Applications of the Notions of Weak Dependence in Statistic

PhD supervised by Paul Doukhan & Jean Marc Bardet

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Outline

Time Series

Let $(X_t)_{t\in\mathbb{Z}}$ $(\in E)$ be a stationary time series. We observe $(X_t)_{1\leq t\leq n}$.

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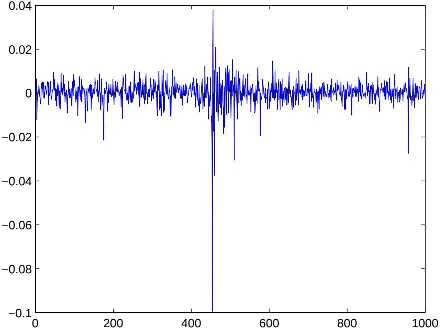

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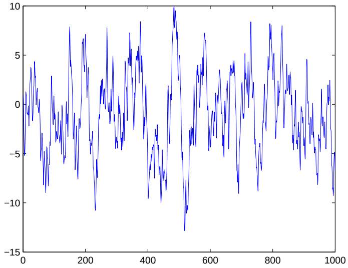

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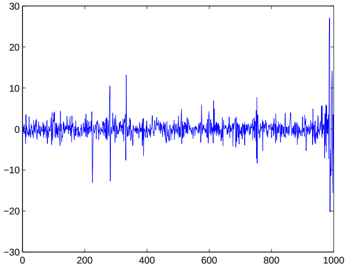

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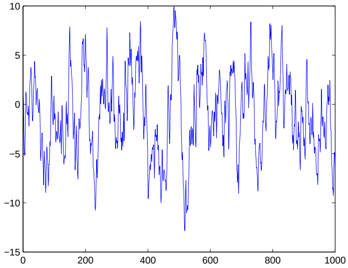

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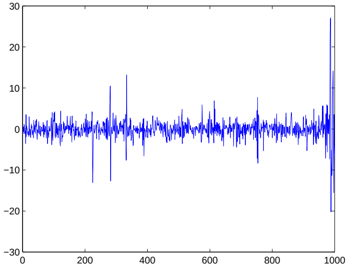

GARCH(1, 1)

- TGARCH(1,1), Zakoïan (1994) $$\begin{cases} X_t &= \sigma_t \xi_t, \\ \sigma_t &= b + b_1^+ \max(X_{t-1}, 0) - b_j^- \min(X_{t-1}, 0) + c_1 \sigma_{t-1}. \end{cases}$$

Causality

ARMA(1, 1) $\quad X_t = b + b_1 X_{t-1} + c_1 \xi_{t-1} + \xi_t$

The stationary solution could be expressed as MA(∞) $\quad X_t = \sum_{j \geq 0} d_j \xi_{t-j}.$

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Linear Models: A noncausal generalization is $\quad X_t = \sum_{j \in \mathbb{Z}} d_j \xi_{t-j}$.

It appears as solutions of NonCausal AR models

$$X_t = b + b_1 X_{t-1} + b_{-1} X_{t+1} + \xi_t.$$

Outline

1. Time Series Analysis
2. Weak Dependence Notions
3. Causal Case
   - Examples
   - Applications
4. Noncausal Case
   - Examples
   - Applications

Independence

Here, the main question is how to weaken the relation

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B) \quad ?$$

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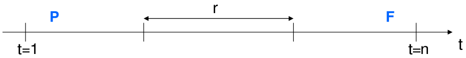

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Bradley (1983), Doukhan (1994), Rio (2000), Shao (1993)

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$$s_1\le\cdots\le s_u,\, t_1\le\cdots\le t_v, \qquad r=t_1-s_u.$$

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Example of non-mixing models

$$X_t=\frac{1}{2}\left(X_{t-1}+\xi_t\right), \xi_t\sim b\left(\frac{1}{2}\right), iid \text{ (Andrews-Rosenblatt, 1984).}$$

