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## Etude du bruit quantique dans les lasers à semiconducteur et à solide

Alberto Bramati

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**LABORATOIRE  
KASTLER BROSSEL**



**Thèse de doctorat de l'Université Pierre et Marie Curie**

**Spécialité : Physique Quantique**

*présentée par*

**Alberto BRAMATI**

*pour obtenir le grade de Docteur de l'Université Pierre et Marie Curie*

Sujet de la Thèse :

**ETUDE DU BRUIT QUANTIQUE DANS LES LASERS  
A SEMICONDUCTEUR ET A SOLIDE**

Soutenue le 16 décembre 1998 devant le jury composé de :

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M. M. DUCLOY	Président
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Mme E. GIACOBINO	Directeur de thèse
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# 1 Introduction

La première indication de la nature quantique de la lumière date de 1900 lorsque Planck introduisit la notion de quantification de l'énergie de l'oscillateur harmonique pour contourner les difficultés rencontrées par les théories classiques dans l'interprétation de la distribution spectrale du rayonnement du corps noir. En 1905 Einstein montra que l'effet photoélectrique peut s'expliquer en supposant que l'énergie contenue dans un faisceau lumineux est répartie en quanta d'énergie : le concept de photon venait de naître. Ensuite, le développement de la mécanique quantique et en particulier la formulation, en 1925 par Heisenberg, des célèbres inégalités qui portent son nom, firent apparaître la notion de *fluctuations quantiques*. L'inégalité de Heisenberg interdit de mesurer simultanément avec une précision infinie les valeurs de deux observables conjuguées (position et impulsion d'une particule, par exemple). Plus précisément elle impose que le produit des variances des observables conjuguées soit borné par une limite inférieure, fixée par la valeur du commutateur des deux variables.

Dans le cadre de la description quantique du champ électromagnétique, l'inégalité de Heisenberg, appliquée aux observables conjuguées du champ électromagnétique, implique l'existence de fluctuations quantiques du champ électromagnétique.

Contrairement à ce qui se passe dans beaucoup d'autres domaines de la physique, il est relativement facile d'observer les fluctuations quantiques en optique. Lorsque la lumière est produite par un laser dont toutes les fluctuations d'origine mécanique, acoustique ou électrique ont été supprimées, les fluctuations quantiques donnent lieu à un bruit résiduel sur l'intensité lumineuse détectée. Ce bruit est connu sous le nom de bruit de grenaille (*shot noise*) ou bruit quantique standard. Dans les années récentes, grâce à l'amélioration des sources lasers et des détecteurs, le bruit quantique standard est de plus en plus souvent atteint dans les mesures de précision, en spectroscopie ou en interférométrie, et il constitue une réelle limitation de la sensibilité de la détection.

On a longtemps considéré que le bruit quantique standard constituait une limite insurmontable. Mais au milieu des années 1980, plusieurs expériences ont montré que,

si les fluctuations quantiques étaient inévitables, leur effet sur les mesures pouvait être contourné. En effet, l'inégalité de Heisenberg impose une condition sur le produit des variances des observables conjuguées : il est donc possible de réduire les fluctuations d'une des observables sous le bruit quantique standard, au prix de voir les fluctuations de l'autre augmentées en conséquence. De tels états sont dits comprimés.

Les techniques qui permettent de produire des états comprimés du rayonnement peuvent être classées en deux catégories. La première a recours à des processus paramétriques utilisant une interaction non linéaire entre matière et lumière. La non linéarité peut être du second ordre ( $\chi^{(2)}$ ) ou du troisième ordre ( $\chi^{(3)}$ ), correspondant au mélange à trois et quatre ondes respectivement.

La première observation expérimentale de la réduction de bruit date de 1985 [1] et a été obtenue par mélange à quatre ondes dans une vapeur de sodium ; une compression des fluctuations quantiques a aussi été produite en utilisant la non linéarité ( $\chi^{(3)}$ ) dans une fibre optique en silice [2,3]. Les effets obtenus avec cette technique sont de l'ordre de 50%.

Beaucoup plus performants se sont révélés les amplificateurs ou les oscillateurs paramétriques qui reposent sur une non linéarité de type ( $\chi^{(2)}$ ). Le matériau non linéaire est un cristal doubleur et la réduction de bruit est obtenue par génération paramétrique ou par génération de second harmonique. Les compressions observées sont très importantes pouvant aller jusqu'à 81% au-dessous du bruit quantique standard [4-6].

La deuxième catégorie s'applique aux lasers et permet de produire des états comprimés en intensité en agissant sur le mécanisme de pompage du laser. Le principe utilisé est celui dit de la pompe régulière [7] et consiste à supprimer le bruit associé au processus de pompage : si la conversion de la puissance de pompage en puissance lumineuse est efficace, la statistique du flux de photons émis par le laser reproduira en partie celle de la pompe. L'application de ce principe aux diodes laser a donné d'excellents résultats arrivant jusqu'à 70% de réduction de bruit d'intensité sous le bruit quantique standard [8].

Ce travail de thèse est consacré à l'étude du bruit quantique dans les lasers et aux méthodes pour le réduire. Nous avons étudié différents types de sources laser : des lasers à semiconducteur (diodes laser à ruban et lasers à microcavité verticale dits *VCSELs*) et des lasers à solide (microlasers Nd:YVO<sub>4</sub>).

Après une introduction rappelant les principes généraux de la réduction du bruit quantique (ch. 2), nous présentons dans le troisième chapitre une étude détaillée de la réduction de bruit observée dans les diodes laser en fonction des configurations

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expérimentales utilisées, à température ambiante et en régime cryogénique. Cette recherche nous a conduit à la compréhension du rôle fondamental joué par les phénomènes d'anticorrélation entre le mode principal et les modes longitudinaux du laser dans la détermination de ses caractéristiques de bruit.

Le quatrième chapitre détaille les résultats expérimentaux obtenus avec les VCSELs : une réduction de bruit d'intensité en régime de fonctionnement multimode du laser est démontrée. Encore une fois le phénomène physique à la base de ce résultat est l'anticorrélation entre les différents modes transverses qui oscillent simultanément. Dans ce cas l'anticorrélation est associée à des distributions spatiales différentes pour différents modes de polarisation et donne lieu à des effets remarquables sur le bruit d'intensité du laser.

Le chapitre suivant est consacré à l'étude des caractéristiques de bruit des microlaser à solide. L'application du principe de la pompe régulière à ces lasers, qui présentent une dynamique similaire à celle des diodes laser, laisse présager la possibilité d'obtenir des états comprimés. Cela est d'un grand intérêt, du fait des nombreuses applications que connaissent ces lasers. Cependant la réalisation de cet objectif est compliqué par la présence, à basse fréquence (quelques megahertz), de l'oscillation de relaxation. Nous avons d'abord étudié les effets du bruit de pompe sur le bruit d'intensité du microlaser. La comparaison des résultats expérimentaux aux prévisions théoriques fournies par un modèle quantique du laser, très satisfaisante, révèle néanmoins l'existence d'un désaccord à basse fréquence où le bruit du microlaser est plus élevé que prévu. Pour éclaircir l'origine de l'excès de bruit constaté dans le spectre de bruit du microlaser, l'approche que nous avons suivie a consisté à utiliser une rétroaction électro-optique rétroagissant sur la pompe afin de réduire l'excès de bruit autour de l'oscillation de relaxation, sans toutefois rétroagir à basse fréquence, pour préserver les propriétés de bruit du microlaser dans cette région. Parallèlement, nous avons développé un modèle quantique du laser en présence de rétroaction. Dans ce cas l'excès de bruit est absent et l'accord entre théorie et expérience est satisfaisant sur toute la bande de fréquences ; cela suggère la présence d'effets non linéaires dus à l'oscillation de relaxation. Une autre technique de réduction de l'oscillation de relaxation est expérimentée : l'injection optique du microlaser. Théorie et expérience sont en bon accord et une forte réduction de l'excès de bruit est observée.

Enfin, le dernier chapitre présente une application des diode laser à bruit d'intensité comprimé à la spectroscopie de haute sensibilité. L'utilisation de la technique de la modulation de fréquence accompagnée des propriétés non-classiques de la lumière émise

par la diode laser conduit à l'amélioration de la sensibilité ultime dans des mesures d'absorption.

## 2 Introduction au bruit quantique

### 2.1 Fluctuations quantiques et états comprimés

De manière simplifiée, on peut voir le bruit quantique observé sur l'intensité d'un faisceau laser idéal comme résultant de l'arrivée aléatoire des photons sur le détecteur. La distribution aléatoire des photons a une statistique de Poisson et peut se mesurer directement en régime de comptage de photons. Lorsque l'on mesure une intensité lumineuse dans un régime de fort flux où le comptage est impossible, on retrouve sur cette intensité la trace de la distribution des photons : à la valeur moyenne se superposent des fluctuations dont la valeur quadratique moyenne est proportionnelle à la racine carrée de l'intensité, ce qui est caractéristique de la distribution Poissonienne.

#### 2.1.1 Champ électromagnétique quantique

Cependant, l'image de l'arrivée aléatoire des photons ne rend pas compte de toutes les caractéristiques du bruit quantique, qui se manifeste non seulement sur l'intensité mais aussi sur la phase ou sur les diverses composantes en quadrature du champ. Pour les représenter correctement, il faut faire appel à une théorie où le champ électromagnétique est quantifié.

Une onde plane de fréquence  $\omega_L$  et de direction de propagation et de polarisation données, correspondant à un mode particulier du champ électromagnétique peut être décrite par son champ électrique [9] :

$$E(t) = E_1 \cos(\omega_L t) + E_2 \sin(\omega_L t) \quad (2.1)$$

où  $E_1$  et  $E_2$  sont les composantes en quadrature du champ. Quand le champ est quantifié, les deux composantes sont des opérateurs qui s'expriment en fonction des opérateurs de création et annihilation  $a^\dagger$  et  $a$  du mode, opérateurs qui ont pour effet d'ajouter ou de retirer un photon :



$$E_1 = E_0(a + a^\dagger) \quad E_2 = -iE_0(a - a^\dagger) \quad (2.2)$$

$$E_0 = \sqrt{\frac{\hbar\omega_L}{2\varepsilon_0V}} \quad (2.3)$$

où  $V$  est le volume dans lequel est quantifié le champ. Notons que  $E_0$  est le champ correspondant à un photon.

Le commutateur de  $a^\dagger$  et  $a$  étant :  $[a, a^\dagger] = 1$ ,  $E_1$  et  $E_2$  ne commutent pas. Ce sont des observables conjuguées, de même que la position  $q$  et l'impulsion  $p$  d'une particule. En conséquence, elles ne peuvent pas être mesurées simultanément avec une précision infinie et le produit des dispersions des deux observables  $\Delta E_1$  et  $\Delta E_2$  obéit à une inégalité de Heisenberg :

$$\Delta E_1 \Delta E_2 \geq E_0^2 \quad (2.4)$$

On peut représenter le champ dans l'espace des phases où les coordonnées sont les deux composantes en quadrature  $E_1$  et  $E_2$ . Dans un tel diagramme, un champ classique est représenté par un vecteur dont le module et l'argument sont l'amplitude et la phase. Pour un champ quantique, le résultat des mesures sur  $E_1$  et  $E_2$  fluctue des quantités  $\Delta E_1$  et  $\Delta E_2$ . L'extrémité du vecteur représentant le champ appartient à une surface dans l'espace des phases dont la dimension est supérieure à la limite donnée par l'équation (2.4).

L'état le plus proche d'un champ classique est celui pour lequel les fluctuations sur les deux quadratures sont égales entre elles et égales à la limite permise par l'équation (2.4). L'aire d'incertitude a alors une forme circulaire de diamètre  $E_0$  dans l'espace des phases comme indiqué sur la figure 1 (sur cette figure, le champ et ses fluctuations ne sont pas à la même échelle). Un tel état d'incertitude minimale est appelé *état cohérent*. Les fluctuations qui lui correspondent sont les fluctuations quantiques standard.

Le champ correspondant à l'état de plus basse énergie d'un mode a une valeur moyenne nulle, mais, à cause du principe d'incertitude de Heisenberg, ne peut avoir des fluctuations nulles. On peut montrer que cet état vide est un état cohérent particulier. Il est représenté dans l'espace des phases par un disque de diamètre  $E_0$  centré sur l'origine.

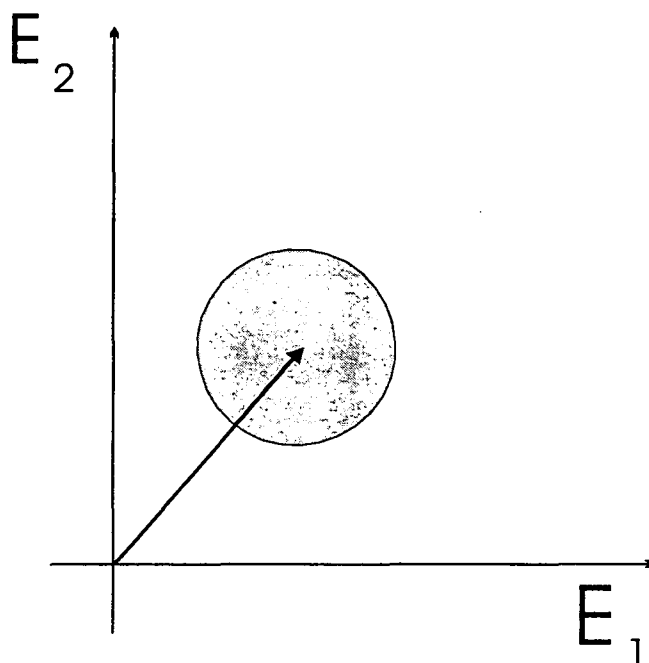


Fig. 1: Représentation d'un état cohérent dans l'espace des phases

### 2.1.2 États comprimés

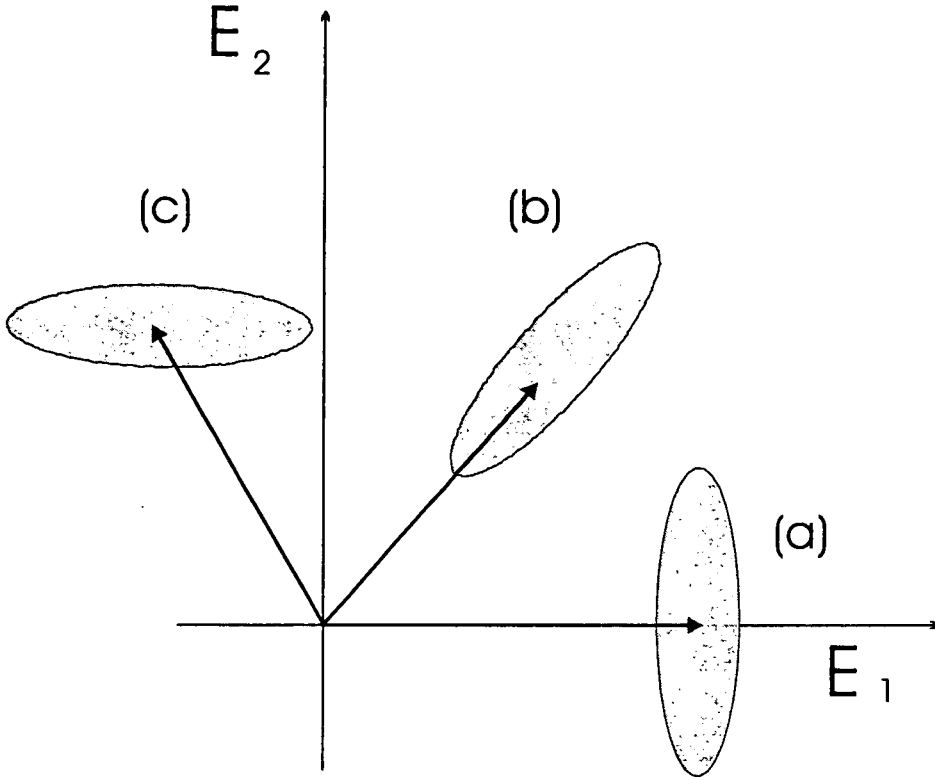
Pour réduire les fluctuations au-dessous de  $E_0$ , il faut briser la symétrie entre les deux quadratures. On peut en effet diminuer  $\Delta E_1$  pourvu que  $\Delta E_2$  augmente au-dessus de  $E_0$ . Des états possédant cette propriété sont dits *comprimés*.

Dans l'espace des phases, un état comprimé a une forme allongée, avec une dispersion plus petite sur une des composantes. Les fluctuations peuvent être comprimées en amplitude (parallèlement au champ moyen) (fig.2(a)) ou en phase (perpendiculairement au champ moyen) (fig.2(b)) ou sur une quadrature quelconque, par exemple (fig.2(c)).

### 2.1.3 Mesures d'intensité

Si le champ moyen est très grand devant les fluctuations, on peut écrire les fluctuations d'intensité dans le cadre d'une approximation linéaire :

$$\Delta I^2 = 4 \langle E_1 \rangle^2 \Delta E_1^2 \quad (2.5)$$



**Fig. 2:** Représentation d'un état comprimé dans l'espace des phases : (a) état comprimé en amplitude, (b) état comprimé en phase, (c) état comprimé en quadrature.

où nous avons supposé que le champ moyen est aligné le long de l'axe  $E_1$ . Le bruit quantique standard correspond au cas où les fluctuations d'amplitude sont égales à celles d'un champ cohérent :  $\Delta E_1^2 = E_0^2$ . Le bruit de photon est donc caractérisé par une variance proportionnelle à l'intensité moyenne :

$$(\Delta I^2)_{st} = 4 \langle E_1 \rangle^2 E_0^2 \quad (2.6)$$

En tenant compte de la relation qui relie l'intensité au nombre de photons  $N$  :

$$I = \frac{2\hbar\omega_L}{\varepsilon_0 V} N \quad (2.7)$$

nous pouvons écrire l'équation (2.6) comme :

$$(\Delta N^2)_{st} = \langle N \rangle \quad (2.8)$$

On retrouve la statistique de Poisson obtenue par le modèle de l'arrivée aléatoire de photons. Le bruit d'intensité peut être réduit par compression des fluctuations de la

composante  $E_1$ , c'est à dire ici de l'amplitude du champ. Plus généralement, dans la plupart des mesures optiques, seule une composante de quadrature du champ est concernée. On peut réduire le bruit sur la mesure en comprimant les fluctuations quantiques du champ sur la quadrature correspondante.

### 2.1.4 Modèle de champ quantique pour un faisceau laser

Jusqu'ici nous avons seulement considéré un champ strictement monomode et ses fluctuations à la même fréquence. De fait, les mesures sont sensibles aux fluctuations dans une certaine bande de fréquence, qui est l'inverse de la constante de temps utilisée. Nous sommes donc amenés à considérer des champs quasi-monomodes, avec un mode de fréquence centrale  $\omega_L$ , dont la valeur moyenne du nombre de photons peut être importante, entouré de modes de fréquences voisines qui ne comportent pas ou presque pas de photons. Ces modes sont les modes de fluctuation qui ont des fréquences non nulles par rapport à la fréquence centrale.

Pour traiter ce cas, il est commode d'introduire un champ  $\mathcal{E}(t)$  qui est une somme de Fourier des modes concernés, avec des fréquences de bruit  $\omega$  faibles par rapport à la fréquence centrale  $\omega_L$  [10] :

$$\mathcal{E}^+(t) = E_{\omega_L} \int \frac{d\omega}{2\pi} a_{\omega_L+\omega} \exp(-i\omega t) \exp(-i\omega_L t) \quad (2.9)$$

$$\mathcal{E}^-(t) = E_{\omega_L} \int \frac{d\omega}{2\pi} a_{\omega_L-\omega}^\dagger \exp(-i\omega t) \exp(i\omega_L t) \quad (2.10)$$

La relation de commutation des opérateurs transformés de Fourier  $a_{\omega_L+\omega}$  et  $a_{\omega_L-\omega}^\dagger$  est :

$$[a_{\omega_L+\omega}, a_{\omega_L-\omega'}^\dagger] = 2\pi\delta(\omega + \omega') \quad (2.11)$$

où  $E_{\omega_L}$  est défini par

$$E_{\omega_L} = \sqrt{\frac{\hbar\omega_L}{2\varepsilon_0 S c}} \quad (2.12)$$

Dans cette expression,  $S$  est la section du faisceau lumineux. On voit que la valeur moyenne du produit  $\langle a_{\omega_L-\omega}^\dagger a_{\omega_L+\omega} \rangle$  est le nombre moyen de photons par unité de temps dans le faisceau. On peut montrer que la relation d'incertitude de Heisenberg pour la dispersion des composantes de quadrature, moyennée sur un certain temps de mesure  $T$  s'écrit maintenant :

$$\Delta E_1 \Delta E_2 \geq \frac{E_{\omega_L}^2}{T} \quad (2.13)$$

Cette expression est similaire à l'expression (2.4) pour un mode de volume  $ScT$ . En utilisant cette relation pour calculer le bruit de photon (eq. 2.6) on retrouve le résultat bien connu que le bruit est réduit si l'on moyenne les mesures sur des temps plus longs.

