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ÉLÉMENTS DE PHYSIQUE DE LA BEAUTÉ

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Introduction

La physique de la beauté est d'une importance cruciale en physique des particules pour plusieurs raisons. Tout d'abord, elle intervient dans un grand nombre de modèles théoriques rencontrés : modèle standard électrofaible (détermination de ses paramètres et prédictions), chromodynamique quantique, étude de la violation de CP, nombreux modèles phénoménologiques, etc. Ensuite, du point de vue expérimental, il existe un grand nombre d'usines à B qui génèrent une source colossale de données pour tester ces modèles, ce qui justifie l'importance des études expérimentales des désintégrations des mésons beaux. Enfin, la physique du B peut potentiellement constituer une source de ce que l'on appelle « nouvelle physique » que ce soit par exemple au travers des théories supersymétriques, ou bien encore des modèles à dimensions supplémentaires en ce qui concerne la théorie, mais aussi au travers de découvertes expérimentales inattendues comme les récentes mises en évidence (voir par exemple [1] et [2]) de nouvelles résonances « bizarres » $X(3872)$ et $X(4260)$ par Belle, Babar, D0 et CDF.

L'objet de ce mémoire est d'adresser un (petit) nombre d'aspects théoriques relatifs à l'ensemble de cette physique. Il est le fruit d'une collaboration avec le Laboratoire de Physique Théorique d'Orsay et pour une bonne partie prolonge naturellement les recherches effectuées lors de ma thèse [3]. Il se présente comme suit : dans un premier chapitre, nous discuterons de l'hypothèse de dualité quark-hadron, pierre angulaire de nombreuses études des désintégrations des B. Puis, dans un deuxième chapitre, nous présenterons un travail relatif à une autre hypothèse fondamentale utilisée lors des études des processus non leptoniques : l'hypothèse de factorisation. Au cours d'un troisième chapitre, nous reprendrons un problème lié à la confrontation théorie $\leftrightarrow$ expérience qui apparaît lorsque l'on étudie les désintégrations des mésons B en mésons D présentant une excitation orbitale $L = 1$ et déja mentionné dans [3]. Enfin, pour essayer de lever ce problème, un dernier chapitre décrira une étude de faisabilité réalisée par calcul de QCD sur réseaux. Ce dernier chapitre contiendra aussi une description du projet européen apeNEXT, projet visant à la création/fabrication de A à Z d'un ordinateur à architecture parallèle optimisé pour les calculs de chromodynamique quantique.

Chapitre 1

Dualité quark-hadron

Où nous allons présenter la notion de dualité quark-hadron et voir que sa réalisation est liée à l'existence de règles de sommes connues ou bien nouvelles.

Du point de vue théorique, l'étude des désintégrations des mésons B est facilitée par la présence du quark lourd b dont l'inverse de la masse peut servir de paramètre de développement perturbatif (HQET) ; par ailleurs, lorsque $m_b \to \infty$, des symétries supplémentaires apparaissent qui contraignent les éléments de matrice de transition. Ces outils ont permis une étude poussée des désintégrations exclusives, principalement semileptoniques, des B dans cette limite de masse infinie ainsi que des premières corrections en puissance de $\frac{\Lambda}{m_b}$; le point commun dans cette approche est que l'objet considéré est le hadron.

D'un autre côté, une approche différente consiste à étudier de façon inclusive les processus en ne se plaçant non plus à l'échelle du hadron mais à l'échelle du quark ($b \to cX$ par exemple). On réalise alors un développement en produits d'opérateurs (OPE) qui, avec la théorie effective de quark lourd, conduit également à un développement limité en puissances de $\frac{\Lambda}{m_b}$. La dualité quark-hadron est alors invoquée pour faire le lien entre les calculs réalisés au niveau des quarks et des gluons et le monde « réel » des hadrons : le taux de désintégration inclusive (c'est-à-dire celui obtenu par OPE) doit coïncider avec la somme des taux de désintégration exclusive (« ce qui se passe au niveau des quarks est l'image de ce qui se passe au niveau des hadrons »).

L'existence de la dualité est indiscutable dans les désintégrations semileptoniques $B \to X_c \ell\nu$; en effet, le quark c, dans la limite $m_b \to \infty$, possède une énergie très grande devant Λ_{QCD}. De fait, il se comporte à l'ordre zéro comme un quark libre et il doit donc s'hadroniser en méson D. La question est alors plutôt de savoir avec quelle précision cette hypothèse de dualité « tient la route » et quelles sont les conditions derrière sa réalisation.

1.1 Quelques rappels

Avant de développer le sujet de ce chapitre, il serait bon de rappeler quelques généralités et de fixer les notations.

1.1.1 Spectroscopie

Tous les mésons lourds M considérés ici sont des états liés $Q\bar{q}$ (Q quark lourd et q quark léger associé). Le moment cinétique total est $\vec{J} = \vec{s}_Q + \vec{\jmath}$ avec, dans le cadre du modèle de quarks, $\vec{\jmath} = \vec{\ell} + \vec{s}_q$ ($\vec{\ell}$ désigne le moment cinétique orbital relatif du quark léger et $\vec{s}$ le spin). Pour des raisons d'invariance par rotation, ce $\vec{J}$ est une quantité conservée. Par ailleurs, grâce aux symétries de quark lourd (SQL), le moment cinétique de spin $\vec{s}_Q$ est aussi une quantité conservée, donc $\vec{\jmath}$ l'est également (on l'appelle « composante légère » dans la littérature). On range alors les diverses possibilités de la façon suivante :

- **Cas $\boldsymbol{\ell = 0}$ (états $\boldsymbol{S}$) :**

multiplet	valeurs J^P	notation générique	exemple
$j^P = \dfrac{1}{2}^-$	0^-	M	B, D
	1^-	M^*	D^*

- **Cas $\boldsymbol{\ell = 1}$ (états $\boldsymbol{P}$) :**

multiplet	valeurs J^P	notation générique	exemple
$j^P = \dfrac{1}{2}^+$	0^+	M^{**}	D_0^*
	1^+		D_1^*
$j^P = \dfrac{3}{2}^+$	1^+	M^{**}	D_1
	2^+		D_2^*

Il est important de noter que les D^{**} appartenant à un même multiplet sont reliés par les symétries de quark lourd.

1.1.2 Notations diverses

Lors de l'étude des désintégrations semileptoniques de quarks lourds, il est coutume d'introduire les facteurs de forme suivants pour décrire les amplitudes de transition dans la limite $m_b \to \infty$:

- **Canaux élastiques $\boldsymbol{1/2^-}$:** il s'agit des transitions du type $\langle D|J|B\rangle$ et $\langle D^*|J|B\rangle$ qui s'expriment en terme d'un seul facteur de forme (au lieu de 6 pour une masse m_b finie) noté $\xi(w)$ et souvent appelé « fonction d'Isgur-Wise » [4, 5, 6].

▶ **Canaux inélastiques $1/2^+$ et $3/2^+$:** dans ce cas, les éléments de matrice du type $\langle D^{**}|J|B\rangle$ s'expriment en terme de 2 facteurs de forme uniquement [7], $\tau_{1/2}(w)$ pour le multiplet $\dfrac{1}{2}^+$ et $\tau_{3/2}(w)$ pour le multiplet $\dfrac{3}{2}^+$ (au lieu des 14 facteurs de forme habituels).

Le paramètre w qui apparaît ici est égal $v \cdot v'$ où v et v' sont les quadrivitesses des mésons initial et final.

La pente de $\xi(w)$ au point $w = 1$, c'est-à-dire lorsque $\vec{v} = \vec{0} = \vec{v}\,'$ (autrement dit lorsque il n'y a pas de recul), est utilisée pour définir le nombre $\rho^2 \equiv -\left(\dfrac{\mathrm{d}\xi}{\mathrm{d}w}\right)_{w=1}$.

1.1.3 Règles de somme de Bjorken

Les facteurs de forme précédemment introduits ne sont pas complètement indépendants ; on montre que [8, 7] :

$$\begin{aligned} 1 = \frac{w+1}{2}|\xi(w)|^2 + (w-1)\Bigg[\sum_{n=1}^{\infty} \frac{w^2-1}{2}|\xi^{(n)}(w)|^2 + 2\sum_{n=1}^{\infty}|\tau_{1/2}^{(n)}(w)|^2 \\ + (w+1)^2\sum_{n=1}^{\infty}|\tau_{3/2}^{(n)}(w)|^2\Bigg] + \cdots\cdots \end{aligned} \tag{1.1}$$

où les exposants (n) représentent les excitations radiales des états P.

On obtient une forme plus connue de la règle de somme de Bjorken en faisant dans (1.1) un développement limité au premier ordre autour de $w = v \cdot v' = 1$ (point de recul nul) qui donne :

$$\sum_n \left(|\tau_{1/2}^{(n)}(1)|^2 \;+\; 2\,|\tau_{3/2}^{(n)}(1)|^2\right) \;=\; \rho^2 \;-\; \frac{1}{4}$$

1.1.4 Règle de somme de Voloshin

Une autre règle de somme se rencontre, qui relie certaines des fonctions précédentes au point de recul nul :

$$\bar{\Lambda} \;=\; \sum_n \left(2\,\Delta_{1/2}^{(n)}\,|\tau_{1/2}^{(n)}(1)|^2 \;+\; 4\,\Delta_{3/2}^{(n)}\,|\tau_{3/2}^{(n)}(1)|^2\right)$$

où $\Delta_j^{(n)} = M_j^{(n)} - M_o$ est la différence d'énergie de l'état j, n avec l'état fondamental.

1.2 Sources possibles de violation de dualité

Nathan Isgur [9] a proposé une analyse générale des causes potentielles pouvant induire des violations de dualité d'ordre $\dfrac{1}{m_Q}$. Nous avons déjà dit que la théorie effective de quark lourd permet de développer, en faisant un OPE, les taux de désintégration inclusifs en une somme de termes en puissance de $\frac{1}{m_Q}$ où le terme d'ordre zéro correspond au cas où le quark lourd est libre et où le terme d'ordre un est absent (théorème de Luke [10]) :

$$\Gamma_{\text{inclusif}}(B \to X_c\,\ell\,\nu) \;=\; \Gamma_{\text{libre}}(b \to c\,\ell\,\nu) \;+\; \mathscr{O}\left(\left(\frac{\Lambda}{m_Q}\right)^2\right)$$

L'argument de N. Isgur procède comme suit : dans la limite *exacte* de masse infinie ($m_b \to \infty$ à $r = \frac{m_b}{m_c}$ fixé), nous avons (E_ℓ désigne l'énergie du lepton) :

$$\frac{\mathrm{d}^2\Gamma^{\text{inclusif}}_{\text{hadron}}}{\mathrm{d}w\,\mathrm{d}E_\ell} \;=\; \frac{\mathrm{d}^2\Gamma^{\text{inclusif}}_{\text{quark}}}{\mathrm{d}w\,\mathrm{d}E_\ell} \cdot \left[\frac{w\,+\,1}{2}\,|\xi(w)|^2 \right.$$
$$\left. +\,2\,(w-1)\left[\sum_n \left(|\tau^{(n)}_{1/2}(1)|^2 \;+\; 2\,|\tau^{(n)}_{3/2}(1)|^2\right)\right] +\; \mathscr{O}\Big((w-1)^2\Big)\right]$$

Alors la règle de Bjorken (1.1) assure que la dualité est bien vérifiée car la diminution du canal élastique $\frac{1}{2}^-$ lorsque $w-1$ augmente est exactement compensée par l'apparition et l'augmentation des canaux inélastiques $\frac{1}{2}^+$ et $\frac{3}{2}^+$ (sachant que le seuil de création des hadrons se situe en $w = 1$).

Cependant, dans le cas où m_b est grande mais finie, cette compensation se trouve retardée car il existe un domaine de valeurs de w autour de 1 pour lequel l'apparition des états $\frac{1}{2}^+$ et $\frac{3}{2}^+$ est cinématiquement interdite. En effet, en introduisant les transferts $t = (q^o)^2 - \vec{q}^{\,2}$, alors les transferts maximaux $t^{**}_{\max} = (m_B - m_{D^{**}})^2$ et $t_{\max} = (m_B - m_D)^2$ ne coïncident plus puisque :

$$\frac{t_{\max}\;-\;t^{**}_{\max}}{2\,m_b\,m_c} \;\simeq\; \left(1\,-\,\frac{m_c}{m_b}\right)\,\frac{m_{D^{**}}\;-\;m_D}{m_c}$$

Il est visible sur cette dernière relation que l'effet est d'ailleurs d'ordre $\frac{\Lambda}{m_Q}$, ce qui semble être en contradiction avec les résultats obtenus par développement en produits d'opérateurs ; en fait, il se trouve que des effets d'ordre $\frac{\Lambda}{m_Q}$ peuvent apparaître lorsque l'énergie dégagée dans les transitions $b \to c$ est de l'ordre de Λ_{QCD} (l'OPE « rate » la dépendance en fonction du tranfert d'impulsion de l'ouverture de certains canaux au voisinage de $w = 1$, et donc peut prendre en compte certaines résonances dans des zones où elles sont cinématiquement interdites – lien avec le rayon de convergence de l'OPE) alors que lorsque l'énergie dégagée est grande ($\gg \Lambda_{\text{QCD}}$), le résultat obtenu par la méthode OPE, c'est-à-dire l'absence de termes d'ordre 1 en $\frac{\Lambda}{m_Q}$, reste valide car des règles de somme « conspirent » pour assurer la dualité.

Remarque. D'un point de vue plus technique [11], les calculs fondés comme ici sur les OPE sont réalisés dans l'espace euclidien : cette procédure est légitime pour des contours d'intégration éloignés de la région physique et l'idée est de relier une intégration dans un domaine non physique à une intégration dans le domaine physique (voir par exemple [12]) ; la difficulté est que bien souvent, le contour se rapproche toujours de la région physique (celle qui contient les pôles ou les coupures) et donc les résultats obtenus par le développement en produits d'opérateurs peuvent ne pas être fiables. Enfin, une fois ces calculs effectués dans l'euclidien, on réalise un prolongement analytique dans l'espace minkowskien pour aboutir aux résultats physiques (grandeurs mesurables, règles de somme, etc...) et c'est grâce à ce prolongement que la dualité entre le calcul hadronique et le calcul au niveau des quarks apparaît [1].

Pour résumer, une violation de la dualité peut physiquement provenir :

- des seuils de production hadronique (pôles du contour)
- des coupures du contour
- de la validité ou non du développement en puissance de $\dfrac{1}{m_Q}$

1.3 Une première étude

Nous nous sommes demandés [13] en utilisant un modèle non relativiste de quark comment la dualité se manifeste lorsque l'on considère la description hadronique et la description « partonique » des désintégrations semileptoniques. Pour cela, nous avons calculé d'une part le taux de désintégration total par sommation des canaux exclusifs et, d'autre part, le taux de désintégration à partir des quarks considérés comme libres. Enfin, nous avons étudié le comportement de $\Gamma_{\text{hadron}} - \Gamma_{\text{quark}}$ pour une masse m_b finie.

1.3.1 Cadre de l'étude

Les hadrons sont décrits par un modèle de quark non relativiste dans lequel les états liés sont des oscillateurs harmoniques ; l'intérêt d'un tel modèle est que les différents niveaux sont complètement connus et surtout que les niveaux $n \geq 2$ interviennent avec une contribution minimale d'ordre $\dfrac{1}{m_b^3}$ donc négligeable pour notre étude : il suffira donc de conserver uniquement les niveaux $n = 0$ et $n = 1$.

Par ailleurs, nous avons pris un hamiltonien dont le terme d'énergie potentielle ne dépend pas de la saveur. Ensuite, le couplage quark-lepton est décrit par un courant vectoriel (nous

1. De ce point de vue, parler d'*hypothèse* de dualité est incorrect car il n'y a aucune hypothèse supplémentaire introduite.

travaillons avec des quarks sans spin) et enfin, nous nous sommes placés dans la limite de Shifman-Voloshin.

1.3.2 Limite de Shifman-Voloshin

La limite de Shifman-Voloshin [14], notée SV désormais, est la situation dans laquelle :

$$\Lambda \ll \delta m = m_b - m_c \ll m_b, m_c\, m_Q \qquad \text{où} \quad \Lambda : \text{échelle hadronique}$$

L'intérêt de cette limite est que les désintégrations ont lieu presque à recul nul ($w \sim 1$) et donc que peu de canaux contribuent à l'ordre dominant en $\dfrac{1}{m_Q}$

1.3.3 Méthode

Considérons le terme : $\qquad \epsilon = \dfrac{\Gamma_{\text{hadron}} - \Gamma_{\text{quark}}}{\Gamma_{\text{quark}}}$

En principe, les premières corrections de masse à ϵ doivent être, conformément aux résultats fournis par OPE, en $\dfrac{1}{m_b^2}$ (en tout cas lorsque $m_b \rightarrow \infty$ avec $r = \dfrac{m_b}{m_c}$ fixé : c'est ce que j'appellerai dans la suite « développement en $1/m_Q$ »). Par ailleurs, le fait de travailler dans la limite SV rajoute un développement en $(1-r)$, c'est-à-dire en $\dfrac{\delta m}{m_b}$ si bien que, globalement, nous obtiendrons un développement dont les termes seront de la forme :

$$\frac{(\delta m)^p}{(m_b)^q} \times \underbrace{\begin{pmatrix} \text{termes associés} \\ \text{aux quarks lé-} \\ \text{gers } d \text{ et/ou } u \end{pmatrix}}_{m_d,\, m_u,\, \Delta}$$

où Δ représente l'énergie du premier niveau excité. De fait, il va nous falloir non seulement prouver l'absence de termes en $\dfrac{1}{m_b}$ dans ϵ, mais aussi faire attention aux termes en $\dfrac{(\delta m)^p}{(m_b)^q}$ qui pourraient correspondre à un terme d'ordre $\dfrac{1}{m_Q}$ dans la dénomination précédente. En effet, les facteurs du type $\dfrac{(\delta m)^p}{m_b^2}$, avec $p > 0$, peuvent contribuer à l'ordre 1 en $\dfrac{1}{m_Q}$ puisque δm est grand, voire d'ordre m_Q, dans la limite $m_b \rightarrow \infty$.

En pratique, nous avons retenu les termes en $\dfrac{\delta m^2}{m_b^2}$ et $\dfrac{\delta m}{m_b^2}$ (ils correspondent respectivement à $\left(\dfrac{1}{m_Q}\right)^0$ et $\dfrac{1}{m_Q}$ dans la limite de masse infinie) et montré qu'ils disparaissent dans la largeur totale[2], en ne laissant que des termes au minimum d'ordre $\frac{1}{m_Q^2}$. Quant aux termes $p > 2$ et ceux en $(\delta m)^p$ et $\frac{(\delta m)^p}{m_b}$, ils n'apparaissent pas dans la façon dont ϵ est calculé.

1.3.4 Importance des règles de somme dans la dualité

Le lecteur intéressé trouvera le détail des calculs dans [13] ; pour que les premières corrections à ϵ soient d'ordre $\frac{1}{m_Q^2}$ afin que la dualité soit satisfaite, il est indispensable d'avoir les règles de somme de Bjorken et de Voloshin. Ce sont elles qui permettent la compensation des termes gênants d'ordre $\frac{\delta m}{m_b^2}$ qui apparaissent dans les expressions des largeurs de désintégration totales du fait que les seuils de production des états excités et fondamental ne coïncident pas comme noté précédemment.

1.4 Une nouvelle règle de somme

L'étude précédente nous a permis d'appréhender le fonctionnement de la dualité par un modèle simple ; néanmoins, l'idéal serait de se placer dans un cadre plus réaliste comme la QCD.

Glenn Boyd *et al.* [15] avaient déjà de façon très approfondie étudié la dualité dans les désintégrations semileptoniques des B (ils se sont placés dans le cadre de la QCD et dans la limite de Shifman-Voloshin) ; cependant, pour le calcul de la largeur totale en tant que somme sur les canaux exclusifs, ils n'ont considéré que les mésons fondamentaux D et D^*. Nous avons repris ce travail et complété le calcul de la partie hadronique en rajoutant les premières excitations orbitales D^{**}. Cependant, nous n'avons pris aucune correction radiative en compte. Nous avons alors comparé la largeur totale Γ obtenue à partir d'un développement en produits d'opérateurs à celle obtenue par sommation des largeurs des canaux $B \to D$, $B \to D^*$, $B \to D^{**}$; si la dualité est vérifiée, alors on s'attend à ce que les termes d'ordre de type $\frac{\delta m^2}{m_b^2}$ et $\frac{\Lambda\, \delta m}{m_b^2}$ (voir le paragraphe 1.3) se compensent, mais avec quelle précision ?

Après calculs [16], nous avons établi que la règle de somme de Bjorken permet effectivement la compensation du terme en $\frac{\delta m^2}{m_b^2}$, et, en utilisant la règle de Voloshin, nous avons réalisé

2. La disparition des termes $\frac{\delta m}{m_b^2}$ assure l'absence de violation de dualité à l'ordre $\frac{1}{m_Q}$ tandis que la compensation des termes en $\frac{\delta m^2}{m_b^2}$ est à l'origine du fait qu'à l'ordre zéro en $\frac{1}{m_Q}$, on obtient le taux de désintégration correspondant au calcul avec les quarks libres.

que la compensation des termes en $\dfrac{\Lambda\,\delta m}{m_b^2}$ impose qu'une nouvelle règle de somme soit vérifiée (avec des notations déjà explicitées) :

$$a_+ \;=\; 4\sum_n \left[\Delta^{(n)}_{1/2}\,|\tau^{(n)}_{1/2}(1)|^2 \;-\; \Delta^{(n)}_{3/2}\,|\tau^{(n)}_{3/2}(1)|^2\right] \tag{1.2}$$

a_+ est un des six facteurs de forme évoqués au paragraphe 1.1 qui s'expriment en fonction de ξ dans la limite de masse infinie (il intervient dans l'écriture des amplitudes de transition $B \to D^*$ par courant axial).

Notons que cette nouvelle règle de somme, associée à des estimations théoriques extérieures des facteurs de forme [17, 18], génère des valeurs numériques permettant une discussion phénoménologique des processus. En particulier, la règle (1.2) produit la hiérarchie suivante :

$$\sum_n |\tau^{(n)}_{1/2}(1)|^2 \;<\sum_n |\tau^{(n)}_{3/2}(1)|^2$$

Nous avions déjà remarqué cette propriété dans le cadre des modèles covariants de facteurs de forme à la Bakamjian-Thomas [19, 3] : les processus $B \to D^{**}_{3/2^+}$ semblent favorisés par rapport au canal $B \to D^{**}_{1/2^+}$. D'un autre côté, les études expérimentales apportent la conclusion radicalement opposée... il y a donc là un mystère à résoudre... (corrections radiatives conséquentes par exemple ?)

Enfin, nous nous sommes aperçus en cours d'étude qu'il était possible d'établir en plus de (1.2) toute une classe de règles de somme supplémentaires et nous en avons donné la méthode.

1.5 Tentative de généralisation

Au vu de l'apparition de ces règles de somme qui « conspirent » pour compenser les termes d'ordre problématique dans les largeurs totales de désintégration, nous avons entrepris une étude plus générale [20] de la dualité dans les désintégrations semileptoniques des mésons B, toujours dans le cadre des modèles de quark non relativistes mais sans particulariser ces modèles comme ce fut le cas au paragraphe 1.3.

1.5.1 Principe de l'étude

Comme précédemment, l'idée est de calculer les largeurs de désintégration totales de deux façons différentes (OPE et somme des canaux exclusifs) puis de vérifier dans quelle mesure elles sont compatibles et à quels ordres.

Description du modèle

Nous avons choisi de considérer les processus semileptoniques $B \to X_c\,\ell\,\nu$ dans la limite SV (pour des raisons déjà évoquées) en identifiant les mésons à des états liés de quarks non relativistes sans spin (courant scalaire) soumis à un potentiel confinant a priori quelconque dans un premier temps. La partie leptonique du processus est, elle, traitée de façon relativiste. Enfin, pour des raisons de simplification, nous nous sommes placés dans le référentiel propre du méson B.

L'intérêt d'un tel modèle est qu'il nous permet d'avoir accès au spectre des états liés (ils sont calculables) de façon assez simple et que les facteurs de forme des transitions hadroniques en terme des fonctions d'onde des mésons sont également connus : de fait, la somme exclusive des largeurs peut être obtenue sans problème.

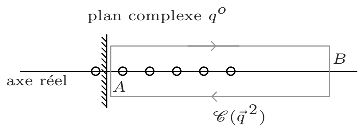
Calcul de Γ par développement en produits d'opérateurs

Pour résumer (le détail de la procédure est explicité dans [20]), le taux de désintégration semileptonique intégré s'obtient à partir de :

$$\Gamma(B \to X_c\,\ell\,\nu) \;=\; \frac{1}{2\,\pi\,i} \int \mathrm{d}\vec{q}^{\,2}\,\|\vec{q}\| \int_{\mathscr{C}(\vec{q}^{\,2})} \mathrm{d}q^o\,\theta\big(q^o > \|\vec{q}\|\big)\,L(q^2)\,T(q^o,\vec{q}^{\,2})$$

sachant que
- $L(q^2)$ représente le tenseur leptonique relativiste
- $T(q^o,\vec{q}^{\,2})$ représente la partie hadronique non relativiste dont l'expression exacte dépend du potentiel de quark considéré
- $\mathscr{C}(\vec{q}^{\,2})$ est un contour d'intégration dans le plan q^o complexe

Remarque. Le contour $\mathscr{C}(\vec{q}^{\,2})$ possède les caractéristiques suivantes :



Les cercles correspondent aux pôles de T (résonances charmées $c\bar{q}$) ; la ligne $\mathscr{R}e(q^o) = \|\vec{q}\|$ sépare la région cinématiquement interdite de la région autorisée (cf le terme en $\theta\big(q^o > \|\vec{q}\|\big)$) et, pour chaque valeur de $\|\vec{q}\|$, le point A est fixé sur l'axe réel tandis que le contour peut être déformé à volonté (point B) là où T est analytique. Notons qu'une partie de $\mathscr{C}(\vec{q}^{\,2})$ se trouve au voisinage de la région physique (pôles et coupure) ce qui peut induire des incertitudes quant à la fiabilité du calcul par OPE (cf. paragraphe 1.2). Enfin, un tel contour sélectionne, pour chaque valeur de $\vec{q}^{\,2}$, les résonances physiques produites lors de la désintégration $B \to X_c\,\ell\,\nu$.

Ensuite, l'amplitude T est développée en produit d'opérateurs :

$$T(q^o,\vec{q}^{\,2}) \;=\; \sum_n C_n(q^o,\vec{q}^{\,2})\,\langle B|\mathscr{O}_n|B\rangle$$

avec $\mathscr{O}_n$: opérateurs locaux de dimension croissante

L'introduction du hamiltonien nous donne alors explicitement cette décomposition pour obtenir un développement de Γ^{OPE} en $\dfrac{\Lambda}{m_c}$ et $\dfrac{\Lambda}{\delta m}$. Enfin, nous utilisons les formes non relativistes des règles de somme (Bjorken, Voloshin et les autres) pour traduire, dans l'expression obtenue pour Γ^{OPE}, les termes contenant les éléments de matrice des opérateurs $\mathscr{O}_n$ en fonction des facteurs de forme utilisés dans la description exclusive des processus (de cette façon, nous pourrons comparer directement Γ^{OPE} à $\sum \Gamma^{\text{exclusif}}$) et l'on obtient finalement une expression de la forme suivante :

$$\Gamma^{\text{OPE}}(B \to X_c\,\ell\,\nu) \;=\; \int \mathrm{d}[\text{qqchse}] \sum_{n=0}^{\infty} \mathscr{F}(q^o, \vec{q}^{\,2}, \delta m, m_c, \ldots) \qquad (1.3)$$

où d[qqchse] représente la mesure de l'intégration et $\mathscr{F}$ est une fonction dont l'expression exacte est donnée dans [20].

Calcul de Γ par sommation sur les canaux exclusifs

Cette largeur de désintégration s'obtient en sommant les différents canaux exclusifs et le résultat s'exprime comme suit :

$$\Gamma(B \to X_c\,\ell\,\nu) \;=\; \int \mathrm{d}[\text{qqchse}] \sum_{n=0}^{n_{\max}(q^2)} \mathscr{F}(q^o, \vec{q}^{\,2}, \delta m, m_c, \ldots) \qquad (1.4)$$

où $\mathscr{F}$ est exactement la *même* fonction que dans la formule (1.3) (ainsi que la mesure d[qqchse]).

Résultats et commentaires

Nous voyons apparaître ici une violation possible de dualité, dans la mesure où, bien que les expressions (1.3) et (1.4) soient similaires, elles diffèrent dans les bornes de la sommation discrète : Γ^{OPE}, borne ∞, et Γ, borne $n_{\max}(q^2)$. Autrement dit, le développement en produit d'opérateurs, à cause de l'introduction des règles de somme, prend en compte pour chaque q^2 toutes les résonances, y-compris celles qui sont cinématiquement interdites pour ce q^2, alors que le calcul exclusif exclut ces états excités comme il se doit : il faudrait tronquer les règles de somme pour avoir la correspondance. Le problème de la violation de la dualité se transfère donc au problème de la convergence de ces séries infinies dans Γ^{OPE}, qui lui-même est lié à la forme du potentiel de quark choisi (comportement à grande et petite distances). De façon plus générale, nous avons établi que l'erreur due à la troncation dans Γ^{OPE} est d'ordre $\dfrac{\Lambda^2}{m_c^2}\left(\dfrac{\Lambda}{\delta m}\right)^b$ où b est un nombre réel positif qui dépend des propriétés du potentiel.

Là encore, le rôle joué par les règles de somme est fondamental pour la compensation des termes, du moins jusqu'à l'ordre où apparaîssent ces termes de violation de dualité :

- Bjorken et Voloshin permettent la compensation des termes en $\frac{\delta m^2}{m_c^2}$ et $\Lambda\frac{\delta m^2}{m_c^2}$
- Les autres règles de somme permettent la compensation des termes en $\frac{\Lambda^2}{m_c^2}$, $\left(\frac{\Lambda}{m_c}\right)^2\frac{1}{\delta m_c}$, etc...

Enfin, il est possible avec le même formalisme d'étudier ce qui se passe au niveau des largeurs différentielles, et non plus totales, afin d'appréhender comment la dualité se manifeste du point de vue *local*. Raisonnons en terme de la variable q^2, variant de façon générale entre 0 et q^2_{max} ; à mesure que l'on se rapproche de la valeur maximale de q^2, les canaux produisant les états excités se ferment les uns après les autres pour ne laisser que l'état fondamental en $q^2 = q^2_{\text{max}}$. Cet effet génère une violation de dualité au voisinage de q^2_{max} d'ordre $\Lambda\frac{\delta m}{m_c^2}$. Il est intéressant de noter que, dans la largeur totale, cet ordre dangereux est compensé par d'autres termes de même contribution provenant de la règle de somme de Voloshin ; il pourrait donc y avoir quelques problèmes si, expérimentalement, on ne réalise les mesures que dans une zone limitée de l'espace des phases au voisinage de q^2_{max}.

Nature du potentiel de quark

Au cours du paragraphe précédent, la nature du potentiel de quark utilisé a été présentée comme directement reliée au problème de la violation de la dualité car influant sur la convergence des séries rencontrées.

Afin de clarifier ce point, nous avons étudié [21] les effets du potentiel non relativiste choisi pour décrire les mésons ; nous avons pris les deux situations où le potentiel présente une singularité à l'origine et celle où il n'en présente pas. Nous avons montré dans ce cadre d'hypothèse que :

- cas régulier : lorsque le potentiel ne présente pas de singularité à l'origine, il existe une violation de dualité qui peut être mise en évidence[3].
- cas singulier : lorsque le potentiel est singulier à l'origine, l'étude de la convergence de l'OPE est problématique car, au-dessus d'un certain ordre dans le développement, les coefficients deviennent infinis. Si l'on tronque la série au dernier terme fini, alors nous n'avons obtenu aucune indication de violation de dualité à cet ordre.

  Ce cas mériterait d'être approfondi car le potentiel effectif de QCD est singulier. Mais nous nous sommes placés dans un modèle non-relativiste ; une vraie théorie relativiste

3. Le terme correspondant est néanmoins exponentiellement petit, d'ordre $\frac{\delta m^2}{m_c^2}\exp(-\frac{\delta m}{\Lambda})$

conduirait peut-être alors à un développement correct, c'est-à-dire ne présentant pas de divergences, qui permettrait de fournir une conclusion plus précise.

1.6 Conclusion

Dans ce chapitre, nous avons étudié certains mécanismes pouvant être à l'origine de la violation de la dualité quark-hadron.

Nous avons établi que, en ce qui concerne les largeurs totales de désintégration, des règles de sommes[4] sont nécessaires qui conspirent pour compenser les termes problématiques.

De plus, nous avons montré l'absence de termes de violation d'ordre $\frac{1}{m_Q}$ en utilisant des modèles de quark, contrairement à l'analyse de N. Isgur [9].

Enfin, la situation est différente si l'on considère les largeurs différentielles pour lesquelles les compensations entre termes sont moins évidentes puisque certains canaux ne sont pas obligatoirement ouverts dans la zone de l'espace de phase considérée.

4. Certaines sont connues telles les règles de Bjorken ou de Voloshin et d'autres sont nouvelles : une méthode a par ailleurs été présentée pour en établir toute une classe.

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One interesting new sum rule extending Bjorken's to order $1/m_Q$

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Abstract

We explicitly check quark-hadron duality to order $(m_b - m_c)\Lambda/m_b^2$ for $b \to c l \nu$ decays in the limit $m_b - m_c \ll m_b$ including ground state and orbitally excited hadrons. Duality occurs thanks to a new sum rule which expresses the subleading HQET form factor ξ_3 or, in other notations, $a_+^{(1)}$ in terms of the infinite mass limit form factors and some level splittings. We also demonstrate the sum rule, which is not restricted to the condition $m_b - m_c \ll m_b$, applying OPE to the longitudinal axial component of the hadronic tensor without neglecting the $1/m_b$ subleading contributions to the form factors. We argue that this method should produce a new class of sum rules, depending on the current, beyond Bjorken, Voloshin and the known tower of higher moments. Applying OPE to the vector currents we find another derivation of the Voloshin sum rule. From independent results on ξ_3 we derive a sum rule which involves only the $\tau_{1/2}^{(n)}$ and $\tau_{3/2}^{(n)}$ form factors and the corresponding level splittings. The latter strongly supports a theoretical evidence that the B semileptonic decay into narrow orbitally-excited resonances dominates over the decay into the broad ones, in apparent contradiction with some recent experiments. We discuss this issue. © 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

It is well known [1] that quark-hadron duality is valid to a good accuracy in b-quark decay and particularly in semileptonic decay. A systematic study of the corrections to duality [2–6] using the powerful tools of Operator Product Expansion (OPE) [7] and Heavy Quark Effective Theory (HQET), in particular Luke's theorem [8], has demonstrated that the first corrections to duality only appear at second order, namely $O(\Lambda^2/m_Q^2)$ where Λ is for the QCD scale and m_Q is one of the heavy quark masses (m_b or m_c). For simplicity we leave aside in this letter the $O(\alpha_s)$ radiative corrections notwithstanding their manifest practical relevance.

The OPE based proof is very elegant and circumvents the detailed calculation of the relevant channels. Precisely this feature has generated some doubts or at least some worries. First of all there is the experimental problem of the Λ_b life time which has not yet been understood within OPE framework. Second it has been asked if OPE could not miss some subtle kinematical effects related with the de-

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lay in the opening of different decay channels [9]. We have shown [10] in a non-relativistic model that the latter effect does not affect the validity of duality.

A numerical calculation of the sum over exclusive channels in the 't Hooft two dimensional QCD model [11] reported a presence of a duality-violating $1/m_Q$ correction in the total width [12]. Later the summation was performed analytically in the case of the massless light quark [13]. Agreement between the OPE and the exact result was found in this case through $1/m_Q^4$ order.

The ''miraculous'' conspiracy of exclusive decay channels to add up to the partonic result and its OPE corrections may be expressed in terms of sum rules which the hadronic matrix elements must satisfy in QCD [14–17]. OPE was first explicitly used to derive Bjorken sum rule in [15].

To leading order in Λ/m_b Bjorken sum rule straightforwardly implies quark hadron duality for the semileptonic widths (the differential and the total widths). The suppression of the $O(\Lambda/m_b)$ corrections is not so direct. The authors of [18] have done a thorough study of the exclusive contributions of the ground state D and D^* mesons up to order $O(\Lambda^2/m_b^2)$. They have chosen the Shifman Voloshin (SV) [19] limit, $\Lambda \ll m_b - m_c \ll m_b$, which drastically simplifies the calculation, but did not consider the orbitally excited states, and therefore could not check the matching between the sum of exclusive channels and the OPE prediction to the order $O(\Lambda(m_b - m_c)/m_b^2)$.

Our first motivation was precisely to complete this part and add the $L=1$ excited states in the sum of exclusive channels. We will discuss in Section 3 why we neglect other excitations.

While performing this task we had a surprise. We found that a *new sum rule*, Eq. (12), was needed beyond Bjorken, Voloshin, and the known tower of higher moment sum rules [14–17] and we found that this new sum rule could be demonstrated from OPE.

We believe that other new sum rules can be derived along the same line. When the form factors are taken at leading order in $1/m_b$, OPE applied to different components of the hadronic tensor, or to different operators, always provides the unique series: Bjorken sum rule, Voloshin sum rule and higher moments. But when the next to leading contribution to the form factors is considered, no such unicity holds anymore. Changing the current operators in the OPE might lead to several other sum rules at order $1/m_b$.

In the following we will simplify our task as much as possible. We will neglect radiative corrections. We will also leave aside terms of order $O(\Lambda^2/m_b^2)$, which implies that operators with higher dimension than identity may be neglected in the OPE and consequently that the inclusive results may be computed only via the partonic contribution.

In the next section we will show how the equality of partonic and inclusive widths to the desired order demands for a new sum rule. In Section 3 we will derive the latter sum rule from OPE applied to the T-product of currents. Finally in Section 4 we show interesting phenomenological consequences of the sum rule. We then conclude.

2. Inclusive semileptonic widths

We work in the SV limit [19], i.e. we assume the following hierarchy

$$\Lambda \ll \delta m \ll m_b \tag{1}$$

where $\delta m \equiv m_b - m_c$ and Λ is any energy scale stemming from QCD, for example the hadron-quark mass difference $\overline{\Lambda} \equiv m_B - m_b = m_D - m_c + O(1/m_b)$ or the excitation energy.

From OPE [3] one expects quark-hadron duality to be valid up to $O(\Lambda^2/m_b^2)$ corrections, i.e. in terms of the double expansion in $\delta m/m_b$ and Λ/m_b, it should be valid to all orders $(\delta m/m_b)^n$ and $(\delta m/m_b)^n \Lambda/m_b$. In fact we will restrict ourselves to check duality up to order $(\delta m/m_b)^2$ and $\delta m \Lambda/m_b^2$. The terms of order $\delta m \Lambda/m_b^2$ will turn out to be the trickiest. Of course, in the preceding sentences we mean orders as compared to the leading contribution. For example the inclusive semileptonic width is of order $(\delta m)^5$, which implies that we will compute it up to order $\overline{\Lambda}(\delta m)^6/m_b^2$. In this letter the symbol $\simeq$ will always refer to neglecting higher orders than those just mentioned. From OPE the partonic semileptonic decay width should equate the explicit sum of the corresponding exclusive decay widths up to $O(\Lambda^2/m_b^2)$ terms, i.e. [18]:

$$\Gamma\left(\overline{B} \to X_c l\nu\right) = \Gamma\left(b \to cl\nu\right) + O\left(\Lambda^2/m_b^2\right) \tag{2}$$

with the semileptonic partonic width

$$\Gamma(b \to cl\nu) = 32\, K\, (\delta m)^5 \times \left[\frac{2}{5} - \frac{3}{5}\frac{\delta m}{m_b} + \frac{9}{35}\frac{(\delta m)^2}{m_b^2} \right] \quad (3)$$

where

$$K = \frac{G_F^2}{192\pi^3} |V_{cb}|^2 \quad (4)$$

Using $M_B \simeq m_b + \overline{\Lambda}$ and $\delta M \equiv M_B - M_D \simeq \delta m$ we get

$$\Gamma\left(\overline{B} \to X_c l\nu\right) \simeq 32\, K\, (\delta M)^5 \times \left[\frac{2}{5} - \frac{3}{5}\frac{\delta M}{M_B} + \frac{9}{35}\frac{(\delta M)^2}{M_B^2} - \frac{21}{35}\frac{\overline{\Lambda}\delta M}{M_B^2} \right] \quad (5)$$

The ground state contribution is [18]

$$\Gamma\left(\overline{B} \to (D + D^*)\, l\nu\right) \simeq 32\, K\, (\delta M)^5 \left[\frac{2}{5} - \frac{3}{5}\frac{\delta M}{M_B} + \frac{11 - 8\rho^2}{35}\frac{(\delta M)^2}{M_B^2} - \frac{1}{10}\frac{a_+^{(1)}\delta M}{M_B^2} \right] \quad (6)$$

Strictly speaking nothing compels $a_+^{(1)}$ to be real and we must read $\Re[a_+^{(1)}]$ everywhere in this letter instead of $a_+^{(1)}$ and $\Re[\xi_3]$ instead of ξ_3. The contribution of the first orbitally excited states may be computed using results in [20]. We get

$$\Gamma\left(\overline{B} \to (D_1 + D_2^*)\, l\nu\right) \simeq 32\, K\, |\tau_{3/2}(1)|^2 \times \left[\frac{16}{35}\frac{(\delta M)^2}{M_B^2} - \frac{56}{35}\frac{\Delta_{3/2}\,\delta M}{M_B^2} \right] \quad (7)$$

for the states with total angular momentum of the light quanta $j = 3/2$ and $\tau_j(w)$ are the infinite mass limit form factors $B \to D^{**}$ as defined in [15]. In all this letter we use for any state n the notation

$$\Delta_n = M_n - M_0\,, \quad (8)$$

where 0 refers to the ground state.

$$\Gamma\left(\overline{B} \to (D_1^* + D_0^*)\, l\nu\right) \simeq 32\, K\, |\tau_{1/2}(1)|^2 \times \left[\frac{8}{35}\frac{(\delta M)^2}{M_B^2} - \frac{49}{35}\frac{\Delta_{1/2}\,\delta M}{M_B^2} \right] \quad (9)$$

for the lowest $j = 1/2$ states. It is often overlooked that $O(\delta m \Lambda / m_Q^2)$ corrections are also provided by $L = 1$ excited states but depending only on asymptotic form factors and level splittings.

To the order considered, quark-hadron duality of the semileptonic decay widths implies the equality of the r.h.s. of Eq. (5) with the sum of the r.h.s's of Eqs. (6), (7) and (9) to which we need to add the $L = 1$ radially excited states. Their contributions are identical to Eqs. (7) and (9) with the replacement $\tau_j \to \tau_j^{(n)}$ and $\Delta_j \to \Delta_j^{(n)}$. The terms proportional to $(\delta M / M_B)^2$ match thanks to Bjorken sum rule [14,15]:

$$\rho^2 - \frac{1}{4} = \sum_n \left[|\tau_{1/2}^{(n)}|^2 + 2|\tau_{3/2}^{(n)}|^2 \right] \quad (10)$$

From now on, unless specified, it is understood that the form factors are taken at $w = 1$. Taking into account Voloshin sum rule [16]

$$\overline{\Lambda} = \sum_n \left[2\, \Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2 + 4\, \Delta_{3/2}^{(n)} |\tau_{3/2}^{(n)}|^2 \right], \quad (11)$$

the matching of the terms of order $\Lambda \delta M / M_B^2$ leads to the requirement

$$a_+^{(1)} = 4 \sum_n \left[\Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2 - \Delta_{3/2}^{(n)} |\tau_{3/2}^{(n)}|^2 \right] \quad (12)$$

The sum rule (12) is the main result of this paper. The preceding lines can be taken as a derivation of the sum rule, since we simply have made explicit the result from OPE, Eq. (5). However, one might feel uncomfortable in view of the peculiarity of the SV kinematics, one might fear that some exception to OPE could happen there. Furthermore, as recalled in the introduction, OPE has been repeatedly submitted to various interrogations. Therefore, we will rederive in the next section the sum rule (12) in a less questionable manner.

Let us note that in the *vector current case,* we do not need the a_+ form factor. In that case, matching of the $(\delta M/M_B)^2$ and $\Lambda \delta M/M_B^2$ terms occurs thanks to Bjorken and Voloshin sum rule only - or conversely we can invoke duality to demonstrate these sum rules. In particular, it gives a demonstration of Voloshin sum rule just from the same duality requirement invoked by Isgur and Wise to derive Bjorken sum rule: the Voloshin sum rule comes from the matching of $\Lambda \delta M/M_B^2$ terms.

It is in the axial case or in the $V-A$ case (which corresponds to the sum of vector and axial contribution) that we need the new sum rule. More precisely, we can separate also the contributions with definite helicity of the lepton pair. In the transverse helicity case, there is still matching from just Bjorken and Voloshin sum rule. In fact the need for a new sum rule occurs in the axial current and for *longitudinal* helicity. We obtain indeed for the $\lambda=0$ helicity of the axial current:

$$\Gamma(b\to cl\nu)_{A,\lambda=0} \simeq 4K(\delta M)^5 \times\left[\frac{4}{3}-2\frac{\delta M}{M_B}+\frac{4}{5}\frac{(\delta M)^2}{M_B^2}-2\frac{\overline{\Lambda}\delta M}{M_B^2}\right] \tag{13}$$

$$\Gamma(\overline{B}\to D^* l\nu)_{A,\lambda=0} \simeq 4K(\delta M)^5\left[\frac{4}{3}-2\frac{\delta M}{M_B} +\left(1-\frac{4}{5}\rho^2\right)\frac{(\delta M)^2}{M_B^2}-\frac{4}{5}\frac{a_+^{(1)}\delta M}{M_B^2}\right] \tag{14}$$

$$\Gamma(\overline{B}\to D^{**} l\nu)_{A,\lambda=0} \simeq 4K(\delta M)^5\left[\frac{4}{5}\sum_n\left[|\tau_{1/2}^{(n)}|^2+2|\tau_{3/2}^{(n)}|^2\right]\frac{(\delta M)^2}{M_B^2} -\frac{28}{5}\sum_n\left[\Delta_{1/2}^{(n)}|\tau_{1/2}^{(n)}|^2+2\,\Delta_{3/2}^{(n)}|\tau_{3/2}^{(n)}|^2\right]\frac{\delta M}{M_B^2} +\frac{24}{5}\sum_n\left[\Delta_{1/2}^{(n)}|\tau_{1/2}^{(n)}|^2\right]\frac{\delta M}{M_B^2}\right] \tag{15}$$

whence we get the Eq. (12) from the matching of $\delta M/M_B^2$ terms.

3. Derivation of the sum rule from OPE

The authors of [21] have derived corrections to Bjorken and Voloshin sum rules and to the resulting *inequalities* on ρ^2. We will follow the same philosophy but including the orbitally excited states in order to derive $O(\Lambda/m_b)$ corrections, within our approximations, to the *equalities* resulting from the sum rules. We will use the differential semileptonic distributions [22].

Defining two currents which at present we take arbitrary:

$$J(x)\equiv(\bar{b}\Gamma c)(x),\quad J'(y)\equiv(\bar{c}\Gamma' b)(y). \tag{16}$$

Their T product is

$$T(q)\equiv i\int d^4x e^{-iqx}\langle\overline{B}|T(J(x)J'(0))|\overline{B}\rangle \tag{17}$$

where the states are normalised according to $\langle p|p'\rangle = (2\pi)^3\delta_3(\boldsymbol{p}'-\boldsymbol{p})$.

Neglecting heavy quarks in the ‘‘sea’’, it is clear that $x<0$ receives contributions from intermediate states with one c quark and light quanta, usually referred to as the direct channel, while $x>0$ receives contributions from intermediate states with $b\bar{c}b$ quarks plus light quanta. This will be referred to as the crossed channel, or Z diagrams. Expanding the r.h.s of (17) on intermediate states X in the B rest frame,

$$T=(2\pi)^3\left[\sum_X\delta_3(\boldsymbol{p}_X+\boldsymbol{q}) \times\frac{\langle\overline{B}|J(0)|X\rangle\langle X|J'(0)|\overline{B}\rangle}{M_B-q_0-E_X} -\sum_{X'}\delta_3(\boldsymbol{p}_{X'}-\boldsymbol{q}) \times\frac{\langle\overline{B}\overline{X}'|J(0)|0\rangle\langle 0|J'(0)|\overline{X}'\overline{B}\rangle}{M_B+q_0-(E_{X'}+2M_B)}\right] \tag{18}$$

where X,X' are charmed states. Let us call $\mathscr{V}$ the typical virtuality of the direct channels, M_B-q_0-

$E_X \simeq \mathscr{V}$, we will take q_0 such that $\Lambda \ll \mathscr{V} \ll M_B$. While the direct channels (X) contribute like $1/\mathscr{V}$ to (18), the crossed channels (X') contribute like $1/(m_D + \mathscr{V})$. In both cases the denominator is $\gg \Lambda$, which allows to use the leading contribution to OPE:

$$T = i\int d^4 x \mathrm{e}^{-iqx} \langle \overline{B}|\bar{b}(x)\Gamma S_c(x,0)\Gamma' b(0)|\overline{B}\rangle + O\left(1/m_c^2\right) \qquad (19)$$

where $S_c(x,0)$ is the free charmed quark propagator as long as $O(\alpha_s)$ corrections are neglected. Assuming as usual that the b quark has a momentum $p_b = m_b v + k$ with $k_\mu = O(\Lambda)$, the charmed quark propagator in (19) has two terms, the positive energy pole with a denominator $m_b v_0 + k_0 - q_0 - E_c \simeq \mathscr{V}$ and the negative energy one with a denominator $m_b v_0 + k_0 - q_0 + E_c \simeq m_c + \mathscr{V}$. Varying $\mathscr{V}$ independently of $m_b \simeq m_c$ one can check that the direct channels sum up to the contribution of the positive energy pole of the charmed quark propagator.

As a result, considering now only resonances among the states X and fixing $\boldsymbol{q}$ in the following, one gets equating the residues

$$\sum_n \langle \overline{B}|J(0)|n\rangle\langle n|J'(0)|\overline{B}\rangle = \langle \overline{B}|\bar{b}\Gamma \frac{\not{v}'_q + 1}{2v'_0}\Gamma' b|\overline{B}\rangle \qquad (20)$$

where all the three-momenta are equal to $-\boldsymbol{q}$ in the B rest frame and

$$v'_q = \frac{1}{m_c}\left(-\boldsymbol{q},\sqrt{\boldsymbol{q}^2 + m_c^2}\right) \qquad (21)$$

It is well known [15] that to leading order this leads to Bjorken sum rule. Considering successive moments, i.e. multiplying T in (17) by $(q_0 - E_0)^n$ (E_0 being the ground state energy) leads to a tower of sum rules [17], Voloshin sum rule when $n = 1$, etc.

In the following *we will stick to the $n = 0$ moment*, but include the $1/m_b$ correction to the residues. Let us insist on this point. One may discover a tower of sum rules by keeping the form factors to leading order but considering successive moments [17]. One may also discover new sum rules by sticking to the lowest moment but considering the higher orders in the form factors. This is not equivalent and leads to different sum rules, the first moment yields Voloshin sum rule Eq. (11), the second adds at least one new sum rule, (12), as we shall demonstrate now. The distinction is important since in practice both sum rules apply to the same order in $1/m_b$. A significant difference between the two types of subleading sum rules is the following: All the currents provide via OPE the same Voloshin sum rule because the form factors are all related by the heavy quark symmetry. On the contrary, when the form factors are taken at subleading order in $1/m_b$, different currents have different corrective terms depending on several independent form factors, and OPE should yield different subleading sum rules. In this letter we only consider Eq. (12) for its physical relevance, leaving other sum rules for a forthcoming study.

We now apply Eq. (20) with J, J' substituted by the vector current V^μ and the axial one A^μ. One may check that Eq. (20) applied to currents projected perpendicularly to the v,v' plane is trivially satisfied, including the $O(\Lambda/m_b)$ order, by Bjorken sum rule. Let us now consider the vector current projected on the B meson four velocity: $V \cdot v$. Among the orbitally excited states only the $J = 1$ states contribute to the wanted order. Dividing both sides of Eq. (20) by $(1 + w)/(2v_0 v'_0)$ one gets using the results of [20] and [23]

$$\frac{1+w}{2}|\xi(w)|^2 + \sum_n (w-1)\left\{2|\tau_{1/2}^{(n)}|^2\left[1 + \frac{\Delta_{1/2}^{(n)}}{m_b}\right] + (w+1)^2|\tau_{3/2}^{(n)}|^2\left[1 + \frac{\Delta_{3/2}^{(n)}}{m_b}\right]\right\} \simeq 1 + (w-1)\frac{\overline{\Lambda}}{m_b} \qquad (22)$$

where we have neglected higher powers of $(w-1)$ and of Λ/m_b than the first [3]. The l.h.s is found by a straightforward application of [23] for the ground state and of [20] for the excited ones. The r.h.s yields

[3] Remember that we take $\overline{\Lambda} \sim \Delta_j \sim \Lambda$

$(1+w_q)/(1+w)$ which has been transformed according to

$$w_q \equiv v.v'_q \simeq w + \boldsymbol{q}^2\left[\frac{1}{2m_c^2} - \frac{1}{2M_D^2}\right]$$
$$\simeq w + \frac{(w^2-1)\overline{\Lambda}}{m_b} \tag{23}$$

The leading terms in Eq. (22) simply reproduce Bjorken sum rule as expected [15], while the $O(\Lambda/m_b)$ terms provide Voloshin sum rule. This is *another derivation of Voloshin sum rule* which does not use higher momenta.

Analogously the axial current projected on the D meson velocity v', $A\cdot v'$ gives, inserted in Eq. (20) and after dividing both sides by $(w-1)/(2v_0v'_0)$,

$$\frac{1+w}{2}|\xi(w)|^2 - \frac{4}{m_b}\xi_3(w)\xi(w)$$
$$+\sum_n\left\{\left[2(w-1) - \frac{6(w+1)\Delta_{1/2}^{(n)}}{m_b}\right]|\tau_{1/2}^{(n)}|^2 + (w-1)(w+1)^2|\tau_{3/2}^{(n)}|^2\right\}$$
$$\simeq 1-(w+1)\frac{\overline{\Lambda}}{m_b} \tag{24}$$

where ξ_3 in the notations of [23] is equal to $-a_+^{(1)}/2$ used in [18]. The matching of the $1/m_b$ terms in Eq. (24) leads to the sum rule

$$\overline{\Lambda} + a_+^{(1)} = L_4(1) = +6\sum_n \Delta_{1/2}^{(n)}|\tau_{1/2}^{(n)}|^2 \tag{25}$$

L_4 being defined according to [23]. In words, this contribution comes from the ''small components'' of the Dirac spinors. Eliminating $\overline{\Lambda}$ from Eqs. (25) and (11) we are left with Eq. (12).

We can check this result by using the method for sum rules developed earlier by Bigi and the Minnesota group [24], which relies on a systematic $1/m_Q$ expansion of the moments of the Lorentz invariants of the imaginary part of the hadronic tensor, w_i. From their Eq. (131), we read

$$\int dq^0 w_2^{AA}(q^0,\boldsymbol{q}^2) \simeq \frac{m_b}{E_c} \tag{26}$$

the terms left over being the power corrections due to higher dimension operators. Computing from [18] and [20] the hadronic contribution to the same integral at $\boldsymbol{q}=0$ i.e. $w=1$, we get the equation (with $r_0 = M_D^*/M_B$, $r_{1/2,3/2} = M_{D^{**}_{1/2,3/2}}/M_B$)

$$\int dq^0 w_2^{AA}(q^0,\boldsymbol{q}^2=0)$$
$$= \frac{1}{r_0}\left\{\frac{f^2}{4M_B^2r_0^2} + \frac{(1-r_0)fa_+}{r_0}\right\}$$
$$+\left\{\frac{(1-r_{3/2})^2}{r_{3/2}^2}\left(\frac{k_{A_1}^2}{24} - \frac{f_A^2}{4}\right)\right.$$
$$+\frac{1}{4r_{1/2}^2}\left(\left[(1+r_{1/2})g_+ - (1-r_{1/2})g_-\right]^2\right.$$
$$\left.\left. -g_A^2\right)\right\} \tag{27}$$

with all form factors taken at $w=1$, and with notations for the $L=1$ form factors $g_+, g_-, g_A, f_A, k_{A_1}$ to be found in [20]. A sum over the $L=1$ excitations is understood. If we now work in the SV limit, we see that we need g_-, f_A, g_A, k_{A_1} only in the HQET limit, i.e. $\tau_{1/2,3/2}$, except for some algebraic factors; as for g_+, it is subleading, but at $w=1$, it is expressible in terms of $\tau_{1/2}$ and we do not need to know any of the new subleading form factors. In the $L=1$ contributions, only the g_+g_- term remains. We finally end with the equation:

$$\frac{M_B}{M_D} - \frac{\delta M}{M_B^2}a_+^{(1)} + 6\frac{\delta M}{M_B^2}\sum_n \Delta_{1/2}^{(n)}|\tau_{1/2}^{(n)}|^2 \simeq \frac{m_b}{m_c} \tag{28}$$

which leads directly to Eq. (25).

In the preceding calculations we have systematically neglected the contributions from higher orbital excitations or $L=0$ radial excitations. This can be justified as follows. The leading B transition to radially excited $L=0$ final states or to $L=2$ final states are suppressed by a factor $\boldsymbol{q}^2/m_b^2$ due to three facts: first, the current operator is proportional at leading order to the identity operator or to $\boldsymbol{\sigma}_b$ [4], second, the orthogonality of the wave functions implies vanishing at $\boldsymbol{q}=0$ in the B rest frame and, third, parity implies an even power in $\boldsymbol{q}$. This sup-

[4] The heavy quark spin may be factorised out thanks to HQS.

pression leads to the well known fact that these terms appear in the Bjorken sum rule or in the differential widths with a $(w-1)^2$ factor as compared to the ground state contribution. On the contrary the $O(\Lambda/m_b)$ contributions to the axial form factors for the same type of transitions are not suppressed as compared to the ground state because the current operator is no more proportional to identity neither to $\boldsymbol{\sigma}_b$. For example the transition to radially or orbitally excited $J^P=1^-$ states other than the D^* are in principle of the same order of magnitude than the $\propto a_+^{(1)}$ terms mentioned above. However, in this letter we have only considered the terms $\propto a_+^{(1)}$ via crossed terms, i.e. via cross products of the leading order terms with the $O(\Lambda/m_b)$ ones, because we have neglected all $O(\Lambda^2/m_b^2)$ contributions. Hence we are left with a suppression of a factor $\boldsymbol{q}^2/m_b^2$ in the hadronic tensors or the differential widths, i.e. a factor $(w-1)$ as compared to the corresponding ground state contribution and we can consequently neglect the $L=0$ radial excitations and the $L=2$ orbital ones. $L=3$ contributions are negligible simply because the total angular momentum $J \geq 2$ again leads to $(w-1)$ factors resulting from angular momentum conservation (D-waves). All other operators which are already negligible for the ground state and the $L=1$ states are even more so for higher excitations.

Turning now to a comparison of our different demonstrations, we should note that it is not really unexpected that we find consistent results according to three approaches: imposing duality to the widths (Section 2), imposing duality to the tensors as in Eqs. (22) and (24) and finally to the invariant tensors Eqs. (26) and (27). Indeed, at fixed q^0 and $\boldsymbol{q}$ there is a linear relation between the tensor components and the invariant tensors. It is as well true that the formula for the decay widths before integrating on the q^0 variable is, for fixed q^0 and $\boldsymbol{q}$, linear in the tensor components.

We might worry about what happens when we apply duality to the sum of the residues. Integration over q^0 leads to a sum of residues multiplied by δ functions and the position of the poles is different for each term in the sum and still different for the quark contribution. As a consequence the projector which projects out w_2 from the tensor residues is different for each term since it depends on q^0. Still this difference does not lead to a collapse of the sum rule thanks to Voloshin sum rule and the tower of higher momenta sum rules: one can expand the difference between the intervening projectors in powers of q^0 and the resulting alteration to the sum rule vanishes. Exactly the same happens when one computes the decay widths with the real kinematics on each term.

4. Phenomenological consequences

Eq. (25) is phenomenologically relevant as it expresses the dominant correction to the zero recoil differential $B \to Dl\nu$ decay width as a function of leading form factors and level spacings. Indeed

$$\frac{d\Gamma(B\to Dl\nu)}{dw} \propto (w^2-1)^{3/2} \times\left[1-2\left(\frac{1}{2m_b}+\frac{1}{2m_c}\right)\frac{M_B-M_D}{M_B+M_D}L_4(1)\right] \quad (29)$$

On the other hand, we may combine our result with an independent estimate of the form factor ξ_3 [25] from QCD sum rules [5]:

$$\frac{\xi_3(1)}{\overline{\Lambda}} = \frac{1}{3}+O(\alpha_s) = 0.6 \pm 0.2,$$
$$\frac{a_+^{(1)}}{\overline{\Lambda}} = -\frac{2}{3}-O(\alpha_s) = -1.2 \pm 0.4 \quad (30)$$

The dispersion formulation of the constituent quark model [26] finds that $\xi_3(1)$ is $1/3$ the average kinetic energy of the light quark. For a light constituent mass of $m_u = 0.25$ GeV it gives

$$\xi_3(1) = 0.17\,\mathrm{GeV}, \quad \overline{\Lambda} = 0.5\,\mathrm{GeV} \quad (31)$$

in perfect agreement with Eq. (30) for $\alpha_s = 0$.