Noncausal Coefficients Doukhan & Louhichi (1999) Doukhan & W. (2007)

$$\epsilon(P,F) = \sup_{\|f\|_\infty, \|g\|_\infty \le 1} \frac{|\mathrm{Cov}(f(P), g(F))|}{\Psi(u, v, \mathrm{Lip}\, f, \mathrm{Lip}\, g)},$$

where f, g are Lipschitz functions and

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First Examples

- $$X_t = H((\xi_{t-j})_{j\in\mathbb{Z}}), \qquad \text{with } (\xi_t)_{t\in\mathbb{Z}} \ (\in \mathbb{R}) \text{ iid } (\xi \sim \mu).$$

  If $H \in \mathbb{L}^1(\mathbb{R}^{\mathbb{Z}}, \mu)$ and for $(\zeta_t)_{t\in\mathbb{Z}}$ and $(\xi_t)_{t\in\mathbb{Z}}$ iid,

  $$\mathbb{E}\left|H\left(\xi_j, j\in\mathbb{Z}\right) - H\left(\xi_j \mathbb{1}_{|j|<r} + \zeta_j \mathbb{1}_{|j|\geq r}, j\in\mathbb{Z}\right)\right| \leq \delta_r \downarrow 0 \ (r\uparrow\infty)$$

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For the Linear Model $X_t = \sum_{j\in\mathbb{Z}} d_j\xi_{t-j}$, $\qquad \eta(r)\leq 2\mathbb{E}|\xi_0|\sum_{|j|\geq r} d_j$.

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- If $(X_t)_{t\in\mathbb{Z}}$ is Gaussian or associated $(\mathrm{Cov}\,(f(P),g(F))\geq 0$ if $f,g\uparrow)$ then

  $$\lambda(r)\leq\sup_{j\geq r}|\mathrm{Cov}(X_0,X_j)|.$$

Causal Coefficient (Coupling)

Dedecker & Prieur (2004)

Here $P = \sigma(X_t, X_{t-1}, \ldots)$ and $X \subset F = \{X_{t+r}, X_{t+r+1}, \ldots\}$.

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Coupling: for Y distributed as X and independent of P then

$$\tau(P, X) \le \mathbb{E}\|X - Y\|.$$

There exists Y^* such that $\tau(P, X) = \mathbb{E}\|X - Y^*\|.$

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Processes with Infinite Memory Doukhan & W. (submitted)

$$X_t = F(X_{t-1}, X_{t-2}, X_{t-3}, \dots ; \xi_t) \text{ a.s.}$$

$(\xi_t)_{t\in\mathbb{Z}}$ iid, $F : E^{\mathbb{N}} \times E' \to E$.

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Existence, Moment and Weak Dependence

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with $a = \sum_{j=1}^{\infty} a_j < 1$ then existence in $\mathbb{L}^m$ of a (essentially) unique stationarity solution and weak dependence hold with:

$$\tau(r) \leq 2\frac{\mu_1}{1-a} \inf_{N>0} \left(\sum_{j\geq N} a_j + a^{-\frac{r}{N}} \right) \to_{r\to\infty} 0.$$

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- ARMA(1, 1) $\quad X_t = b + b_1 X_{t-1} + c_1 \xi_{t-1} + \xi_t.$

  If $|b_1|, |c_1| < 1$, $\|\xi_0\|_m < \infty$, existence in $\mathbb{L}^m$,
  and $\quad \tau(r) \le C e^{-\sqrt{\alpha\beta r}}, \quad \begin{cases} \alpha = \ln(1 + c_1) - \ln(b_1 + c_1), \\ \beta = - \ln(c_1). \end{cases}$

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  If $\|\xi_0\|_m\sqrt{b_1} + \sqrt{c_1} < 1$, existence in $\mathbb{L}^m$
  and $\quad \tau(r) \leq Ce^{-\sqrt{\alpha\beta r}}$, $\quad \begin{cases} \alpha &= \ln(1-\sqrt{c_1}) - \ln(\|\xi_0\|_m\sqrt{b_1}), \\ \beta &= -\ln(c_1)/2. \end{cases}$

General NonMarkov Nonlinear Models

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- LARCH(∞) Process, Doukhan *et al.* (2006)

$$X_t = \xi_t \Big(a + \sum_{j \geq 1} b_j X_{t-j} \Big) \text{ a.s.}$$

exists, belongs in $\mathbb{L}^m$ and is τ-weakly dependent if $\|\xi_0\|_m \sum_{j\geq 1} \|b_j\| < 1$.

