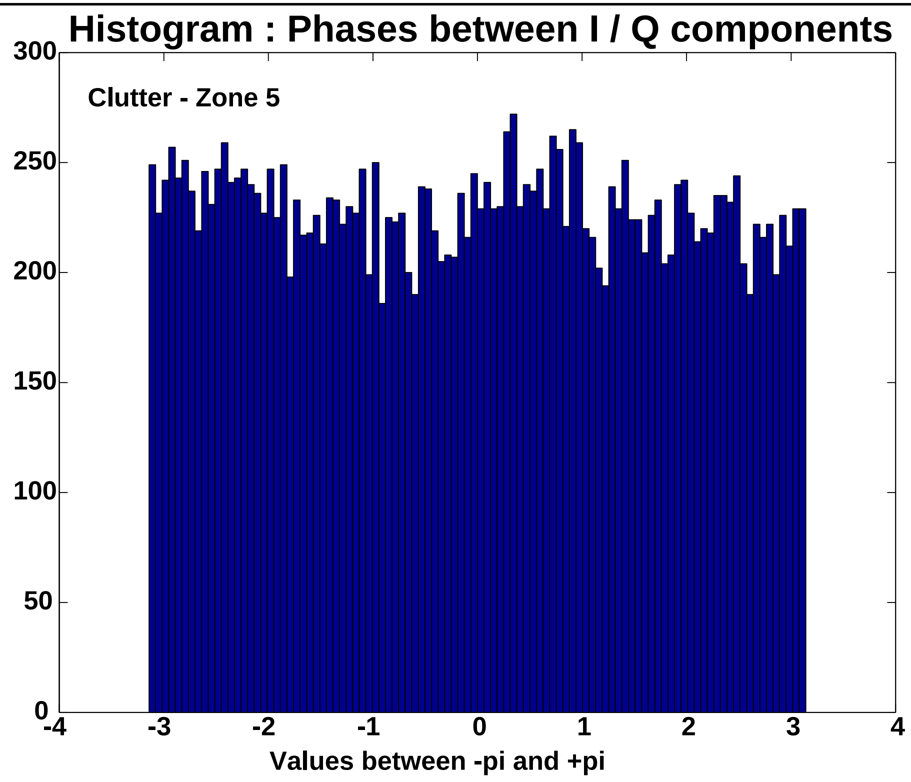


4 - The BORD, applied to experimental data

Data phases between I and Q components : Assumption uniformely distributed : verified