Dans la suite, nous nous intéresserons aux composantes de Fourier des fluctuations du champ, et à leur spectre de bruit qui est directement accessible à l'expérience à l'aide d'un analyseur de spectre. La mesure de ces fluctuations fait intervenir un système optique très simple, la lame séparatrice, que nous allons présenter rapidement.

## 2.2 Bruit quantique à la sortie d'une lame séparatrice

Quand un faisceau laser passe à travers une lame semi-transparente (fig.3), en entrant par l'entrée  $A$ , les deux sorties  $C$  et  $D$  voient leur bruit quantique modifié. Cela peut se comprendre comme une conséquence de la nature corpusculaire de la lumière. Chaque photon a une probabilité  $1/2$  d'être transmis ou réfléchi. Par suite du caractère aléatoire de ce processus de transmission/réflexion, il apparaît un bruit de partition qui est indépendant de la statistique de photon du faisceau entrant. On peut retrouver cette propriété de manière plus quantitative en calculant les fluctuations des champs aux deux sorties de la lame.

### 2.2.1 Détection homodyne et détection équilibrée

Considérons un dispositif de détection homodyne comprenant une lame séparatrice qui mélange les champs entrant par les voies  $A$  et  $B$ .

Les coefficients de transmission et de réflexion en amplitude de la lame sont respectivement  $t$  et  $r$  (avec  $r^2 + t^2 = 1$ ). Les relations entre les champs quantiques entrants et sortants sont les mêmes que celles données par l'optique classique :

$$E_C = tE_A + rE_B \quad (2.14)$$

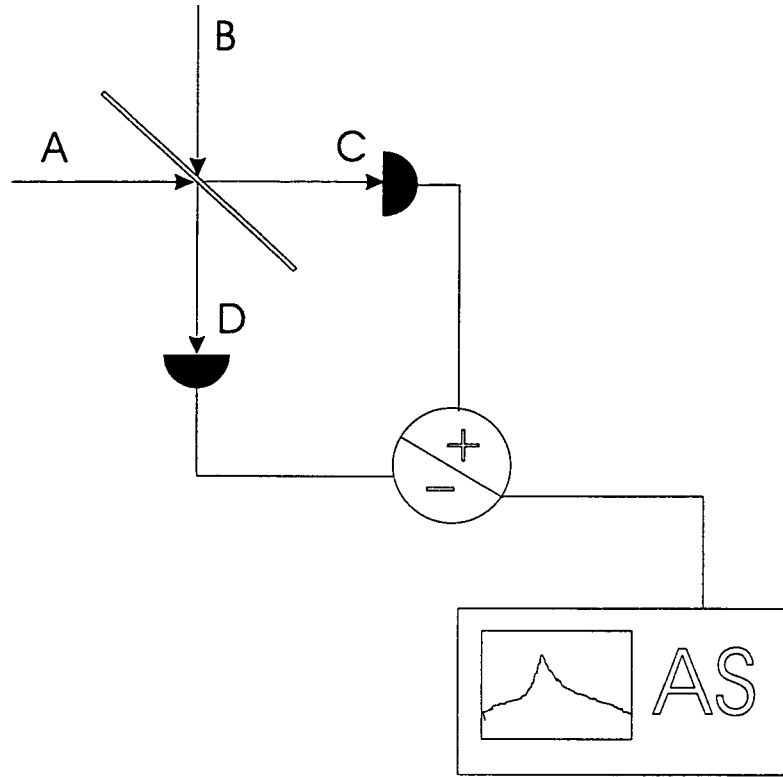


Fig. 3: Détection homodyne

$$E_D = -rE_A + tE_B \quad (2.15)$$

Le signe  $-$  dans l'équation (2.15) vient du déphasage de  $\pi$  entre les réflexions air-verre et verre-air.

Considérons tout d'abord une lame semi-transparente. Dans ce cas, on a:

$$r = t = \frac{1}{\sqrt{2}} \quad (2.16)$$

Dans la suite, nous utiliserons la méthode "semi-classique" [11] pour traiter les champs quantiques : les champs considérés sont la somme d'un champ classique de valeur égale à la valeur moyenne du champ quantique (éventuellement nulle) et d'un champ fluctuant dont la valeur quadratique moyenne est égale à celle des fluctuations quantiques. Ainsi, si on n'envoie aucun champ à l'entrée d'un système optique, il faudra supposer que le champ entrant est en fait le champ fluctuant du vide. Avec ces hypothèses on montre que l'on peut appliquer les lois de l'optique classique pour déterminer le champ sortant d'un système optique.

Nous utilisons maintenant ces principes pour calculer les intensités  $I_C$  et  $I_D$  des champs en sortie de la lame semi-transparente. En supposant que le champ  $E_A$  est beaucoup plus intense que le champ  $E_B$  nous obtenons :

$$I_C = \frac{1}{2} (|E_A|^2 + 2 \langle E_A \rangle E_{B1}) \quad (2.17)$$

$$I_D = \frac{1}{2} (|E_A|^2 - 2 \langle E_A \rangle E_{B1}) \quad (2.18)$$

La différence  $I_-$  des intensités  $I_C$  et  $I_D$  s'écrit :

$$I_- = I_C - I_D = 2 \langle E_A \rangle E_{B1} \quad (2.19)$$

Cette différence d'intensité est un signal résultant de l'homodynage par le champ  $E_A$  (oscillateur local) de la composante de quadrature du champ  $E_B$  alignée avec  $E_A$ . Lorsque l'on calcule les fluctuations de  $I_-$ , les fluctuations de l'oscillateur local disparaissent et on obtient :

$$\Delta I_-^2 = 4 \langle E_A \rangle^2 \Delta E_{B1}^2 \quad (2.20)$$

Supposons maintenant que le champ entrant en  $B$  est nul. Cela implique que les fluctuations du vide entrent en  $B$  et sont homodynées par l'oscillateur local. Le bruit sur la différence d'intensité est alors donné par :

$$\Delta I_-^2 = 4 \langle E_A \rangle^2 E_0^2 \quad (2.21)$$

Comparant ce résultat avec l'équation (2.6), on voit que ce bruit est le même que le bruit quantique standard du champ  $E_A$  de l'oscillateur local. Cette expérience procure donc une mesure directe du bruit quantique standard d'un champ quelconque.

Si maintenant on envoie sur l'entrée  $B$  de la lame semi-transparente un champ qui n'est plus vide mais qui est constitué de "vide comprimé", le bruit sur  $I_-$  n'est plus le bruit quantique standard. Si la quadrature comprimée de  $E_B$  est alignée avec  $E_A$ , le bruit sur  $I_-$  est inférieur au bruit quantique standard. La détection homodyne permet de mesurer le bruit associé à la quadrature en phase avec l'oscillateur local. En faisant varier la phase de l'oscillateur local, on peut donc mesurer le bruit dans une quadrature quelconque du champ.

Par la suite nous serons principalement intéressés à la mesure du bruit d'intensité, qui correspond au bruit de la quadrature en phase avec le champ moyen. Nous

considérons le montage de détection homodyne représenté en fig.3 avec  $E_B$  nul. En utilisant les équations (2.17) et (2.18), nous obtenons pour la somme des intensités  $I_C$  et  $I_D$  et pour ses fluctuations :

$$I_+ = |E_A|^2 \quad (2.22)$$

$$\Delta I_+^2 = 4 \langle E_A \rangle^2 \Delta E_A^2 \quad (2.23)$$

Les fluctuations de la somme des intensités sont donc proportionnelles au bruit d'intensité du champ  $E_A$ . Nous avons déjà montré (eq. (2.21)) que, dans les mêmes conditions,  $\Delta I_-^2$  est proportionnel au bruit quantique standard du champ  $E_A$ , donc le rapport  $\Delta I_+^2 / \Delta I_-^2$  fournit directement le bruit d'intensité du champ  $E_A$  normalisé au bruit quantique standard. Notons que la fiabilité de la mesure du bruit quantique standard est directement liée à la qualité de l'équilibrage entre les deux voies  $C$  et  $D$ .

### 2.2.2 Effets des pertes sur la lumière comprimée

Les pertes optiques ont un effet destructeur sur la lumière comprimée. On peut modéliser les pertes optiques linéaires qui se produisent à la traversée d'un milieu matériel par une lame séparatrice ayant un coefficient de transmission en intensité  $T$ . Nous reprenons la lame de la figure 3 et nous supposons qu'un champ non nul entre par l'entrée  $A$  tandis que le vide entre par l'entrée  $B$ . Les fluctuations du champ transmis  $E_C$  valent alors (voir eq.(2.14)) :

$$E_C^2 = t^2 \Delta E_A^2 + r^2 E_0^2 \quad (2.24)$$

La lame semi-transparente mélange les fluctuations des deux champs. Si les fluctuations du champ entrant sont égales au bruit quantique standard, on trouve que les fluctuations du champ sortant sont également au bruit quantique standard car  $r^2 + t^2 = 1$ . Cependant, si le champ entrant a des fluctuations inférieures au bruit quantique standard, les pertes tendent à ramener ses fluctuations au bruit quantique standard et donc à détruire la compression de bruit. Notons que le même raisonnement est vrai pour un champ ayant un fort excès de bruit, qui est également ramené au bruit quantique standard.

Dans un système de détection, une efficacité quantique inférieure à 1 a le même effet, puisqu'un détecteur de rendement quantique  $T$  peut être considéré comme un



détecteur de rendement quantique 1 précédé d'une lame de coefficient de transmission  $T$ .

### 2.3 Réduction de bruit dans les lasers

Depuis un peu plus de 10 ans, un certain nombre de méthodes ont été mises en oeuvre pour produire de la lumière comprimée. En dépit de la diversité des méthodes et des milieux utilisés, on peut distinguer deux types de processus : les processus dans lesquels un milieu non linéaire modifie les fluctuations quantiques de la lumière entrante, c'est en particulier le cas des processus paramétriques du second et troisième ordre et de la génération du second harmonique ; les processus lasers dans lesquels l'émission lumineuse peut être manipulée en agissant sur le processus de pompage. C'est à ce deuxième type de processus que nous nous intéresserons ici.

Dans les lasers habituels, la statistique du mécanisme de pompage des atomes dans le niveau excité (soit par injection d'atomes excités dans une cavité, comme pour les lasers à colorant, soit par excitation des atomes dans la cavité laser par pompage optique ou électrique) peut être décrite, avec une très bonne approximation, par une distribution Poissonienne pour le nombre d'atomes excités dans un intervalle de temps donné. La plupart des modèles théoriques du laser utilisent, de façon explicite ou implicite, ce type de distribution. Mais une étude plus détaillée des mécanismes d'excitation et émission montre que les fluctuations de l'intensité sortante dépendent du processus de pompage. Les premiers travaux théoriques sur les lasers avec une pompe régulière (pour laquelle le flux d'atomes pompés est constant) sont dus à Golubev et Sokolov en 1984 [7]. La première observation expérimentale de la réduction des fluctuations d'intensité d'une source lumineuse grâce à la régularisation du mécanisme de pompe a été effectuée par Teich et Saleh en 1985 [12]. Ils ont observé un état comprimé du rayonnement produit par une lampe à vapeur de mercure pompée par un faisceau d'électrons dont les fluctuations sont limitées par l'effet de charge d'espace. Cet effet, donné par l'accumulation de la charge électrique entre anode et cathode d'une lampe à décharge, était bien connu dans les tubes à vide : à cause de la distribution de charge dans l'espace, les électrons se repoussent et arrivent sur l'anode de façon très régulière. Tapster et al. (1987) ont observé une réduction du bruit d'intensité dans le faisceau émis par une diode LED alimentée par un courant électrique régulier [13]. Cependant, dans ces expériences, l'efficacité de conversion électrons-photons est très basse et, par conséquent, la compression de bruit observée très faible.

En principe on peut éviter cet inconvénient en utilisant l'émission stimulée dans un oscillateur laser (par exemple une diode laser) pour la conversion électrons-photons. Supposons que l'on dispose d'un laser avec une efficacité quantique égale à 1. Cela signifie que chaque électron de pompe sera, tôt ou tard, converti en photon cohérent sortant de la cavité. Pour que cette condition soit réalisée il est nécessaire que le taux de désexcitation des électrons par émission stimulée soit très grand devant le taux de désexcitation non radiative ou par émission spontanée, et que le taux de décroissance des photons dû au couplage à l'extérieur de la cavité soit très grand devant celui dû aux pertes internes. Il reste que l'émission stimulée et le couplage à l'extérieur de la cavité sont des processus aléatoires et le retard entre l'injection d'un électron (par le processus de pompage) et l'émission d'un photon cohérent est variable. Cependant, si la statistique des photons est mesurée dans un intervalle de temps suffisamment long devant les constantes de temps caractéristiques, la probabilité que l'électron injecté reste à l'intérieur de la cavité est négligeable. Si le nombre d'électrons pompés dans un intervalle de temps donné est régulier, il en sera de même pour le nombre de photons.

Finalement, trois conditions sont nécessaires pour produire un état comprimé en intensité à l'aide d'un laser :

- 1) les fluctuations de la pompe doivent être supprimées sous le bruit quantique standard,
- 2) le laser doit présenter une efficacité quantique très élevée,
- 3) le temps de mesure doit être grand devant les constantes de temps qui caractérisent le laser.

Ces arguments heuristiques sont confirmés par la description quantique du laser avec suppression du bruit de pompe (voir par exemple [14]).

Les premières observations de la réduction du bruit d'intensité à l'aide de lasers à semiconducteur alimentés par un courant régulier ont été réalisées par Yamamoto et collaborateurs [15, 16] en 1987. Dans ce travail, nous avons montré que les conditions décrites par Yamamoto sont nécessaires mais non suffisantes et nous avons mis en évidence les phénomènes qu'il convient de prendre en compte pour obtenir une réduction du bruit d'intensité à la sortie des lasers à semiconducteur. Nous avons également appliqué le principe de la pompe régulière à des minilasers solides. Cette méthode conduit à une réduction appréciable du bruit en sortie, même si elle ne permet pas pour l'instant d'observer de lumière comprimée.

## 3 Production d'états comprimés à l'aide des lasers à semiconducteur

### 3.1 Introduction

Depuis la première réalisation expérimentale d'un état comprimé au moyen des diodes laser, obtenue par le groupe de Yamamoto (cf. 2.3), les performances n'ont cessé de progresser, et plusieurs groupes ont participé au processus de perfectionnement et de mise au point qui a permis d'atteindre le niveau actuel [17–21]. En 1995, à température ambiante, notre groupe a obtenu une compression de 28 % (41 % corrigé des pertes) sur un laser injecté [22]. Récemment, Steel et Kilper ont obtenu 75 % de compression (corrigée des pertes) sur un laser injecté et refroidi à la température de l'Helium liquide [8]. La production d'états comprimés au moyen de diodes laser est apparue dès ses débuts comme un enjeu considérable [16] en raison des taux de compression qu'on pouvait en attendre (qui ne dépendent *a priori* que de l'efficacité quantique du semiconducteur), et de la possibilité de couvrir un large domaine spectral, celui couvert par les lasers à semiconducteurs : de 0,7 à 10  $\mu m$ . De plus la lumière peut être comprimée sur une très large bande de fréquence ; enfin les lasers à semiconducteur sont en général légers et maniables et offrent de ce fait des perspectives d'application intéressantes de la lumière comprimée qu'ils produisent.

Dans la première partie de ce chapitre (cf. 3.2), nous allons décrire les caractéristiques de diodes utilisées, les éléments essentiels du montage et les techniques expérimentales qui permettent d'obtenir une compression de bruit. La deuxième partie sera consacrée à la présentation de résultats expérimentaux que nous avons obtenus sur la réduction de bruit à température ambiante (cf. 3.3) et à la mise en évidence d'un processus très important pour la réduction de bruit : les anticorrélations entre modes. La troisième partie détaille l'interprétation des résultats dans le cadre

des modèles théoriques communément utilisés pour décrire les propriétés de bruit des lasers [14, 23, 24].

## 3.2 La mise en oeuvre expérimentale

### 3.2.1 Les diodes laser utilisées

Nous avons utilisé les diodes lasers de la série 5400 produites par Spectra Diode Laboratories. Ces diodes sont fabriquées à partir de GaAlAs par croissance épitaxiale selon le procédé de déposition par vapeur chimique organométallique. Cette série couvre le domaine spectral de 810 à 860 *nm*. Les diodes sont dotées d'une structure à puits quantiques avec un guide d'onde obtenu par variation de l'indice de réfraction. La face arrière est pourvue d'un traitement réfléchissant à 95% et d'une photodiode interne. La face avant a subi un traitement antireflet à large bande et ne réfléchit que 4% de la lumière émise.

La puissance maximale émise est de 100 *mW* (SDL-5410) à 150 *mW* (SDL-5420). Les qualités optiques du faisceau sont caractérisées par un faible astigmatisme, un profil gaussien en champ lointain et un comportement monomode transverse. Les dimensions de la surface émettrice sont de  $1 \times 3 \mu m$ . La divergence du faisceau est de 30 ° perpendiculairement à la direction de polarisation principale et de 10 ° parallèlement à cette direction. Dans le montage que nous utilisons au laboratoire, le faisceau lumineux divergent est collimaté par une optique de collimation Melles Griot 06GLC002/D, de focale 8 mm. La tache formée en sortie du collimateur est elliptique, le rapport du grand axe au petit axe est de 3.

Le courant de seuil de ces diodes est assez faible, de l'ordre de 20 à 30 *mA* à 25 °C, le courant maximal est de l'ordre de 160 à 170 *mA*. Leur efficacité quantique différentielle est de 65% à 70%. Ceci permet d'atteindre, pour des puissances modérées, des régimes de fonctionnement dont l'efficacité quantique est satisfaisante.

Ces diodes ont un fonctionnement monomode longitudinal garanti jusqu'à 100 *mW*, avec une atténuation des modes latéraux de l'ordre de 25 *dB* par rapport au mode principal. La largeur spectrale est d'environ 15 *MHz* à 100 *mW*. Le taux de polarisation augmente avec la puissance de sortie ; il est à peu près de l'ordre de 1000 pour un courant d'une centaine de milliampères.

### 3.2.1.1 Alimentation

L'alimentation doit présenter des caractéristiques bien précises pour garantir un fonctionnement correct de la diode laser : en particulier le bruit du courant qu'elle produit doit être très faible devant le shot noise de façon à se conformer au principe de la pompe régulière et à éviter les problèmes supplémentaires induits par un courant d'alimentation bruyant (augmentation de la largeur de raie). Nous avons utilisé des alimentations stabilisées fabriquées au laboratoire. Elles emploient des composants électroniques à faible bruit et le bruit du courant qu'elles délivrent est très inférieur au shot noise. Il est de plus atténué par un filtre placé en sortie de l'alimentation. Le filtre est composé d'une forte capacité en parallèle avec la sortie de l'alimentation, qui détourne les hautes fréquences vers la masse, suivie, en série avec la diode, d'une forte impédance (résistance ou inductance selon les régimes du courant d'alimentation). L'ensemble alimentation-filtre forme une source de courant de haute impédance qui fournit une source de pompage sans bruit à la diode laser.

Typiquement pour des régimes de courant modéré -jusqu'à 80 mA- nous avons retenu un filtre  $RC$  avec  $R = 100 \Omega$  et  $C = 1 \mu F$ , et pour les régimes de fort courant un filtre  $LC$ , avec  $L = 1 mH$  et  $C = 1 \mu F$ . La résistance interne de la diode laser est de l'ordre de  $3 \Omega$ .

L'alimentation dispose d'une entrée pour la modulation. La capacité de moduler la puissance de sortie d'une diode laser en modulant son courant d'alimentation est une caractéristique importante pour les diodes lasers, que nous avons exploitée dans les expériences présentées ici : l'optimisation du seuil de la diode sur réseau, l'optimisation du taux de réjection de la détection équilibrée, la mesure de la réponse en fréquence du cristal de  $Nd:YVO_4$  (cf. ch.5) et la réalisation d'une expérience de spectroscopie d'absorption (cf ch. 6).

Une modulation basse fréquence (quelques  $kHz$ ) peut être produite à partir de l'entrée prévue à cet effet sur le circuit d'alimentation. Une modulation à plus haute fréquence exige une prise directe sur la diode avec un dispositif de protection adéquat. Celui ci est constitué d'une capacité et une résistance de  $47 \Omega$  placées en série avec la diode.

### 3.2.1.2 Régulation de température

L'environnement thermique est un élément important touchant à la durée de vie et au bruit d'intensité des diodes laser.