Combining (11), (12) and (30), assuming $\alpha_s = 0$ since we have neglected radiative corrections all along this letter, we get

$$\frac{\sum_n \Delta_{1/2}^{(n)}\,|\tau_{1/2}^{(n)}|^2}{\sum_n \Delta_{3/2}^{(n)}\,|\tau_{3/2}^{(n)}|^2} = \tfrac{1}{4}, \quad \text{for} \quad \alpha_s = 0 \quad (32)$$

[5] The definitions of ξ_3 differ by a factor $\overline{\Lambda}$ in [23] and [25]. We use the notations of [23].

and

$$\sum_n \Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2 = \frac{1}{18}\overline{\Lambda},$$

$$\sum_n \Delta_{3/2}^{(n)} |\tau_{3/2}^{(n)}|^2 = \frac{2}{9}\overline{\Lambda} \qquad (33)$$

Notice that if we had, somehow inconsistently, taken $\xi_3(1)/\overline{\Lambda} = 0.6$ the result would not be qualitatively different.

Since in all spectroscopic models the mass differences between the $j = 1/2$ and $j = 3/2$ states turn out to be not so large, we conclude that the $\sum_n |\tau_{1/2}^{(n)}|^2$ *are significantly smaller than the* $\sum_n |\tau_{3/2}^{(n)}|^2$.

Interestingly enough, this hierarchy $|\tau_{1/2}^{(0)}|^2 < |\tau_{3/2}^{(0)}|^2$ was a clear outcome of a class of covariant quark models [27]. In [27] four different potentials had been used within the Bakamjian-Thomas covariant quark model framework. The potentials labeled ISGW, VD, CCCN, and GI potentials in [27] give respectively for the ratio $|\tau_{1/2}^{(0)}|^2/|\tau_{3/2}^{(0)}|^2$ the values 0.33, 0.09, 0.01 and 0.17. As a result these models predict a dominance of the $B \to D_{j=3/2} l\nu$ semileptonic decay widths by one order of magnitude over the $B \to D_{j=1/2} l\nu$. We will comment this prediction later. The same models [27] give for the l.h.s. of Eq. (32) 0.39, 0.166, 0.151 and 0.247 respectively for the ISGW, VD, CCCN, and GI potentials, in reasonable agreement with $1/4$. It might not be mere luck if the GI model, which fits the spectrum in the most elaborate way, yields an almost too good agreement with the expectation (32) [6]. From Eq. (30) we expect the r.h.s. of Eq. (12) divided by that of Eq. (11) to be close to $-2/3$. We have tested this with the numerical calculations of [27]. In all cases we find that the sums in the r.h.s of Eqs. (11) and (12) saturate very fast to their symptotic values. At $n = 3$ they are at less than 3% in all cases. For the ratios $a_+^{(1)}/\Lambda$ computed from the r.h.s of Eqs. (11) and (12) one finds -0.51, -0.77, -0.79, -0.67 respectively for the ISGW, VD, CCCN, and GI models. This agreement with (30) is quite striking, and again GI is embarrassingly good.

In more general terms, the prediction [27] that the B meson decays dominantly into the narrow resonances $j = 3/2$ was comforted by a study within a constituent quark-meson model [28] as well as by a semi-relativistic study [29]. A QCD sum rule analysis [30] predicted rather a rough equality between these form factors contrarily to another one [31] which concluded to an overwhelming dominance of the $j = 3/2$ semileptonic decay over the $j = 1/2$.

It is fair to say that the general trend of theoretical models is to predict $3/2$ dominance and a total semileptonic branching ratio into the orbitally excited states exceeding hardly 1 %. It is well known that the $j = 3/2$ are expected to be relatively narrow and are identified with the observed narrow resonances $D_1(2422)$ and $D_2^*(2459)$. As far as the decay widths into the latter narrow resonances is considered, experimental results [32] are in rough agreement with [27] for the $B \to D_1(2422) l\nu$ and rather below [27] for $B \to D_2^*(2422) l\nu$. In brief, experiment is rather below the theoretical models for $B \to D_{3/2} l\nu$. The $j = 1/2$ states are not easy to isolate, being very broad. But thorough studies have been done of the channels $B \to D^{(*)}\pi l\nu$ and the resulting branching fraction is very large: $3.4 \pm 0.52 \pm 0.32\%$ by DELPHI [33] and $2.26 \pm 0.29 \pm 0.33\%$ by ALEPH.

These experimental results are both welcome and puzzling. Welcome because these $B \to D^{(*)}\pi l\nu$ fill the gap between the inclusive semileptonic decay branching fraction of 10 - 11 % and the sum $B \to (D + D^*) l\nu \simeq 7\%$. They are puzzling when one tries to understand which channels contribute to them. As we have just said, the $j = 3/2$ channels provide no more than 1 %. The remaining 2 % can come from the $j = 1/2$, from higher excitations or from a non-resonant continuum. Higher excitations are unlikely to contribute very much, being suppressed both by dynamics and phase space. In [33] the quoted $b \to D^{**} l\nu$ branching fractions are very large, exceeding by far what is expected for example in [27].

The results presented in this letter are doubly relevant in the above discussion. First Eq. (32) seems to confirm the models which find a dominance of the $3/2$ channels. Of course it is mathematically possi-

[6] We should nevertheless remember that the potentials used in [27] contain a Coulombic part which implies that some part of the $O(\alpha_s)$ corrections might be implicit in these models.

ble that that Eq. (32) is satisfied while $|\tau^{(0)}_{1/2}| > |\tau^{(0)}_{3/2}|$, the higher excitations compensating for the sum rule. Admittedly such a situation would look rather queer, and as mentioned above, the models [27], which agree rather well with the new sum rule Eq. (12), also yield $|\tau^{(0)}_{1/2}|^2 < 0.35|\tau^{(0)}_{3/2}|^2$.

It is then hard to understand how the $b \to D^{**} l\nu$ branching fractions can be as large as quoted in [33] in view of the smallness of the experimental $B \to D_{3/2} l\nu$ branching fractions. However, the second lesson from our study is that $1/m_c$ corrections may play an important role, and a further study of their effect is wanted.

The most serious caveat to our present derivation of a narrow resonance dominance comes from the fact that we have neglected radiative corrections. A priori we expect radiative corrections to provide only corrections and our present estimate to yield the general trend. This is unhappily not always true. As a counterexample see the discussion which follows Eq. (7.8) in [18]. It is argued that some radiative corrections to the parameter K are parametrically larger than the $\alpha_s = 0$ estimate. A careful study of radiative corrections to our present sum rule and its consequences would be welcome.

It is not excluded that an important fraction of the $B \to D^{(*)}\pi l\nu$ decays observed at LEP are non-resonant. Unluckily theoretical works addressing non-resonant decays are rare, [34] find in the soft-pion domain a resonance dominance while Isgur [35] predicts that no more than 5 % of the semileptonic decay is non-resonant. Furthermore, if such a continuum contributes significantly, it should also be included in the sum rules [35] and we might fear that at the end of the day the paradox would still be there.

Finally another experimental result [36] seems to contradict our theoretical expectation: the branching ratio for $B \to D_1(j=1/2)\pi^-$ is found to be $\simeq 1.5$ times larger than that of $B \to D_1(2420)\pi^-$. Of course the experimental error is still large, and the relation between nonleptonic decays and the semileptonic ones assumes factorisation.

But still *there is a puzzle*: on one side an increasing amount of theoretical evidence in favor of the narrow resonances dominance, and on the other side an increasing amount of experimental evidence in the opposite direction!

5. Conclusion and outlook

We have explicitly checked quark-hadron duality in the SV limit to order $\delta m \Lambda / m_b^2$ including ground state final hadrons and $L=1$ orbitally excited states. We have shown that this duality implied a new sum rule Eq. (12) which we have also demonstrated from OPE applied to T-product of axial currents.

We have shown that this sum rule combined with some theoretical estimates of ξ_3 lead to the conclusion that very probably the B decay into narrow $L=1$ resonances was dominant over the one into broad resonances. This remark seems to contradict recent experimental claims that the broad resonances dominate. We have discussed this situation which needs urgently further theoretical and experimental work.

Beyond understanding this experimental puzzle, further theoretical work is needed. For example we might wonder if some proof of the new sum rule along the line of [14] is possible. Some progress has been done in this direction. The effect of radiative corrections should also be studied.

Last but not least, other new sum rules derived along the same line with other currents or other components of the currents should be considered.

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Duality in the non-relativistic harmonic oscillator quark model in the Shifman–Voloshin limit: A pedagogical example

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Abstract

The detailed way in which duality between sum of exclusive states and the free quark model description operates in semileptonic total decay widths, is analysed. It is made very explicit by the use of the non relativistic harmonic oscillator quark model in the SV limit, and a simple interaction current with the lepton pair. In particular, the Voloshin sum rule is found to eliminate the mismatches of order $\delta m/m_b^2$. © 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

Discussions have recently arisen about the possibility that expectations from OPE for some types of semi-leptonic rates may be violated by terms of order $1/m_Q$. The argument of Nathan Isgur [1] is founded on general considerations; namely the duality is obtained in the infinite mass limit through cancellation between the falloff of the ground state contribution and the rise of the excitations (the Bjorken sum rule indeed relates the derivatives of these contributions with respect to w, near $w=1$), but at finite mass there is some mismatch near zero recoil, which could be of order $1/m_Q$. Indeed, in terms of t, the quadri-dimensional transfer[3]:

$$t = \left(q^0\right)^2 - \boldsymbol{q}^2, \tag{1}$$

the respective $t_{\max}$ do not coincide anymore. The argument is then given by the author further likeliness by some calculations within a very simple 'toy' model: the non relativistic harmonic oscillator (HO) potential model.

In the present letter, we will not discuss directly the issue about QCD (see our article [3]). We simply stick to the very model used in [1], and show that within this model, calculating the *total integrated* rate $\Gamma_{\text{inclusive}}$ by summation on the relevant final (bound) states, duality with free quark decay rate is

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[3] t is we use the old standard notation t, to avoid confusion with the tridimensional $|\boldsymbol{q}|^2$, which will be used extensively in this non relativistic (NR) context.

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in fact satisfied, in the SV (Shifman–Voloshin [2]) limit[4]; this means that the difference $\Gamma_{\text{inclusive}} - \Gamma_{\text{free quark}}$ comes out as expected, which implies in particular (as discussed below) cancellation of terms of relative order $(\delta m)^2/m_b^2$ and $\delta m/m_b^2$ (by relative, we mean with respect to the free quark decay rate; note that such terms correspond to $(1/m_Q)^0$, $(1/m_Q)^1$ in the usual $1/m_Q$ expansion). Our argument is for integrated decay rates, so we do not claim anything on possible effects in differential or partially integrated rates. Also, of course, we cannot exclude by such argument that something odd may happen in QCD.

One very interesting point raised in the discussion of [1] is about the very specific cancellations which are necessary for duality to hold, and about the contributions of the various regions of phase space. We try to analyze through our demonstration how such cancellations occur in subleading order for total widths. An interesting consequence of the analysis is that to find the required cancellations, one needs not to consider only the sum rule of Bjorken; one has to take into account in addition the Voloshin sum rule (the fact that one needs the sum rules has been suggested by the Minnesota group in their discussion with N. Isgur [1], but is made here quite explicit; for related discussions in QCD by the same group, see [4]). In fact, the Voloshin sum rule is exactly what is needed for cancellation of terms of relative order $\delta m/m_b^2$ in the difference $\Gamma_{\text{inclusive}} - \Gamma_{\text{free quark}}$. The sum rules are trivially satisfied in the HO model, but it is not so trivial in general. Our conclusion is not in contradiction with the mismatch occuring near zero recoil, considered in [1], because the latter is very small parametrically with respect to the terms we consider in the total width.

Note that the use of SV limit is not essential to demonstrate duality in this way, and neither is the use of an HO potential. Their choice is pedagogical. Indeed we have also done the demonstration for an arbitrary potential ([5]) and also for fixed m_c/m_b ratio. Nevertheless, the particular case considered here is of pedagogical interest, because on the one hand the discussion in the SV limit is much simpler, and the similar discussion in QCD can hardly be made beyond the SV limit, and because on the other hand, within HO model, we can give explicit expressions. Moreover, we are able to give a complete proof that in the HO model $1/m_Q$ terms are absent in the ratio $\Gamma_{\text{inclusive}}/\Gamma_{\text{free quark}}$ beyond the SV limit (article to appear [6]). Note also that the demonstration is independent of the leptonic tensor, as we have also shown elsewhere, but we choose here one specific for illustration. On the other hand, the coefficient of the terms of order $\frac{1}{R^2 m_b^2}$, which we also evaluate, is model-dependent (in particular it depends on the choice of the leptonic tensor; we choose here one for illustration).

2. Model

● The model for hadrons is the non relativistic harmonic oscillator quark model (the motion of quarks both internal and due to overall hadron are both treated non relativistically), describing the initial (quarks b and d) and final (c and d) hadrons. The potential is assumed to be *flavor independent*, which is crucial for the demonstration. The great advantage of the harmonic oscillator, which appears in the summation on final states, is that very few states contributes to the transition rates in the limited expansion in $\frac{1}{m_b}$ which we perform (see next section). Energy levels, for a state labelled by (n_x, n_y, n_z), $n = n_x + n_y + n_z$, write:

$$E_n = m_{b,c} + m_d + \left(\tfrac{3}{2} + n\right) \frac{1}{\mu_{b,c} R_{b,c}^2}, \qquad (2)$$

where $\mu_{b,c}$ are the reduced masses $\frac{m_{b,c} m_d}{m_{b,c} + m_d}$ and the radii $R_{b,c}^2$ can be written as:

$$R_{b,c}^2 = \sqrt{\frac{m_d}{\mu_{b,c}}}\ R_\infty^2. \qquad (3)$$

[4] By SV limit, we do not mean simply that the recoil velocity is small, but also, as in the original paper [2], that $\frac{\delta m}{m}$ is small; in addition, δm is taken large with respect to light quark parameters; in non relativistic quantum mechanics, we assume: $\Delta = \frac{1}{m_d R^2} \ll \delta m$ and $m_d, \delta m \ll m_b, m_c$, Δ being the level spacing.

R_∞ being the radius in the infinite mass limit. We will often denote the first level excitation energy in the infinite mass limit as:

$$\Delta = \frac{1}{m_d R_\infty^2}. \tag{4}$$

For simplicity, from now on, we denote:

$$R_\infty = R. \tag{5}$$

● Quarks are then coupled to lepton pairs: $b \to c\ell\nu$, through a quark vector current $j^0 = 1$, $\boldsymbol{j} = 0$[5] (or equivalently we can speak of spinless quarks), and a leptonic tensor, which will be described by functions denoted generically through letter L and some arguments and indices. $\boldsymbol{P}$ and $\boldsymbol{P}'$ are the initial and final hadron momenta; the total energies of hadrons are $P^0 = E + \boldsymbol{P}^2/2(m_b + m_d), P'^0 = E' + \boldsymbol{P}'^2/2(m_c + m_d)$, with E, E' the energies at rest; but, in practice, we will always work in the initial hadron rest frame: $\boldsymbol{P} = 0$; $\boldsymbol{P}$ and $\boldsymbol{P}'$ are the initial and final quark momenta. We denote $\boldsymbol{q} = \boldsymbol{P} - \boldsymbol{P}' = -\boldsymbol{P}'$. The basic equation is then, in the initial state rest frame:

$$\Gamma_{\text{inclusive}} = K \sum_n \int_0^{|\boldsymbol{q}|_{max,n}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_n(|\boldsymbol{q}|) \times \sum_{n=n_x+n_y+n_z} |j_{0\to(n_x,n_y,n_z)}|^2. \tag{6}$$

The constant K depends only on the decay interaction strength. The constant K will be omitted in the rest of the letter. $\sum_{n=n_x+n_y+n_z} |j_{0\to(n_x,n_y,n_z)}|^2$ only depends on $|\boldsymbol{q}|$. The angular integration has been performed. The notations $L_n(|\boldsymbol{q}|)$ and $|\boldsymbol{q}|_{max,n}$ are explained now. A priori, after angular integration, the leptonic tensor appears through a function of energy loss q^0 and $\boldsymbol{q}^2$, $L(q^0, |\boldsymbol{q}|^2)$. However, for the decay from the ground state to a h.o. state labelled by (n_x, n_y, n_z), by energy conservation, $q^0 = P^0 - P'^0$ is just a function of $|\boldsymbol{q}|$ and (n_x, n_y, n_z). Moreover, the energy loss q^0 will depend only on $n = n_x + n_y + n_z$. and we then denote as $L_n(|\boldsymbol{q}|)$ the result of $L(q^0, |\boldsymbol{q}|^2)$, when the energy loss q^0 is assumed to be calculated for the corresponding n, as a function of $|\boldsymbol{q}|$. Indeed, for a state with degree of excitation n:

$$q^0(n, |\boldsymbol{q}|) = m_b - m_c + \frac{3}{2}\left(\frac{1}{\mu_b R_b^2} - \frac{1}{\mu_c R_c^2}\right) - n\frac{1}{\mu_c R_c^2} - \frac{|\boldsymbol{q}|^2}{2(m_c + m_d)}. \tag{7}$$

Now $q_{\max}$ is determined by the equation $t = (q^0)^2 - |\boldsymbol{q}|^2 = 0$, $q^0(|\boldsymbol{q}|) = |\boldsymbol{q}|$:

$$|\boldsymbol{q}|_{max,n} = \frac{2(m_c + m_d)(\delta E)_n}{2(m_c + m_d) + \sqrt{(m_c + m_d)^2 + 2(m_c + m_d)(\delta E)_n}} \tag{8}$$

where

$$(\delta E)_n = q^0(n, \boldsymbol{q} = 0) = m_b - m_c + \frac{3}{2}\left(\frac{1}{\mu_b R_b^2} - \frac{1}{\mu_c R_c^2}\right) - n\frac{1}{\mu_c R_c^2}. \tag{9}$$

$|\boldsymbol{q}|_{\max}$ just depends on n. $L(q^0, |\boldsymbol{q}|^2)$ can be taken as an arbitrary function without spoiling any of the general statements made below, but for definiteness we will henceforth choose:

$$L(q^0, \boldsymbol{q}^2) = 3(q^0)^2 - |\boldsymbol{q}|^2, \tag{10}$$

inspired by a static quark approximation of the V-A current. The corresponding free quark decay rate is:

$$\Gamma_{\text{free}} = K \int_0^{|\boldsymbol{q}|_{\max,\text{free}}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L(q^0, |\boldsymbol{q}|^2), \tag{11}$$

with:

$$q^0(\text{free}, |\boldsymbol{q}|) = m_b - m_c - \frac{|\boldsymbol{q}|^2}{2m_c}, \tag{12}$$

$$|\boldsymbol{q}|_{\max,\text{free}} = \frac{2m_c\,\delta m}{\left(m_c + \sqrt{m_c^2 + 2m_c\,\delta m}\right)}, \tag{13}$$

with $\delta m = m_b - m_c$.

[5] Note that we do not claim to make a systematic non relativistic *expansion* of a relativistic theory, but only to consider a non relativistic Hamiltonian for the bound states; we can choose freely the weak interaction current. The essential point is then to treat consistently the matrix elements according to the chosen interactions, in the specified SV expansion.

3. SV expansion and demonstration of duality

● We have then to consider the expansion of

$$\epsilon = \frac{\Gamma_{\text{inclusive}} - \Gamma_{\text{free}}}{\Gamma_{\text{free}}}, \tag{14}$$

in powers of $\frac{1}{m_b}$, and the aim is in principle to show that it begins with order $\frac{1}{m_b^2}$ only, as expected from a formal OPE (the NR version of OPE will be explained in the more developed article). More precisely this holds in the limit $m_b \to \infty$ with $r = \frac{m_c}{m_b}$ fixed, for which we reserve for clarity the term 'usual $1/m_Q$ expansion'. However, we will work in the SV (Shifman–Voloshin) limit, which corresponds to making in addition an expansion in $1 - r$. Namely, with:

$$\delta m = m_b - m_c, \tag{15}$$

we write $m_c = m_b - \delta m$ and we expand in powers of $\frac{1}{m_b}$, keeping δm fixed, as well as the light quark parameters, m_d, $1/R$; then, we make a second limited expansion, taking $\Delta = \frac{1}{m_d R^2}$ small with respect to δm. The terms have the form $\frac{(m_b)^k}{(\delta m)^{k'}}$ times light quark factors. But then the aim must be more than just showing the absence of powers[6] $\frac{1}{(m_b)^k}$, $k < 2$ in ϵ.

Indeed, if it is true, this would not in principle preclude terms of the type $\frac{x(\delta m)^{k'}}{m_b^2}$ $(k' > 0)$ in ϵ. Such terms would be large in practice, since δm is not so small. And in fact, they would correspond, in terms of the usual $1/m_Q$ expansion, to terms of order $(1/m_Q)^0$,$(1/m_Q)$, since δm would be then $\propto m_Q$. Such terms are not expected from OPE. We must therefore show that such terms do not exist in the final result, and we show it in fact. More precisely, we show that potentially large terms of the type $\frac{(\delta m)^2}{m_b^2}$, $\frac{m_d \delta m}{m_b^2}$, which appear in particular contributions, do finally cancel out, leaving us with terms of the type $\frac{1}{R^2 m_b^2}$ (terms with $k' > 2$ simply do not appear in the way we calculate ϵ, neither do terms with power $\frac{1}{(m_b)^0}$ or $\frac{1}{m_b}$ - in fact, the delicate part consists in showing the cancellation of $\frac{m_d \delta m}{m_b^2}$ terms). This is all that is required by duality with free quarks, as concerns the terms with power $(\frac{1}{m_b})^k$, $k \le 2$. We will calculate the terms of type $\frac{1}{R^2 m_b^2}$, which do not vanish in general. Note that such terms are small with respect to $\frac{m_d \delta m}{m_b^2}$ by a factor $\frac{\Delta}{\delta m}$. In the usual $1/m_Q$ expansion they correspond to order $1/m_Q^2$. On the other hand, we will not calculate in the expansion of ϵ smaller terms having also the power $\frac{1}{m_b^2}$, but which contain still additional powers of $\frac{\Delta}{\delta m}$ with respect to $\frac{1}{R^2 m_b^2}$, corresponding in $\Gamma_{\text{inclusive}} - \Gamma_{\text{free}}$ to terms like $\frac{(\delta m)^4 \times \Delta}{m_b^2}$, $\frac{(\delta m)^3 \times \Delta^2}{m_b^2}$, etc.. and retain only the terms proportional to $\Gamma_{\text{free}} \propto (\delta m)^5$. The neglected terms correspond to terms of relative order $1/m_Q^3$ or beyond in the $1/m_Q$ expansion. For sake of simplicity, we will neither examine further checks of duality in terms of the type $\frac{(\delta m)^{k'}}{(m_b)^k}$, with $k > 2$.

In any case, we see that we do have to calculate terms with a power $\frac{1}{m_b^2}$ and not $\frac{1}{m_b}$ only, since the terms with a power $\frac{1}{m_b^2}$ may correspond to terms of the order $(1/m_Q)^0$,$(1/m_Q)^1$ in the usual expansion. The method precisely consists in writing the difference $\Gamma_{\text{inclusive}} - \Gamma_{\text{free}}$ as a sum of terms which contain already a power $\frac{1}{m_b^2}$, and then to demonstrate the above additional cancellations.

● The advantage of harmonic oscillator (HO) model is that the level $n = 1$ (which corresponds to $L = 1$ states) appears only with a power $\frac{1}{m_b^2}$, and that higher levels come only with a power $\frac{1}{m_b^3}$ at least. Since we keep terms with a power $\frac{1}{m_b^i}$, $i \le 2$, we only need consider $n = 0{,}1$ states. For sake of simplicity, we denote their respective contributions $\Gamma_{0,1}$.

[6] Note that, in the present letter, we term generically as *power* $\frac{1}{(m_b)^k}$ all the terms which contain the factor $\frac{1}{(m_b)^k}$, whatever the powers of δm and light quantities.

We have at this order, by expanding the matrix elements:

$$\Gamma_0 \simeq \int_0^{|\boldsymbol{q}|_{\max,0}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{n=0}(|\boldsymbol{q}|)\left(1-\rho^2\frac{|\boldsymbol{q}|^2}{m_b^2}\right), \tag{16}$$

where $\rho^2 = \frac{1}{2}m_d^2 R^2$ is the standard slope of the ground state form factor with respect to w ($w-1 \simeq \frac{1}{2}\frac{|\boldsymbol{q}|^2}{m_b^2}$); note that effect of non complete overlapping between hadrons with b and c quarks is completely negligible here, since it contributes at order $1/R^2\frac{(\delta m)^2}{m_b^4}$. For $L=1$ states:

$$\Gamma_1 \simeq \int_0^{|\boldsymbol{q}|_{\max,1}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{n=1}(|\boldsymbol{q}|)\,\tau^2\,\frac{|\boldsymbol{q}|^2}{m_b^2}, \tag{17}$$

with $\tau = \frac{m_d R}{\sqrt{2}}$ corresponding to the $\tau_{1/2,3/2}(w=1)\times\sqrt{3}$. The other excitations do not contribute at this order, because the matrix element $< n|\boldsymbol{r}|0 >$ is non zero only if $n=1$. From the explicit expressions, we have the relations:

$$\rho^2 - \tau^2 = 0, \tag{18}$$

$$\Delta\tau^2 = \frac{m_d}{2}, \tag{19}$$

(Δ being the level spacing, Eq. (4)) as non relativistic analogues of the Bjorken and Voloshin sum rules. These sum rules could then be used to generalise the present analysis. In fact, we will try as much as possible not to specify separately Δ,ρ,τ, but to use only the above sum rules and expressions for $\Gamma_{0,1}$.

● The strategy is to note that the difference between $\Gamma_0+\Gamma_1$ and Γ_{free} can be reexpressed by successive steps:

1) Decomposition into the same difference with $L_{0,1}(|\boldsymbol{q}|)$ substituted by their free counterpart $L_{\text{free}}(|\boldsymbol{q}|)$ (contribution (I)) plus a $\frac{1}{m_b^2}$ term (contribution (II)).
2) Then the first contribution (I) is rewritten trivially as a difference between two contributions having a power $\frac{1}{m_b^2}$ relative to the free quark decay integrand, further shown to be of relative order $\frac{1}{R^2 m_b^2}$ or smaller.
3) It is also shown that in contribution (II), which contains manifestly a power $\frac{1}{m_b^2}$, there are only terms of the type $\frac{1}{R^2 m_b^2}$ or smaller.

● In a first step, using the respective expressions given above for the $q^0(|\boldsymbol{q}|)$'s, and expanding it to the required order, we find:

$$q^0(\text{free},|\boldsymbol{q}|) \simeq \delta m - \frac{|\boldsymbol{q}|^2}{2m_c}, \tag{20}$$

$$q^0(n=0,|\boldsymbol{q}|) \simeq \delta m\left(1-\frac{3}{4m_b^2R^2}\right) - \frac{|\boldsymbol{q}|^2}{2(m_c+m_d)}, \tag{21}$$

$$q^0(n=1,|\boldsymbol{q}|) \simeq \delta m - \Delta. \tag{22}$$

Note that in our expansion, the main term in the three quantities is δm. The main term of $q_{\max}$ is then also δm ($q_{\max} = q^0$ at $t=0$). We use these expansions to make, in the integrals for $\Gamma_{0,1}$:

$$L_0(|\boldsymbol{q}|) \simeq L_{\text{free}}(|\boldsymbol{q}|) + 6\delta m\left(-\frac{3\delta m}{4R^2m_b^2} + \frac{m_d|\boldsymbol{q}|^2}{2m_b^2}\right), \tag{23}$$

$$L_1(|\boldsymbol{q}|) \simeq L_{\text{free}}(|\boldsymbol{q}|) + 6\delta m(-\Delta) + 3\Delta^2. \tag{24}$$

The second terms in the r.h.s. come from the difference between the respective q^0, as a function of $|\boldsymbol{q}|$. Note that in the expansion of $L_1(|\boldsymbol{q}|)$, one can neglect terms in $\frac{1}{m_b^2}$ because Γ_1 has already a power $\frac{1}{m_b^2}$. We get $\Gamma_{\text{inclusive}} - \Gamma_{\text{free}} \simeq \delta\Gamma_I + \delta\Gamma_{II}$ with:

$$\delta\Gamma_I = \int_0^{|\boldsymbol{q}|_{\max,0}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{\text{free}}(|\boldsymbol{q}|)\left(1-\rho^2\frac{|\boldsymbol{q}|^2}{m_b^2}\right) + \int_0^{|\boldsymbol{q}|_{\max,1}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{\text{free}}(|\boldsymbol{q}|)\,\tau^2\frac{|\boldsymbol{q}|^2}{m_b^2} - \int_0^{|\boldsymbol{q}|_{\max,\text{free}}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{\text{free}}(|\boldsymbol{q}|), \tag{25}$$

and

$$\delta\Gamma_{II} = \int_0^{|\boldsymbol{q}|_{\max,0}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, 6\delta m\left(-\frac{3\delta m}{4R^2 m_b^2} + \frac{m_d|\boldsymbol{q}|^2}{2m_b^2}\right) + \int_0^{|\boldsymbol{q}|_{\max,1}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\left(6\delta m(-\Delta) + 3\,\Delta^2\right) \times \left(\tau^2\,\frac{|\boldsymbol{q}|^2}{m_b^2}\right). \tag{26}$$

● Contribution I. One can write it as the difference of two integrals which *have already manifestly a factor* $\frac{1}{m_b^2}$, i.e. the terms with power $\frac{1}{(m_b)^0}$ or $\frac{1}{m_b}$ are already cancelled (this amounts to using $\rho^2 - \tau^2 = 0$, which is the particular form of the Bjorken sum rule in the model):

$$\delta\Gamma_I = \int_{|\boldsymbol{q}|_{\max,\text{free}}}^{|\boldsymbol{q}|_{\max,0}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{\text{free}}(|\boldsymbol{q}|) - \int_{q_{\max,1}}^{|\boldsymbol{q}|_{\max,0}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{\text{free}}(|\boldsymbol{q}|)\,\tau^2\,\frac{|\boldsymbol{q}|^2}{m_b^2}. \tag{27}$$

We first expand each integral. - One has:

$$|\boldsymbol{q}|_{\max,0} - |\boldsymbol{q}|_{\max,\text{free}} \simeq \delta m\left(\frac{1}{2}\,\frac{m_d\delta m}{m_b^2} - \frac{3}{4}\,\frac{1}{R^2 m_b^2}\right), \tag{28}$$

whence

$$\int_{|\boldsymbol{q}|_{\max,\text{free}}}^{|\boldsymbol{q}|_{\max,0}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{\text{free}}(|\boldsymbol{q}|) \simeq \delta m\left(\frac{1}{2}\,\frac{m_d\delta m}{m_b^2} - \frac{3}{4}\,\frac{1}{R^2 m_b^2}\right) \times (\delta m)^2\, L_{\text{free}}(|\boldsymbol{q}|_{\max} = \delta m). \tag{29}$$

One can make $|\boldsymbol{q}| \simeq \delta m$ in the integrand, because the integration interval contains already a power $1/m_b^2$, and the difference between $|\boldsymbol{q}|$ and δm contains a further $1/m_b$ factor.

- The second integral is more delicate, because the integration interval has not a factor $1/m_b^2$; it is just $\simeq \Delta$; the variation of $|\boldsymbol{q}|$ is not negligible. One must do a limited expansion of the integrand in powers of $\frac{\Delta}{\delta}$, so as to retain at least terms of the type $\frac{1}{R^2 m_b^2}$. It is there that the second expansion,in powers of $\frac{\Delta}{\delta}$, enters the game :

$$\int_{|\boldsymbol{q}|_{\max,1}}^{|\boldsymbol{q}|_{\max,0}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{\text{free}}(|\boldsymbol{q}|)\,\tau^2\,\frac{|\boldsymbol{q}|^2}{m_b^2} \simeq \int_{\delta m-\Delta}^{\delta m} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{\text{free}}(|\boldsymbol{q}|)\,\tau^2\,\frac{|\boldsymbol{q}|^2}{m_b^2} \simeq \Delta(\delta m)^2\, L_{\text{free}}(|\boldsymbol{q}| = \delta m)\,\tau^2\,\frac{(\delta m)^2}{m_b^2} - \frac{\Delta^2}{2}\,\frac{\tau^2}{m_b^2}\,\frac{d}{d|\boldsymbol{q}|}\left(|\boldsymbol{q}|^4\, L_{\text{free}}(|\boldsymbol{q}|)\right)(|\boldsymbol{q}| = \delta m). \tag{30}$$

Let us note that to estimate the relative order of the different terms, one has to divide by a reference rate, which will be taken to be the free quark decay rate; now $(\delta m)^3\, L_{\text{free}}(|\boldsymbol{q}| = \delta m)$, as well as $\frac{d}{d|\boldsymbol{q}|}(|\boldsymbol{q}|^4$-$L_{\text{free}}(|\boldsymbol{q}|))(|\boldsymbol{q}| = \delta m)$, are of the order of the free quark decay rate (with our choice $L_{\text{free}}(|\boldsymbol{q}| = \delta m) \propto (\delta m)^2$).

Then, one can first observe that in fact not only all the terms written in Eq. (29), (30) have a relative power $1/m_Q^2$, but that they are more precisely of relative order $m_d\delta m/m_b^2$ at most; terms of relative order $(\delta m)^2/m_b^2$ are already cancelled. This will be obtained more generally thanks to Bjorken sum rule. Now, the term of relative order $m_d\delta m/m_b^2$ encountered in the r.h.s. of the first integral (29) is cancelled by the first term in the r.h.s. of the second integral (30), just using Voloshin sum rule (19), i.e. $\Delta\tau^2 = m_d/2$. All the remaining contributions are of the type $(\delta m)^5\,\frac{1}{R^2 m_b^2}$. We can evaluate them readily and find them to cancel too *for the particular choice* made for $L(q^0,|\boldsymbol{q}|)$. Finally:

$$\delta\Gamma_I = \int_{|\boldsymbol{q}|_{\max,\text{free}}}^{|\boldsymbol{q}|_{\max,0}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{\text{free}}(|\boldsymbol{q}|) - \int_{|\boldsymbol{q}|_{\max,1}}^{|\boldsymbol{q}|_{\max,0}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{\text{free}}(|\boldsymbol{q}|)\,\rho^2\,\frac{|\boldsymbol{q}|^2}{m_b^2} \simeq 0 \tag{31}$$

It must be emphasized that the cancellation can occur because the difference between $|\boldsymbol{q}|_{\max,n}$ and $|\boldsymbol{q}|_{\max,\text{free}}$ is changing sign between the ground state and the excitations. With our assumption $\Delta \ll \delta m$, one has $|\boldsymbol{q}|_{\max,1} < |\boldsymbol{q}|_{\max,\text{free}} < |\boldsymbol{q}|_{\max,0}$

● Contribution II. It is also obvious that it contains already a power $\frac{1}{m_b^2}$. On factorising $(\delta m)^5$, one sees that $\frac{m_d \delta m}{m_b^2}$ terms are present in the first integral (second term of the bracket in the integrand): $\int_0^{|\boldsymbol{q}|_{\max,0}} dq\, q^2\,(6\delta m \frac{m_d q^2}{2m_b^2}))$ and in the second one (first term of the bracket in the integrand): $\int_0^{|\boldsymbol{q}|_{\max,0}} dq$-$q^2\,(6\delta m(-\Delta)(\tau^2 \frac{|\boldsymbol{q}|^2}{m_b^2})))$, the rest being smaller. It is easily seen that these $\frac{m_d \delta m}{m_b^2}$ terms cancel at this order, just using Voloshin sum rule $\Delta\tau^2 = m_d/2$, to leave a smaller contribution, which is only of order $\frac{1}{R^2 m_b^2}$; the latter is found by performing a limited expansion of the integrand as above Eq. (29) (the interval is once more $\mathcal{O}(\Delta)$), in powers of $\frac{\Delta}{\delta m}$:

$$3\frac{m_d\,\delta m}{m_b^2}\int_{|\boldsymbol{q}|_{\max,1}}^{|\boldsymbol{q}|_{\max,0}} d|\boldsymbol{q}|\,|\boldsymbol{q}|^4 \simeq 3\,(\,\delta m)^5\,\frac{1}{R^2 m_b^2}\,. \tag{32}$$

The other terms in the integrals are already manifestly of this order, and one ends with:

$$\delta\Gamma_{II} = \tfrac{9}{5}\,(\,\delta m)^5\,\frac{1}{R^2 m_b^2}\,. \tag{33}$$

This result has been checked by a systematic expansion using Mathematica.

Finally, with $\Gamma_{\text{free}} \simeq \frac{4}{5}(\delta m)^5$:

$$\epsilon = \frac{\Gamma_0 + \Gamma_1 - \Gamma_{\text{free}}}{\Gamma_{\text{free}}} \simeq \frac{\frac{9}{5}(\,\delta m)^5\,\frac{1}{R^2 m_b^2}}{\frac{4}{5}\,(\,\delta m)^5} = \frac{9}{4}\,\frac{1}{R^2 m_b^2}\,. \tag{34}$$

Let us reinsist that it is of the order expected from OPE, unlike terms of the type $\frac{(\delta m)^2}{m_b^2}$ or $\frac{m_d \delta m}{m_b^2}$, which duely cancel, as has been shown.

4. Relative magnitude of Isgur contribution

● Let us now return briefly to the very discussion raised by Ref. [1]. One could be worried why it is found there some duality violating effect, while we do not. The contradiction is only apparent. The answer seems to be that in *totally integrated widths*, the effect considered in [1] is finally relatively small parametrically with respect to the ones we have considered. Let us show that. The mismatch near zero recoil considered in [1] is the integral of the ground state contribution over $w_0(t) = \frac{m_B^2 + m_D^2 - t}{2 m_B m_D}$ between $w_0(t=(m_B - m_D)^2) = 1$ and the threshold for the excited state production $w_0(t=(m_B - m_{D^{**}})^2)$ (the variable w for the ground state contribution is considered as a function of t, $w_0(t)$). Let us pass through the variable $\boldsymbol{q}$, which is more adapted to the NR problem, and denote as $|\boldsymbol{q}|_n(t)$ the value of $|\boldsymbol{q}|$ which corresponds to some t for a state n; the total ground state contribution can be decomposed into two parts:

$$\begin{aligned}\Gamma_0 &\simeq \int_{|\boldsymbol{q}|_0(t=(m_B-m_D)^2)}^{|\boldsymbol{q}|_0(t=0)} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2 \\ &\quad\times L_{n=0}(|\boldsymbol{q}|)\left(1-\rho^2\,\frac{|\boldsymbol{q}|^2}{m_b^2}\right) \\ &= \int_{|\boldsymbol{q}|_0(t=(m_B-m_D)^2)}^{|\boldsymbol{q}|_0(t=0)} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\,L_{n=0}(|\boldsymbol{q}|) \\ &\quad - \int_{|\boldsymbol{q}|_0(t=(m_B-m_D)^2)}^{|\boldsymbol{q}|_0(\,t=(0)} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\,L_{n=0}(|\boldsymbol{q}|) \\ &\quad \times\left(\rho^2\,\frac{|\boldsymbol{q}|^2}{m_b^2}\right).\end{aligned} \tag{35}$$

In the infinite mass limit, $m_B - m_{D^{**}} \simeq m_B - m_D \simeq m_b - m_c$ and the functions $|\boldsymbol{q}|_0(t)$ and $|\boldsymbol{q}|_1(t)$, as well as $|\boldsymbol{q}|_{\text{free}}(t)$, become identical, and the functions $L(q^0, \boldsymbol{q}^2)$ become also identical for all states. Then, the first contribution equates the free quark decay rate, while the second one:

$$\begin{aligned}\delta\Gamma_0 &\simeq \int_{|\boldsymbol{q}|_0(t=(m_B-m_D)^2)}^{|\boldsymbol{q}|_0(t=0)} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\,L_{n=0}(|\boldsymbol{q}|) \\ &\quad\times\left(-\rho^2\,\frac{|\boldsymbol{q}|^2}{m_b^2}\right),\end{aligned} \tag{36}$$

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is exactly cancelled by the excited state contribution:

$$\Gamma_1 \simeq \int_{|\boldsymbol{q}|_1(t=(m_B-m_{D^{**}})^2)}^{|\boldsymbol{q}|_1(t=0)} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{n=1}(|\boldsymbol{q}|)\,\tau^2\,\frac{|\boldsymbol{q}|^2}{m_b^2}, \tag{37}$$

due to Bjorken sum rule. Whence duality. However, when quark masses are finite, there is a small part of the integral (36) which is uncancelled, in spite of the Bjorken sum rule, by the corresponding excited state contribution, in particular because $t=(m_B-m_{D^{**}})^2$ now differs from $t=(m_B-m_D)^2$. We estimate the mismatch as:

$$\delta\Gamma \simeq \int_{|\boldsymbol{q}|_0(t=(m_B-m_D)^2)}^{|\boldsymbol{q}|_0(t=(m_B-m_{D^{**}})^2)} d|\boldsymbol{q}|\,|\boldsymbol{q}|^2\, L_{n=0}(|\boldsymbol{q}|) \times\left(-\rho^2\frac{|\boldsymbol{q}|^2}{m_b^2}\right). \tag{38}$$

In this calculation, following [1], we disregard all other sources of difference, in particular the fact that the leptonic tensor functions are no more equal, and neither are the functions $|\boldsymbol{q}|_n(t)$ for $n=0$ and $n=1$ respectively, and that also the first contribution in 35 no longer equates the free quark decay rate. Then, our point is that this mismatch of total widths is very small with respect to the terms we have retained. Indeed, the integral runs over a small part of the phase space, but in addition the integrand is much smaller near zero recoil, where the mismatch takes place, first because of the leptonic factor $L(q^0,\boldsymbol{q}^2)=3(q^0)^2-|\boldsymbol{q}|^2$, second because of the factor $(-\rho^2\frac{|\boldsymbol{q}|^2}{m_b^2})$. Since $L(q^0,\boldsymbol{q}^2)=3(q^0)^2-|\boldsymbol{q}|^2$, using $|\boldsymbol{q}|_0(t=(m_B-m_{D^{**}})^2)\simeq\sqrt{\frac{2\Delta}{\delta m}}|\boldsymbol{q}|_{0,\max}$:

$$\delta\Gamma \simeq \frac{\rho^2}{m_b^2}3(\delta m)^2\,\frac{\left(\frac{2\Delta}{\delta m}\right)^{\frac{5}{2}}}{5}|\boldsymbol{q}|^5_{0,\max}, \tag{39}$$

and, relative to the free quark decay rate (i.e. contribution to ϵ):

$$\frac{\delta\Gamma}{\Gamma_{\text{free}}} \simeq \frac{\rho^2}{m_b^2}\,\frac{3}{4}\,(\delta m)^2\left(\frac{2\Delta}{\delta m}\right)^{\frac{5}{2}}, \tag{40}$$

which is parametrically small, because of the factor $(\frac{2\Delta}{\delta m})^{\frac{5}{2}}$ (since $\Delta \ll \delta m$ in the SV limit). In fact, in our calculation we have not retained such terms.

Numerically too, we find it very small, with real physical masses. It is true, as noticed in Ref. [1], that numerically the region of Dalitz plot which is concerned is physically not very small, because one is far from the SV limit; with our approximative formula, we find around 20% of the free decay rate in this region of phase space, not far from the 30% estimated in Ref. [1]; but the factors considered above nevertheless combine to yield a very small effect for $\frac{\delta\Gamma}{\Gamma_{\text{free}}}$, around $10^{-3}\rho^2$. This is due to the fact that the factor $(-\rho^2\frac{|\boldsymbol{q}|^2}{m_b^2})$ is very small in this region of phase space.

5. Conclusion

Stimulated by the worries raised by N. Isgur, we have noticed mismatches between the sum of exclusive decays and the free quark total decay rate, which, considered separately, could convey the impression that quark hadron duality between total widths is violated at order $\delta m/m_b^2$, because all these mismatches are of this order. Let us recapitulate them:

1) The upper limit in terms of $|\boldsymbol{q}|$ (corresponding to $t=0$) of the integrals for the ground state and the excited states contributions do not coincide. Therefore, the contributions from the falloff of ground state and rise of excited states do not cancel near $|\boldsymbol{q}|_{\max}$ $(t=0)$.

2) The upper limit in $|\boldsymbol{q}|$ of the integrals for the ground state contribution and the free quark decay do not coincide for similar reasons.

3) The leptonic tensors of the various contributions are different, because the function $q^0(|\boldsymbol{q}|)$ depends on the transition considered.

At order $\mathcal{O}(\frac{\delta m}{m_b^2})$, 1) and 2) cancel between each other, while 3) has a zero net effect, by internal cancellation of the differences of leptonic tensors, when integrated (taking into account the difference in upper limits of integration in $|\boldsymbol{q}|$, near maximum recoil, is once more necessary).

It must be emphasized that even in this simple model and in the SV limit, it is by no means trivial to check duality, because the check requires to take into account detailed effects, such as the dependence of ground state binding energy on the heavy quark masses through their different radii, which itself reflects the flavor independence of the quark potential, etc..

In both cases, the cancellation occurs because of *Voloshin sum rule*. The consideration of it is absolutely necessary, in addition to Bjorken one, to demonstrate duality of total widths through summation of exclusive states at subleading order. Of course, in the simple model considered, the two sum rules are trivially satisfied. Note that, if we have an independent mean to demonstrate duality, for example by a rigorous demonstration of OPE to the required order, we can use the result on the sum of exclusive states, on the reverse, to demonstrate these sum rules in more general potentials.

Finally a short comment should be made on the practical relevance of the model used here, having however in mind that our aim is not phenomenological, but pedagogical. It is of course an oversimplification in many respects, first of all because it is not a field theory, and not even a relativistic model, but also for more concrete reasons concerning the potential. The harmonic oscillator model is indeed not devised to give an accurate description of the spectrum, because there is no spin force, but also because the spin-independent potential is known to be smoother (something like $\log r$). Another set of simplifications concerns the current operator; it is taken to be static, which is crude for transitions to spatial (orbital or radial) excitations; yet we have tried to account somewhat for the $V-A$ structure by our choice of the leptonic tensor, which leads to an overall reasonable total decay rate. In summary, even with an optimised choice of the two parameters, the light mass and R^2, it is certainly a crude model as regards phenomenological predictions.

This being said, one must emphasize that this 'toy' model is doing relatively well in that it combines two aspects of the model approach which are not often simultaneously present: it is both a not too unrealistic modelling of QCD states and transitions, and one which allows for exact statements on duality, not bound to further approximations or adjustments. Indeed, the two parameters of the H.O. potential being given, and chosen arbitrarily – therefore we could even choose a true non relativistic situation – hadronic widths are calculated without any approximation, and the duality derives straightforwardly. Such exact statements can also be checked numerically, as we have done, with very high accuracy due the simplicity of the eigenfunctions.

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Duality in semileptonic inclusive B-decays in potential models: regular versus singular potentials

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Abstract

Making use of the nonrelativistic potential model for the description of mesons, and working in the Shifman–Voloshin limit, we compare the integrated rate $\Gamma(B \to X_c l\nu)$ calculated as a sum of the individual decay rates to the quantum-mechanical analog of the OPE. In the case of a potential regular at the origin, we find a well-defined duality violation, which is, however, exponentially small. It corresponds to the charm resonances kinematically forbidden in the decay process, but apparently picked up by the OPE. For singular potentials, we do not obtain a full OPE series, but only a limited Taylor expansion, since the coefficients become infinite beyond some order. In this case, we do not find an indication of duality violation: the difference is smaller than the last term of the limited expansion. This emphasizes that the case of singular potentials, which may be relevant for QCD, deserves further study. © 2001 Published by Elsevier Science B.V.

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The theoretical framework based on the Operator Product Expansion (OPE) determines in QCD the heavy meson inclusive decay rate as series in inverse powers of the heavy quark mass, with the coefficients proportional to the meson matrix elements of the local operators of increasing dimensions [1,2]. The calculation is based on representing the decay rate as the contour integral in the complex q_0-plane. The OPE makes the contour integrals easily calculable term by term and provides the decay rate as a $1/m_Q$ series.

There are, however, potentially dangerous points in this calculation:

(i) the OPE series is at best asymptotically convergent even for large absolute values of the complex q_0;

(ii) the integration contour for the decay rate contains a segment near the physical region, where the OPE cannot be justified [1].

This might lead to the violation of duality for the decay rate, i.e., to the difference between the OPE-calculated decay rate and the result of summing the individual decay rates of the opened channels. This issue was also discussed by N. Isgur [3].

In this Letter we discuss the semileptonic decay rate in the small velocity limit and use the nonrelativistic potential model for the description of mesons. We perform a short-time expansion in operators of increasing dimensions which we call OPE and which has indeed some common features (but also important dif-

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ferences) with the OPE expansion in the field theory. We consider the two cases: regular confining potentials [1] and singular potentials. Note that, in the past, the nonrelativistic potential model has been widely used in the discussion of QCD sum rules [4,5]. The question of the distinction between regular (harmonic) and singular (Coulomb) potentials has been emphasised by Novikov et al. [4] (in the context of heavy quark QCD sum rules).

We work in the Shifman–Voloshin (SV) limit $\Lambda \ll \delta m = m_b - m_c \ll m_c, m_b$. In this limit both the amplitude and the decay rate can be formally obtained as a *double expansion* in $1/m_c$ and $1/\delta m$. We consider lowest orders in $1/m_c$, up to $1/m_c^2$, and *all orders in* $1/\delta m$. Note that this involves terms of much higher order than usually done when one expands in $1/m_Q$ with m_b/m_c, or as well $\delta m/m_Q$, fixed. Our double expansion allows on the contrary to go much further in $1/\delta m$, and this might allow to display subtle duality violations.

For the regular potential we obtain the full $1/\delta m$ expansion, which is only asymptotic to the physical width expanded to the same order in $1/m_c$. The difference [2] is of order $\delta m/m_c^2 \exp(-\delta m/\Lambda)$, which means exponentially small duality violation.

For the singular potential we do not obtain the full $1/\delta m$ expansion: following the same procedure as for the regular potential leads to infinite coefficients beyond some order in $1/\delta m$. In this case, we find that the truncated expansion satisfies duality up to this order.

We consider the inclusive semileptonic decay $B \to X_c l\nu$ in the SV limit and treat mesons as nonrelativistic bound states of spinless quarks in a confining potential (a detailed calculation is given in [6]). This model maximally simplifies both constructing the OPE series and calculating the sum of the exclusive channels. For the sake of argument we consider the case of leptons coupled to hadrons through the scalar current. In this case the leptonic tensor is reduced to a scalar function $L(q^2)$. The amplitude T depends on the two variables, and we choose them as q_0 and $\vec{q}^{\,2}$ in

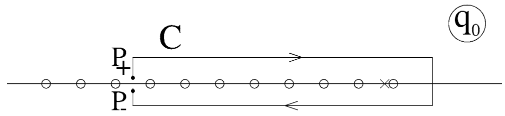

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Fig. 1. Singularities of the amplitude $T(q_0, \vec{q}^{\,2})$ in the complex q_0-plane. Circles are poles, corresponding to low-lying charm states, and the cross marks the location of the pole in the free $b \to c$ quark transition.

the B-rest frame:

$$T(q_0, \vec{q}^{\,2}) = \frac{1}{i} \int dx\, e^{-iqx} \langle B | T\big(J(x), J^{+}(0)\big) | B \rangle = \sum_X \frac{|\langle B|J|X(-\vec{q})\rangle|^2}{M_B - E_X(-\vec{q}) - q_0}. \qquad (1)$$

The sum in (1) runs over all hadron states with the appropriate quantum numbers. The states are normalized as follows $\langle \vec{p}|\vec{p}\,'\rangle = (2\pi)^3 \delta(\vec{p} - \vec{p}\,')$, and $E_X(-\vec{q})$ is the energy of the state X with the total 3-momentum $-\vec{q}$.

At fixed $\vec{q}^{\,2}$, $T(q_0, \vec{q}^{\,2})$ has a cut in the complex q_0-plane along the real axis for $q_0 < M_B - M_D - \vec{q}^{\,2}/2(m_c + m_d)$, see Fig. 1.

A part of this cut for $|\vec{q}| < q_0 < M_B - M_D - \vec{q}^{\,2}/2(m_c + m_d)$ corresponds to the decay process. The decay rate can be represented as the contour integral in the complex q_0-plane over the contour $C(\vec{q}^{\,2})$ (Fig. 1)

$$\Gamma(B \to X_c l\nu) = \int d\vec{q}^{\,2} |\vec{q}| \int_{C(\vec{q}^{\,2})} \frac{dq_0}{2\pi i} L(q^2) T(q_0, \vec{q}^{\,2}). \qquad (2)$$

The contour $C(\vec{q}^{\,2})$ selects at any given $\vec{q}^{\,2}$ only states kinematically allowed in the decay $B \to X_c l\nu$. It is tightly attached to the points $P_\pm$ with the coordinates $(|\vec{q}|, \pm i0)$, otherwise it can be freely deformed in the region where the function $T_0(q_0, \vec{q}^{\,2})$ is analytic.

The amplitude can be expanded in a series

$$T(q_0, \vec{q}^{\,2}) = \sum_i c_i(q_0, \vec{q}^{\,2}) \langle B|\widehat{\mathcal{O}}_i|B\rangle, \qquad (3)$$

where $\widehat{\mathcal{O}}_i$ are operators of increasing dimensions and $c_i(q_0, \vec{q}^{\,2})$ are the c-number coefficients. Introducing the expansion (3) into (2) gives the integrated rate as an OPE series. [3]

[1] A regular potential is a potential which is an analytic function of $\vec{r}$ at $r = 0$. For example, the potential $V(r) \simeq |\vec{r}|$ falls out of this class.

[2] As we shall see this expansion contains only a finite number of nonzero terms.

[3] The OPE series in the potential model has an important distinctions from the Wilsonian scheme in the field theory where

1. The model

Let us proceed along the lines of Ref. [6]. We treat the leptonic part relativistically, but for the description of mesons as bound states of spinless quarks use the nonrelativistic potential model with a confining potential. We consider the decay in the B-rest frame. The Hamiltonian of the $b\bar{q}$ system at rest has the form

$$\widehat{H}_{bd} = m_b + m_d + \hat{h}_{bd},$$
$$\hat{h}_{bd} = \vec{k}^2/2m_b + \vec{k}^2/2m_d + V_{bd}(r),$$

such that

$$(\hat{h}_{bd} - \epsilon_B)|B\rangle = 0,$$
$$(\widehat{H}_{bd} - M_B)|B\rangle = 0,$$
$$M_B = m_b + m_d + \epsilon_B.$$

The Hamiltonian of the $c\bar{q}$ system produced in the semileptonic $b \to cl\nu$ decay reads

$$\widehat{H}_{cd}(\vec{q}) = m_c + m_d + (\vec{k}+\vec{q})^2/2m_c + \vec{k}^2/2m_d + V_{cd}(r).$$

The eigenstates of this Hamiltonian are $|D_n(\vec{q})\rangle$ such that

$$\big(\widehat{H}_{cd}(\vec{q}) - E_{D_n}(\vec{q})\big)\big|D_n(\vec{q})\big\rangle = 0,$$

where

$$E_{D_n}(\vec{q}) = M_{D_n} + \vec{q}^2/2(m_c+m_d),$$
$$M_{D_n} = m_c + m_d + \epsilon_{D_n}.$$

The $Q\bar{q}$ potential can be expanded as follows:

$$V_{Qq} = V_0 + V_1/2m_Q + V_2/2m_Q^2 + \cdots.$$

contributions of distances below the scale $1/\mu$ are referred to the Wilson coefficients, while contributions of distances above this scale are referred to the matrix elements. As a result, both the Wilson coefficients and the matrix elements acquire the μ-dependence. In the potential model, the T-product of the two currents is also expanded in a series of operators of increasing dimensions, but the resulting c-number coefficients and the matrix elements in Eq. (3) are scale independent.

2. Sum rules

The relationship between the sum over the individual channels and the meson matrix elements of the operators is established by the sum rules. Let us introduce $\delta_n(\vec{q})$ through the relation ($\delta_n(\vec{q}) = \epsilon_{D_n} - \epsilon_B - \vec{q}^2 m_d/2m_c(m_c+m_d)$)

$$M_B - q_0 - E_{D_n}(\vec{q}) = \delta m - q_0 - \vec{q}^2/2m_c - \delta_n(\vec{q}). \tag{4}$$

The $\delta_n(\vec{q})$ is the eigenvalue of the operator $\delta H(\vec{q})$

$$M_B - \widehat{H}_{cd}(\vec{q}) = \delta m - \vec{q}^2/2m_c - \delta H(\vec{q}) \tag{5}$$

with $|D_n(\vec{q})\rangle$ the corresponding eigenstates. The sum rules are obtained by inserting the full system of the eigenstates $|D_n(\vec{q})\rangle$ into $\langle B|\,(\delta H(\vec{q}))^i\,|B\rangle$:

$$\langle B|\big(\delta H(\vec{q})\big)^i|B\rangle = \sum_{n=0}^{\infty}\big|F_n(\vec{q})\big|^2\big(\delta_n(\vec{q})\big)^i, \tag{6}$$

where $F_n(\vec{q}) = \langle B|D_n(\vec{q})\rangle$ is the $B \to D_n$ transition form factor. This relation represents the sum over all $c\bar{d}$ resonances in terms of the B-meson matrix element of the operators $(\delta H(\vec{q}))^i$. For the potential regular at the origin $r=0$ the sum over n is convergent for any i, whereas for the singular potential both sides of Eq. (6) are convergent for small i and diverge for large i. At the moment we proceed formally and discuss this problem in more detail in Section 5.

3. Duality relation for the amplitude

Making use of the sum rules (6), we represent the amplitude as a sum of the operators:

$$T(q_0,\vec{q}) = \sum_{n=0}^{\infty}\frac{|F_n(\vec{q})|^2}{M_B - q_0 - E_n(\vec{q})} \tag{7}$$

$$= \frac{1}{\delta m - \frac{\vec{q}^2}{2m_c} - q_0}\sum_{n=0}\sum_{i=0}\frac{|F_n(\vec{q})|^2(\delta_n(\vec{q}))^i}{\big(\delta m - \frac{\vec{q}^2}{2m_c} - q_0\big)^i} \tag{8}$$

$$= \frac{1}{\delta m - \frac{\vec{q}^2}{2m_c} - q_0}\sum_{i=0}\frac{\langle B|(\delta H(\vec{q}))^i|\rangle B}{\big(\delta m - \frac{\vec{q}^2}{2m_c} - q_0\big)^i}. \tag{9}$$

This expression is the duality relation for the amplitude: the sum (7) runs over the infinite number of the charm resonances, and the sum (8) runs over the infinite number of the operators of the increasing dimensions (the OPE series). In fact, the location of singularities in the complex q_0-plane in the series (7) and (9) is quite different: in (7) it is an infinite set of single poles at the different locations corresponding to different charm resonances, and in (8) it is an infinite set of poles of the increasing order at the same point.

However, this set of equations is only a formal one; in fact, (7) is a summable series leading to a finite result in all cases; on the other hand, the situation of Eq. (9) is more subtle. In the singular case, the coefficients are infinite beyond some order, and one must accordingly truncate the series. In the regular case, the Eq. (9) is only an asymptotic series: notice that the geometric sum over i in Eq. (8) has a domain of convergence which is repelled to infinity with n.

Let us illustrate it with a simple example: assume that $F_n^2 \simeq e^{-n}$ and $E_n \simeq n$. Then the analog of the above equations takes the form

$$\sum_{n=0}^{\infty} \frac{e^{-n}}{z-n} = \sum_{n=0} \frac{e^{-n}}{z} \sum_{i=0} \left(\frac{n}{z}\right)^i \simeq \frac{1}{z} \sum_{i=0} \frac{i!}{z^i}. \tag{10}$$

The last step is obtained by changing the order of summation and using the relation $\sum_{n=0}^{\infty} e^{-n} n^i \simeq i!$ The series (10) in i is only asymptotic and not even Borel summable.

Such a factorial divergence appears in the example of M.A. Shifman [7]. In this example, the residues are constant with the excitation number. This would imply that the direct method of our Letter will give formally infinite coefficients for the expansion, which means that it fails completely. In our example, on the contrary, the residues are rapidly decreasing, and all the coefficients are finite. The interest of our example is to show that nevertheless a factorial divergence is also appearing.

From the amplitude T under the form Eq. (7) or Eq. (9), respectively, by integration over the same contour C, we can obtain either the width as a sum over the exclusive final states, or as the OPE series. The expression (9) is an accurate approximation to (7) only when q_0 is far from the singularities of $T(q_0, \vec{q})$. The contour C can be deformed away from the singularities except near its fixed end points. When integrating over q_0 this is a possible source of discrepancy, i.e., of duality violation. Consequently, we are now going to estimate the integral of expression (7), i.e. the sum over the exclusive channels, and the integral of expression (9), i.e., the OPE prediction, and compare both results.

4. The OPE calculation of the decay rate

Let us first proceed with the amplitude in the form (9) and obtain the OPE expression for the decay rate. We consider the leptonic tensor of the general form $L(q^2) = (q^2)^N$. For technical reasons, it is convenient to isolate h_{bd} in the expression for $\delta H(\vec{q})$ as follows

$$\delta H(\vec{q}) = h_{bd} - \epsilon_B + \frac{\vec{k}\vec{q}}{m_c} + \left(\frac{1}{m_c} - \frac{1}{m_b}\right)\frac{\vec{k}^2 + V_1}{2}. \tag{11}$$

Substituting (11) in (9) and performing the necessary integrations gives a series in $1/m_c$ [6]

$$\begin{aligned} &\frac{\Gamma^{\mathrm{OPE}}(B \to X_c l\nu)}{\Gamma(b \to c l\nu)} \\ &\quad = 1 + \frac{\langle B|\vec{k}^2|B\rangle}{2m_c^2} - (2N+3)\frac{\langle B|V_1|B\rangle}{2m_c^2} \\ &\qquad + \sum_{i=1}^{2N+3} (-1)^i \frac{C_{2N+5}^{i+2}}{2N+5} \frac{\langle B|\widehat{O}_i|B\rangle}{m_c^2 \delta m^i} + O\left(\frac{\Lambda^2 \delta m}{m_c^3}\right), \end{aligned} \tag{12}$$

with $C_n^i = \frac{n!}{i!(n-i)!}$ and $\widehat{O}_i = \vec{k}(h_{bd} - \epsilon_B)^i \vec{k}$. An important feature of the OPE series (12) is that the leading-order term reproduces the free-quark decay rate, and the first correction emerges only in the $1/m_c^2$ order (cf. [1,2]).