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- Affine Models

$$X_t = M(X_{t-1}, X_{t-2}, \dots)\cdot \xi_t + f(X_{t-1}, X_{t-2}, \dots), \text{ a.s.}$$

exists, belongs in $\mathbb{L}^m$ and is τ-weakly dependent if

$$\sum_{j=1}^{\infty}\alpha_j(f) + \|\xi_0\|_m\Big(\sum_{j=1}^{\infty}\alpha_j(M)\Big) := a < 1$$

with for all finitely non-zero sequences $x = (x_j)_{j\geq 1}$, $y = (y_j)_{j\geq 1}$

$$\begin{aligned}\|f(x) - f(y)\| &\leq \textstyle\sum_{j=1}^{\infty}\alpha_j(f)\|x_j - y_j\|,\\ \|M(x) - M(y)\| &\leq \textstyle\sum_{j=1}^{\infty}\alpha_j(M)\|x_j - y_j\|.\end{aligned}$$

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- Limit Results similar to the independent case.

Parametric Estimation

The true parameter $\theta_0 \in C$ compact $\subset \mathbb{R}^d$.

$$X_t = M_{\theta_0}(X_{t-1}, X_{t-2}, \ldots) \cdot \xi_t + f_{\theta_0}(X_{t-1}, X_{t-2}, \ldots), \text{ a.s.}$$

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If $\|f_\theta(0,0,\ldots)\|_C + \|M_\theta(0,0,\ldots)\|_C < \infty$ ($\|\cdot\|_C = \sup_C \|\cdot\|$) and

$$\sum_{j=1}^{\infty} \alpha_j(f) + \|\xi_0\|_m \Big(\sum_{j=1}^{\infty} \alpha_j(M) \Big) < 1$$

with for all finitely non-zero sequences $(x_j)_{j\geq 1}$, $(y_j)_{j\geq 1}$

$$\begin{array}{lcl} \|f_\theta(x) - f_\theta(y)\|_C & \leq & \sum_{j=1}^{\infty} \alpha_j(f) \|x_j - y_j\|, \\ \|M_\theta(x) - M_\theta(y)\|_C & \leq & \sum_{j=1}^{\infty} \alpha_j(M) \|x_j - y_j\|. \end{array}$$

Quasi Maximum Likelihood Estimator

$$\widehat{\theta}_n = \arg\max_{\theta \in C} \widehat{L}_n(\theta)$$

where

$$\widehat{L}_n(\theta) := -\frac{1}{2} \sum_{t=1}^{n} \left[\left(X_t - \widehat{f}_\theta^t\right)' \left(\widehat{H}_\theta^t\right)^{-1} \left(X_t - \widehat{f}_\theta^t\right) - \frac{1}{2} \ln\left(\det\left(\widehat{H}_\theta^t\right)\right) \right].$$

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First prove that ($\mathcal{F}_t = \sigma(X_t, X_{t-1}, \dots)$)

$$\begin{aligned} \widehat{f}_{\theta_0}^t &:= f_{\theta_0}(X_{t-1}, \dots, X_1, 0, \dots) \xrightarrow[t\to\infty]{\mathbb{P}} \mathbb{E}(X_t / \mathcal{F}_t), \\ \widehat{H}_{\theta_0}^t &:= M_{\theta_0}(X_{t-1}, \dots, X_1, 0, \dots) M'_{\theta_0}(X_{t-1}, \dots, X_1, 0, \dots) \xrightarrow[t\to\infty]{\mathbb{P}} \mathrm{Var}(X_t / \mathcal{F}_t). \end{aligned}$$