Le dispositif de régulation thermique comporte deux éléments principaux : l'élément Peltier et la thermistance intégrés à la diode ; la régulation de chauffage du boîtier externe qui contient la diode. Un module électronique de régulation de température (PID), pilote l'élément Peltier afin de stabiliser la température de la diode entre  $-10^{\circ}\text{C}$  et  $40^{\circ}\text{C}$ . La température à l'intérieur du boîtier est maintenue à  $30^{\circ}$  au moyen de transistors régulés électroniquement. La stabilisation atteinte est de l'ordre de  $0.01^{\circ}\text{C}$ .

### 3.2.2 Le système de détection

La presque totalité de nos mesures de bruit sur les diodes laser s'est faite par **détection équilibrée**. Cependant, nous avons eu recours dans certains cas à la détection directe du faisceau avec calibration indépendante, notamment dans le cas du montage cryogénique et dans les mesures de bruit faites après le monochromateur (cf 3.3.2 et 3.5.2). Dans la suite nous appellerons ce deuxième type de détection **détection calibrée**.

#### 3.2.2.1 La détection équilibrée

La procédure est la suivante : on partage à l'aide d'un séparateur optique le faisceau initial en deux parties égales que l'on détecte avec deux chaînes de mesure identiques. Comme nous l'avons indiqué en 2.2.1 la somme des deux signaux permet de reconstituer les propriétés du faisceau initial, en particulier son spectre de bruit, tandis que la différence donne le bruit quantique standard d'un faisceau d'intensité égale à celle du faisceau incident. Comme la réponse de la chaîne de détection dépend de la fréquence, le bruit quantique standard mesuré n'est pas un bruit blanc. La détection équilibrée permet de comparer presque instantanément le bruit de la source étudiée au bruit quantique standard, pour chaque fréquence de bruit en réduisant considérablement les sources d'erreurs par rapport à la détection calibrée. La compression de bruit de certains dispositifs pouvant être très faible, l'équilibrage de la détection est un point critique du montage.

Une fois le faisceau initial partagé en deux faisceaux de puissance égale, il s'agit de détecter ces deux faisceaux au moyen d'une chaîne de mesure parfaitement symétrique qui permette d'additionner ou de soustraire l'un de l'autre le bruit d'intensité des deux faisceaux. Les éléments de la chaîne sont les suivants :

- séparation du faisceau : lame demi onde et cube polariseur
- détection des photons : photodiodes
- amplificateurs de photodiodes: sortie continue et haute fréquence

- sommateur soustracteur
- amplificateur (éventuellement)

**Les photodiodes** Les photodiodes doivent présenter les caractéristiques suivantes : une excellente efficacité quantique, car tout défaut d'efficacité quantique ramène le bruit vers le bruit quantique standard et réduit la compression mesurée. Il est important de travailler dans des domaines d'intensité lumineuse qui ne produisent pas de saturation, ce que l'on vérifie en mesurant la réponse en continu et à diverses fréquences du signal qu'elles délivrent pour plusieurs intensités incidentes. Les photodiodes que nous avons utilisées sont des FND-100 de EG&G, de très large bande (350  $MHz$ ), de faible capacité interne (8 à 10 pF), et dont la surface photosensible est un disque de diamètre 1 mm. Nous les avons polarisées en 75 V. Leur efficacité quantique est très élevée (90%). Nous n'avons observé aucune saturation en DC pour des puissances détectées jusqu'à 45  $mW$ . Cependant, la saturation de la réponse AC est observée pour des puissances beaucoup plus faibles, de l'ordre de 10  $mW$ .

Enfin, les deux photodiodes utilisées pour la détection équilibrée doivent présenter une réponse aussi similaire que possible. Parmi les photodiodes disponibles nous sélectionnons les deux qui présentent les efficacités quantiques les plus proches. La réponse en continu des photodiodes que nous avons utilisées était identique à 1%. En revanche, la réponse en fréquence peut être assez différente et ce déséquilibre doit être compensé au niveau des amplificateurs des photodiodes.

Le montage amplificateur sépare en deux voies distinctes la partie continue du courant (voie DC) et la partie haute fréquence (voie HF). Le continu est envoyé sur une résistance de charge de précision (1%) qui convertit le courant en une tension restituée par un OP27 monté en suiveur, avec un gain global de 100  $mV/mA$ .

La partie haute fréquence, comporte un filtre passe-haut puis un amplificateur à faible bruit de type CLC425 en montage transimpédance. La tension délivrée par l'amplificateur est finalement filtrée à nouveau par un filtre passe-haut afin de supprimer toute composante continue résiduelle (dommageable pour l'analyseur de spectre) et la sortie de l'amplificateur est adaptée 50  $\Omega$ . Typiquement les ordres de grandeur des fréquences de coupure des filtres présents sur la partie haute fréquence sont les suivants :

- filtre passe-haut d'entrée : moins de 10  $kHz$
- filtre passe-haut de sortie : 10  $kHz$
- filtre passe-bas dû au montage transimpédance de l'amplificateur ( $\sim 30 MHz$ ).

L'équilibrage des amplificateurs se fait de la manière suivante : il s'obtient en continu par l'utilisation de résistances de précision ; pour les hautes fréquences on envoie une modulation de faible amplitude sur les résistances de charge de chaque amplificateur et on enregistre les courbes de réponse obtenues à l'analyseur de spectre en faisant varier la fréquence de la modulation. On vérifie que les deux chaînes donnent des courbes de gain parallèles (si ces courbes ne sont pas parallèles on peut être amené à changer un des amplificateurs). Puis on ajuste les gains à des valeurs égales en ajustant une des résistances de gain.

**Soustracteur-additionneur** Le soustracteur-additionneur a deux entrées qui reçoivent les signaux envoyés par les deux photodiodes et une sortie qui donne soit la somme soit la différence des signaux d'entrée. Il doit présenter un faible bruit, une large bande, et pas de saturation.

Nous avons utilisé des modules soustracteurs de marque MiniCircuit. Les pertes d'insertion sont de l'ordre de  $3,6 \text{ dB}$  pour une utilisation en sens inverse. Ces modules ont une très grande bande passante (de 1 à  $200 \text{ MHz}$ ) et fonctionnent avec une adaptation d'impédance à  $50 \Omega$ .

Nous nous sommes servis de trois modules soustracteurs pour réaliser la chaîne de détection. Le principe du montage est le suivant : le signal de chaque photodiode amplifié est envoyé sur une des bornes d'entrée de deux soustracteurs ; un bouchon  $50 \Omega$  est placé sur l'autre borne. Les deux signaux de sortie sont envoyés chacun sur les bornes d'entrée du troisième soustracteur. Selon que les signaux des photodétecteurs sont reliés à la même borne des soustracteurs ou non, on obtient un signal proportionnel à la différence ou à la somme de ces signaux. Le taux de rejection mesuré du soustracteur-additionneur est supérieur à  $40 \text{ dB}$  sur une bande de  $10 \text{ kHz}$  à  $30 \text{ MHz}$ . Pour compenser les pertes d'environ  $6 \text{ dB}$  introduites par le sommateur-soustracteur, nous avons placé deux amplificateurs de type ZHL de  $9 \text{ dB}$  de gain en sortie du montage.

**Chaîne complète** Pour vérifier l'équilibrage du montage complet (cube polariseur, photodiodes, amplificateurs, sommateur-soustracteur) nous mesurons le taux de rejection en modulant l'intensité du faisceau émis par le laser (par exemple en modulant le courant d'alimentation de la diode) et en observant l'atténuation du pic de modulation sur le signal – par rapport au signal +. Nous sommes parvenus à un taux de rejection de  $-38 \text{ dB}$  sur la bande  $10 \text{ kHz} - 20 \text{ MHz}$ .



### 3.2.2.2 La détection directe calibrée

Le signal obtenu par détection directe à l'aide d'un seul système photodiode et amplificateur donne le bruit d'intensité du faisceau. Il faut pouvoir le comparer avec le bruit quantique standard correspondant. Pour cela deux méthodes ont été utilisées. L'une à référence fixe, l'autre à référence simultanée.

La méthode à référence fixe consiste à mesurer avec l'analyseur de spectre le bruit d'une source lumineuse au bruit quantique standard (faisceau laser atténué ou lampe halogène) à l'aide de la même chaîne de détection que pour le faisceau à étudier. On enregistre sur fichier informatique cette référence de bruit quantique standard. Sachant que le bruit quantique standard évolue linéairement avec l'intensité de la source, on peut déduire du spectre précédent le spectre correspondant à une intensité quelconque de la source. Une telle méthode donne des résultats reproductibles dans un intervalle de confiance de l'ordre de quelques %.

Le principe de la détection à référence simultanée est le suivant : nous utilisons les deux photodiodes et l'électronique de la chaîne de détection équilibrée en configuration sommatrice. Une des photodiodes est placée devant le faisceau à mesurer, l'autre est placée devant le faisceau d'une diode laser sur réseau fonctionnant au bruit quantique standard et d'intensité ajustée pour que les signaux continus donnés par les deux photodiodes soient égaux. En masquant le premier faisceau on a le bruit d'intensité de la source étudiée, et en masquant le second on a la référence de bruit quantique standard correspondant au même photocourant. Nous pouvons de plus contrôler, en mettant les photodiodes dans la configuration de la détection équilibrée, que le faisceau de référence est effectivement au bruit quantique standard. Nous estimons la précision relative à environ 3% sur l'écart mesuré par rapport au bruit quantique standard.

### 3.2.3 Les techniques d'affinement spectral

La simple application du principe de la pompe régulière à la diode laser est insuffisante pour obtenir des états comprimés du rayonnement à température ambiante (la situation est différente en régime cryogénique). L'utilisation de techniques d'affinement spectral telles que l'insertion de la diode dans une cavité étendue et l'injection optique permet de franchir la limite quantique standard et d'observer une réduction du bruit d'intensité (*amplitude squeezing*). Nous présentons ici les principales caractéristiques de ces techniques.

### 3.2.3.1 La diode sur réseau

Le montage d'une diode laser sur réseau fait partie des techniques classiques de réduction de largeur de raie par implantation du laser en cavité étendue [25]. La diode et le réseau sont montés en configuration de Littrow : l'ordre 1 du réseau est renvoyé sur la diode (24% de la puissance incidente), et l'ordre 0 (60%) constitue le faisceau de sortie. La cavité est donc formée par la face arrière de la diode et par le réseau. Ce montage permet d'abaisser le seuil de la diode, de réduire considérablement la largeur de raie et de balayer la longueur d'onde de la diode.

Typiquement le seuil de la diode sur réseau est de 13 – 15 *mA* alors que, dans les mêmes conditions de température, la diode libre présente un seuil de 18 – 20 *mA*. La largeur de raie peut se réduire encore avec une cavité plus longue [26]. Ainsi, il est possible, au moyen d'une cavité de quelques centimètres de réduire la largeur de raie d'une diode commerciale de 40 *MHz* à moins de 10 *kHz*. L'utilisation du réseau permet en outre de balayer un large domaine de longueurs d'onde avec des sauts d'un mode à l'autre (rotation du réseau dans un plan orthogonal à la direction de propagation du faisceau laser), mais également de balayer continûment la fréquence de la diode sur plusieurs *GHz* (variation de la longueur de la cavité à l'aide de la cale piézo-électrique solidaire au réseau) [27]. La plage de balayage d'une diode sur réseau est de l'ordre de  $\pm 10$  *nm* autour de la longueur d'onde nominale de la diode libre (déterminée par la température de fonctionnement et le courant d'alimentation).

### 3.2.3.2 L'injection optique

Le montage d'un laser en injection répond à un schéma très simple mettant en jeu deux lasers. L'un, le laser esclave, est celui dans lequel est injecté le signal émis par l'autre, le laser maître. Plusieurs conditions expérimentales doivent être remplies pour réaliser l'injection. Les deux lasers doivent être susceptibles d'osciller exactement à la même fréquence, leur recouvrement spatial doit être correct et le laser maître ne doit pas être déstabilisé par le retour du signal qu'il envoie vers l'esclave, ce qui exige l'emploi d'isolateurs optiques. Lorsque l'injection fonctionne, la phase de l'esclave se bloque sur celle du maître.

Le montage que nous avons utilisé est le suivant: le laser maître est constitué par une diode sur réseau, dont la longueur d'onde nominale est, à quelques nanomètres près, égale à celle de l'esclave. Le faisceau maître, après avoir traversé une lame demi-onde, une paire de prismes anamorphoseurs et un isolateur optique qui le protège du retour de lumière, est envoyé vers l'esclave à travers la fenêtre latérale d'un isolateur

optique. Le faisceau de sortie du laser esclave passe par la fenêtre centrale de l'isolateur et est détecté. Une lame demi-onde, placée juste avant l'isolateur, permet d'ajuster la puissance injectée dans l'esclave. La puissance injectée dans l'esclave est assez faible, puisqu'elle ne représente pas plus de  $5 \text{ mW}$ . L'avantage de ce type de montage consiste dans le fait que le couplage spatial et de polarisation entre laser maître et esclave se font automatiquement.

L'injection exige également un bon accord en fréquence entre les deux lasers. Le laser esclave oscillera sur celui de ses modes longitudinaux le plus proche du mode du laser maître. Ce mode longitudinal n'est donc pas nécessairement le mode principal. Cependant, il ne doit pas se situer trop loin de celui-ci sur la courbe de gain de la diode. A température ambiante, il est possible d'asservir sur une plage de l'ordre de  $1 \text{ nm}$  de part et d'autre de la longueur d'onde du laser libre. De plus, il est possible de modifier légèrement la fréquence du laser maître d'un  $\text{GHz}$  environ, sans détruire l'asservissement, de sorte que le laser injecté possède une réelle capacité de balayage en fréquence, suffisante notamment pour couvrir des raies d'absorption atomiques.

### 3.3 Résultats expérimentaux

Nous avons étudié le bruit d'intensité à température ambiante de plusieurs diodes laser avec une longueur d'onde d'émission de  $810$  ou  $850 \text{ nm}$ . Des réductions de bruit ont été observées dans le cas de la diode sur réseau et de la diode injectée. Les résultats obtenus sont présentés dans l'article qui suit. Dans cet article est aussi analysé le bruit de phase des diodes laser en présence des techniques d'affinement spectral. Les mesures concernant le bruit de phase ont été effectuées à l'Institut d'Optique d'Orsay par le groupe de Philippe Grangier.

#### 3.3.1 Reproduction de l'article : "Quantum noise of free-running and externally-stabilized laser diodes" (Quantum Semiclass. Opt. 7, 601 (1995))

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## Quantum noise of free-running and externally-stabilized laser diodes

T-C Zhang†§, J-Ph Poizat†, P Grelu†, J-F Roch†, P Grangier†, F Marin†, A Bramati†, V Jost†, M D Levenson†|| and E Giacobino†

† Institut d'Optique, BP 147, F91403 Orsay Cedex, France

‡ Laboratoire Kastler Brossel, Université Pierre et Marie Curie, F-75252 Paris Cedex 05, France

**Abstract.** We have investigated the intensity and phase noise of single-mode laser diodes, either free-running or using different types of line narrowing techniques at room temperature, namely feedback from an external grating and injection locking. We have measured an intensity squeezing of 1.2 dB in the first case, and 1.4 dB in the second case (respectively, 1.6 dB and 2.3 dB inferred at the laser output). We have observed that the intensity noise of a free-running 'single-mode' laser diode actually results from a cancellation effect between large anticorrelated fluctuations of the main mode and of weak longitudinal side modes. It is also shown that free-running diodes exhibit very large excess phase noise, typically more than 80 dB above shot noise at 10 MHz, which can be significantly reduced by the above-mentioned techniques.

### 1. Introduction

Quantum noise reduction in laser emission based on pump-noise suppression was first predicted in 1984 [1]. Semiconductor lasers are particularly well suited for the implementation of this property since it is possible to drive them with a current whose noise is well below shot noise. If the quantum efficiency of the carrier-to-photon conversion is high enough, the electron statistics of the pumping can be transferred to the light emission, yielding sub-Poissonian operation of the laser. Quantum noise in the intensity of constant-current-driven laser diodes was observed for the first time by Machida *et al* in 1987 [2], and further improved to 8.3 dB in 1991 [3]. This result was obtained in a measurement at 4 K, where the detector was closely coupled to the laser. The difficulties encountered by other groups in reproducing this result suggested that factors other than the constant current supply could be important for the noise reduction. In 1993, it was shown by Steel and his group [4, 5] that line-narrowing techniques (cf [6] and references therein) greatly helped in the noise reduction by further suppressing the weak but very noisy longitudinal side modes. Intensity squeezing of 3 dB (4.3 dB if corrected for detection efficiency) at 10 K and of 1.8 dB (2.0 dB corrected) at room temperature was obtained by injection locking the laser [5] or by feedback from an external grating [4], respectively.

We have investigated intensity noise but also more generally quadrature noise of laser diodes, using various methods for line narrowing, including injection-locking with another diode laser or a Ti:Sapphire laser, and feedback from an external grating. The best intensity squeezing at room temperature was 1.4 dB (2.3 dB when corrected for the detection efficiency), and was obtained with injection-locking. As far as quadrature noise is concerned,

§ Institute of optoelectronics, Shanxi University, Taiyuan 030006, China.

|| Levenson Consulting, 19868 Bonnie Ridge Way, Saratoga CA 95070, USA.

we have shown that the large excess phase noise of semiconductor lasers can be partially reduced by these line-narrowing techniques.

## 2. Experimental set-up. General features

The laser diodes we have used are index-guided quantum well GaAlAs laser diodes (model SDL 5422-H1 and SDL 5411-G1), operating at 850 and 810 nm. The rear facet reflection coefficient is 95%, the front facet is AR coated with a reflection coefficient of about 4%. The laser diodes are temperature stabilized and carefully electromagnetically shielded. Appropriate electrical filtering is used on the power supply†. The free-running laser diodes have a rather low threshold of 18 mA and a differential quantum efficiency (slope above threshold) of 66%. The operating current in the experiments described below is typically 5–7 times larger than the threshold current, and the resulting high overall quantum efficiency is at the origin of the squeezing. No squeezing was found in similar experiments performed on laser diodes with higher threshold (80 mA), which operate only 2 times above their threshold.

The quantum noise in the intensity is measured in the standard way with balanced detection [7]. The beam going out of the laser is split in two equal parts by a beamsplitter. Each output of the beamsplitter is sent into a high efficiency (90%) photodiode (EG&G model FND100 or C30809E). The DC parts of the photodiode currents are filtered out while the AC parts are amplified using 20 MHz bandwidth amplifiers. The amplifiers' outputs, proportional to the noise signals, are either subtracted or added by a RF +/– power combiner. When set on the difference position, the circuit gives a signal proportional to the shot noise, while in the sum position, it gives the full intensity noise of the beam impinging on the beamsplitter. The output of the +/– power combiner is sent to a spectrum analyser and noise spectra are recorded for the sum and the difference signals. The electronic noise is then subtracted on each recording.

Consistency between the shot noise of a laser diode measured in this way and the noise of a white light source was carefully checked. The beamsplitter is either a 50/50 coated plate or a polarizing beamsplitter preceded by a half-wave plate. In the latter case, a polarizer must be placed at the output of the laser diode, in order to eliminate the small component of polarization perpendicular to the main polarization direction [5, 8]. Otherwise, the interference between the two components, which are mixed by the polarizing beamsplitter, may give rise to errors in the noise measurements.

When biased with a high voltage ( $V > 70$  V), the photodiodes do not exhibit any DC saturation, for detected powers up to at least 45 mW. The AC response has a slight linear dependence on the detected DC power. However, the response is the same for the sum and difference positions of the power combiner, and so the balanced detection is not affected by the change in the AC response. At high powers, heating of the photodiodes causes a small decrease in the noise signal when the detectors have been illuminated for a few tens of seconds. In such cases, noise measurements were performed using short time intervals. Finally, consistency checks for high values of detected power were done by measuring the noise reduction as a function of the value of a neutral density filter, inserted in the beam before the photodiodes.

Two types of set-ups were investigated to achieve line narrowing: cavity extension with an external grating and injection-locking with another laser.

† We have used RC filters ( $R = 47 \Omega$ ,  $C = 1 \mu\text{F}$ ) for a driving current less than 100 mA, and LC filters ( $L = 1 \text{ mH}$ ,  $C = 1 \mu\text{F}$ ) for higher currents.

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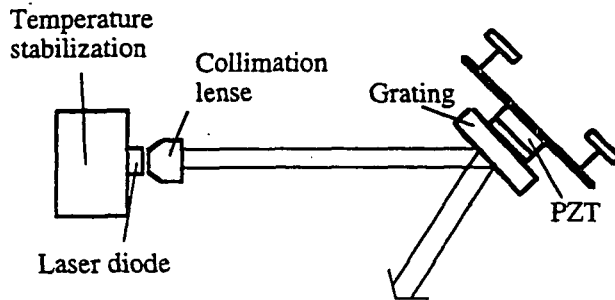


Figure 1. External grating stabilization scheme.