5. Summation of the exclusive channels

Now let us sum the rates of the exclusive channels. The $B \to D_n$ transition form factors have the form [6]

$$\begin{aligned} F_0^2(\vec{q}) &= 1 - \rho_0^2 \vec{q}^{\,2}/m_c^2 + O\big(\vec{q}^{\,4}/m_c^4\big) \\ &\quad + O\big(\delta m^2 \beta^2/m_c^4\big), \\ F_n^2(\vec{q}) &= \rho_n^2 \vec{q}^{\,2}/m_c^2 + O\big(\vec{q}^{\,4}/m_c^4\big) + O\big(\delta m^2 \beta^2/m_c^4\big). \end{aligned}$$

Since $|\vec{q}| \leqslant \delta m$ in the decay region, these expressions allow calculating the decay rate to the accuracy

$\delta m^2/m_c^2$. Explicitly, we obtain [6]:

$$\frac{\Gamma(B\to D_0 l\nu)}{\Gamma(b\to cl\nu)} = 1-\frac{3\rho_0^2}{2N+5}\frac{\delta m^2}{m_c^2}+\frac{3}{2}\frac{m_d}{1+m_d/m_c}\frac{\delta m}{m_c^2} - (2N+3)\frac{\langle B|\vec{k}^2+V_1|B\rangle}{2m_c^2},$$

$$\frac{\Gamma(B\to D_n l\nu)}{\Gamma(b\to cl\nu)} = \frac{3\rho_n^2}{2N+5}\frac{\delta m^2}{m_c^2}-3\big(\rho_n^2\Delta_n\big)\frac{\delta m}{m_c^2} + \frac{1}{m_c^2}\sum_{i=2}^{2N+5}\frac{(-1)^i C_{2N+5}^i}{2N+5}\frac{(3\rho_n^2\Delta_n^i)}{\delta m^{i-2}}, \tag{13}$$

where $\Delta_n = \epsilon_{D_n}-\epsilon_{D_0}$.

The main contribution is given by the $B\to D_0$ transition. Excited states contribute only starting from the $(\delta m)^2/m_c^2$ order in the SV limit. Notice that *each of the exclusive rates contains terms of the order $\delta m^2/m_c^2$ and $\Lambda\delta m/m_c^2$ which are absent in the OPE series*.

Summing over all opened exclusive channels gives

$$\frac{\Gamma(B\to X_c l\nu)}{\Gamma(b\to cl\nu)} = 1-\frac{\delta m^2}{m_c^2}\frac{3}{2N+5}\left(\rho_0^2-\sum_{n=1}^{n_{\max}}\rho_n^2\right) + 3\frac{\delta m}{m_c^2}\left(\frac{m_d/2}{1+m_d/m_c}-\sum_{n=1}^{n_{\max}}\rho_n^2\Delta_n\right)-(2N+3) \times\frac{\langle B|\vec{k}^2+V_1|B\rangle}{2m_c^2} + \sum_{i=2}^{2N+5}\frac{(-1)^i C_{2N+5}^i}{2N+5}\frac{3\big(\sum_{n=1}^{n_{\max}}\rho_n^2\Delta_n^i\big)}{m_c^2\delta m^{i-2}}. \tag{14}$$

The sum over the charm resonances is truncated at $n_{\max}$, which is the total number of the resonance levels opened at $q^2=0$. For the confining potential and in the SV limit $n_{\max}$ is found from the relation $\Delta_{n_{\max}}\simeq\delta m$.

6. Check of duality for regular potentials

The transition radii in the expression (14) are not independent and related to each other through the sum rules. These sum rules can be obtained from (6). Expanding both sides of (6) in powers of $1/m_Q$ and taking the linear $\vec{q}^2$ term gives the set of the sum rules [6]: for $i=0$ one finds the Bjorken sum rule [8], for $i=1$ — the Voloshin sum rule [9], for $i\geqslant 2$ — higher moment sum rules:

$$i=0:\quad \sum_{n=1}^{\infty}\rho_n^2=\rho_0^2,$$

$$i=1:\quad \sum_{n=1}^{\infty}\rho_n^2\Delta_n=\frac{m_d/2}{1+m_d/m_c},$$

$$i\geqslant 2:\quad \sum_{n=1}^{\infty}\rho_n^2\Delta_n^i=\frac{1}{3}\langle B|\vec{k}(h_{bd}-\epsilon_B)^{i-2}\vec{k}|B\rangle. \tag{15}$$

Using these relations to rewrite the OPE result (12) as the sum over hadronic resonances, the difference between the OPE and the exclusive sum (the duality-violating contribution) explicitly reads

$$\delta_\Gamma \equiv \frac{\Gamma^{\mathrm{OPE}}(B\to X_c l\nu)-\Gamma(B\to X_c l\nu)}{\Gamma(b\to cl\nu)} = 3\frac{\delta m^2}{m_c^2}\sum_{i=0}^{2N+5}\frac{(-1)^i C_{2N+5}^i}{(2N+5)\delta m^i}\sum_{n>n_{\max}}\rho_n^2(\Delta_n)^i + O\left(\frac{\Lambda^2\delta m}{m_c^3}\right) = \frac{\delta m^2}{m_c^2}\frac{3}{2N+5}\sum_{n>n_{\max}}\rho_n^2\left(1-\frac{\Delta_n}{\delta m}\right)^{2N+5} + O\left(\frac{\Lambda^2\delta m}{m_c^3}\right). \tag{16}$$

Quite remarkably, δ_Γ happens to be equal to the sum of the extrapolated widths for charm states beyond the kinematical limit. A similar expression is found in QCD_2 [12]. Clearly, the duality-violating effect is connected with the charm states forbidden kinematically in the decay process. Notice that $\Gamma^{\mathrm{OPE}}(B\to X_c l\nu)-\Gamma(B\to X_c l\nu)<0$, because $\Delta_n>\delta m$ for $n>n_{\max}$, and $2N+5$ is odd.

To estimate the size of the duality-violation effects, the behavior of the *transition radii* and the relation between Δ_n and $n_{\max}$, which will be given by the behaviour of the *excitation energies* also at large n, are needed. For quite a general form of the confining potential we can write the following relations for Δ_n at large n $\Delta_n \geqslant \Lambda C n^a$ for $n > n_{\max}$ and $\Delta_{n_{\max}} = \Lambda C (n_{\max})^a \simeq \delta m$, with C and a some positive numbers. In particular, this estimate is valid for the confining potentials with a power behavior at large r. This estimate for Δ_n is only depending on the behaviour of the potential at large distances.

The behavior of the radii ρ_n^2 at large n are then connected with the finiteness of the r.h.s. of the sum rules (15): for a potential regular at $r = 0$, the matrix elements in the r.h.s. of the sum rules are finite for any i, which means that the radii ρ_n^2 are decreasing with n faster than any power. Essentially this means that $\rho_n^2 \simeq \exp(-n)$, and, therefore, the duality-violating effect in the decay rate in (16) is of order $\delta_\Gamma \simeq \delta m^2/m_c^2 \exp(-\delta m/\Lambda)$. One of such examples, the harmonic oscillator potential, is discussed in [10].

7. Singular potentials

However, if the potential is singular at $r = 0$, the situation changes dramatically. First, only a few first number of the matrix elements $\langle B|\widehat{O}_i|B\rangle$ are finite. [4]

We can try to proceed along the same lines but then have to truncate the series in $1/\delta m$ at the last finite term. We want to estimate the difference between this truncated series and the exclusive sum.

Let us illustrate this considering a potential with a Coulomb behavior at small r, $V \simeq -\alpha/r$, and confining at large r. Then $\langle B|\vec{k}(h_{bd} - \epsilon_B)^i \vec{k}|B\rangle$ are finite for $i \leqslant 1$, but diverge starting from $i = 2$. We then find that

$$\rho_n^2 \leqslant \frac{1}{n^{1+\varepsilon}} \left(\frac{1}{n^a}\right)^3. \tag{17}$$

Such ρ_n^2 lead to the estimate

$$\delta_\Gamma \simeq \frac{\Lambda^2}{m_c^2} \left(\frac{\Lambda}{\delta m}\right)^{(1+\varepsilon/a)}. \tag{18}$$

More generally, if the above matrix element begins to diverge for some value $i = K + 1$, the formulas are to be replaced by:

$$\rho_n^2 \leqslant \frac{1}{n^{1+\varepsilon}} \left(\frac{1}{n^a}\right)^{K+2}, \tag{19}$$

$$\delta_\Gamma \simeq \frac{\Lambda^2}{m_c^2} \left(\frac{\Lambda}{\delta m}\right)^{(K+\varepsilon/a)}. \tag{20}$$

Notice that this δ_Γ is smaller than the last retained term in the OPE series which is of order $\frac{\Lambda^2}{m_c^2}\left(\frac{\Lambda}{\delta m}\right)^K$. Therefore, the 'duality violation' is just smaller than the last retained term as for the asymptotic series. This means in fact that there is *no indication of duality violation at this computable order*. This is independent of a, therefore of the *large distance behavior of the potential*.

8. Conclusion

Summarizing our results, the amplitude $T(q_0, \vec{q}^2)$ (the T-product, Eq. (1)) can be expanded in inverse powers of $\delta m - \vec{q}^2/2m_c - q_0$, the so-called OPE expansion. Exact duality would mean that the OPE series was convergent and equal to T. Actually, this is not exactly the case. Even in the favourable case of the *regular potentials* (at $\vec{r} = 0$), the OPE series is not convergent, it is only asymptotic to the actual T. For *singular potentials*, the coefficients are simply infinite beyond a certain order.

Besides these problems concerning the amplitude, additional problems appear for the expansion of the width, which is given by a contour integral of T in the q_0 complex plane: the OPE expansion is accurate far from the singularities in q_0, while the contour has fixed end points in the complex plane close to the singularities (Fig. 1). In view of this situation, we have computed explicitly the difference between the OPE and the actual width. For singular potentials, the series

[4] The appearance of infinite coefficients in the OPE series is probably due to a breakdown of the power series expansion, for instance by fractional powers or logarithms of m_Q as seems to be the case in the pure Coulomb case [11]. For similar phenomena in a perturbation expansion, see [13].

In particular, it has been noted by Novikov et al. [4] that, in the case of the Coulomb potential, half integral powers of the expansion parameter occur in the calculation of the moments of the ratio R. However, in our case, preliminary work shows that there are not only half integer powers of $\Lambda/\delta m$, but also logarithms of this quantity. In fact, we note in this respect that the full Schwinger formula for the ratio R contains also logarithms in addition to half integer powers [14].

must be truncated, and the difference is found smaller than the last retained term.

As to the perspectives opened by this work, we must first emphasize that singular potentials seem more interesting than regular ones. Indeed, in QCD the effective quark potential is singular, a smoothed Coulomb singularity. Moreover, in QCD_2, one can suspect some similarity with a linear potential $|\vec{r}|$, which is also singular at the origin in the sense of this Letter. For a singular potential, we have seen that the entire series must be truncated at some order, because the coefficients become eventually infinite. We think that such infinite coefficients in an entire series expansion correspond to the fact that the correct expansion is not entire but must include fractional powers and/or logarithms in the expansion parameter, i.e., δm. In QCD, one can argue that the operator matrix elements are finite due to renormalisation, but nevertheless the coefficients still contain logarithms of heavy masses. In the nonrelativistic case, the object of the present Letter, the method which has been followed does not lead to definite conclusions as regards duality for singular potentials: namely, to the order we are able to calculate in this Letter, we find that there is no duality violation, but this leaves open the question of duality violation at some higher order.[5] To proceed further, one would have to devise new methods to obtain the above conjectured generalized expansions.

[5] In the context of QCD_2, one has demonstrated duality up to the order $1/m_Q^4$ and it may be believed that duality has been fully demonstrated in higher orders [12]. However, a comment is in order here. In [12], it was shown that the matrix element of the leading operator $\langle B|\overline{Q}Q|B\rangle$ is dual to the sum of the widths of the full tower of resonances. Therefore, one can suspect that there is a difference between the actual width and the OPE, that is of higher order $1/m_Q^5$, corresponding to the extrapolated width of the kinematically forbidden states. This difference, however, has the same order $1/m_Q^5$ as the matrix elements of the higher dimension operators [12]. It was then *assumed* that both quantities are dual to each other, but the corresponding OPE coefficients were not calculated and we have not found where this assumption was demonstrated.

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Semileptonic inclusive heavy meson decay: Duality in a nonrelativistic potential model in the Shifman-Voloshin limit

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The quark-hadron duality in the inclusive semileptonic decay $B \to X_c l \nu$ in the Shifman-Voloshin limit $\Lambda \ll \delta m = m_b - m_c \ll m_b, m_c$ is studied within a nonrelativistic potential model. The integrated semileptonic decay rate is calculated in two ways: first, by constructing the operator product expansion, and second by a direct summation of the exclusive channels. Sum rules (Bjorken, Voloshin, etc.) for the potential model are derived, providing a possibility to compare the two representations for $\Gamma(B \to X_c l \nu)$. An explicit difference between them referred to as the duality-violation effect is found. The origin of this effect is related to higher charm resonances which are kinematically forbidden in the decay process but are nevertheless picked up by the OPE. Within the considered $1/m_c^2$ order the OPE and the sum over exclusive channels match each other, up to the contributions of higher resonances, by virtue of the sum rules. In particular this is true for the terms of order $\delta m^2/m_c^2$ and $\Lambda\, \delta m/m_c^2$ which are present in each of the decay channels and cancel in the sum of these channels due to the Bjorken and Voloshin sum rules, respectively. The size of the duality violation effects is estimated to be of the order $O(\Lambda^{2+b}/m_c^2 \delta m^b)$ with $b>0$ depending on the details of the potential. Constraints for a better accuracy are discussed.

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I. INTRODUCTION

The interest in inclusive decays of heavy mesons is twofold: experimental study of such decays can provide important information on the weak mixing angles of heavy mesons, and a theoretical treatment of such processes which includes also nonperturbative effects is possible. The theoretical framework based on combining the operator product expansion (OPE) and heavy quark (HQ) expansion provides decay rates and differential distributions as series in inverse powers of the heavy quark mass with the coefficients proportional to the matrix elements of the operators of a proper dimension [1–4]. A remarkable property of this expansion is that in the leading-order this is just the free-quark decay, and the first correction appears only at order $1/m_Q^2$.

On the other hand, it is understood that the quark-hadron duality technically implemented through the OPE is an approximate framework [5]. For example, the calculation based on OPE does not take into account all the details of the hadron spectrum which lead to the dependence of the set of open decay channels on the momentum transfer. The OPE ignores this fact and this inevitably yields some errors in the OPE results [6].

The theoretical description based on the OPE represents the decay rate as a contour integral in the complex q^0-plane (for details see the next section). The OPE can be justified only in regions of the complex q^0-plane away from the physical region, whereas in the case of the calculation of the decay rate (both differential and integrated) the contour always involves a segment which is close to the physical region [1]. This can lead to duality-violating effects, i.e., the difference between the exact and the OPE based results.

However, it is not easy to estimate the errors arising in the OPE, since the exact hadron spectrum in QCD is complicated and not exactly known. So, testing directly the accuracy of the quark-hadron duality is only possible in few exceptional cases. Examples discussed in the literature are QCD in the Shifman-Voloshin (SV) limit [7], and the 't Hooft model [8].

In the 't Hooft model (2-dimensional QCD with $N_c \to \infty$) the spectrum is reduced to an infinite number of single bound states and known precisely so that the direct summation of exclusive channels is possible. First numerical analysis of the sum over exclusive channels reported the presence of the duality-violating $1/m_Q$ correction for the total width [9]. Later the summation was performed analytically for the case of a massless light quark [10]. The result of the OPE calculation agreed with the exact result in this case through $1/m_Q^4$ order.

Duality in QCD in the SV limit [7] has been studied in [11,12]. This limit requires $\Lambda_{\rm QCD} \ll \delta m = m_b - m_c \ll m_Q$. A peculiar feature of the SV limit is that a summation over exclusive channels becomes possible due to kinematical reasons: the process occurs near the zero recoil and thus only few decay channels contribute in the leading $1/m_Q$ order. The expansion of the relevant transition form factors in this kinematical region is known and the sum over exclusive channels can be evaluated. The absence of $\Lambda_{\rm QCD}/m_Q$ corrections to the free-quark result in the semileptonic (SL) decay rate has been demonstrated in [11]. However, to check the

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absence of $\Lambda_{\rm QCD}/m_Q$ and $\delta m/m_Q$ corrections within the SV kinematics is not enough to ensure duality in the $1/m_Q$ order in the general case, beyond the SV limit. Namely, one should also check that potentially large terms of order $O(\Lambda_{\rm QCD}\delta m^n/m_Q^{n+1})$ which are present in individual decay rates cancel in the sum over exclusive channels. The analysis of the $\Lambda_{\rm QCD}\delta m/m_Q^2$ terms in the exclusive sum was performed in [12] for QCD in the $V-A$ case. It was found that the duality within this accuracy requires a new sum rule. The full comparison to higher orders has not yet been performed.

We study the quark-hadron duality in the SV limit within a nonrelativistic potential model. The model has several features which make it especially suitable for this purpose: the model is self-consistent in the SV limit; the spectrum of bound states is relatively simple and can be calculated; the exact representations of the transition form factors in terms of the hadron wave functions are known. These features provide a possibility to calculate the exclusive sum. We adopt a technical simplification of a Lorentz scalar current instead of the $V-A$ current, like it is done in Ref. [6].

The main purpose of our analysis is to check whether or not the OPE result calculated to some order is equal to the sum over exclusive channels expanded to the same order. Both series are double expansions in powers of Λ/m_c and $\Lambda/\delta m$. They are asymptotic series [10], and the question of their convergence is left for a later publication [13].

Our main results are as follows.

We construct the expansion of the T-product of the two currents in a series of local operators (the OPE) in the potential model for a general form of the quark potential. Technically this is done by the expansion of the Lippmann-Schwinger equation. We consider the expansion to all orders in $\Lambda/\delta m$ but neglect terms $\sim\Lambda^n/m_c^n$ with $n\geqslant 3$. This OPE series provides the expansion of the differential and integrated semileptonic decay rates in powers of Λ/m_c and $\Lambda/\delta m$.

Let us point out that the OPE series in the potential model has an important distinction from the Wilsonian scheme in the field theory: Namely, in QCD (perturbative) contributions of small distances below the scale $1/\mu$ is referred to the Wilson coefficients while contributions of large distances above this scale is referred to the matrix elements of the local operators. As a result both the c-number Wilson coefficients and the matrix elements of the local operators acquire the μ-dependence. In the potential model we also expand the average of the T-product of the two current operators over the B meson in a series of local operators, but the resulting c-number coefficents as well as the average values of the local operators [see Eq. (8)] do not have a scale dependence.

The OPE and the sum over exclusive channels are related to each other by sum rules, similar to the Bjorken [14], the Voloshin [15], and the whole tower of higher moments [16]. We derive these sum rules. They involve an infinite sum of terms corresponding to all hadronic excitations, with each term having a well-defined heavy mass expansion. The question of the heavy mass expansion of the sum (in other words, of the uniform convergence of the series) has not been tackled in this paper. If the contribution of higher excitations vanishes rapidly enough the uniform convergence is expected.

The OPE provides a heavy mass expansion for the inclusive semileptonic decay rate. To compare it with the result of summation over exclusive semileptonic decay channels we make use of the sum rules. An explicit difference between the two expressions is found, both for the integrated and the differential rates. This difference corresponds to the contribution of the resonances kinematically forbidden in the decay process which are picked up by the OPE. This ''unphysical'' contribution is related to the poles in the complex q^0-plane outside the physical region which however contribute to the OPE result. The size of this duality-violation cannot be estimated in all generality since it depends on the potential and on the convergence properties of the sums over resonances.

For the integrated decay rate the OPE prediction and the sum over exclusive channels match, *up to the duality-violating contributions of higher resonances*, within the $1/m_c^2$ order: Terms of order $\delta m^2/m_Q^2$, $\delta m\Lambda/m_Q^2$, which are present in any individual decay rate cancel in the sum over all channels thanks to the Bjorken [14] and Voloshin [15] sum rules, respectively. For terms of order Λ^2/m_Q^2, $\Lambda^3/m_Q^2\delta m$, etc., the agreement (*again up to contributions of higher resonances*) is provided by the higher moment sum rules. The duality-violation induced by the kinematical truncation of these higher resonances in general has the order $O(\Lambda^{2+b}/m_c^2\delta m^b)$ where b depends on the details of the potential $V(r)$ both at large and small r.

For the smeared differential distributions near maximal q^2 the violation of the *local duality* is found at the order $\Lambda\,\delta m/m_c^2$.

We make an explicit proof of the present results for the special case of the harmonic oscillator potential in Ref. [17]. This is important since some demonstrations given below are rather formal.

In the next section we present some details of the kinematics and discuss the analytical properties of the decay amplitude. In Sec. III the $1/m_Q$ expansion of the quark propagator is performed and the OPE series for the SL decay rate in nonrelativistic quantum mechanics is constructed. In Sec. IV we consider the HQ expansion of the exclusive form factors in the potential model, and derive the inclusive sum rules (Bjorken, Voloshin, etc.) which are crucial for comparing the exclusive sum and the OPE result. In Sec. V we provide an analytic expression for the duality-violation contribution and identify its origin. We estimate the accuracy of the OPE both for the integrated rate and the smeared distribution near zero recoil. A special emphasis is laid on discussing the role of different inclusive sum rules in establishing the relationship between the OPE and the sum over the exclusive channels. A conclusion summarizes our results.

II. KINEMATICS AND THE ANALYTICAL PROPERTIES OF THE DECAY AMPLITUDE

We consider the inclusive SL decay $B\to X_c l\nu$. The rate of this process reads

$$\Gamma(B\to X_c l\nu)=\frac{1}{2M_B}\int\frac{d^4q}{2\pi}\theta(q^0>|\vec{q}|)L(q)W(q), \quad (1)$$

where L is the leptonic tensor, and the hadronic tensor W is defined as follows:

$$W=\sum_X\int d^4p_X\theta(p_X^0)\delta(p_X^2-M_X^2)\langle B|J|X(\vec{p})\rangle$$
$$\times\langle X(\vec{p})|J^+|B\rangle\delta_4(p_B-p_X-q). \quad (2)$$

Here the relativistic normalization of states is implied:

$$\langle p|p'\rangle=2p^0(2\pi)^3\delta(\vec{p}-\vec{p}\,'). \quad (3)$$

For the sake of clarity we assume the technical simplification that the leptons are coupled to hadrons through the *scalar current*.[1] In this case the leptonic tensor is a scalar function of only one variable, q^2, and the hadronic tensor W depends on the two invariant variables $\nu=P_B\cdot q/M_B$ and q^2. In the rest frame of the B-meson these are q^0 and q^2. At $q^0>0$ and fixed q^2 the sum in Eq. (2) runs over the hadronic states with masses $M_X<M_B-\sqrt{q^2}$. The decay rate can be written as follows:

$$\Gamma(B\to X_c l\nu)=\frac{1}{2M_B}\int dq^2dq^0|\vec{q}|\theta$$
$$\times(q^0>|\vec{q}|)L(q^2)W(q^0,q^2), \quad (4)$$

with $q^2=(q^0)^2-\vec{q}^2$.

Equivalently, we can use q^0 and $\vec{q}^2$. Let us consider the $W(q^0,\vec{q}^2)$ as an analytical function of q^0 at fixed $\vec{q}^2$. One can write the following relation:

$$\frac{1}{2M_B}W(q^0,\vec{q}^2)=\frac{1}{2\pi i}\mathrm{disc}_{q^0}T(q^0,\vec{q}^2), \quad (5)$$

where

$$T(q^0,\vec{q}^2)=\frac{1}{2M_B}\int dx\exp^{-iqx}\langle B|T[J(x),J^+(0)]|B\rangle$$
$$=\frac{1}{2M_B}\sum_X\frac{\langle B|J|X(-\vec{q})\rangle|^2}{M_X-E_X(-\vec{q})-q^0}, \quad (6)$$

$E_X(-\vec{q})=\sqrt{\vec{q}^2+M_X^2}$ is the energy of the state with the mass M_X and the total 3-momentum $-\vec{q}$. The sum over X in Eq. (6) for T runs over *all* hadron states with the appropriate quantum numbers. The selection of the states kinematically allowed in the decay process is made by the proper choice of the integration contour in the complex q^0 plane. Namely, the

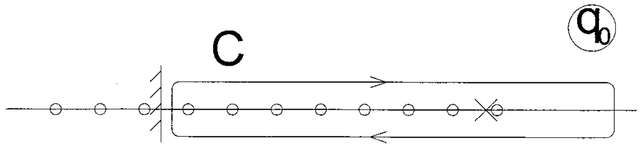

FIG. 1. Singularities of the amplitude $T(q^0,\vec{q}^2)$ in the complex q^0 plane. Circles are hadronic $(\overline{c}q)$ poles which are only singularities in the confined potential model, and the cross stands for the free $b\to c$ quark process. The vertical line $\mathrm{Re}\,(q^0)=|\vec{q}|$ separates the kinematically allowed region of the real axis from the kinematically forbidden region. The contour $C(\vec{q}^2)$ embraces all states in the allowed region. Poles at the left of the boundary correspond to kinematically forbidden bound states.

decay rate $\Gamma(B\to X_c l\nu)$ can be represented as the contour integral in the complex q^0 plane over the contour $C(\vec{q}^2)$ which depends on the value of $\vec{q}^2$ (Fig. 1) as follows:

$$\Gamma(B\to X_c l\nu)=\frac{1}{2\pi i}\int d\vec{q}^2|\vec{q}|\int_{C(\vec{q}^2)}dq^0\theta(q^0>|\vec{q}|)$$
$$\times L((q^0)^2-\vec{q}^2)T(q^0,\vec{q}^2). \quad (7)$$

It is important that the left crossing of the contour $C(\vec{q}^2)$ with the real axis in the complex q^0 plane occurs at the point $q_0=|\vec{q}|$, otherwise the contour can be freely deformed in the region where the function $T_0(q_0,\vec{q}^2)$ is analytic. We symbolically mark this constraint with a θ-function in the integrand. The integration over such contour selects at any given $\vec{q}^2$ only physical states which can be produced in the decay $B\to X_c l\nu$, i.e. states with the invariant masses such that $\sqrt{M_X^2+\vec{q}^2}<M_B-|\vec{q}|$. Notice that whereas the left crossing of the contour with the real axis is tightly fixed at the point $\mathrm{Re}\,(q^0)=|\vec{q}|$, $\mathrm{Im}\,(q^0)=0$, the right crossing of the contour with the real axis can be safely moved to the right. In the general relativistic case there are cuts which correspond to other physical processes. In the SV limit these cuts are separated from the physical decay cut by windows of the width $O(m_Q)$. In the potential quark model such cuts are absent.

The amplitude $T(q^0,\vec{q}^2)$ is given by the T-product of the two local current operators, which is the classical case for performing the OPE. Namely, one has

$$T(q^0,\vec{q}^2)=\frac{1}{2M_Bi}\int dx\exp(-iqx)\langle B|T(J(x),J^+(0))|B\rangle$$
$$=\sum_n C_n(q^0,\vec{q}^2)\langle B|\hat{\mathcal{O}}_n|B\rangle, \quad (8)$$

where $\hat{\mathcal{O}}_n$ are local operators and $C_n(q^0,\vec{q}^2)$ are the c-number coefficients. Introducing the expansion (8) into Eq. (7) gives the integrated rate as a sum over various local operators.

We shall obtain the integrated SL decay rate within our model by two means: first, we construct the OPE series for $T(q^0,q^2)$, and second, we calculate directly the sum over exclusive channels.

[1]Recall that for the case of the $V-A$ current and massless leptons, the leptonic tensor has the form $L_{\mu\nu}\sim g_{\mu\nu}q^2-q_\mu q_\nu$, and for the scalar current $L\sim q^2$. We consider throughout the paper the leptonic tensor of the generalized form $L=(q^2)^N$.

III. THE MODEL

We consider this decay in the SV limit,

$$\Lambda \ll \delta m = m_b - m_c \ll m_c, m_b. \tag{9}$$

Notice that in a non-relativistic model, Λ refers to a fixed energy scale proportional to the light quark mass m_d, to the average quark momentum in the hadron rest frame $(\langle B|\vec{k}^2|B\rangle)^{1/2}$, to the parameter β defined in Eq. (24) and h_{bd} or ϵ_B to be defined in Eq. (11). These parameters may be strongly hierarchized, for example a genuine nonrelativistic situation implies $(\langle B|\vec{k}^2|B\rangle)^{1/2} \ll m_d$, but all these quantities remain constant as $m_c, m_b \to \infty$, they remain proportional to some fixed hadronic scale which we call Λ by analogy with QCD. This is to be distinguished from δm which is taken as an independent parameter. Thus we consider the double limit $\delta m/m_c \to 0$, and $\Lambda/\delta m \to 0$. Notice finally that $|\vec{q}|$ is of order δm.

To avoid confusion, it is important to stress that the standard OPE expansion assumes $\delta m/m_Q$ constant, even if small. So the order of a term $O(\Lambda^{n+m}/m_c^n(\delta m)^m)$ in this paper corresponds to the order $O(\Lambda^{n+m}/m_Q^{n+m})$ of the standard OPE expansion.

We treat the leptonic part relativistically, but for the calculation of the hadronic tensor we use the nonrelativistic potential model. The nonrelativistic treatment of the hadronic tensor is consistent within the SV kinematics and can be used as a tool for studying some of the aspects of quark-hadron duality. We shall make use the fact that in the nonrelativistic potential model we know the structure of the hadron spectrum and have an exact representation for the hadronic matrix elements of the quark currents.

It is convenient to use the nonrelativistic normalization of states (which is used hereafter)

$$\langle p|p'\rangle = (2\pi)^3 \delta(\vec{p} - \vec{p}\,'), \tag{10}$$

and consider the process in the rest frame of the decaying B-meson. The B meson is the ground eigenstate of the Hamiltonian $\hat{H}_{bd}$,

$$\hat{H}_{bd}|B\rangle = M_B|B\rangle = (m_b + m_d + \epsilon_B)|B\rangle. \tag{11}$$

In the B-rest frame this Hamiltonian has the form

$$\hat{H}_{bd} = m_b + m_d + \frac{\vec{k}^2}{2m_b} + \frac{\vec{k}^2}{2m_d} + V_{bd}(r) \equiv m_b + m_d + h_{bd}. \tag{12}$$

For the $B \to X_c$ transition we need the $c\bar{d}$ bound states with the total 3-momentum $-\vec{q}$, which we denote $D_n(-\vec{q})$. These are eigenstates of the Hamiltonian

$$\hat{H}_{cd}(\vec{q}) = m_c + m_d + \frac{(\vec{k}+\vec{q})^2}{2m_c} + \frac{\vec{k}^2}{2m_d} + V_{cd}(r), \tag{13}$$

such that

$$\hat{H}_{cd}(\vec{q})|D_n(-\vec{q})\rangle = E_{D_n}(\vec{q})|D_n(-\vec{q})\rangle. \tag{14}$$

In this equation $E_{D_n}(\vec{q})$ is the nonrelativistic energy of the bound state D_n with the 3-momentum $-\vec{q}$

$$E_{D_n}(\vec{q}) = M_{D_n} + \frac{\vec{q}^2}{2(m_c+m_d)}, \quad M_{D_n} = m_c + m_d + \epsilon_{D_n}. \tag{15}$$

The expression (6) for the decay amplitude T now takes the form

$$T(q^0,\vec{q}) = \sum_n^{\infty} |F_n(\vec{q})|^2 \frac{1}{M_B - E_{D_n}(\vec{q}) - q^0}, \tag{16}$$

where $F_n(\vec{q})$ is the $B \to D_n$ transition form factor,

$$F_n(\vec{q}) = \langle B|J|D_n(-\vec{q})\rangle, \quad M_B = m_b + m_d + \epsilon_B,$$

and the sum runs over all $c\bar{d}$ resonances. The expression (16) can be also written as

$$T(q^0,\vec{q}) = \langle B|J \frac{1}{M_B - q^0 - \hat{H}_{cd}(\vec{q})} J^+|B\rangle, \tag{17}$$

where $[\hat{H}_{cd}(\vec{q}) - E]^{-1} = G_{cd}(\vec{q},E)$ is the full off-energy-shell Green function (propagator) of the $c\bar{d}$ system. The B decay amplitude is thus given by an average of the Green function $\hat{G}_{cd}(\vec{q},E)$ at the point $E = M_B - q^0$ over the B meson.

Let us specify the transition current operator $\hat{J}_{b\to c}$. For the sake of argument we neglect the quark spin effect and consider spinless nonrelativistic quarks and choose the quark current in the form

$$\hat{J}_{b\to c} = \int d\vec{k} d\vec{k}' \hat{b}(\vec{k}')\hat{c}^+(\vec{k}), \tag{18}$$

where $\hat{c}(\hat{b})$ is the annihilation operator of the $c(b)$ quark.[2]

For the quark current (18) the $B \to D_n$ transition form factor in the rest frame of the B-meson reads

$$F_n(\vec{q}) = \int d\vec{k}_q \psi_B(\vec{k}_q)\psi_{D_n}\left(\vec{k}_q + \frac{m_d}{m_c+m_d}\vec{q}\right), \tag{19}$$

where $\vec{k}_q$ is the momentum of the light spectator.

[2]Notice that the standard scalar current reads $\hat{J}_{b\to c} = \int (d\vec{k}/2k^0)(d\vec{k}'/2k'^0)\hat{b}(\vec{k}')\hat{c}^+(\vec{k})$ and in the nonrelativistic limit takes the form $\simeq \int d\vec{k}\hat{c}^+(\vec{k})d\vec{k}'\hat{b}(\vec{k}')(1-\vec{k}^2/4m_c^2)(1-\vec{k}'^2/4m_b^2)$. Neglecting the factor $(1-\vec{k}^2/4m_c^2)(1-\vec{k}'^2/4m_b^2)$ as done in Eq. (18) leads to technical simplifications both in the OPE and in the exclusive sum. It can be easily realized that a particular choice of the current however does not touch any arguments related to duality.

Similarly, for the current (18) the expression (17) takes the form

$$T(q^0,\vec{q})=\langle B|\frac{1}{M_B-q^0-\hat{H}_{cd}(\vec{q})}|B\rangle. \tag{20}$$

IV. THE OPE OF THE DECAY RATE

The main idea in constructing the OPE series for T, Eq. (8), is to single out $\hat{H}_{bd}$ from $\hat{H}_{cd}(\vec{q})$ in the denominator in Eq. (20) and to use the eigenvalue equation (11). First, let us introduce the operator $\delta H(\vec{q})$ which measures the difference of the denominator of Eq. (20) from the inverse Green function of the free-quark transition

$$M_B-q^0-\hat{H}_{cd}(\vec{q})=\left(\delta m-\frac{\vec{q}^2}{2m_c}-q^0\right)-\delta H(\vec{q}). \tag{21}$$

Explicitly, one finds

$$\delta H(\vec{q})=\hat{H}_{cd}(\vec{q})-m_c-m_d-\frac{\vec{q}^2}{2m_c}-\epsilon_B. \tag{22}$$

Next, isolating h_{bd} in $\delta H(\vec{q})$ we obtain

$$\delta H(\vec{q})=(h_{bd}-\epsilon_B)+\frac{1}{2}\left(\frac{1}{m_c}-\frac{1}{m_b}\right)(\vec{k}^2+V_1)+\frac{\vec{k}\cdot\vec{q}}{m_c}+O\left(\frac{\beta^3\delta m}{m_c^3}\right), \tag{23}$$

where the scale β is provided by the hadronic matrix elements

$$\beta^2\simeq\langle B|\vec{k}^2|B\rangle\simeq\langle B|\hat{V}_1|B\rangle. \tag{24}$$

As already mentioned, β is of the order of Λ. The quantity V_1 here is a part of the expansion of the potential V_{Qq} in powers of $1/m_Q$

$$V_{Qq}=V_0+\frac{1}{2m_Q}V_1+\frac{1}{2m_Q^2}V_2+O\left(\frac{\Lambda^4}{m_Q^3}\right). \tag{25}$$

Equations (20) and (21) allow us to construct the expansion of $T(q^0,\vec{q}^2)$ in inverse powers of $\delta m-\vec{q}^2/2m_c-q^0$ as follows:

$$T(q_0,\vec{q}^2)=\frac{1}{\delta m-\frac{\vec{q}^2}{2m_c}-q^0}\sum_{i=0}^{\infty}\frac{\langle B|(\delta H(\vec{q}))^i|B\rangle}{\left(\delta m-\frac{\vec{q}^2}{2m_c}-q^0\right)^i}. \tag{26}$$

Making use of Eq. (23) we obtain

$$T(q_0,\vec{q}^2)=\langle B|B\rangle\frac{1}{\delta m-\frac{\vec{q}^2}{2m_c}-q^0}+\langle B|(h_{bd}-\epsilon_B)|B\rangle\frac{1}{\left(\delta m-\frac{\vec{q}^2}{2m_c}-q^0\right)^2}+\langle B|\left[\frac{\delta m}{2m_c^2}(\vec{k}^2+V_1)-\frac{\vec{k}\cdot\vec{q}}{m_c}\right]|B\rangle$$
$$\times\frac{1}{\left(\delta m-\frac{\vec{q}^2}{2m_c}-q^0\right)^2}+\sum_{i,j=1}^{3}\sum_{n=0}^{\infty}q_iq_j\langle B|\left[\frac{k_i(h_{bd}-\epsilon_B)^nk_j}{m_c^2}\right]|B\rangle\frac{1}{\left(\delta m-\frac{\vec{q}^2}{2m_c}-q^0\right)^{3+n}}+O(\Lambda^2/m_c^3). \tag{27}$$

The remainder has the order $O(\Lambda^2/m_c^3)$ if we keep $\vec{q}^2\simeq\delta m^2$ and q_0 fixed.

Finally, using Eq. (11) and the relations $\langle B|k_i|B\rangle=0$ and $\langle B|k_ik_j|B\rangle=\frac{1}{3}\delta_{ij}\langle B|\vec{k}^2|B\rangle$ we find the following OPE series:

$$T(q^0,\vec{q}^2)=\frac{1}{\delta m-\frac{\vec{q}^2}{2m_c}-q^0}+\frac{\delta m}{2m_c^2}\langle B|\vec{k}^2+V_1|B\rangle\frac{1}{\left(\delta m-\frac{\vec{q}^2}{2m_c}-q^0\right)^2}$$
$$+\sum_{n=2}^{\infty}\frac{\langle B|\hat{O}_{n-2}|B\rangle}{3m_c^2}\vec{q}^2\frac{1}{\left(\delta m-\frac{\vec{q}^2}{2m_c}-q^0\right)^{n+1}}+O(\Lambda^2/m_c^3), \tag{28}$$

where $\hat{O}_n=\Sigma_{j=1}^3k_j(h_{bd}-\epsilon_B)^nk_j$. Hereafter the Σ symbol is omitted. We denote $\langle B|\vec{k}^2+V_1|B\rangle=\beta_0^2$, $\beta_0\simeq\Lambda$.

The series (28) is a double expansion of $T(q^0,q^2)$ in Λ/m_c and $\Lambda/(\delta m-\vec{q}^2/2m_c-q^0)$, limited to second order in Λ/m_c and expanded to all orders in $\Lambda/(\delta m-\vec{q}^2/2m_c-q^0)$. The poles are at

$$q_c^0(\vec{q}^2)=\delta m-\frac{\vec{q}^2}{2m_c}. \tag{29}$$

The *first term* in Eq. (28) gives *the free quark decay amplitude*. A remarkable feature of this series is that the $\Lambda/\delta m$ and Λ/m_c corrections to the free-quark decay are absent thanks to Eq. (11) and the relation $\langle B|k_i|B\rangle=0$. The expansion (28) substitutes the whole set of hadron poles by a complicated quark singularity at the point $q^0=q_c^0(\vec{q}^2)$.

Let us treat the series (28) *formally* and calculate the integrated rate which is obtained as a double expansion in Λ/m_c and $\Lambda/\delta m$.

Let us rewrite the expression (28) as follows:

$$T(q^0,\vec{q}^2)=\left(1-\frac{\langle B|\vec{k}^2+V_1|B\rangle}{2m_c^2}\delta m\frac{\partial}{\partial\delta m}+\vec{q}^2\sum_{n=2}^{\infty}\frac{\langle B|\hat{O}_{n-2}|B\rangle}{3m_c^2}\frac{1}{n!}\left(-\frac{\partial}{\partial\delta m}\right)^n\right)\frac{1}{\left(\delta m-\frac{\vec{q}^2}{2m_c}-q^0\right)}. \tag{30}$$

This representation is very convenient for the calculation of the decay rate Eq. (7): the integration over q^0 is now easily performed since

$$\int_{C(\vec{q}^2)}dq^0L((q^0)^2-\vec{q}^2)\theta(q^0>|\vec{q}|)\frac{1}{\left(\delta m-\frac{\vec{q}^2}{2m_c}-q^0\right)}=L(q_c^0(\vec{q}^2),\vec{q}^2)\,\theta(|\vec{q}|<-m_c+\sqrt{m_c^2+2m_c\delta m}), \tag{31}$$

where the θ-function $\theta(|\vec{q}|<-m_c+\sqrt{m_c^2+2m_c\delta m})$ reflects the fact that the left crossing of the contour with the real axis in the complex q^0 plane should always happen at the point $\mathrm{Re}(q^0)=|\vec{q}|$. The integrated rate is then given by the expression[3]

$$\Gamma^{OPE}(B\to X_cl\nu)=\left(1-\frac{\langle B|\vec{k}^2+V_1|B\rangle}{2m_c^2}\delta m\frac{\partial}{\partial\delta m}\right)I_1(\delta m,m_c)+\sum_{n=2}^{\infty}\frac{\langle B|\hat{O}_{n-2}|B\rangle}{3m_c^2}\frac{1}{n!}\left(-\frac{\partial}{\partial\delta m}\right)^nI_3(\delta m,m_c), \tag{32}$$

where

$$I_n(\delta m,m_c)=\int_0^{-m_c+\sqrt{m_c^2+2m_c\delta m}}d\vec{q}^2|\vec{q}|^nL([q_c^0(\vec{q}^2)]^2-\vec{q}^2). \tag{33}$$

For the free quark decay one finds

$$\Gamma(b\to cl\nu)=I_1(\delta m,m_c). \tag{34}$$

Let us consider the leptonic tensor of the general form $L(q^2)=(q^2)^N$. For semileptonic decays to massless spin 1/2 leptons $N=1$. The case $N=0$ corresponds to scalar leptons. Since the leptonic tensor is proportional to q^2, it is now convenient to introduce a new integration variable q^2 as follows:

$$q^2=[q_c^0(\vec{q}^2)]^2-\vec{q}^2. \tag{35}$$

Then the integrated rate takes the form

[3]A remark is in order here. When computing the decay rate in Eq. (32) we have interchanged the derivation with respect to δm and the integration over dq^0. We can also directly integrate the expression Eq. (28) over dq^0. In this case we should take into account that the contour $C(\vec{q}^2)$ in the complex q^0-plane always lies on the right-hand side (RHS) of the line $\mathrm{Re}(q^0)=|\vec{q}|$. If we erroneously do not take this condition into account then multiple poles in Eq. (28) do not contribute at all since the complex integral vanishes when the multiple poles are inside the contour as well as when they are outside (see also discussion in [2]). To proceed correctly, one should replace a multiple pole, say a double pole, by an equivalent set of two neighboring poles. Then the crossing of the boarder gives a nonvanishing result when one of the two poles is inside the contour and the other one is outside. Integrating before taking the derivative with respect to δm as in Eq. (32) corresponds to treating in a specific way the crossing of the boarder by the multiple poles. We show elsewhere [13] that both treatments lead to the same result.

$$\Gamma^{OPE}(B\to X_c l\nu)=\Gamma(b\to cl\nu)-\frac{\langle B|\vec{k}^2+V_1|B\rangle}{2m_c^2}\delta m\frac{\partial}{\partial\delta m}I_1(\delta m,m_c)+\sum_{n=2}^{\infty}\frac{\langle B|\hat{O}_{n-2}|B\rangle}{3m_c^2}\frac{1}{n!}\left(-\frac{\partial}{\partial\delta m}\right)^n I_3(\delta m,m_c), \quad (36)$$

where we have taken into account that $I_1(\delta m,m_c)$ gives the exact free-quark decay rate. A simple algebraic exercise gives to the $1/m_c^2$ accuracy

$$I_1(\delta m,m_c)=(\delta m)^{2N+3}\left[A_{1/2}^N\left(1-\frac{3}{2}\frac{\delta m}{m_c}+\frac{15}{8}\frac{\delta m^2}{m_c^2}\right)+\frac{5}{8}A_{3/2}^N\frac{\delta m^2}{m_c^2}+O\left(\frac{\delta m^3}{m_c^3}\right)\right],$$
$$I_3(\delta m,m_c)=(\delta m)^{2N+5}\left[A_{3/2}^N+O\left(\frac{\delta m}{m_c}\right)\right], \quad (37)$$

where

$$A_m^N=\int_0^1 dx x^m(1-x)^N=B(N+1,m+1),\quad A_{3/2}^N=\frac{3}{2N+5}A_{1/2}^N, \quad (38)$$

$B(p,q)$ being the Euler function. Finally, we come to the relation

$$\frac{\Gamma(B\to X_c l\nu)}{\Gamma(b\to cl\nu)}=1+\frac{\langle B|\vec{k}^2|B\rangle}{2m_c^2}-(2N+3)\frac{\langle B|V_1|B\rangle}{2m_c^2}+\sum_{n=3}^{2N+5}\frac{(-1)^nC_{2N+5}^n}{2N+5}\frac{\langle B|\hat{O}_{n-2}|B\rangle}{m_c^2\delta m^{n-2}}+O\left(\frac{\Lambda^2\delta m}{m_c^3}\right), \quad (39)$$

with $C_n^k=n!/k!(n-k)!$. Notice that the coefficient of the term $\langle B|\vec{k}^2|B\rangle$ does not depend on N, i.e. it does not depend on the form of the leptonic tensor.

Summing up, the OPE predicts the following features of the inclusive SL decay rate.

The LO term reproduces the rate of the free-quark decay process $b\to c$.

The $1/m_c$ and $1/\delta m$ corrections are absent. This is due to the fact that the average over the B-state of the operator of the relevant dimension vanishes.

Lowest-order corrections to the free-quark process emerge in the $1/m_c^2$ order. A main part of these corrections is due to the average values of the dimension-2 operators $\langle B|\vec{k}^2|B\rangle$ and $\langle B|\vec{k}^2+V_1|B\rangle$. Also the operators $\hat{O}_n=k_j(h_{bd}-\epsilon_B)^n k_j$ contribute in the $1/m_c^2$ order. Their contribution is however suppressed with the additional powers of δm.

In the next section we shall analyze the accuracy of the OPE predictions.

V. HEAVY QUARK EXPANSION AND THE HADRONIC SUM RULES

Before proceeding with the direct summation of the exclusive channels one by one we derive hadronic sum rules which are important for the comparison of the exact result with the OPE analysis.

A. Heavy quark expansion of the form factors in the potential model

The wave function of the $Q\bar{q}$ bound state has the form

$$\Psi_{\vec{p}}(\vec{k}_Q,\vec{k}_q)=\delta(\vec{p}-\vec{k}_Q-\vec{k}_q)\psi\left(\frac{m_Q m_q}{m_Q+m_q}\left(\frac{\vec{k}_q}{m_q}-\frac{\vec{k}_Q}{m_Q}\right)\right)$$
$$=\delta(\vec{p}-\vec{k}_Q-\vec{k}_q)\psi\left(\vec{k}_q-\frac{m_q}{m_Q+m_q}\vec{p}\right), \quad (40)$$

where $\vec{p}$ is the momentum of the bound state.

The $B\to D_n$ transition form factor is the average over the meson states of the operator $\Omega_{bc}(\vec{q})$ given by the following kernel:

$$\langle\vec{k}_b|\Omega_{bc}(\vec{q})|\vec{k}_c\rangle=\delta(\vec{k}_b-\vec{k}_c-\vec{q}). \quad (41)$$

So the transition form factor is defined by the following expression:

$$\langle B(\vec{p}_B)|\Omega_{bc}(\vec{q})|D_n(\vec{p}_n)\rangle=\delta(\vec{p}_B-\vec{p}_n-\vec{q})F_n((\vec{v}_B-\vec{v}_n)^2), \quad (42)$$

with $\vec{v}_B=\vec{p}_B/(m_b+m_q)$ and $\vec{v}_n=\vec{p}_n/(m_c+m_q)$ and

$$F_n(\vec{v}_B-\vec{v}_n)=\int d\vec{k}_q\psi_B\left(\vec{k}_q-\frac{m_q}{m_b+m_q}\vec{p}_B\right)$$
$$\times\psi_{D_n}\left(\vec{k}_q-\frac{m_q}{m_c+m_q}\vec{p}_n\right). \quad (43)$$

A simple change of variables $\vec{k}_q\to\vec{k}_q+m_q/(m_b+m_q)\vec{p}_B$ makes it obvious that the decay form factor depends on the square of the relative 3-velocities of the initial and final mesons, and not on the relative 3-momentum squared as the elastic form factor. Nevertheless in the B rest frame we write

$$F_n(\vec{q}) = \int d\vec{k}_q \psi_B(\vec{k}_q)\psi_{D_n}\left(\vec{k}_q + \frac{m_q}{m_c+m_q}\vec{q}\right). \quad (44)$$

The wave function $\psi_{Q\bar{q}}$ is an eigenstate of the Hamiltonian

$$h_{Q\bar{q}} = \frac{\vec{k}^2}{2}\left(\frac{1}{m_Q} + \frac{1}{m_q}\right) + V_{Qq}(r), \quad (45)$$

where $\vec{k}$ is the conjugate variable to $\vec{r}$. The transition form factors F_n has some general properties independent of the details of the potential $V_{Q\bar{q}}$. Such properties of the transition form factors are derived by performing the HQ expansion of the Hamiltonian. To this end we apply the usual quantum mechanical perturbation theory.

For our purposes it is convenient to consider h_{bd} as the full Hamiltonian, h_{cd} as a nonperturbed Hamiltonian, and $\hat{U} = h_{bd} - h_{cd}$ as the perturbation. The perturbation has the form

$$\begin{aligned}\hat{U} &= \frac{1}{2}\left(\frac{1}{m_b} - \frac{1}{m_c}\right)(\vec{k}^2 + V_1) + O\left(\beta^3 \frac{\delta m}{m_c^3}\right)\\ &= -\frac{\delta m}{2m_c^2}(\vec{k}^2 + V_1) + O\left(\beta^2 \frac{\delta m^2}{m_c^3}\right), \quad (46)\end{aligned}$$

where we assume the following expansion of the $Q\bar{q}$ potential

$$V_{Q\bar{q}} = V_0 + \frac{1}{2m_Q}V_1 + \frac{1}{2m_Q^2}V_2 + \cdots . \quad (47)$$

The perturbation has the order $\delta m/m_c^2$ such that we can construct the HQ expansion of the wave functions and the binding energies. Let us remind the standard formulas: Let $\{\psi_{D_n}\}$ be the full system of eigenstates of the h_{cd}, and the $\{\epsilon_{D_n}\}$ the corresponding eigenvalues. Then, the mass of the nth excitation in the $c\bar{d}$ system reads $M_{D_n} = m_c + m_d + \epsilon_{D_n}$. Let $\{\psi_{B_n}\}$ be the full system of eigenstates of the h_{bd}, and the $\{\epsilon_{B_n}\}$ the corresponding eigenvalues.

The standard formulas give

$$\psi_{B_n} = \psi_{D_n} + \sum_{m\neq n} \frac{U_{mn}}{\epsilon_{D_n} - \epsilon_{D_m}}\psi_{D_m} + O(\delta m^2\beta^2/m_c^4) \quad (48)$$

and

$$\epsilon_{B_n} = \epsilon_{D_n} + U_{nn} + \sum_{m\neq n}\frac{|U_{mn}|^2}{\epsilon_{D_n} - \epsilon_{D_m}} + \cdots, \quad (49)$$

where[4]

[4] ϵ_{B_0} is just ϵ_B defined in Eq. (11). We also usually write B instead of B_0.

$$U_{mn} = -\frac{\delta m}{2m_c^2}\langle\psi_{D_n}|\vec{k}^2 + V_1|\psi_{D_m}\rangle + O\left(\beta^2\frac{\delta m^2}{m_c^3}\right). \quad (50)$$

The excitation energies satisfy the relation

$$\epsilon_{D_n} - \epsilon_{D_m} \simeq (n-m)\lambda_{nm}, \quad \lambda_{mn} \simeq \beta. \quad (51)$$

In terms of the wave functions, the transition form factor (44) takes a simple form:

$$F_n(\vec{q}) = \langle\psi_{B_0}|\psi_{D_n}(-\vec{q})\rangle. \quad (52)$$

The expansion of the wave function $\psi_{B_0} \equiv \psi_B$ reads

$$\begin{aligned}\psi_{B_0} &= \psi_{D_0} + \sum_{m\neq 0}\frac{1}{2}\left(\frac{1}{m_b} - \frac{1}{m_c}\right)\frac{\langle\psi_{D_0}|\vec{k}^2 + V_1|\psi_{D_m}\rangle}{\epsilon_{D_n} - \epsilon_{D_m}}\psi_{D_m}\\ &\quad + O(\delta m^2\beta^2/m_c^4), \quad (53)\end{aligned}$$

such that

$$\begin{aligned}F_n(\vec{q}) &= \langle\psi_{D_0} + \sum_{m\neq 0}\frac{1}{2}\left(\frac{1}{m_b} - \frac{1}{m_c}\right)\frac{\langle\psi_{D_0}|\vec{k}^2 + V_1|\psi_{D_m}\rangle^*}{\epsilon_{D_0} - \epsilon_{D_m}}\\ &\quad \times \psi_{D_m}|\psi_{D_n}(\vec{q})\rangle + O(\delta m^2\beta^2/m_c^4)\\ &= f_{0n}(\vec{q}) + \sum_{m\neq 0}\left(-\frac{\delta m}{2m_c^2}\right)\frac{\langle\psi_{D_0}|\vec{k}^2 + V_1|\psi_{D_m}\rangle^*}{\epsilon_{D_0} - \epsilon_{D_m}}f_{mn}(\vec{q})\\ &\quad + O(\delta m^2\beta^2/m_c^4), \quad (54)\end{aligned}$$

where $f_{mn}(\vec{q}) = \langle\psi_{D_m}|\psi_{D_n}(\vec{q})\rangle$. By virtue of Eq. (44) one obtains

$$\begin{aligned}f_{nn}^2(\vec{q}) &= 1 - r_{nn}^2\frac{\vec{q}^2}{m_c^2} + O(\vec{q}^4/m_c^4),\\ f_{nm}^2(\vec{q}) &= r_{nm}^2\frac{\vec{q}^2}{m_c^2} + O(\vec{q}^4/m_c^4), \quad m\neq n, \quad (55)\end{aligned}$$

with r_{mn} being numbers of order unity plus higher order $1/m_c$ corrections. We shall use the notation $r_n = r_{n0}$. Notice that the radii r_n describe the form factors of the transitions between different levels in the $c\bar{d}$ system ($D_0 \to D_n$) and so know nothing about δm.

We now rewrite Eq. (54) as follows:

$$\begin{aligned}F_0(\vec{q}) &= 1 - r_0^2\frac{\vec{q}^2}{2m_c^2} + \sum_{m\neq 0}\left(-\frac{\delta m}{2m_c^2}\right)\frac{\langle\psi_{D_0}|\vec{k}^2 + V_1|\psi_{D_m}\rangle^*}{\epsilon_{D_0} - \epsilon_{D_m}}\\ &\quad \times f_{m0}(\vec{q}) + O\left(\frac{\delta m^2\beta^2}{m_c^4}\right). \quad (56)\end{aligned}$$

At $\vec{q}^2 = 0$ we thus come to the relation

$$F_0(0)=1+O(\delta m^2\beta^2/m_c^4). \tag{57}$$

One can see that the $O(\delta m\beta/m_c^2)$ term in $F_n(0)$ is absent. This is a nonrelativistic analog of the Luke theorem [18].

For the squares of the form factors we obtain the following important relations:[5]

$$F_0^2(\vec q)=1-\rho_0^2\frac{\vec q^{\,2}}{m_c^2}+O\left(\frac{\delta m^2\beta^2}{m_c^4}\right),\quad \rho_0^2=r_0^2+O\left(\frac{\beta\delta m}{m_c^2}\right),$$

$$F_n^2(\vec q)=\rho_n^2\frac{\vec q^{\,2}}{m_c^2}+O\left(\frac{\delta m^2\beta^2}{m_c^4}\right),\quad \rho_n^2=r_n^2+O\left(\frac{\beta\delta m}{m_c^2}\right). \tag{58}$$

As we shall see later the radii ρ_n (as well as r_n) are not independent and satisfy certain sum rules. The relations (58) are the main result of this section. They are necessary for the calculation of the decay rates.

B. Inclusive hadronic sum rules

To obtain a nonrelativistic equivalent of the whole tower of sum rules [16], i.e., the Bjorken sum rule, the Voloshin and the higher moments, we consider the following set of quantities $(i=0,1,\dots)$:

$$S_i(\vec q)=\langle B|(\delta H(\vec q))^i|B\rangle, \tag{59}$$

where $\delta H(\vec q)$ is defined in Eq. (21). Notice that $S_i(\vec q)$ appear in the expansion for $T(q^0,\vec q^{\,2})$, Eq. (26). We shall derive two different representations for $S_i(\vec q)$ and obtain sum rules equating these representations.

The first representation is obtained by inserting the full system of the eigenstates $|D_n(-\vec q)\rangle$ of the Hamiltonian $H_{cd}(\vec q)$ in Eq. (59). The $|D_n(-\vec q)\rangle$ are also eigenstates of the operator $\delta H(\vec q)$ that is made obvious using $\delta H_{cd}(\vec q)$ in the form Eq. (22):

$$\delta H(\vec q)|D_n(-\vec q)\rangle=\delta_n(\vec q)|D_n(-\vec q)\rangle,$$

$$\delta_n(\vec q)=\epsilon_{D_n}-\epsilon_B+\frac{\vec q^{\,2}}{2(m_c+m_d)}-\frac{\vec q^{\,2}}{2m_c}. \tag{60}$$

As a result of inserting the full system we find

$$S_i(\vec q)=\sum_{n=1}^{\infty}|F_n(\vec q)|^2(\delta_n(\vec q))^i. \tag{61}$$

Equation (49) gives the following expansion for $\delta_n(\vec q)$:

$$\delta_n(\vec q)=\Delta_n+\frac12\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\langle D_0|\vec k^2+V_1|D_0\rangle-\frac{m_d\vec q^{\,2}}{2m_c(m_c+m_d)}+O\left(\frac{\delta m^2\beta^3}{m_c^4}\right), \tag{62}$$

where

$$\Delta_n\equiv\epsilon_{D_n}-\epsilon_{D_0}. \tag{63}$$

Notice that within the leading-order accuracy we can replace $\langle D_0|\vec k^2+V_1|D_0\rangle$ with $\langle B|\vec k^2+V_1|B\rangle$.

Another representation for $S_i(\vec q)$ is obtained by using $\delta H(\vec q)$ in the form (23):

$$S_i(\vec q)=\langle B|(\delta H(\vec q))^i|B\rangle=\langle B|\left(\hat h_{bd}-\epsilon_B+\frac{\vec k^2+V_1}{2}\left(\frac{1}{m^c}-\frac{1}{m^b}\right)+\frac{\vec k\cdot\vec q}{m_c}+O\left(\frac{\beta^3\delta m}{m_c^3}\right)\right)^i|B\rangle. \tag{64}$$

This formula gives $S_i(\vec q)$ in terms of the matrix elements of various operators over the B-meson.

The representations (61) and (64) for $S_i(\vec q)$ provide the LHS and the RHS of the sum rules, respectively. Let us notice that terms denoted by $O(\beta^3\delta m/m_c^3)$ in Eqs. (62) and (64) do not depend on $\vec q$. All $\vec q$-dependent terms are shown explicitly.

Using Eq. (11) and the relations $\langle B|k_i|B\rangle=0$ and $\langle B|k_ik_j|B\rangle=\frac13\delta_{ij}\langle B|\vec k^2|B\rangle$ we come to the set of sum rules. In fact each of these sum rules is equivalent to an infinite number of relations at different powers of $\vec q^{\,2}$ and $1/m_c$.

i=0:

$$S_0=\sum_n^{\infty}|F_n(\vec q)|^2=1. \tag{65}$$

Obviously the RHS does not depend on $\vec q$. At $\vec q^{\,2}=0$ this is an identity. Using the definition (58) and comparing the term linear in $\vec q^{\,2}$ we find the *NR Bjorken sum rule* [14]

$$\rho_0^2=\sum_{n=1}^{\infty}\rho_n^2. \tag{66}$$

i=1: The RHS of this sum rule reads

$$S_1=\frac12\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\langle B|\vec k^2+V_1|B\rangle+O\left(\frac{\beta^3\delta m}{m_c^3}\right), \tag{67}$$

where we have used Eqs. (11) and (12). The RHS of this sum rule is also independent of $\vec q$. From the definition (59) and using Eq. (62) as well as the SR (65), we rewrite Eq. (67) as follows:

[5] At any n states with angular momenta $L=0,\dots,n$ exist. The form factors $f_n(\vec q^{\,2})$ and $F_n(\vec q^{\,2})$ are thus understood as properly normalized sums $\Sigma_{L=0}^n f_{nL}(\vec q)$ and $\Sigma_{L=0}^n F_{nL}(\vec q)$, respectively.

$$\sum_{n=0}^{\infty} F_n^2(\vec{q})\Delta_n = \frac{\vec{q}^2 m_d}{2m_c(m_c+m_d)} + O\left(\frac{\beta^3 \delta m}{m_c^3}\right). \qquad (68)$$

Notice that the terms $O(\beta^2 \delta m/m_c^2)$ cancel between RHS amd LHS. Comparing the linear in $\vec{q}^2$ term yields the *NR Voloshin sum rule* [15]

$$\sum_{n=1}^{\infty} \rho_n^2 \Delta_n = \frac{m_d}{2} \frac{1}{1+\frac{m_d}{m_c}}. \qquad (69)$$

Let us notice that the RHS of the Eq. (69) does not contain higher-order $1/m_c$ corrections.