Thermal noise zone	**Clutter zone**
8×1617 **phases**	8×1441 phases





4 - The BORD, applied to experimental data

Data Correlation

Thermal noise : $\widehat{\rho}_1 = 0.0855$

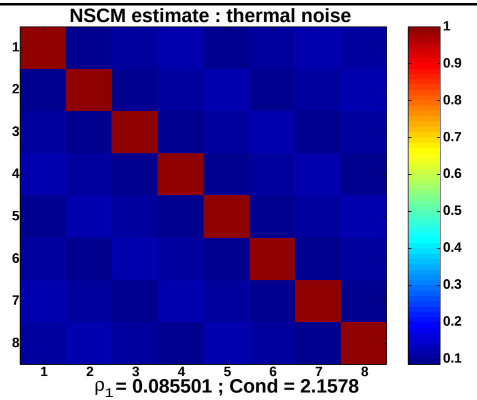
$$Cond(\widehat{\mathbf{M}}) = 2.1578$$

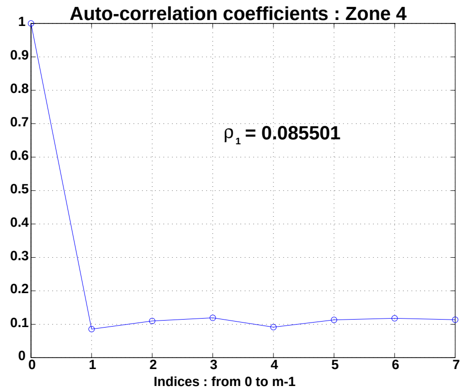
Matrix well conditioned

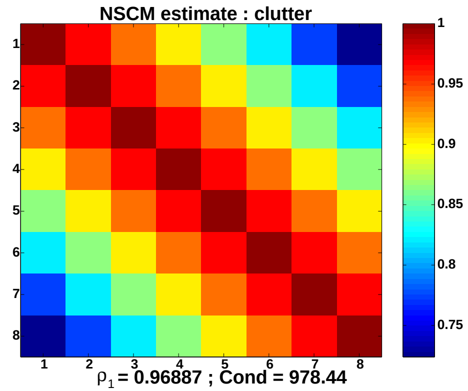
Clutter : $\widehat{\rho}_1 = 0.96887$