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Then, under $\inf_{\theta \in C} \det\left(H_\theta(x_1, \dots)\right) \geq \underline{H} > 0$, using **Ergodic Arguments**

$$\left\| \frac{\widehat{L}_n(\theta)}{n} - L_{X_0/\mathcal{F}_0}(\theta) \right\|_C \xrightarrow[n\to\infty]{\mathbb{P}} 0, \text{ if } \xi_0 \text{ is a Gaussian random variable.}$$

Asymptotic Behavior of the QMLE Bardet & W. (in preparation)

Strong Consistency

If $m \geq 2$, identifiability of the model and

$$\alpha_j(f) + \alpha_j(M) = O\big(j^{-\ell}\big) \quad \text{with} \quad \ell > 3/2, \tag{1}$$

then

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Asymptotic Normality (using **Central Limit Theorem for Martingale Differences**)

Moreover, if $m \geq 4$ and under conditions on 2-first derivatives of f and M,

$$\sqrt{n}\big(\widehat{\theta}_n - \theta_0\big) \xrightarrow[n\to\infty]{\mathcal{D}} \mathcal{N}_d\big(0\ ,\Gamma(\theta_0)\big),$$

where the matrix $\Gamma(\theta_0)$ exists.

More general but less sharp than results of Berkes *et al* (2003).

Limit Theorems Dedecker & Doukhan (2003), Dedecker & Prieur (2004)

$$\|F(x_1, x_2, x_3, \ldots; \xi_t) - F(y_1, y_2, y_3, \ldots; \xi_t)\|_m \leq \sum_{j=1}^{\infty} a_j \|x_j - y_j\|.$$

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Limit Theorems when $a_j \leq cj^{-\beta}$ with $\beta > 1$

If $m = 1$ and $\beta > 2$, $$\frac{1}{n}\sum_{i=1}^{n}(X_i - \mathbb{E}X_0) \underset{n\to\infty}{\longrightarrow} 0, \qquad \text{a.s.}$$

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If $2 > m > p > 1$ and $\beta > 1 + \dfrac{(p-1)(m-1)}{m-p}$, $\displaystyle n^{-\frac{1}{p}}\sum_{i=1}^{n}(X_i - \mathbb{E}X_0) \underset{n\to\infty}{\longrightarrow} 0,$ a.s.

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$\displaystyle \sum_{i=1}^{n}(X_i-Y_i)=o(\sqrt{n\ln\ln n}),$ a.s. where $(Y_t)_{t\in\mathbb{Z}}$ iid Gaussian random variables.

Outline

Noncausal Bernoulli Shifts

Doukhan & W. (2007)

$$X_t = H((\xi_{t-j})_{j\in\mathbb{Z}}),$$

with $(\xi_t)_{t\in\mathbb{Z}}$ $(\in \mathbb{R})$ iid $(\xi \sim \mu)$ $\Rightarrow$ $(X_t)_{t\in\mathbb{Z}}$ $\eta-$weakly dependent.

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Heredity

If $\mathbb{E}|\xi_0|^{m'} < \infty$, and $x_j = y_j$ for $j \neq s$,

$$|H((x_j)_{j\in\mathbb{Z}}) - H((y_j)_{j\in\mathbb{Z}})| \le b_s\big(\sup_{j\neq s}|x_j|^\ell \vee 1\big)|x_s - y_s|$$

then $\sum_s sb_s < \infty$, implies existence in $\mathbb{L}^m$, if $\ell m + 1 \le m'$.