Combining the Bjorken and the Voloshin sum rules provides a simple constraint on the parameter ρ_0^2 which is in fact the slope of the Isgur-Wise function. Namely,

$$\rho_0^2 = \sum_{n=1}^{\infty} \rho_n^2 = \frac{1}{\Delta_1}\sum_{n=1}^{\infty} \rho_n^2 \Delta_1 < \frac{1}{\Delta_1}\sum_{n=1}^{\infty} \rho_n^2 \Delta_n$$
$$= \frac{m_d}{2\Delta_1}\frac{1}{1+m_d/m_c} < \frac{m_d}{2\Delta_1}, \qquad (70)$$

where Δ_n are defined in Eq. (63)

i=2: The RHS of this sum rule reads

$$S_2 = \frac{\vec{q}^2\langle B|\vec{k}^2|B\rangle}{3m_c^2} + O\left(\frac{\beta^4 \delta m^2}{m_c^4}\right). \qquad (71)$$

Using Eqs. (65) and (67) yields for the LHS

$$\sum_{n=0}^{\infty} F_n^2(\vec{q})\Delta_n^2 = \frac{\vec{q}^2\langle B|\vec{k}^2|B\rangle}{3m_c^2}\left(1+O\left(\frac{\beta^2 \delta m}{m_c^3}\right)\right)$$
$$+ \frac{\vec{q}^4 m_d^2}{4m_c^2(m_c+m_d)^2} + O\left(\frac{\beta^4 \delta m^2}{m_c^4}\right). \qquad (72)$$

The linear $\vec{q}^2$ term yields

$$\sum_{n=1}^{\infty} \rho_n^2 \Delta_n^2 = \frac{1}{3}\langle B|\vec{k}^2|B\rangle\left(1+O\left(\frac{\beta^2 \delta m}{m_c^3}\right)\right). \qquad (73)$$

$i \geqslant 3$:

For i=3 we find for the RHS

$$S_3 = \frac{1}{3}\frac{\vec{q}^2\langle B|k_j(h_{bd}-\epsilon_B)k_j|B\rangle}{m_c^2}\left(1+O\left(\frac{\beta \delta m}{m_c^2}\right)\right)$$
$$+ O\left(\frac{\beta^6 \delta m^3}{m_c^6}\right). \qquad (74)$$

Using Eqs. (65)–(71) yields for the LHS

$$\sum_{n=0}^{\infty} F_n^2(\vec{q})\Delta_n^3 = \frac{\vec{q}^2\langle B|k_j(h_{bd}-\epsilon_B)k_j|B\rangle}{3m_c^2}\left(1+O\left(\frac{\beta \delta m}{m_c^2}\right)\right)$$
$$+ \frac{\vec{q}^4}{m_c^4} O\left(\frac{m_d^2 \beta^2 \delta m}{m_c^2}\right) + \frac{\vec{q}^6}{m_c^6} O(m_d^3)$$
$$+ O\left(\frac{\beta^5 \delta m^2}{m_c^4}\right). \qquad (75)$$

The linear $\vec{q}^2$ term yields

$$\sum_{n=1}^{\infty} \rho_n^2 \Delta_n^3 = \frac{\langle B|k_j(h_{bd}-\epsilon_B)k_j|B\rangle}{3}\left(1+O\left(\frac{\beta \delta m}{m_c^2}\right)\right)$$
$$= \frac{1}{3}\langle B|\hat{O}_1|B\rangle\left(1+O\left(\frac{\beta \delta m}{m_c^2}\right)\right). \qquad (76)$$

Similarly at higher $i \geqslant 3$ one obtains at the $\beta \delta m/m_c^2$ accuracy

$$\sum_{n=1}^{\infty} \rho_n^2 \Delta_n^i = \frac{1}{3}\langle B|k_j(h_{bd}-\epsilon_B)^{i-2}k_j|B\rangle = \frac{1}{3}\langle B|\hat{O}_{i-2}|B\rangle. \qquad (77)$$

These sum rules are used in the next section for comparison of the exact decay rate with the OPE result and for analyzing the duality-violation effects.

VI. SUMMATION OVER THE EXCLUSIVE CHANNELS

We now proceed to the summation of the exclusive channels. As the first step, let us show that there is an explicit difference between the exclusive sum and the OPE series.

A. The origin of duality violation

Proceeding with the sum over the exclusive channels we write

$$\begin{aligned}\Gamma(B\to X_c l\nu) &= \frac{1}{2\pi i}\sum_n^{\infty}\int dq^2 L(q^2)\int_{C(q^2)} dq^0|\vec{q}|\frac{|F_n(\vec{q})|^2}{M_b-q^0-E_n(\vec{q})}\\ &= \frac{1}{2\pi i}\sum_n^{\infty}\int dq^2 L(q^2)\int_{C(q^2)} dq^0|\vec{q}|\frac{|F_n(\vec{q})|^2}{\delta m-\frac{\vec{q}^2}{2m_c}-q^0+\delta_n(\vec{q})}\\ &= \frac{1}{2\pi i}\sum_n^{\infty}\int dq^2 L(q^2)\int_{C(q^2)} dq^0|\vec{q}|\frac{|F_n(\vec{q})|^2}{\delta m-\frac{\vec{q}^2}{2m_c}-q^0}\left[1-\frac{\delta_n(\vec{q})}{\delta m-\frac{\vec{q}^2}{2m_c}-q^0}+\cdots\right]\\ &= \frac{1}{2\pi i}\int dq^2 L(q^2)\sum_n^{n(q^2)}\int_{C(q^2)} dq^0|\vec{q}|\frac{|F_n(\vec{q})|^2}{\delta m-\frac{\vec{q}^2}{2m_c}-q^0}\left[1-\frac{\delta_n(\vec{q})}{\delta m-\frac{\vec{q}^2}{2m_c}-q^0}+\cdots\right], \end{aligned} \tag{78}$$

where in the RHS $|\vec{q}|=\sqrt{(q^0)^2-q^2}$. Notice that the sum is truncated at the proper $n(q^2)$ which is the maximal number of hadron resonances kinematically allowed at a given value of q^2, i.e., resonances satisfying the relation $M_n<M_B-\sqrt{q^2}$. The contour $C(q^2)$ is responsible for this selection, since only the resonances enclosed by the contour contribute into the sum. All states which are beyond this contour do not contribute.

Finally, the series (78) can be written in the form

$$\begin{aligned}\Gamma(B\to X_c l\nu) &= \int dq^2 L(q^2)\,\theta(q^2)\int dq^0 d\vec{q}^2|\vec{q}|\,\delta((q^0)^2-q^2-\vec{q}^2)\sum_n^{n(q^2)}|F_n(\vec{q})|^2\\ &\quad\times\left(1+\delta_n(\vec{q}^2)\frac{\partial}{\partial\delta m}+\frac{1}{2}\delta_n^2(\vec{q}^2)\frac{\partial^2}{\partial\delta m^2}+\cdots\right)\delta\left(q^0-\frac{\vec{q}^2}{2m_c}-\delta m\right).\end{aligned} \tag{79}$$

On the other hand, the sum rules (67)–(74) allow us to rewrite the decay rate (32) in the form

$$\begin{aligned}\Gamma^{OPE}(B\to X_c l\nu) &= \int dq^2 L(q^2)\,\theta(q^2)\int dq^0 d\vec{q}^2|\vec{q}|\,\delta((q^0)^2-q^2-\vec{q}^2)\sum_n^{\infty}|F_n(\vec{q})|^2\\ &\quad\times\left(1+\delta_n(\vec{q}^2)\frac{\partial}{\partial\delta m}+\frac{1}{2}\delta_n^2(\vec{q}^2)\frac{\partial^2}{\partial\delta m^2}+\cdots\right)\delta\left(q^0-\frac{\vec{q}^2}{2m_c}-\delta m\right).\end{aligned} \tag{80}$$

It is easy to see that the exact result and the result of the OPE are different due to contributions of highly excited states: *at any q^2 the OPE picks up also the contribution of the resonances forbidden kinematically at this q^2*. Thus the accuracy of duality is determined by the accuracy of violating the sum rules connected with the truncation of the exclusive sum, and is therefore connected with the convergence of these sums.

B. Sum of the exclusive channels and the accuracy of the OPE

We now calculate the individual decay rates keeping terms of order $(\Lambda^2/m_c^2)(\delta m/\Lambda)^n$ in the decay rates but neglecting higher orders $(\Lambda^3/m_c^3)(\delta m/\Lambda)^n$. The necessary expressions with the relevant accuracy are given below.

$|\vec{q}|$ in the free-quark decay $b\to c l\nu$ at q^2 has the form

$$|\vec{q}|=\sqrt{\delta m^2-q^2}\left(1-\frac{\delta m}{2m_c}+\frac{3}{8}\frac{\delta m^2}{m_c^2}+\frac{\delta m^2-q^2}{8m_c^2}\right). \tag{81}$$

The general expression for $|\vec{q}|$ in the $B\to D_n l\nu$ transition at q^2 reads

$$\begin{aligned}|\vec{q}|_n &= \sqrt{\delta M_n^2-q^2}\left(1-\frac{\delta M_n}{2(m_c+m_d)}+\frac{3}{8}\frac{\delta M_n^2}{(m_c+m_d)^2}\right.\\ &\quad\left.+\frac{\delta M_n^2-q^2}{8(m_c+m_d)^2}\right),\end{aligned} \tag{82}$$

where $\delta M_n=M_B-M_n\simeq\delta m-\Delta_n-\delta m\beta_0/(2m_c)$ from Eqs. (50) and (63).

The necessary accuracy for the transition into the ground state is

$$|\vec{q}|_{n=0}=\sqrt{\delta m^2\left(1-\frac{\beta_0^2}{2m_c^2}\right)^2-q^2}\left(1-\frac{\delta m}{2(m_c+m_d)}\right.$$
$$\left.+\frac{3}{8}\frac{\delta m^2}{m_c^2}+\frac{\delta m^2-q^2}{8m_c^2}\right). \qquad (83)$$

Recall that $\beta_0^2=\langle B|\vec{k}^2+V_1|B\rangle$.

For $|\vec{q}|_{n\neq 0}$ less accuracy is enough since the contribution of the $D_n, n\neq 0$ into the SL decay rate is suppressed by the additional factor $\vec{q}^2/m_c^2$:

$$|\vec{q}|_{n\neq 0}=\sqrt{(\delta m-\Delta_n)^2-q^2}. \qquad (84)$$

With these formulas for $|\vec{q}|$ the decay rates of the exclusive channels take the following form.

Free quark decay $b\to cl\nu$:

$$\frac{1}{L(q^2)}\frac{d\Gamma(b\to cl\nu)}{dq^2}$$
$$=|\vec{q}|\left(1-\frac{\delta m}{m_c}+\frac{3}{2}\frac{\delta m^2}{m_c^2}-\frac{q^2}{2m_c^2}\right)$$
$$=\sqrt{\delta m^2-q^2}\left(1-\frac{3}{2}\frac{\delta m}{m_c}+\frac{15}{8}\frac{\Delta m^2}{m_c^2}+\frac{5}{8}\frac{\delta m^2-q^2}{m_c^2}\right). \qquad (85)$$

The $B\to D_0 l\nu$ channel

$$\frac{1}{L(q^2)}\frac{d\Gamma(B\to D_0 l\nu)}{dq^2}=|\vec{q}|_{n=0}\left(1-\frac{\delta m}{m_c+m_d}+\frac{3}{2}\frac{\delta m^2}{m_c^2}\right.$$
$$\left.-\frac{q^2}{2m_c^2}\right)-\frac{\rho_0^2}{m_c^2}|\vec{q}|^3_{n=0}, \qquad (86)$$

using the definition (56).

The $B\to D_n l\nu$ $(n\neq 0)$ channel:

$$\frac{1}{L(q^2)}\frac{d\Gamma(B\to D_n l\nu)}{dq^2}=\frac{\rho_n^2}{m_c^2}((\delta m-\Delta_n)^2-q^2)^{3/2}. \qquad (87)$$

Now everything is ready for the calculation of the integrated SL decay rate. We again consider $L=(q^2)^N$.

1. The integrated rate and the global duality

It is convenient to represent the results for the partial decay rates in terms of their ratios to the free quark decay rate. The latter has the form

$$\Gamma(b\to cl\nu)=(\delta m)^{2N+3}\left[A^N_{1/2}\left(1-\frac{3}{2}\frac{\delta m}{m_c}+\frac{15}{8}\frac{\delta m^2}{m_c^2}\right)\right.$$
$$\left.+\frac{5}{8}A^N_{3/2}\frac{\delta m^2}{m_c^2}+O\left(\frac{\delta m^3}{m_c^3}\right)\right]. \qquad (88)$$

Making use of the relation (38) we find

$$\frac{\Gamma(B\to D_0 l\nu)}{\Gamma(b\to cl\nu)}=1-\frac{3\rho_0^2}{2N+5}\frac{\delta m^2}{m_c^2}+\frac{3}{2}\frac{m_d}{1+m_d/m_c}\frac{\delta m}{m_c^2}$$
$$-(2N+3)\frac{\beta_0^2}{2m_c^2}, \qquad (89)$$

$$\frac{\Gamma(B\to D_n l\nu)}{\Gamma(b\to cl\nu)}=\frac{\delta m^2}{m_c^2}\frac{3\rho_n^2}{2N+5}\left(1-\frac{\Delta_n}{\delta m}\right)^N$$
$$=\frac{3\rho_n^2}{2N+5}\frac{\delta m^2}{m_c^2}-3(\rho_n^2\Delta_n)\frac{\delta m}{m_c^2}+\frac{1}{m_c^2}$$
$$\times\sum_{k=2}^{2N+5}(-1)^k\frac{1}{2N+5}C^k_{2N+5}\frac{(3\rho_n^2\Delta_n^k)}{\delta m^{k-2}}. \qquad (90)$$

Some remarks are in order.

(1) The main part of the OPE (i.e., the free quark decay) is reproduced by the $\Gamma(B\to D_0 l\nu)$, within the leading and the subleading $1/m_c$ orders. The excited states contribute only within the $(\delta m)^2/m_c^2$ and $\Lambda\delta m/m_c^2$ orders in the SV limit.

(2) Nevertheless, each of the individual exclusive channels contains potentially large terms of the order $\delta m^2/m_c^2$ and $\Lambda\delta m/m_c^2$ which are absent in the OPE series.

Now summing over all exclusive channels we find

$$\frac{\Gamma(B\to X_c l\nu)}{\Gamma(b\to cl\nu)}=1-\frac{\delta m^2}{m_c^2}\left(\rho_0^2-\sum_{n=1}^{n_{max}}\rho_n^2\right)\frac{3}{2N+5}+3\frac{\delta m}{m_c^2}\left(\frac{1}{2}\frac{m_d}{1+m_d/m_c}-\sum_{n=1}^{n_{max}}\rho_n^2\Delta_n\right)$$
$$-(2N+3)\frac{\langle B|\vec{k}^2+V_1|B\rangle}{2m_c^2}+(2N+4)\frac{\left(\sum_{n=1}^{n_{max}}\rho_n^2\Delta_n^2\right)}{2m_c^2}+\frac{1}{m_c^2\delta m}\sum_{k=3}^{2N+5}\frac{(-1)^kC^k_{2N+5}}{2N+5}\frac{\left(3\sum_{n=1}^{n_{max}}\rho_n^2\Delta_n^k\right)}{\delta m^{k-3}}. \qquad (91)$$

The sum over the charm resonance levels is truncated at n_{max}, which is the number of the resonance levels opened at $q^2=0$. For the confining potential and in the SV limit n_{max} is found from the relation $\Delta_{n_{max}} \simeq \delta m$.

Using the sum rules (66)–(77) to rewrite the OPE result (39) as the sum over hadronic resonances, the difference between the OPE and the exclusive sum (the duality-violating contribution) explicitly reads

$$\frac{\Gamma^{OPE}(B\to X_c l\nu)-\Gamma(B\to X_c l\nu)}{\Gamma(b\to cl\nu)} = \frac{\delta m^2}{m_c^2}\sum_{k=0}^{2N+5}\frac{(-1)^k C_{2N+5}^k}{2N+5}\frac{\delta^{(k)}}{\delta m^k}+O(\Lambda^2\delta m/m_c^3), \tag{92}$$

where

$$\delta^{(k)} \equiv \sum_{n=n_{max}+1}^{\infty}\rho_n^2(\Delta_n)^k = \sum_{n=n_{max}+1}^{\infty}[r_n^2+O(\Lambda\,\delta m/m_c^2)](\Delta_n)^k. \tag{93}$$

As expected, this duality-violating contribution is connected with the charm resonance states forbidden kinematically in the decay process. This kinematical truncation of the higher resonances induces a violation of duality equal to $[(\delta m)^{2-k}/m_c^2]\delta^{(k)}$ for every $k<2N+5$.

To estimate the error induced by the truncation and thus the size of the duality-violation effects, we need to know the behavior of the excitation energies and the transition radii at large n.

(1) For quite a general form of the confining potential we can write the following relations for Δ_n for large n (recall that in the SV limit $\Delta_{n_{max}} \simeq \delta m$):

$$\Delta_{n_{max}} = \Lambda C(n_{max})^a = \delta m, \qquad \Delta_n \geqslant \Lambda C n^a, \quad n>n_{max}, \tag{94}$$

with C and a some positive numbers. In particular, this estimate is valid for the confining potentials with a power behavior at large r.

This estimate for Δ_n is only depending on the behavior of the potential at large distances (the infrared region).

(2) The transition radii r_n^2 satisfy sum rules similar to sum rules for ρ_n^2 in Sec. V, namely[6]

$$\sum_{n=1}^{\infty} r_n^2 = r_0^2,$$

$$\sum_{n=1}^{\infty} r_n^2\Delta_n = \frac{m_d}{2(1+m_d/m_c)},$$

$$\sum_{n=1}^{\infty} r_n^2(\Delta_n)^{(k+2)} = \frac{1}{3}\langle D_0|k_j(h_{cd}-\epsilon_{D_0})^k k_j|D_0\rangle, \qquad k=0,1,2,\ldots . \tag{95}$$

Hence, the behavior of the radii r_n^2 at large n are connected with the finiteness of the RHS of the sum rules. We can guarantee this for the Bjorken and Voloshin sum rules, where finite values stand in the RHS (the ground state radius r_0 is finite for the confining potential). In general, the finiteness of the matrix elements of the operators $k_j(h_{cd}-\epsilon_0)^k k_j$ (such as, e.g., the kinetic energy of quarks in the ground state) depend on the properties of the potential at small r (*the ultraviolat behavior*) and probably also at large r (*the infrared behavior*).[7] We have assumed throughout the paper that the average kinetic energy of the light spectator quark in the ground state is finite, i.e., $\langle D_0|\vec{k}^2|D_0\rangle \simeq \Lambda^2$. This already restricts some properties of the potential at small r and provides convergency of one more sum rule in Eq. (95). If, in addition to this, we assume that the average values of the operators $k_j(h_{cd}-\epsilon_{D_0})^k k_j$ for $k=1,\ldots,K$ over the ground state are finite, then combining with the behavior of the energies at large n we come to the following estimate:

$$r_n^2 \lesssim \frac{1}{n^{1+\varepsilon}}\left(\frac{1}{n^a}\right)^{2+K}, \qquad \varepsilon>0. \tag{96}$$

This allows us to obtain the duality-violation originating from the truncation of the various sum rules:

$$\text{Bjorken:} \quad \frac{\delta m^2}{m_c^2}\delta^{(0)} = \frac{\delta m^2}{m_c^2}\sum_{n_{max}} r_n^2 \lesssim \frac{\delta m^2}{m_c^2}\left(\frac{1}{n_{max}^a}\right)^{K+2} \simeq \frac{\Lambda^2}{m_c^2}\left(\frac{\Lambda}{\delta m}\right)^K,$$

$$\text{Voloshin:} \quad \frac{\delta m}{m_c^2}\delta^{(1)} = \frac{\delta m}{m_c^2}\sum_{n_{max}} r_n^2\Delta_n \lesssim \frac{\delta m}{m_c^2}\Lambda\left(\frac{1}{n_{max}^a}\right)^{K+1} \simeq \frac{\Lambda^2}{m_c^2}\left(\frac{\Lambda}{\delta m}\right)^K. \tag{97}$$

Similar estimates can be done for higher moment sum rules. One can see that the truncation error in any of the sum rules leads to the duality-violation of the same order $O(\Lambda^{2+K}/m_c^2\delta m^K)$. An interesting feature about these esti-

[6]Notice that these relations are exact and do not have any $1/m_c$ corrections.

[7]We do not have a classical Wilsonian scheme where the ultraviolet region is referred to the Wilson coefficients and the infrared region is referred to the matrix elements of the operators, so we can have these regions mixed.

mates is that the dependence on a has disappeared from the final result. Hence, *the estimates are independent of the details of the potential at large r*, provided the potential guarantees the confinement, i.e., a is positive.

These are however rather crude estimates which do not take into account further possible suppressions (due, e.g., to the orthogonality of the wave functions of the ground $n=0$ and the excited states $n>0$). In such a case the real accuracy is better, and might depend on the details of the potential also at large r. In general, we can state that the truncation (duality-violaiton) error occurs at the order

$$\frac{\Lambda^2}{m_c^2}\left(\frac{\Lambda}{\delta m}\right)^b, \tag{98}$$

where the exponent $b>0$ depends on the properties of the potential (in general, both at short and long distances). A more detailed analysis of which potentials satisfy the above requirements is beyond the scope of this paper and is left for another publication [13].

If we would like to have the truncation error of a higher order in $1/m_c$, e.g. in $O(\Lambda^3/m_c^3)$, this is not so straight. Namely, in this case we need

$$\frac{\delta^{(k)}}{\delta m^k}\simeq O\left(\frac{\Lambda^3}{\delta m^2 m_c}\right). \tag{99}$$

As we have noticed, the series for $\delta^{(k)}$ in the main part does not depend on m_c, so the only possibility to have the relation (99) fulfilled in the SV limit, is to have $r_n^2=0$ starting from some number n. (Exactly this situation takes place for the HO potential where all $D_0\to D_n$ transition radii for $n>1$ are equal zero [17]). In this case for large enough values of δm, the term proportional to r_n^2 in Eq. (93) disappears and the second term provides the truncation error of the order $O(\delta m^2\Lambda^2/m_c^4)$.

As we are going to show elsewhere, the accuracy of duality of order Λ^3/m_c^3 can be achieved if we keep a fixed ratio $\delta m/m_c$ when $m_c\to\infty$. One can proceed exactly along the same lines, but technically a bit different treatment is necessary: namely, at several places throughout the paper terms of the order $\delta m^3/m_c^3$ have been omitted, and they should be kept if the limit $\delta m/m_c=$const is considered. This analysis will be presented in [13].

Finally, it is interesting to notice that all resonance levels opened at $q^2=0$ are contributing on equal footing to the sum rules and therefore to the decay rate. So, a considerable delay in opening channels with large n compared to the channels with small n with the increasing recoil does not matter at all. This is a very important feature which basically determines a high accuracy of the OPE calculation of the integrated decay rate (cf. [6,12]).

2. The smeared q^2 distribution and the local duality

The situation however differs considerably if we consider the differential decay widths. We find it more physical to use here the four vector q^2 variable. The region near q^2_{max} (zero recoil) is special: as we move to higher q^2, the excited channels close one after another leaving ultimately only the D_0 ground state opened.

Let us consider a partially integrated decay rate in the q^2-region above the threshold of the $D_{n=1}$ channel. In this case the relation between the OPE and the exact result (which is reduced in this case to the exclusive $B\to D_0 l\nu$ decay) reads

$$\int_{(\delta m-\Delta_1)^2}^{\delta m^2} dq^2\frac{d\Gamma(B\to X_c l\nu)}{dq^2} = \int_{(\delta m-\Delta_1)^2}^{\delta m^2} dq^2\frac{d\Gamma(b\to c l\nu)}{dq^2}\left[1+O\left(\frac{\Delta_1\delta m}{m_c^2}\right)\right]. \tag{100}$$

In this formula we have neglected a difference between the upper boundaries of the quark and hadron channels of the order Λ^2/m_c^2. Equation (100) means that local duality near maximal q^2 is violated at order $O(\Lambda_1\delta m/m_c^2)$. As we have seen, the dangerous terms of this order are cancelled in the integrated rate against similar contributions of other channels due to the Voloshin sum rule. However, the $O(\Lambda\delta m/m_c^2)$ violation of the local duality might have negative consequences for the application of the method to the analysis of the experimental results. For example this happens if one observes only a small part of the phase space near maximal q^2 [6].

VII. CONCLUSION

We have studied quark-hadron duality in decays of heavy mesons in the SV limit using the nonrelativistic potential model for the description of mesons as $q\bar{q}$ bound states. Our main results are as follows:

(1) The OPE is constructed and the following Λ/m_c and $\Lambda/\delta m$ double series is found for the integrated decay rate:

$$\begin{aligned}\frac{\Gamma^{OPE}(B\to X_c l\nu)}{\Gamma(b\to c l\nu)} &= 1+C_0\frac{\langle B|\vec{k}^2+V_1|B\rangle}{2m_c^2} \\ &\quad +(1-C_0)\frac{\langle B|\vec{k}^2|B\rangle}{2m_c^2} \\ &\quad +\sum_{k=1}^{k_0} C_k\frac{\langle B|\vec{k}(h_{bd}-\epsilon_B)^k\vec{k}|B\rangle}{2m_c^2(\delta m)^k} \\ &\quad +O\left(\frac{\Lambda^2\delta m}{m_c^3}\right),\end{aligned}$$

where C_k are calculable constants and k_0 depends on the leptonic tensor.

(2) The HQ expansion of the transition form factors in the nonrelativistic potential model is performed. A nonrelativistic analog of the Luke theorem for the exclusive transition form factor between the ground states is obtained.

It is shown that the sum of the squares of the $B \to D_n l \nu$ transition form factors are expressed through the expectation values of the operators emerging in the OPE series. These nonrelativistic analogs of the Bjorken, Voloshin, and higher order sum rules provide a bridge between the sum over exclusive channels and the OPE series.

(3) The integrated decay rate is calculated by direct summation of exclusive channels. For the comparison of this directly calculated $\Gamma(B \to X_c l \nu)$ and the corresponding $\Gamma^{OPE}(B \to X_c l \nu)$ the sum rules are necessary. A difference (duality-violation) between the two expressions is observed. As shown explicitly by the use of the sum rules, this difference is connected with the higher $c\bar{d}$ resonances which are forbidden kinematically in the decay process but are implicitly taken into account in the OPE approach. Therefore the accuracy of the OPE is directly related to the error induced by the kinematical truncation in the sum rules (Bjorken, Voloshin, etc). The actual error depends on the convergence of the series, i.e., on the nature of the potential. We have discussed the constraints on the latter convergence which lead to the duality violation of order $O(\Lambda^{2+b}/m_c^2 \delta m^b)$ with b depending on the behavior of the potential both at the short and long distances.

Up to the mentioned duality-violation, the agreement between the OPE and the exclusive sum is achieved within different $1/m_c$ orders due to different reasons:

The leading order and the subleading $\delta m/m_c$ and Λ/m_c orders the free quark integrated decay rate $\Gamma(b \to c l \nu)$ is equal to the rate of the transition into the ground state D_0. This is due to the specific behavior of the transition form factor between the ground states near the zero recoil (Luke theorem). Also part of the Λ^2/m_Q^2 correction in the OPE result proportional to the $\langle B|\vec{k}^2 + V_1|B\rangle$ matches the contribution of the ground state D_0 in the exclusive sum.

For higher order terms the agreement between the OPE and the exclusive sum is a collective effect due to subtle cancellations in the sum over exclusive channels:

Namely, each of the individual decay rates $\Gamma(B \to D_n l \nu)$ contain potentially large terms of the order $\delta m^2/m_c^2$ and $\Lambda \delta m/m_c^2$. These terms cancel in the exclusive sum due to the Bjorken and Voloshin sum rules, respectively. The higher order sum rules allow us to represent the contribution of exclusive channels in terms of the average values of the operators O_i over the B-meson state.

(4) If the differential semileptonic decay widths are considered near maximum q^2, the violation of the local duality occurs at order $O(\Lambda \delta m/m_c^2)$.

Clearly, in QCD the situation is more complicated because of the multiparticle X_c states, pion emission, hybrid and multiquark exotic D mesons, radiative corrections. Nevertheless the duality violation due to the kinematical truncation of the series should be quite similar to the case of nonrelativistic quantum mechanics. Also similar is the role of the inclusive sum rules in obtaining the duality relations.

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Chapitre 2

Hypothèses de factorisation

> Ce chapitre est dédié aux désintégrations non leptoniques des mésons B : nous présenterons l'hypothèse de factorisation, le modèle QCDF qui est une amélioration apportée à cette hypothèse et nous finirons par une confrontation d'un ensemble de données expérimentales aux prédictions de ce modèle.

Au cours du chapitre précédent, nous nous sommes uniquement intéressés aux désintégrations semileptoniques des mésons B. Une autre classe de réactions est formée par l'ensemble des désintégrations non leptoniques pour laquelle il est tout aussi important d'avoir des outils capables de la décrire car les réactions de désintégration des B sont extrêmement bien étudiées et analysées du point de vue expérimental (un thème d'étude important concerne bien sûr la violation de CP et la mesure des éléments de matrice CKM du modèle standard). La difficulté théorique derrière la modélisation de ces désintégrations non leptoniques est le calcul des éléments de matrice de transition des opérateurs à quatre fermions : historiquement, une méthode énormément utilisée fut l'hypothèse de factorisation, appelée « factorisation naïve » dans la suite (cf. paragraphe 2.2) puis, plus récemment, une amélioration a été apportée ([22, 23, 24]) qui a conduit au modèle de « factorisation QCD » (paragraphe 2.3).

Ce modéle de « factorisation QCD » (ou QCDF) a été appliqué aux procesus $B \to PP$ (où P désigne un méson pseudoscalaire) et nous avons réalisé une étude similaire pour les désintégrations $B \to PV$ (V désigne un méson vectoriel), dans le but de confronter les prédictions théoriques de ce modèle amélioré à un ensemble exhaustif de données accessibles fournies par la communauté expérimentale (paragraphe 2.4 et suivants).

2.1 Dynamique de la désintégration

Pour décrire les désintégrations $B \to PV$, il nous faut calculer les éléments de matrice $\langle PV|\mathscr{H}_{\text{eff}}|B\rangle$ sachant que $\mathscr{H}_{\text{eff}}$ est le hamiltonien effectif habituel :

$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1\, Q_1^p + C_2\, Q_2^p + \sum_{i=3,\ldots,10} C_i\, Q_i + C_{7\gamma}\, Q_{7\gamma} + C_{8g}\, Q_{8g} \right) + \text{h.c.}$$

où C_i sont les coefficients de Wilson qui contiennent les contributions à courte distance, λ_p s'expriment en terme des éléments de matrice CKM selon $\lambda_p = V_{pb}V_{ps}^*$ ou $V_{pb}V_{pd}^*$ et Q_i désignent les opérateurs locaux :

→ *arbre*

$$Q_1^p = (\bar{p}b)_{V-A}(\bar{s}p)_{V-A} \qquad\qquad Q_2^p = (\bar{p}_i b_j)_{V-A}(\bar{s}_j p_i)_{V-A}$$

→ *pingouins*

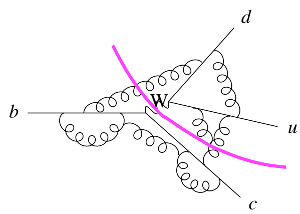
$$\begin{aligned}
Q_3 &= (\bar{s}b)_{V-A} \textstyle\sum_q (\bar{q}q)_{V-A} & Q_4 &= (\bar{s}_i b_j)_{V-A} \textstyle\sum_q (\bar{q}_j q_i)_{V-A} \\
Q_5 &= (\bar{s}b)_{V-A} \textstyle\sum_q (\bar{q}q)_{V+A} & Q_6 &= (\bar{s}_i b_j)_{V-A} \textstyle\sum_q (\bar{q}_j q_i)_{V+A} \\
Q_7 &= (\bar{s}b)_{V-A} \textstyle\sum_q \frac{3}{2} e_q (\bar{q}q)_{V+A} & Q_8 &= (\bar{s}_i b_j)_{V-A} \textstyle\sum_q \frac{3}{2} e_q (\bar{q}_j q_i)_{V+A} \\
Q_9 &= (\bar{s}b)_{V-A} \textstyle\sum_q \frac{3}{2} e_q (\bar{q}q)_{V-A} & Q_{10} &= (\bar{s}_i b_j)_{V-A} \textstyle\sum_q \frac{3}{2} e_q (\bar{q}_j q_i)_{V-A}
\end{aligned}$$

→ *dipôle*

$$Q_{7\gamma} = \frac{-e}{8\pi^2}\, m_b\, \bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b \qquad Q_{8g} = \frac{-g_s}{8\pi^2}\, m_b\, \bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b$$

2.2 La factorisation "naïve"

Considérons la désintégration générique $B \to M_1 M_2$ (M_1 et M_2 désignent les mésons de l'état final). Alors, dans cette hypothèse, les éléments de transition des opérateurs à quatre fermions sont écrits comme le produit de deux éléments de transition à deux fermions $(B \to M_1) \times (\text{vide} \to M_2)$; autrement dit, les éléments de l'état final « ne parlent pas » (décorrélation de couleur), par échange de gluons, entre eux et avec le reste. Par exemple :



$$\langle \pi\, D|(\bar{u}\,\gamma_\mu\, d)(\bar{c}\,\gamma^\mu\, b)|\bar{B}^o\rangle \;=\; \langle D|\bar{c}\,\gamma^\mu\, b|\bar{B}^o\rangle \,\times\, \langle \pi|\bar{u}\,\gamma_\mu\, d|0\rangle$$

Cette hypothèse produit en général des prédictions convenables mais il est connu qu'elle présente plusieurs défauts comme une mauvaise dépendance en échelle de renormalisation des résultats (due aux dimensions anomales des éléments de matrice qui sont incorrectes).

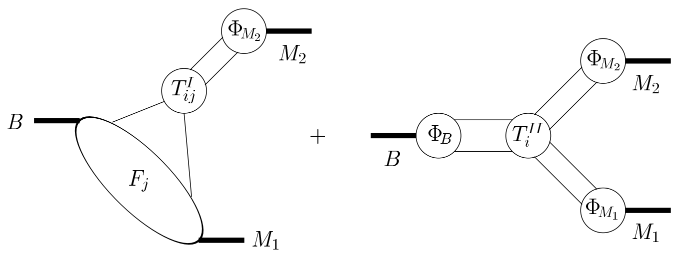
2.3 La factorisation QCD (QCDF)

QCDF repose sur la considération suivante : lors de la désintégration $B \to M_1 M_2$, où l'un des deux mésons finaux est un méson léger, la masse du quark b est grande devant l'échelle d'énergie de l'interaction forte si bien que les contributions dues aux échanges de gluons mous dans l'état final deviennent négligeables. En conséquence, les transitions $B \to M_1 M_2$ s'écrivent à l'aide d'éléments de matrice factorisés et de coefficients « non-factorisables », c'est-à-dire des termes correctifs en fait, qui peuvent être calculés de façon perturbative. Autrement dit, QCDF correspond à un double développement perturbatif en :

$$\begin{cases} \alpha_s : \text{corrections QCD (gluons durs) calculables perturbativement} \\ \dfrac{\Lambda_{QCD}}{m_b} : \text{corrections de masse lourde (peu connues)} \end{cases}$$

(Le terme d'ordre zéro redonne la factorisation « naïve »)

Plus précisément, QCDF permet d'écrire l'élément de matrice d'un opérateur local Q_i comme une somme de deux termes :

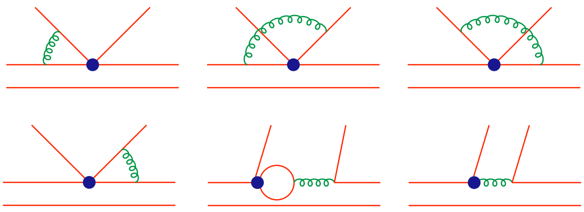


$$\langle M_1\, M_2 | Q_i | B\rangle \;=\; F_1^{B\to P}\, T^{\mathrm{I}}_{V,i} \star f_V \Phi_V \;+\; A_0^{B\to V}\, T^{\mathrm{I}}_{P,i} \star f_P \Phi_P \;+\; T^{\mathrm{II}}_i \star f_B \Phi_B \star f_V \Phi_V \star f_P \Phi_P$$

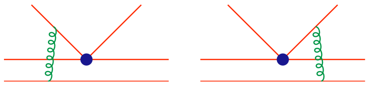
avec les notations suivantes :

- $F^{B\to P}$, $A^{B\to V}$: facteurs de forme des transitions $B \to P$ et $B \to V$
- Φ_i : amplitude de distribution sur le cône de lumière des mésons B, V et P
- $\star$: désigne une intégration sur les fractions de quantité de mouvement des quarks constituants des mésons (en variables cône de lumière)
- f_i : constantes de désintégration des mésons B, V et P
- $T^{(I)}$ et $T^{(II)}$: noyaux de diffusion calculés perturbativement en α_s ; $T^{(I)}$ correspond aux diagrammes de corrections de vertex et de pingouin et $T^{(II)}$ correspond à l'interaction par gluon dur avec le quark spectateur du méson B

Une représentation graphique de ces noyaux est donnée dans la figure suivante :



noyau $T^{(I)}$



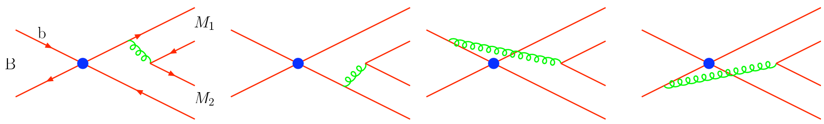
noyau $T^{(II)}$

Finalement, en revenant aux désintégrations $B \to PV$, les éléments de matrice dans la limite de masse infinie s'écrivent :

$$\mathscr{A}(B \to PV) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_{i=1}^{10} \lambda_p \; a_i^p \; \langle P\, V | \mathcal{O}_i | B \rangle_{\rm nf} \tag{2.1}$$

où a_i contient les effets « non factorisables » (développement en α_s) et $\langle P\, V|\mathcal{O}_i|B\rangle_{\rm nf}$ est l'élément de matrice factorisé au sens de la « factorisation naïve »

Remarque. À cette expression, il faut aussi rajouter le terme suivant (appelé terme d'annihilation) qui, bien que d'ordre $\frac{\Lambda_{QCD}}{m_b}$, peut néanmoins recevoir une contribution non négligeable des corrections de QCD :



$$\mathscr{A}^{\rm ann}(B \to PV) \propto f_B \; f_P \; f_V \sum \lambda_p \; b_i$$

où b_i sont des paramètres caractéristiques de l'annihilation.

Il existe encore d'autres termes qui sont formellement d'ordre $\frac{\Lambda_{QCD}}{m_b}$ mais qui doivent être pris en compte : il se trouve en effet que, dans (2.1), certains a_i multiplient des éléments de matrice qui sont supprimés par des puissances de rapports de la forme (par exemple pour le canal $B \to K\,\pi$) :

$$r_\chi^K \;=\; \frac{2\, m_K^2}{m_b\,(m_q + m_s)} \qquad q = u, d$$

Or, bien que r_χ^K soit littéralement d'ordre $\mathscr{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$, il est numériquement plus proche de l'ordre $\mathscr{O}(1)$ car le facteur $\frac{1}{m_b}$ est compensé par le terme en $\frac{m_K^2}{m_q + m_s}$.

Autre remarque. Les noyaux T^{II} présentent des singularités de bord qui proviennent d'intégrales de la forme $\int_0^1 \frac{\mathrm{d}y}{1-y}$ (y est relié à l'impulsion de l'antiquark du méson formé à partir du quark spectateur du méson B initial) qui divergent logarithmiquement. On s'attend à ce que des effets non-perturbatifs adoucissent ces singularités mais... on ne sait pas les calculer. Aussi, nous paramétrons pour chaque méson ces intégrales par $X_H = (1 + \rho_H\, e^{i\,\varphi_H}) \ln \frac{m_B}{\Lambda_h}$ ce qui introduit deux nouveaux paramètres dans le modèle ($\rho_H \leq 1$ et la phase arbitraire φ_H).

Notons enfin qu'un phénomène similaire se produit pour les termes d'annihilation $\mathscr{A}^{\text{ann}}$ et que l'on introduit également $X_A = (1 + \rho_A\, e^{i\,\varphi_A}) \ln \frac{m_B}{\Lambda_h}$.

2.4 Analyse et premiers résultats

2.4.1 Principe de l'analyse

Afin de confronter QCDF aux données, nous avons commencé par compiler un ensemble exhaustif de résultats expérimentaux fournis par les collaborations Babar, Belle et CLEO ; en les combinant, nous en avons déduit une valeur centrale ainsi que les erreurs correspondantes pour chaque canal possible.

Dans le même temps, nous avons entré dans Mathematica toutes les relations du modèle présenté dans la section précédente permettant de calculer tous les rapports d'embranchement possibles (les formules utilisées sont recensées dans [25]). Les paramètres importants du modèle sont :

→ les facteurs de forme $A^{B\to\rho}$, $A^{B\to\omega}$, $A^{B\to K^*}$, $F^{B\to\pi}$ et $F^{B\to K}$

→ la constante de désintégration f_B

→ les facteurs CKM R_u, R_c et γ

→ la masse m_s, l'échelle d'énergie μ

→ ρ_A and φ_A (nous avons pris $X_H = X_A$ pour tous les mésons, ce qui constitue une hypothèse simplificatrice grossière et pas vraiment justifiable physiquement)

→ λ_B (amplitudes de distribution)

Enfin, nous avons réalisé l'analyse proprement dite :

1° Tout d'abord, nous avons utilisé les valeurs expérimentales pour ajuster les paramètres du modèle (par minimisation de χ^2) en contraignant les valeurs possibles de ces paramètres théoriques dans des intervalles bien définis (le choix de ces intervalles provient de la littérature, cf. références dans [25]). Puis, à partir de ces paramètres qui donnent le meilleur ajustement, nous avons calculé nos prédictions théoriques (taux d'embranchement, asymétries CP).

2° Enfin, nous avons réalisé un « test de qualité », relatif à l'accord entre les mesures et nos prédictions, fondé sur un test de type « goodness-of-fit » à base de Monte-Carlo :

- les erreurs expérimentales de chaque mesure sont utilisées pour générer de nouvelles valeurs distribuées autour des rapports d'embranchement et des asymétries CP obtenus dans l'ajustement précédent (avec une probabilité gaussienne)
- un nouvel ajustement, du même type que celui exécuté sur les données expérimentales réelles, est réalisé en utilisant ces nouvelles données simulées : le χ^2 obtenu est enregistré dans un histogramme $H(\chi^2)$
- finalement, on compare le $\chi^2_{\rm data}$ obtenu à partir des mesures expérimentales avec le χ^2 que l'on obtiendrait si nos prédictions étaient correctes et un niveau de confiance CL est calculé suivant :

$$CL = \frac{\displaystyle\int_{\chi^2>\chi^2_{\rm data}} H(\chi^2)\,\mathrm{d}\chi^2}{\displaystyle\int_{\chi^2>0} H(\chi^2)\,\mathrm{d}\chi^2}$$

3° Nous n'avons pas pris en compte les canaux faisant intervenir des processus à deux gluons (donc rejeté les canaux contenant des mésons η').

2.4.2 Présentation des résultats : premier scénario

Dans ce premier scénario, tous les paramètres du modèle sont laissés libres ; alors, l'ajustement avec les valeurs expérimentales produit le résultat suivant :

Paramètre	**Domaine**	**Scénario n° 1**
γ (deg)	libre	99.955
m_s (GeV)	[0.085,0.135]	0.085
μ (GeV)	[2.1,8.4]	3.355
ρ_A	[−1,1]	1.000
ϕ_A(deg)	[−180,180]	−22.928
λ_B (GeV)	[0.2,0.5]	0.500
f_B (GeV)	[0.14,0.22]	0.220
R_u	[0.35,0.49]	0.350
R_c	[0.018,0.025]	0.018
$A_0^{B\to\rho}$	[0.3162,0.4278]	0.373
$F_1^{B\to\pi}$	[0.23,0.33]	0.330
$A_0^{B\to\omega}$	[0.25,0.35]	0.350
$A_0^{B\to K^*}$	[0.3995,0.5405]	0.400
$F_1^{B\to K}$	[0.28,0.4]	0.333

Notons que beaucoup de paramètres prennent l'une de leurs deux valeurs extrêmes. À partir de ces paramètres, les prédictions théoriques calculées sont regroupées dans le tableau suivant :

Canaux	**Expérience**	**Scénario n° 1** Prédiction	χ^2
$\mathcal{BR}(\overline{B}^0 \to \rho^0 \pi^0)$	2.07 ± 1.88	0.132	1.1
$\mathcal{BR}(\overline{B}^0 \to \rho^+ \pi^-)$		11.023	
$\mathcal{BR}(\overline{B}^0 \to \rho^- \pi^+)$		18.374	
$\mathcal{BR}(\overline{B}^0 \to \rho^\pm \pi^\mp)$	25.53 ± 4.32	29.397	0.8
$\mathcal{BR}(B^- \to \rho^0 \pi^-)$	9.49 ± 2.57	9.889	0.0
$\mathcal{BR}(B^- \to \omega \pi^-)$	6.22 ± 1.7	6.002	0.0
$\mathcal{BR}(B^- \to \Phi \pi^-)$		0.004	
$\mathcal{BR}(B^- \to \rho^- \pi^0)$		9.646	
$\mathcal{BR}(B^- \to K^{*-} K^0)$		0.457	
$\mathcal{BR}(B^- \to K^{*0} K^-)$		0.490	
$\mathcal{BR}(\overline{B}^0 \to \rho^0 \overline{K}^0)$		5.865	
$\mathcal{BR}(\overline{B}^0 \to \omega \overline{K}^0)$	6.34 ± 1.82	2.318	4.9
$\mathcal{BR}(\overline{B}^0 \to \rho^+ K^-)$	15.88 ± 4.65	6.531	4.0
$\mathcal{BR}(\overline{B}^0 \to K^{*-} \pi^+)$	19.3 ± 5.2	9.760	3.4
$\mathcal{BR}(B^- \to K^{*-} \pi^0)$	7.1 ± 11.4	7.303	0.0
$\mathcal{BR}(\overline{B}^0 \to \Phi \overline{K}^0)$	8.72 ± 1.37	8.360	0.1
$\mathcal{BR}(B^- \to \overline{K}^{*0} \pi^-)$	12.12 ± 3.13	7.889	1.8
$\mathcal{BR}(B^- \to \rho^0 K^-)$	8.92 ± 3.6	1.882	3.8
$\mathcal{BR}(B^- \to \rho^- \overline{K}^0)$		7.140	
$\mathcal{BR}(B^- \to \omega K^-)$	2.92 ± 1.94	2.398	0.1
$\mathcal{BR}(B^- \to \Phi K^-)$	8.88 ± 1.24	8.941	0.0
$\mathcal{BR}(\overline{B}^0 \to \overline{K}^{*0} \eta)$	16.41 ± 3.21	22.807	4.0
$\mathcal{BR}(B^- \to K^{*-} \eta)$	25.4 ± 5.6	17.855	1.8

Nous observons que le modèle fonctionne bien pour les canaux non-étranges et qu'il y a des problèmes pour les canaux étranges (à part $B \to \Phi K$).

Si nous retirons de l'étude les canaux contenant un K^*, alors il y a accord global entre QCDF et les valeurs expérimentales (cette conclusion est en accord avec [26].).

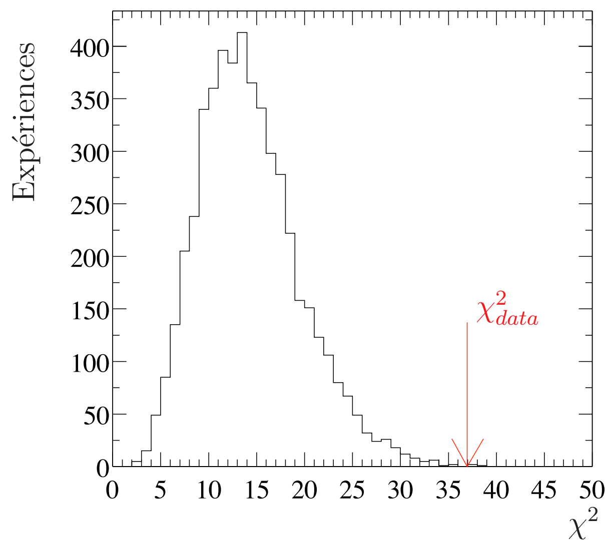
Pour ce qui est des asymétries CP, nous obtenons les résultats regroupés dans le tableau ci-après :

	Expérience	**Scénario n° 1** Prédiction	χ^2
$\Delta\,\mathcal{C}_{\rho\pi}$	0.38 ± 0.23	0.250	8.1/4
$\mathcal{C}_{\rho\pi}$	0.45 ± 0.21	0.019	
$\mathcal{A}_{CP}^{\rho\,\pi}$	-0.22 ± 0.11	-0.015	
$\mathcal{A}_{CP}^{\rho\,K}$	0.19 ± 0.18	0.060	
$\mathcal{A}_{CP}^{\omega\,\pi^-}$	-0.21 ± 0.19	-0.072	0.5
$\mathcal{A}_{CP}^{\omega\,K^-}$	-0.21 ± 0.28	0.029	0.7
$\mathcal{A}_{CP}^{\eta\,K^{*-}}$	-0.05 ± 0.3	-0.138	0.1
$\mathcal{A}_{CP}^{\eta\,\overline{K}^{*0}}$	0.17 ± 0.28	-0.186	1.6
$\mathcal{A}_{CP}^{\phi\,K^-}$	-0.05 ± 0.2	0.006	0.1
			36.9

	Expérience	**Scénario n° 1**
$\mathcal{A}_{CP}^{\rho^+\,\pi^-}$	$-0.82 \pm 0.31 \pm 0.16$	-0.04
$\mathcal{A}_{CP}^{\rho^-\,\pi^+}$	$-0.11 \pm 0.16 \pm 0.09$	-0.0002

Il apparaît que les asymétries CP directes sont sous-estimées (surtout le canal $B \to \rho^+\,\pi^-$ dont l'expérience prédit une contribution plutôt importante).

Enfin, le test « goodness of fit » nous donne :



et le niveau de confiance du modèle calculé est inférieur à 0.1% ce qui semble dire que les données expérimentale excluent le modèle...

2.4.3 Résumé

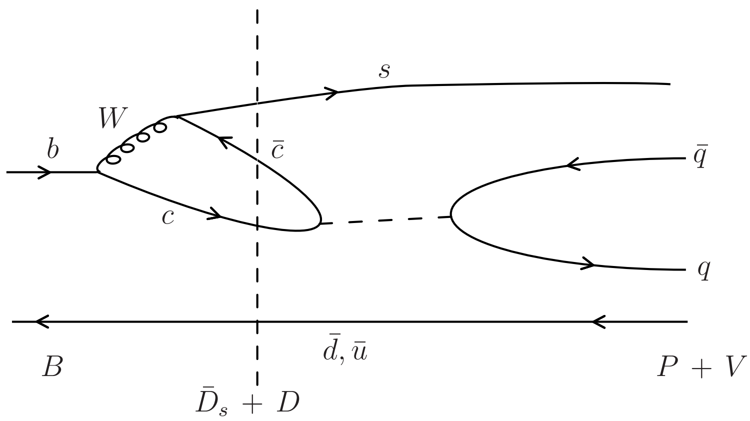
Il y a des problèmes avec les canaux faisant intervenir un quark étrange : ils sont sous-estimés par le modèle QCDF. De même, les asymétries CP directes sont aussi sous-estimées dans les canaux non-étranges. Et enfin, il y a accord avec les données expérimentales lorsque les canaux K^* ne sont pas pris en compte.

2.5 Les “pingouins charmés”

Pour tenter de faire mieux coller les prédictions de QCDF aux données expérimentales, il faudrait trouver un mécanisme non-perturbatif qui augmente les contributions des diagrammes pingouins $|P|$ dans les canaux étranges (cela permettrait d'augmenter les rapports d'embranchement) ainsi que le rapport $|P|/|T|$ dans les canaux non étranges (ce qui augmenterait également les asymétries CP). Aussi avons-nous utilisé un modèle d'interaction à longue distance : le modèle des pingouins charmés.

2.5.1 Philosophie du modèle

L'idée de ce modèle est que le quark c n'est pas si lourd que cela et donc qu'il peut se propager.... d'où un opérateur correspondant non local :



Il y a création d'un état hadronique intermédiaire (le système $\bar{D}_s + D$) qui peut se propager à longue distance puis, finalement, la paire $c\bar{c}$ s'annihile par interaction forte en une paire de quarks légers pour donner à la fin deux mésons légers.

En pratique, cela revient à ajouter à l'élément de matrice de transition $B \to PV$ une amplitude constante, qui devra être ajustée ; typiquement, cette amplitude est de la forme :

$$\mathscr{A}^{\mathrm{LD}}(B \to PV) = \frac{G_F}{\sqrt{2}} m_B^2 \lambda_c (CG^P \mathcal{A}^P + CG^V \mathcal{A}^V)$$

(CG : coefficients de Clebsch-Gordan de $SU(3)$ de saveur)

Deux paramètres supplémentaires sont donc rajoutés aux modèle initial : $\mathcal{A}^P$ et $\mathcal{A}^V$

2.5.2 Nouveaux résultats : second scénario

Les nouvelles valeurs des paramètres ajustés sont alors :

Paramètre	**Domaine**	**Scénario n° 2**
γ (deg)	[34°,82°]	81.933
m_s (GeV)	[0.085,0.135]	0.085
μ (GeV)	[2.1,8.4]	5.971
ρ_A	[−1,1]	1.000
ϕ_A(deg)	[−180,180]	−87.907
λ_B (GeV)	[0.2,0.5]	0.500
f_B (GeV)	[0.14,0.22]	0.203
R_u	[0.35,0.49]	0.350
R_c	[0.018,0.025]	0.018
$A_0^{B\to\rho}$	[0.3162,0.4278]	0.377
$F_1^{B\to\pi}$	[0.23,0.33]	0.301
$A_0^{B\to\omega}$	[0.25,0.35]	0.326
$A_0^{B\to K^*}$	[0.3995,0.5405]	0.469
$F_1^{B\to K}$	[0.28,0.4]	0.280
Re$[\mathcal{A}^P]$	[−0.01,0.01]	0.00253
Im$[\mathcal{A}^P]$	[−0.01,0.01]	−0.00181
Re$[\mathcal{A}^V]$	[−0.01,0.01]	−0.00187
Im$[\mathcal{A}^V]$	[−0.01,0.01]	0.00049

Nous notons là encore que certains des paramètres prennent une de leurs valeurs extrêmes. Par ailleurs, les valeurs des termes du modèle des pingouins charmés sont plutôt petites (principalement à cause du canal $B \to \Phi K$ qui est en bon accord avec QCDF [1] et donc qui limite énormément l'introduction d'un terme de longue distance).

Quant aux rapports d'embranchement, ils sont rassemblés dans le tableau suivant :

Canaux	**Expérience**	**Scénario n° 2**	
		Prédiction	χ^2
$\mathcal{BR}(\overline{B}^0 \to \rho^0\,\pi^0)$	2.07 ± 1.88	0.177	1.0
$\mathcal{BR}(\overline{B}^0 \to \rho^+\,\pi^-)$		10.962	
$\mathcal{BR}(\overline{B}^0 \to \rho^-\,\pi^+)$		17.429	
$\mathcal{BR}(\overline{B}^0 \to \rho^\pm\,\pi^\mp)$	25.53 ± 4.32	28.391	0.4

1. Bien que les canaux étranges soient généralement sous-estimés par QCDF, les canaux $s\bar{s}s$ constituent une exception.

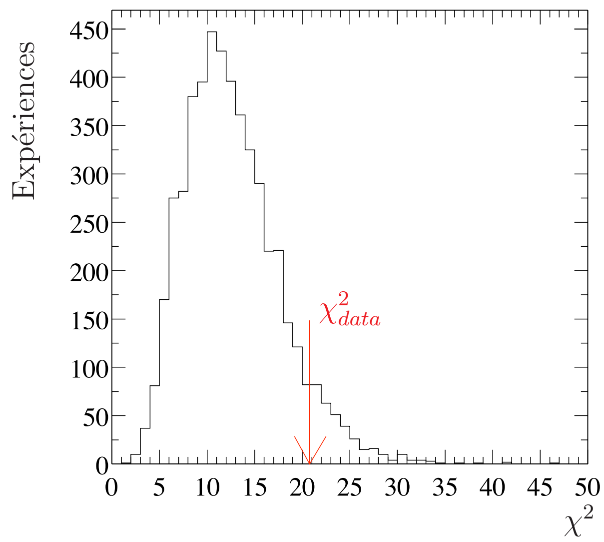
Canaux	**Expérience**	**Scénario n° 2** Prédiction	χ^2
$\mathcal{BR}(B^- \to \rho^0\,\pi^-)$	9.49 ± 2.57	7.879	0.4
$\mathcal{BR}(B^- \to \omega\,\pi^-)$	6.22 ± 1.7	5.186	0.4
$\mathcal{BR}(B^- \to \Phi\,\pi^-)$		0.003	
$\mathcal{BR}(B^- \to \rho^-\,\pi^0)$		11.404	
$\mathcal{BR}(B^- \to K^{*-}\,K^0)$		0.788	
$\mathcal{BR}(B^- \to K^{*0}\,K^-)$		0.494	
$\mathcal{BR}(\overline{B}^0 \to \rho^0\,\overline{K}^0)$		8.893	
$\mathcal{BR}(\overline{B}^0 \to \omega\,\overline{K}^0)$	6.34 ± 1.82	5.606	0.2
$\mathcal{BR}(\overline{B}^0 \to \rho^+\,K^-)$	15.88 ± 4.65	14.304	0.1
$\mathcal{BR}(\overline{B}^0 \to K^{*-}\,\pi^+)$	19.3 ± 5.2	10.787	2.7
$\mathcal{BR}(B^- \to K^{*-}\,\pi^0)$	7.1 ± 11.4	8.292	0.0
$\mathcal{BR}(\overline{B}^0 \to \Phi\,\overline{K}^0)$	8.72 ± 1.37	8.898	0.0
$\mathcal{BR}(B^- \to \overline{K}^{*0}\,\pi^-)$	12.12 ± 3.13	11.080	0.1
$\mathcal{BR}(B^- \to \rho^0\,K^-)$	8.92 ± 3.6	5.655	0.8
$\mathcal{BR}(B^- \to \rho^-\,\overline{K}^0)$		14.006	
$\mathcal{BR}(B^- \to \omega\,K^-)$	2.92 ± 1.94	6.320	3.1
$\mathcal{BR}(B^- \to \Phi\,K^-)$	8.88 ± 1.24	9.479	0.2
$\mathcal{BR}(\overline{B}^0 \to \overline{K}^{*0}\,\eta)$	16.41 ± 3.21	18.968	0.6
$\mathcal{BR}(B^- \to K^{*-}\,\eta)$	25.4 ± 5.6	15.543	3.1

et pour les asymétries CP, nous obtenons :

	Expérience	**Scénario n° 2** Prédiction	χ^2
$\Delta\,\mathcal{C}_{\rho\pi}$	0.38 ± 0.23	0.228	3.9/4
$\mathcal{C}_{\rho\pi}$	0.45 ± 0.21	0.092	
$\mathcal{A}_{CP}^{\rho\,\pi}$	-0.22 ± 0.11	-0.115	
$\mathcal{A}_{CP}^{\rho\,K}$	0.19 ± 0.18	0.197	
$\mathcal{A}_{CP}^{\omega\,\pi^-}$	-0.21 ± 0.19	-0.198	0.0
$\mathcal{A}_{CP}^{\omega\,K^-}$	-0.21 ± 0.28	0.189	2.0
$\mathcal{A}_{CP}^{\eta\,K^{*-}}$	-0.05 ± 0.3	-0.217	0.3
$\mathcal{A}_{CP}^{\eta\,\overline{K}^{*0}}$	0.17 ± 0.28	-0.158	1.4
$\mathcal{A}_{CP}^{\phi\,K^-}$	-0.05 ± 0.2	0.005	0.1
			20.8

	Expérience	Scénario n° 2
$\mathcal{A}_{CP}^{\rho^+\pi^-}$	$-0.82 \pm 0.31 \pm 0.16$	-0.23
$\mathcal{A}_{CP}^{\rho^-\pi^+}$	$-0.11 \pm 0.16 \pm 0.09$	0.04

L'accord avec l'expérience est amélioré (comme il était prévu) et nous avons bien obtenu une augmentation des asymétries CP. Enfin, le test « goodness of fit » nous donne :



correspondant à un niveau de confiance inférieur à 7.7%. Le modèle ne peut donc pas être exclu par les données expérimentales.