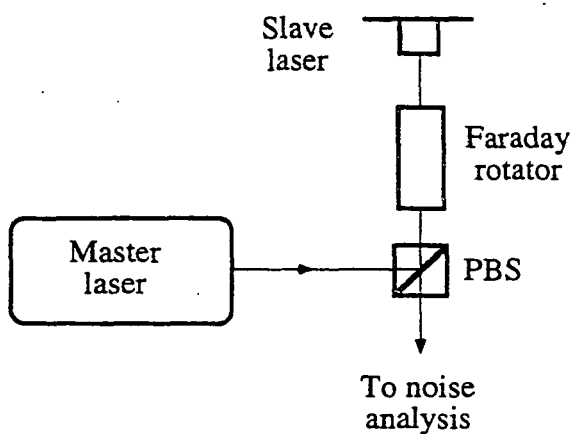


Figure 2. Injection-locking scheme. The Faraday rotator rotates the linear polarization by  $45^\circ$ , PBS is a polarizing beam splitter. The master laser is either a grating-extended laser diode (cf figure 1) or a frequency stabilized Ti:Sapphire laser.

The extended-cavity laser diode is shown in figure 1. The beam going out of the laser diode is collimated with a  $f = 8$  mm objective placed in front of the output facet of the diode. The cavity is extended to 10 cm with a reflection holographic grating (Jobin-Yvon, 1200 lines/mm) reflecting the first order into the cavity, while the zero order goes out of the cavity (Littrow configuration). The grating is glued on a piezoelectric transducer, which is mounted on a finely orientable mirror mount. The efficiency of the grating is 60% in the zero order (output coupling) and 24% in the first order (feedback to the laser), with 16% losses. The alignment of the grating is critical. When it is achieved, the threshold of the laser is lowered from 18–13 mA and the DC power of the side modes goes down to  $-60$  dB below the DC power of the main mode, while the total intensity noise is decreased below the shot-noise level.

The injection-locking scheme is depicted in figure 2. The master laser is either an external-grating diode laser or a Ti:Sapphire laser. It is injected into the slave laser by means of an optical isolator. The master beam enters through the escape port of the polarizer placed after the Faraday rotator. Locking is observed over a rather broad power range† of the master laser, from 1–4 mW. The direction of the master laser must be carefully adjusted for optimum noise reduction.

The detection scheme for the quadrature noise measurement is described in the corresponding section (section 4).

† It should be mentioned that only a small fraction (a few per cent) of this injected power is actually coupled to the lasing mode of the diode due to the imperfect mode overlap.

### 3. Intensity squeezing

#### 3.1. Experimental results

We have investigated intensity squeezing in the two cases described above. Noise spectra were recorded for various supply currents. Squeezing was observed for currents higher than 50 mA ( $I/I_{th} = 2.8$ ) for the injected laser and 30 mA ( $I/I_{th} = 2.4$ ) for the extended cavity laser, at noise frequencies from 1–30 MHz (limited by our detection bandwidth). The noise, measured with a resolution bandwidth of 1 MHz, was nearly constant from 7–30 MHz. The optimum squeezing was observed in the injection-locking scheme. At 7 MHz, with a driving current of 130 mA, we obtained a noise reduction of 27%, i.e. 1.4 dB. Taking into account the total detection quantum efficiency of 65% from the laser output power to the photodiode current (through the optical isolator), we infer a value of 2.3 dB at the output of the laser diode. The best squeezing obtained with the grating-extended cavity is 25% (1.2 dB) at 30 MHz and 110 mA, from which we infer a 1.6 dB noise reduction at the output of the

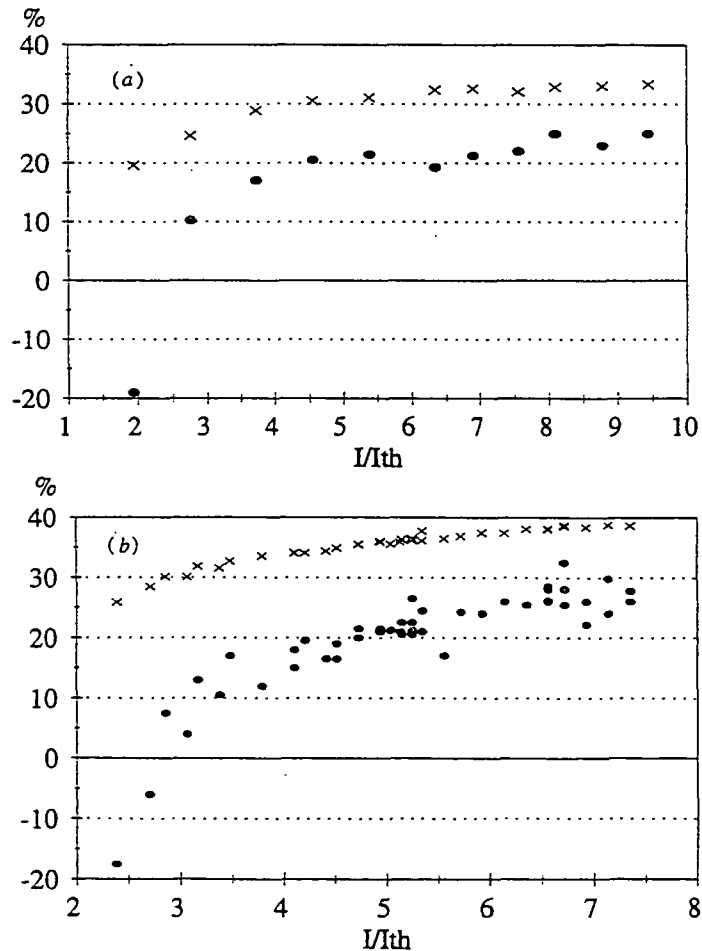


Figure 3. Noise reduction (full circles) and ratio of the detector current to the driving current (crosses) versus driving current normalized to threshold, (a) for the grating and (b) injection-locked.

grating. The fact that the squeezing is better with the injection-locking scheme can be attributed to the large losses due to the grating.

These numbers are close to those of [4, 5]. They are below the theoretical maxima expected from the quantum efficiency of the laser, which are, respectively, 58% (3.8 dB) at 130 mA for the injected laser and 42% (2.4 dB) at 110 mA for the grating-extended cavity. To check the dependence of the noise reduction on the laser diode quantum efficiency, in figure 3 we have plotted the intensity squeezing and the ratio of the detector current to the driving current against the driving current for the grating-extended laser (figure 3(a)) and for the injection-locked laser (figure 3(b)). It can be noticed that the ratio between the intensity squeezing and the current-to-current efficiency goes towards a maximum asymptotic value of 0.75, instead of the expected value of unity. Steel and coworkers, using similar laser diodes, obtained comparable values for this ratio: 0.83 for an injection-locked laser at 10 K [5] and 0.72 for a room temperature laser with external grating feedback [4]. This non-unity value can be attributed to additional noise sources in the semiconductor devices which are not included in the simple theoretical prediction mentioned above. Let us note that Richardson *et al* [3] observed a squeezing of 85% with a current-to-current efficiency of only 48%. This was attributed to the existence of another non-lasing junction, connected in parallel to the lasing one, and to the fact that 'electrical splitting' does not introduce partition noise [9, 10]. These various observations show that a comprehensive theoretical model of the quantum noise of laser diodes is still needed.

### 3.2. Role of the longitudinal side modes

The free-running laser diode apparently operates on a single mode. However, the longitudinal side modes have a non-negligible power, the closest ones being only  $-10$  to  $-25$  dB below the main mode (figure 4). The arguments given in [4, 5] to explain why

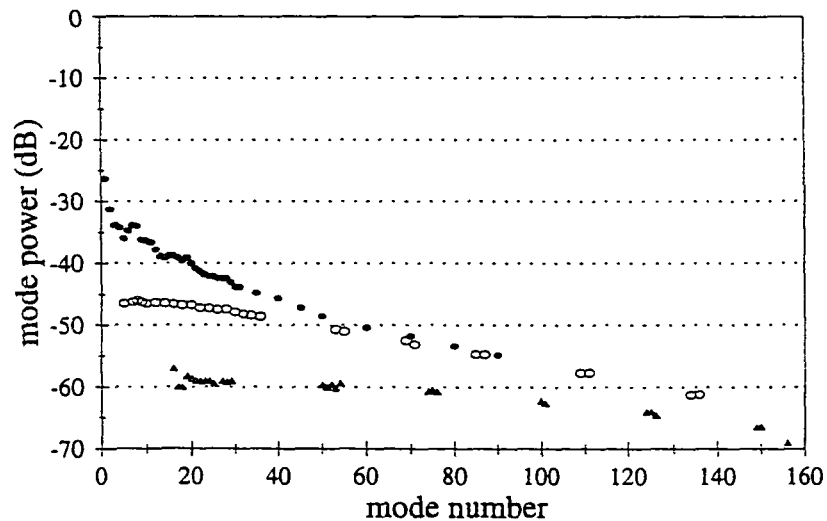


Figure 4. Power of individual longitudinal modes (optical power in dB, with respect to the main mode), measured with a high-resolution monochromator, for a driving current of 80 mA. On the x-axis each mode is labelled by a number, the number 0 corresponding to the main mode. The full circles are for the free-running laser, the open circles for the injection-locked laser, and the full triangles for the external grating configuration.



side-mode suppression reduces the total intensity noise to below the shot-noise level, tend to suggest that because these side modes are very noisy, the less powerful they are the less they will contribute to the total intensity noise. In order to explore these arguments more precisely, we have investigated the noise properties of individual modes by sending the laser beam through a high-resolution spectrometer. We have observed that the intensity noise of the main mode of a free-running laser diode is much larger (40 dB above shot noise) than the total intensity noise (2 dB above shot noise). This low value of the total intensity noise is then explained by very strong anticorrelations between the intensity noise of the main mode and the one of the whole set of side modes [11]. This effect will be analysed experimentally and theoretically in a forthcoming publication [12].

#### 4. Phase noise

##### 4.1. Quadrature noise detection scheme

The investigation of the phase noise of a laser beam requires a phase-to-amplitude converter, i.e. a device whose complex transmission  $T$  depends on the frequency  $\omega$ . In this paper, for this purpose we use the reflection off an empty detuned Fabry-Pérot cavity [13] as shown in figure 5. When the rear mirror is highly reflecting, this system has the advantage over a Mach-Zehnder interferometer that the mean-field transmission  $|T(\omega = 0)|$  does not depend on the cavity detuning and is always equal to unity. This makes the shot-noise reference level independent of the quadrature analysed. Phase noise analysis is then carried out conveniently for frequencies in the range of the cavity bandwidth.

Explicit expressions of the quadrature rotation after reflection off a detuned Fabry-Pérot cavity are given in appendix A. A simple way to understand this effect is to have in mind that in Fourier space, the quadrature component  $X(\omega)$  can be written as  $X(\omega) = (a(\omega) + a^\dagger(\omega))/\sqrt{2} = (a(\omega) + [a(-\omega)]^\dagger)/\sqrt{2}$ . The key point which yields a quadrature rotation is that the various frequency components at 0 (mean field),  $\omega$  and  $-\omega$  do not undergo the same phase shift when the laser is scanned across the resonance peak of the cavity. The quadrature rotation is zero in two cases: when the laser is tuned exactly

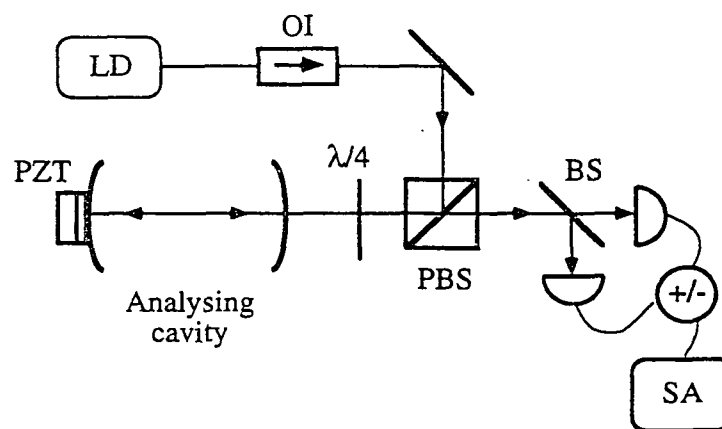


Figure 5. Phase noise detection set-up. Great care has been given in order to avoid any feedback from the analysing cavity to the laser, and optical isolation (oi) of about 80 dB has been used. The rear mirror is a high reflector and its position is controlled by a piezo electrical transducer (PZT).

on resonance, where the phase shifts for both frequency components  $\pm\omega$  cancel out, and when it is tuned far outside the peak, where all frequency components undergo the same phase shift of 0 or  $\pi$ .

In our set-up the Fabry-Pérot cavity has a half-width at half-maximum (HWHM) of 8 MHz and a finesse of  $\mathcal{F} = 125$ . The rear mirror is highly reflecting, but its small leaks nevertheless allow us to monitor the intracavity intensity to adjust the mode matching. One of the mirrors is mounted on a piezo-electrical transducer, so that the length of the cavity can be scanned.

#### 4.2. Experimental results

We have measured the quadrature noise of a laser diode in the same three configurations which were used for the intensity noise measurements described above. These results are presented in figure 6. The phase noise (quadrature angle  $\pi/2$  with respect to the mean field)

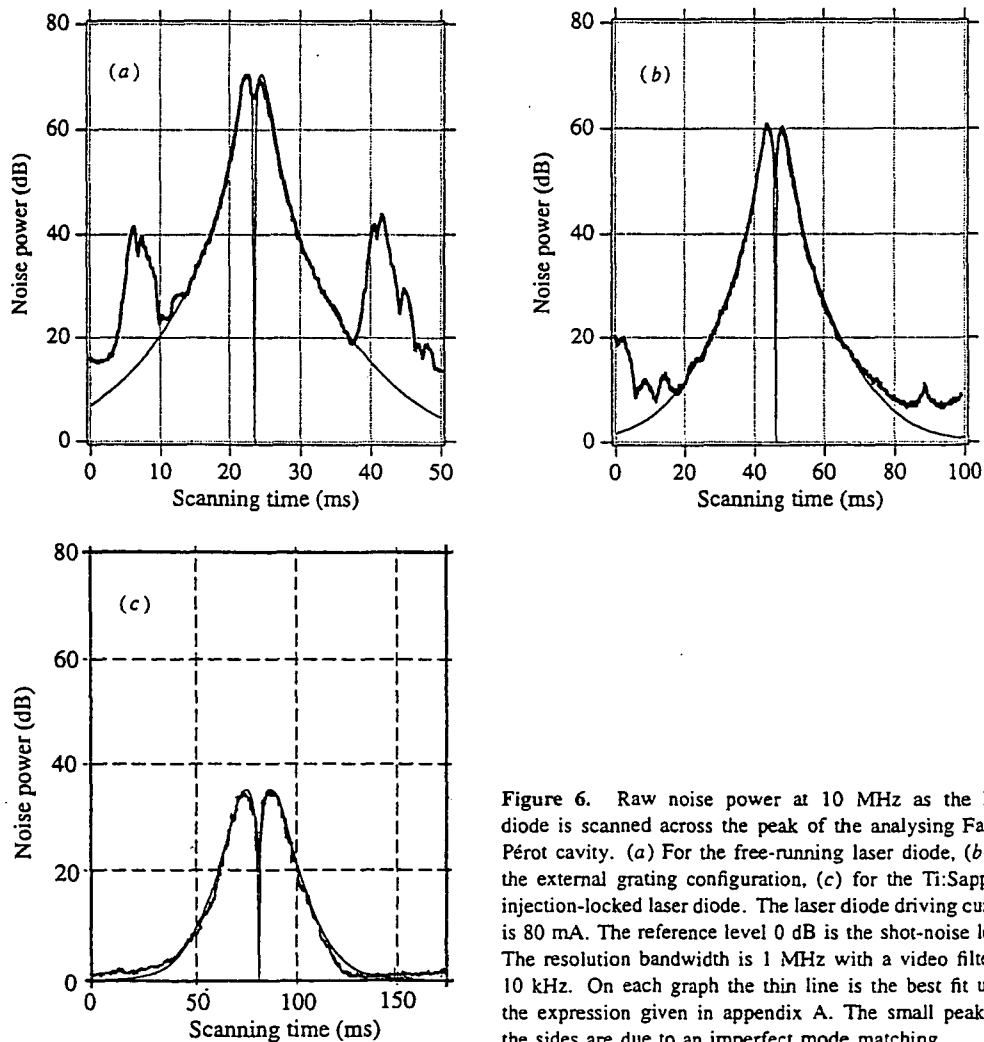


Figure 6. Raw noise power at 10 MHz as the laser diode is scanned across the peak of the analysing Fabry-Pérot cavity. (a) For the free-running laser diode, (b) for the external grating configuration, (c) for the Ti:Sapphire injection-locked laser diode. The laser diode driving current is 80 mA. The reference level 0 dB is the shot-noise level. The resolution bandwidth is 1 MHz with a video filter of 10 kHz. On each graph the thin line is the best fit using the expression given in appendix A. The small peaks on the sides are due to an imperfect mode matching.

is inferred from the experimental curves by fitting them with a simple model (see appendix A). This model has a single adjustable parameter which is the excess phase noise. This value has then to be corrected for various losses: propagation from the output of the laser to the detectors (3 dB), scattering losses inside the analysing cavity (3 dB on resonance), imperfect mode matching to the cavity (1 dB).

The phase noise inferred at the laser output for the free-running diode, the external grating configuration and the injection-locked scheme are of 82 dB, 72 dB and 46 dB, respectively, above the shot-noise level.

Let us compare these experimental results with the prediction given by the Schawlow–Townes model [14] (see appendix B). Within this model, the phase noise normalized to the shot-noise level at a noise angular frequency  $\omega = 2\pi f$  is

$$V_{\phi}(\omega) = 1 + \frac{8DI_o}{\omega^2}(1 + \alpha^2) = 1 + \frac{2\kappa^2(1 + \alpha^2)}{\omega^2} \quad (1)$$

where  $I_o$  is the flow of photon outside of the laser (photons/sec),  $\kappa$  is the cavity decay rate for intensity,  $\alpha$  is the line enhancement factor [15] (also called the phase–amplitude coupling coefficient), and  $D$  is the Schawlow–Townes phase diffusion coefficient defined in (B17). The first term is the contribution of the vacuum fluctuation (shot noise) and the second term is due to the phase diffusion assuming a random walk of the phase in the laser.

Using the value of  $\kappa$  deduced from the experiment† one can calculate a theoretical estimation of the phase noise if the factor  $(1 + \alpha^2)$  is known. Conversely, by using the experimental value of the phase noise in (1), one can deduce a value of  $(1 + \alpha^2) = 10$ , which is in agreement with other measurements. However, the linewidth of the laser diode was also measured directly by sending the light through a Fabry–Pérot cavity with a linewidth (HWHM) of 2 MHz. We obtained  $D(1 + \alpha^2)/(2\pi) = 2$  MHz (HWHM linewidth). Using the value  $I_o = 2.5 \times 10^{17}$  photons/sec corresponding to 60 mW laser output, the above model predicts  $D(1 + \alpha^2)/(2\pi) = \kappa^2(1 + \alpha^2)/(8\pi I_o) = 50$  kHz, which is significantly smaller than the measured value. This discrepancy could be attributed to jitter of the laser frequency due to power supply noise and thermal fluctuations.

In the injection locking case, the phase noise reduction mechanism relies on the fact that the slave laser locks its phase to that of the master laser [16]. The phase noise of this master laser is therefore of great importance. In this experiment we have used a frequency-stabilized Ti:Sapphire laser, which has a linewidth of 500 kHz and is both phase and intensity shot-noise limited at 10 MHz. We have observed a very significant phase noise reduction, from 82 to 46 dB for an injected power of 2 mW (see figure 6(c)).

Finally, let us emphasize that the quadrature noise detection scheme that we used is expected to work well only for a true single-mode laser. As was discussed previously (section 3.2), this is not the case for so-called ‘single-mode’ laser diodes, for which weak longitudinal side modes are very noisy and can therefore play an important role in the overall noise behaviour. As long as the intensity noise power in the main mode is small with respect to the total phase noise power, which is generally the case in the results described above, these effects can be neglected. However, one has to be cautious in some cases. For instance, it can be noticed that the experimental trace of figure 6(c) exhibits a slight asymmetry around its basis. This effect can be modelled simply with the equations of appendix A, using an input covariance matrix such that the main axis of the noise ellipse is not exactly the phase axis (quadrature angle  $\pi/2$ ) but is slightly tilted. In our experiments,

† The quantity  $1/\kappa$  is the lifetime of the photon in the laser diode cavity, calculated from the measured free spectral range of  $\Delta\lambda = 0.12$  nm, and from the transmission coefficient of the output mirrors ( $R_1 = 95\%$  and  $R_2 = 4\%$ ). This yields  $\kappa = (c\Delta\lambda/\lambda^2) \ln(1/(R_1R_2)) = 1.8 \times 10^{11} \text{ s}^{-1}$ .

this small rotation effect has only been observed for the injection-locked laser, decreases as the driving current increases, and the dip on the right-hand side was always above the shot noise [17, 18]. It is likely that a detailed analysis of this effect should include the contributions of the small modes, since intensity-phase correlations are essential in this process.