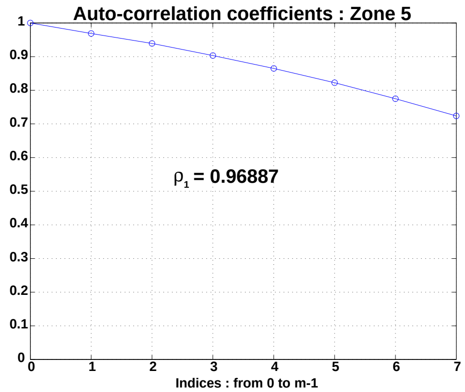
$$Cond(\widehat{\mathbf{M}}) = 978.44$$

Matrix bad conditioned







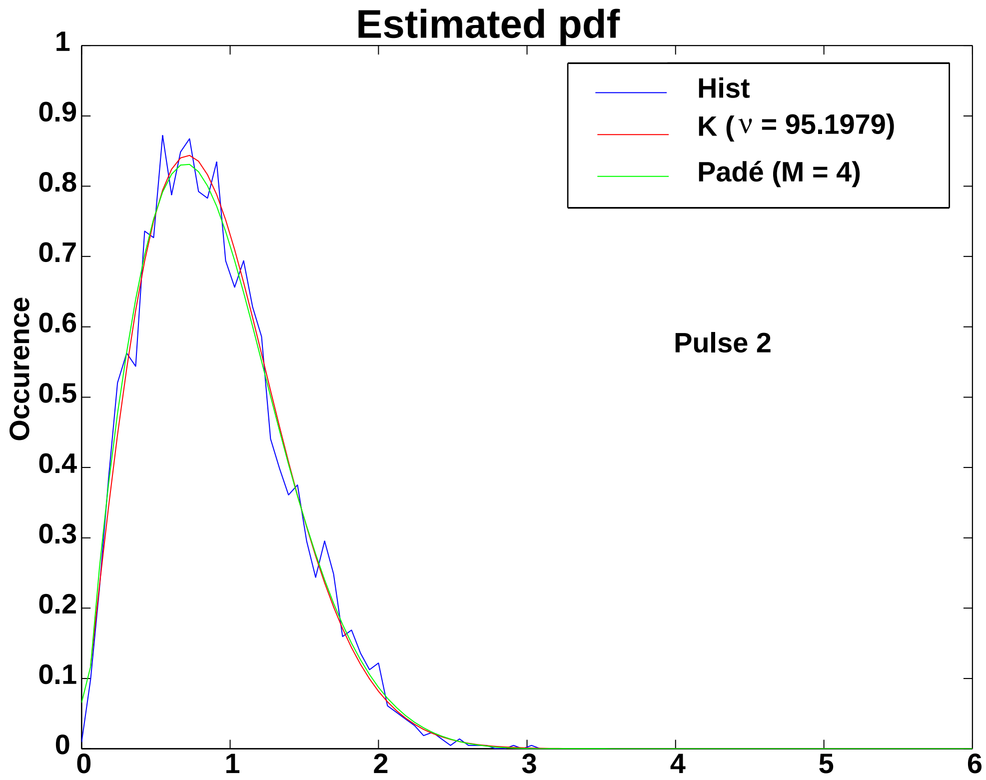


4 - The BORD, applied to experimental data

Comparison with K-distribution

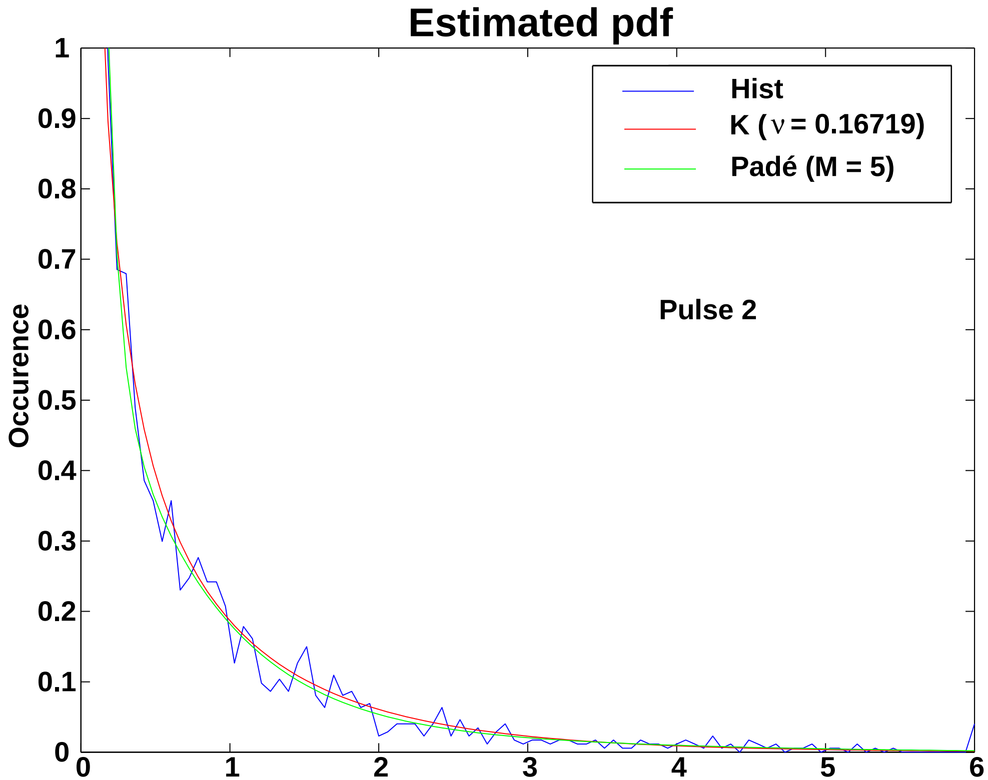
Thermal noise (Zone 4)

Histogram / Padé ($M = 4$ ou $M = 5$)



Clutter (Zone 5)

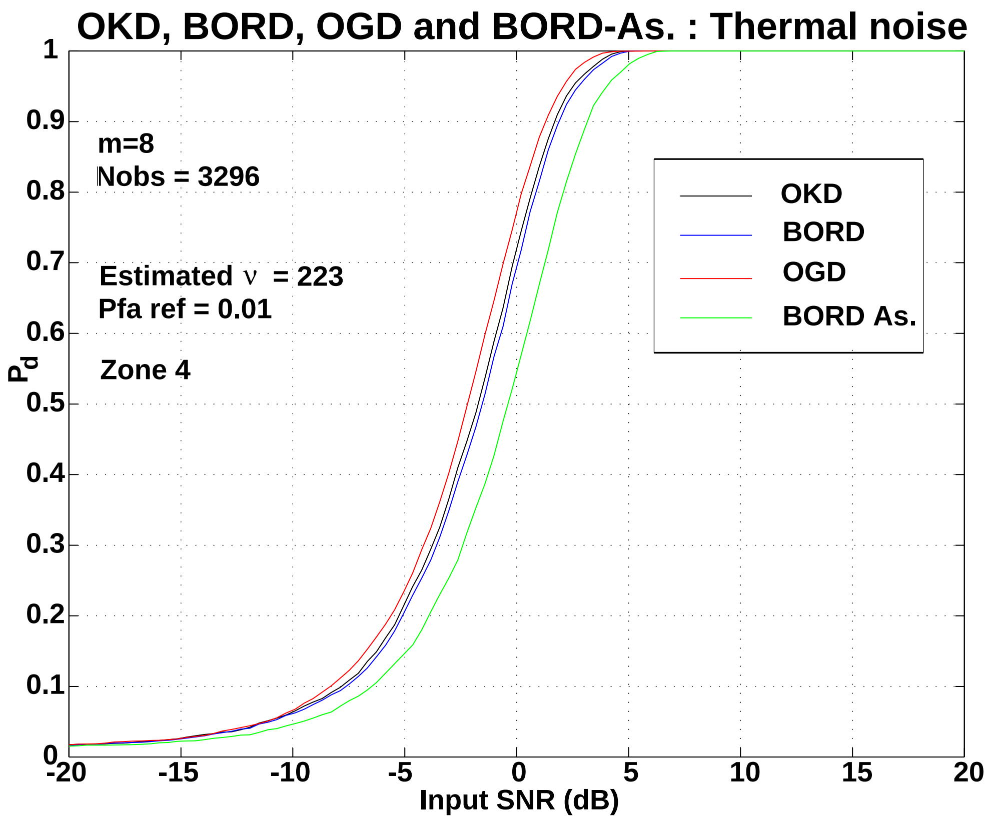
Histogram / Padé ($M = 5$ ou $M = 6$)