2.6 Conclusions

Au vu de l'étude précédente, il apparaît qu'il est impossible de faire un ajustement correct des données expérimentales avec le modèle de factorisation amélioré QCDF ; la raison à l'origine de cet échec est une estimation trop faible par QCDF des canaux étranges (excepté le canal $B \to \Phi K$) ainsi qu'une valeur pour les asymétries CP directes non-étranges $\overline{B} \to \rho^+\pi^-$ bien plus petite que la valeur expérimentale. Par ailleurs, le test « goodness of fit » rejette le modèle (0.1%) dont les valeurs ajustées ont d'ailleurs tendance à vouloir sortir de leur domaine de variation autorisé. Un ajustement réalisé sans les canaux K^* améliore, de fait, les résultats de manière significante.

Afin d'augmenter les amplitudes P par rapport aux amplitudes T, dans le but de corriger le problème précédemment mentionné, nous avons rajouté des termes d'interaction à longue distance (les pingouins charmés) ; les modifications introduites ne permettent plus vraiment d'exclure le modèle (niveau de confiance du test « goodness of fit » de l'ordre de 8%) mais un certain nombre de paramètres prennent encore une de leurs valeurs limites et les contributions des pingouins charmés ajustées sont relativement faibles. Même si le modèle a été amélioré, ce n'est pas encore très convaincant.

En résumé, la situation pour QCDF vis-à-vis des données expérimentales ne semble pas très bonne et peut-être faudra-t-il considérer l'évolution des mesures fournies par les expérimentateurs pour affiner le test ; en effet, la prise en compte de données plus récentes concernant les canaux $B \to \rho\pi$ et $B \to \rho K$ de la collaboration Babar ont augmenté le niveau de confiance de QCDF jusqu'à 1% et diminué celui du modéle « pingouins charmés » à 3%...

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Testing QCD factorization and charming penguin diagrams in charmless $B \to PV$

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We try a global fit of the experimental branching ratios and CP asymmetries of the charmless $B \to PV$ decays according to QCD factorization. We find it impossible to reach satisfactory agreement, the confidence level (C.L.) of the best fit being smaller than 0.1%. The main reason for this failure is the difficulty to accommodate several large experimental branching ratios of the strange channels. Furthermore, experiment was not able to exclude a large direct CP asymmetry in $\overline{B^0} \to \rho^+ \pi^-$, which is predicted to be very small by QCD factorization. Trying a fit with QCD factorization complemented by a charming-penguin-diagram-inspired model we reach a best fit which is not excluded by experiment (C.L. of about 8%) but is not fully convincing. These negative results must be tempered by the remark that some of the experimental data used are recent and might still evolve significantly.

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I. INTRODUCTION

It is an important theoretical challenge to master the nonleptonic decay amplitudes and particularly B nonleptonic decay. It is not only important *per se*, in view of the many experimental branching ratios which have been measured recently with increasing accuracy by BaBar [1–10], Belle [11–15], and CLEO [16–21], but it is also necessary in order to get control over the measurement of CP violating parameters and particularly the so-called angle α of the unitarity triangle. It is well known that extracting α from measured indirect CP asymmetries needs a sufficient control of the relative size of the so-called tree (T) and penguin (P) amplitudes.

However, the theory of nonleptonic weak decays is a difficult issue. Lattice QCD gives predictions for semileptonic or purely leptonic decays but not directly for nonleptonic ones. For a while, one has used what is now called "naive factorization" which replaces the matrix element of a four-fermion operator in a heavy-quark decay by the product of the matrix elements of two currents, one semileptonic matrix element and one purely leptonic. It has been noticed for a while that naive factorization did provide reasonable results, although it was impossible to derive it rigorously from QCD except in the $N_c \to \infty$ limit. It is also well known that the matrix elements computed via naive factorization have a wrong anomalous dimension.

Recently an important theoretical progress has been performed [22,23] which is commonly called "QCD factorization." It is based on the fact that the b quark is heavy compared to the intrinsic scale of strong interactions. This allows one to deduce that nonleptonic decay amplitudes in the heavy-quark limit have a simple structure. It implies that corrections termed "nonfactorizable," which were thought to be intractable, can be calculated rigorously. The anomalous dimension of the matrix elements is now correct to the order at which the calculation is performed. Unluckily the subleading $\mathcal{O}(\Lambda/m_b)$ contributions cannot in general be computed rigorously because of infrared singularities, and some of these which are chirally enhanced are not small, of order $\mathcal{O}(m_\pi^2/m_b(m_u+m_d))$, which shows that the inverse m_b power is compensated by $m_\pi/(m_u+m_d)$. In the seminal papers of [22,23], these contributions are simply bounded according to a qualitative argument which could as well justify a significantly larger bound with the risk of seeing these unpredictable terms become dominant. It is then of utmost importance to check experimentally QCD factorization (QCDF).

Since a few years, it has been applied to $B \to PP$ (two charmless pseudoscalar mesons) decays. The general feature is that the decay to nonstrange final states is predicted to be slightly larger than experiment while the decay to strange final states is significantly underestimated. In [23] it is claimed that this can be cured by a value of the unitarity-triangle angle γ larger than generally expected, larger maybe than $90°$. Taking also into account various uncertainties the authors conclude positively as for the agreement of QCD factorization with the data. In [24,25] it was objected that the large branching ratios for strange channels argued in favor of the presence of a specific nonperturbative contribution called "charming penguin diagrams" [25–30]. We will return to this approach later.

The $B \to PV$ (charmless pseudoscalar + vector mesons) channels are more numerous and allow a more extensive check. In Ref. [31] it was shown that naive factorization implied a rather small $|P|/|T|$ ratio, for the $\overline{B^0} \to \rho^\pm \pi^\mp$ decay channel, to be compared to the larger one for the B

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$\to\pi^+\pi^-$. This prediction is still valid in QCD factorization where the $|P|/|T|$ ratio is of about 3% (8%) for the $\overline{B^0} \to\rho^+\pi^-$ ($\overline{B^0}\to\rho^-\pi^+$) channel against about 20% for the $\overline{B^0}\to\pi^+\pi^-$ one. If this prediction were reliable, it would put the $\overline{B^0}\to\rho^+\pi^-$ channel in a good position to measure the Cabibbo-Kobayashi-Maskawa (CKM) angle α via indirect CP violation. This remark triggered the present work: we wanted to check QCD factorization in the $B\to PV$ sector to estimate the chances for a relatively easy determination of the angle α.

The noncharmed $B\to PV$ amplitudes have been computed in naive factorization [32], in some extension of naive factorization including strong phases [33], in QCD factorization [34–36], and some of them in so-called perturbative QCD [37,38]. In [39], a global fit to $B\to PP,PV,VV$ was investigated using QCDF in the heavy-quark limit and a plausible set of soft QCD parameters has been found that, apart from three pseudoscalar vector channels, fit well the experimental branching ratios. Recently [36] it was claimed from a global fit to $B\to PP,PV$ that the predictions of QCD factorization are in good agreement with experiment when one excludes some channels from the global fit. When this paper appeared we had been for some time considering this question and our feeling was significantly less optimistic. This difference shows that the matter is far from trivial mainly because experimental uncertainties can still be open to some discussion. We would like in this paper to understand better the origin of the difference between our unpublished conclusion and the one presented in [36] and try to settle the present status of the comparison of QCD factorization with experiment.

One general remark about QCD factorization is that it yields predictions which do not differ so much from naive factorization ones. This is expected since QCD factorization makes a perturbative expansion the zeroth order of which being naive factorization. As a consequence, QCD factorization predicts very small direct CP violation in the nonstrange channels. Naive factorization predicts vanishing direct CP violation. Indeed, direct CP violation needs the occurrence of two distinct strong contributions with a strong phase between them. It vanishes when the subdominant strong contribution vanishes and also when the relative strong phase does as is the case in naive factorization. In the case of nonstrange decays, the penguin (P) and tree (T) contributions being at the same order in the Cabibbo angle, the penguin diagram is strongly suppressed because the Wilson coefficients are suppressed by at least one power of the strong coupling constant α_s and the strong phase in QCD factorization is generated by a $\mathcal{O}(\alpha_s)$ correction. Having both P/T and the strong phase small, the direct CP asymmetries are doubly suppressed. Therefore a sizable experimental direct CP asymmetry in $\overline{B^0}\to\rho^+\pi^-$ which is not excluded by experiment [9] would be at odds with QCD factorization. We will discuss this later on. Notice that this argument is independent of the value of the unitarity angle γ, contrarily to arguments based on the value of some branching ratios which depend on γ [23].

The perturbative QCD (PQCD) predicts larger direct CP asymmetries than QCDF due to the fact that penguin contributions to annihilation diagrams, claimed to be calculable in PQCD, contribute to a larger amount to the amplitude and have a large strong phase. In fact, in PQCD, this penguin annihilation diagram is claimed to be of the same order, $\mathcal{O}(\alpha_s)$, than the dominant naive factorization diagram while in QCDF it is also $\mathcal{O}(\alpha_s)$ but smaller than the dominant naive factorization which is $\mathcal{O}(1)$. Hence, in PQCD, this large penguin contribution with a large strong phase yields a large CP asymmetry [40–42].

If QCD factorization is concluded to be unable to describe the present data satisfactorily, while there is to our knowledge no theoretical argument against it, we have to incriminate nonperturbative contributions which are larger than expected. One could simply enlarge the allowed bound for those contributions which are formally subleading but might be large. However, a simple factor of 2 on these bounds makes these unpredictable contributions comparable in size with the predictable ones, if not larger. This spoils the predictivity of the whole program.

A second line is to make some model about the nonperturbative contribution. The "charming penguin diagram" approach [27,30] starts from noticing the underestimate of strange-channel branching ratios by the factorization approaches. This will be shown to apply to the PV channels as well as to the PP ones. This has triggered us to try a charming-penguin-diagram-inspired approach. It is assumed that some hadronic contribution to the penguin loop is nonperturbative, in other words that weak interactions create a charm-anticharm intermediate state which turns into noncharmed final states by strong rescattering. In order to make the model as predictive as possible we will use not more than two unkown complex numbers and use flavor symmetry in strong rescattering.

In Sec. II we will recall the weak-interaction effective Hamiltonian. In Sec. III we will recall QCD factorization. In Sec. IV we will compare QCD factorization with experimental branching ratios and direct CP asymmetries. In Sec. V we will propose a model for nonperturbative contribution and compare it to experiment. We will then conclude.

II. EFFECTIVE HAMILTONIAN

The effective weak Hamiltonian for charmless hadronic B decays consists of a sum of local operators Q_i multiplied by short-distance coefficients C_i given in Table I and products of elements of the quark mixing matrix, $\lambda_p = V_{pb}V^*_{ps}$ or $\lambda'_p = V_{pb}V^*_{pd}$. Below we will focus on $B\to PV$ decays, where P and V hold for pseudoscalar and vector mesons, respectively. Using the unitarity relation $-\lambda_t = \lambda_u + \lambda_c$, we write

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \Bigg(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\dots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \Bigg) + \text{H.c.}, \tag{1}$$

where $Q^p_{1,2}$ are the left-handed current–current operators arising from W-boson exchange, $Q_{3,\dots,6}$ and $Q_{7,\dots,10}$ are

TABLE I. Wilson coefficients C_i in the NDR scheme. Input parameters are $\Lambda^{(5)}_{\overline{\rm MS}}=0.225$ GeV, $m_t(m_t)=167$ GeV, $m_b(m_b)=4.2$ GeV, $M_W=80.4$ GeV, $\alpha=1/129$, and $\sin^2\theta_W=0.23$ [23].

NLO	C_1	C_2	C_3	C_4	C_5	C_6
$\mu=m_b/2$	1.137	−0.295	0.021	−0.051	0.010	−0.065
$\mu=m_b$	1.081	−0.190	0.014	−0.036	0.009	−0.042
$\mu=2m_b$	1.045	−0.113	0.009	−0.025	0.007	−0.027
	C_7/α	C_8/α	C_9/α	C_{10}/α	$C_{7\gamma}^{\rm eff}$	$C_{8g}^{\rm eff}$
$\mu=m_b/2$	−0.024	0.096	−1.325	0.331	—	—
$\mu=m_b$	−0.011	0.060	−1.254	0.223	—	—
$\mu=2m_b$	0.011	0.039	−1.195	0.144	—	—
LO	C_1	C_2	C_3	C_4	C_5	C_6
$\mu=m_b/2$	1.185	−0.387	0.018	−0.038	0.010	−0.053
$\mu=m_b$	1.117	−0.268	0.012	−0.027	0.008	−0.034
$\mu=2m_b$	1.074	−0.181	0.008	−0.019	0.006	−0.022
	C_7/α	C_8/α	C_9/α	C_{10}/α	$C_{7\gamma}^{\rm eff}$	$C_{8g}^{\rm eff}$
$\mu=m_b/2$	−0.012	0.045	−1.358	0.418	−0.364	−0.169
$\mu=m_b$	−0.001	0.029	−1.276	0.288	−0.318	−0.151
$\mu=2m_b$	0.018	0.019	−1.212	0.193	−0.281	−0.136

QCD and electroweak penguin operators, and $Q_{7\gamma}$ and Q_{8g} are the electromagnetic and chromomagnetic dipole operators. They are given by

$$Q_1^p=(\bar p b)_{V-A}(\bar s p)_{V-A}, \quad Q_2^p=(\bar p_i b_j)_{V-A}(\bar s_j p_i)_{V-A},$$

$$Q_3=(\bar s b)_{V-A}\sum_q (\bar q q)_{V-A},$$

$$Q_4=(\bar s_i b_j)_{V-A}\sum_q (\bar q_j q_i)_{V-A},$$

$$Q_5=(\bar s b)_{V-A}\sum_q (\bar q q)_{V+A},$$

$$Q_6=(\bar s_i b_j)_{V-A}\sum_q (\bar q_j q_i)_{V+A},$$

$$Q_7=(\bar s b)_{V-A}\sum_q \tfrac{3}{2}\, e_q(\bar q q)_{V+A},$$

$$Q_8=(\bar s_i b_j)_{V-A}\sum_q \tfrac{3}{2}\, e_q(\bar q_j q_i)_{V+A},$$

$$Q_9=(\bar s b)_{V-A}\sum_q \tfrac{3}{2}\, e_q(\bar q q)_{V-A},$$

$$Q_{10}=(\bar s_i b_j)_{V-A}\sum_q \tfrac{3}{2}\, e_q(\bar q_j q_i)_{V-A},$$

$$Q_{7\gamma}=\frac{-e}{8\pi^2}m_b\bar s\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b,$$

$$Q_{8g}=\frac{-g_s}{8\pi^2}m_b\bar s\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b, \tag{2}$$

where $(\bar q_1 q_2)_{V\pm A}=\bar q_1\gamma_\mu(1\pm\gamma_5)q_2$, i,j are color indices, e_q are the electric charges of the quarks in units of $|e|$, and a summation over $q=u,d,s,c,b$ is implied. The definition of the dipole operators $Q_{7\gamma}$ and Q_{8g} corresponds to the sign convention $iD^\mu=i\partial^\mu+g_sA_a^\mu t_a$ for the gauge-covariant derivative. The Wilson coefficients are calculated at a high scale $\mu\sim M_W$ and evolved down to a characteristic scale $\mu\sim m_b$ using next-to-leading order renormalization-group equations. The essential problem obstructing the calculation of nonleptonic decay amplitudes resides in the evaluation of the hadronic matrix elements of the local operators contained in the effective Hamiltonian.

III. QCD FACTORIZATION IN $B\to PV$ DECAYS

When the QCDF method is applied to the decays $B\to PV$, the hadronic matrix elements of the local effective operators can be written as

$$\langle PV|\mathcal{O}_i|B\rangle=F_1^{B\to P}(0)T^{\rm I}_{V,i}\star f_V\Phi_V+A_0^{B\to V}(0)T^{\rm I}_{P,i}\star f_P\Phi_P + T_i^{\rm II}\star f_B\Phi_B\star f_V\Phi_V\star f_P\Phi_P, \tag{3}$$

where Φ_M are leading-twist light-cone distribution amplitudes, and the $\star$-products imply an integration over the light-cone momentum fractions of the constituent quarks inside the mesons. A graphical representation of this result is shown in Fig. 1.

Here $F_1^{B\to P}$ and $A_0^{B\to V}$ denote the form factors for $B\to P$ and $B\to V$ transitions, respectively. $\Phi_B(\xi)$, $\Phi_V(x)$, and $\Phi_P(y)$ are the light-cone distribution amplitudes (LCDAs) of

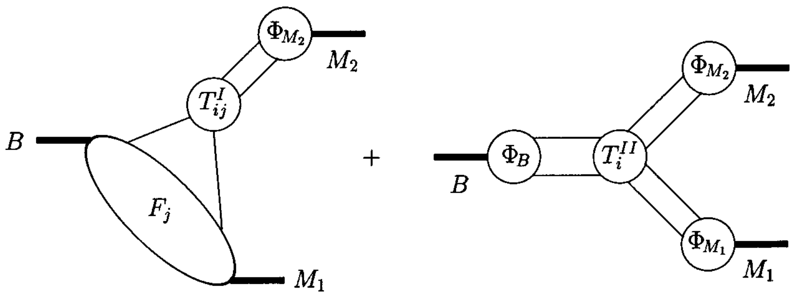

FIG. 1. Graphical representation of the factorization formula. Only one of the two form-factor terms in Eq. (3) is shown for simplicity.

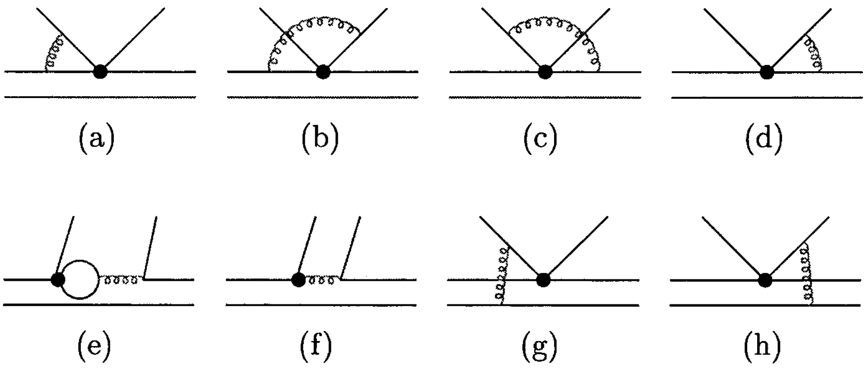

FIG. 2. Order α_s corrections to the hard-scattering kernels. The upward quark lines represent the emitted mesons from b quark weak decays. These diagrams are commonly called vertex corrections, penguin corrections, and hard-spectator-scattering diagrams for (a)–(d), (e),(f), and (g),(h), respectively.

valence quark Fock states for B, vector, and pseudoscalar mesons, respectively. $T_i^{I,II}$ denote the hard-scattering kernels, which are dominated by hard gluon exchange when the power suppressed $\mathcal{O}(\Lambda_{QCD}/m_b)$ terms are neglected. So they are calculable order by order in perturbation theory. The leading terms of T^I come from the tree level and correspond to the naive factorization (NF) approximation. The order of α_s terms of T^I can be depicted by vertex-correction diagrams Figs. 2(a)–2(d) and penguin-correction diagrams Figs. 2(e),(f). T^{II} describes the hard interactions between the spectator quark and the emitted meson M_2 when the gluon virtuality is large. Its lowest order terms are $\mathcal{O}(\alpha_s)$ and can be depicted by hard-spectator-scattering diagrams, Figs. 2(g)–2(h). One of the most interesting results of the QCDF approach is that, in the heavy-quark limit, the strong phases arise naturally from the hard-scattering kernels at the order of α_s. As for the nonperturbative part, they are, as already mentioned, taken into account by the form factors and the LCDAs of mesons up to corrections which are power suppressed in $1/m_b$.

With the above discussions on the effective Hamiltonian of B decays Eq. (1) and the QCDF expressions of hadronic matrix elements, Eq. (3), the decay amplitudes for $B\to PV$ in the heavy-quark limit can be written as

$$\mathcal{A}(B\to PV)=\frac{G_F}{\sqrt{2}}\sum_{p=u,c}\sum_{i=1}^{10}\lambda_p a_i^p\langle PV|\mathcal{O}_i|B\rangle_{nf}. \quad (4)$$

The above $\langle PV|\mathcal{O}_i|B\rangle_{nf}$ are the factorized hadronic matrix elements, which have the same definitions as those in the NF approach. The "nonfactorizable" effects are included in the coefficients a_i which are process dependent. The coefficients a_i are collected in Sec. III A, and the explicit expressions for the decay amplitudes of $B\to PV$ can be found in Appendix A.

According to the arguments in [22], the contributions of weak annihilation to the decay amplitudes are power suppressed, and they do not appear in the QCDF formula, Eq. (3). But as emphasized in [40–42], the contributions from weak annihilation could give large strong phases with QCD corrections, and hence large CP violation could be expected, so their effects cannot simply be neglected. However, in the QCDF method, the annihilation topologies (see Fig. 3) violate factorization because of the end-point divergence. There is similar end-point divergence when considering the chirally enhanced hard-spectator scattering. One possible way is to treat the endpoint divergence from different sources as different phenomenological parameters [23]. The corresponding price is the introduction of model dependence and extra numerical uncertainties in the decay amplitudes. In this work, we will follow the treatment of Ref. [23] and express the weak annihilation topological decay amplitudes as

$$\mathcal{A}^a(B\to PV)\propto f_B f_P f_V \sum \lambda_p b_i, \quad (5)$$

where the parameters b_i are collected in Sec. III B, and the expressions for the weak annihilation decay amplitudes of $B\to P\ V$ are listed in Appendix B.

A. QCD coefficients a_i

We express the QCD coefficients a_i [see Eq. (4)] in two parts: i.e., $a_i=a_{i,I}+a_{i,II}$. The first term $a_{i,I}$ contains the naive factorization and the vertex corrections which are described by Figs. 2(a)–2(f), while the second part $a_{i,II}$ corresponds to the hard-spectator-scattering diagrams Figs. 2(g),(h).

There are two different cases according to the final states. Case I is that the recoiled meson M_1 is a vector meson, and the emitted meson M_2 corresponds to a pseudoscalar meson, and vice versa for case II. For case I, we sum up the results for a_i as follows:

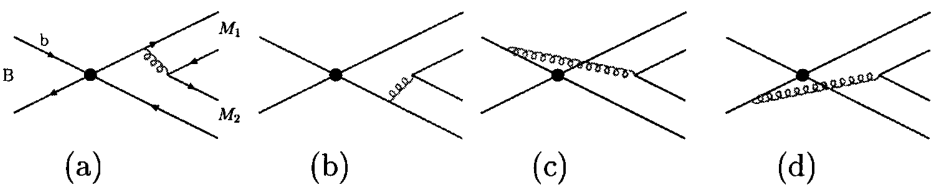

FIG. 3. Order α_s corrections to the weak annihilations of charmless decays $B\to PV$.

$$a_{1,I}=C_1+\frac{C_2}{N_c}\left[1+\frac{C_F\alpha_s}{4\pi}V_M\right],$$

$$a_{1,II}=\frac{\pi C_F\alpha_s}{N_c^2}C_2H(BM_1,M_2),$$

$$a_{2,I}=C_2+\frac{C_1}{N_c}\left[1+\frac{C_F\alpha_s}{4\pi}V_M\right],$$

$$a_{2,II}=\frac{\pi C_F\alpha_s}{N_c^2}C_1H(BM_1,M_2),$$

$$a_{3,I}=C_3+\frac{C_4}{N_c}\left[1+\frac{C_F\alpha_s}{4\pi}V_M\right],$$

$$a_{3,II}=\frac{\pi C_F\alpha_s}{N_c^2}C_4H(BM_1,M_2),$$

$$a_{4,I}^p=C_4+\frac{C_3}{N_c}\left[1+\frac{C_F\alpha_s}{4\pi}V_M\right]+\frac{C_F\alpha_s}{4\pi}\frac{P_{M,2}^p}{N_c},$$

$$a_{4,II}=\frac{\pi C_F\alpha_s}{N_c^2}C_3H(BM_1,M_2),$$

$$a_{5,I}=C_5+\frac{C_6}{N_c}\left[1-\frac{C_F\alpha_s}{4\pi}V'_M\right],$$

$$a_{5,II}=-\frac{\pi C_F\alpha_s}{N_c^2}C_6H'(BM_1,M_2),$$

$$a_{6,I}^p=C_6+\frac{C_5}{N_c}\left[1-6\frac{C_F\alpha_s}{4\pi}\right]+\frac{C_F\alpha_s}{4\pi}\frac{P_{M,3}^p}{N_c},$$

$$a_{6,II}=0,\quad a_{7,I}=C_7+\frac{C_8}{N_c}\left[1-\frac{C_F\alpha_s}{4\pi}V'_M\right],$$

$$a_{7,II}=-\frac{\pi C_F\alpha_s}{N_c^2}C_8H'(BM_1,M_2),$$

$$a_{8,I}^p=C_8+\frac{C_7}{N_c}\left[1-6\frac{C_F\alpha_s}{4\pi}\right]+\frac{\alpha}{9\pi}\frac{P_{M,3}^{p,ew}}{N_c},$$

$$a_{8,II}=0,\quad a_{9,I}=C_9+\frac{C_{10}}{N_c}\left[1+\frac{C_F\alpha_s}{4\pi}V_M\right],$$

$$a_{9,II}=\frac{\pi C_F\alpha_s}{N_c^2}C_{10}H(BM_1,M_2),$$

$$a_{10,I}^p=C_{10}+\frac{C_9}{N_c}\left[1+\frac{C_F\alpha_s}{4\pi}V_M\right]+\frac{\alpha}{9\pi}\frac{P_{M,2}^{p,ew}}{N_c},$$

$$a_{10,II}=\frac{\pi C_F\alpha_s}{N_c^2}C_9H(BM_1,M_2), \tag{6}$$

where $C_F=(N_c^2-1)/2N_c$, and $N_c=3$. The vertex parameters V_M and V'_M result from Figs. 2(a)–2(d); the QCD penguin parameters $P_{M,i}^p$ and the electroweak penguin parameters $P_{M,i}^{p,ew}$ result from Figs. 2(e)–2(f).

The vertex corrections are given by

$$V_M=12\ln\frac{m_b}{\mu}-18+\int_0^1 dx g(x)\Phi_M(x),$$

$$V'_M=12\ln\frac{m_b}{\mu}-6+\int_0^1 dx g(1-x)\Phi_M(x),$$

$$g(x)=3\left(\frac{1-2x}{1-x}\ln x-i\pi\right)+\left[2\,\mathrm{Li}_2(x)-\ln^2x+\frac{2\ln x}{1-x}\right.$$
$$\left.-(3+2i\pi)\ln x-(x\leftrightarrow 1-x)\right], \tag{7}$$

where $\mathrm{Li}_2(x)$ is the dilogarithm function, whereas the constants 18 and 6 are specific to the naive dimensional reduction (NDR) scheme.

The penguin contributions are

$$P_{M,2}^p=C_1\left[\frac{4}{3}\ln\frac{m_b}{\mu}+\frac{2}{3}-G_M(s_p)\right]+\left(C_3-\frac{1}{2}C_9\right)$$
$$\times\left[\frac{8}{3}\ln\frac{m_b}{\mu}+\frac{4}{3}-G_M(0)-G_M(1)\right]$$
$$+\sum_{q=q'}\left(C_4+C_6+\frac{3}{2}e_qC_8+\frac{3}{2}e_qC_{10}\right)$$
$$\times\left[\frac{4}{3}\ln\frac{m_b}{\mu}-G_M(s_q)\right]-2C_{8g}^{\mathrm{eff}}\int_0^1 dx\frac{\Phi_M(x)}{1-x},$$

$$P_{M,3}^p=C_1\left[\frac{4}{3}\ln\frac{m_b}{\mu}+\frac{2}{3}-\hat{G}_M(s_p)\right]+\left(C_3-\frac{1}{2}C_9\right)$$
$$\times\left[\frac{8}{3}\ln\frac{m_b}{\mu}+\frac{4}{3}-\hat{G}_M(0)-\hat{G}_M(1)\right]$$
$$+\sum_{q=q'}\left(C_4+C_6+\frac{3}{2}e_qC_8+\frac{3}{2}e_qC_{10}\right)$$
$$\times\left[\frac{4}{3}\ln\frac{m_b}{\mu}-\hat{G}_M(s_q)\right]-2C_{8g}^{\mathrm{eff}}, \tag{8}$$

and the electroweak penguin parameters $P_{M,i}^{p,ew}$,

$$
\begin{aligned}
P_{M,2}^{p,ew} = & (C_1+N_cC_2)\left[\frac{4}{3}\ln\frac{m_b}{\mu}+\frac{2}{3}-G_M(s_p)\right]-(C_3+N_cC_4)\\
& \times\left[\frac{4}{3}\ln\frac{m_b}{\mu}+\frac{2}{3}-\frac{1}{2}G_M(0)-\frac{1}{2}G_M(1)\right]\\
& +\sum_{q=q'}(N_cC_3+C_4+N_cC_5+C_6)\\
& \times\frac{3}{2}e_q\left[\frac{4}{3}\ln\frac{m_b}{\mu}-G_M(s_q)\right]-N_cC_{7\gamma}^{\mathrm{eff}}\int_0^1 dx\frac{\Phi_M(x)}{1-x},
\end{aligned}
$$

$$
\begin{aligned}
P_{M,3}^{p,ew} = & (C_1+N_cC_2)\left[\frac{4}{3}\ln\frac{m_b}{\mu}+\frac{2}{3}-\hat{G}_M(s_p)\right]-(C_3+N_cC_4)\\
& \times\left[\frac{4}{3}\ln\frac{m_b}{\mu}+\frac{2}{3}-\frac{1}{2}\hat{G}_M(0)-\frac{1}{2}\hat{G}_M(1)\right]\\
& +\sum_{q=q'}(N_cC_3+C_4+N_cC_5+C_6)\\
& \times\frac{3}{2}e_q\left[\frac{4}{3}\ln\frac{m_b}{\mu}-\hat{G}_M(s_q)\right]-N_cC_{7\gamma}^{\mathrm{eff}},
\end{aligned}
\tag{9}
$$

where $s_q=m_q^2/m_b^2$, and where q' in the expressions for $P_{M,i}^p$ and $P_{M,i}^{p,ew}$ runs over all the active quarks at the scale $\mu=\mathcal{O}(m_b)$, i.e., $q'=u,d,s,c,b$. The functions $G_M(s)$ and $\hat{G}_M(s)$ are given, respectively, by

$$
G_M(s)=\int_0^1 dx G(s-i\epsilon,1-x)\Phi_M(x),
\tag{10}
$$

$$
\hat{G}_M(s)=\int_0^1 dx G(s-i\epsilon,1-x)\Phi_M^p(x),
\tag{11}
$$

$$
\begin{aligned}
G(s,x) & =-4\int_0^1 du\,u(1-u)\ln[s-u(1-u)x]\\
& =\frac{2(12s+5x-3x\ln s)}{9x}\\
& \quad -\frac{4\sqrt{4s-x}(2s+x)}{3x^{3/2}}\arctan\sqrt{\frac{x}{4s-x}}.
\end{aligned}
\tag{12}
$$

The parameters $H(BM_1,M_2)$ and $H'(BM_1,M_2)$ in $a_{i,II}$, which originate from hard gluon exchanges between the spectator quark and the emitted meson M_2, are written as

$$
\begin{aligned}
H(BV,P) = & \frac{f_Bf_V}{m_B^2A_0^{B\to V}(0)}\int_0^1 d\xi\int_0^1 dx\\
& \times\int_0^1 dy\frac{\Phi_B(\xi)}{\xi}\frac{\Phi_P(x)}{\bar{x}}\frac{\Phi_V(y)}{\bar{y}},
\end{aligned}
$$

$$
\begin{aligned}
H'(BV,P) = & \frac{f_Bf_V}{m_B^2A_0^{B\to V}(0)}\int_0^1 d\xi\int_0^1 dx\\
& \times\int_0^1 dy\frac{\Phi_B(\xi)}{\xi}\frac{\Phi_P(x)}{x}\frac{\Phi_V(y)}{\bar{y}}.
\end{aligned}
\tag{13}
$$

For case II (vector meson emitted) except for the parameters of $H(BM_1,M_2)$ and $H'(BM_1,M_2)$, the expressions for a_i are similar to those in case I. However, we would like to point out that, because $\langle V|(\bar{q}q)_{S\pm P}|0\rangle=0$, the contributions of the effective operators $\mathcal{O}_{6,8}$ to the hadronic matrix elements vanish; i.e., the terms that are related to $a_{6,8}$ disappear from the decay amplitudes for case II. As to the parameters $H(BM_1,M_2)$ and $H'(BM_1,M_2)$ in $a_{i,II}$, they are defined as

$$
\begin{aligned}
H(BP,V) = & \frac{f_Bf_P}{m_B^2F_1^{B\to P}(0)}\int_0^1 d\xi\int_0^1 dx\int_0^1 dy\frac{\Phi_B(\xi)}{\xi}\\
& \times\frac{\Phi_V(x)}{\bar{x}}\left[\frac{\Phi_P(y)}{\bar{y}}+\frac{2\mu_P}{m_b}\frac{\bar{x}}{x}\frac{\Phi_P^p(y)}{\bar{y}}\right],
\end{aligned}
$$

$$
\begin{aligned}
H'(BP,V) = & -\frac{f_Bf_P}{m_B^2F_1^{B\to P}(0)}\int_0^1 d\xi\int_0^1 dx\int_0^1 dy\frac{\Phi_B(\xi)}{\xi}\\
& \times\frac{\Phi_V(x)}{x}\left[\frac{\Phi_P(y)}{\bar{y}}+\frac{2\mu_P}{m_b}\frac{x}{\bar{x}}\frac{\Phi_P^p(y)}{\bar{y}}\right].
\end{aligned}
\tag{14}
$$

The parameter $\mu_P=m_P^2/(m_1+m_2)$, where $m_{1,2}$ are the current quark masses of the meson constituents, is proportional to the chiral quark condensate.

B. Annihilation parameters b_i

The parameters of b_i in Eq. (5) correspond to weak-annihilation contributions. Now we give their expressions, which are analogous to those in [23]:

$$
b_1(M_1,M_2)=\frac{C_F}{N_c^2}C_1A_1^i(M_1,M_2),
$$

$$
b_2(M_1,M_2)=\frac{C_F}{N_c^2}C_2A_1^i(M_1,M_2),
$$

$$
\begin{aligned}
b_3(M_1,M_2) = & \frac{C_F}{N_c^2}\{C_3A_1^i(M_1,M_2)+C_5A_3^i(M_1,M_2)\\
& +[C_5+N_cC_6]A_3^f(M_1,M_2)\},
\end{aligned}
$$

$$
b_4(M_1,M_2)=\frac{C_F}{N_c^2}\{C_4A_1^i(M_1,M_2)+C_6A_2^i(M_1,M_2)\},
$$

$$b_3^{ew}(M_1,M_2)=\frac{C_F}{N_c^2}\{C_9A_1^i(M_1,M_2)+C_7A_3^i(M_1,M_2)+[C_7+N_cC_8]A_3^f(M_1,M_2)\},$$

$$b_4^{ew}(M_1,M_2)=\frac{C_F}{N_c^2}\{C_{10}A_1^i(M_1,M_2)+C_8A_2^i(M_1,M_2)\}. \tag{15}$$

Here the current-current annihilation parameters $b_{1,2}(M_1,M_2)$ arise from the hadronic matrix elements of the effective operators $\mathcal{O}_{1,2}$, the QCD penguin annihilation parameters $b_{3,4}(M_1,M_2)$ from $\mathcal{O}_{3-6}$, and the electroweak penguin annihilation parameters $b_{3,4}^{ew}(M_1,M_2)$ from $\mathcal{O}_{7-10}$. The parameters of b_i are closely related to the final states; they can also be divided into two different cases according to the final states. Case I is that M_1 is a vector meson and M_2 is a pseudoscalar meson (here M_1 and M_2 are tagged in Fig. 3). Case II is that M_1 corresponds to a pseudoscalar meson and M_2 corresponds to a vector meson. For case I, the definitions of $A_k^{i,f}(M_1,M_2)$ in Eq. (15) are

$$A_{1,2}^f(V,P)=0,$$

$$A_3^f(V,P)=\pi\alpha_s\int_0^1 dx\int_0^1 dy\Phi_V(x)\Phi_P^p(y)\frac{2\mu_P}{m_b}\frac{2(1+\bar{x})}{\bar{x}^2y},$$

$$A_1^i(V,P)=\pi\alpha_s\int_0^1 dx\int_0^1 dy\Phi_V(x)\Phi_P(y)\times\left[\frac{1}{y(1-\bar{x}y)}+\frac{1}{\bar{x}^2y}\right],$$

$$A_2^i(V,P)=-\pi\alpha_s\int_0^1 dx\int_0^1 dy\Phi_V(x)\Phi_P(y)\times\left[\frac{1}{\bar{x}(1-\bar{x}y)}+\frac{1}{\bar{x}y^2}\right],$$

$$A_3^i(V,P)=\pi\alpha_s\int_0^1 dx\int_0^1 dy\Phi_V(x)\Phi_P^p(y)\times\frac{2\mu_P}{m_b}\frac{2\bar{y}}{\bar{x}y(1-\bar{x}y)}. \tag{16}$$

For case II,

$$A_{1,2}^f(P,V)=0,$$

$$A_3^f(P,V)=-\pi\alpha_s\int_0^1 dx\int_0^1 dy\Phi_P^p(x)\Phi_V(y)\times\frac{2\mu_P}{m_b}\frac{2(1+y)}{\bar{x}y^2},$$

$$A_1^i(P,V)=\pi\alpha_s\int_0^1 dx\int_0^1 dy\Phi_P(x)\Phi_V(y)\times\left[\frac{1}{y(1-\bar{x}y)}+\frac{1}{\bar{x}^2y}\right],$$

$$A_2^i(P,V)=-\pi\alpha_s\int_0^1 dx\int_0^1 dy\Phi_P(x)\Phi_V(y)\times\left[\frac{1}{\bar{x}(1-\bar{x}y)}+\frac{1}{\bar{x}y^2}\right],$$

TABLE II. Experimentally known data of CP-averaged branching ratios for the charmless $B\to PV$ decay modes, used as input for the global fit. The channels containing the η' meson have been excluded.

$\mathcal{B}R$ $(\times 10^6)$	BaBar [1–10]	Belle [11–15]	CLEO [16–21]	Average
$B^0\to\pi^\pm\rho^\mp$	$28.9\pm5.4\pm4.3$	$20.8^{+6.0}_{-6.3}{}^{+2.8}_{-3.1}$	$27.6^{+8.4}_{-7.4}\pm4.2$	25.53 ± 4.32
$B^+\to\pi^+\rho^0$	$24\pm8\pm3(<39)$	$8.0^{+2.3}_{-2.0}\pm0.7$	$10.4^{+3.3}_{-3.4}\pm2.1$	9.49 ± 2.57
$B^0\to\pi^0\rho^0$	$3.6\pm3.5\pm1.7(<10.6)$	<5.3	$1.6^{+2.0}_{-1.4}\pm0.8(<5.5)$	2.07 ± 1.88
$B^+\to\pi^+\omega$	$6.6^{+2.1}_{-1.8}\pm0.7$	$4.2^{+2.0}_{-1.8}\pm0.5$	$11.3^{+3.3}_{-2.9}\pm1.4$	6.22 ± 1.70
$B^0\to K^+\rho^-$	–	$15.8^{+5.1}_{-4.6}{}^{+1.7}_{-3.0}$	$16.0^{+7.6}_{-6.4}\pm2.8(<32)$	15.88 ± 4.65
$B^+\to K^+\rho^0$	$10\pm6\pm2(<29)$	–	$8.4^{+4.0}_{-3.4}\pm1.8(<17)$	8.92 ± 3.60
$B^+\to K^+\omega$	$1.4^{+1.3}_{-1.0}\pm0.3(<4)$	$9.2^{+2.6}_{-2.3}\pm1.0$	$3.2^{+2.4}_{-1.9}\pm0.8(<7.9)$	2.92 ± 1.94
$B^0\to K^0\omega$	$5.9^{+1.7}_{-1.5}\pm0.9$	–	$10.0^{+5.4}_{-4.2}\pm1.4(<21)$	6.34 ± 1.82
$B^0\to K^{*+}\pi^-$	–	$26.0\pm8.3\pm3.5$	$16^{+6}_{-5}\pm2$	19.3 ± 5.2
$B^+\to K^{*0}\pi^-$	$15.5\pm3.4\pm1.8$	$19.4^{+4.2}_{-3.9}\pm2.1^{+3.5}_{-6.8}$	$7.6^{+3.5}_{-3.0}\pm1.6(<16)$	12.12 ± 3.13
$B^+\to K^{*-}\pi^0$	–	–	$7.1^{+11.4}_{-7.1}\pm1.0(<31)$	7.1 ± 11.4
$B^+\to K^{*+}\eta$	$22.1^{+11.1}_{-9.2}\pm3.3$	$26.5^{+7.8}_{-7.0}\pm3.0$	$26.4^{+9.6}_{-8.2}\pm3.3$	25.4 ± 5.6
$B^0\to K^{*0}\eta$	$19.8^{+6.5}_{-5.6}\pm1.7$	$16.5^{+4.6}_{-4.2}\pm1.2$	$13.8^{+5.5}_{-4.6}\pm1.6$	16.41 ± 3.21
$B^+\to K^+\phi$	$9.2\pm1.0\pm0.8$	$10.7\pm1.0^{+0.9}_{-1.6}$	$5.5^{+2.1}_{-1.8}\pm0.6$	8.58 ± 1.24
$B^0\to K^0\phi$	$8.7^{+1.7}_{-1.5}\pm0.9$	$10.0^{+1.9}_{-1.7}{}^{+0.9}_{-1.3}$	$5.4^{+3.7}_{-2.7}\pm0.7(<12.3)$	8.72 ± 1.37

TABLE III. Experimental measured data of direct CP asymmetries for the charmless $B\to PV$ decay modes, used as input for the global fit.

$\mathcal{A}_{CP}$	BaBar [1–10]	Belle [11–15]	CLEO [16–21]	Average
$B^-\to\pi^-\omega$	$-0.01^{+0.29}_{-0.31}\pm0.03$	–	$-0.34\pm0.25\pm0.02$	-0.21 ± 0.19
$B^-\to K^-\omega$	–	$-0.21\pm0.28\pm0.03$	–	-0.21 ± 0.28
$B^-\to K^{*-}\eta$	–	$-0.05^{+0.25}_{-0.30}\pm0.01$	–	-0.05 ± 0.30
$\bar{B}^0\to\bar{K}^{*0}\eta$	–	$0.17^{+0.28}_{-0.25}\pm0.01$	–	0.17 ± 0.28
$B^-\to K^-\phi$	$-0.05\pm0.20\pm0.03$	–	–	-0.05 ± 0.20

$$A_3^i(P,V)=\pi\alpha_s\int_0^1dx\int_0^1dy\Phi_P^p(x)\Phi_V(y)\times\frac{2\mu_P}{m_b}\frac{2x}{\bar{x}y(1-\bar{x}y)}. \qquad (17)$$

Here our notation and convention are the same as those in [23]. The superscripts i and f on $A^{i,f}$ correspond to the contributions from Figs. 3(a),(b) and 3(c),(d), respectively. The subscripts $k=1$, 2, and 3 on $A_k^{i,f}$ refer to the Dirac structures $(V-A)\otimes(V-A)$, $(V-A)\otimes(V+A)$, and $(-2)(S-P)\otimes(S+P)$, respectively. $\Phi_V(x)$ denotes the leading-twist LCDAs of a vector meson, and $\Phi_P(x)$ and $\Phi_P^p(x)$ denote twist-2 and twist-3 LCDAs of a pseudoscalar meson, respectively.

Note that assuming SU(3) flavor symmetry implies symmetric LCDAs of light mesons (under $x\leftrightarrow\bar{x}$), whence $A_1^i=-A_2^i$. In this approximation the weak-annihilation contributions (for case I) can be parametrized as

$$A_1^i(V,P)\simeq18\pi\alpha_s\left(X_A-4+\frac{\pi^2}{3}\right),$$

$$A_3^i(V,P)\simeq\pi\alpha_s r_\chi[2\pi^2-6(X_A^2+2X_A)],$$

$$A_3^f(V,P)\simeq6\pi\alpha_s r_\chi(2X_A^2-X_A), \qquad (18)$$

where $X_A=\int_0^1dx/x$ parametrizes the divergent end-point integrals and $r_\chi=2\mu_P/m_b$ is the so-called *chirally enhanced* factor. We can get similar forms to Eq. (18) for case II, but with $A_3^f(P,V)=-A_3^f(V,P)$. In our calculation, we will treat X_A as a phenomenological parameter and take the same value for all annihilation terms, although this approximation is crude and there is no known physical argument justifying this assumption. We shall see below that X_A gives large uncertainties in the theoretical prediction.

TABLE IV. Experimental results and correlation matrix for the various asymmetries measured in the channels $\rho^\pm\pi^\mp/\rho^\pm K^\mp$. The notation is explained in [9].

		Correlation coefficient (%)			
$B^0\to\rho^\pm\pi^\mp$	Measurement	$\mathcal{A}_{CP}^{\rho\pi}$	$\mathcal{A}_{CP}^{\rho K}$	$\mathcal{C}_{\rho\pi}$	$\Delta\mathcal{C}_{\rho\pi}$
$\mathcal{A}_{CP}^{\rho\pi}$	$-0.22\pm0.08\pm0.07$	–	3.4	-11.8	-10.4
$\mathcal{A}_{CP}^{\rho K}$	$0.19\pm0.14\pm0.11$	3.4	–	-1.3	-1.1
$\mathcal{C}_{\rho\pi}$	$0.45^{+0.18}_{-0.19}\pm0.09$	-11.8	-1.3	–	23.9
$\Delta\mathcal{C}_{\rho\pi}$	$0.38^{+0.19}_{-0.20}\pm0.11$	-10.4	-1.1	23.9	–

IV. QCD FACTORIZATION VERSUS EXPERIMENT

In order to propose a test of QCD factorization with respect to experiment, a compilation of various charmless branching fractions and direct CP asymmetries was performed and is given in Tables II, III, and IV. This compilation includes the latest results from BaBar, Belle, and CLEO. The measurements were combined into a single central value and error, which may be compared with the theoretical prediction. First, the total error from each experiment was computed by summing quadratically the statistic and systematic error: this approach is valid in the limit that the systematic error is not so large with respect to the statistic error. Second, when the experiment provides an asymmetric error $^{+\sigma_1}_{-\sigma_2}$, a conservative symmetric error was assumed in the calculation by using $\pm\mathrm{Max}(\sigma_1,\sigma_2)$. In case of a disagreement between several experiments for a given measurement, the total error was increased by a ''scale factor'' computed from a χ^2 combining the various experiments, using the standard procedure given by the Particle Data Group (PDG) [43].

In order to compare the theoretical predictions $\{y\}$ with the experimental measurements $\{x\pm\sigma_x\}$, the following χ^2 was defined:

$$\chi^2=\sum\left(\frac{x-y}{\sigma_x}\right)^2.$$

In the case when a correlation matrix between several measurements is given by the experiment, as in the case of the $\rho^+\pi^-/\rho^+K^-$ measurements, the χ^2 was corrected to account for it. The above χ^2 was then minimized using MINUIT [44], letting free all theoretical parameters in their allowed range. The quality of the minimum yielded by MINUIT was assessed by replacing it with an *ad hoc* minimizer scanning the entire parameter space. The theoretical predictions, with the theoretical parameters yielding the best fits, are compared to experiment in Table V for two scenarios to be explained below. The asymmetries of the $\rho^\pm\pi^\mp$ channels can be expressed [9] in terms of the quantities reported in Table IV. The comparison between their theoretical predictions and experiment is reported in Table VI.

Scenario 1 refers to a fit according to QCD factorization, varying all theoretical parameters in the range presented in

TABLE V. Best-fit values using the global analysis of $B\to PV$ decays in QCDF with free γ (scenario 1) and QCDF+charming penguin diagrams (scenario 2) with constrained γ. The CP-averaged branching ratios are in units of 10^{-6}.

	Experiment	Scenario 1 Prediction	Scenario 1 χ^2	Scenario 2 Prediction	Scenario 2 χ^2
$\mathcal{BR}(\overline{B}^0\to\rho^0\pi^0)$	2.07 ± 1.88	0.132	1.1	0.177	1.0
$\mathcal{BR}(\overline{B}^0\to\rho^+\pi^-)$		11.023		10.962	
$\mathcal{BR}(\overline{B}^0\to\rho^-\pi^+)$		18.374		17.429	
$\mathcal{BR}(\overline{B}^0\to\rho^\pm\pi^\mp)$	25.53 ± 4.32	29.397	0.8	28.391	0.4
$\mathcal{BR}(B^-\to\rho^0\pi^-)$	9.49 ± 2.57	9.889	0.0	7.879	0.4
$\mathcal{BR}(B^-\to\omega\pi^-)$	6.22 ± 1.7	6.002	0.0	5.186	0.4
$\mathcal{BR}(B^-\to K^{*-}K^0)$		0.457		0.788	
$\mathcal{BR}(B^-\to K^{*0}K^-)$		0.490		0.494	
$\mathcal{BR}(B^-\to\Phi\pi^-)$		0.004		0.003	
$\mathcal{BR}(B^-\to\rho^-\pi^0)$		9.646		11.404	
$\mathcal{BR}(\overline{B}^0\to\rho^0\overline{K}^0)$		5.865		8.893	
$\mathcal{BR}(\overline{B}^0\to\omega\overline{K}^0)$	6.34 ± 1.82	2.318	4.9	5.606	0.2
$\mathcal{BR}(\overline{B}^0\to\rho^+K^-)$	15.88 ± 4.65	6.531	4.0	14.304	0.1
$\mathcal{BR}(\overline{B}^0\to K^{*-}\pi^+)$	19.3 ± 5.2	9.760	3.4	10.787	2.7
$\mathcal{BR}(B^-\to K^{*-}\pi^0)$	7.1 ± 11.4	7.303	0.0	8.292	0.0
$\mathcal{BR}(\overline{B}^0\to\Phi\overline{K}^0)$	8.72 ± 1.37	8.360	0.1	8.898	0.0
$\mathcal{BR}(B^-\to\overline{K}^{*0}\pi^-)$	12.12 ± 3.13	7.889	1.8	11.080	0.1
$\mathcal{BR}(B^-\to\rho^0K^-)$	8.92 ± 3.6	1.882	3.8	5.655	0.8
$\mathcal{BR}(B^-\to\rho^-\overline{K}^0)$		7.140		14.006	
$\mathcal{BR}(B^-\to\omega K^-)$	2.92 ± 1.94	2.398	0.1	6.320	3.1
$\mathcal{BR}(B^-\to\Phi K^-)$	8.88 ± 1.24	8.941	0.0	9.479	0.2
$\mathcal{BR}(\overline{B}^0\to\overline{K}^{*0}\eta)$	16.41 ± 3.21	22.807	4.0	18.968	0.6
$\mathcal{BR}(B^-\to K^{*-}\eta)$	25.4 ± 5.6	17.855	1.8	15.543	3.1
$\Delta\mathcal{C}_{\rho\pi}$	0.38 ± 0.23	0.250	8.1/4	0.228	3.9/4
$\mathcal{C}_{\rho\pi}$	0.45 ± 0.21	0.019		0.092	
$\mathcal{A}_{CP}^{\rho\pi}$	-0.22 ± 0.11	-0.015		-0.115	
$\mathcal{A}_{CP}^{\rho K}$	0.19 ± 0.18	0.060		0.197	
$\mathcal{A}_{CP}^{\omega\pi^-}$	-0.21 ± 0.19	-0.072	0.5	-0.198	0.0
$\mathcal{A}_{CP}^{\omega K^-}$	-0.21 ± 0.28	0.029	0.7	0.189	2.0
$\mathcal{A}_{CP}^{\eta K^{*-}}$	-0.05 ± 0.3	-0.138	0.1	-0.217	0.3
$\mathcal{A}_{CP}^{\eta K^{*0}}$	0.17 ± 0.28	-0.186	1.6	-0.158	1.4
$\mathcal{A}_{CP}^{\phi K^-}$	-0.05 ± 0.2	0.006	0.1	0.005	0.1
			36.9		20.8

Table VII. Even the unitarity triangle angle γ is varied freely and ends up not far from $90°$. We have taken $X_A=X_H$ in the range proposed in Ref. [23]:

$$X_{A,H}=\int_0^1\frac{dx}{x}=\ln\frac{m_B}{\Lambda_h}(1+\rho_{A,H}e^{i\phi_{A,H}}). \tag{19}$$

These parameters label our ignorance of the nonperturbatively calculable subdominant contribution to the annihilation and hard scattering, defined in Eqs. (16),(17) and Eqs. (13),(14), respectively. They do not need to have the same value for all PV channels but we have nevertheless assumed one common value since a fit would become impossible with too many unknown parameters.

Scenario 2 in Table V refers to a fit adding a charming-penguin-diagram-inspired long-distance contribution which

TABLE VI. Values of the CP asymmetries for $B\to\pi\rho$ decays in QCDF (scenario 1) and QCDF+charming penguin diagrams (scenario 2). The notation is explained in [9].

	Experiment	Scenario 1	Scenario 2
$\mathcal{A}_{CP}^{\rho^+\pi^-}$	$-0.82\pm0.31\pm0.16$	-0.04	-0.23
$\mathcal{A}_{CP}^{\rho^-\pi^+}$	$-0.11\pm0.16\pm0.09$	-0.0002	0.04

TABLE VII. Various theoretical inputs used in our global analysis of $B\to PV$ decays in QCDF. The parameter ranges have been taken from literature [23,34,35,45]. The two last columns give the best fits of both scenarios.

Input	Range	Scenario 1	Scenario 2
γ (deg)		99.955	81.933
m_s (GeV)	[0.085,0.135]	0.085	0.085
μ (GeV)	[2.1,8.4]	3.355	5.971
ρ_A	[−1,1]	1.000	1.000
ϕ_A (deg)	[−180,180]	−22.928	−87.907
λ_B (GeV)	[0.2,0.5]	0.500	0.500
f_B (GeV)	[0.14,0.22]	0.220	0.203
R_u	[0.35,0.49]	0.350	0.350
R_c	[0.018,0.025]	0.018	0.018
$A_0^{B\to\rho}$	[0.3162,0.4278]	0.373	0.377
$F_1^{B\to\pi}$	[0.23,0.33]	0.330	0.301
$A_0^{B\to\omega}$	[0.25,0.35]	0.350	0.326
$A_0^{B\to K^*}$	[0.3995,0.5405]	0.400	0.469
$F_1^{B\to K}$	[0.28,0.4]	0.333	0.280
$\mathrm{Re}[\mathcal{A}^P]$	[−0.01,0.01]		0.00253
$\mathrm{Im}[\mathcal{A}^P]$	[−0.01,0.01]		−0.00181
$\mathrm{Re}[\mathcal{A}^V]$	[−0.01,0.01]		−0.00187
$\mathrm{Im}[\mathcal{A}^V]$	[−0.01,0.01]		0.00049

will be presented and discussed in Sec. V. In this fit γ is constrained within the range [34°,82°].

The values of the theoretical parameters found for the two best fits is given in Table VII: many parameters are found to be at the edge of their allowed range.[1] In order to estimate the quality of the agreement between measurements and predictions, the standard Monte Carlo based ‘‘goodness of fit’’ test was performed.

(i) The best-fit values of the theoretical parameters were used to make predictions for the branching ratios and CP asymmetries.

(ii) The total experimental error from each measurement was used to generate new experimental values distributed around the predictions with a Gaussian probability.

(iii) The full fit previously performed on real measurements is now run on this simulated data, and the χ^2 of this fit is saved in a histogram H.

It is then possible to compare the χ^2_{data} obtained from the measurement with the χ^2 one would obtain if the predictions were true. Additionally, one may compute the confidence level of the tested model by using

$$\mathrm{C.L.} \leqslant \frac{\int_{\chi^2>\chi^2_{\mathrm{data}}} H(\chi^2)\,d\chi^2}{\int_{\chi^2>0} H(\chi^2)\,d\chi^2}.$$

The results of the ‘‘goodness of fit’’ tests are given in Fig. 4. From these tests, one may quote an upper limit for the confidence level in scenario 1, C.L.$\leqslant$0.1% and, in the case of scenario 2, C.L.$\leqslant$7.7%.

In Tables II (III) we give the experimental CP-averaged branching ratios (direct CP asymmetries) which we have used in our fits. We have also used the quantities reported in Table IV which are related to the branching ratios and CP asymmetries of the $B\to\rho^\pm\pi^\mp$ channels.

For the sake of definiteness let us recall that the branching ratios for any charmless B decays, $B\to PV$ channel, in the rest frame of the B meson, is given by

$$\mathcal{BR}(B\to PV) = \frac{\tau_B}{8\pi}\frac{|p|}{m_B^2}|\mathcal{A}(B\to PV)+\mathcal{A}^a(B\to PV) + \mathcal{A}^{\mathrm{LD}}(B\to PV)|^2, \tag{20}$$

where τ_B represents the B-meson lifetime (charged or uncharged according to the case). The amplitudes $\mathcal{A},\mathcal{A}^a$ and $\mathcal{A}^{\mathrm{LD}}$ are defined in Appendixes A, B and in Eqs. (24) and (25), respectively. In the case of pure QCD factorization (scenario 1) we take of course $\mathcal{A}^{\mathrm{LD}}=0$. The kinematical factor $|p|$ is written as

$$|p| = \frac{\sqrt{[m_B^2-(m_P+m_V)^2][m_B^2-(m_P-m_V)^2]}}{2m_B}. \tag{21}$$

Comparison with Du *et al.*

Our negative conclusion about the QCD factorization fit of the $B\to PV$ channels is at odds with the conclusion of the authors of Ref. [36], who have performed a successful fit of both $B\to PP$ and $B\to PV$ channels using the same theoretical starting point. These authors have excluded from their fits the channels containing a K^* in the final state, arguing that these channels seemed questionable to them. We have thus made a fit without the channels containing the K^*, and indeed we find as the authors of Ref. [36] that the global agreement between QCD factorization and experiment was satisfactory. Notice that this fit was done without discarding the channels $B^+\to\omega\pi^+(K^+)$ as done by Du *et al.*

Notice also that the parameters $C_{\rho\pi}$ and the $A_{CP}^{\rho\pi}$ have been kept in this fit. The disagreement between QCDF and experiment for these quantities was not enough to spoil the satisfactory agreement of the global fit because the experimental errors are still large on these quantities.

The conclusion of this subsection is that the difference between the ‘‘optimistic’’ conclusion about QCDF of Du *et al.* and our rather pessimistic one comes from their choice of discarding the channels containing the K^*'s. In other words the conclusion about the status of QCDF in the $B\to PV$ channels depends on the confidence we give to the

[1]Table VII shows that the fit value of ρ_A appears at the edge of the input range, $\rho_A=1$. However enlarging the range of ρ_A, such as $|\rho_A|\leqslant 10$, brings a large annihilation contribution $(\rho_A,\phi_A)=(2.3,-41°)$ for scenario 1 and $(4.4,-108°)$ for scenario 2. With so large values of $|\rho_A|$ the unpredictable contributions would dominate the total result making the whole exercise void of signification.

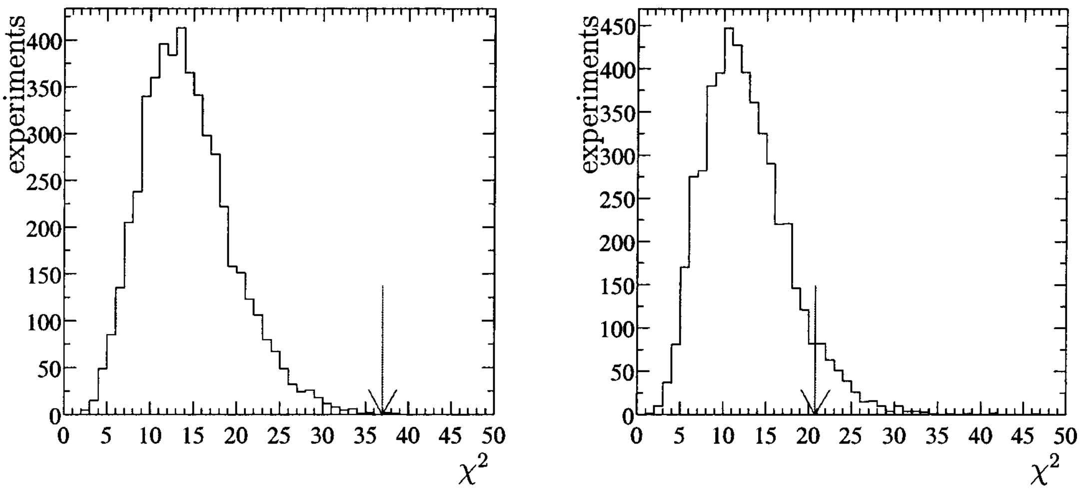

FIG. 4. Goodness-of-fit test of the two proposed theoretical models: the arrow points at the value $\chi^2_{\rm data}$ found from the measurements, and the histogram shows the values allowed for χ^2 in the case that the models predictions are correct.

published results on these channels.

V. SIMPLE MODEL FOR LONG-DISTANCE INTERACTIONS

As seen in Table V the failure of our overall fit with QCDF can be traced to two main facts. First the strange branching ratios are underestimated by QCDF. Second the direct CP asymmetries in the nonstrange channels might also be underestimated. *A priori* this could be cured if some nonperturbative mechanism were contributing to $|P|$. Indeed, first, in the strange channels, $|P|$ is Cabibbo enhanced and such a nonperturbative contribution could increase the branching ratios, and second, increasing $|P|/|T|$ in the nonstrange channels with nonsmall strong phases could increase significantly the direct CP asymmetries as already discussed. We have therefore tried a charming-penguin-diagram-inspired model. We wanted nevertheless to avoid to add too many new parameters which would make the fit void of signification. We have therefore tried a model for long-distance penguin contributions which depends only on two fitted complex numbers.