## 5. Conclusions

In this paper we have reported on a detailed experimental analysis of both intensity and phase noise of commercial laser diodes at room temperature. We have studied the free-running diode and two other configurations using different line-narrowing techniques (injection-locking or external grating).

We have measured an intensity noise of 1.2 dB below the shot-noise level (1.6 dB inferred at the laser output) in the external grating configuration, and an intensity noise of 1.4 dB below shot noise (2.3 dB inferred at the laser output) for the injection-locked laser. Also, preliminary results show that these low-noise properties of quasi-single-mode laser diodes are actually due to cancellation of the large excess noise of the main mode by the anticorrelated noise of many weak but very noisy longitudinal side modes [12].

Concerning the measurements on quadrature noise, the main result is that laser diodes exhibit a very large excess phase noise (up to 80 dB above shot noise for the free-running laser), which can be partially decreased by line-narrowing techniques. The passive feedback from an external grating reduces the spectral width of the emitted light, thereby decreasing the phase noise from 82–72 dB above shot noise. In the injection-locking scheme, the phase noise reduction mechanism also involves the master laser, and using a shot-noise limited frequency stabilized Ti:Sapphire laser, we observed a reduction of the phase noise from 82 to 46 dB above shot noise.

We believe that these results have important practical implications for spectroscopy and quantum optics experiments involving laser diodes.

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## Appendix A. Quadrature rotation after reflection on a detuned cavity

In this appendix we give the explicit input-output expression for the fluctuating amplitude and phase quadrature components at an analysis frequency  $\omega$  (respectively,  $\delta p^{in,out}(\omega)$  and  $\delta q^{in,out}(\omega)$ ) of a field reflecting off a detuned cavity. We define

$$\delta c^{in,out}(\omega) = \begin{pmatrix} \delta p^{in,out} \\ \delta q^{in,out} \end{pmatrix}. \quad (\text{A1})$$

We have then

$$\delta c^{out}(\omega) = R(\omega)\delta c^{in}(\omega) \quad (\text{A2})$$

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with

$$\begin{aligned}
 R_{11}(\omega) = R_{22}(\omega) &= \frac{4\omega^2\kappa^2 + \kappa^4 - 16\omega^2\lambda^2 + 8\kappa^2\lambda^2 + 16\lambda^4}{(\kappa^2 + 4\lambda^2)(4\omega^2 - 4i\omega\kappa - \kappa^2 - 4\lambda^2)} \\
 R_{21}(\omega) = -R_{12}(\omega) &= \frac{16\omega^2\kappa\lambda}{(\kappa^2 + 4\lambda^2)(4\omega^2 - 4i\omega\kappa - \kappa^2 - 4\lambda^2)}
 \end{aligned} \tag{A3}$$

where  $\kappa$  is the cavity decay rate and  $\lambda$  the cavity detuning.

The angle  $\beta$  of the quadrature rotation is given by

$$\tan \beta = \frac{R_{12}(\omega)}{R_{11}(\omega)}. \tag{A4}$$

Let us now give the expression we used for the fits of figure 6, displaying the noise at a given analysis angular frequency  $\omega$  of a laser light exhibiting an excess phase noise as the length of the cavity is scanned around a resonance peak. We define the input and output noise covariance matrix  $V^{in}$ ,  $V^{out}$  in the  $p$ ,  $q$  basis as

$$V^{in,out}(\omega) = \frac{1}{2}(V_{(+)}^{in,out} + V_{(-)}^{in,out}) \quad \text{where} \quad V_{(\pm)}^{in,out} = \delta c^{in,out}(\pm\omega)[\delta c^{in,out}(\mp\omega)]^T. \tag{A5}$$

These two matrices are then linked using (A2), and we obtain

$$V_{(\pm)}^{out}(\omega) = R(\pm\omega)V_{(\pm)}^{in}[R(\mp\omega)]^T. \tag{A6}$$

The quantity plotted in the fits of figure 6 is the coefficient  $V_{11}^{out}$  versus the cavity detuning  $\lambda$  at a given  $\omega$ . The input noise covariance matrix used for these plots is the one of a field whose amplitude noise is at the shot-noise level and whose phase noise is  $v_q^{in}$  time above shot noise ( $v_q^{in}$  is the adjustable parameter in the fits):

$$V^{in}(\omega) = \begin{pmatrix} 1 & 0 \\ 0 & v_q^{in} \end{pmatrix}. \tag{A7}$$

It can be mentioned that the use of non-zero off-diagonal elements for  $V^{in}$  models the asymmetry resulting from a phase-amplitude coupling very well in the injection-locking case (figure 6(c)).

### Appendix B. Link between phase noise, frequency noise and linewidth

In this appendix, we wish to recall some definitions and to present formulae linking together quantities used in the main part of the paper, such as phase and frequency noise spectra, and the linewidth of a single-mode laser.

In a semiclassical model, the slowly varying part of the electric field of a single-mode electromagnetic field can be written as

$$\alpha(t) = (I(t))^{1/2} e^{i(\phi_o + \phi(t))} \tag{B1}$$

where  $I(t)$  is the (eventually fluctuating) intensity,  $\phi_o$  is the phase origin, and  $\phi(t)$  is a fluctuating phase with  $\langle \phi(t) \rangle = 0$ . In this expression, as everywhere else in the paper, the term  $e^{i\omega_o t}$  oscillating at the optical frequency has been taken out, and  $\omega = \omega_{opt} - \omega_o$  is a RF angular frequency. In the following, we will also consider only stationary random processes, and brackets will denote ensemble averaging (or time averaging, which is the same assuming ergodicity). The field variable  $\alpha(t)$  will be considered as a  $c$ -number, but quantum noise can be readily included using standard techniques in quantum optics. If the  $P$  representation (normal ordering) is used, the vacuum noise contribution in the

correlation functions is zero, and shot noise has to be included using the standard theory of photodetection [20]. On the other hand, it is also possible to use the Wigner representation (symmetrical ordering), which directly includes vacuum noise contributions, and therefore shot noise effects in the spectra [21]. In the following, we will rather use normal ordering, and shot noise will appear only at the end of the calculation.

The phase noise power spectrum at  $\omega = 2\pi f$ , where  $f$  is the RF noise analysis frequency, is given by

$$S_{\phi}(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} e^{i\omega t} \langle \phi(t)\phi(0) \rangle dt. \quad (\text{B2})$$

On the other hand, the instantaneous angular frequency  $\omega(t)$  is given by

$$\omega(t) = \frac{d[\phi_o + \phi(t)]}{dt} = \dot{\phi}(t) \quad (\text{B3})$$

and therefore the frequency noise spectrum is given by

$$S_{\dot{\phi}}(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} e^{i\omega t} \langle \dot{\phi}(t)\dot{\phi}(0) \rangle dt. \quad (\text{B4})$$

A simple calculation yields the well known formula

$$S_{\dot{\phi}}(\omega) = \omega^2 S_{\phi}(\omega). \quad (\text{B5})$$

Let us also define the quantity  $\Delta_{\tau}\phi(t) = \phi(t + \tau) - \phi(t)$ , whose noise spectrum is

$$S_{\Delta_{\tau}\phi}(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} e^{i\omega t} \langle \Delta_{\tau}\phi(t)\Delta_{\tau}\phi(0) \rangle dt. \quad (\text{B6})$$

It is then straightforward to show that

$$S_{\Delta_{\tau}\phi}(\omega) = 2(1 - \cos \omega\tau) S_{\phi}(\omega). \quad (\text{B7})$$

The quantity  $S_{\Delta_{\tau}\phi}(\omega)$  is useful in order to relate the quantities defined above to the spectral lineshape (one has  $\omega = \omega_{opt} - \omega_o$ , hence the lineshape is centred on zero instead of  $\omega_o$ ):

$$L(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} e^{i\omega\tau} \langle \alpha^*(\tau)\alpha(0) \rangle d\tau. \quad (\text{B8})$$

Assuming that the intensity noise is negligible (i.e. that the orthoradial fluctuations of the Fresnel vector are much larger than the radial ones), and denoting  $I_o$  the average intensity expressed in photons/sec, we have

$$\langle \alpha^*(t + \tau)\alpha(t) \rangle = I_o \langle \exp(-i\Delta_{\tau}\phi(t)) \rangle. \quad (\text{B9})$$

The imaginary part of  $\langle \exp(i\Delta_{\tau}\phi) \rangle$  vanishes for symmetry reasons. A standard assumption at this point is that  $\Delta_{\tau}\phi$  is a stationary Gaussian random variable [19]. Its variance, which will be denoted by  $\sigma_{\tau}^2$ , is given by the integral of the spectrum

$$\sigma_{\tau}^2 = (2\pi)^{-1/2} \int_{-\infty}^{\infty} S_{\Delta_{\tau}\phi}(\omega) d\omega. \quad (\text{B10})$$

We can then write

$$\begin{aligned} \langle \alpha^*(\tau)\alpha(0) \rangle &= I_o (2\pi\sigma_{\tau}^2)^{-1/2} \int_{-\infty}^{\infty} e^{\left(-\frac{(\Delta_{\tau}\phi)^2}{2\sigma_{\tau}^2}\right)} \cos(\Delta_{\tau}\phi) d(\Delta_{\tau}\phi) \\ &= I_o e^{\left(-\frac{\sigma_{\tau}^2}{2}\right)} \end{aligned} \quad (\text{B11})$$

from which the expression of the spectral lineshape (B8) can be readily deduced.

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In the case where  $\Delta_\tau\phi(t)$  is generated by a random walk process, it is possible to derive analytical expressions for all these quantities. The fact that  $\Delta_\tau\phi(t)$  is a random walk process means that the phase derivatives at different times are not correlated, i.e.

$$\forall t_1, t_2, \langle \dot{\phi}(t_1)\dot{\phi}(t_2) \rangle = 2D\delta(t_1 - t_2) \quad (\text{B12})$$

where  $D$  is a constant, and  $\delta$  the Dirac function. The frequency noise spectrum (cf equation (B4)) is then given by

$$S_{\dot{\phi}}(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} e^{i\omega t} 2D\delta(t) dt = \frac{2D}{\sqrt{2\pi}} \quad (\text{B13})$$

and is thus independent of  $\omega$  (white frequency noise). Using equation (B5), the phase noise spectrum can be written

$$S_{\phi}(\omega) = \frac{1}{\sqrt{2\pi}} \frac{2D}{\omega^2}. \quad (\text{B14})$$

Furthermore, it is a well known result [22] that the variance  $\sigma_\tau^2$  of a random walk process is proportional to  $\tau$ , and it is easy to show, using (B7), (B5) and (B10), that

$$\sigma_\tau^2 = 2D|\tau|. \quad (\text{B15})$$

The lineshape  $L(\omega)$  can then be expressed explicitly, using its definition (B8) and (B11),

$$\begin{aligned} L(\omega) &= (2\pi)^{-1/2} \int_{-\infty}^{\infty} I_0 e^{i\omega\tau} e^{-D|\tau|} d\tau \\ &= I_0 \frac{D(2/\pi)^{1/2}}{D^2 + \omega^2} \end{aligned} \quad (\text{B16})$$

which is the usual Lorentzian shape of the laser linewidth (Schawlow–Townes formula [14]). Identifying equation (B16) with the Schawlow–Townes expression, we can relate the coefficient  $D$  to the parameters of the laser which generated this light, and we have, for a laser far above threshold [21]:

$$2D = \frac{\kappa^2}{2I_0} \quad (\text{B17})$$

where  $\kappa$  is the laser cavity (intensity) decay rate.

As was said at the beginning of this appendix, these calculations have been done using normal ordering. The contribution of phase vacuum fluctuations (to the linewidth for example) is therefore zero, but the shot noise will appear in the detected phase noise power, calculated using the standard methods quoted at the beginning. We finally obtain the total phase noise power that would be read on a spectrum analyser at frequency  $f = \omega/(2\pi)$ :

$$\begin{aligned} V_{\phi}(\omega) &= S_{\phi}^{\text{vac}}(\omega) + S_{\phi}(\omega) \\ &= \frac{1}{\sqrt{2\pi}} \left( \frac{1}{4I_0} + \frac{2D}{\omega^2} \right) \\ &= \frac{1}{\sqrt{2\pi}} \frac{1}{4I_0} \left( 1 + \frac{2\kappa^2}{\omega^2} \right). \end{aligned} \quad (\text{B18})$$

At a given angular frequency  $\omega$ , the phase noise is therefore the sum of the shot noise and of a term proportional to the laser linewidth. In equation (1) of the text, this expression has been normalized to the shot-noise level, and the linewidth enhancement factor  $(1 + \alpha^2)$  has been added in the second term.

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### 3.3.2 Rôle des modes longitudinaux

Nous avons vu dans l'article reproduit dans la section précédente que la possibilité de produire des états comprimés en intensité est liée à l'utilisation des techniques d'affinement spectral qui modifient le comportement modal de la diode. Pour mieux comprendre les mécanismes qui sont à la base de la réduction de bruit dans ces laser, nous avons effectué une analyse spectrale très détaillée de la radiation émise par une diode laser fonctionnant en trois configurations expérimentales différentes : diode libre, diode injectée et diode sur réseau.

Le schéma expérimental adopté (décrit par la suite) permet d'analyser la puissance associée à chaque mode longitudinal ainsi que ses propriétés de bruit.

#### 3.3.2.1 Comportement modal

Afin d'étudier les modes longitudinaux de la diode, nous avons envoyé le faisceau dans un monochromateur Jobin-Yvon de haute résolution ( $0,03 \text{ nm}$ ), capable d'isoler distinctement les modes de la diode, eux-mêmes séparés de  $1,2 \text{ \AA}$ . A la sortie de cet instrument, nous avons pris des spectres sur table traçante montrant, sur plusieurs nanomètres, la répartition de la puissance entre les modes.

En condition de fonctionnement normale, la diode libre présente le comportement modal suivant : la puissance des premiers modes longitudinaux est typiquement de  $-25 \text{ dB}$  par rapport au mode principal et la puissance totale contenue dans l'ensemble des modes longitudinaux (plus de 150 modes sont détectables) correspond à environ  $-18 \text{ dB}$  par rapport au mode principal. Dans la plupart des applications de telles performances autoriseraient à parler de diode monomode. En revanche, nous verrons que, pour la réduction de bruit, les effets des modes longitudinaux jouent encore un rôle très important à ces niveaux de puissance.

Dans la diode injectée les modes longitudinaux sont beaucoup plus atténués que dans la diode libre. La résolution instrumentale du monochromateur (limitée par la courbe de diffraction qu'on obtient en envoyant un laser monomode dans le monochromateur) ne nous permet de mesurer la puissance qu'à partir du cinquième mode longitudinal. Nous avons mesuré une puissance inférieure à  $-45 \text{ dB}$  par rapport au mode principal avec une puissance totale dans les modes longitudinaux d'environ  $-30 \text{ dB}$ .

Dans la configuration sur réseau, la puissance des premiers modes détectables (autour du quinzième mode longitudinal) est de  $-55 \text{ dB}$  par rapport au mode principal ; la puissance dans tous les modes longitudinaux descend à  $-35 \text{ dB}$ . Monter la diode sur réseau se révèle la technique la plus efficace pour avoir un laser monomode.

### 3.3.2.2 Mesures de bruit et mise en évidence des anticorrélations

Les propriétés de bruit de chaque mode longitudinal ont été mesurées avec un montage analogue au précédent : le faisceau laser est envoyé dans le monochromateur et détecté après la fente de sortie. Les mesures de bruit ont été faites soit en installant la détection équilibrée à la sortie du monochromateur, soit en procédant à une détection calibrée à référence simultanée, où la référence est fournie par la diode laser elle-même.

Pour les mesures avec détection équilibrée, nous mesurons successivement le bruit du faisceau avant l'entrée dans le monochromateur, et après la sortie. Pour les mesures à référence simultanée, on mesure avec la détection équilibrée le bruit et le shot noise du faisceau sur la table où se trouve la diode puis on retire une des deux photodiodes, que l'on place en sortie du monochromateur. La photodiode restée sur la table du montage permet de contrôler que les conditions de mesure soient les mêmes pour la mesure du bruit de l'intensité totale effectuée sur la table et pour celle du bruit du faisceau après analyse spectrale (effectuée en sortie du monochromateur). Les spectres fournis simultanément par la seconde photodiode, en sortie du monochromateur, sont normalisés au bruit quantique standard mesuré avec la détection équilibrée, en appliquant les corrections nécessaires.

Le monochromateur introduit des pertes, de l'ordre de 70 à 85 % de la puissance lumineuse envoyée. Le signal transmis permet néanmoins d'évaluer (en appliquant la correction correspondante) le rapport du bruit d'intensité au bruit quantique standard, tant que ce rapport est supérieur à 5 – 10%.

Considérons d'abord la diode libre : le bruit en intensité sur le faisceau total (mesuré avant le monochromateur) est généralement assez faible, de l'ordre de 2 à 4 *dB* au-dessus du bruit quantique standard. En revanche, le bruit du mode principal (mesuré à la sortie du monochromateur) présente un excès d'environ 40 *dB* au-dessus du bruit quantique standard. Le bruit d'intensité des modes longitudinaux doit donc être comparable à celui du mode principal, bien que leur puissance soit beaucoup plus faible, comme nous l'avons vu dans la section précédente. Cela a été observé expérimentalement comme en témoigne la figure 4 qui montre le bruit du mode principal et des quatre premiers modes longitudinaux.

De plus, si l'on ouvre progressivement la fente de sortie du monochromateur, on constate que l'excès de bruit tombe à 32 *dB*, puis à 30 *dB*, montrant des échelons très nets pour des largeurs de la fente qui correspondent à la détection des modes longitudinaux adjacents au mode principal (modes  $\pm 1$ ,  $\pm 2$  et ainsi de suite). Ces observations démontrent clairement que le bruit d'intensité de la diode libre est la résultante de la forte anticorrélation entre les fluctuations du mode principal et celles des modes longi-

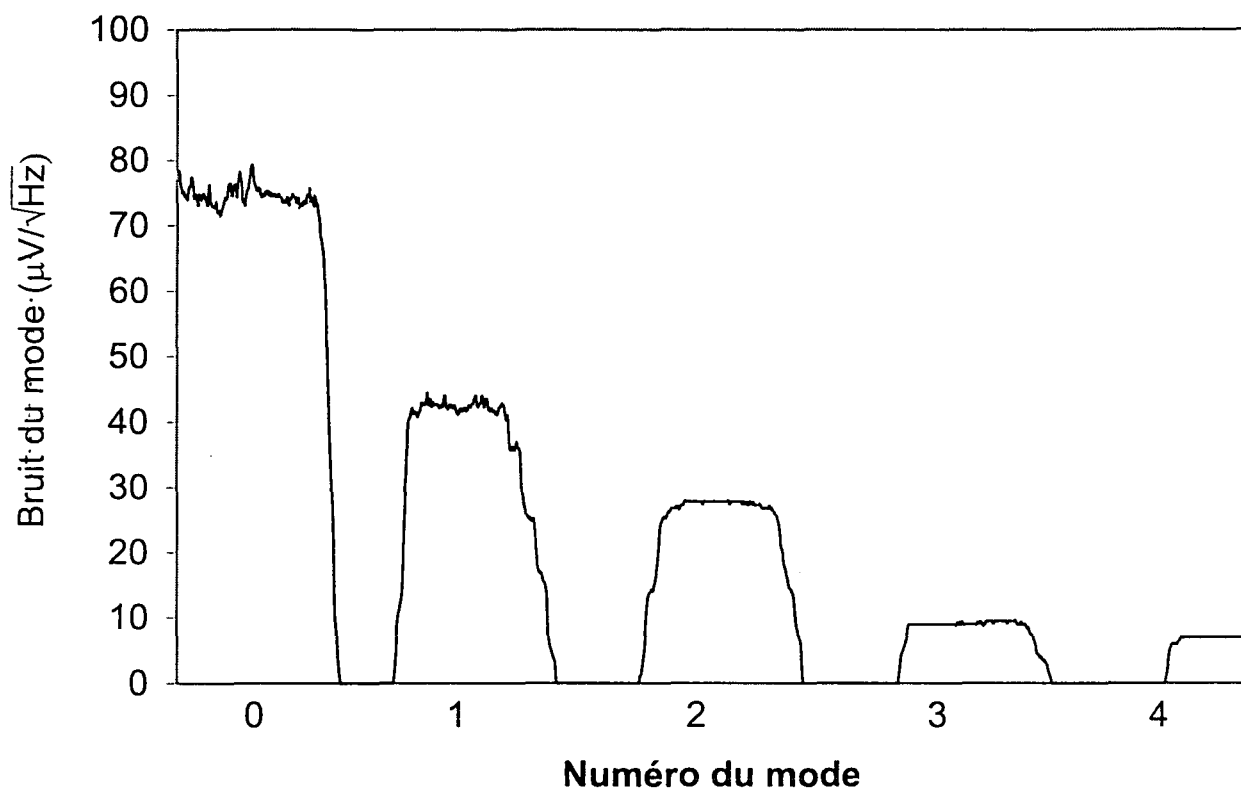


Fig. 4: Bruit d'intensité du mode principal et des premiers modes latéraux

tudinaux. Quand la fente est complètement ouverte, on peut estimer à une quinzaine le nombre de modes parvenant à la détection. Pourtant, le bruit reste encore bien plus élevé qu'à l'entrée du monochromateur, ce qui montre que l'anticorrélation qui permet aux modes latéraux de compenser le bruit du mode principal met en jeu l'ensemble des 160 modes détectables par le monochromateur.