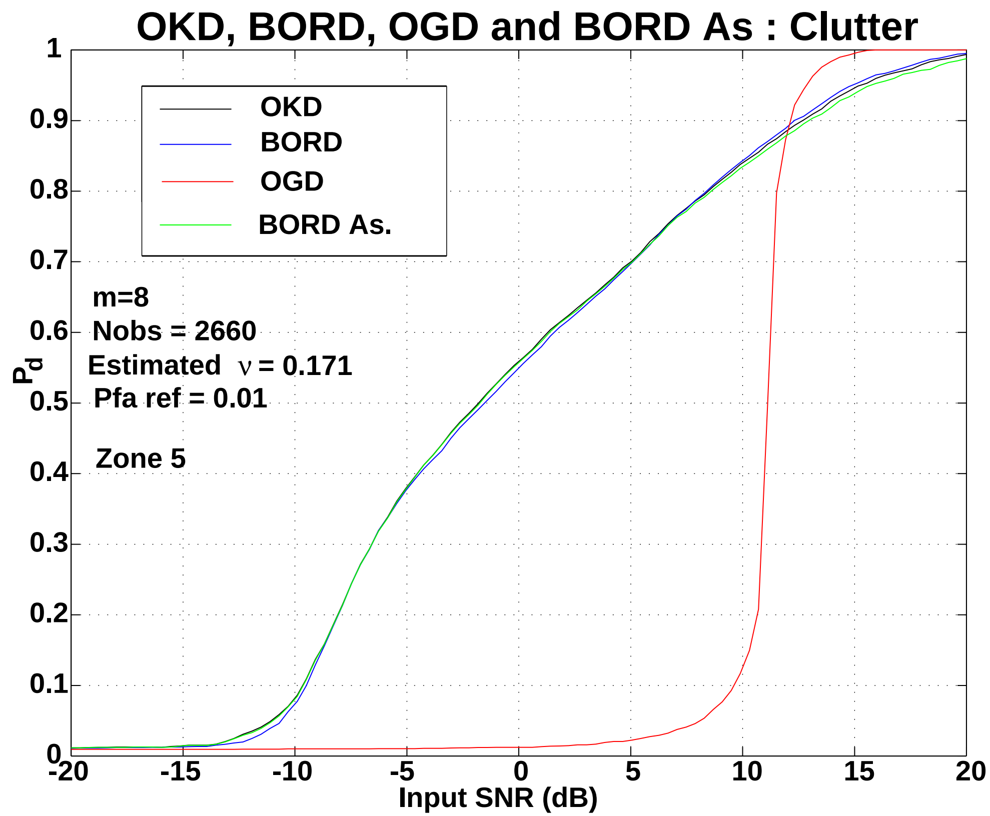
4 - The BORD, applied to experimental data

Detection performances

Thermal noise : Zone 4



Clutter : Zone 5



5 - Conclusions

Contribution of my thesis : Bayesian approach to characterize SIRV environment

---

- **The "ideal" PEOD :**Like a **"Reference"** Detector if the optimum detector is unknown,
- **The "non-ideal" adaptive PEOD : Robust** if a large number of references is available,
- **The BORD : Adaptive, CFAR** / *texture* pdf, **Optimal**, Asymptotically convergent,
- **The Asymptotic BORD :** coincide with ALQ, test pdf identified, directely linked to M.F.,
  Adaptive, CFAR / *texture* pdf, **Theoritical performances**, **Immediate implementation,**
- **Experimentally : Validation** of the SIRV models and of the related detectors.

BORD generalises all the existing SIRV detectors : Adaptive to the environment

Work included in the PEA TRA conducted with TAD (Thalès Air Defence)

Outlook

◇ **Theoritical studies :**

- Adaptive empirical *prior*,
- Asymptotic BORD pdf : case where the matrix is singular, random (Wishart)

Presentiment : expression dependent on the matrix rank

◇ **Numerical studies :**

- Use of numerical methods for the regularisation of the covariance matrix, in order to reach lower values of P_{fa} (Importance sampling, weighted sampling),

◇ **Experimental studies :**

- Extension of the work to sea clutter, to sea/ground transition, to fluctuating targets, to SAR imaging for segmentation / classification of ground zones...
- Implementation of BORD and Asymptotic BORD using a small number of references (like CFAR using small windows).