Let us start by describing our charming-penguin-diagram-inspired model for strange final states. In the "charming penguin diagram" picture the weak decay of a $\overline{B}^0$ (B^-) meson through the action of the operator Q_1^c [see notation in Eqs. (1) and (2)] creates an hadronic system containing the quarks $s,\overline{d}(\overline{u}),c,\overline{c}$, for example $\overline{D}_s^{(*)}+D^{(*)}$ systems. This system goes to long distances, the $c,\overline{c}$ eventually annihilate, a pair of light quarks are created by a nonperturbative strong interaction and one is left with two light mesons. Let us here restrict ourselves to the case of a PV pair of mesons; i.e., one of the final mesons is a light pseudoscalar (π,K,η) and the other a light vector meson (ρ,ω,ϕ,K^*). In this paper we leave aside the η' which is presumably quite special.

We will picture now this hadronic system as a coherent state which decays into the two final mesons with total strangeness -1. This state has a total angular momentum $J=0$. Its flavor $s\overline{d}$ is that of a member of an octet of flavor-SU(3) symmetry. We will assume flavor-SU(3) symmetry in the decay amplitude of this hadronic state. This still leaves four SU(3)-invariant amplitudes since both P and V can have an octet and a singlet component and that there exist two octets in the decomposition of 8×8. We make a further simplifying assumption based on the Okubo-Zweig-Iizuka (OZI) rule. Let us give an example: we assume that $V=(s\overline{q})$ where q is any of the light quarks u,d,s, and that $P=(q\overline{d})$. Then we compute the contractions between

$$\langle(s\overline{q})(q\overline{d})|s(\overline{u}u+\overline{d}d+\overline{s}s)\overline{d}\rangle=1. \tag{22}$$

The meaning of this rule is simple. We add to the $s\overline{d}$ quarks in our hadronic state an SU(3) singlet $\overline{u}u+\overline{d}d+\overline{s}s$ and compute an "overlap" making contractions so that the quarks in the singlet go into two different mesons. This latter constraint is the OZI rule. This is why the overlap in Eq. (22) is 1 even if $q=d$ since it is forbidden to have both d quarks from the singlet in the same final meson. As an example, the decay $B\to\overline{K}^0\rho^0$ gives the following overlap coefficient:

$$\left\langle (s\overline{d})\frac{(u\overline{u}-d\overline{d})}{\sqrt{2}}\,\middle|\, s(\overline{u}u+\overline{d}d+\overline{s}s)\overline{d}\right\rangle=-\frac{1}{\sqrt{2}}. \tag{23}$$

For the η meson we will use the decomposition in [32]. The overlap coefficients thus computed play the role of SU(3) Clebsch-Gordan (CG) coefficients computed in a simple way. These coefficients are assumed to be multiplied by an universal complex amplitude to be fitted from experiment. Up to now we have assumed that the active quark (here, s) ended up in the vector meson. We need another universal amplitude for the case where the active quark ends up in the pseudoscalar meson.

We are thus left with two theoretically independent and unknown amplitudes: one with $V=(s\overline{q})$, $P=(q\overline{d})$, one with $P=(s\overline{q})$, $V=(q\overline{d})$. We shall write them respectively as $\mathcal{A}^{\mathcal{P}}$ ($\mathcal{A}^{\mathcal{V}}$) when the active quark ends up in the pseudoscalar (vector) meson.

TABLE VIII. Flavor-SU(3) Clebsch-Gordan coefficient for long-distance penguin-diagram-like contributions. Notice that the channel $B^-\to\Phi\pi^-$ vanishes due to the OZI rule.

	Cl^P	Cl^V
$\mathcal{BR}(\overline{B}^0\to\rho^0\pi^0)$	0.5	0.5
$\mathcal{BR}(\overline{B}^0\to\rho^+\pi^-)$	1.0	0.
$\mathcal{BR}(\overline{B}^0\to\rho^-\pi^+)$	0.	1.0
$\mathcal{BR}(B^-\to\rho^0\pi^-)$	$1/\sqrt{2}$	$-1/\sqrt{2}$
$\mathcal{BR}(B^-\to\omega\pi^-)$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\mathcal{BR}(B^-\to K^{*-}K^0)$	1.0	0.
$\mathcal{BR}(B^-\to K^{*0}K^-)$	0.	1.0
$\mathcal{BR}(B^-\to\Phi\pi^-)$	0.	0.
$\mathcal{BR}(B^-\to\rho^-\pi^0)$	$-1/\sqrt{2}$	$1/\sqrt{2}$
$\mathcal{BR}(\overline{B}^0\to\rho^0\overline{K}^0)$	$-1/\sqrt{2}$	0.
$\mathcal{BR}(\overline{B}^0\to\omega\overline{K}^0)$	$1/\sqrt{2}$	0.
$\mathcal{BR}(\overline{B}^0\to\rho^+K^-)$	1.0	0.
$\mathcal{BR}(\overline{B}^0\to K^{*-}\pi^+)$	0.	1.0
$\mathcal{BR}(B^-\to K^{*-}\pi^0)$	0.	$1/\sqrt{2}$
$\mathcal{BR}(\overline{B}^0\to\Phi\overline{K}^0)$	0.	1.0
$\mathcal{BR}(B^-\to\overline{K}^{*0}\pi^-)$	0.	1.0
$\mathcal{BR}(B^-\to\rho^0K^-)$	$1/\sqrt{2}$	0.
$\mathcal{BR}(B^-\to\rho^-\overline{K}^0)$	1.0	0.
$\mathcal{BR}(B^-\to\omega K^-)$	$1/\sqrt{2}$	0.
$\mathcal{BR}(B^-\to\Phi K^-)$	0.	1.0
$\mathcal{BR}(\overline{B}^0\to\overline{K}^{*0}\eta)$	-0.665	0.469
$\mathcal{BR}(B^-\to K^{*-}\eta)$	-0.665	0.469

Concerning the $\overline{B}$ decay into a pseudoscalar + vector meson of vanishing total strangeness, we apply the same recipe with the same amplitudes $\mathcal{A}^{\mathcal{P}}$ and $\mathcal{A}^{\mathcal{V}}$, replacing the s quark by a d quark and, of course, the corresponding replacement of the CKM factor $V_{cb}V_{cs}^*$ by $V_{cb}V_{cd}^*$.

To summarize, the long-distance term is given by two universal complex amplitudes multiplied by a CG coefficient computed simply by the overlap factor in Eq. (23); see Table VIII.

In practice, to the amplitudes described in the Appendixes we add the long-distance amplitudes, given by

$$\mathcal{A}^{\mathrm{LD}}(B\to PV)=\frac{G_F}{\sqrt{2}}m_B^2\lambda_c'(Cl^P\mathcal{A}^P+Cl^V\mathcal{A}^V) \qquad (24)$$

for the nonstrange channels and

$$\mathcal{A}^{\mathrm{LD}}(B\to PV)=\frac{G_F}{\sqrt{2}}m_B^2\lambda_c(Cl^P\mathcal{A}^P+Cl^V\mathcal{A}^V) \qquad (25)$$

for the strange channels. In Eqs. (24) and (25), $\mathcal{A}^P$ and $\mathcal{A}^V$ are two complex numbers which are fitted in the global fit of scenario 2 and Cl^P and Cl^V are the flavor-SU(3) Clebsch-Gordan coefficients which are given in Table VIII. For both channels containing the η we have used the formulas

$$Cl^V=\frac{\cos\theta_8}{\sqrt{6}}-\frac{\sin\theta_0}{\sqrt{3}},\quad Cl^P=-2\frac{\cos\theta_8}{\sqrt{6}}-\frac{\sin\theta_0}{\sqrt{3}}, \qquad (26)$$

with $\theta_0=-9.1°$ and $\theta_8=-22.2°$.

The fit with long-distance penguin contributions is presented in Table V under the label "Scenario 2." The agreement with experiment is improved, and it should be so, but not in such a fully convincing manner. The goodness of the fit is about 8% which implies that this model is not excluded by experiment. However, a look at Table VII shows that several fitted parameters are still stuck at the end of the allowed range of variation. In particular $\rho_A=1$ means that the uncalculable subleading contribution to QCDF is again stretched to its extreme.

Finally the fitted complex numbers which fix the size of the long-distance penguin contribution (last four lines in Table VII) are small. To make this statement quantitative, assuming the long-distance amplitude were alone, the values for $\mathcal{A}^P$ and $\mathcal{A}^V$ in Table V correspond to branching ratios which reach at their maximum 6×10^{-6} but are more generally in the vicinity of 2×10^{-6}. In part, this is due to the fact that, if some strange channels want a large nonperturbative contribution to increase their branching ratios, some other strange channels and particularly the $B\to K\phi$ channels which are in good agreement with QCDF cannot accept the addition of a too large nonperturbative penguin contribution. This last point should be stressed: if the strange channels show a general tendency to be underestimated by QCDF, there is the striking exception of the $s\overline{s}s$ channels which agree very well with QCDF and make the case for charming penguins diagrams rather difficult.

VI. CONCLUSION

We have made a global fit according to QCD factorization of published experimental data concerning charmless $B\to PV$ decays including CP asymmetries. We have only excluded from the fit the channels containing the η' meson. Our conclusion is that it is impossible to reach a good fit. As can be seen in scenario 1 of Table V, the reason for this failure is that the branching ratios for the strange channels are predicted to be significantly smaller than experiment except for the $B\to\phi K$ channels, and in Table VI it can be seen that the direct CP asymmetry of $\overline{B}\to\rho^+\pi^-$ is predicted very small while experiment gives it very large but only two sigmas from zero. Not only is the "goodness of the fit" smaller than 0.1%, but the fitted parameters show a tendency to evade the allowed domain of QCD factorization. One might wonder if we were not too strict in imposing the same scale μ in all terms since the value of μ, representing the effect of unknown higher order corrections, could be different in different classes of channels.[2] We have performed several tests relaxing this unicity of μ and concluded that it affected very little the outcome of our fit.

[2]We thank Gerhard Buchalla for raising this question.

For the sake of comparison with the authors of Ref. [36] we have tried a fit without the channels containing a K^*. The result improves significantly. The only lesson we can receive from this is that one must look carefully at the evolution of the experimental results, many of them being recent, before drawing a final conclusion.

Both the small predicted branching ratios of the strange channels and the small predicted direct CP asymmetries in the nonstrange channels could be blamed on too small P amplitudes with too small ''strong phases'' relatively to the T amplitudes. We have therefore tried the addition of two ''charming-penguin''-diagram-inspired long-distance complex amplitudes combined, in order to make the model predictive enough, with exact flavor-SU(3) symmetry and the OZI rule. This fit is better than the pure QCDF one: with a goodness of fit of about 8% the model is not excluded by experiment. But the parameters show again a tendency to reach the limits of the allowed domain and the best fit gives rather small value to the long-distance contribution. The latter fact is presumably due to the $B\to\phi K$ which are well predicted by QCDF and thus deliver a message which contradicts the other strange channels. This seems to be the reason of the moderate success of our ''charming-penguin''-diagram-inspired model.

Altogether, the present situation is unpleasant. QCDF seems to be unable to comply to experiment. QCDF implemented by an *ad hoc* long-distance model is not fully convincing. No clear hint at the origin of this problem is provided by the total set of experimental data. PQCD, also called k_T factorization, would predict larger direct CP asymmetries, but we do not know if their sign would fit experiment either if an overall agreement of the branching ratios with data can be achieved.

Maybe, however, the coming experimental data will move enough to resolve, at least partly, this discrepancy. We would like to insist on the crucial importance of direct CP asymmetries in nonstrange channels. If they confirm the tendency to be large, this would make the case for QCDF really difficult.

Finally we do not know yet the answer to our initial question: are we in a good position to study the unitarity-triangle angle α from indirect CP asymmetries thanks to small penguin diagrams. If experimental data evolve so as to provide a better support to QCDF, one could become bold enough to use it in estimating α and this would reduce the errors. Else, only model-independent bounds [46] could be used but they are not very constraining in part because of discrete ambiguities.

Note added in proof. Since this paper was submitted, one of us (P.-F.G.) has finalized a new experimental estimate of the $\to\rho\pi$ and $B\to\rho K$ channels for the Babar Collaboration and redone the fits presented in this paper [47]. As a result, the confidence level raises to 1% for the QCDF and drops to 3% for the charming penquins.

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APPENDIX A: THE DECAY AMPLITUDES FOR $B\to PV$

Following Ref. [32], we give the decay amplitudes for the following $B\to PV$ decay processes.

1. $b\to d$ processes

$$\mathcal{A}(\overline{B^0}\to\rho^-\pi^+)=\frac{G_F}{\sqrt{2}}m_B^2 f_\rho F_1^{B\to\pi}(m_\rho^2)\{\lambda_u' a_1+(\lambda_u'+\lambda_c')[a_4+a_{10}]\},\tag{A1}$$

$$\mathcal{A}(\overline{B^0}\to\rho^+\pi^-)=\frac{G_F}{\sqrt{2}}m_B^2 f_\pi A_0^{B\to\rho}(m_\pi^2)\{\lambda_u' a_1+(\lambda_u'+\lambda_c')[a_4+a_{10}-r_\chi^\pi(a_6+a_8)]\},\tag{A2}$$

$$\begin{aligned}\mathcal{A}(\overline{B^0}\to\pi^0\rho^0)=-\frac{G_F}{2\sqrt{2}}m_B^2\Bigg(& f_\pi A_0^{B\to\rho}(m_\pi^2)\left\{\lambda_u' a_2-(\lambda_u'+\lambda_c')\left[a_4-\frac{1}{2}a_{10}-r_\chi^\pi\left(a_6-\frac{1}{2}a_8\right)+\frac{3}{2}(a_7-a_9)\right]\right\}\\ &+f_\rho F_1^{B\to\pi}(m_\rho^2)\left\{\lambda_u' a_2-(\lambda_u'+\lambda_c')\left(a_4-\frac{1}{2}a_{10}-\frac{3}{2}(a_7+a_9)\right)\right\}\Bigg),\end{aligned}\tag{A3}$$

$$\mathcal{A}(B^-\to\pi^-\rho^0)=\frac{G_F}{2}m_B^2\Big[f_\pi A_0^{B\to\rho}(m_\pi^2)\{\lambda_u'a_1+(\lambda_u'+\lambda_c')[a_4+a_{10}-r_\chi^\pi(a_6+a_8)]\}$$
$$+f_\rho F_1^{B\to\pi}(m_\rho^2)\left\{\lambda_u'a_2+(\lambda_u'+\lambda_c')\left(-a_4+\frac{1}{2}a_{10}+\frac{3}{2}(a_7+a_9)\right)\right\}\Big], \tag{A4}$$

$$\mathcal{A}(B^-\to\rho^-\pi^0)=\frac{G_F}{2}m_B^2\Big(f_\pi A_0^{B\to\rho}(m_\pi^2)\left\{\lambda_u'a_2+(\lambda_u'+\lambda_c')\left[-a_4+\frac{1}{2}a_{10}-r_\chi^\pi\left(-a_6+\frac{1}{2}a_8\right)+\frac{3}{2}(a_9-a_7)\right]\right\}$$
$$+f_\rho F_1^{B\to\pi}(m_\rho^2)\{\lambda_u'a_1+(\lambda_u'+\lambda_c')[a_4+a_{10}]\}\Big), \tag{A5}$$

$$\mathcal{A}(B^-\to\pi^-\omega)=\frac{G_F}{2}m_B^2\Big(f_\pi A_0^{B\to\omega}(m_\pi^2)\{\lambda_u'a_1+(\lambda_u'+\lambda_c')[a_4+a_{10}-r_\chi^\pi(a_6+a_8)]\}$$
$$+f_\omega F_1^{B\to\pi}(m_\omega^2)\left\{\lambda_u'a_2+(\lambda_u'+\lambda_c')\left[a_4+2(a_3+a_5)+\frac{1}{2}(a_7+a_9-a_{10})\right]\right\}\Big), \tag{A6}$$

2. $b\to s$ processes

$$\mathcal{A}(\overline{B^0}\to K^{*-}\pi^+)=\frac{G_F}{\sqrt{2}}m_B^2 f_{K*}F_1^{B\to\pi}(m_{K*}^2)\{\lambda_u a_1+(\lambda_u+\lambda_c)[a_4+a_{10}]\}, \tag{A7}$$

$$\mathcal{A}(\overline{B^0}\to K^-\rho^+)=\frac{G_F}{\sqrt{2}}m_B^2 f_K A_0^{B\to\rho}(m_K^2)\{\lambda_u a_1+(\lambda_u+\lambda_c)[a_4+a_{10}-r_\chi^K(a_6+a_8)]\}, \tag{A8}$$

$$\mathcal{A}(\overline{B^0}\to\overline{K^0}\rho^0)=\frac{G_F}{2}m_B^2\Big\{f_K A_0^{B\to\rho}(m_{K^0}^2)(-\lambda_u-\lambda_c)\left[a_4-\frac{1}{2}a_{10}-r_\chi^K\left(a_6-\frac{1}{2}a_8\right)\right]$$
$$+f_\rho F_1^{B\to K}(m_\rho^2)\left[\lambda_u a_2+(\lambda_u+\lambda_c)\times\frac{3}{2}(a_9+a_7)\right]\Big\}, \tag{A9}$$

$$\mathcal{A}(B^-\to K^{*-}\pi^0)=\frac{G_F}{2}m_B^2\Big[f_\pi A_0^{B\to K^*}(m_\pi^2)\left\{\lambda_u a_2+(\lambda_u+\lambda_c)\times\frac{3}{2}(a_9-a_7)\right\}$$
$$+f_{K*}F_1^{B\to\pi}(m_{K*}^2)\{\lambda_u a_1+(\lambda_u+\lambda_c)(a_4+a_{10})\}\Big], \tag{A10}$$

$$\mathcal{A}(B^-\to K^-\rho^0)=\frac{G_F}{2}m_B^2\Big[f_K A_0^{B\to\rho}(m_K^2)\{\lambda_u a_1+(\lambda_u+\lambda_c)[a_4+a_{10}-r_\chi^K(a_6+a_8)]\}$$
$$+f_\rho F_1^{B\to K}(m_\rho^2)\left\{\lambda_u a_2+(\lambda_u+\lambda_c)\times\frac{3}{2}(a_9+a_7)\right\}\Big], \tag{A11}$$

$$\mathcal{A}(\overline{B^0}\to\overline{K^0}\omega)=\frac{G_F}{2}m_B^2\Big(f_K A_0^{B\to\omega}(m_{K^0}^2)(\lambda_u+\lambda_c)\left[a_4-\frac{1}{2}a_{10}-r_\chi^K\left(a_6-\frac{1}{2}a_8\right)\right]$$
$$+f_\omega F_1^{B\to K}(m_\omega^2)\left\{\lambda_u a_2+(\lambda_u+\lambda_c)\left[2(a_3+a_5)+\frac{1}{2}(a_9+a_7)\right]\right\}\Big), \tag{A12}$$

$$\mathcal{A}(B^-\to K^-\omega)=\frac{G_F}{2}m_B^2\Big[f_K A_0^{B\to\omega}(m_K^2)\{\lambda_u a_1+(\lambda_u+\lambda_c)[a_4+a_{10}-r_\chi^K(a_6+a_8)]\}$$
$$+f_\omega F_1^{B\to K}(m_\omega^2)\left\{\lambda_u a_2+(\lambda_u+\lambda_c)\left(2(a_3+a_5)+\frac{1}{2}(a_9+a_7)\right)\right\}\Big], \tag{A13}$$

$$\mathcal{A}(B^-\to K^{*-}\eta^{(\prime)})=\frac{G_F}{\sqrt{2}}m_B^2\Bigg[f_{K^{(\prime)}}F_1^{B\to\eta^{(\prime)}}(m_K^2)\{\lambda_u a_1+(\lambda_u+\lambda_c)(a_4+a_{10})\}+f^u_{\eta^{(\prime)}}A_0^{B\to K^*}(m^2_{\eta^{(\prime)}})$$
$$\times\Bigg(\lambda_u a_2+\lambda_c a_2\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}+(\lambda_u+\lambda_c)\Bigg\{2(a_3-a_5)+\frac{1}{2}(a_9-a_7)+r_\chi^{\eta^{(\prime)}}\left(a_6-\frac{1}{2}a_8\right)$$
$$+(a_3-a_5+a_9-a_7)\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}+\left[a_3-a_5-\frac{1}{2}(a_9-a_7)+a_4-\frac{1}{2}a_{10}-r_\chi^{\eta^{(\prime)}}\left(a_6-\frac{1}{2}a_8\right)\right]\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}\Bigg\}\Bigg)\Bigg],\tag{A14}$$

$$\mathcal{A}(\overline{B}^0\to\overline{K}^{*0}\eta^{(\prime)})=\frac{G_F}{\sqrt{2}}m_B^2\Bigg[f_{K^*}F_1^{B\to\eta^{(\prime)}}(m_K^2)(\lambda_u+\lambda_c)\left[a_4-\frac{1}{2}a_{10}\right]+f^u_{\eta^{(\prime)}}A_0^{B\to K^*}(m^2_{\eta^{(\prime)}})$$
$$\times\Bigg(\lambda_u a_2+\lambda_c a_2\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}+(\lambda_u+\lambda_c)\Bigg\{2(a_3-a_5)+\frac{1}{2}(a_9-a_7)+r_\chi^{\eta^{(\prime)}}\left(a_6-\frac{1}{2}a_8\right)$$
$$+(a_3-a_5+a_9-a_7)\frac{f^c_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}+\left[a_3-a_5-\frac{1}{2}(a_9-a_7)+a_4-\frac{1}{2}a_{10}-r_\chi^{\eta^{(\prime)}}\left(a_6-\frac{1}{2}a_8\right)\right]\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}\Bigg\}\Bigg)\Bigg],\tag{A15}$$

with $r_\chi^{\eta^{(\prime)}}=2m^2_{\eta^{(\prime)}}/(m_b+m_s)(m_s+m_s)$.

3. Pure penguin processes

$$\mathcal{A}(B^-\to\pi^-\overline{K}^{*0})=\frac{G_F}{\sqrt{2}}m_B^2 f_{K^*}F_1^{B\to\pi}(m^2_{K^*})(\lambda_u+\lambda_c)\left[a_4-\frac{1}{2}a_{10}\right],\tag{A16}$$

$$\mathcal{A}(B^-\to\rho^-\overline{K}^0)=\frac{G_F}{\sqrt{2}}m_B^2 f_K A_0^{B\to\rho}(m^2_{K^0})(\lambda_u+\lambda_c)\left[a_4-\frac{1}{2}a_{10}-r_\chi^K\left(a_6-\frac{1}{2}a_8\right)\right],\tag{A17}$$

$$\mathcal{A}(B^-\to K^-K^{*0})=\frac{G_F}{\sqrt{2}}m_B^2 f_{K^*}F_1^{B\to K}(m^2_{K^*})(\lambda'_u+\lambda'_c)\left[a_4-\frac{1}{2}a_{10}\right],\tag{A18}$$

$$\mathcal{A}(B^-\to K^{*-}K^0)=\frac{G_F}{\sqrt{2}}m_B^2 f_K A_0^{B\to K^*}(m^2_{K^0})(\lambda'_u+\lambda'_c)\left[a_4-\frac{1}{2}a_{10}-r_\chi^K\left(a_6-\frac{1}{2}a_8\right)\right],\tag{A19}$$

$$\mathcal{A}(B^-\to\pi^-\phi)=-\frac{G_F}{2}m_B^2 f_\phi F_1^{B\to\pi}(m^2_\phi)(\lambda'_u+\lambda'_c)\left[a_3+a_5-\frac{1}{2}(a_7+a_9)\right],\tag{A20}$$

$$\mathcal{A}(B^-\to K^-\phi)=\mathcal{A}(\overline{B}^0\to\overline{K}^0\phi)$$
$$=\frac{G_F}{\sqrt{2}}m_B^2 f_\phi F_1^{B\to K}(m^2_\phi)(\lambda_u+\lambda_c)\left[a_3+a_4+a_5-\frac{1}{2}(a_7+a_9+a_{10})\right].\tag{A21}$$

APPENDIX B: THE ANNIHILATION AMPLITUDES FOR $B\to PV$

We give in this section the following annihilation amplitudes for $B\to PV$ already given in Ref. [35] but with different notations.

1. $b \to d$ processes

$$\mathcal{A}^a(\overline{B^0}\to\pi^-\rho^+)=\frac{G_F}{\sqrt{2}}f_Bf_\pi f_\rho\Bigg\{\lambda_u' b_1(\rho^+,\pi^-)+(\lambda_u'+\lambda_c')\Bigg[b_3(\pi^-,\rho^+)+b_4(\rho^+,\pi^-)+b_4(\pi^-,\rho^+)-\frac{1}{2}b_3^{ew}(\pi^-,\rho^+)+b_4^{ew}(\rho^+,\pi^-)-\frac{1}{2}b_4^{ew}(\pi^-,\rho^+)\Bigg]\Bigg\}, \tag{B1}$$

$$\mathcal{A}^a(\overline{B^0}\to\pi^+\rho^-)=\frac{G_F}{\sqrt{2}}f_Bf_\pi f_\rho\Bigg\{\lambda_u' b_1(\pi^+,\rho^-)+(\lambda_u'+\lambda_c')\Bigg[b_3(\rho^-,\pi^+)+b_4(\pi^+,\rho^-)+b_4(\rho^-,\pi^+)-\frac{1}{2}b_3^{ew}(\rho^-,\pi^+)+b_4^{ew}(\pi^+,\rho^-)-\frac{1}{2}b_4^{ew}(\rho^-,\pi^+)\Bigg]\Bigg\}, \tag{B2}$$

$$\mathcal{A}^a(\overline{B^0}\to\pi^0\rho^0)=\frac{G_F}{2\sqrt{2}}f_Bf_\pi f_\rho\Bigg\{\lambda_u'[b_1(\rho^0,\pi^0)+b_1(\pi^0,\rho^0)]+(\lambda_u'+\lambda_c')\Bigg\{b_3(\rho^0,\pi^0)+b_3(\pi^0,\rho^0)+2b_4(\pi^0,\rho^0)+2b_4(\rho^0,\pi^0)+\frac{1}{2}[-b_3^{ew}(\rho^0,\pi^0)-b_3^{ew}(\pi^0,\rho^0)+b_4^{ew}(\pi^0,\rho^0)+b_4^{ew}(\rho^0,\pi^0)]\Bigg\}\Bigg\}, \tag{B3}$$

$$\mathcal{A}^a(B^-\to\pi^-\rho^0)=\frac{G_F}{2}f_Bf_\pi f_\rho\{\lambda_u'[b_2(\pi^-,\rho^0)-b_2(\rho^0,\pi^-)]+(\lambda_u'+\lambda_c')[b_3(\pi^-,\rho^0)-b_3(\rho^0,\pi^-)+b_3^{ew}(\pi^-,\rho^0)-b_3^{ew}(\rho^0,\pi^-)]\}, \tag{B4}$$

$$\mathcal{A}^a(B^-\to\pi^0\rho^-)=\frac{G_F}{2}f_Bf_\pi f_\rho\{\lambda_u'[b_2(\rho^-,\pi^0)-b_2(\pi^0,\rho^-)]+(\lambda_u'+\lambda_c')[b_3(\rho^-,\pi^0)-b_3(\pi^0,\rho^-)+b_3^{ew}(\rho^-,\pi^0)-b_3^{ew}(\pi^0,\rho^-)]\}, \tag{B5}$$

$$\mathcal{A}^a(B^-\to\pi^-\omega)=\frac{G_F}{2}f_Bf_\pi f_\omega\{\lambda_u'[b_2(\pi^-,\omega)+b_2(\omega,\pi^-)]+(\lambda_u'+\lambda_c')[b_3(\pi^-,\omega)+b_3(\omega,\pi^-)+b_3^{ew}(\pi^-,\omega)+b_3^{ew}(\omega,\pi^-)]\}, \tag{B6}$$

2. $b \to s$ processes

$$\mathcal{A}^a(\overline{B^0}\to\pi^+K^{*-})=\frac{G_F}{\sqrt{2}}f_Bf_\pi f_{K*}\Bigg\{(\lambda_u+\lambda_c)\Bigg[b_3(K^{*-},\pi^+)-\frac{1}{2}b_3^{ew}(K^{*-},\pi^+)\Bigg]\Bigg\}, \tag{B7}$$

$$\mathcal{A}^a(\overline{B^0}\to K^-\rho^+)=\frac{G_F}{\sqrt{2}}f_Bf_K f_\rho\Bigg\{(\lambda_u+\lambda_c)\Bigg[b_3(K^-,\rho^+)-\frac{1}{2}b_3^{ew}(K^-,\rho^+)\Bigg]\Bigg\}, \tag{B8}$$

$$\mathcal{A}^a(\overline{B^0}\to\overline{K^0}\rho^0)=-\frac{G_F}{2}f_Bf_K f_\rho\Bigg\{(\lambda_u+\lambda_c)\Bigg[b_3(\overline{K^0},\rho^0)-\frac{1}{2}b_3^{ew}(\overline{K^0},\rho^0)\Bigg]\Bigg\}, \tag{B9}$$

$$\mathcal{A}^a(\overline{B^0}\to\overline{K^0}\omega)=\frac{G_F}{2}f_Bf_K f_\omega\Bigg\{(\lambda_u+\lambda_c)\Bigg[b_3(\overline{K^0},\omega)-\frac{1}{2}b_3^{ew}(\overline{K^0},\omega)\Bigg]\Bigg\}, \tag{B10}$$

$$\mathcal{A}^a(B^-\to K^-\omega)=\frac{G_F}{2}f_Bf_K f_\omega\{\lambda_u b_2(K^-,\omega)+(\lambda_u+\lambda_c)[b_3(K^-,\omega)+b_3^{ew}(K^-,\omega)]\}. \tag{B11}$$

$$\mathcal{A}^a(B^-\to\pi^0K^{*-})=\frac{G_F}{2}f_Bf_\pi f_{K*}\{\lambda_u b_2(K^{*-},\pi^0)+(\lambda_u+\lambda_c)[b_3(K^{*-},\pi^0)+b_3^{ew}(K^{*-},\pi^0)]\}, \tag{B12}$$

$$\mathcal{A}^a(B^-\to K^-\rho^0)=\frac{G_F}{2}f_Bf_Kf_\rho\{\lambda_u b_2(K^-,\rho^0)+(\lambda_u+\lambda_c)[b_3(K^-,\rho^0)+b_3^{ew}(K^-,\rho^0)]\}, \tag{B13}$$

$$\mathcal{A}^a(\overline{B^0}\to\eta^{(\prime)}\overline{K}^{*0})=\frac{G_F}{\sqrt{2}}f_Bf^u_{\eta^{(\prime)}}f_{K^*}\Bigg\{(\lambda_u+\lambda_c)\Bigg[b_3(\overline{K}^{*0},\eta^{(\prime)})-\frac{1}{2}b_3^{ew}(\overline{K}^{*0},\eta^{(\prime)}) + \frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}\bigg(b_3(\eta^{(\prime)},\overline{K}^{*0})-\frac{1}{2}b_3^{ew}(\eta^{(\prime)},\overline{K}^{*0})\bigg)\Bigg]\Bigg\}, \tag{B14}$$

$$\mathcal{A}^a(B^-\to\eta^{(\prime)}K^{*-})=\frac{G_F}{\sqrt{2}}f_Bf^u_{\eta^{(\prime)}}f_{K^*}\Bigg\{\lambda_u\Bigg[b_2(K^{*-},\eta^{(\prime)})+\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}b_2(\eta^{(\prime)},K^{*-})\Bigg] + (\lambda_u+\lambda_c)\Bigg[b_3(K^{*-},\eta^{(\prime)})+b_3^{ew}(K^{*-},\eta^{(\prime)})+\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}(b_3(\eta^{(\prime)},K^{*-})+b_3^{ew}(\eta^{(\prime)},K^{*-}))\Bigg]\Bigg\}. \tag{B15}$$

3. Pure penguin diagram processes

$$\mathcal{A}^a(B^-\to\pi^-\overline{K}^{*0})=\frac{G_F}{\sqrt{2}}f_Bf_\pi f_{K^*}\{\lambda_u b_2(\overline{K}^{*0},\pi^-)+(\lambda_u+\lambda_c)[b_3(\overline{K}^{*0},\pi^-)+b_3^{ew}(\overline{K}^{*0},\pi^-)]\}. \tag{B16}$$

$$\mathcal{A}^a(B^-\to\overline{K}^0\rho^-)=\frac{G_F}{\sqrt{2}}f_Bf_Kf_\rho\{\lambda_u b_2(\overline{K}^0,\rho^-)+(\lambda_u+\lambda_c)[b_3(\overline{K}^0,\rho^-)+b_3^{ew}(\overline{K}^0,\rho^-)]\}, \tag{B17}$$

$$\mathcal{A}^a(B^-\to K^-K^{*0})=\frac{G_F}{\sqrt{2}}f_Bf_Kf_{K^*}\{\lambda'_u b_2(K^{*0},K^-)+(\lambda'_u+\lambda'_c)[b_3(K^{*0},K^-)+b_3^{ew}(K^{*0},K^-)]\}, \tag{B18}$$

$$\mathcal{A}^a(B^-\to K^0K^{*-})=\frac{G_F}{\sqrt{2}}f_Bf_Kf_{K^*}\{\lambda'_u b_2(K^0,K^{*-})+(\lambda'_u+\lambda'_c)[b_3(K^0,K^{*-})+b_3^{ew}(K^0,K^{*-})]\}, \tag{B19}$$

$$\mathcal{A}^a(B^-\to\pi^-\phi)=\mathcal{A}^a(\overline{B^0}\to\pi^0\phi)=0. \tag{B20}$$

$$\mathcal{A}^a(B^-\to K^-\phi)=\frac{G_F}{\sqrt{2}}f_Bf_Kf_\phi\{\lambda_u b_2(\phi,K^-)+(\lambda_u+\lambda_c)[b_3(\phi,K^-)+b_3^{ew}(\phi,K^-)]\}, \tag{B21}$$

$$\mathcal{A}^a(\overline{B^0}\to\overline{K}^0\phi)=\frac{G_F}{\sqrt{2}}f_Bf_Kf_\phi\bigg\{(\lambda_u+\lambda_c)\bigg[b_3(\phi,\overline{K}^0)-\frac{1}{2}b_3^{ew}(\phi,\overline{K}^0)\bigg]\bigg\}. \tag{B22}$$

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Chapitre 3

Le puzzle des états P

Où nous discuterons de l'incohérence entre les prédictions théoriques et les mesures expérimentales relatives aux canaux $B \to D^{**}$

Comme il l'a déjà été annoncé dans le cours du chapitre 1, un problème se pose entre les prédictions théoriques et les mesures expérimentales relatives aux désintégrations des mésons B en mesons D orbitalement excités $L = 1$ (états P notés D^{**}). Dans la suite de ce chapitre, nous allons présenter deux éléments de recherche supplémentaires spécifiques à ce problème après avoir rappelé quelques généralités sur la construction d'amplitudes de transition covariantes à la façon de Bakamjian-Thomas.

3.1 Philosophie de la construction de Bakamjian-Thomas

L'objectif de cette construction [27, 28, 29] est d'écrire, dans le cadre des modèles de quarks, l'amplitude de transition de n'importe quel processus hadronique de façon covariante, et cela quelle que soit la dynamique décrivant les états liés de quarks.

3.1.1 Hypothèses

Comme l'on veut étudier un processus du type $B \to D^{(*),(**)}$, il nous faut calculer des amplitudes du style $\langle \Psi' | J | \Psi \rangle$ qui, puisque nous nous plaçons dans le cadre des modèles de quarks constituants pour les mésons, peut s'écrire selon :

$$\langle \Psi' | J | \Psi \rangle = \int \frac{d\vec{p}_1^{\,\prime}}{(2\pi)^3} \frac{d\vec{p}_1}{(2\pi)^3} \frac{d\vec{p}_2}{(2\pi)^3} \sum_{s_1',s_1,s_2} \Psi_{s_1',s_2}(\vec{p}_1^{\,\prime},\vec{p}_2)^* J(\vec{p}_1^{\,\prime},\vec{p}_1)_{s_1',s_1} \Psi_{s_1,s_2}(\vec{p}_1,\vec{p}_2) \qquad (3.1)$$

où l'indice 1 repère le quark actif, l'indice 2 le quark spectateur du méson et s_i désigne le spin du quark i. Le courant J n'agit que sur le quark actif 1 et décrit l'interaction subie lors de la désintégration. Notons que les variables $\vec{p_i}$ et s_i sont définies dans le référentiel du laboratoire (qui peut être quelconque en fait).

3.1.2 Construction à la Bakamjian-Thomas

La construction de Bakamjian-Thomas permet d'imposer une description covariante pour un système formé d'un nombre fini et fixe de constituants en interaction. Pour cela, l'idée est d'imposer les bonnes propriétés de groupe (le groupe de Poincaré en fait) dans l'écriture des fonctions d'onde $\Psi_{s_1,s_2}(\vec{p}_1,\vec{p}_2)$; plus précisément, on utilise les générateurs du groupe de Poincaré pour relier les fonctions d'onde qui apparaissent dans (3.1) écrites en termes de variables $\vec{p_i}$ et s_i à des fonctions d'onde qui ne dépendent que de variables internes [3] (grâce aux rotations de Wigner qui permettent de passer du référentiel du laboratoire au référentiel interne). À la fin, nous obtenons une écriture covariante des amplitudes $\langle\Psi'|J|\Psi\rangle$ reposant uniquement sur des arguments cinématiques et de théorie des groupes.

3.1.3 Comportement dans la limite de masse infinie

Ce formalisme a été utilisé pour étudier les désintégrations semileptoniques $B \to D^{(*),(**)}\ell\nu$ ainsi que les constantes de désintégration des états S et P dans la limite de masse infinie. Nous avons montré [19, 3] que :

- le comportement d'échelle est correctement et naturellement reproduit : nous avons obtenu des expressions pour les différents facteurs de forme ξ, $\tau_{1/2}$ et $\tau_{3/2}$
- la règle de somme de Bjorken est également naturellement satisfaite dans le cadre de ce formalisme, ainsi qu'une règle de somme « maison » (appelée LOPR dans [3]) qui fait intervenir les constantes de désintégration (voir la section 3.3 ci-après).

Enfin, un choix de modèle dynamique (nous en avons utilisé quatre) pour décrire les états liés mésoniques nous a permis de réaliser plusieurs prédictions théoriques relatives à la phénoménologie de ce type de réactions.

3.1.4 Apparition du puzzle des états P

Parmi les prédictions théoriques mentionnées ci-dessus, nous avons établi que les désintégrations $B \to D^{**}_{3/2}\ell\nu$ étaient favorisées par rapport aux canaux $B \to D^{**}_{1/2}\ell\nu$, et ce pour les quatre modèles considérés. D'un autre côté, les mesures expérimentales disponibles semblent plutôt aller vers la conclusion opposée : c'est le « puzzle des états P ». Dans [3], il nous est apparu par calcul direct des $\tau_{1/2}$ et $\tau_{3/2}$ à partir des quatre modèles dynamiques , puis comme conséquence de la règle de Bjorken.

3.2 Règle de somme d'Uraltsev

Une nouvelle règle de somme a été établie par Uraltsev [30] de façon complètement générale en QCD dans la limite de masse infinie :

$$\sum_n |\tau^{(n)}_{3/2}(1)|^2 \; - \; \sum_n |\tau^{(n)}_{1/2}(1)|^2 \; = \; \frac{1}{4} \tag{3.2}$$

Cette régle de somme fait intervenir des facteurs de forme pour lesquels nous possédons des expressions exactes dans le cadre de notre modèle à la Bakamjian-Thomas [19, 3] ; en substituant dans le membre de gauche de (3.2) les fonctions $\tau_{1/2}$ et $\tau_{3/2}$ par nos expressions, nous avons vérifié [31] que notre formulation covariante des facteurs de forme satisfait également *naturellement* et sans hypothèse supplémentaire cette nouvelle relation.

Par ailleurs, là-encore, apparaît le puzzle des états P : la règle de somme suggère fortement en effet que $\sum_n |\tau^{(n)}_{3/2}(1)|^2 > \sum_n |\tau^{(n)}_{1/2}(1)|^2$.

Notons enfin que cette conclusion n'est donc pas uniquement une conséquence des modèles uitlisant la construction de Bakamjian-Thomas puisque (3.2) a été établie à partir de la limite de masse infinie de QCD.

3.3 Une complication supplémentaire

L'étude des désintégrations non leptoniques de mésons lourds demande de connaître un certain nombre de constantes de désintégration et, dans la limite de masse infinie, il suffit de connaître les constantes[1] $f^{(n)}$, $f^{(n)}_{1/2}$ et $f^{(n)}_{3/2}$ où n désigne l'excitation radiale considérée. En imposant la dualité, deux nouvelles règles de somme peuvent être démontrées qui relient les constantes f et les facteurs de formes dans la limite de masse infinie [32] :

$$\sum_n \frac{f^{(n)}}{f^{(0)}}\, \xi^{(n)}(w) = 1 \qquad\qquad \sum_n \frac{f^{(n)}_{1/2}}{f^{(0)}}\, \tau^{(n)}_{1/2}(w) = \frac{1}{2} \tag{3.3}$$

Ces relations sont valables en toute généralité dans la limite $m_Q \to \infty$ et pour n'importe quelle valeur de w ce qui constitue une contrainte importante. De plus, toujours dans cette limite, les constantes $f^{(n)}_{3/2}$ doivent s'annuler.

Dans une première étude il y a quelques temps [33, 3], nous avons utilisé la construction des amplitudes de transition façon Bakamjian-Thomas pour obtenir les expressions exactes des constantes ce qui nous a permis de vérifier que les règles de somme (3.3) étaient correctement satisfaites par cette construction.

1. Ce sont respectivement les constantes de désintégrations des états S, des états P avec $j = 1/2$ et des états P avec $j = 3/2$.

À partir de ces constantes et en utilisant l'hypothèse de factorisation « naïve » mentionée dans le chapitre précédent, il est également possible de faire des prédictions relatives à certains canaux non leptoniques dans la limite de masse infinie. Dans [34], nous avons donc poursuivi notre étude des constantes de désintégration pour aboutir aux résultats suivants :

- comme les $f^{(n)}_{3/2}$ s'annulent dans la limite $m_Q \to \infty$, il faut s'attendre à ce que les canaux non semi-leptoniques faisant intervenir l'émission[2] d'une résonance P excitée $j = 3/2$ soient défavorisés par rapport à ceux émettant une résonance $j = 1/2$; par exemple :

$$\Gamma(\bar{B}_d \to \bar{D}_{s,\,3/2}\, D) \;\ll\; \Gamma(\bar{B}_d \to \bar{D}_{s,\,1/2}\, D)$$

  en utilisant l'hypothèse – grossière – de factorisation naïve. Nous tombons ici sur une hiérarchie inverse de celle générée par les facteurs de forme d'Isgur-Wise $\tau_{1/2}$ et $\tau_{3/2}$.
- en appliquant l'inégalité de Schwartz aux relations (3.3), les sommes suivantes divergent :

$$\sum_n \left(\frac{f^{(n)}}{f^{(0)}}\right)^2 \qquad\qquad \sum_n \left(\frac{f^{(n)}_{1/2}}{f^{(0)}}\right)^2$$

  Cette propriété de divergence de ces sommes semblent être vérifiée par les prédictions numériques de notre modélisation à la Bakamjian-Thomas.

3.4 Bilan

Nous avons montré qu'il existe un problème relatif à l'étude des canaux faisant intervenir la production d'états excités P dans la limite de masse infinie : les prédictions théoriques, que ce soient des règles de somme sur les facteurs de forme ou des prédictions numériques dans les modèles à la Bakamjian-Thomas, vont dans le sens d'une hiérarchie $\Gamma(\bar{B}_d \to D^{**}_{1/2}\,\ell\nu) \ll \Gamma(\bar{B}_d \to D^{**}_{3/2}\,\ell\nu)$. D'un autre côté, les mesures expérimentales correspondantes suggèrent plutôt la hiérarchie inverse. Par ailleurs, si l'on considère les canaux d'émission d'états excités P, on observe du point de vue théorique la hiérarchie également inverse de celle obtenue pour les canaux de production.

Afin de lever ce paradoxe, il serait utile d'avoir accès aux paramètres théoriques d'une façon plus fondamentale avec le minimum d'approximation : une piste évidente serait de pouvoir réaliser les calculs des facteurs de forme d'Isgur-Wise $\tau_{1/2}$ et $\tau_{3/2}$ directement à partir des théories fondamentales, d'où l'étude présentée dans le chapitre suivant portant sur les calculs de QCD sur réseau.

2. Car la résonance en question est couplée au vide. Cette notion d'*émission* s'oppose à celle de *production* pour laquelle la résonance est couplée à l'état initial comme dans les désintégrations semi-leptoniques par exemple.

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Uraltsev sum rule in Bakamjian–Thomas quark models

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Abstract

We show that the sum rule recently proved by Uraltsev in the heavy quark limit of QCD holds in relativistic quark models à la Bakamjian and Thomas, that were already shown to satisfy Isgur–Wise scaling and Bjorken sum rule. This new sum rule provides a *rationale* for the lower bound of the slope of the elastic IW function $\rho^2 \geqslant 3/4$ obtained within the BT formalism some years ago. Uraltsev sum rule suggests an inequality $|\tau_{3/2}(1)| > |\tau_{1/2}(1)|$. This difference is interpreted in the BT formalism as due to the Wigner rotation of the light quark spin, independently of a possible LS force. In BT models, the sum rule convergence is very fast, the $n = 0$ state giving the essential contribution in most of the phenomenological potential models. We underline that there is a serious problem, in the heavy quark limit of QCD, between theory and experiment for the decays $B \to D^*_{0,1}(\text{broad})\ell\nu$, independently of any model calculation. © 2001 Published by Elsevier Science B.V.

1. Introduction

Recently, Uraltsev [1] has established, in the heavy quark limit of QCD, a new sum rule. The demonstration of the sum rule (SR) follows from the OPE applied to the scattering amplitude $T(\varepsilon, \mathbf{v}, \mathbf{v} - \mathbf{v}')$ in the Shifman–Voloshin limit. The function $T(\varepsilon, \mathbf{v}, \mathbf{v} - \mathbf{v}')$, where ε is the energy variable ($\varepsilon = 0$ for elastic transitions of a free quark), is the Fourier transform of the expectation value

$$\langle B^*(\mathbf{v} - \mathbf{v}') | T(J^+(0) J(x)) | B^*(0) \rangle, \tag{1}$$

where the initial state is at rest and the final state has a momentum $m_Q(\mathbf{v} - \mathbf{v}')$, $-m_Q\mathbf{v}$ being the momentum transfer carried by the intermediate states. The novelty in Uraltsev procedure is to allow a momentum for the final state in (1). Then, the function $T(\varepsilon, \mathbf{v}, \mathbf{v} - \mathbf{v}')$ can be decomposed into symmetric and antisymmetric parts $h_\pm(\varepsilon)$ in $\mathbf{v}, \mathbf{v}'$. The zero order moment of $h_+(\varepsilon)$ leads to Bjorken SR [2] involving ρ^2, the slope of the elastic IW function $\xi(w)$:

$$\rho^2 = \frac{1}{4} + \sum_n |\tau_{1/2}^{(n)}(1)|^2 + 2\sum_n |\tau_{3/2}^{(n)}(1)|^2 \tag{2}$$

while the zero order moment of $h_-(\varepsilon)$ leads to the new SR [1]:

$$\sum_n |\tau_{3/2}^{(n)}(1)|^2 - \sum_n |\tau_{1/2}^{(n)}(1)|^2 = \frac{1}{4}. \tag{3}$$

From (2) and (3) one gets the lower bound

$$\rho^2 \geqslant \frac{3}{4}. \tag{4}$$

The simple relations that come out immediately from (2) and (3),

$$\sum_n |\tau_{3/2}^{(n)}(1)|^2 = \frac{\rho^2}{3}, \tag{5}$$

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$$\sum_n \left|\tau_{1/2}^{(n)}(1)\right|^2 = \frac{1}{3}\left(\rho^2 - \frac{3}{4}\right) \tag{6}$$

deserve a comment. One can see that $\sum_n |\tau_{3/2}^{(n)}(1)|^2$ is proportional to ρ^2 and that $\sum_n |\tau_{1/2}^{(n)}(1)|^2$ is proportional to the *deviation* of ρ^2 from the lower bound $3/4$. Then, there is little room left for $\sum_n |\tau_{1/2}^{(n)}(1)|^2$, as it has been pointed out recently from a SR obtained for the subleading function $\xi_3(1)$ [3]:

$$\xi_3(1) = 2\left[\sum_n \Delta E_{3/2}^{(n)}\left|\tau_{3/2}^{(n)}(1)\right|^2 - \sum_n \Delta E_{1/2}^{(n)}\left|\tau_{1/2}^{(n)}(1)\right|^2\right]. \tag{7}$$

This sum rule, combined with Voloshin sum rule [4]

$$\bar{\Lambda} = 2\sum_n \Delta E_{1/2}^{(n)}\left|\tau_{1/2}^{(n)}(1)\right|^2 + 4\sum_n \Delta E_{3/2}^{(n)}\left|\tau_{3/2}^{(n)}(1)\right|^2 \tag{8}$$

yields

$$\sum_n \Delta E_{3/2}^{(n)}\left|\tau_{3/2}^{(n)}(1)\right|^2 = \frac{1}{6}[\bar{\Lambda} + \xi_3(1)], \tag{9}$$

$$\sum_n \Delta E_{1/2}^{(n)}\left|\tau_{1/2}^{(n)}(1)\right|^2 = \frac{1}{6}[\bar{\Lambda} - 2\xi_3(1)]. \tag{10}$$

Ignoring short distance QCD corrections, QCD sum rules predict, independently of all sum rule parameters [5]

$$\xi_3(1) = \frac{\bar{\Lambda}}{3} \tag{11}$$

giving

$$\frac{\sum_n \Delta E_{1/2}^{(n)}|\tau_{1/2}^{(n)}(1)|^2}{\sum_n \Delta E_{3/2}^{(n)}|\tau_{3/2}^{(n)}(1)|^2} = \frac{1}{4}. \tag{12}$$

Since the LS coupling is small, we see that we have the same trend of inequality between $\sum_n |\tau_{3/2}^{(n)}(1)|^2$ and $\sum_n |\tau_{1/2}^{(n)}(1)|^2$ as in Eqs. (5) and (6).

2. Uraltsev sum rule in Bakamjian–Thomas quark models

One of the aims of this note is to show that the SR (3) follows within quark models à la Bakamjian and Thomas. Quark models of hadrons with a fixed number of constituents, based on the Bakamjian–Thomas (BT) formalism [6,7], yield form factors that are covariant and satisfy Isgur–Wise (IW) scaling [8] in the heavy mass limit. In this class of models, the lower bound (4) was predicted some years ago [6]. Moreover, this approach satisfies the Bjorken SR that relates the slope of the IW function to the P-wave IW functions $\tau_{1/2}(w)$, $\tau_{3/2}(w)$ at zero recoil [9]. In this approach were also computed the P-wave meson wave functions and the corresponding inelastic IW functions [10], and a numerical study of ρ^2 in a wide class of models of the meson spectrum was performed (each of them characterized by an ansatz for the mass operator M, i.e., the dynamics of the system at rest) [11], together with a phenomenological study of the elastic and inelastic IW functions and the corresponding rates for $B \to D, D^*, D^{**}\ell\nu$. Moreover, the calculation of decay constants of heavy mesons within the same approach was also performed [12].

The first demonstration of Uraltsev SR within the BT quark models is rather short, relying on formulas established in Ref. [10]. Two other demonstrations will follow that will exhibit the underlying physics. The starting point is [10]:

$$\tau_j^{(n)}(1) = \int \frac{p^2\,dp}{(2\pi)^2}\varphi_j^{(n)*}(p)F_j(p), \tag{13}$$

where

$$F_{1/2}(p) = -\frac{1}{3\sqrt{3}}\left\{\varphi(p)\frac{p^2}{m+p_0}\left(3+\frac{m}{p_0}\right) + 2pp_0\frac{d\varphi}{dp}\right\},$$
$$F_{3/2}(p) = -\frac{1}{3\sqrt{3}}\left\{\varphi(p)\frac{p^2}{m+p_0}\frac{m}{p_0} + 2pp_0\frac{d\varphi}{dp}\right\} \tag{14}$$

with the radial part of the $L = 1$ wave functions normalized according to

$$\frac{1}{6\pi^2}\int p^2\,dp\left[p\varphi_j^{(n)}(p)\right]^2 = 1 \tag{15}$$

and m, $p = |\mathbf{p}|$ and $p_0 = \sqrt{p^2+m^2}$ are the mass, momentum and energy of the spectator quark.

From (13), using closure in the sectors of definite $j = 1/2,\ 3/2$ one finds (page 325 of Ref. [10]):

$$\sum_n \left|\tau_j^{(n)}(1)\right|^2 = \frac{3}{8\pi^2}\int dp\,|F_j(p)|^2. \tag{16}$$

From (14)–(16), the expression for the difference in the left-hand side of (3) can be integrated by parts, yielding, after some algebra:

$$\sum_n \left|\tau_{3/2}^{(n)}(1)\right|^2 - \sum_n \left|\tau_{1/2}^{(n)}(1)\right|^2 = \frac{1}{8\pi^2}\int p^2\,dp\,[\varphi(p)]^2 = \frac{1}{4}, \tag{17}$$

where the last equality follows from the ground state wave function normalization [6].

Therefore, the SR (17) within the BT quark models provides a *rationale* for the lower bound $\rho^2 \geqslant 3/4$ that was found within this class of models [6]. The sum rule also establishes that the sum over the $j = 3/2$ states dominates over the one over the $j = 1/2$.

The second demonstration, that follows more closely Uraltsev proof, will illustrate quark–hadron duality. Let us first remind the proof of Bjorken SR that was given in [9]. It was shown that the *spin averaged* hadronic tensor in the BT formalism is, *in the heavy quark limit* for the active quark, identical to the free quark hadronic tensor:

$$\bar{h}_{\mu\nu}(\mathbf{v},\mathbf{v}') = \bar{h}_{\mu\nu}^{\text{free quark}}(\mathbf{v},\mathbf{v}'). \tag{18}$$

From this relation, Bjorken SR follows. In Eq. (18), the free quark tensor is

$$\bar{h}_{\mu\nu}^{\text{free quark}}(\mathbf{v},\mathbf{v}') = \frac{1}{2}\sum_{s_1,s_1'}\left[\bar{u}_{s_1'}(\mathbf{v}')\gamma_\mu u_{s_1}(\mathbf{v})\right] \times \left[\bar{u}_{s_1'}(\mathbf{v}')\gamma_\nu u_{s_1}(\mathbf{v})\right]^* \tag{19}$$

and the hadronic tensor writes

$$\bar{h}_{\mu\nu}(\mathbf{v},\mathbf{v}') = \frac{1}{2J+1}\sum_\lambda\sum_n \langle \mathbf{P},\lambda|J_\nu|n,\mathbf{P}'\rangle \times \langle n,\mathbf{P}'|J_\mu|\mathbf{P},\lambda\rangle, \tag{20}$$

where J, λ are the spin and spin projection of the hadron of momentum $\mathbf{P}$.

In BT models, the hadronic tensor can be written [9]:

$$\bar{h}_{\mu\nu}(\mathbf{v},\mathbf{v}') = \frac{1}{2J+1}\sum_\lambda \sum_{s_{1f},s_1',s_{1i}} \left[\bar{u}_{s_1'}(\mathbf{v}')\gamma_\mu u_{s_{1i}}(\mathbf{v})\right] \times \left[\bar{u}_{s_1'}(\mathbf{v}')\gamma_\nu u_{s_{1f}}(\mathbf{v})\right]^* f_{s_{1f}s_{1i}}^{\lambda\lambda}, \tag{21}$$

where $f_{s_{1f}s_{1i}}^{\lambda\lambda}$ is the hadronic overlap:

$$f_{s_{1f}s_{1i}}^{\lambda\lambda} = \sum_{s_2}\int d^3\mathbf{p}_2\,\psi_{s_{1f}s_2}^{\lambda *}(\mathbf{P}-\mathbf{p}_2,\mathbf{p}_2) \times \psi_{s_{1i}s_2}^{\lambda}(\mathbf{P}-\mathbf{p}_2,\mathbf{p}_2) \tag{22}$$

and (18) follows from (21) and (22). The wave function $\psi_{s_1,s_2}^{\lambda}(\mathbf{P}-\mathbf{p}_2,\mathbf{p}_2)$ is the internal moving ground state wave function, with the active quark labelled 1 and λ being the spin projection along some axis. It is defined by deleting the momentum conserving δ-function from the total wave function. In the BT model, it is obtained from a P-depending transformation on the *rest* internal wave function.

To proceed like Uraltsev, one must generalize the hadronic tensor, allowing for different velocities and angular momentum projections. Let us consider the *polarized* hadronic tensor:

$$h_{\mu\nu}^{\lambda_i\lambda_f}(\mathbf{v}_i,\mathbf{v}_f,\mathbf{v}') = \sum_n \langle \mathbf{P}_f,\lambda_f|J_\nu|n,\mathbf{P}'\rangle \times \langle n,\mathbf{P}'|J_\mu|\mathbf{P}_i,\lambda_i\rangle. \tag{23}$$

In the BT formalism, this tensor writes, using closure and heavy mass limit [9]: [2]

$$h_{\mu\nu}^{\lambda_i\lambda_f}(\mathbf{v}_i,\mathbf{v}_f,\mathbf{v}') = \sum_{s_{1f},s_1',s_{1i}} \left[\bar{u}_{s_1'}(\mathbf{v}')\gamma_\mu u_{s_{1i}}(\mathbf{v}_i)\right] \times \left[\bar{u}_{s_1'}(\mathbf{v}')\gamma_\nu u_{s_{1f}}(\mathbf{v}_f)\right]^* f_{s_{1f}s_{1i}}^{\lambda_f\lambda_i}(\mathbf{P}_i,\mathbf{P}_f) \tag{24}$$

with the hadronic overlap

$$f_{s_{1f}s_{1i}}^{\lambda_f\lambda_i}(\mathbf{P}_i,\mathbf{P}_f) = \sum_{s_2}\int d^3\mathbf{p}_2\,\psi_{s_{1f}s_2}^{\lambda_f *}(\mathbf{P}_f-\mathbf{p}_2,\mathbf{p}_2) \times \psi_{s_{1i}s_2}^{\lambda_i}(\mathbf{P}_i-\mathbf{p}_2,\mathbf{p}_2). \tag{25}$$

In this expression $\psi_{s_{1i}s_2}^{\lambda_i}(\mathbf{P}_i-\mathbf{p}_2,\mathbf{p}_2)$ ($i \to f$ likewise) is the internal moving ground state meson wave function, and the active quark is labelled 1.

Let us choose, like Uraltsev, the vector meson B^* as initial and final state, with $\mathbf{P}_i = 0$, $\lambda_i = 0$, $\lambda_f = +1$, and the vector current with $\mu = \nu = 0$. We are thus

[2] The states $|n,\mathbf{P}'\rangle$ form a complete set of eigenfunctions at fixed $\mathbf{P}'$: $\sum_n |n,\mathbf{P}'\rangle\langle n,\mathbf{P}'| = 1$.

considering the object

$$h_{00}^{0,+1}(\mathbf{v}_i,\mathbf{v}_f,\mathbf{v}') = \sum_{s_{1f},s'_1,s_{1i}} \left[\bar{u}_{s'_1}(\mathbf{v}')\gamma_0 u_{s_{1i}}(0)\right] \times \left[\bar{u}_{s'_1}(\mathbf{v}')\gamma_0 u_{s_{1f}}(\mathbf{v}_f)\right]^* \times f^{+1,0}_{s_{1f}s_{1i}}(0,\mathbf{P}_f) \tag{26}$$

to first order in $\mathbf{v}'$, $\mathbf{v}_f$. There are, in principle, two kinds of terms contributing to this quantity:

(1) Spin-flip term coming from the active quark, i.e., from the quark current matrix element at the desired order $\bar{u}_{s'_1}(\mathbf{v}')\gamma_0 u_{s_{1f}}(\mathbf{v}_f) \sim \mathbf{v}' \times \mathbf{v}_f$ while $\bar{u}_{s'_1}(\mathbf{v}')\gamma_0 u_{s_{1i}}(0)$ cannot give a spin flip because $\mathbf{v}_i = 0$. At the desired order, one can also take the hadronic overlap at $\mathbf{P}_i = \mathbf{P}_f = 0$:

$$h_{00}^{0,+1}(0,\mathbf{v}_f,\mathbf{v}') = [\bar{u}_{-1/2}(\mathbf{v}')\gamma_0 u_{-1/2}(0)] \times [\bar{u}_{-1/2}(\mathbf{v}')\gamma_0 u_{+1/2}(\mathbf{v}_f)]^* \times f^{+1,0}_{s_{1f}s_{1i}}(0,0). \tag{27}$$

One obtains

$$h_{00}^{0,+1}(0,\mathbf{v}_f,\mathbf{v}') = \frac{1}{4\sqrt{2}}\big(\langle\downarrow|i\sigma_1\cdot(\mathbf{v}'\times\mathbf{v}_f)|\uparrow\rangle\big)^*, \tag{28}$$

where the factor $1/\sqrt{2}$ comes from the hadronic overlap, and 1 labels the active quark.

(2) Terms without spin-flip of the active quark. Then, to have a contribution to (26), one needs to appeal to a Wigner rotation of the spectator quark 2, giving a contribution $\sim \mathbf{p}_2 \times \mathbf{P}_f$. But, by integration, this term is zero, because there is no other hadron momentum than $\mathbf{P}_f$—in the hadronic overlap there is no dependence on $\mathbf{P}'$.