La même procédure expérimentale a été appliquée à la diode injectée. Typiquement l'intensité totale de la diode injectée présente une réduction de bruit de  $-2.3 \text{ dB}$  sous le bruit quantique standard. En revanche, comme pour la diode libre, le bruit du mode principal montre un excès de bruit qui peut varier de 1 à 10  $\text{dB}$  au-dessus du bruit quantique standard selon les conditions d'alignement, la puissance injectée, la fréquence du maître, tandis que la réduction de bruit sur l'intensité totale reste la même. C'est donc l'anticorrélation du bruit des modes longitudinaux qui compense l'excès de bruit du mode principal et permet la génération d'états comprimés. La

diode injectée constitue un cas remarquable de laser multimode comprimé sous le bruit quantique standard.

L'analyse spectrale du faisceau émis par la diode sur réseau n'a révélé aucune différence entre le bruit de l'intensité totale et le bruit du mode principal : dans ce cas, les modes longitudinaux sont effectivement négligeables et nous pouvons parler d'état comprimé monomode (*single-mode squeezing*).

Les expérimentations que nous avons menées ont donc contribué à éclaircir de manière définitive le rôle des modes longitudinaux et des techniques d'affinement spectral pour la production d'états comprimés. L'obligation d'utiliser de telles techniques pour obtenir une réduction de bruit sous le bruit quantique standard est liée au fait que les anticorrélations entre le bruit du mode principal et celui des nombreux modes longitudinaux, bien que très fortes, ne sont pas parfaites. Cela explique la dépendance du bruit de l'intensité totale par rapport au taux de réjection des modes longitudinaux. En effet, pour des anticorrélations parfaites, le bruit d'intensité est indépendant de la puissance de modes longitudinaux et est idéalement le même que pour un laser monomode. En réalité, à défaut d'une complète anticorrélation, les modes longitudinaux très bruyants sont responsables, dès que leur puissance devient non négligeable, de l'excès de bruit observé sur l'intensité totale, comme dans le cas de la diode libre. L'atténuation considérable des modes longitudinaux, réalisée grâce aux techniques précédemment décrites, permet de s'affranchir de leur effet néfaste, et d'atteindre des réductions de bruit selon les modalités que nous avons présentées.

Un résumé de résultats obtenus, ainsi que leur interprétation qualitative à l'aide d'un modèle phénoménologique multimode basé sur les équations de Langevin (développé par le groupe de Philippe Grangier) sont contenus dans l'article que nous reproduisons au paragraphe suivant.

### 3.3.3 Reproduction de l'article : "Squeezing and Intermode Correlations in Laser Diodes" (Phys. Rev. Lett., 75, 4606 (1995))

## Squeezing and Intermode Correlations in Laser Diodes

F. Marin, A. Bramati, and E. Giacobino

*Laboratoire Kastler Brossel, Université Pierre et Marie Curie, F-75252 Paris Cedex 05, France*

T.-C. Zhang,\* J.-Ph. Poizat, J.-F. Roch, and P. Grangier

*Institut d'Optique, B.P. 147, F-91403 Orsay Cedex, France*

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We demonstrate experimentally that the intensity noise of so-called "free-running single-mode" laser diodes results from a cancellation between very large anticorrelated fluctuations of the main mode, on one hand, and of many weak longitudinal side modes, on the other hand. When line narrowing techniques are used, intensity squeezing can be observed at room temperature, but this noise reduction is not always single-mode squeezing. These experimental results are in agreement with a simple phenomenological model using Langevin-type equations.

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Laser diodes have proved in the past two decades to be a very powerful and convenient tool in the field of telecommunications [1] and spectroscopy [2]. Their main advantages are compactness, energy efficiency, tunability, and low intensity noise. This last property has been brought into the quantum domain by Yamamoto and co-workers [3,4] about 10 years ago. The physical idea is that the intensity noise of a laser is related to the noise of the pumping process [5], and that appropriate control of the driving current in laser diodes allows one to generate sub-Poissonian light through pump-noise suppression. Very large noise reductions down to  $-8.3$  dB below the shot-noise level (SNL) were observed for free-running laser diodes cooled to 66 K [6]. However, the very mechanisms capable of explaining why some laser diodes and not others were generating sub-shot-noise light remained unclear.

Recently, intensity squeezing was observed with so-called "single-mode" commercial laser diodes using line-narrowing techniques such as injection locking or feedback from an external grating [7,8]. These experiments shed new light on the understanding of the noise reduction mechanisms by putting forward the existence of weak longitudinal side modes, and their importance concerning the intensity noise behavior of such "quasi-single-mode" lasers. The arguments given in Refs. [7,8] tended to suggest that the less powerful these side modes are, the less they will contribute to the total intensity noise. However, this argument ignores possible correlations between the modes, which were demonstrated for instance by Inoue *et al.* [9] (see also Refs. [10–12]) for multimode laser diodes (several modes above threshold).

In order to explore these arguments more precisely, we have investigated the noise properties of the individual modes by sending the beam of various types of single-mode laser diodes through a high resolution spectrometer, which enabled us to analyze not only the intensities but also the noise of the individual side modes (see Fig. 1). In this Letter, we demonstrate by this method that the

intensity noise of these so-called "free-running single-mode" lasers, which is typically a few dB above the SNL at room temperature, results from a cancellation effect between very large anticorrelated fluctuations of the main mode, on one hand, and of many weak longitudinal side modes, on the other hand. When line-narrowing techniques are used, the total intensity noise goes below the shot-noise level [7,8,12,13], but we show that the sub-Poissonian character of the light can still be due to a cancellation effect between large anticorrelated noises of the various modes. This points out that sub-shot-noise operation of these lasers is actually not always single-mode squeezing. We believe that these results could have important practical implications, since for most applications, in particular, when the laser diodes are to be used in spectroscopy, single-mode and multimode properties should be clearly discriminated.

The experimental apparatus is shown in Fig. 1. We have used several samples of quantum well AlGaAs laser diodes (SDL 5422-H1) operating at 800 or 852 nm with high impedance source pumping. The free-running diodes have a rather low threshold of 18 mA and a differential quantum efficiency (slope above threshold) of 66%. We

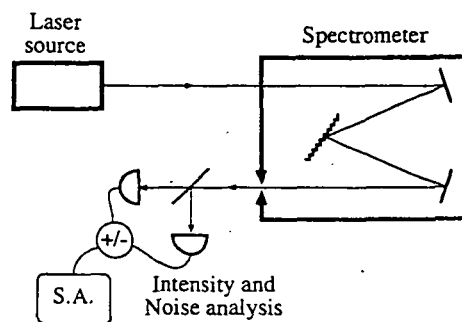


FIG. 1. Longitudinal side mode investigation scheme. S.A. is for spectrum analyzer.

have investigated the performances of free-running lasers, extended cavity lasers using feedback from an external grating, and injection-locked lasers. In the last case, the master laser was a semiconductor laser of the same kind in the external grating configuration.

The amplitude noise was measured by means of high efficiency *p-i-n* photodiodes (EG&G FND 100). The detected intensity noise was flat in the frequency region from 7 to 30 MHz. For an accurate comparison of the noise in different conditions, all data presented in this paper were measured at 7 MHz, with a detection bandwidth of 1 MHz. Great care was taken to avoid feedback to the laser, using several stages of optical isolation [14]. A polarizing cube was used in order to avoid spurious interference effects between the two orthogonal polarizations [15]. A more detailed description of the experimental apparatus can be found in [13].

By focusing the laser beam on the input slit, the overall transmission of the monochromator (Jobin-Yvon HR1000) was 25%. Its resolution was 0.03 nm, which was enough to resolve the laser modes separated by 0.12 nm. The negligible level of scattered light within the spectrometer allowed us to separate the different modes (all of them for the free-running laser and starting from the 5th–10th for the injected laser and from the 15th–20th for the grating stabilized laser). We have measured the noise either sequentially before and after the spectrometer with a standard balanced detection [16] or in parallel by splitting the beam and sending only one beam in the spectrometer. The second method used simultaneously two previously calibrated detectors, respectively, located before the spectrometer on one beam and after it on the other one, in order to ensure that the measurements of the total intensity noise and of the spectrally resolved noise were performed under the same laser conditions. The noise measurements have been confirmed using another grating with a higher efficiency (60% transmission), but poorer resolution and optical quality.

For the free-running laser, the power of one of the first side modes is typically  $-25$  dB lower than the one of the main mode (see Fig. 2), and the total power in the side modes is about  $-18$  dB below the main mode. The novel and rather unexpected result that we obtained concerns the noise of the individual modes: We have observed that the intensity noise of the main mode alone is much higher than the total intensity noise. For example, for a driving current of 80 mA the main mode exhibits an excess noise of  $+39$  dB, while the total intensity noise, measured before the spectrometer, is only 2 dB above the SNL (all noises are referred to the laser output). The intensity noise of the side modes is then expected to be comparable to the intensity noise of the main mode despite their much weaker power, and this is what was observed. Indeed, if the output slit is kept centered on the main mode, and is progressively opened, the intensity noise decreases, showing clear steps as symmetrical side modes enter the

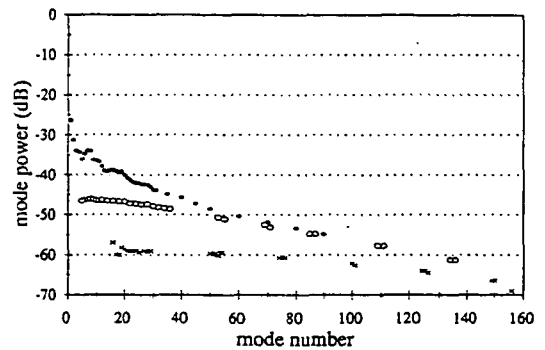


FIG. 2. Power of individual longitudinal modes (optical power in dB, with respect to the main mode), measured with a high resolution monochromator, for a driving current of 80 mA. On the x axis each mode is labeled by a number, the number 0 corresponding to the main mode. (•: free-running laser. ◦: injection-locked laser. ×: extended cavity laser.)

detector (Fig. 3). The step size corresponds to the noise of the respective modes: For instance, when the noise in the main mode was  $2.5 \mu\text{V}/\sqrt{\text{Hz}}$ , the total noise in the two first adjacent modes was about  $2.15 \mu\text{V}/\sqrt{\text{Hz}}$ , and the noise for all three modes was  $0.3 \mu\text{V}/\sqrt{\text{Hz}}$ . This clearly demonstrates that the observed total intensity fluctuations result from a cancellation effect between the very large anticorrelated fluctuations of the main mode and of the side modes. In fact, all of the 160 side modes displayed in Fig. 2 contribute to some extent to this cancellation effect. This is demonstrated by the fact that the noise level after the spectrometer, with the output slit fully opened (about 15 modes detected), is still much higher than the total intensity noise level.

As seen from Fig. 2, the power of the first side modes of the injection-locked laser is reduced to less than  $-45$  dB

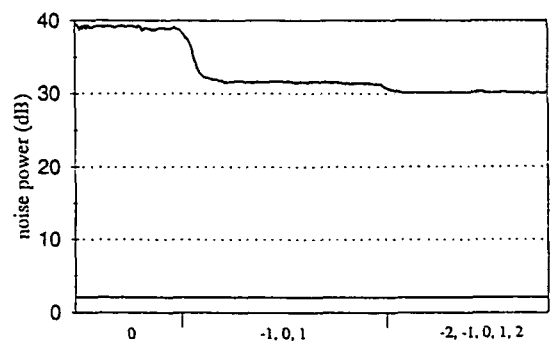


FIG. 3. Intensity noise of the free-running laser diode, referred to the shot noise, as the output slit is opened up. In the first section, only the main mode is detected, while the two steps correspond to the entrance of the two couples of side modes ( $-1.1$ ) and ( $-2.2$ ). The straight line at 2 dB shows the total intensity noise level (measured before the spectrometer).

below the main mode, while the total power in the side modes is  $-30$  dB below the main mode. The total intensity noise referred at the laser output is now squeezed by  $-2.3$  dB below the SNL (see [13]), while the intensity noise of the main mode alone is still well above the quantum limit. The level of this excess noise depends critically on the injection parameters, such as alignment, injected power, or master laser frequency, whereas the total intensity squeezing is unchanged (cf. the remark at the end of the theoretical discussion below). Optimization of these parameters does not only reduce the excess main mode noise, typically from 10 to 1 dB above the SNL, but also the power in the side modes, from  $-44$  to  $-49$  dB below the main mode for the first side mode. The total intensity noise of the injection-locked laser again results from a cancellation effect among anticorrelated fluctuations of the main and side modes. Note that in this case the sub-Poissonian intensity noise is not single-mode squeezing.

For the laser in the extended cavity configuration, the side modes are suppressed further down, to about  $-55$  dB below the main mode (see Fig. 2), which corresponds to a total side mode power of  $-35$  dB below the main mode. In that case, we have noticed virtually no difference between the total intensity noise and the noise of the main mode alone. For instance, at 80 mA, we have obtained a squeezing of  $-1.85 \pm 0.05$  dB for the total intensity, and of  $-1.6 \pm 0.3$  dB for the main mode alone (both referred to the laser output), and the noise in the side modes was too low to be detected (less than 7.5% of the SNL). In this case, and only in this case, it can be concluded that the side modes are actually negligible, and that true single-mode squeezing is generated.

In order to build a theoretical model for the observed correlations, a first possibility is to couple all the modes to the same excited carrier population [9]. This would be a correct description if a predominantly homogeneous behavior of the lasing junction is assumed. However, in a fully homogeneous gain medium, the anticorrelations between the modes should be perfect and the total intensity noise should always be squeezed, depending eventually on the quantum efficiency but not on the size of the side modes. This is not what we have observed. Therefore, for a description of our experimental results, we have modified an homogeneous multimode Langevin model, as introduced by Inoue *et al.* [9], by bringing in some inhomogeneity as suggested by Wang *et al.* [8]. A simple model for inhomogeneous noise behavior is to introduce a small self-saturation of each mode by its own fluctuations, which adds up to the homogeneous saturation process due to the excited carrier population. It will be shown below that the correlations between the main and side modes are then degraded as the power of the side modes gets larger (case of the free-running diode), resulting in an increase of the total intensity noise.

The quasi-single-mode laser diode is described by three modes (one main mode, labeled "1" and two side modes,

labeled "2" and "3") coupled to a common carriers population (homogeneous behavior) [9]. The dynamic variables are the photon number  $n_i(t)$  of the  $i$ th mode and the total number of excited carriers  $N_c(t)$ . The equations obeyed by these quantities are then

$$\frac{dn_i(t)}{dt} = -\frac{n_i(t)}{\tau_i^{(p)}} + A_i N_c(t) [n_i(t) + 1] + S_i(t) + G_i(t) + g_i(t) + f_i(t). \quad (1)$$

The quantity  $1/\tau_i^{(p)} = 1/\tau_i^{(po)} + 1/\tau^{(pe)}$  is the photon decay rate of mode  $i$ , decomposed into internal losses  $1/\tau_i^{(po)}$  and output coupling losses  $1/\tau^{(pe)}$ . The coefficient  $A_i$  is the spontaneous emission rate into the corresponding lasing mode, given by  $A_i = \beta_i/\tau^{(sp)}$ , where  $\beta_1$  and  $\beta_{2,3} = m\beta_1$  (with  $m < 1$ ) are the spontaneous emission coefficients in mode  $i$ , and  $\tau^{(sp)}$  is the spontaneous electron lifetime. The term  $S_i(t)$  describes self-saturation of mode  $i$ , and is taken equal to  $\tau_i^{(p)} S_i(t) = -s_i[\langle n_i \rangle / (P\tau_i^{(p)})] \delta n_i(t)$ , with  $s_i$  being a small adjustable parameter,  $P$  the pumping rate [see Eq. (2) below], and  $\delta n_i(t) = n_i(t) - \langle n_i \rangle$  the fluctuations of the photon numbers around their mean values. The last three terms are Langevin noise terms, respectively, associated with the stimulated-emission gain [correlation  $\langle G_i(t)G_i(t') \rangle = \delta(t-t')A_i\langle N_c \rangle \langle n_i \rangle$ ], the internal losses [correlation  $\langle g_i(t)g_i(t') \rangle = \delta(t-t')\langle n_i \rangle / \tau_i^{(po)}$ ], and the output coupling [correlation  $\langle f_i(t)f_i(t') \rangle = \delta(t-t')\langle n_i \rangle / \tau^{(pe)}$ ].

The equation of motion for the total excited carrier number  $N_c(t)$  is

$$\frac{dN_c(t)}{dt} = P - \frac{N_c(t)}{\tau^{(sp)}} - \sum_i A_i [n_i(t) + 1] N_c(t) + \Gamma^{(p)}(t) + \Gamma^{(sp)}(t) + \Gamma(t). \quad (2)$$

The last three terms are Langevin noises. The first one  $\Gamma^{(p)}(t)$  is associated with the pump noise, and for a pump-noise-suppressed laser its correlation function is  $\langle \Gamma^{(p)}(t)\Gamma^{(p)}(t') \rangle = 0$ . The term  $\Gamma^{(sp)}(t)$  is associated with spontaneous noise and its correlation is  $\langle \Gamma^{(sp)}(t)\Gamma^{(sp)}(t') \rangle = \delta(t-t')\langle N_c \rangle / \tau^{(sp)}$ , and  $\Gamma(t)$  is associated with stimulated noise and its correlation is  $\langle \Gamma(t)\Gamma(t') \rangle = \delta(t-t')\sum_j A_j \langle N_c \rangle \langle n_j \rangle$ . Finally, due to their same physical origin, the noise terms associated with the stimulated gain for the photons and stimulated emission for the electrons are perfectly anticorrelated and have cross correlations  $\langle G_i(t)\Gamma(t') \rangle = -\delta(t-t')A_i\langle N_c \rangle \langle n_i \rangle$ .

By neglecting the noise terms and the operator correlations in Eqs. (1) and (2), and by setting the time derivatives to zero, one obtains the average numbers of photons  $\langle n_i \rangle$  in each mode  $i$ . These numbers depend on the two ratios  $m = \beta_{2,3}/\beta_1$  (corresponding to the relative gains) and  $p = \tau_{2,3}^{(po)}/\tau_1^{(po)}$  (corresponding to the relative losses). The value of  $m$  can be determined from the value of the free-running diode, and then the mode

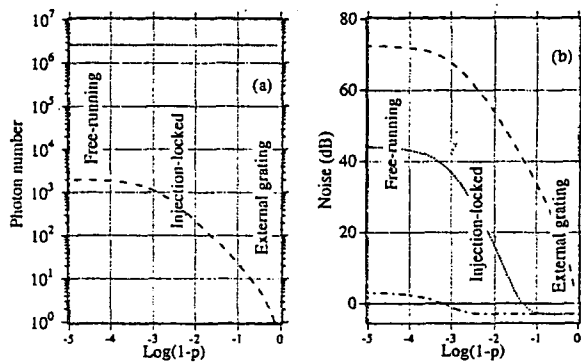


FIG. 4. (a) Calculated photon number in the main mode (dotted line) and in one side mode (dashed line) versus the deviation from unity of the relative optical losses  $p$ , for a given gain ratio  $m = 0.9995$ . (b) Noise power in the main mode (dotted line), in one side mode (dashed line), and in all the modes (dash-dotted line) versus  $1 - p$ . The 0 dB level is the SNL of each individual mode. The pumping rate is  $R = I/I_{th} - 1 = 7$ , the spontaneous coefficient is  $\beta = 2.7 \times 10^{-6}$ , the lifetimes are  $\tau^{(up)} = 10^{-9}$  s,  $\tau^{(pe)} = 5.6 \times 10^{-12}$  s,  $\tau_1^{(po)} = 1.5 \times 10^{-11}$  s, and the self-saturation parameters are  $s_1 = s_2 = s_3 = 0.015$ .

selection process can be modeled by decreasing  $p$ . The average numbers of photons in each mode obtained using this procedure are plotted in Fig. 4(a) as a function of  $\log(1 - p)$ , for fixed  $m = 0.9995$ . It can be seen that the number of photons in the main mode varies very little with  $p$ , but the number of photons in the side modes exhibits much larger variations. In Fig. 4(b) are plotted the noise levels obtained from the above equations after a standard linearization procedure. Three regions appear clearly on this graph: on the left hand side, there is no squeezing; in the center, the total intensity is squeezed, but the individual modes are not; on the right hand side, both the main mode and the total intensity are squeezed, while the side modes still exhibit excess noise, but have negligible intensity. These three regions are in quite good agreement with the experimental results described previously, respectively, for free-running, injection-locked, and grating-stabilized diodes. Note that the behavior observed while optimizing the alignment in the injection-locked laser is nicely reproduced by the model. Indeed, in the central region, a change in the  $p$  parameter modifies the power in the side modes and the noise of the main mode, without changing the total intensity noise.