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then $\sum_s sb_s < \infty$, implies existence in $\mathbb{L}^m$, if $\ell m + 1 \le m'$.
If $b_s \le Cs^{-b}$,

$$\eta_\xi(r) \le Cr^{-\eta} \quad \Rightarrow \quad \eta(r) \le C' r^{-\eta\left(1-\frac{1}{b-1}\right)\frac{m'-2}{m'-\ell-1}},$$
$$\lambda_\xi(r) \le Cr^{-\lambda} \quad \Rightarrow \quad \lambda(r) \le C' r^{-\lambda\left(1-\frac{2}{b}\right)\left(1-\frac{\ell}{m'-1}\right)}.$$

Some Applications

Due to the complexity of our models, a main problem is how to identify them ?
Parametric methods on the dynamic do not work. We work on the marginal law.

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- Moment Estimation (Central Limit Theorem, CLT),
- Kernel Density Estimation,
- Functional Thresholded Estimation.

Lindeberg CLT

Doukhan & W. (2007)

A C.L.T. is proved by using Bernstein blocks

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A C.L.T. is proved by using Bernstein blocks

CLT

$(X_t)_{t\in\mathbb{Z}}$ stationary 0-mean, with m-order moments ($m>2$). If $(X_t)_{t\in\mathbb{Z}}$ is λ-weakly dependent with $\lambda(r)=O(r^{-\lambda})$ when $r\to\infty$, with $\lambda>4+\dfrac{2}{m-2}$, then

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{[nt]} X_i \xrightarrow[n\to\infty]{\mathcal{D}} \sigma W_t \text{ for some } \sigma^2\geq 0.$$

Under τ-weak dependence, conditions $m>2$ and $\tau(r)=O(r^{-\tau})$ with $\tau>1+1/(m-2)$ are sufficient.

Kernel Density Estimation Tsybakov (2004)

$(X_t)_{t\in\mathbb{Z}}$ stationary with marginal density f. $K:\mathbb{R}\to\mathbb{R}$ bounded, Lipschitz function with compact support, $\int_{-\infty}^{\infty} K(t)\,dt = 1$

$$\widehat{f}_n(x) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h_n}K\left(\frac{x-X_i}{h_n}\right) \quad \text{for } x\in\mathbb{R},\ h_n \underset{n\to\infty}{\longrightarrow} 0,\ nh_n \underset{n\to\infty}{\longrightarrow} \infty$$

Kernel Density Estimation

Ragache & W. (2006)

Rates of Convergence

If $|f| < \infty$, $\sup_{i \neq j} \|f_{i,j}\|_\infty < \infty$ (joint marginal densities) and $f \in \mathcal{C}_s$. If $(X_t)_{t\in\mathbb{Z}}$ is η-weakly dependent with $\eta(r) = O(r^{-\eta})$ when $r \to \infty$, with $\eta > \max\left(1 + 2/d + (d+1)/\rho, 2 + 1/d\right)$. If K satisfies a moment condition, then h_n can be chosen (depending on s) such that

$$\mathbb{E}|\hat{f}_n(x) - f(x)|^m = O\left(n^{-ms/(2s+d)}\right) \quad \text{for all } 0 < m \leq m_0 = 2\left\lceil(\eta - 1)/2\right\rceil.$$

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If $(X_t)_{t\in\mathbb{Z}}$ is η-weakly dependent with $\eta(r) = O(\exp(-\eta r^b))$ $(0 < \eta)$ when $r \to \infty$,

$$\sup_{\|x\|\le 1} |\hat{f}_n(x) - f(x)| = O\left(\left(\frac{\ln^{2(b+1)/b}(n)}{n}\right)^{s/(d+2s)}\right) \text{ a.s. or in } \mathbb{L}^m.$$

Adaptive Estimator

Donoho *et al.* (1996)

A function $f \in \mathbb{L}^2([0,1])$ can be decomposed as

$$f = \sum_{k=0}^{2^{j_0}-1} \alpha_{j_0,k}\phi_{j_0,k} + \sum_{j=j_0}^{\infty}\sum_{k=0}^{2^j-1} \beta_{j_0,k}\psi_{j_0,k},$$

where $\alpha_{j,k} = \int_0^1 f(x)\phi_{j,k}(x)dx$, $\beta_{j,k} = \int_0^1 f(x)\psi_{j,k}(x)dx$.