We are then left with expression (28), that means that we have exact duality, just like in the unpolarized, $\mathbf{P}_i = \mathbf{P}_f$ case:

$$h_{00}^{0,+1}(0,\mathbf{v}_f,\mathbf{v}') = \left[h_{00}^{0,+1}(0,\mathbf{v}_f,\mathbf{v}')\right]_{\text{free quark}}. \tag{29}$$

We need now to compute the same hadronic tensor (23) in terms of the phenomenological Isgur–Wise functions $\tau_j(w)$, within the same approximations. After a good deal of algebra, we find, using the definitions of [14], and taking into account that the states are not normalized according to the usual normalization $\langle\mathbf{v}'|\mathbf{v}\rangle = \sqrt{4v^0v'^0}\,\delta(\mathbf{v}-\mathbf{v}')$ but by $\langle\mathbf{v}'|\mathbf{v}\rangle = \delta(\mathbf{v}-\mathbf{v}')$,

$$h_{00}^{0,+1}(\mathbf{v}_f,\mathbf{v}',\mathbf{v}_i) \cong v_f^z \frac{1}{\sqrt{2}}\big(v'^x - iv'^y\big) \times \left[C\big(0^+, j=\tfrac{1}{2}\big) + C\big(1^+, j=\tfrac{1}{2}\big) + C\big(1^+, j=\tfrac{3}{2}\big) + C\big(2^+, j=\tfrac{3}{2}\big)\right], \tag{30}$$

where the different contributions are (a sum over a radial quantum number is implicit)

$$C\big(0^+, j=\tfrac{1}{2}\big) = 0,$$
$$C\big(1^+, j=\tfrac{1}{2}\big) = -|\tau_{1/2}(1)|^2,$$
$$C\big(1^+, j=\tfrac{3}{2}\big) = -\frac{1}{2}|\tau_{3/2}(1)|^2,$$
$$C\big(1^+, j=\tfrac{3}{2}\big) = \frac{3}{2}|\tau_{3/2}(1)|^2 \tag{31}$$

and the ground state does not contribute. One obtains,

$$h_{00}^{0,+1}(\mathbf{v}_f,\mathbf{v}',\mathbf{v}_i) \cong v_f^z \frac{1}{\sqrt{2}}\big(v'^x - iv'^y\big) \times \left[\sum_n \big|\tau_{3/2}^{(n)}(1)\big|^2 - \sum_n \big|\tau_{1/2}^{(n)}(1)\big|^2\right]. \tag{32}$$

Identifying the expressions (28) and (32), Uraltsev SR follows.

Some words of caution about the general scope and limitations of Bakamjian–Thomas quark models are in order here. Both zero order moment sum rules, the ones of Bjorken [9] and Uraltsev are satisfied by this class of models. However, higher moment sum rules as Voloshin sum rule [4] are not satisfied. These higher moments sum rules seem to be specific to the gauge nature of QCD. Anyhow, one limitation of BT models is the following, as exposed in [6]. The Bakamjian–Thomas scheme was formulated to describe relativistic bound states with a fixed number of constituents, that form representations of the Poincaré group. However, when one considers matrix elements of currents with one active quark (the simplest ansatz), these matrix elements are not covariant in general, although a main result of the formalism is that they are covariant in the heavy quark limit. In the fact, one does not obtain a covariant expression for the Voloshin sum $4\sum_n \Delta E_{3/2}^{(n)}|\tau_{3/2}^{(n)}(1)|^2 + 2\sum_n \Delta E_{1/2}^{(n)}|\tau_{1/2}^{(n)}(1)|^2$, reflecting the non-covariance outside the heavy quark limit, by contrast to the Bjorken and Uraltsev ones, that are covariant.

3. The role of spectator quark Wigner rotations

Within quark models à la BT, the difference between $\tau_{3/2}^{(0)}(1)$ and $\tau_{1/2}^{(0)}(1)$ follows from formulas (13), (14) (with a suitable phase convention, $\tau_{3/2}^{(0)}, \tau_{1/2}^{(0)} \geqslant 0$)

$$\tau_{3/2}^{(0)}(1) - \tau_{1/2}^{(0)}(1) \cong \frac{1}{(2\pi)^2\sqrt{3}} \int p^2\, dp \left[p\varphi_{L=1}^{(0)}(p)\right]^* \frac{p}{p_0+m}\varphi(p), \tag{33}$$

where $\varphi_{1/2}(p) \cong \varphi_{3/2}(p) = \varphi_{L=1}^{(0)}(p)$ (assuming small LS coupling) are the internal hadron wave functions at rest. We assume, as it is natural, that for the ground state $\varphi_{L=1}^{(0)}(p)$ is positive. One finds that $\tau_{3/2}^{(0)}(1)$ is larger than $\tau_{1/2}^{(0)}(1)$ even in the limit of vanishing LS coupling. The difference (33) has a simple physical interpretation, outlined in Ref. [11]: it is essentially due to the relativistic structure of the matrix elements in terms of the wave functions. More precisely, it is due to the light spectator *quark Wigner rotations*, i.e., a relativistic effect due to the center-of-mass boost, and not due to the difference coming from the spin-orbit force between the 1/2 and 3/2 internal wave functions at rest, which is small and has a rather moderate effect. On the contrary, the difference (33) is quite large, at least for the lowest $L = 1$ states, since for a constituent quark mass $m \cong 0.3$ GeV, the quantity $p/(p_0+m)$ is of O(1).

Expression (33), that comes from a specific relativistic effect, is to be contrasted with the equality for any non-relativistic quark model with spin-orbit independent potential [13], also used in Ref. [14], that analizes $1/m_Q$ corrections:

$$\tau_{3/2}^{(n)}(1) = \tau_{1/2}^{(n)}(1). \tag{34}$$

Let us see how, in terms of internal wave functions *at rest*, the Wigner rotation of the *spectator quark* is responsible for the difference between $\tau_{3/2}^{(n)}$ and $\tau_{1/2}^{(n)}$ and for the non-vanishing of the r.h.s. of Uraltsev SR (17) within the BT formalism. In the previous demonstration of Uraltsev SR, the Wigner rotations were hidden in the moving internal wave functions, which themselves disappeared using completeness relations. We will now make those explicit by using the internal wave functions at rest, that gives a feeling of how the difference $|\tau_{3/2}^{(n)}|^2 - |\tau_{1/2}^{(n)}|^2$ comes out in the l.h.s. of Uraltsev SR. Consider a meson with the active heavy quark labelled 1 and the spectator quark labelled 2. In terms of internal wave functions, the current matrix element in the BT formalism writes (formula (27) of Ref. [9]):

$$\begin{aligned}\langle \mathbf{v}'|V_\mu(0)|\mathbf{v}\rangle &= \sum_{s_1' s_1} \bar{u}_{s_1'}\gamma_\mu u_{s_1} \int d\mathbf{p}_2\, \frac{\sqrt{(p_i\cdot v)(p_i'\cdot v')}}{p_2^0} \\ &\quad \times \sum_{s_2' s_2} \varphi'^*_{s_1' s_2'}(\mathbf{k}_2')\left[D\left(R_2'^{-1}R_2\right)\right]_{s_2' s_2}\varphi_{s_1 s_2}(\mathbf{k}_2).\end{aligned} \tag{35}$$

In this expression we see the basic ingredients of the model. There is a change of variables of the quark momenta, e.g., for the initial state $(\mathbf{p}_1, \mathbf{p}_2) \to (\mathbf{P}, \mathbf{k}_2)$, where $\mathbf{P}$ is the center-of mass momentum, and $\mathbf{k}_2$ the internal relative momentum, and likewise for the final state $(\mathbf{p}_1', \mathbf{p}_2') \to (\mathbf{P}', \mathbf{k}_2')$. The first term under the integral comes from the Jacobian of this change of variables. The matrix element $u_{s_1'}\gamma_\mu u_{s_1}$ expresses the fact that the quark 1 is the active heavy quark. The relation between, e.g., k_2 and p_2 is given by the boost $k_2^0 = v^0 p_2^0 - v^z p_2^z$, $k_2^z = v^0 p_2^z - v^z p_2^0$, $k_2^{x,y} = p_2^{x,y}$, v being the four-velocity of the initial state. The wave functions φ and φ' are the initial and final internal wave functions at rest, dependent only on the relative momenta and Pauli spinors. Finally, the matrix $D(R_2'^{-1}R_2)$ is the Wigner rotation acting on the spin of the spectator quark 2 due to the product of the boosts on the initial and final states. Formula (35) leads to the difference (33) and to the r.h.s. of Uraltsev SR (17). Expanding the fourth component vector current matrix element between the ground state and $L = 1$ states up to the first power of $\mathbf{v}$, $\mathbf{v}'$ gives, from (35) (formula (29) of Ref. [9]):

$$\begin{aligned}\langle n(\mathbf{v}')|V_0(0)|0(\mathbf{v})\rangle &\cong \frac{1}{2}(\mathbf{v}'-\mathbf{v}) \\ &\quad \times \langle n| - i\left(p_2^0\mathbf{r}_2 + \mathbf{r}_2 p_2^0\right) + \frac{i(\sigma_2\times\mathbf{p}_2)}{p_2^0+m}|0\rangle,\end{aligned} \tag{36}$$

where $|0(\mathbf{v})\rangle$ stands for the ground state wave function in motion and likewise $|0)$ for the internal ground state at rest in terms of Pauli spinors. The first operator $-i(p_2^0\mathbf{r}_2+\mathbf{r}_2p_2^0)$, where $\mathbf{r}_2$ is the operator $i\,\partial/\partial\mathbf{p}_2$, comes from the variation of the Jacobian factor and the variation of the argument $\mathbf{k}$ of the wave function, while the second operator $i(\sigma_2\times\mathbf{p}_2)/(p_2^0+m)$ is the Wigner rotation. Eq. (36) becomes, in the nonrelativistic limit, the matrix element of the electric dipole operator, and leads to the difference (33) through the latter spin-dependent term. To demonstrate Uraltsev SR, we are interested in the hadronic tensor

$$h_{00}^{+10}(\mathbf{v}_f,\mathbf{v}',\mathbf{v}_i)=\sum_n\big\langle B^{*(+1)}(\mathbf{v}_f)\big|V_0(0)\big|n(\mathbf{v}')\big\rangle \times\big\langle n(\mathbf{v}')\big|V_0(0)\big|B^{*(0)}(\mathbf{v}_i)\big\rangle. \tag{37}$$

The ground state does not contribute to the sum rule over intermediate states in (37), in HQET and likewise in BT quark models, that satisfy HQET. We have indeed demonstrated in Ref. [6] (formulas (26)–(29)) that BT quark models in the heavy quark limit satisfy HQET relations for all ground state form factors. More specifically, in BT quark models, as follows after some algebra from (35), the contributions of the active quark (28) cancels with the one of the spectator quark for the ground state. We are then left with the $L=1$ intermediate states for which we apply formula (36).

Defining the frame $v_i=(1,0,0,0)$, $v_f=(v_f^0,0,0,v_f^z)$, the hadronic tensor can then be written, at first order in the velocities $\mathbf{v}_f$ and $\mathbf{v}'$,

$$\begin{aligned}
&h_{00}^{+10}(\mathbf{v}_f,\mathbf{v}',\mathbf{v}_i)\\
&\quad\cong\frac{1}{4}\big(B^{*(+1)}\big|\Bigg\{-v_f^z\Bigg[-i\big(p_2^0z_2+z_2p_2^0\big)+\frac{i(\sigma_2\times\mathbf{p}_2)_z}{p_2^0+m}\Bigg]\Bigg\}^+|n)\\
&\quad\times(n|\Bigg\{v'^x\Bigg[-i\big(p_2^0x_2+x_2p_2^0\big)+\frac{i(\sigma_2\times\mathbf{p}_2)_x}{p_2^0+m}\Bigg]+v'^y\Bigg[-i\big(p_2^0y_2+y_2p_2^0\big)+\frac{i(\sigma_2\times\mathbf{p}_2)_y}{p_2^0+m}\Bigg]\Bigg\}\\
&\quad\times\big|B^{*(0)}\big),
\end{aligned}\tag{38}$$

where the $|n)$ states are $L=1$. The spin flip $B^{*(0)}\to B^{*(+1)}$ can occur because of the Wigner rotation on the spectator light quark. Using completeness $\sum_n|n)(n|=1$, two kinds of terms contribute: crossed terms between a Wigner rotation and a spin-independent operator, and products of two Wigner rotations. After some algebra, the final result reads:

$$h_{00}^{+10}(\mathbf{v}_f,\mathbf{v}',\mathbf{v}_i)\cong\frac{1}{4}v_f^z\frac{1}{\sqrt{2}}\big(v'^x-iv'^y\big). \tag{39}$$

Making explicit the states $|n)$, Eq. (38) shows that the $L=1$ states contribute to the left-hand side of Uraltsev sum rule (Eq. (32)), since the operators in brackets are $\Delta L=1$.

It may seem surprising that only a spectator quark operator appears in Eq. (38), giving the same result as the previous calculation (28), where only the active quark appeared. This is due to the fact that the right-hand side of Eq. (28) or (39) comes out from a combination of three terms: $S_1+S_2+P_2$, where S (P) means the S-wave (P-wave) contribution and 1 (2) the active (spectator) quark. It turns out that $S_1=-S_2=P_2$, showing that one gets the same r.h.s. of the SR within both formalisms. The first demonstration underlines duality, since the hadronic tensor is identical to the active quark tensor. The second demonstration underlines the physical interpretation of the SR through the Wigner rotations, since the crossed terms $\Delta L=1$, $\Delta S=1$ in (38) provide the l.h.s. of the SR, giving the difference between $j=3/2$ and $j=1/2$.

4. Phenomenological remarks

From the calculations of Ref. [11] in the BT formalism for a wide class of potentials, one can see from Table 1 that Uraltsev SR converges rapidly, as well as Bjorken's one, and are almost saturated by the $n=0$ states. [3]

The Godfrey and Isgur potential [15] is the one that describes the meson spectrum in the most complete way, from light meson spectroscopy to heavy quarkonia. The agreement of the contribution of lowest $n=0$ states with the right-hand side of the SR (17) is quite striking. Within the BT class of quark models, one gets [11] a value $\rho^2\cong1$, not inconsistent with present experimental data on the $\xi(w)$ slope, and also, consistently, with small values for $\tau_{1/2}^{(n)}(1)$.

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[3] This fast convergence of the sum rules has also been observed in QCD_2 in the $N_c\to\infty$ limit [19].

Table 1
Contribution of the lowest $L = 1$ states to the Bjorken and Uraltsev sum rules and the slope of elastic IW function in BT quark models for different potentials

Quark–antiquark potential	Godfrey, Isgur [15] $(Q\overline{Q}, Q\bar{q}, q\bar{q})$	Cea, Colangelo, Cosmai, Nardulli [16]	Isgur, Scora, Grinstein, Wise [17]
$\lvert\tau_{1/2}^{(0)}(1)\rvert^2$	0.051	0.004	0.117
$\lvert\tau_{3/2}^{(0)}(1)\rvert^2$	0.291	0.265	0.305
$\frac{1}{4} + \lvert\tau_{1/2}^{(0)}(1)\rvert^2 + 2\lvert\tau_{3/2}^{(0)}(1)\rvert^2$	0.882	0.790	1.068
ρ^2	1.023	0.98	1.283
$\lvert\tau_{3/2}^{(0)}(1)\rvert^2 - \lvert\tau_{1/2}^{(0)}(1)\rvert^2$	0.240	0.261	0.233

Table 2
Branching ratios in BT quark models for different potentials. The experimental BR for $B \to D_2(3/2)\ell\nu$ and $B \to D_1(3/2)\ell\nu$ come from ALEPH (a), DELPHI (b) and CLEO (c) data [18], with the errors added in quadrature. The last entry corresponds to DELPHI data for the wide states

Quark–antiquark potential	Godfrey–Isgur	Cea et al.	Isgur et al.	Experimental
$B \to D\ell\nu$	2.36%	2.45%	1.94%	(2.1 ± 0.2)%
$B \to D^*\ell\nu$	6.86 %	7.02%	6.07%	(5.3 ± 0.8)%
$B \to D_2(\frac{3}{2})\ell\nu$	7.0×10^{-3}	6.5×10^{-3}	7.7×10^{-3}	(a) $(2.4 \pm 1.1) \times 10^{-3}$
				(b) $(4.4 \pm 2.4) \times 10^{-3}$
				(c) $(3.0 \pm 3.4) \times 10^{-3}$
$B \to D_1(\frac{3}{2})\ell\nu$	4.5×10^{-3}	4.2×10^{-3}	4.9×10^{-3}	(a) $(7.0 \pm 1.6) \times 10^{-3}$
				(b) $(6.7 \pm 2.1) \times 10^{-3}$
				(c) $(5.6 \pm 1.6) \times 10^{-3}$
$B \to D_1(\frac{1}{2})\ell\nu$	7×10^{-4}	4×10^{-5}	1.3×10^{-3}	$(2.3 \pm 0.7) \times 10^{-2}$
$B \to D_0(\frac{1}{2})\ell\nu$	6×10^{-4}	4×10^{-5}	1.1×10^{-3}	$[D_0(\frac{1}{2}) + D_1(\frac{1}{2})]$

It is interesting to remark that, among the three potential models quoted in Table 1, only the more complete one by Godfrey and Isgur contains a LS coupling. There are indeed in this case LS splittings ($M_{3/2}^{(n)}$ different from $M_{1/2}^{(n)}$), and the wave functions are perturbed also by this piece of the interaction, giving a different behavior for the wave functions $\varphi_{3/2}^{(n)}(p)$ and $\varphi_{1/2}^{(n)}(p)$. The other models have neglected the LS splitting, although, due to the Wigner rotations, $\tau_{3/2}^{(n)}(w)$ is, of course, different from $\tau_{1/2}^{(n)}(w)$ even for these latter potentials. However, even in the case of the Godfrey–Isgur potential, the LS force is small.

In Table 2 we compare the predictions of the BT quark models for the different semileptonic decays. While the BR for the modes $B \to D_2\ell\nu$ and $B \to D_1(3/2)\ell\nu$ have the right order of magnitude, and are consistent with experiment within 1σ, the trend of the ratio $D_1(3/2)/D_2(3/2)$ is opposite to experiment. This moderate disagreement could be explained by $1/m_Q$ corrections [20]. However, in the case of the $j = 1/2$ the disagreement is very strong. QCD in the heavy quark limit predicts, according to Uraltsev SR, that the $j = 3/2$ states are dominant over the $j = 1/2$. This general trend could be hardly reversed by the small hard QCD corrections to Uraltsev [1] and

Bjorken [20] sum rules. As to the $1/m_Q$ corrections [14], their magnitude is poorly known, since the numerical estimate of Ref. [14], although the formalism is completely general, relies on a large number of dynamical hypotheses.

Another strong experimental indication of large branching ratios of a broad resonance $D_1(1/2)$ is the non-leptonic decay $B \to D_1^0(1/2)\pi$ which is found larger than the $B \to D_J(3/2)\pi$ [21]. Factorization is reasonable in such a mode and, consequently, once again, this experimental result seems to contradict that $|\tau_{3/2}(1)| > |\tau_{1/2}(1)|$.

The serious problem for the decays $B \to D_{0,1}(1/2)\ell\nu$ goes beyond the specific BT quark models and appears to be, more generally, a problem between experiment and the heavy quark limit of QCD.

5. Conclusion

We have shown that the sum rule proved recently by Uraltsev in the heavy quark limit of QCD holds in relativistic quark models à la Bakamjian and Thomas. Its physical interpretation is the Wigner rotation of the spectator light quark spin, and not a possible LS perturbation. We have underlined that, since $|\tau_{3/2}(1)| > |\tau_{1/2}(1)|$ [22], there is a serious problem between theory and experiment for the decays $B \to D^*_{0,1}(\text{broad})\ell\nu$. This problem goes beyond the BT quark models and appears to be a general one, within the heavy quark limit of QCD.

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On P-wave meson decay constants in the heavy quark limit of QCD

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Abstract

In previous work it has been shown that, either from a sum rule for the subleading Isgur–Wise function $\xi_3(1)$ or from a combination of Uraltsev and Bjorken SR, one infers for P-wave states $|\tau_{1/2}(1)| \ll |\tau_{3/2}(1)|$. This implies, in the heavy quark limit of QCD, a hierarchy for the *production* rates of P-states $\Gamma(\overline{B}_d \to D(1/2)\ell\nu) \ll \Gamma(\overline{B}_d \to D(3/2)\ell\nu)$ that seems at present to be contradicted by experiment. It was also shown that the decay constants of $j = 3/2$ P-states vanish in the heavy quark limit of QCD, $f_{3/2}^{(n)} = 0$. Assuming the *model* of factorization in the decays $\overline{B}_d \to \overline{D}_s^{**} D$, one expects the opposite hierarchy for the *emission* rates $\Gamma(\overline{B}_d \to \overline{D}_s(3/2)D) \ll \Gamma(\overline{B}_d \to \overline{D}_s(1/2)D)$, since $j = 1/2$ P-states are coupled to vacuum. Moreover, using Bjorken SR and previously discovered SR involving heavy–light meson decay constants and IW functions, one can prove that the sums $\sum_n (f^{(n)}/f^{(0)})^2$, $\sum_n (f_{1/2}^{(n)}/f^{(0)})^2$ (where $f^{(n)}$ and $f_{1/2}^{(n)}$ are the decay constants of S-states and $j = 1/2$ P-states) are divergent. This situation seems to be realized in the relativistic quark models à la Bakamjian and Thomas, that satisfy HQET and predict decays constants $f^{(n)}$ and $f_{1/2}^{(n)}$ that do not decrease with the radial quantum number n. © 2001 Published by Elsevier Science B.V.

In previous work [1,2] we have pointed out a problem between experiment and the heavy quark limit of QCD for the semi-leptonic decays of B mesons to $L = 1$ excited states $D_{0,1}(1/2)$, $D_{1,2}(3/2)$. In a few words, the argument is as follows.

From Bjorken SR [3,4]

$$\rho^2 = \frac{1}{4} + \sum_n \left|\tau_{1/2}^{(n)}(1)\right|^2 + 2\sum_n \left|\tau_{3/2}^{(n)}(1)\right|^2 \tag{1}$$

and the recently demonstrated Uraltsev SR [5]

$$\sum_n \left|\tau_{3/2}^{(n)}(1)\right|^2 - \sum_n \left|\tau_{1/2}^{(n)}(1)\right|^2 = \frac{1}{4}, \tag{2}$$

one obtains,

$$\sum_n \left|\tau_{3/2}^{(n)}(1)\right|^2 = \frac{\rho^2}{3}, \tag{3}$$

$$\sum_n \left|\tau_{1/2}^{(n)}(1)\right|^2 = \frac{1}{3}\left(\rho^2 - \frac{3}{4}\right). \tag{4}$$

One can see that $\sum_n \left|\tau_{3/2}^{(n)}(1)\right|^2$ is proportional to ρ^2 and that $\sum_n \left|\tau_{1/2}^{(n)}(1)\right|^2$ is proportional to the *deviation* of ρ^2 from the lower bound $3/4$. Then, there is little

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room left for $\sum_n |\tau_{1/2}^{(n)}(1)|^2$, as it has been pointed out recently from a SR obtained for the subleading function $\xi_3(1)$ [3]. The expected hierarchy for the form factors $|\tau_{3/2}(1)| \gg \tau_{1/2}(1)$, that can be inferred from the precedent equations, implies that $\overline{B}_d \to D^{**}\ell\nu$ and $\overline{B}_d \to D^{**}\pi$ (assuming factorization, a reasonable assumption in view of the recent papers on nonleptonic decays with emission of a light meson) are dominated by the narrow resonances

$$\Gamma\left(\overline{B}_d \to D_{1,2}\left(\tfrac{3}{2}\right)\ell\nu\right) \gg \Gamma\left(\overline{B}_d \to D_{0,1}\left(\tfrac{1}{2}\right)\ell\nu\right), \quad (5)$$

$$\Gamma\left(\overline{B}_d \to D_{1,2}\left(\tfrac{3}{2}\right)\pi\right) \gg \Gamma\left(\overline{B}_d \to D_{0,1}\left(\tfrac{1}{2}\right)\pi\right). \quad (6)$$

On the other hand, it was demonstrated that the decay constants of $j = 3/2$ P-wave mesons vanish, $f_{3/2}^{(n)} = 0$, in the heavy quark limit of QCD [6,7]. This result can be summarized in the statement that we expect the "broad" D^{**} resonances ($j = 1/2$) to have a much larger decay constant than the "narrow" ones ($j = 3/2$). This to be contrasted to the opposite expected hierarchy for form factors, $|\tau_{3/2}| \gg |\tau_{1/2}|$ [1], that can be inferred from Eqs. (3), (4). This hierarchy in the *production* is expected to be opposite to the one due to the selection rule $f_{3/2}^{(n)} = 0$,

$$\Gamma\left(\overline{B}_d \to \overline{D}_{s\,1,2}\left(\tfrac{3}{2}\right)D\right) \ll \Gamma\left(\overline{B}_d \to \overline{D}_{s\,0,1}\left(\tfrac{1}{2}\right)D\right), \quad (7)$$

where there is *emission* of $\overline{D}_{sJ}(j)$. Of course, this is only a qualitative statement, because factorization in the decays (7) is just a model and is not in a firm status as in the light meson emission case (6) [8]. To summarize, "broad" resonances ($j = 1/2$) are expected to be suppressed in the production, while "narrow" resonances ($j = 3/2$) are expected to be suppressed in the emission. The BaBar experiment has started looking at $B_d \to (\overline{D}K)D$ [9] where such decays with emitted excited states might be seen, but the statistics has to be improved.

Let us now make some further remarks on decay constants of excited states. Imposing duality to $\Delta\Gamma$ in the B_s^0–$\overline{B}_s^0$ system in the heavy b, c quark limit, the following sum rules have been demonstrated, in the heavy quark limit of QCD [6]:

$$\sum_n \frac{f^{(n)}}{f^{(0)}} \xi^{(n)}(w) = 1, \quad (8)$$

$$\sum_n \frac{f_{1/2}^{(n)}}{f^{(0)}} \tau_{1/2}^{(n)}(w) = \frac{1}{2}. \quad (9)$$

The decay constants for S-states $f^{(n)}$ and for $j = 1/2$ P-states $f_{1/2}^{(n)}$ are properly defined in Ref. [6]. The sum rules (8), (9) are strong constraints, as they hold for any value of w.

The w-dependence in Eqs. (8), (9) was obtained by considering the two-body intermediate states of the type $D_s\overline{D}_s$ (ground state and excited states) for the diagrams of the spectator quark type. As explained in Ref. [6], in this kind of diagrams the transition $\overline{B}_s \to D_s\overline{D}_s \to B_s$ occurs by $\overline{D}_s$ emission and D_s production by the $\overline{B}_s$ ($\bar{s}$ quark being spectator), followed by D_s emission and $\overline{D}_s$ production by the B_s. In the heavy quark limit, the expression for the contribution of one intermediate state is proportional to a quantity of the type $f_{D_s}^2[\xi(w_c)]^2$ where f_{D_s} is a generic ground state or excited D_s meson decay constant and $\xi(w_c)$ is a generic Isgur–Wise function $\xi^{(n)}(w)$, $\tau_{1/2}^{(n)}(w)$ or $\tau_{3/2}^{(n)}(w)$ [4], taken at the fixed value of $q^2 = m_c^2$:

$$w_c = \frac{m_b^2 + m_c^2 - q^2}{2m_b m_c} = \frac{m_b}{2m_c}. \quad (10)$$

Varying the ratio m_b/m_c, the w-dependence in the IW functions appears. Identifying the sum over the exclusive modes with the contribution to the quark box diagram having the same topology [10], one obtains, considering the vector or the axial weak current, the sum rules (8) and (9).

In an earlier paper [11], we had demonstrated that indeed duality for $\Delta\Gamma_{B_s}$ occurs for $N_c = 3$ in the heavy quark limit in the particular Shifman–Voloshin conditions $\Lambda_{\mathrm{QCD}} \ll m_b - m_c \ll m_b, m_c$ [12].

On the other hand, the sum rules (8), (9) were studied [13] within the relativistic quark models of the Bakamjian–Thomas (BT) type [14], that satisfy HQET relations for form factors and decay constants. The sum rules are satisfied for different values of w, although the numerical convergence becomes worst as w increases.

From (8), (9), using Schwartz inequality, one can obtain the lower bounds:

$$\left[\sum_n [\xi^{(n)}(w)]^2\right]\left[\sum_n \left(\frac{f^{(n)}}{f^{(0)}}\right)^2\right] \geqslant \left(\sum_n \frac{f^{(n)}}{f^{(0)}}\xi^{(n)}(w)\right)^2 = 1, \quad (11)$$

$$\left[\sum_n \left[\tau_{1/2}^{(n)}(w)\right]^2\right]\left[\sum_n \left(\frac{f_{1/2}^{(n)}}{f^{(0)}}\right)^2\right] \geqslant \left(\sum_n \frac{f_{1/2}^{(n)}}{f^{(0)}}\tau_{1/2}^{(n)}(w)\right)^2 = \frac{1}{4}. \quad (12)$$

Considering first these inequalities at $w = 1$, one can see, from $\xi^{(n)}(1) = 0$ for $n \neq 0$, that (11) does not provide any useful constraint. However, from (12) at $w = 1$ and (4) one obtains the bound for the $j = 1/2$ P-wave decay constants

$$\sum_n \left(\frac{f_{1/2}^{(n)}}{f^{(0)}}\right)^2 \geqslant \frac{3}{4\rho^2 - 3} \quad (13)$$

that contains the IW slope ρ^2 in the right-hand side. Although the bound (13) is very weak, it deserves a few comments. Its r.h.s., and also its l.h.s. diverge as $\rho^2 \to 3/4$ and it reflects the fact that the excited P-wave mesons $D_{0,1}(j = 1/2)$ do indeed couple to vacuum, respectively, through the vector and axial currents for $J = 0, 1$, as was already proved from the sum rule (9). This is to be contrasted with the selection rule $f_{3/2}^{(n)} = 0$ [6,7] that applies to the excited mesons $D_{1,2}(j = 3/2)$. In the example of the nonrelativistic quark model, also given as an illustration in Ref. [6], both sides of the inequality (13) are of $O(v^2/c^2)$, since $f_{1/2}^{(n)}$ is of $O(v/c)$ (formula (17) of [6]) and ρ^2 is of $O(c^2/v^2)$ (formula (52) of [6]). In this case, as can be easily seen using completeness, the l.h.s. of (13) is infinite, proportional to the divergent integral $\int_0^\infty p^4\,dp$, two powers of p coming from the current and p^2 from the measure $d\vec{p}$.

Actually, one can demonstrate this divergence also in field theory, in the heavy quark limit of QCD by considering arbitrary large w. From Bjorken SR for any value of w [4],

$$\frac{w+1}{2}\sum_n \left|\xi^{(n)}(w)\right|^2 + (w-1)\left[2\sum_n \left|\tau_{1/2}^{(n)}(w)\right|^2 + (w+1)^2\sum_n \left|\tau_{3/2}^{(n)}(w)\right|^2\right] + \cdots = 1 \quad (14)$$

one obtains, since all terms in this sum are definite positive

$$\sum_n \left|\xi^{(n)}(w)\right|^2 \leqslant \frac{2}{w+1},$$

$$\sum_n \left|\tau_{1/2}^{(n)}(w)\right|^2 \leqslant \frac{1}{2(w-1)}. \quad (15)$$

From (11) and (12) for any w one gets

$$\sum_n \left(\frac{f^{(n)}}{f^{(0)}}\right)^2 \geqslant \frac{w+1}{2},$$

$$\sum_n \left(\frac{f_{1/2}^{(n)}}{f^{(0)}}\right)^2 \geqslant \frac{w-1}{2}. \quad (16)$$

Since the l.h.s. of these inequalities is independent of w, that can be made arbitrarily large, one concludes that the sums $\sum_n (f^{(n)}/f^{(0)})^2$, $\sum_n (f_{1/2}^{(n)}/f^{(0)})^2$ must diverge.

This situation seems to be realized in quark models à la Bakamjian and Thomas. The decay constants $f^{(n)}$ and $f_{1/2}^{(n)}$ were computed within the BT quark models for different quark–antiquark potentials up to $n = 10$, and the convergence of the SR (8) and (9) was studied for different values of w [13]. The error on the decay constants induced by a truncation procedure in the calculation increases strongly with n. The decay constants $f^{(n)}$ and $f_{1/2}^{(n)}$ do not decrease with increasing n. For the most sophisticated Godfrey–Isgur potential, that describes all $q\bar{q}$, $q\overline{Q}$ and $Q\overline{Q}$ quarkonia, including spin dependent effects [15], one obtains the decay constants of Table 1 for the S-states and the $j = 1/2$ P-states [13].

In view of the values of the decay constants in Table 1, besides the qualitative hierarchy (7), one expects for the $n = 0$ states, assuming factorization, neglecting Penguin diagrams and also the mass differences

Table 1
Decay constants $f^{(n)}$, $f_{1/2}^{(n)}$ for radial excitations S-states and $j = 1/2$ P-states in the GI spectroscopic model. The estimated error is in parentheses [13], and M is the bound state mass

Radial excitation	$\sqrt{M} f^{(n)}$ (GeV$^{3/2}$)	$\sqrt{M} f_{1/2}^{(n)}$ (GeV$^{3/2}$)
$n = 0$	0.67(2)	0.64(2)
$n = 1$	0.73(4)	0.66(4)
$n = 2$	0.76(5)	0.71(5)
$n = 3$	0.78(9)	0.73(8)
$n = 4$	0.80(10)	0.76(11)

between the P states and the ground state:

$$\frac{\Gamma\left(\overline{B}_d \to D_{s0}^-(\frac{1}{2})D^+\right)}{\Gamma(\overline{B}_d \to D_s^- D^+)} \cong \frac{\Gamma\left(\overline{B}_d \to D_{s0}^-(\frac{1}{2})D^{*+}\right)}{\Gamma(\overline{B}_d \to D_s^- D^{*+})} \cong 1,$$
$$\frac{\Gamma\left(\overline{B}_d \to D_{s1}^-(\frac{1}{2})D^+\right)}{\Gamma(\overline{B}_d \to D_s^{*-} D^+)} \cong \frac{\Gamma\left(\overline{B}_d \to D_{s1}^-(\frac{1}{2})D^{*+}\right)}{\Gamma(\overline{B}_d \to D_s^{*-} D^{*+})} \cong 1. \tag{17}$$

These relations follow from the approximate numerical equality $f^{(0)} \cong f_{1/2}^{(0)}$ in Table 1 and the heavy quark relations among, respectively, the decay constants of D, D^* and $D_0(1/2)$, $D_1(1/2)$ mesons [6].

In conclusion, we have underlined a disymmetry in the prediction of the rates of production and emission of P-wave heavy–light mesons, and we have undertaken a theoretical discussion of decay constants of excited heavy mesons.

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Chapitre 4

Calculs sur réseau

Où seront développés le projet apeNEXT, ordinateur à architecture parallèle dédié aux calculs de QCD sur réseau, ainsi qu'une étude préliminaire nécessaire pour la levée du paradoxe « $1/2 > 3/2$ » lors des désintégrations $B \to D^{**}\ell\nu$

Nous avons déjà évoqué dans le premier chapitre que tous les modèles théoriques phénoménologiques (modèles de quark, formulation covariante d'amplitudes façon Bakamjian-Thomas, règles de sommes, etc...) semblent converger vers le résultat suivant : dans l'étude des désintégrations semileptoniques des mésons B en mésons D^{**} (premières excitations radiales des D) et dans la limite de masse infinie, les taux d'embranchement des canaux $B \to D^{**}_{3/2}\ell\nu$ sont plus élevés que ceux des canaux $B \to D^{**}_{1/2}\ell\nu$; malheureusement, les données expérimentales ne confirment absolument pas cette propriété mais vont plutôt dans l'autre sens : c'est le paradoxe « $1/2 > 3/2$ ».

Afin d'essayer de comprendre voire de résoudre ce paradoxe, il est nécessaire de calculer directement à partir d'une théorie fondamentale (QCD) les facteurs de forme $\tau_{1/2}$ et $\tau_{3/2}$; une façon de faire serait de réaliser le calcul sur réseau (seule façon à l'heure actuelle de calculer complètement de façon non perturbative des quantités physiques).

Cela m'a conduit à m'intéresser à ce domaine que sont les calculs sur réseau, d'abord au travers du projet apeNEXT (paragraphe 4.1) et ensuite au travers de l'étude proprement dite du puzzle des états D^{**} (paragraphe 4.2).

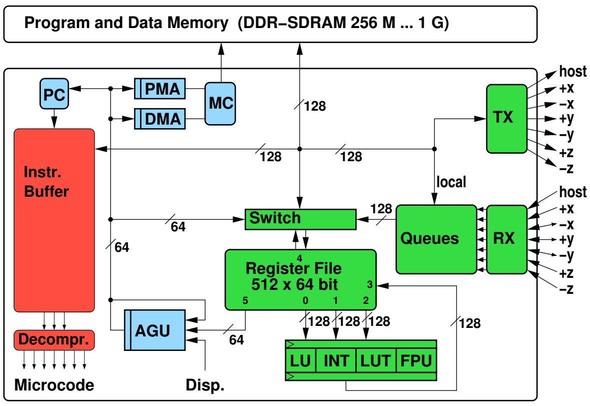
4.1 Projet apeNEXT

apeNEXT est la quatrième génération d'ordinateur du projet « Array Processor Experiment » (APE) : il s'agit d'une machine optimisée spécifiquement pour QCD et capable de développer une puissance de calcul soutenue de plusieurs teraflops (1 flop = 1 opération par seconde – float operation per second).

4.1.1 Description du hardware

apeNEXT est un ordinateur 64 bit à architecture parallèle de type SPMD[1], c'est-à-dire que chaque nœud de la machine possède sa propre mémoire, ses propres unités de calcul et de gestion des flux de données, etc.., et exécute le même programme que les autres nœuds mais en manipulant des données différentes[2] et cela de façon asynchrone : la synchronisation est nécessaire uniquement lorsque des échanges de données entre nœuds doivent avoir lieu[3]. Ensuite, toujours dans un but d'optimisation, les opérations arithmétiques ainsi que la communication entre nœuds peuvent avoir lieu simultanément au cours d'un même cycle d'horloge. De plus, un système de queues permet le stockage de donnée (locales au nœud ou bien distantes) et leur accès avec un temps de latence nul ce qui permet également d'optimiser les accès mémoire avec les calculs et les communications inter-nœud.

Un module de base apeNEXT est constitue du processeur, nommé $J\&T$, et de sa banque mémoire privée. Rapidement, le processeur $J\&T$ se compose des éléments suivants :



- module arithmétique : réalise tous les calculs (FPU : traite les opérations sur les nombres flottants – double précision, 64 bits –, INT : même chose pour les entiers, LU : réalise les manipulations logiques et LUT : fournit des fonctions comme les racines carrées, les exponentielles, etc...) ; les nombres sont codés sur 128 bits (2 fois 64 bits).
- module registres (register file) : fournit les opérandes pour le module arithmétique et stocke les résultats des calculs.
- module AGU : génère les adresses pour les accès mémoire.

1. Single Program Multiple Data
2. Le programme et les données sont stockées dans les banques mémoires privées de chaque nœud.
3. À ce sujet, le temps de latence associé à ce type d'échange est deux à trois fois plus court que le temps de latence associé à un accès à la mémoire locale si bien que ce qui limite en pratique le taux d'échange de données entre nœuds est la bande passante.

- module accès mémoire : la mémoire contient à la fois les données ainsi que les instructions du programme ; ces dernières sont comprimées et la décompression est réalisée en temps réel par le hardware et un buffer dédié permet un pré-chargement du code comprimé (dans le but d'éviter les conflits entre les données manipulées et le programme lui-même).
- interface réseau : cette interface contient sept liens (pour la transmission – TX – et la réception – RX –) dont six sont utilisés pour la connection avec les nœuds adjacents et un pour les opérations d'entrée/sortie. La bande passante théorique est de 200 MByte par seconde et par lien ($\leq$ 180 en pratique) et le temps de latence du réseau est de l'ordre de 0.1 μs.
- gestion des queues : les queues contiennent les données en provenance de la mémoire ou du module registres jusqu'à ce que le réseau puisse les envoyer aux autres nœuds (TX) car les accès mémoires sont beaucoup plus rapides que les accès réseau ; elles permettent aussi le stockage des données en provenance du réseau (RX) jusqu'à ce que le processeur en ait besoin ; enfin elles garantissent que les paquets de données sont envoyés dans le bon ordre.

La table suivante résume les principales caractéristiques du module *J&T* :

Fréquence d'horloge	200 MHz
Performance de crête	1.6 GFlops
Mémoire	256 - 1024 MByte/nœud
Bande passante mémoire	3.2 GByte/seconde
Bande passante réseau	0.2 GByte/seconde/lien
Module registres	512 registres
Buffer d'instructions	4096 mots

Une carte apeNEXT contient 16 modules *J&T* arrangés topologiquement selon la structure tridimensionnelle $4 \times 2 \times 2$[4]. Un bloc apeNEXT de base héberge un ensemble de 16 cartes apeNEXT (des systèmes plus gros peuvent être réalisés en assemblant plusieurs de ces blocs avec des câbles). Enfin, un (ou plusieurs) PC joue le rôle d'interface entre l'utilisateur et une tour apeNEXT.

L'opération aritmétique de base exécutée par cette machine est l'opération, dite *normale*, $a \times b + c$ où a, b et c sont des nombres complexes 128 bits. Une opération normale peut être réalisée à chaque coup d'horloge ; de fait, la performance de crête[5] est de 1.6 GFlops par nœud (voir la table ci-dessus).

4. 4 nœuds dans la direction x et 2 dans chacune des directions y et z.

5. On appelle « performance de crête » la situation idéale où une opération normale est réalisée à chaque coup d'horlage.

En pratique, la puissance de calcul attendue pour un système apeNEXT est de l'ordre de 8 à 20 TFlops.

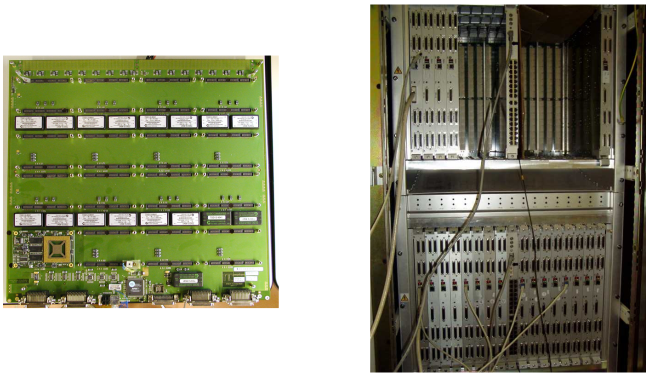
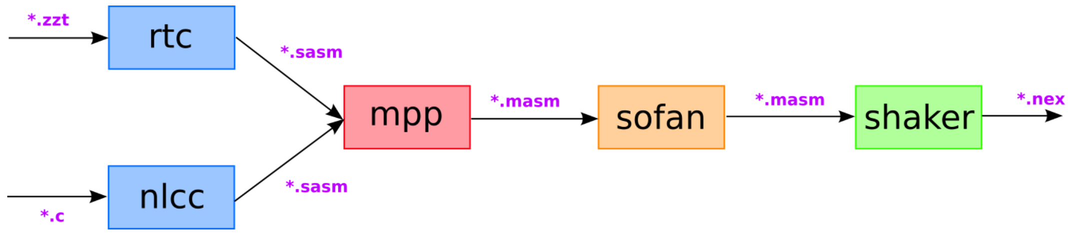


FIG. 4.1 – *La photographie de gauche représente une carte apeNEXT munie d'un module J&T et celle de droite illustre un bloc apeNEXT de base avec ses 16 cartes dans la partie basse.*

4.1.2 Software

Deux langages de haut niveau sont disponibles : le TAO, qui est le langage historique des ordinateurs de la série APE, et le C muni d'extensions spécifiques à la parallélisation et à apeNEXT. La chaîne de compilation depuis le langage de haut niveau jusqu'au microcode est résumée par le graphe suivant :



rtc est le compilateur TAO et *nlcc* le compilateur *C* modifié qui génèrent à partir du programme écrit en langage compréhensible par les humains une série d'instructions en langage assembleur encore de haut niveau, c'est-à-dire utilisant des mnémoniques pas encore fondamentales. Puis ce fichier *.sasm est traité par un préprocesseur (*mpp*) qui traduit le code assembleur de haut niveau en code assembleur de bas niveau (fichier *.masm). Ensuite, ce nouveau fichier passe dans un premier optimiseur (*sofan*) spécifique à l'architecture apeNEXT qui améliore le code machine en retirant par exemple les codes « morts » superflus, ou bien en réordonnant certaines instructions relatives aux calculs arithmétiques ou à la gestion des adresses. Enfin, le *shaker* s'occupe du fichier *.masm créé par sofan en remettant en ordre, comme son nom l'indique, les instructions restantes comme la construction des boucles, la résolution des « labels », l'allocation des registres.... Le résultat est un fichier microcode « digérable » par l'apeNEXT ou bien par un simulateur fonctionnel de apeNEXT[6].

4.1.3 Contribution personnelle

Ce projet apeNEXT est une collaboration italienne, allemande et française. L'équipe française est associée à la partie software. Pour ma part, j'ai d'abord participé aux tests de la modélisation VHDL du processeur apeNEXT (et en particulier ce qui concerne les accès mémoire) en écrivant des programmes en assembleur de bas niveau. J'ai également été impliqué dans les débuts de l'optimiseur sofan développé à l'INRIA de Rennes et, depuis l'existence physique de machines apeNEXT, aux tests de la chaîne de compilation mentionnée au paragraphe précédent.

4.2 Facteurs de forme $\tau_{1/2}$ et $\tau_{3/2}$

Comme annoncé précédemment, il est indispensable d'obtenir de façon non perturbative les facteurs de forme qui interviennent dans la description des réactions $B \to D^{**}\ell\nu$ pour comprendre le paradoxe des états P ; aussi, nous avons proposé une méthode [35] pour calculer sur le réseau les facteurs de forme $\tau^{(0)}_{1/2}(1)$ et $\tau^{(0)}_{3/2}(1)$ (pas d'excitation radiale).

4.2.1 Principe de la méthode

Nous nous sommes placés dans l'hypothèse où le quark lourd des mésons B et D^{**} a une masse infinie ; sur le réseau, cela signifie que le quark est statique. Il nous faut donc trouver l'élément de matrice à partir duquel nous pourrons extraire $\tau_{1/2}$ et $\tau_{3/2}$. Malheureusement, dans la limite où le quark lourd est au repos, les éléments de matrice entre B et D^{**} d'un courant électrofaible est nul sur le réseau ; pour s'en sortir, nous avons considéré les éléments de matrice d'opérateurs suivants, qui font intervenir la dérivée covariante D_μ (d'après [36]):

$$\langle D_0^*(v)|\bar{h}(v)\,\gamma_i\gamma_5\,D_j\,h(v)|B(v)\rangle = i\,g_{ij}\left(M_{D_0^*} - M_B\right)\tau_{1/2}(1)$$
$$\langle D_2^*(v)|\bar{h}(v)\,\gamma_i\gamma_5\,D_j\,h(v)|B(v)\rangle = -i\sqrt{3}\left(M_{D_2^*} - M_B\right)\tau_{3/2}(1)\epsilon^*_{ij}$$

(ϵ^*_{ij} est le tenseur de polarisation et $h(v)$ désigne le champ de quark lourd.)

L'extraction des facteurs de forme à partir de ces relations nécessite le calcul des éléments de matrice mais aussi des termes $M_{D^{**}} - M_B$:

- dérivée covariante : nous avons pris une forme discrétisée symétrisée habituelle de la dérivée covariante

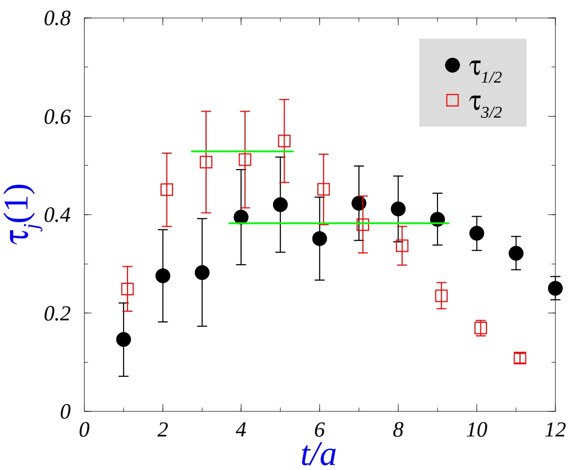
6. Il s'agit d'une émulation sur PC de l'architecture apeNEXT, la vitesse en moins.

- champs interpolants : pour décrire les états (notation J^P) 0^-, 0^+ et 2^+, les champs interpolants utilisés sont obtenus à partir des représentations irréductibles adéquates du groupe de symétrie du réseau[7] (la base de l'espace des représentations utilisée est celle formée par les liens reliant deux nœuds consécutifs du réseau – mais d'autres bases peuvent être également utilisées) ; lorsque plusieurs solutions sont possibles, nous avons pris celle qui correspond à ce que donne le modèle de quarks (combinaison de moment cinétique de spin et moment cinétique orbital). Pour ce qui est du champ de quark $q(x)$ associé au quark léger, nous avons introduit une fonction de « smearing » utile pour séparer le mieux possible l'état fondamental 0^- des états excités $L = 1$ et pour améliorer les champs interpolants.
- quarks lourds : les propagateurs des quarks lourds sont représentés par des liens « hypés[8] »

Les différences de masse entre les états $L = 0$ et $L = 1$ ont été obtenues à partir des fonctions de Green à deux points pour ces états et les fonctions de Green à trois points donnent alors accès aux facteurs de forme (voir [35] pour les aspects techniques).

4.2.2 Résultats

Les calculs ont été réalisés à Orsay sur une machine APEmille (la génération précedant apeNEXT), dans l'approximation "quenched", avec un volume du réseau de $16^3 \times 40$ et un coefficient $\beta = 6.0$. Le paramètre de "hopping" pour le calcul du propagateur du quark léger a été pris à $\kappa = 0.1334$. Les fonctions $\tau_{1/2}$ et $\tau_{3/2}$ au point $w = 1$ obtenues sont représentées ci-après :



7. Comme nous considérons des quarks lourds statiques, le groupe de symétrie considéré n'est plus le groupe hypercubique $H(4)$ complet mais le produit O_h du groupe O des rotations tri-dimensionnelles avec le groupe des réflexions spatiales (symétries du cube 3D).

8. L'action HQET utilisée pour décrire les quarks lourds fait intervenir des liens de jauge construits à partir d'un « blocking hypercubique » [37]. Le principe de la méthode est le suivant : le lien « hypé » $U_\mu^{\mathrm{HYP}}(x)$ est un mélange de liens de jauge appartenant aux hypercubes accrochés au lien initial considéré.

Les plateaux prédisent les valeurs suivantes :

$$\tau^{(0)}_{1/2}(1) = 0.38(4)(?) \qquad \tau^{(0)}_{3/2}(1) = 0.53(8)(?) \qquad {\tau^{(0)}_{3/2}}^2(1) - {\tau^{(0)}_{1/2}}^2(1) = 0.13(8)(?)$$

où les points d'interrogation remplacent les erreurs sytématiques (volume fini, taille de la maille, masse du quark léger,...) non calculées.

Rappelons les résultats obtenus par la construction façon Bakamjian-Thomas en utilisant le modèle dynamique le plus complet [19, 3] :

$$\tau^{(0)}_{1/2}(1) \in [0.1, 0.23] \qquad \tau^{(0)}_{3/2}(1) \in [0.43, 0.54]$$

Le résultat sur le réseau pour $\tau_{1/2}$ est plus grand et celui pour $\tau_{3/2}$ est compatible. D'un autre côté, la prédiction de $\tau_{1/2}$ est certainement plus fiable que celle de $\tau_{3/2}$: il suffit de regarder la taille des plateaux sur la figure précédente pour s'en convaincre : le signal pour le méson D_2^* est moins bon que celui du méson D_0^*. Néanmoins, il ne s'agit que d'une étude de faisabilité et beaucoup de chose pourraient être améliorées.

Enfin, remarquons encore que la règle de somme d'Uraltsev (3.2) est partiellement saturée par les termes fondamentaux calculés sur le réseau et que, à moins d'un comportement tout à fait anormal des excitations radiales, le puzzle des états P tient toujours.

4.3 Renormalisation de l'opérateur de dérivation

Au cours de l'étude précédente, nous n'avons pas vraiment discuté des problèmes de renormalisation des quantités introduites. En particulier, dans [35], nous avons calculé les éléments de matrice d'un opérateur de dimension 4 ($\bar{h}(v)\,\gamma_i\gamma_5\,D_j\,h(v)$) ; cet opérateur ne se mélange pas avec des opérateurs de dimension 3 pour des raisons de parité[9] : il n'y a donc pas de divergence en puissance. Il ne devrait également pas exister de divergences logarithmiques car les courants vectoriels et axiaux en HQET ne présentent pas de dimension anomale à $w = 1$. Mais il y a, a priori, des effets de renormalisation finie et c'est ce que nous avons calculé dans [38]. Parallèlement, nous avons également voulu savoir si l'utilisation de liens « hypés » pour le propagateur du quark lourd a une influence sur les prédictions faites.

Pour ce faire, nous devons renormaliser puis effectuer le matching de $\bar{h}^B(v)\,\gamma_i\gamma_5\,D_j\,h^B(v)$, écrit en terme du champ nu de quark lourd sur le réseau, dans le continu $a \to 0$ (nous avons calculé les corrections radiatives à une boucle et nous avons choisi le schéma de renormalisation MOM). La marche à suivre est, somme toute, classique :

1° après avoir rappelé l'action utilisée pour décrire le quark lourd sur le réseau dans la limite HQET, nous avons établi les règles de Feynman correspondantes. Le caractère

9. Les éléments de matrice d'un opérateur de dimension 3 entre deux états de parités opposées, ce qui est le cas ici, sont nuls.

« hypé » ou non apparaît dans les valeurs des paramètres qui interviennent dans ces règles.

2° nous avons calculé la self-energie du quark lourd pour accéder au propagateur.

3° finalement, avec les résultats précédemment établis, nous avons renormalisé l'opérateur de dérivation (notons que ce dernier met en jeu des liens non « hypés »).

Numériquement, nous avons montré que les corrections radiatives induisent une augmentation des quantités renormalisées lorsque les quarks lourds sont décrits par des liens « hypés » alors que, en l'absence de cette hypothèse, les corrections radiatives diminuent les quantités renormalisées. Une nouvelle estimation des facteurs de forme des états P nous donne alors :

$$\tau_{1/2}^{(0)}(1) = 0.41(5)(?) \qquad \tau_{3/2}^{(0)}(1) = 0.57(10)(?) \qquad {\tau_{3/2}^{(0)}}^2(1) - {\tau_{1/2}^{(0)}}^2(1) = 0.15(10)(?)$$

La saturation de la règle d'Uraltsev est meilleure mais l'augmentation des facteurs $\tau_{1/2}$ et $\tau_{3/2}$ rend les prédictions des modèles à la Bakamjian-Thomas plus discutables.

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Lattice measurement of the Isgur–Wise functions $\tau_{1/2}$ and $\tau_{3/2}$

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Abstract

We propose a method to compute the Isgur–Wise form factors $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ for the decay of B mesons into orbitally excited (P wave) D^{**} charmed mesons on the lattice in the static limit. We also present the result of a quenched exploratory numerical simulation which shows that the signal/noise ratio allows for a more dedicated computation.

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1. Introduction

The scalar heavy-light mesons and more generally the first orbital excitations D^{**} have attracted attention since years and they still remain somehow mysterious. The recent discovery of a $c\bar{s}$-scalar meson significantly lighter than expected has renewed the interest in these states [1,2]. There have been several lattice studies of this spectrum [3,4] and a recent rather complete one compares quenched and unquenched [5] computations. Recently the $H_0^* \to H\pi$ transition (scalar–pseudoscalar–pion) have also been considered [6,7].

The transitions of the type $B \to D^{**} l\nu$ raise a serious problem. In the infinite mass limit these decays are described by the Isgur–Wise form factors $\tau_{1/2}$ and $\tau_{3/2}$ [8]. To make a long story short, a series of sum rules [9–14] have been derived from QCD, all indicating that $\tau_{3/2}$ should be significantly larger than $\tau_{1/2}$. These sum rules relate the τ_j form factors, as well as form factors related to excitations, to derivatives of the ground state Isgur–Wise function ξ and allow to bound the latter derivatives in an efficient and useful way [15–17]. Not only does

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the slope of ξ verify $\rho^2 > 3/4$ but also the curvature and even higher derivatives are bound. The limit in which $\tau_{1/2} = 0$ has been baptised "BPS" by Uraltsev [18,19] and was proven to provide interesting hints.

However, the theoretical prediction that $\tau_{3/2}^{(0)} > \tau_{1/2}^{(0)}$ and hence that the decay $B \to D_2^*$ should be significantly larger than the $B \to D_0^*$ is not verified by experiment [2,20]. This is the '1/2 > 3/2' paradox [21]. One might incriminate the corrections to the infinite mass limit. Another possibility could be that the sum rules are fulfilled by higher excitations and that the ground state obeys an opposite hierarchy, i.e., $\tau_{3/2}^{(0)} < \tau_{1/2}^{(0)}$.[1]

To answer this question one needs to compute directly $\tau_{3/2}^{(0)}$ and $\tau_{1/2}^{(0)}$. Here we propose a lattice method to do that. We will work in the static quark limit, $m_{b,c} \to \infty$, with the four vectors $v' = v = (1, 0, 0, 0)$, and we will exhibit operators whose matrix elements allow to measure these form factors.

This Letter is meant to propose this new method and to make a feasibility study. We do not intend at this stage to provide accurate results for these form factors but merely to describe the principle of the method and to show with preliminary simulations that there is good hope to make the precision calculation.

2. Principle of the calculation

We are concerned with the matrix element of an electroweak current between a pseudoscalar or vector heavy-light meson $H^{(*)}$ and an orbitally excited one H^{**}. However, in the conditions of the infinite mass limit on the lattice with the heavy quarks at rest, both in the initial and final state ($v_\mu = v'_\mu$), this matrix element vanishes.

The way out is to use a series of relations derived in Ref. [22]. In that paper it has been shown that in the case of a matrix element which vanishes linearly with the difference $v' - v$, when $v' \to v$, there are non-vanishing forward matrix elements (for $v' = v$) involving the covariant derivative operator D_μ. These matrix elements are proportional to the infinite mass limit form factors $\tau_{1/2}(1)$ or $\tau_{3/2}(1)$.

Let us summarise their proof using different notations: for simplicity we take $v' = (1, 0, 0, 0)$, and $v = v' + v_\perp$, where $v_\perp$ is spatial up to higher orders in the difference $v' - v$. We assume that for some Dirac matrix Γ_l

$$\langle H^{**}(v') | \bar{h}(v') \Gamma_l h(v) | H^{(*)}(v) \rangle = t_l^m v_{\perp m} \tau_j(w) + \cdots, \tag{1}$$

where $w \equiv v \cdot v'$, $j = 1/2, 3/2$, and $l, m = 1, 2, 3$ are spatial indices, t^{lm} is a tensor which depends on the final state (H^{**}) and the initial state (H^* or H). The dots represent higher orders in $v' - v$. From translational invariance in the time direction,

$$\begin{aligned} &-i\partial_0 \langle H^{**}(v') | \bar{h}(v') \Gamma_l h(v) | H^{(*)}(v) \rangle \\ &\quad = -i \langle H^{**}(v') | \bar{h}(v') [\Gamma_l \overrightarrow{D}^0 + \overleftarrow{D}^0 \Gamma_l] h(v) | H^{(*)}(v) \rangle = t_l^m v_{\perp m} \tau_j(w)(M_{H^{**}} - M_H) + \cdots. \end{aligned} \tag{2}$$

The authors of Ref. [22] use the field equation: $(v \cdot D)h(v) = 0$, which implies that

$$D^0 h(v') = 0, \qquad D^0 h(v) = -(D \cdot v_\perp) h(v), \tag{3}$$

whence from (2)

$$i \langle H^{**}(v') | \bar{h}(v') \Gamma_l (D \cdot v_\perp) h(v) | H^{(*)}(v) \rangle = t_l^m v_{\perp m} \tau_j(w)(M_{H^{**}} - M_H) + \cdots, \tag{4}$$

which has a finite limit when $v_\perp \to 0$, namely

$$i \langle H^{**}(v) | \bar{h}(v) \Gamma_l D^m h(v) | H^{(*)}(v) \rangle = t_l^m \tau_j(1)(M_{H^{**}} - M_H). \tag{5}$$

[1] There is no mathematical impossibility for the sum rules to be fulfilled with a reversed hierarchy for the ground state, but it does not seem very likely and is not seen in models.