The basic physics of our observations is thus explained by strong mode competition due to the homogeneous broadening of the transition (which yields large anticorrelated noises), plus a small amount of inhomogeneities degrading the anticorrelation between the modes when the rejection rate of the side modes is not sufficiently large. The final amount of noise in the individual and total intensities depends therefore on this degree of rejection, which

is described in the model by the modal gain and loss parameters  $m$  and  $p$ . This simple model is in good agreement with the experimental data, and could provide us with some useful hints for more detailed calculations.

As a conclusion, we have shown that a thorough analysis of the quantum noise of laser diodes should distinguish between true single mode squeezing and sub-Poissonian light involving contributions from a large number of weak side modes. This distinction could allow one to understand better the underlying physics of the observed noise reductions in laser diodes, and therefore to progress towards the use of such devices in quantum optics experiments.

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\*Present address: Institute of Optoelectronics, Shanxi University, Taiyuan 030006, China.

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### 3.4 Comparaison des résultats expérimentaux avec les prévisions théoriques

Nous avons établi, dans la section précédente, que les techniques d'affinement spectral utilisées permettent de négliger l'effet des modes longitudinaux sur le bruit d'intensité du laser. Nous pouvons donc considérer le laser comme monomode. Dans ce cas, la réduction de bruit maximale attendue est égale à l'efficacité quantique du laser, si on suppose que l'on supprime complètement le bruit de pompe. En effet, très au-dessus du seuil, le bruit lié à l'émission spontanée devient négligeable et seuls les processus aléatoires dus à l'imparfaite conversion des électrons de pompe en photons lasant peuvent contribuer à ramener le bruit du faisceau émis vers le bruit quantique standard. Ce simple raisonnement est en bon accord avec les résultats expérimentaux obtenus pour des forts taux de pompage (environ 75 fois au-dessus du seuil) [8].

Pour des taux de pompage plus faibles, comme dans notre cas, d'autres sources de bruit sont à considérer : notamment le bruit lié à l'émission spontanée et stimulée. De plus, il est intéressant de tenir compte du bruit du mécanisme de pompe. Dans ces conditions, nous avons comparé les résultats expérimentaux avec les prévisions théoriques des différents modèles quantiques du laser.

La comparaison entre théorie et expérience est établie dans l'article que nous reproduisons dans le paragraphe suivant et met en évidence la nécessité de développer un modèle plus complet pour prédire correctement les propriétés du bruit quantique dans les lasers à semiconducteur.

#### 3.4.1 Reproduction de l'article : "Quantum noise models for semiconductor lasers: is there a missing noise source?" (J. of Mod. Opt., 44, 1929 (1997))

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### Quantum noise models for semiconductor lasers: is there a missing noise source?

A. BRAMATI, V. JOST, F. MARIN and E. GIACOBINO

Laboratoire Kastler Brossel, Université Pierre et Marie Curie, Case 74,  
4 Place Jussieu, F-75252 Paris Cedex 05, France

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**Abstract.** Experimental results on intensity noise reduction in semiconductor lasers are compared with the predictions of different models. This procedure allows us to check the validity of the models and points to their weaknesses.

Semiconductor lasers have been the subject of numerous theories and modeling since the concept was first introduced by Basov in 1961. Nevertheless, their behaviour and in particular their noise properties are far from being completely understood. Surprisingly enough, in spite of their complexity, semiconductor lasers have been the only ones where the idea of quantum noise reduction in the emission based on pump noise suppression, proposed by Golubev and Sokolov [1], has been implemented to date. The noise reduction relies on the possibility of having a noiseless pump current, as predicted [2, 3] and demonstrated [4] by Yamamoto and co-workers. Very large noise reduction, down to 8.3 dB below the shot noise level, was observed for a free-running laser diode cooled to 66 K, with the detector closely coupled to the laser [5]. However, the squeezing measured on the collimated beam was limited to 1.3 dB [6].

More recently, Steel and co-workers demonstrated intensity squeezing by combining high impedance suppression of pump noise with line-narrowing techniques. By means of injection-locking, they obtained a squeezing of 3 dB (4.3 dB corrected for the detection efficiency) from AlGaAs diode lasers kept at 10 K [7]. The present state of the art for noise reduction in the intensity of a collimated beam of a single mode semiconductor laser at the front laser facet is 2.3 dB below shot noise at room temperature [8] and 5.9 dB at low temperature [9], using either feedback from an external grating or injection locking. In previous papers [8, 10], we have investigated the role of these line-narrowing techniques on the intensity and phase noise of diode lasers. The fact that these techniques allow us to decrease the small but very noisy longitudinal side modes is now rather well understood. The side modes should exhibit perfect anticorrelation [11] with the main mode, analogous to the antiphase dynamics for classical fluctuations in multimode lasers [12]. Imperfect noise anticorrelation was shown to be at the origin of the excess noise in the laser intensity [10, 13]. Anticorrelation was also found between the main mode and a small mode of orthogonal polarization, the full noise reduction being only obtained when the total intensity of the two modes was detected [9].

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Once the problem of anticorrelated noise is taken into account and eventually suppressed by depressing the small side modes, the lasers can be considered to be single mode. It is generally accepted that the maximum achievable noise reduction is equal to the quantum efficiency, if the pump current is fully noiseless. Indeed, far above threshold, spontaneous and stimulated emission noises are expected to disappear, and randomness arises only from the failure to convert electrons into outgoing laser photons. This simplistic reasoning only holds for fluctuations with frequencies within the cavity bandwidth (or, more precisely, well below the relaxation oscillation), but it is in good agreement with some experimental results [9].

When the laser is not operated very far above threshold, more elaborate models are required to evaluate the noise. For example, with a pumping rate 10 times above threshold there is still a significant contribution from the noise linked to stimulated and spontaneous emission [14]. The model should then properly account for the dipole fluctuations and also for the pump noise. In this paper, we have compared the predictions of several versions of fully quantum laser models with experimental observations. This study points to the need to include additional noise sources in the models.

A model allowing the derivation of the outgoing fluctuations of semiconductor lasers was developed by Yamamoto and co-workers [2, 3, 15]. It includes a specific treatment of the carrier noise in a semiconductor junction showing that current noise suppression obtained with a high impedance power supply actually suppresses noise in the pumping process. This model has been widely used to account for the noise measured in semiconductor lasers. Recently, Gardiner and Eschmann showed that a master equation approach for semiconductor lasers [16] leads to similar results for the statistics of the output light.

The basic model by Yamamoto and co-workers is a two-level one. It describes the behaviour of the quantum fluctuations in a semiconductor laser with a set of two quantum Langevin equations, one for the field and one for the population difference between the conduction and the valence bands. The polarization, which has a very fast relaxation rate, has been adiabatically eliminated. The equations for the field operator  $\hat{A}$  and for the total population difference operator  $\hat{N}$  between the conduction and the valence bands are written as

$$\frac{d\hat{A}}{dt} = -\frac{\kappa + \kappa'}{2}\hat{A} + \frac{(\hat{E}_{CV} - \hat{E}_{VC})}{2}\hat{A} + \hat{G} + \hat{g} + \hat{f}, \quad (1)$$

$$\frac{d\hat{N}}{dt} = R - \gamma\hat{N} - (\hat{E}_{CV} - \hat{E}_{VC})\hat{A}^\dagger\hat{A} + \hat{I}_P + \hat{I}_{SP} + \hat{I}, \quad (2)$$

where  $\kappa$  is the photon decay rate of the cavity due to the output coupling,  $\kappa'$  is the photon decay rate due to internal losses,  $R$  is the pump rate,  $\gamma$  is the decay rate of the population difference,  $\hat{E}_{CV} - \hat{E}_{VC}$  is the gain operator,  $\hat{G}$ ,  $\hat{g}$  and  $\hat{f}$  are noise operators corresponding respectively to the dipole fluctuations and to the vacuum field fluctuations entering through the output coupling mirror and through other lossy parts of the cavity. In the same way  $\hat{I}_P$ ,  $\hat{I}_{SP}$  and  $\hat{I}$  are noise operators corresponding to the noise of the pumping process, to the spontaneous emission noise and to dipole fluctuations. The correlation functions of these noise operators can be found in [2]. Let us note that  $\hat{G}$  and  $\hat{I}$ , having the same origin, have a non-zero cross-correlation function.

The steady state solution for the output photon number per second,  $M_0 = \kappa \langle \hat{A}^+ \hat{A} \rangle$ , is obtained as a function of the population difference  $N_0$ , by setting the noise parts equal to zero:

$$M_0 = \frac{\kappa}{\kappa + \kappa'} (R - \gamma N_0). \quad (3)$$

The steady state population difference can be related to the gain  $G_0 = \langle \hat{E}_{CV} - \hat{E}_{VC} \rangle$ , with a more detailed treatment of the stimulated emission and absorption processes in the semiconductor [17]. The differential quantum efficiency is defined by

$$\eta = \frac{dM_0}{dR}. \quad (4)$$

Independently of the gain process, it can be seen that the intrinsic differential quantum efficiency of the laser (obtained for  $\kappa' = 0$ ) is equal to 1, a property which is also found in more elaborate models [16, 17].

These equations can be compared to the ones derived from a three-level atomic model, in which the possibility of pumping with reduced noise is taken into account [18]. In this model, decay of the populations of the lasing levels to another level is assumed. As shown by Walls and co-workers [19], this type of model yields a different noise to that of the two-level one close to threshold (around twice the threshold). Comparison of the predicted noise with experimental data is then a very sensitive test of the validity of the models.

Assuming that the dipole decays at a much faster rate  $\gamma_{ab}$  than both the field and the populations, the evolution equations for the field and for the populations of the upper and lower levels a and b,  $\hat{N}_a$  and  $\hat{N}_b$ , can be written as

$$\frac{d\hat{A}}{dt} = -\frac{\kappa + \kappa'}{2} \hat{A} + \frac{g^2}{\gamma_{ab}} \hat{N} \hat{A} + \hat{G}_{dip} + \hat{g} + \hat{f}, \quad (5)$$

$$\frac{d\hat{N}_a}{dt} = R - (\gamma_a + \gamma'_a) \hat{N}_a - \frac{2g^2}{\gamma_{ab}} \hat{A}^+ \hat{N} \hat{A} + \hat{I}_R + \hat{I}_a + \hat{I}_{a,dip}, \quad (6)$$

$$\frac{d\hat{N}_b}{dt} = -\gamma_b \hat{N}_b + \gamma'_a \hat{N}_a + \frac{2g^2}{\gamma_{ab}} \hat{A}^+ \hat{N} \hat{A} + \hat{I}_b + \hat{I}_{b,dip}, \quad (7)$$

where  $\gamma'_a$  is the spontaneous decay rate of level a on the lasing transition,  $\gamma_a$  is the decay rate of level a to other levels,  $\gamma_b$  is the total decay rate of level b,  $\hat{N}$  is the population difference operator,  $\hat{G}_{dip}$ ,  $\hat{I}_{a,dip}$  and  $\hat{I}_{b,dip}$  are the noise operators coming from the adiabatic elimination of the dipole operator, and  $\hat{I}_R$ ,  $\hat{I}_a$ ,  $\hat{I}_b$  are the noise operators coming respectively from the pumping and from the decay of the populations of the two levels, including spontaneous emission. The correlation functions of the noise operators can be found in [18].

It is easily verified that equations (1) and (2) can be obtained from equations (5), (6) and (7) by assuming  $\gamma_b \gg \gamma_a, \gamma'_a$ . However, when the decay rate of the lower level, b, is not much larger than that of the upper level, the steady state equation for the population difference becomes:

$$0 = R \left( 2 \frac{\gamma_b - \gamma'_a}{\gamma_a + \gamma_b} \right) - N_0 \left( 2\gamma_b \frac{\gamma_a + \gamma'_a}{\gamma_a + \gamma_b} \right) - \frac{4g^2}{\gamma_{ab}} N \langle \hat{A}^+ \hat{A} \rangle. \quad (8)$$

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Using the steady state solution of equations (5) and (8), it can be seen that the differential quantum efficiency is equal to

$$\eta = \frac{\kappa}{\kappa + \kappa'} \frac{\gamma_b - \gamma'_a}{\gamma_a + \gamma_b}. \quad (9)$$

Let us first consider the solutions for the mean values, so as to compare them to the experimental behaviour of the lasers. Above threshold, all the lasers we have studied have a linear intensity dependence on the pumping current, with a differential efficiency  $\eta$  which is about 65%.

In the first model, the non-unit quantum efficiency can only be accounted for with internal losses.

$$\eta = \frac{\kappa}{\kappa + \kappa'}. \quad (10)$$

The decay rate  $\kappa$  of the intensity in the cavity due to the output coupling is given by the manufacturer. For the SDL 5422H1 lasers we have studied, we will use the value  $\kappa = 1.8 \times 10^{11} \text{ s}^{-1}$ . The  $\kappa'$  parameter has then to be adjusted to match the measured quantum efficiency. We take  $\kappa' = 9.7 \times 10^{10} \text{ s}^{-1}$ . The decay rate of the population difference  $\gamma$  which is the electron hole recombination rate is taken to be  $5 \times 10^8 \text{ s}^{-1}$  according to [2].

In the second model, the imperfect quantum efficiency can be related to the values of the relaxation parameters of the upper and lower levels. If internal losses are assumed to be zero, we get:

$$\eta = \frac{\gamma_b - \gamma'_a}{\gamma_b + \gamma_a}. \quad (11)$$

For consistency with the previous model, the relaxation rate  $\gamma_a$  is taken to be  $5 \times 10^8 \text{ s}^{-1}$  and we assume  $\gamma'_a \ll \gamma_a$ . The relaxation rate  $\gamma_b$  is then adjusted to yield the measured value of  $\eta$ .

In both models, the equations are linearized around the steady state mean values to derive the quantum fluctuations. The results are given in analytic form in [18].

We are interested in the noise at frequencies that are low compared to the response time of the cavity and to the frequency of the relaxation oscillation. The noise is then calculated at zero frequency. The pumping statistic is characterized by the parameter  $p$ : a Poissonian statistic corresponds to  $p = 0$ , while a regular pumping corresponds to  $p = 1$ .

Far above threshold, for the model related to equations (1)–(2), one finds for the noise power at zero frequency (normalized to shot noise):

$$V(0) = 1 - \frac{\kappa}{\kappa + \kappa'} p. \quad (12)$$

For the same conditions, for the model related to equations (5)–(7), one obtains

$$V(0) = 1 - \frac{\gamma_b - \gamma'_a}{\gamma_b + \gamma_a} p + 2 \frac{\gamma_a + \gamma'_a}{\gamma_b + \gamma_a} \frac{\gamma'_a}{\gamma_b - \gamma'_a}. \quad (13)$$

Except for the third term in equation (13), it is found that far above threshold and in the absence of pump noise ( $p = 1$ ), both models give a noise reduction which is equal to the quantum efficiency. The third term in equation 13 is an

excess noise term proportional to the spontaneous emission rate  $\gamma'_a$  and is small when  $\gamma'_a$  is small compared to  $\gamma_b$ , which is the case in most lasers.

The limit values are only reached for pumping rates that are several tens of times above threshold. The laser diodes SDL 5422-H1 at 850 nm have been operated in this regime at low temperature (10 K), at which the threshold is much lower compared to its value at room temperature. The noise is actually reduced as predicted by equation (12) or (13) with  $p = 1$  (no pump noise) [9].

Our measurements were performed at room temperature where the diodes can only be operated about 10 times above threshold and where the noise coming from the optical dipole and from the decay must be taken into account. To get a more detailed insight into the validity of the models we have chosen to study the ratio of the noise reduction to the total quantum efficiency:

$$r = \frac{1 - V(0)}{\eta_{\text{tot}}} \quad (14)$$

as a function of the normalized pump intensity  $R_n$ . The total quantum efficiency is given by

$$\eta_{\text{tot}} = \eta \frac{R_n - 1}{R_n}. \quad (15)$$

Experimental and theoretical values of  $r$  are shown in figure 1. Curves (a) and (c) corresponding respectively to the two-level and three-level models are obtained with perfectly noiseless pumping. It can be seen that the three-level model, while correctly predicting the limit high above threshold does not reproduce the variation of  $r$  with pumping. This result agrees with the fact that semiconductor lasers are usually modelled as 2-level lasers [17].

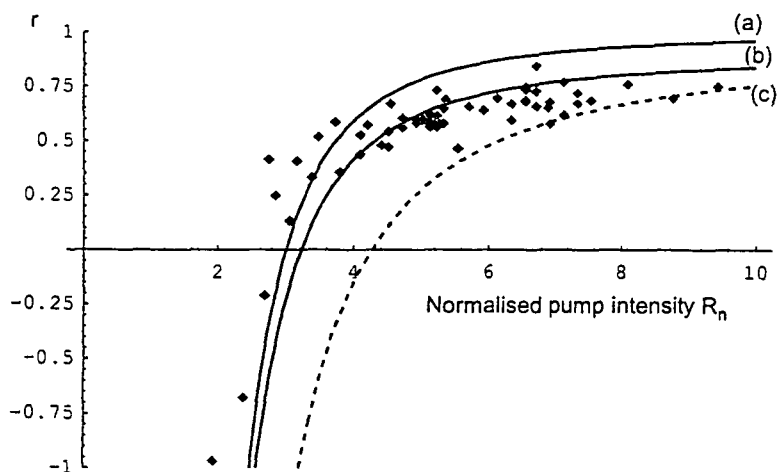


Figure 1. Ratio  $r$  between intensity noise reduction and total quantum efficiency versus pump intensity  $R_n$  (normalized to threshold pump intensity) for 850 nm SDL semiconductor lasers. Curves (a) and (b) correspond to the two-level model, with no pump noise ( $p = 1$ ) for curve (a) and  $p = 0.9$  for curve (b). Curve (c) corresponds to the three-level model with  $p = 1$ . Dots correspond to measurements.

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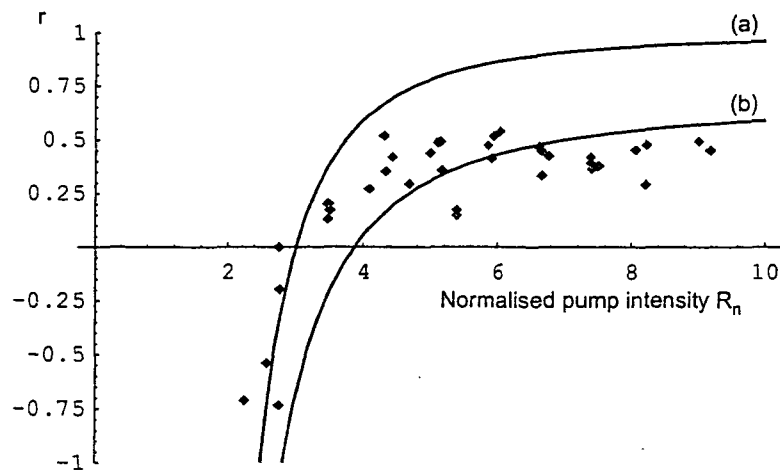
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Figure 2. Ratio  $r$  between intensity noise reduction and total quantum efficiency versus normalized pump intensity  $R_n$  for 810 nm SDL semiconductor lasers. Curves (a) and (b) correspond to the two-level model, with no pump noise ( $p = 1$ ) for curve (a) and  $p = 0.7$  for curve (b). Dots correspond to measurements.

Using the two-level model, a better fit of the experimental points is obtained by including some pump noise. This can be seen on curve (b), corresponding to  $p = 0.9$ .

We have performed the same study on laser diodes of the same type as the previous ones, SDL 5422-H1, operating at 810 nm. Very surprisingly, these laser diodes, which have specifications quite similar to the first ones (same threshold, same quantum efficiency, same operating conditions) consistently exhibit much less quantum noise reduction when the same methods are applied, i.e. high impedance power supply and line narrowing techniques.

In this case, the experimental points (corresponding to two different lasers) can only be fitted by a theoretical curve obtained when assuming  $p = 0.7$ , as shown in figure 2: curve (b), to be compared with curve (a) where a noiseless pump is assumed. This is in contradiction with the principle of high impedance pump noise suppression used in the experimental set-up to generate squeezed states in semiconductor lasers, and clearly indicates the necessity for a more complete theoretical model to predict the quantum noise features of laser diodes.