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where $\alpha_{j,k} = \int_0^1 f(x)\phi_{j,k}(x)dx$, $\beta_{j,k} = \int_0^1 f(x)\psi_{j,k}(x)dx$.

The thresholded estimator is

$$\hat{f}_n = \sum_{k=0}^{2^{j_0}-1} \hat{\alpha}_{j_0,k}\phi_{j_0,k} + \sum_{j=j_0}^{j_1}\sum_{k=0}^{2^j-1} T_{\lambda_j}(\hat{\beta}_{j,k})\psi_{j,k},$$

where $\hat{\alpha}_{j,k} = 1/n\sum_{i=1}^n \phi_{j,k}(X_i)$ and $\hat{\beta}_{j,k} = 1/n\sum_{i=1}^n \psi_{j,k}(X_i)$ are the empirical estimators of the coefficients $\alpha_{j,k}$ and $\beta_{j,k}$ and $T_\lambda(\beta) = \beta 1\!\!1_{|\beta|>\lambda}$ for $\lambda > 0$.

Quasi Optimality of the Estimator Gannaz & W. (submitted)

Quasi Optimality

Suppose that $f \in \mathcal{B}^s_{\pi,r}(M)$ and that $(X_t)_{t\in\mathbb{Z}}$ is η-weakly dependent with $\eta(r) = O(\exp(-\eta r^b))$ $(0 < \eta)$ when $r \to \infty$.

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$$\mathbb{E}\left[\int_0^1 |\hat{f}_n(x) - f(x)|^m dx\right] \leq C \begin{cases} \left(\frac{\ln^{2(b+1)/b} n}{n}\right)^{m\alpha} & \text{if } \epsilon \neq 0 \\ \left(\frac{\ln^{2(b+1)/b} n}{n}\right)^{m\alpha} (\ln n)^{(1-\pi/r)_+} & \text{if } \epsilon = 0 \end{cases}$$

where the optimal rate α and ϵ are given

$$\alpha = \begin{cases} s/(1+2s) & \epsilon \geq 0, \\ (s - 1/\pi + 1/m)/(1 + 2s - 2/\pi) & \epsilon \leq 0, \end{cases} \qquad \text{where } \epsilon = s\pi - (m-\pi)/2.$$

The Threshold Level

Gannaz & W. (submitted)

$$\begin{cases} 2^{j_0} & \simeq & n^{1/(1+N)}, \\ 2^{j_1} & \simeq & n/\log n, \\ \lambda_j & = & Kj^{1+1/b}/\sqrt{n}, \text{ for a well chosen constant } K > 0. \end{cases}$$

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Donoho Johnstone Kerkyacharian Picard (1996): $\lambda_j = K\sqrt{j}/\sqrt{n}$.

$\Rightarrow$ The same rates with $\ln n$ instead of $\ln^{2(1+1/b)} n$.

Papers

Available on my web page http://wintenberger.fr

- N. Ragache, O. Wintenberger (2006). 'Convergence rates for density estimators of weakly dependent time series', Dependence in Probability and Statistics. *Lecture Notes in Statistics. P. Bertail, P. Doukhan and Ph. Soulier editors*, vol 187, Springer Verlag.
- P. Doukhan, O. Wintenberger. 'An invariance principle for weakly dependent stationary general model' (2007). accepted in *Probability and Mathematical Statistics*.
- I. Gannaz, O. Wintenberger. 'Adaptive density estimation under dependence', submitted to *ESAIM P&S*.
- P. Doukhan, O. Wintenberger. 'Weakly dependent chains with infinite memory', submitted to *Stochastic Processes and their Applications*.
- J.M. Bardet, O. Wintenberger. 'Asymptotic normality of the Quasi Maximum Likelihood Estimator for multidimensional conditionally heteroscedastic processes' in preparation.