Applying Eq. (1) to the $J=0$ H_0^* state we get from Ref. [8]:

$$\langle H_0^*(v')|\bar h(v')\gamma_i\gamma_5 h(v)|H(v)\rangle \equiv -\tau_{1/2}(w)v_{\perp i}, \tag{6}$$

where our states normalisation is $1/\sqrt{2M}$ times the one used in Ref. [8]. From Eq. (6) it results that

$$\langle H_0^*(v)|\bar h(v)\gamma_i\gamma_5 D_j h(v)|H(v)\rangle = i g_{ij}(M_{H_0^*}-M_H)\tau_{1/2}(1). \tag{7}$$

Analogously for the $J=2 H_2^*$ state we have

$$\langle H_2^*(v')|\bar h(v')\gamma_i\gamma_5 h(v)|H(v)\rangle \equiv \sqrt{3}\tau_{3/2}(w)\epsilon_i^{*j}v_{\perp j}+\cdots, \tag{8}$$

where ϵ_{ij}^* is the polarisation tensor, whence

$$\langle H_2^*(v)|\bar h(v)\gamma_i\gamma_5 D_j h(v)|H(v)\rangle = -i\sqrt{3}(M_{H_2^*}-M_H)\tau_{3/2}(1)\epsilon_{ij}^*. \tag{9}$$

3. Lattice calculations

To compute the matrix elements in Eqs. (7) and (9) on the lattice we first need a discretized expression for the covariant derivative. We choose the symmetrised form

$$D_i h(\vec x,t) \to \frac{1}{2a}\big(U_i(\vec x,t)h(\vec x+\hat\imath,t) - U_i^\dagger(\vec x-\hat\imath,t)h(\vec x-\hat\imath,t)\big), \tag{10}$$

where $U_i(\vec x,t)$ is the link variable of the lattice.

3.1. Interpolating fields

The interpolating fields for orbitally excited states have been studied in Ref. [24]. Smearing is used not only to improve the signal/noise ratio by better isolating the ground state, but also to produce convenient interpolating fields for the 0^-, 0^+ and 2^+ states. Inspired by Ref. [25] we replace the quark fields $q(x)$ by

$$\begin{aligned} q(x) \to \sum_{r=0}^{R_{\max}}\left(r+\frac{1}{2}\right)^2\phi(r)\sum_{i=x,y,z}\Bigg\{&\left[\prod_{k=1}^{r}U_i^F\big(x+(k-1)\hat\imath\big)\right]q(x+r\hat\imath)\begin{matrix}(r\hat\imath)\,\delta_{il}\\ 1\end{matrix}\\ &+\left[\prod_{k=1}^{r}U_i^{F\dagger}(x-k\hat\imath)\right]q(x-r\hat\imath)\begin{matrix}(-r\hat\imath)\,\delta_{il}\\ 1\end{matrix}\Bigg\}, \end{aligned} \tag{11}$$

where the upper (lower) expressions generate negative (positive) parity smearing functions. The vector $(\pm r\hat\imath)\,\delta_{il}$ is introduced to generate an orbital excitation in the direction l. The wave function $\phi(r)$ is a radial function chosen to optimise the overlap with the ground state. We take $\phi(r)=e^{-r/R_b}$, where R_b is a parameter which is fixed by requiring the smearing to be optimal. Note that it is not necessary to normalise the wave function since the normalisation factors cancel in the computation of matrix elements. The smearing also includes the so-called fuzzing, see Ref. [23]. For convenience, we will use the following notation for the interpolating fields:

$$\left\{\bar h(x)W[x,x+\vec r\,]\begin{matrix}\gamma_k r^l(x)\\ \Gamma\end{matrix}q(x+\vec r\,)\right\}, \tag{12}$$

where $W[x,x+\vec r\,]$ represents the combination of Wilson lines expressed in Eq. (11) which links the location of the heavy quark field, x, to the light quark's one, $x+\vec r$. It ensures gauge invariance. $r^l(x)$ indicates the presence of $(\pm r\hat\imath)\,\delta_{il}$ in Eq. (11), $\vec r$ is a generic vector for the distance between the light and heavy quark field and $\Gamma=1$ ($\Gamma=\gamma_5$) for the 0^+ (0^-) meson.

Using the smeared quark fields of Eq. (11) we now define the interpolating fields. We concentrate on the 0^+ (2^+) states which correspond to $j = 1/2$ ($j = 3/2$). The 0^+-state can be described according to two distinct interpolating fields:

$$\begin{aligned}
&\text{(a)} \quad \bar{h}(x)W[x, x+\vec{r}\,]q(x+\vec{r}\,) \quad \text{and} \\
&\text{(b)} \quad \frac{1}{\sqrt{3}}\bar{h}(x)W[x, x+\vec{r}\,]\big(\vec{\gamma}\cdot\vec{r}(x)\big)q(x+\vec{r}\,).
\end{aligned} \tag{13}$$

These two interpolating fields differ in that the Dirac matrix is diagonal (antidiagonal) for 1 ($\vec{\gamma}\cdot\vec{r}\,$) inducing the coupling of the heavy quark to the "small" ("large") component of the light quark field. The latter is just the quark model combination of quark-spin 1 with orbital momentum 1 to generate $J = 0^+$.

Concerning the 2^+ states, the same duality of interpolating fields exists. In this Letter we only consider the quark-model type ones. This gives the five $J = 2$ states [24] which we may write as follows:

$$\begin{aligned}
&\text{(a)} \quad -\frac{1}{\sqrt{2}}\bar{h}(x)W[x, x+\vec{r}\,]\big[\gamma_i r_j(x) + \gamma_j r_i(x)\big]q(x+\vec{r}\,), \quad i \neq j, \\
&\text{(b)} \quad -\frac{1}{\sqrt{2}}\bar{h}(x)W[x, x+\vec{r}\,]\big[\gamma_1 r_1(x) - \gamma_2 r_2(x)\big]q(x+\vec{r}\,), \\
&\text{(c)} \quad \frac{1}{\sqrt{6}}\bar{h}(x)W[x, x+\vec{r}\,]\big[\gamma_1 r_1(x) + \gamma_2 r_2(x) - 2\gamma_3 r_3(x)\big]q(x+\vec{r}\,).
\end{aligned} \tag{14}$$

These are, as expected, symmetric traceless tensors.

3.2. *Two-point Green functions and* 1/2–3/2 *mass splitting*

The 0^+ two-point Green function is written as

$$C^{1}_{2;0} = \left\langle \sum_{\vec{x}} \mathrm{Tr}\left[P^{\vec{0}}_{0,t_x}\frac{1+\gamma_0}{2}S\big(\vec{r}(0), 0; \vec{x}+\vec{r}(x), t_x; U\big)\right]\right\rangle_U, \tag{15}$$

when we use the interpolating field in Eq. (13)(a). P^{0}_{0,t_x} is a temporal Wilson line[2] corresponding to the Eichten–Hill action for the static quark [26]:

$$P^{\vec{x}}_{t_x,t_y} = \delta(\vec{x}-\vec{y})\prod_{t_z=t_x}^{t_y-1} U^{\mathrm{hyp}}_t(x+t_z\hat{t}\,), \tag{16}$$

using hypercubic blocking [27–29].

The two-point Green functions with $\gamma_i r_j$ interpolating fields allow an interesting comparison between the $j = 1/2$ and the $j = 3/2$ cases. They will contain terms of the general form

$$C^{ijkl}_{2;J}(0, t_x) = \left\langle \sum_{\vec{x}} \mathrm{Tr}\left[\gamma_i r_j(0)P^{\vec{0}}_{0,t_x}\frac{1+\gamma_0}{2}\gamma_k r_l(x)S\big(\vec{r}(0), 0; \vec{x}+\vec{r}(x), t_x; U\big)\right]\right\rangle_U, \tag{17}$$

where $(i, j) = (k, l)$ for the case of interpolating field (14)(a) or $i = j, k = l$ for the case (14)(b), (c) and for the 0^+ case. J stands for the total angular momentum ($J = 0, 2$).

[2] Indeed we compute the two point function using the interpolating fields in Eqs. (13) and (14) properly shifted in space so as to have the light propagator ending at the origin.

After some simple algebra we can write

$$-C^{ijkl}_{2;J}(0,t_x)=\delta_{ij}\delta_{kl}\left\langle\sum_{\vec{x}}\mathrm{Tr}\,P^{\vec{0}}_{0,t_x}\left[\left(\delta_{jl}+i\epsilon^{jlm}\sigma_m\right)r_j(0)\frac{1-\gamma_0}{2}r_l(x)S\left(\vec{r}(0),0;\vec{x}+\vec{r}(x),t_x;U\right)\right]\right\rangle_U,\quad\text{or}$$

$$-C^{ijkl}_{2;J}(0,t_x)=\delta_{il}\delta_{jk}\left\langle\sum_{\vec{x}}\mathrm{Tr}\,P^{\vec{0}}_{0,t_x}\left[\left(\delta_{jl}-i\epsilon^{jlm}\sigma_m\right)r_j(0)\frac{1-\gamma_0}{2}r_l(x)S\left(\vec{r}(0),0;\vec{x}+\vec{r}(x),t_x;U\right)\right]\right\rangle_U.\qquad(18)$$

Let us denote by $C_{2,\delta(r_j(0),r_j(x))}$ and $C_{2,\epsilon^m(r_j(0),r_l(x))}$, respectively, the two terms in Eq. (18). It is easy to see, using the interpolating field in Eq. (13)(b) that the 0^+ two-point Green function writes as

$$\begin{aligned}-C^{\vec{\gamma}\cdot\vec{r}}_{2;0}&=\frac{1}{3}\sum_{j=1,3}C_{2,\delta(r_j(0),r_j(x))}+\frac{i}{3}\sum_{i,j,k}\left[C_{2,\epsilon^k(r_i(0),r_j(x))}+C_{2,\epsilon^k(r_j(0),r_i(x))}\right]\\&=C_{2,\delta(r_1(0),r_1(x))}+i\left[C_{2,\epsilon^3(r_1(0),r_2(x))}-C_{2,\epsilon^3(r_2(0),r_1(x))}\right],\end{aligned}\qquad(19)$$

where i,j,k are in cyclic order and where we have taken advantage of the hypercubic symmetry in the r.h.s.

Taking now any of the 2^+ meson interpolating fields and using again the cubic symmetry we get

$$-C_{2;2}=C_{2,\delta(r_1(0),r_1(x))}-\frac{i}{2}\left[C_{2,\epsilon^3(r_1(0),r_2(x))}-C_{2,\epsilon^3(r_2(0),r_1(x))}\right].\qquad(20)$$

The difference between the $j=1/2$ and the $j=3/2$ state is thus related to the relative sign and coefficient of the ϵ-term compared to the direct one. The effective energy is obtained by taking minus the time derivative of the logarithm of the two-point function. The energy difference between $j=1/2$ and $j=3/2$ is thus proportional to

$$-i\left\langle\left[r_1(0)\dot{r}_2(x)-r_2(0)\dot{r}_1(x)\right]\mathrm{Tr}\,P^{\vec{0}}_{0,t_x}\left[\sigma_3\frac{1-\gamma_0}{2}S\left(\vec{r}(0),0;\vec{x}+\vec{r}(x),t_x;U\right)\right]\right\rangle_U.\qquad(21)$$

In a non-relativistic limit $\dot{\vec{r}}=i\vec{p}/m$. This imaginary velocity comes from the derivation versus the imaginary time. Then the expression in Eq. (21) is reminiscent of a LS-term: $(\vec{r}\times\vec{p})\cdot\vec{\sigma}$ except that the operators $\vec{r}$ and $\vec{p}$ are not taken at the same time. σ in Eq. (21) acts on the heavy quark but the trace will make it also act on the light quark. It is interesting that the coefficients of the last terms in Eqs. (19) and (20) are in the ratio (1), $(-1/2)$ which is exactly the ratio of the LS-eigenvalues for $j=1/2,3/2$, built up from the combination of $L=1$ and $S=1/2$:

$$2\langle\vec{L}\cdot\vec{S}\rangle=j(j+1)-\frac{3}{4}-2=\begin{matrix}-2\\1\end{matrix},\quad\text{for } j=\begin{matrix}1/2\\3/2\end{matrix}.\qquad(22)$$

From Eqs. (19) and (20) it is obvious that if these LS-type terms did vanish the two-point correlators $C_{2;0}$ and $C_{2;2}$ would be equal, which would then imply $M_{H_2^*}=M_{H_0^*}$. In this limit the normalisation of the interpolating fields in Eqs. (13) and (14) has further ensured the equality of the multiplicative constants $\mathcal{Z}_{2;0}$ and $\mathcal{Z}_{2;2}$, where the $\mathcal{Z}_{2;J}$ are defined from

$$C_{2;J}(t_x)=(\mathcal{Z}_{2;J})^2e^{-M_{H_J^*}t_x},\qquad(23)$$

at large time t_x.

3.3. Three-point Green functions and $\tau_{1/2}$–$\tau_{3/2}$ splitting

The three-point Green functions of the axial current using interpolating fields with $\gamma_i r_j$ will contain terms of the general form

$$
\begin{aligned}
&C^{ijkl}_{3,JA_k5}(0,t_y,t_x) \\
&\quad = \frac{1}{2}\left\langle \sum_{\vec{x},\vec{y}} \mathrm{Tr}\left[\gamma_i r_j(0) P^{\vec{0}}_{0,t_y} \frac{1+\gamma_0}{2}\gamma_k\gamma_5\left\{U_l(0,t_y)P^{\hat{l}}_{t_y,t_x} S\left(\vec{r}(0),0;\vec{x}+\vec{r}(x)+\hat{l},t_x;U\right)\right.\right.\right. \\
&\qquad \left.\left.\left. - U^\dagger_l(-\hat{l},t_y)P^{-\hat{l}}_{t_y,t_x} S\left(\vec{r}(0),0;\vec{x}+\vec{r}(x)-\hat{l},t_x;U\right)\right\}\gamma_5\right]\right\rangle_U,
\end{aligned}
\tag{24}
$$

where we have used Eq. (10) in units of a. Writing for short the term in the curly bracket as $D_l(y)\cdots\gamma_5$, we find

$$
-C^{ijkl}_{3,JA_k5}(0,t_y,t_x) = \frac{1}{2}\left\langle \sum_{\vec{x},\vec{y}} \mathrm{Tr}\left[\left(\delta_{jl} \pm i\epsilon^{jlm}\sigma_m\right)\gamma_5 r_j(0) P^{\vec{0}}_{0,t_y}\frac{1+\gamma_0}{2} D_l(y)\cdots\gamma_5\right]\right\rangle_U, \tag{25}
$$

where either $(i,j)=(k,l)$ for the $0^- \to 2^+$ transition, Eq. (14) (a), or $i=j, k=l$ for the $0^- \to 0^+$ one and $0^- \to 2^+$ with Eqs. (14) (b) and (c). Let us define $C_{3,\delta(r_j(0),D_j(x))}$ and $C_{3,\epsilon^m(r_j(0),D_l(x))}$, respectively the two terms in Eq. (25). From the interpolating field in Eq. (13) (b), using the fact that $i=j$ and $k=l$, and choosing for simplicity $k=3$, one can derive,

$$
-C^{\vec{\gamma}\cdot\vec{r}}_{3;05} = \frac{1}{\sqrt{3}} C_{3,\delta(r_3(0),D_3(x))} + \frac{i}{\sqrt{3}}\left[C_{3,\epsilon^1(r_2(0),D_3(x))} + C_{3,\epsilon^2(r_1(0),D_3(x))}\right]. \tag{26}
$$

The axial matrix element is then given in Euclidean metric by

$$
\left\langle H_0^*\middle|\bar{h}(v)\gamma_3\gamma_5 D_3 h(v)\middle|H\right\rangle = \frac{\mathcal{Z}_{2;0}\mathcal{Z}_{2;5}C_{3;05}(0,t_y,t_x)}{C_{2;0}(0,t_y)\,C_{2;5}(0,t_x-t_y)} = (M_{H_0^*}-M_H)\tau_{1/2}(1), \tag{27}
$$

where we have used Eq. (7). This leads, using cubic symmetry, to

$$
(M_{H_0^*}-M_H)\sqrt{3}\tau_{1/2}(1) = \mathcal{Z}_{2;0}\mathcal{Z}_{2;5}\frac{C_{3,\delta(r_3(0),D_3(x))} + 2iC_{3,\epsilon^1(r_2(0),D_3(x))}}{C_{2;0}(0,t_y)C_{2;5}(0,t_x-t_y)}, \tag{28}
$$

plus all terms deduced by cubic symmetry.

From Eq. (9), in Euclidean metric, we get

$$
\frac{1}{2}\left\langle H_2^*\middle|\bar{h}(v)[\gamma_1\gamma_5 D_2 + \gamma_2\gamma_5 D_1]h(v)\middle|H\right\rangle = \frac{\mathcal{Z}_{2;2}\mathcal{Z}_{2;5}C_{3;25}(0,t_y,t_x)}{C_{2;2}(0,t_y)C_{2;5}(0,t_x-t_y)} = \sqrt{\frac{3}{2}}(M_{H_2^*}-M_H)\tau_{3/2}(1), \tag{29}
$$

where we have used the polarisation tensor

$$
\epsilon = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{30}
$$

for the 2^+ state in Eq. (14)(a) with $(i,j)=(1,2)$. From the interpolating field in Eq. (14)(a) using the fact that $(i,j)=(k,l)=(1,2)$ one can derive,

$$
-C_{3;25} = -\frac{1}{2\sqrt{2}}\sum_{l=1,2} C_{3,\delta(r_l(0),D_l(x))} + \frac{i}{2\sqrt{2}}\left[C_{3,\epsilon^3(r_1(0),D_2(x))} + C_{3,\epsilon^3(r_2(0),D_1(x))}\right], \tag{31}
$$

and using cubic symmetry and Eq. (29)

$$
(M_{H_2^*}-M_H)\sqrt{3}\tau_{3/2}(1) = \mathcal{Z}_{2;2}\mathcal{Z}_{2;5}\frac{C_{3,\delta(r_1(0),D_1(x))} - iC_{3,\epsilon^1(r_2(0),D_3(x))}}{C_{2;2}(0,t_y)C_{2;5}(0,t_x-t_y)}. \tag{32}
$$

It can be checked that all the other states in Eq. (17) lead to the same formula (32) up to a cubic rotation. The numerators in Eqs. (28) and (32) exhibit identical $C_{3,\delta}$ terms and $C_{3,\epsilon}$, the latter differing only by multiplicative

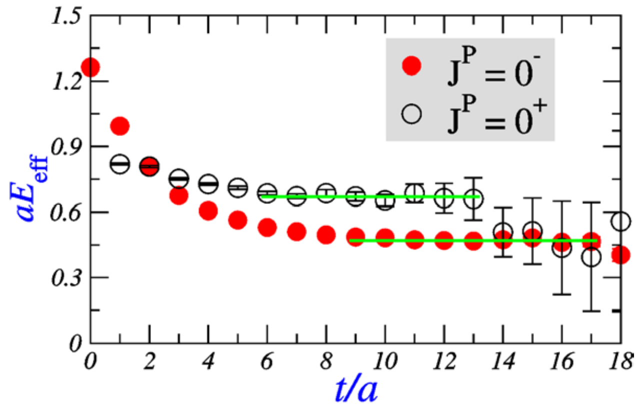

Fig. 1. Signals for the effective binding energies for the pseudoscalar and the scalar heavy-light mesons.

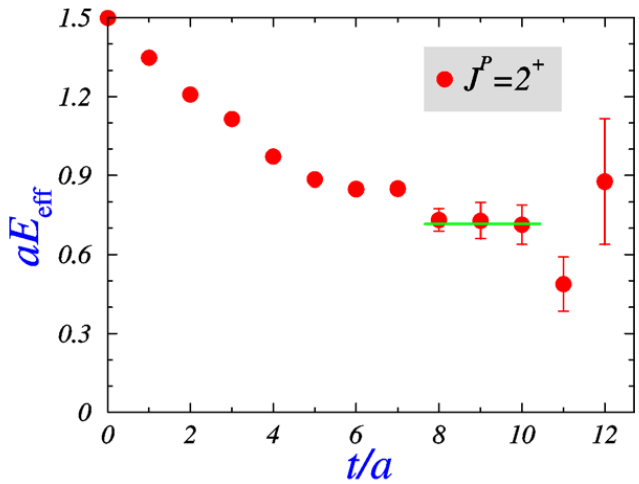

Fig. 2. Signal for the effective binding energies E_{eff} for the 2^+ heavy-light mesons.

coefficients which, once more, are proportional to the LS eigenvalues given in Eq. (22). Combining the results of Eqs. (19), (20), (28) and (32) we may conclude that, if the $C_{2,\epsilon}$ and $C_{3,\epsilon}$ terms did vanish, we would get $M_{H_2^*} = M_{H_0^*}$ and $\tau_{3/2} = \tau_{1/2}$.

4. Condition and results of the simulation

Results presented for Isgur–Wise functions $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ are obtained from the quenched simulation on a $16^3 \times 40$ lattice at $\beta = 6.0$. We collected 600 independent $SU(3)$ gauge configurations using the non-perturbatively $\mathcal{O}(a)$ improved Wilson fermion action with $C_{SW} = 1.769$. The light-quark propagator is computed with the hopping parameter $\kappa = 0.1334$, which corresponds to a pseudoscalar "light-meson" mass of 800 MeV. For the static quark we use the "hyp" links as written previously. In Fig. 1 we plot the binding energy for the scalar and the pseudoscalar meson and in Fig. 2 we plot the binding energy for the 2^+ heavy-light meson.

The scalar meson has been computed using the interpolating field[3] in Eq. (13)(a): $\bar{h}(x)W[x, x+\vec{r}\,]q(x+\vec{r}\,)$. The tensor meson has been computed using the properly averaged interpolating fields in Eq. (14). We get $\Delta \equiv m_{H_0^*} -$

[3] This choice shows a better signal than the one using Eq. (13)(b). A comparison of these signals has been performed in [7].

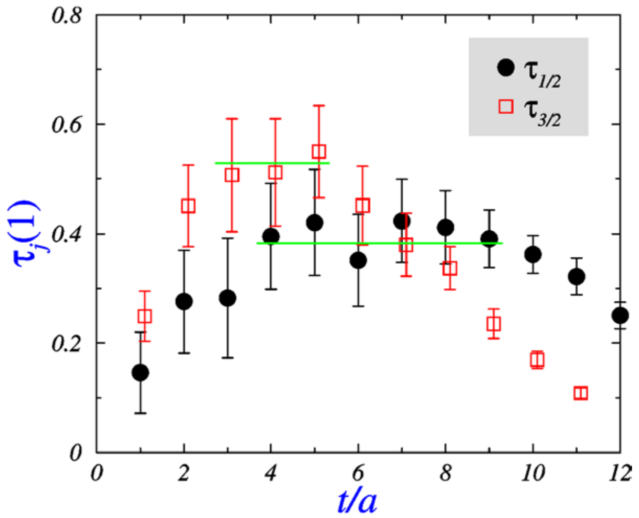

Fig. 3. Signals for the ratios defined in Eq. (33); from the fit in $t/a \in [4, 9]$ and $t/a \in [3, 5]$, respectively, we obtain the value of $\tau_{1/2}$ and $\tau_{3/2}$.

$m_H = 400(12) - 411(16)$ MeV at $\beta = 6.0$. Only statistical errors are considered. It agrees reasonably with Ref. [5] where $\Delta \sim 400(40)$ MeV. Our present signal for the tensor-meson effective mass is still very poor, $m_{H_2^*} - m_H = 0.50(8)$ GeV, which leads to $m_{H_2^*} - m_{H_0^*} = 0.10(8)$ GeV. The large relative error reflects the poor quality of the plateau in Fig. 2. Clearly a more refined simulation is needed here. In particular, we have not yet optimised the wave function for the smearing of the tensor meson. Our result agrees with the result of [5] where we read from Table 2 (Q_3): $m_{H_2^*} - m_H = 0.48(2)$ GeV, and $m_{H_2^*} - m_{H_0^*} = 0.08(4)$ GeV.

Experimentally the situation is not yet clear: whereas Belle [2] reported $m_{D_2^*} - m_{D_0^*} = 153(36)$ MeV,[4] FOCUS [30] finds $m_{D_2^*} - m_{D_0^*} = 61(41)$ MeV. Anyway large $1/m_c$ corrections are expected.

In Fig. 3, we plot the ratios

$$\tau_{1/2}(1) = \frac{1}{(M_{H_0^*} - M_H)} \frac{\mathcal{Z}_{2;0}\mathcal{Z}_{2;5}C_{3;05}(0, t_y, t_x)}{C_{2;0}(0, t_y)C_{2;5}(0, t_x - t_y)},$$

$$\tau_{3/2}(1) = \sqrt{\frac{2}{3}} \frac{1}{(M_{H_2^*} - M_H)} \frac{\mathcal{Z}_{2;2}\mathcal{Z}_{2;5}C_{3;25}(0, t_y, t_x)}{C_{2;2}(0, t_y)C_{2;5}(0, t_x - t_y)}, \tag{33}$$

where the source operator has been fixed at $t_x = 13a$. These equalities are valid on the plateaus.

5. Results, discussion and conclusions

In this Letter we propose a method to compute on the lattice, in the infinite mass limit, the zero recoil Isgur–Wise form factors $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ relevant to the decay of a heavy pseudoscalar meson into orbitally excited states. The main feature of the method is contained in Eqs. (7) and (9). It uses matrix elements of the axial current multiplied by covariant derivatives.

We have also performed an exploratory lattice study in order to estimate if this method is practically usable. We find that the signal/noise ratio is encouraging if one considers that there is still room for improvement.

---

[4] Note, however, that in Belle experiment the narrow and broad $J^P = 1^+$ resonances, usually interpreted as $j = 3/2$, $j = 1/2$, respectively, are practically degenerate in mass: $m_{D_1^0} = 2421(2)$ MeV and $m_{D_1'^0} = 2427(50)$.

Our results are

$$\tau_{1/2}(1) = 0.38(4)(?) \quad \text{and} \quad \tau_{3/2}(1) = 0.53(8)(?), \tag{34}$$

where the question marks represent yet unknown systematic errors. We have also:

$$\tau_{3/2}(1)^2 - \tau_{1/2}(1)^2 = 0.13(8)(?). \tag{35}$$

Within 1.5σ it saturates the Uraltsev sum rule [10]: $\sum_n |\tau_{3/2}^{(n)}(1)|^2 - |\tau_{1/2}^{(n)}(1)|^2 = 1/4$. Note that an approximate saturation of the sum rule by the ground states is seen in several models [31,33] although there is no strong theoretical reason for that.

The result for $\tau_{1/2}(1) = 0.38(4)(?)$ is presumably more reliable than the one on $\tau_{3/2}(1)$ since the two-point signal for the 0^+ meson is much better than the one for the 2^+ meson, see Figs. 1 and 2. Our result for $\tau_{1/2}(1)$ is somewhat larger than the predictions of the covariant quark models a la Bakamjian–Thomas (BT) [31] which predict, for the preferred potentials,[5] $\tau_{1/2}(1) \in [0.1, 0.23]$ and $\tau_{3/2}(1) \in [0.43, 0.54]$. The latter agrees well with Eq. (34). Both numbers of Eq. (34) are compatible with a recent calculation based on a covariant light-front approach with simple harmonic oscillator wave functions which are not derived from a potential:[6] $\tau_{1/2}(1) = 0.31$ and $\tau_{3/2}(1) = 0.61$ [33]. It is interesting to note that both in Ref. [31] and in [33], $\tau_{3/2}(1) - \tau_{1/2}(1) \simeq 0.3$, which might be a general feature of BT covariant quark models (see Eq. (5.1) in Ref. [31]). This is somewhat larger than the difference between central values of (34). We also agree with an older QCD sum rule estimate [32]: $\tau_{1/2}(1) = 0.35(8)$.

An interesting sum rule by Uraltsev which is free from $1/m_c$ corrections (but of course not from $1/m_b$ ones) leads to the following bound [34]:

$$\mu_\pi^2 - \mu_G^2 > 9\Delta^2 \tau_{1/2}^2(1), \tag{36}$$

where μ_π^2 and μ_G^2 are scale-dependent expectation values of the kinetic and chromomagnetic dimension-five operators and Δ is the 0^+–0^- mass splitting.

With our values for Δ (400 MeV) and $\tau_{1/2}(1)$ computed on the lattice, we find 0.22(6) for the r.h.s. of Eq. (36). An estimate by BaBar [35] found $\lambda_1 = -0.26 \pm 0.06 \pm 0.04 \pm 0.04\ \text{GeV}^2$, i.e., $\mu_\pi^2(1\ \text{GeV}) = 0.43 \pm 0.09\ \text{GeV}^2$, i.e. Using $\mu_G^2 = 0.35\ \text{GeV}^2$ from m_{B^*}–m_B hyperfine splitting it leads to 0.08 for the l.h.s. of Eq. (36). This pleads for a lower value of $\tau_{1/2}^2(1)$. Even worst, a previous value $\mu_\pi^2 = 0.31\ \text{GeV}^2$ was derived from Delphi [36], leading to a vanishing $\tau_{1/2}(1)$. At this stage we should not worry too much, since large errors and uncertainties are still present both for the experimental data and the theoretical predictions. Attention to this problem will be maintained.

One important question is whether some renormalisation of our result is needed. This issue is subtle and needs more work. We have computed the matrix elements of operators of dimension 4, of the type $\bar{h}(v)\gamma_i\gamma_5 D_j h(v)$. These operators do not mix with dimension 3 ones due to their parity: all dimension 3 operators have vanishing matrix elements between positive and negative parity states whence no power divergences are to be feared. We do not either believe that logarithmic divergences are to be feared since the vector and axial currents in HQET have a vanishing anomalous dimension at zero recoil ($w = 1$): they are related to generators for the $SU(2)$ flavor symmetry in the effective theory. More precisely, from [37,38], the scale dependence of the matrix element in Eq. (4), $1 + \mathcal{O}(\ln(\alpha_s(\mu))|\vec{v}_\perp|)$, vanishes when the limit $|\vec{v}_\perp| \to 0$ is taken. In general some finite renormalisation must be present. To settle this, a perturbative calculation is under way. We wish also to understand if the hypercubic

[5] It should be stressed that the BT method provides a framework in which different potentials can be used. The physics prediction depends, of course, on the chosen potential.

[6] The covariant light-front framework is equivalent, in the infinite mass limit, to the BT one. The practical predictions depend, however, on the chosen parameters and shape of the wave function. We worry about the use of Gaussian wave functions, whose validity may be questioned as it neglects the short distance potential.

treatment of the Wilson line can have some effect on the discretized covariant derivative we use. Finally a complete control of systematic effects is also needed: finite volume, mass of the light quark, finite lattice spacing. In Section 3.1 a duality of interpolating fields has been pointed out. A systematic comparison of the their predictions is still missing.

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Lattice renormalization of the static quark derivative operator

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Abstract

We give the analytical expressions and the numerical values of radiative corrections to the covariant derivative operator on the static quark line, used for the lattice calculation of the Isgur–Wise form factors $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$. These corrections induce an enhancement of renormalized quantities if an hypercubic blocking procedure is used for the Wilson line, while there is a reduction without such a procedure.

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1. Introduction

In a previous paper [1] we proposed a method to compute on the lattice, in the static limit of HQET, the Isgur–Wise form factors $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ which parameterize decays of B mesons into orbitally excited (P wave) D^{**} charmed mesons. Keep in mind that the zero recoil is the only definite limit of HQET on the lattice, because the Euclidean effective theory with a nonvanishing spatial momentum of the heavy quark is not lower bounded. Then it reveals impossible to calculate directly the τ_j's from the currents because the matrix elements vanish at zero recoil. To compute $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$, we proposed to evaluate on the lattice the matrix elements $\langle H_0^*(v)|\bar{h}(v)\gamma_i\gamma_5 D_j h(v)|H(v)\rangle$ and $\langle H_2^*(v)|\bar{h}(v)\gamma_i\gamma_5 D_j h(v)|H(v)\rangle$, using the following equalities:

$$\left\langle H_0^*(v)\middle|\bar{h}(v)\gamma_i\gamma_5 D_j h(v)\middle|H(v)\right\rangle = i g_{ij}(M_{H_0^*} - M_H)\tau_{\frac{1}{2}}(1), \tag{1}$$

$$\left\langle H_2^*(v)\middle|\bar{h}(v)\gamma_i\gamma_5 D_j h(v)\middle|H(v)\right\rangle = -i\sqrt{3}(M_{H_2^*} - M_H)\tau_{\frac{3}{2}}(1)\epsilon^*_{ij}, \tag{2}$$

where D_i is the covariant derivative ($D_i = \partial_i + i g A_i$), M_H, $M_{H_0^*}$ and $M_{H_2^*}$ are the mass of the 0^-, 0^+ and 2^+ states respectively, and ϵ^*_{ij} is the polarization tensor. These relations are defined between renormalized quantities. Then we have to renormalize the matrix element of the derivative operator computed on the lattice. We explained that power and logarithmic divergences are not to be feared in the zero recoil limit. However finite renormalization is present and we want to establish the one-loop contributions to the derivative operator with the hypercubic blocking [2] of the Wilson line.

We have to renormalize and to match onto the continuum the bare operator $O^B_{ij} = \bar{h}^B\gamma_i\gamma^5 D_j h^B$, where h^B is the bare heavy quark field. We choose the MOM scheme whose renormalization conditions are: (1) the renormalized heavy quark propagator is equal to the free one, and (2) the renormalized vertex function taken between renormalized external legs is the tree level one.

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The r.h.s. of Eqs. (1) and (2) are independent of the renormalization scale μ. Indeed, on the one hand, $M_{H_0^*} - M_H \equiv \bar{\Lambda}_{0^+} - \bar{\Lambda}_{0^-}$ and $M_{H_2^*} - M_H \equiv \bar{\Lambda}_{2^+} - \bar{\Lambda}_{0^-}$ are physical quantities. On the other hand,

$$\frac{\langle D_0|\bar{c}\gamma^\mu\gamma^5 b|B\rangle}{\sqrt{m_B m_{D_0}}} \equiv g^+(v+v')^\mu + g^-(v-v')^\mu \equiv -\tau_{1/2}(\mu,w)\sqrt{w-1}F^\mu,$$
$$F^\mu = \sqrt{w+1}C_1^5(\mu,w)a^\mu + \sqrt{w-1}\left[C_2^5(\mu,w)v^\mu + C_3^5(\mu,w)v'^\mu\right],$$

where $\sqrt{w^2-1}\,a^\mu = (v-v')^\mu$, and the C_i^5's are the matching coefficients between the QCD operator $\bar{c}\gamma^\mu\gamma^5 b$ and the HQET operators $\bar{c}_{v'}\gamma^\mu\gamma^5 b_v$, $\bar{c}_{v'}v^\mu\gamma^5 b_v$ and $\bar{c}_{v'}v'^\mu\gamma^5 b_v$, respectively,

$$\frac{\langle D_0|\bar{c}\gamma^\mu\gamma^5 b|B\rangle}{\sqrt{w-1}\sqrt{m_B m_{D_0}}} = -\tau_{1/2}(\mu,w)F^\mu \underset{w\to 1}{\longrightarrow} -\tau_{1/2}(\mu,1)\sqrt{2}C_1^5(\mu,1)a^\mu. \tag{3}$$

$C_1^5(\mu,1) \equiv C_1^5(1)$ [3] ($C_1^5(1) \equiv \eta_A = 0.986 \pm 0.005$ [4]) and the l.h.s. of (3) is also independent of μ: thus $\tau_{1/2}(\mu,1) \equiv \tau_{1/2}(1)$. We can use the same argument to prove the scale independence of $\tau_{3/2}(\mu,1)$. Consequently the scale μ will be omitted in the following.

It is well known that the heavy quark self-energy diverges linearly in $1/a$ [5], so we introduce a mass counterterm δm to cancel this divergence. Numerically it is canceled nonperturbatively in the ratio between the three-point function and the two-point functions to obtain a matrix element, or in the difference between binding energies of heavy-light mesons.

The bare heavy propagator on the lattice is

$$S^B(p) = \frac{a}{1-e^{-ip_4a} + a\delta m + a\Sigma(p)} = \frac{a}{1-e^{-ip_4a}}\sum_n \left(\frac{-a[\delta m + \Sigma(p)]}{1-e^{-ip_4a}}\right)^n \equiv Z_{2h}S^R(p). \tag{4}$$

By choosing the renormalization conditions

$$\left(S^R\right)^{-1}(p)\big|_{ip_4\to 0} = ip_4, \qquad \delta m = -\Sigma(p_4=0),$$

the constant Z_{2h} is then

$$Z_{2h} = 1 - \frac{d\Sigma}{d(ip_4)}\bigg|_{ip_4\to 0}.$$

The bare vertex function $V_{ij}^B(p)$ is defined as

$$V_{ij}^B(p) = \left(S^B\right)^{-1}(p)\sum_{x,y} e^{ip\cdot(x-y)}\left\langle h^B(x)O_{ij}^B(0)\bar{h}^B(y)\right\rangle\left(S^B\right)^{-1}(p)$$
$$= \frac{Z_{\mathcal{D}}}{Z_{2h}}\left(S^R\right)^{-1}(p)\sum_{x,y} e^{ip\cdot(x-y)}\left\langle h^R(x)O_{ij}^R(0)\bar{h}^R(y)\right\rangle\left(S^R\right)^{-1}(p), \tag{5}$$

where

$$O_{ij}^B(0) = Z_{\mathcal{D}}O_{ij}^R(0).$$

We will see below that $V_{ij}^B(p)$ can be written as

$$V_{ij}^B(p) = (1+\delta V)\bar{u}(p)\gamma_i\gamma^5 p_j u(p) \equiv (1+\delta V)V_{ij}^R(p). \tag{6}$$

δV is given by all the 1PI one-loop diagrams containing the vertex.

We obtain $\langle H^{**}|O_{ij}^R|H\rangle = Z_{\mathcal{D}}^{-1}\langle H^{**}|O_{ij}^B|H\rangle$ where $Z_{\mathcal{D}} = Z_{2h}(1+\delta V)$ and $\langle H^{**}|O_{ij}^B|H\rangle$ was computed on the lattice.

This Letter is organized as follows: in Section 2 we recall the action we use for the heavy quark, we give the corresponding Feynman rules and we clarify our notations; in Section 3 we give the analytical expression for the heavy quark self-energy, in Section 4 we give the analytical expression for radiative corrections to the derivative operator in lattice HQET. We briefly conclude in Section 5.

2. Heavy quark action and Feynman rules

The lattice HQET action for the static heavy quark is

$$S^{\text{HQET}} = a^3\sum_n\left\{h^\dagger(n)\left[h(n) - U_4^{\dagger,\text{HYP}}(n-\hat{4})h(n-\hat{4})\right] + a\delta m h^\dagger(n)h(n)\right\}, \tag{7}$$

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where $U_4^{\rm HYP}(n)$ is a link built from an hypercubic blocking.

We will use in the rest of the Letter the following notations taken from [6–8]:

$$\int_p \equiv \int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4}, \qquad \int_{\vec p} \equiv \int_{-\pi/a}^{\pi/a} \frac{d^3p}{(2\pi)^3}, \qquad a^4\sum_n e^{ipn} = \delta(p), \qquad \int_k \equiv \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4}, \qquad \int_{\vec k} \equiv \int_{-\pi}^{\pi} \frac{d^3k}{(2\pi)^3},$$

$$h(n) = \int_p e^{ipn} h(p),$$

$$U_\mu(n) = e^{iag_0 A_\mu^a(n)T^a} = 1 + iag_0 A_\mu^a(n)T^a - \frac{a^2g_0^2}{2!}A_\mu^a(n)A_\mu^b(n)T^aT^b + \mathcal{O}(g_0^3),$$

$$U_\mu^{\rm HYP}(n) = e^{iag_0 B_\mu^a(n)T^a} = 1 + iag_0 B_\mu^a(n)T^a - \frac{a^2g^2}{2!}B_\mu^a(n)B_\mu^b(n)T^aT^b + \mathcal{O}(g_0^3),$$

$$A_\mu^a(n) = \int_p e^{ip(n+\frac{a}{2})}A_\mu^a(p), \qquad B_\mu^a(n) = \int_p e^{ip(n+\frac{a}{2})}B_\mu^a(p), \qquad \Gamma_\lambda = \sin ak_\lambda,$$

$$c_\mu = \cos\left(\frac{a(p+p')_\mu}{2}\right), \qquad s_\mu = \sin\left(\frac{a(p+p')_\mu}{2}\right), \qquad M_\mu = \cos\left(\frac{k_\mu}{2}\right), \qquad N_\mu = \sin\left(\frac{k_\mu}{2}\right),$$

$$W = 2\sum_\lambda \sin^2\left(\frac{k_\lambda}{2}\right).$$

In the Fourier space, the action is given at the order of $\mathcal{O}(g_0^2)$ by

$$S^{\rm HQET} = \int_p a^{-1}h^\dagger(p)\big(1-e^{-ip_4a}\big)h(p) + \delta m h^\dagger(p)h(p) + ig_0\int_p\int_{p'}\int_q \delta(q+p'-p)h^\dagger(p)B_4^a(q)T^ah(p')e^{-i(p_4+p_4')\frac{a}{2}}$$
$$+ \frac{ag_0^2}{2!}\int_p\int_{p'}\int_q\int_r \delta(q+r+p'-p)h^\dagger(p)B_4^a(q)B_4^b(r)T^aT^bh(p')e^{-i(p_4+p_4')\frac{a}{2}}. \tag{8}$$

The block gauge fields B_μ^a can be expressed in terms of the usual gauge fields

$$B_\mu = \sum_{n=1}^{\infty} B_\mu^{(n)},$$

where $B_\mu^{(n)}$ contains n factors of A. At NLO, it was shown that we only need $B_\mu^{(1)}$ [9]:

$$B_\mu^{(1)}(k) = \sum_\nu h_{\mu\nu}(k)A_\nu(k), \qquad h_{\mu\nu}(k) = \delta_{\mu\nu}D_\mu(k) + (1-\delta_{\mu\nu})G_{\mu\nu}(k),$$

$$D_\mu(k) = 1 - d_1\sum_{\rho\neq\mu}N_\rho^2 + d_2\sum_{\rho<\sigma,\rho,\sigma\neq\mu}N_\rho^2N_\sigma^2 - d_3N_\rho^2N_\sigma^2N_\tau^2,$$

$$G_{\mu\nu}(k) = N_\mu N_\nu\left(d_1 - d_2\frac{N_\rho^2+N_\sigma^2}{2} + d_3\frac{N_\rho^2N_\sigma^2}{3}\right),$$

$$d_1 = (2/3)\alpha_1\big(1+\alpha_2(1+\alpha_3)\big), \qquad d_2 = (4/3)\alpha_1\alpha_2(1+2\alpha_3), \qquad d_3 = 8\alpha_1\alpha_2\alpha_3.$$

Two sets of α_i's have been chosen: (1) $\alpha_1 = 0.75, \alpha_2 = 0.6, \alpha_3 = 0.3$ (which has been chosen in our simulation and has been motivated in [2]) and (2) $\alpha_1 = 1.0, \alpha_2 = 1.0, \alpha_3 = 0.5$, motivated in [10]. We will label these two sets respectively by HYP1 and HYP2.

The Feynman rules can be easily deduced (they must be completed by the application of $h_{\mu\nu}$):

heavy quark propagator	$a\big(1-e^{-ip_4a}+\epsilon\big)^{-1}$,
vertex $V_{\mu,hhg}^a(p,p')$	$-ig_0T^a\delta_{\mu4}e^{-i(p_4+p_4')\frac{a}{2}}$,
vertex $V_{\mu\nu,hhgg}^{ab}(p,p')$	$-\frac{1}{2}ag_0^2\delta_{\mu4}\{T^a,T^b\}e^{-i(p_4+p_4')\frac{a}{2}}$,
gluon propagator in the Feynman gauge	$a^2\delta_{\mu\nu}\delta^{ab}\big(2W+a^2\lambda^2\big)^{-1}$.

Note that p' and p are the in-going and the out-going fermion momenta, respectively. We also introduce an infrared regulator λ for the gluon propagator. We symmetrize the vertex $V^{ab}_{\mu\nu,hhgg}$ by introducing the anticommutator of the $SU(3)$ generators, normalized by a factor $\frac{1}{2}$. The gluon propagator and the vertices are defined with the A field. The coefficient $\sum_{i=1}^{3} h_{4i}^2 \equiv H(N_4)$ will enter as a global multiplicative factor in all the integrals expressed below. We have chosen the Feynman gauge: since one calculates the renormalization of a gauge-invariant operator, the renormalization factor $Z_{\mathcal{D}}$ is gauge invariant.

3. Heavy quark self-energy

From (4) we have $\Sigma(p) = -(F_1 + F_2)$, where F_1 and F_2 correspond to the diagrams shown in Fig. 1(a) and (b):

$$
\begin{aligned}
F_1 &= -\frac{4}{3a} g_0^2 \int\limits_k \frac{H(N_4)}{2W + a^2\lambda^2} \frac{e^{-i(k_4+2ap_4)}}{1 - e^{-i(k_4+ap_4)} + \epsilon} = -\frac{1}{3a} g_0^2 \int\limits_k \frac{H(N_4)}{N_4^2 + E^2} \frac{e^{-i(k_4+2ap_4)}}{1 - e^{-i(k_4+ap_4)} + \epsilon} \\
&= -\frac{1}{3a} g_0^2 \int\limits_k \frac{H(N_4)}{(N_4 + iE)(N_4 - iE)} \frac{e^{-i(k_4+2ap_4)}}{1 - e^{-i(k_4+ap_4)} + \epsilon} = -\frac{1}{3a} g_0^2 \int\limits_{\vec{k}} \frac{1}{E} \frac{H(-iE)}{\sqrt{1+E^2}} \frac{e^{-2iap_4}}{e^{E'} - e^{-iap_4}}.
\end{aligned}
\tag{9}
$$

Note that Latin indices are spatial and

$$
\begin{aligned}
E^2 &= \sum_i N_i^2 + \frac{a^2\lambda^2}{4}, \\
H(N_4) &= \left(1 - d_1 \sum_i N_i^2 + d_2 \sum_{i<j} N_i^2 N_j^2 - d_3 N_1^2 N_2^2 N_3^2\right)^2 + N_4^2 \sum_i N_i^2 \left(d_1 - \frac{d_2}{2} \sum_{j\neq i} N_j^2 + \frac{d_3}{3} \prod_{j\neq i} N_j^2\right)^2, \\
E' &= 2\,\mathrm{argsh}(E).
\end{aligned}
$$

In (9) we have eliminated properly the noncovariant pole $k_4 = -ap_4 + i\epsilon$ by closing the integration contour in the complex plane $\Im(k_4) < 0$ where there is the single pole $N_4 = -iE$. Furthermore the integrals along the lines $k_4 = \pm\pi + ik_4'$ are equal, because the integrand is 2π-periodic.

Finally we have in the limit $ap_4 \to 0$:

$$
F_1 = \frac{4}{3a} g_0^2 \int\limits_{\vec{k}} \frac{H(-iE)}{4E\sqrt{1+E^2}} \frac{1}{1 - e^{E'}} + \frac{4}{3} g_0^2 i p_4 \int\limits_{\vec{k}} \frac{H(-iE)}{2E\sqrt{1+E^2}} \left[\frac{1}{e^{E'} - 1} + \frac{1}{2} \frac{1}{(e^{E'} - 1)^2}\right].
\tag{10}
$$

We find

$$
F_1 \equiv -\frac{g_0^2}{12\pi^2} \left\{ f_1(\alpha_i)/a + i p_4 \left[2 \ln\left(a^2\lambda^2\right) + f_2(\alpha_i)\right]\right\}.
\tag{11}
$$

The tadpole diagram F_2 is

$$
F_2 = -\frac{1}{2} \frac{4g_0^2}{3a} e^{-iap_4} \int\limits_k \frac{H(N_4)}{2W} \underset{ap_4\to 0}{=} -\frac{1}{2} \frac{4g_0^2}{3} (1/a - ip_4) \int\limits_k \frac{H(N_4)}{2W} \equiv -\frac{g_0^2}{12\pi^2} (1/a - ip_4) f_3(\alpha_i).
\tag{12}
$$

The factor $1/2$ is introduced to compensate the over-counting of the factor 2 in the Feynman rule of the 2-gluon vertex when a closed gluonic loop is computed. We can point out that the divergent part

$$
\Sigma_0(\alpha_i) = \frac{g_0^2}{12\pi^2 a} \sigma_0(\alpha_i), \qquad \sigma_0 = f_1 + f_3
\tag{13}
$$

of the self-energy is smaller with the sets HYP1 and HYP2 of the α_i's than with the corresponding contribution without "hypercubic" links [5], as shown in Table 1: $\sigma_0(\alpha_i = 0) = 19.95$, $\sigma_0(\text{HYP1}) = 5.76$ and $\sigma_0(\text{HYP2}) = 4.20$, in good agreement with computations made by the ALPHA Collaboration [10], which compares the pseudoscalar heavy-light meson effective energy with

Fig. 1. Self-energy corrections: (a) sunset diagram; (b) tadpole diagram.

Table 1
Numerical values of the parameters f_1, f_2, f_3, f_4, σ_0, z_2, z_d defined in Eqs. (11), (12), (18), (13), (14) and (19), respectively

	$\alpha_i = 0$	HYP1	HYP2
f_1	7.72	1.64	−1.76
f_2	−12.25	1.60	9.58
f_3	12.23	4.12	5.96
f_4	−12.68	−10.32	−8.18
σ_0	19.95	5.76	4.20
z_2	24.48	2.52	−3.62
z_d	11.80	−7.80	−11.80

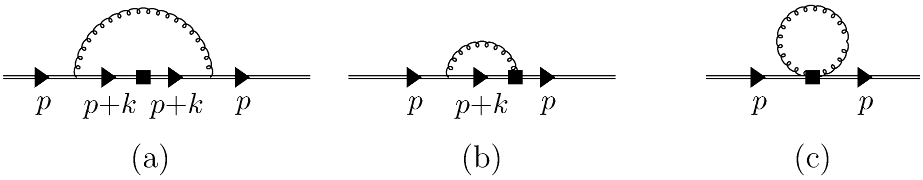

Fig. 2. Operator corrections.

different static heavy quark actions, and by Hasenfratz et al. [11]. Qualitatively, one expects that the hypercubic blocking reduces the radiative corrections since it amounts roughly to introduce an additional cut-off in the integrals.

The wave function renormalization Z_{2h} is

$$Z_{2h}(\alpha_i) = 1 + \frac{g_0^2}{12\pi^2}\left[-2\ln\left(a^2\lambda^2\right) + z_2(\alpha_i)\right], \quad z_2 = f_3 - f_2. \tag{14}$$

$|z_2|$ is also reduced by the hypercubic blocking, as indicated on Table 1: $z_2(\alpha_i = 0) = 24.48$ [5], $z_2(\mathrm{HYP1}) = 2.52$ and $z_2(\mathrm{HYP2}) = -3.62$.

4. Derivative operator in lattice HQET

We have to renormalize the operator $O_{ij}^B = \bar{h}^B \gamma_i \gamma^5 D_j h^B$. Following [6],

$$\begin{aligned}
a^4 \sum_n O_{ij}^B(n) &= a^4 \frac{1}{2a} \sum_n \bar{h}^B(n)\gamma_i\gamma^5 U_j(n) h^B(n+\hat{j}) - \bar{h}^B(n)\gamma_i\gamma^5 U_j^\dagger(n-\hat{j}) h^B(n-\hat{j}) \\
&= \int_p \int_{p}^{\prime} a^{-1}\delta(p-p')\bar{h}^B(p)\left(i\gamma_i\gamma^5 s_j\right)h^B(p') + ig_0 \int_p \int_{p'} \int_q \delta(q+p'-p)\bar{h}^B(p)\gamma_i\gamma^5 c_i A_j^a(q) T^a h^B(p') \\
&\quad - \frac{iag_0^2}{2!} \int_p \int_{p'} \int_q \int_r \delta(q+r+p'-p)\bar{h}^B(p) T^a T^b \gamma_i \gamma^5 s_j A_j^a(q) A_j^b(r) h^B(p').
\end{aligned} \tag{15}$$

Note that we have chosen to not submit the covariant derivative to the hypercubic blocking.

The vertex function V_{ij}^B is obtained by writing $V_{ij}^B = V_{ij}^0 + V_{ij}^1 + V_{ij}^2$, corresponding to the diagrams (a), (b) and (c) in Fig. 2; $V_{ij}^k(\alpha_i) = \bar{u}(p)\gamma_i\gamma^5 u(p) V_j^k(\alpha_i)$, $k = 0, 1, 2$. The contribution V_{ij}^0 is then given by computing

$$\begin{aligned}
V_j^0(\alpha_i) &= -\frac{4i}{3a} g_0^2 \int_k \frac{H(N_4)}{2W + a^2\lambda^2} \sin(k+ap)_j \frac{e^{-i(k_4+2ap_4)}}{(1-e^{-i(k_4+ap_4)}+\epsilon)^2} \\
&= -\frac{4i}{3a} g_0^2 \int_k \frac{H(N_4)}{2W + a^2\lambda^2} (\Gamma_j + ap_j \cos k_j) e^{-iap_4} \left(\frac{e^{-i\frac{k_4+ap_4}{2}}}{1-e^{-i(k_4+ap_4)}+\epsilon}\right)^2 \\
&= -\frac{4i}{3a} g_0^2 \int_k \frac{H(N_4)}{2W + a^2\lambda^2} (\Gamma_j + ap_j \cos k_j)(1 - iap_4) \frac{1}{[2i\sin(\frac{k_4+ap_4}{2}) + e^{i\frac{k_4+ap_4}{2}}\epsilon]^2}.
\end{aligned} \tag{16}$$

We can get rid of the integrand proportional to Γ_j, because it is an odd term. It remains

$$V_j^0(\alpha_i) = -\frac{4}{3} i g_0^2 p_j \int_k \frac{H(N_4)}{2W + a^2\lambda^2} \frac{\cos k_j}{(2iN_4 + \epsilon M_4)^2}.$$

The integrand has poles at $N_4 = \pm iE$, $k_4 = 2i\,\mathrm{argth}(\frac{\epsilon}{2})$. Once again we close the integration contour around the single pole $N_4 = -iE$. Since j is spatial, the "sail" diagram drawn on Fig. 2(b) does not give any contribution, thus $V_{ij}^1 = 0$. There is finally the tadpole contribution V_{ij}^2 which is given by computing

$$V_j^2(\alpha_i) = -\frac{1}{2!}\frac{4}{3} i g_0^2 p_j \int_k \frac{1}{2W} = -\frac{i g_0^2}{12\pi^2} p_j\, 12.23. \tag{17}$$

We have finally $\langle H^{**}|O_{ij}^R|H\rangle = Z_{\mathcal{D}}^{-1}(\alpha_i)\langle H^{**}|O_{ij}^B|H\rangle(\alpha_i)$ where

$$Z_{\mathcal{D}}(\alpha_i) = Z_{2h}(\alpha_i)\big[1 + \delta V(\alpha_i)\big],$$

$$\delta V(\alpha_i) = -\frac{4g_0^2}{3}\int_k \left(\frac{H(N_4)\cos k_j}{(2W + a^2\lambda^2)(2iN_4 + \epsilon M_4)^2} + \frac{1}{4W}\right) = -\frac{4g_0^2}{3}\frac{1}{2}\int_{\vec{k}} \frac{H(-iE)\cos k_j}{(2E)^3\sqrt{1+E^2}} - \frac{g_0^2}{12\pi^2}12.23$$

$$\equiv \frac{g_0^2}{12\pi^2}\big[2\ln(a^2\lambda^2) + f_4(\alpha_i)\big], \tag{18}$$

$$Z_{\mathcal{D}}(\alpha_i) = 1 + \frac{g_0^2}{12\pi^2} z_d(\alpha_i), \quad z_d = z_2 + f_4. \tag{19}$$

The numerical values of z_d are indicated in Table 1.

Remark that infrared divergences appearing in Z_{2h} and $1 + \delta V$ cancel and there is no dependence on a, a further consequence of the μ independence of the matrix element $\langle H_0^*(v')|\bar{h}(v')\gamma_i\gamma_5 D_j h(v)|H(v)\rangle$ at zero recoil. Note also that as already mentioned the tadpole diagram of the operator vertex is not smoothed by the hypercubic blocking; therefore it is quite large. On the other hand the tadpole contribution to the self-energy is smoothed by the blocking. The final result is a large positive correction to the renormalized matrix element. By fixing $g_0 = 1$ ($\beta = 6.0$), this gives $Z_D^{-1}(\mathrm{HYP1}) = 1.07$ and $Z_D^{-1}(\mathrm{HYP2}) = 1.10$, thus the discrepancy between $Z_D^{-1}(\mathrm{HYP1})$ and $Z_D^{-1}(\mathrm{HYP2})$ is small (3%); without hypercubic blocking one would obtain $Z_D^{-1}(\alpha_i = 0) = 0.90$. We can think of applying a boosting procedure; in the pure HYP case the boosting plaquette correction is very small [8] (equation 19 and below). On the other hand we have to take into account that the covariant derivative operator involves links without hypercubic blocking, therefore one should employ a different prescription for the diagram drawn on Fig. 2(c), possibly leading to a larger tadpole contribution from the operator to $Z_D^{-1}(\alpha_i \neq 0)$, and therefore a larger positive correction. Anyhow this kind of recipe would not lead to $Z_D^{-1}(\alpha_i \neq 0)$ significantly larger than 1.1.

With our exploratory lattice study and taking account $Z_{\mathcal{D}}^{-1}(\mathrm{HYP1})$, we find $\tau_{\frac{1}{2}}(1) = 0.41(5)(?)$, $\tau_{\frac{3}{2}}(1) = 0.57(10)(?)$ and $\tau_{\frac{3}{2}}^2(1) - \tau_{\frac{1}{2}}^2(1) = 0.15(10)$, where systematics are unknown; one is then not too far (within 1σ) from saturating by ground states the Uraltsev sum rule [12] $\sum_n |\tau_{\frac{3}{2}}^{(n)}(1)|^2 - |\tau_{\frac{1}{2}}^{(n)}(1)|^2 = \frac{1}{4}$. However the relation $\mu_\pi^2 - \mu_G^2 > 9\Delta^2\tau_{\frac{1}{2}}^2(1)$ [13], with $\Delta \equiv M_{H_0^*} - M_H = 0.4$ GeV [1], $\mu_G^2 = 0.35\ \mathrm{GeV}^2$, leads to μ_π^2 larger than $0.6\ \mathrm{GeV}^2$, which is significantly above experimental determination by moments.

5. Conclusion

In this Letter we have calculated the radiative corrections to the covariant derivative operator $\bar{h}\gamma_i\gamma^5 D_j h$ in lattice HQET with an hypercubic blocking of the Wilson line defining the heavy quark propagator. This determines the renormalization of the operator which is used to estimate the Isgur–Wise functions between the ground state and the $L = 1$ excitations at zero recoil. We find that there is a global, but moderate, enhancement of $\tau_{\frac{1}{2}}(1)$ and $\tau_{\frac{3}{2}}(1)$ with respect to the bare quantities computed on the lattice in the case where one introduces fat timelike links, while there is a reduction with a simple Wilson line.

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Chapitre 5

Liste des Publications

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▶ A. Le Yaouanc, D. Melikhov, V. Morénas, L. Oliver, O. Pène et J.C. Raynal, « *One Interesting New Sum Rule Extending Bjorken's to order* $1/m_Q$ », Physics Letters B **480** (1,2) : 119–128 (2000).

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Conclusion et perspectives

Tout au long de ce mémoire, nous avons abordé plusieurs aspects liés à la physique de la beauté : l'hypothèse de dualité quark-hadron (pour laquelle une étude fondée sur des modèles de quark a montré entre autres que, dans le cas des largeurs totales de désintégration, aucun terme de violation d'ordre $1/m_Q$ n'est présent et que, dans le cas des largeurs différentielles de désintégration, la situation n'est pas aussi claire), l'hypothèse de factorisation, introduite lors de l'étude des désintégrations non-leptoniques des mésons B (dont une version « améliorée » a été confrontée à un ensemble de données expérimentales avec des résultats mitigés) et finalement la production d'états orbitalement excités D^{**} (où nous avons établi l'existence d'une hiérarchie dans les taux de production, hiérarchie non vérifiée expérimentalement).

Ce dernier point est particulièrement intriguant dans la mesure où il dénonce une incompatibilité entre les prédictions théoriques et les données expérimentales : en fait, tous les modèles théoriques phénoménologiques semblent prédire une situation alors que les mesures semblent annoncer son contraire. C'est pourquoi nous avons entamé une étude montrant qu'il est possible d'obtenir les facteurs de forme, mis en jeu dans ces processus, de façon fondamentale et non perturbative, par la QCD sur réseau. Cette étude de faisabilité a été réalisée sur des machines dédiées APEmille du projet APE, dont la génération actuelle est l'apeNEXT. Il serait donc très intéressant de pouvoir affiner cette étude sur cette dernière génération d'ordinateur à architecture parallèle pour lever ce paradoxe ; malheureusement, la situation actuelle du système scientifique français semble compromettre tous les efforts de voir une telle étude achevée dans la mesure où les équipes « lattice » risquent de n'avoir aucune puissance de calcul suffisante qui leur soit propre pour produire le moindre résultat...... et ce malgré le fait que nous faisons partie de la collaboration à l'origine de la conception/réalisation de ces machines parallèles.

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Résumé

La physique du B constitue un pan important de la physique des particules, aussi bien du point de vue théorique qu'expérimental. L'objectif de ce mémoire est d'adresser un certain nombre d'aspects relatifs à cette physique et, dans un premier temps, certaines hypothèses théoriques utilisées : - hypothèse de dualité : la plupart des calculs inclusifs sur les transitions entre états hadroniques repose implicitement sur le fait que ces processus peuvent être décrits en ne faisant intervenir que les transitions entre les quarks constituant ces hadrons. Nous avons testé cette hypothèse dans le cadre des désintégrations semileptoniques des mésons B et cherché les causes d'éventuelles violations. - hypothèse de factorisation : cette hypothèse simplificatrice, couramment employée dans le domaine, permet par exemple l'étude des désintégrations non-leptoniques des mésons B en deux autres mésons. Nous avons réalisé une étude exhaustive comparative entre les données expérimentales disponibles et une version améliorée de cette hypothèse. Ensuite, nous présentons un problème lié à la production des états de moment cinétique orbital l=1 lors des désintégrations des mésons B : les données expérimentales vont à contre sens de tous les modèles théoriques phénoménologiques ; nous proposons alors une tentative de résolution par QCD sur réseau de ce puzzle. Enfin, nous décrivons le projet APEnext (projet européen d'ordinateur à architecture parallèle dédié aux calculs sur réseau).