In conclusion, by means of an accurate comparison between the experimental results from different kinds of semiconductor lasers and the theoretical predictions of full quantum models of lasers, we are able to check the consistency of these models and to show their defects.

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### 3.5 Régime cryogénique

Le régime cryogénique est potentiellement très intéressant pour l'amélioration des performances des diodes laser. En effet, les diodes lasers présentent une forte dépendance du courant de seuil par rapport à la température de fonctionnement. La loi, de type exponentiel est la suivante :

$$I_{th2} = I_{th1} \exp\left(\frac{T_2 - T_1}{T_0}\right) \quad (3.1)$$

où  $T_0$  est une constante qui vaut environ  $110 K$ . La diminution du seuil attendue aux basses températures devrait permettre de faire fonctionner les diodes laser à des taux de pompage de l'ordre de 50 fois au-dessus du seuil et donc nettement supérieurs à ceux normalement atteints à température ambiante (typiquement autour de 10 fois au-dessus du seuil). Cela laisse présager la possibilité d'observer une réduction de bruit proche de l'efficacité quantique du laser (cf. 3.4), et donc bien meilleure que celle obtenue à température ambiante. Le groupe de Steel a pu observer une réduction de bruit de  $5.9 dB$  sous le bruit quantique standard avec une diode refroidie à la température de  $15 K$  [8].

#### 3.5.1 Montage cryogénique

Pour tester cet effet, nous avons donc adapté à un cryostat à azote liquide, d'une contenance de plusieurs litres, un boîtier conçu au laboratoire, et renfermant la diode laser, les cablages élémentaires, et l'optique de collimation, ainsi qu'une thermistance insérée entre la diode et son support en cuivre. Le support de cuivre est composé d'une plaque, sur laquelle sont fixés la diode et son optique de collimation, et d'un cylindre creux dans le prolongement de la plaque, destiné à enserrer le doigt froid du cryostat. Les contacts thermiques sont améliorés au moyen de graisse silicone éventuellement mélangée avec de la poudre de cuivre très fine. Le faisceau sort du boîtier par une fenêtre à l'angle de Brewster.

Afin d'éviter toute condensation sur la fenêtre de la diode et pour obtenir durablement une basse température, il faut réaliser un vide de l'ordre de  $10^{-5} torr$  dans le cryostat. La température atteinte varie de  $-160^\circ C$  à  $-180^\circ C$ . Nous avons utilisé pour la mesurer une thermistance au Platine qui fonctionne dans le domaine de température allant de  $-200^\circ C$  à  $+ 290^\circ C$ . Par sa disposition, elle donne avant tout une indication sur la température du support de la diode et donc seulement une estimation de la température de la diode elle-même. Nous n'avons pas essayé d'utiliser l'élément Peltier

ou même la thermistance interne de la photodiode à ces températures. Il est donc probable que la diode fonctionnant sans sa régulation de température habituelle doit connaître des dérives thermiques plus importantes que lorsqu'elle fonctionne en boîtier classique. Cependant les faibles températures atteintes laissent penser que l'équilibre thermique s'établit rapidement.

### 3.5.2 Résultats expérimentaux

Nous avons testé plusieurs diodes en régime cryogénique, en appliquant les mêmes techniques que celles utilisées à température ambiante (diode sur réseau et injectée). L'évolution du seuil d'oscillation observée expérimentalement est compatible avec les ordres de grandeur donnés par l'équation (3.1). Typiquement les valeurs du seuil passent de 18  $mA$  à température ambiante à 3  $mA$  pour la diode refroidie ; l'efficacité quantique, en revanche, est peu modifiée et présente une légère augmentation : sa valeur typique se situe entre 70 et 72% (à comparer avec 65 – 67% à température ambiante) ; la longueur d'onde d'émission varie considérablement et de façon pas toujours reproductible (par exemple, une diode à 851  $nm$  à température ambiante tombait à 817  $nm$  en régime cryogénique, alors que deux diodes à 848  $nm$  à température ambiante émettaient à 805  $nm$  une fois refroidies). Nous allons maintenant examiner les propriétés de bruit des diodes refroidies dans les différentes configurations expérimentales que nous avons essayées.

#### 3.5.2.1 Diode libre

Pour les diodes libres, le régime des basses températures ne s'est généralement pas montré favorable au fonctionnement en régime de faible bruit. Sur la plupart des diodes que nous avons refroidies, nous avons constaté que les basses températures favorisaient un comportement fortement multimode et très bruyant. On observe certaines plages, entre 10 et 30  $mA$  sur lesquelles le bruit est moins fort que pour les autres courants, mais reste néanmoins élevé, de l'ordre de 16  $dB$ , avec cinq à six modes longitudinaux d'amplitude comparable. En dehors de ces plages, le bruit augmente très rapidement avec le courant et le comportement de la diode devient très multimode avec un peigne d'une trentaine de modes s'étendant sur plusieurs nanomètres. Le comportement multimode du laser est favorisé en régime cryogénique par la diminution du seuil : la condition d'oscillation est alors plus facilement atteinte par plusieurs modes de la cavité. De plus, à basse température, on tend à passer d'un régime d'élargissement homogène à un régime d'élargissement inhomogène de la courbe de gain.

Cependant, alors que ce comportement a été observé sur plusieurs diodes dont la longueur d'onde nominale était de 848 nm, une autre diode a présenté un comportement très différent : il s'agit d'une diode de longueur d'onde nominale 852 nm probablement endommagée, dont l'efficacité quantique différentielle à température ambiante n'était que de 44,3%, mais de 63% à basse température. Le bruit de cette diode a été mesuré par détection directe avec référence simultanée. Nous avons ainsi trouvé une plage de fonctionnement procurant plus de 9% de compression du bruit d'intensité en dessous du bruit quantique standard et un fonctionnement monomode longitudinal du laser.

### 3.5.2.2 Diode injectée et diode sur réseau

Sur la base du comportement modal de la diode libre à basses températures, caractérisé par un régime d'oscillation fortement multimode, nous avons décidé d'appliquer les techniques déjà utilisées à température ambiante afin de supprimer les modes longitudinaux et atteindre un fonctionnement monomode du laser.

L'atténuation des modes longitudinaux obtenue en régime cryogénique pour la diode injectée est tout à fait comparable à celle observée à température ambiante : la puissance des premiers modes longitudinaux (à partir du quinzième) par rapport au mode principal se situe entre  $-45$  et  $-50$  dB. Le réseau, en revanche, à basses températures, se révèle moins performant et le taux de réjection est de  $-45$  dB pour les premiers modes longitudinaux (à comparer avec celui mesuré à température ambiante de  $-55$  dB). En outre, nous remarquons qu'il est de plus en plus difficile d'obtenir un laser bien monomode pour des forts courants d'alimentation ; cela nous a été impossible pour des taux de pompage supérieurs à 15 fois au-dessus du seuil.

Les meilleurs résultats obtenus sont les suivants : pour la diode injectée la compression du bruit d'intensité mesurée est de 20% correspondant à 33% à la sortie du laser (après correction des pertes optiques totales de la chaîne de détection) ; les phénomènes d'anticorrélations entre modes ont été observés, comme dans le cas à température ambiante. La diode sur réseau a permis d'obtenir 16% de compression de bruit d'intensité (25% à la sortie du laser). L'évolution de la compression de bruit en fonction du taux de pompage pour les deux configurations expérimentales est montrée en figure 5. Les résultats sont donc légèrement meilleurs que ceux obtenus à température ambiante pour les diodes sur réseau mais ils restent inférieurs à la limite donnée par l'efficacité quantique de la diode.

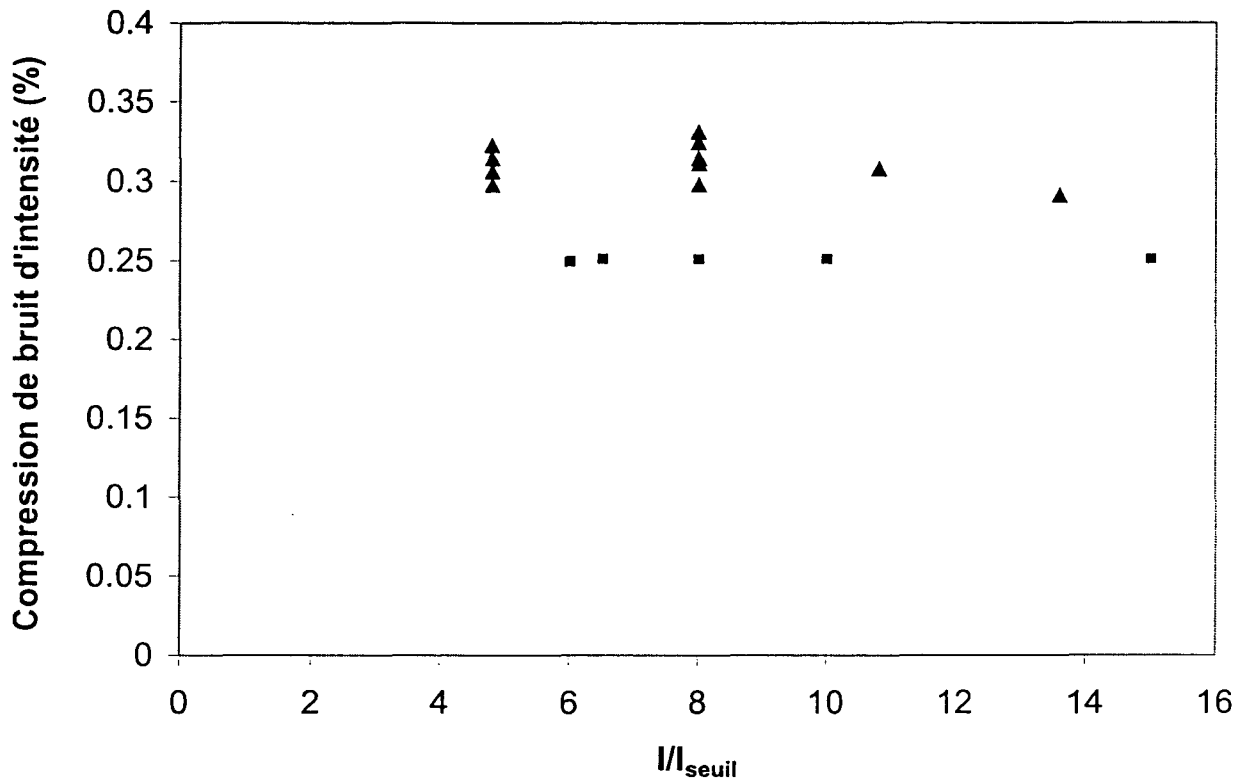


Fig. 5: Compression du bruit d'intensité pour une diode injectée (triangles) et pour une diode sur réseau (carrés) en fonction du taux de pompage

Contrairement à ce qu'on a constaté à température ambiante (cf. 3.3.1), en régime cryogénique, la compression de bruit observée est indépendante du taux de pompage (diode sur réseau) ou bien diminue légèrement quand le taux de pompage est augmenté (diode injectée). Ce comportement, à première vue assez surprenant, et contradictoire avec les prévisions théoriques présentées dans le paragraphe 3.3.1, trouve sa justification dans le comportement modal de la diode en fonction du courant (voir ci-dessus) : l'amélioration des performances attendue très au-dessus du seuil est masquée par la moindre efficacité des techniques d'affinement spectral dans ces conditions.

### 3.5.2.3 Mise en évidence des anticorrélations entre modes de polarisation orthogonale

Jusqu'à présent, nous n'avons pris en considération que les anticorrélations entre modes longitudinaux de même polarisation. Si cela est tout à fait justifié pour les diodes à température ambiante qui présentent un taux de polarisation (défini par le rapport de la puissance de la polarisation principale à la puissance de la polarisation

orthogonale) de l'ordre de 1000, il n'en est pas de même à basses températures, où le taux de polarisation est moins fort à cause de l'augmentation du gain et de la présence de biréfringence et est typiquement de l'ordre de 100. De plus, les résultats obtenus aux Etats Unis par le groupe de Steel [8] sur des diodes laser SDL en régime cryogénique (15 – 80 K) ont montré que le rôle joué par la polarisation secondaire sur le bruit d'intensité totale est important. En le prenant en compte correctement, comme nous l'expliquons plus loin, une considérable amélioration de la compression de bruit a été observée. Le phénomène physique à la base de cette amélioration est encore une fois l'anticorrélation entre les fluctuations de la polarisation principale et de la secondaire. Nous avons donc décidé d'étudier plus en détail ce problème pour nos lasers.

**Configuration expérimentale** Nous avons choisi la configuration de la diode injectée qui s'est là aussi révélée la plus intéressante. L'étude des effets des polarisations sur le bruit d'intensité de la diode nécessite, de façon préalable, un certain nombre de modifications du montage expérimental et du système de détection que nous avons précédemment décrit (cf. 3.2). En effet, dans le montage habituel pour les mesures de bruit à température ambiante le faisceau laser traverse plusieurs éléments polarisants avant d'être détecté par les photodiodes, ce qui empêche la détection de la polarisation secondaire. Il faut donc prendre soin d'éliminer chaque élément susceptible de privilégier une polarisation par rapport à l'autre. En particulier, cela interdit l'utilisation d'isolateurs optiques, ce qui rendra la diode beaucoup plus sensible au retour de lumière parasite. La fenêtre optique du cryostat précédemment à l'angle de Brewster, en verre non traité (cf. 3.5) a été remplacée par une fenêtre traitée antireflet, montée en incidence normale. Le laser maître est constitué par une diode sur réseau et son faisceau, après avoir traversé un isolateur optique (40 dB d'isolation), est superposé à celui du laser esclave grâce à une lame partiellement réfléchissante ; la polarisation de la lumière injectée est soigneusement contrôlée à l'aide d'une lame demi-onde et peut ainsi être alignée avec la polarisation principale du laser esclave, de manière à supprimer les modes longitudinaux associés à la polarisation principale, et atteindre le régime de compression de bruit.

Quelques mots méritent d'être consacrés à l'effet de la lame partiellement réfléchissante : son coefficient de transmission dépend de la direction de la polarisation de la lumière incidente par rapport au plan d'incidence et donc elle introduit des pertes différentes pour les deux polarisations orthogonales. Cela d'une part, comme toute perte, ramène le bruit vers le bruit quantique standard, et d'autre part, change le rapport des puissances des deux modes de polarisation orthogonale dans le faisceau et

donc leur contribution au bruit total. La réflectivité de la lame que nous avons utilisée est de 13% pour la polarisation principale et de 7% pour la secondaire. Les résultats expérimentaux que nous montrerons se rapportent à la sortie du laser et sont corrigés des pertes de détection introduites par ce type de composant.

Quant à la détection équilibrée, elle peut encore être utilisée pour étudier le bruit d'intensité du faisceau, mais avec certaines précautions. La mesure du bruit d'intensité totale peut se faire suivant la procédure habituelle mais la mesure du bruit quantique standard doit être effectuée en prenant soin d'interposer un polariseur pour éliminer la polarisation secondaire [19, 21]. En effet, le signal correspondant à la différence des photocourants dans la détection équilibrée est donné par l'équation (2.21) et ne correspond au bruit quantique standard que si le champ qui entre par la voie  $B$  de la lame est le vide. Or, dans le schéma de détection équilibrée que nous utilisons, la séparation du faisceau est réalisée au moyen d'éléments polarisants (lame demi-onde suivie du cube polariseur) : on peut donc considérer que le vide entre par la lame demi-onde, porté par la polarisation orthogonale à celle du faisceau à mesurer. Ce sont donc les fluctuations d'amplitude de la lumière qui entrent par cette voie du séparateur, que nous homodynons avec le champ moyen qui entre par l'autre voie. Il est important d'éliminer toute lumière sur la voie définie par la polarisation orthogonale à la polarisation du faisceau principal. Si nous ne prenons pas de précautions, au lieu du bruit quantique standard, c'est le bruit de la composante de polarisation orthogonale à celle du faisceau principal que nous mesurerons.

**Mesures** Les résultats obtenus, corrigés des pertes de détection ( $\sim 30\%$ ), sont montrés dans la figure 6. Les paramètres expérimentaux correspondant à cette mesure sont les suivants : le courant d'alimentation de la diode est de 24,3 mA (environ 10 fois au-dessus du seuil), le taux de polarisation vaut 90, la puissance injectée 4 mW.

La figure 6 porte les puissances de bruit en  $dBm/Hz$  en fonction de la fréquence de bruit, pour différentes situations. La courbe (a) représente le bruit quantique standard correspondant à l'intensité totale, la courbe (c) correspond au bruit d'intensité totale du faisceau laser (on détecte les deux polarisations) : la compression de bruit obtenue est d'environ 2 dB sous le bruit quantique standard ( $\sim 37\%$ ) sur une bande de 2 à 20 MHz. Les courbe (b) et (d) représentent respectivement le bruit de la polarisation principale et de la polarisation secondaire. Quand on place devant la détection un polariseur (nous utilisons un prisme de Glan avec un rapport d'extinction de  $5 \times 10^3$ ) et on l'aligne afin de transmettre et détecter seulement la polarisation principale, le

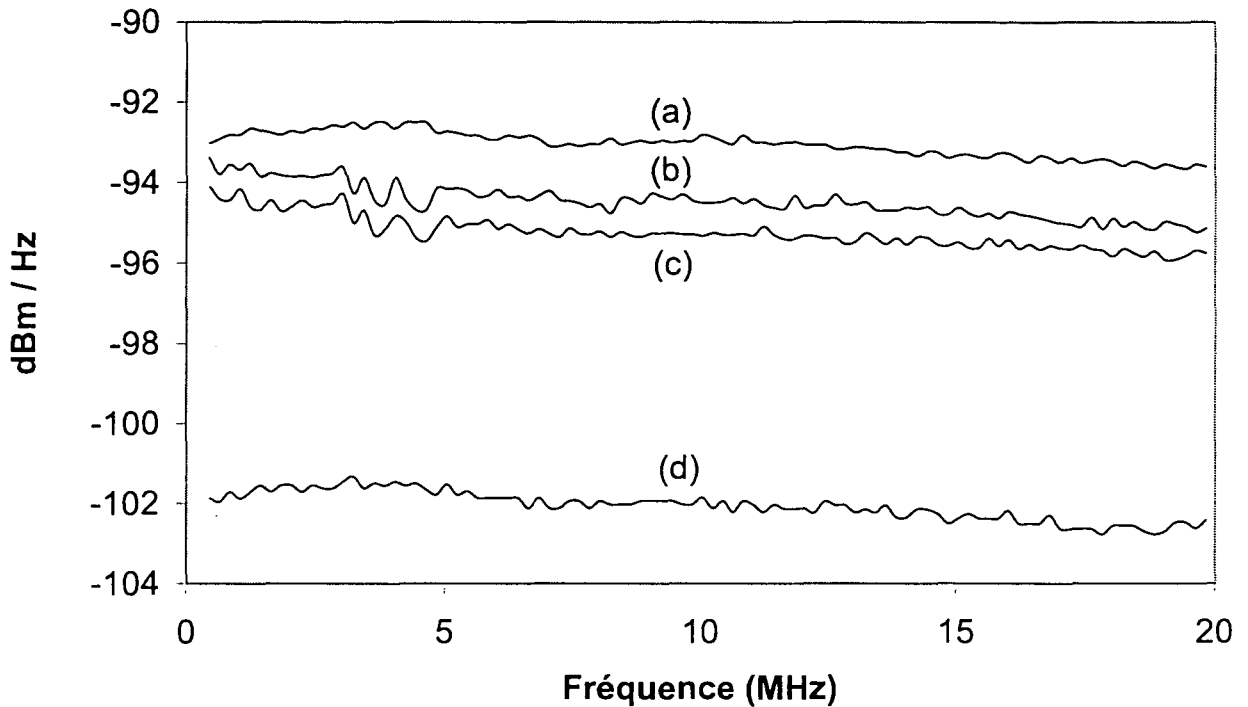


Fig. 6: Mise en évidence des anticorrélations entre les modes de polarisation orthogonale

bruit augmente d'environ  $0,4 \text{ dB}$ , comme on le constate en comparant les courbes (b) et (c) sur la figure 6. Cet effet est la signature claire de l'existence de phénomènes d'anticorrélations entre les fluctuations des modes de polarisation orthogonales. Il est donc nécessaire d'utiliser un système de détection qui préserve les deux polarisations pour obtenir des réductions de bruit plus importantes. Quand le polariseur est tourné pour transmettre la polarisation secondaire la trace (d) est enregistrée : remarquons que, compte tenu de sa faible puissance ( $1/90$  de la puissance de la polarisation principale), la polarisation secondaire présente un excès de bruit d'environ  $11 \text{ dB}$  par rapport à son propre bruit quantique standard. La réduction de bruit de  $37\%$  obtenue sur l'intensité totale constitue une réelle amélioration par rapport aux meilleures performances fournies par l'ancien montage (avec éléments polarisants) limitées autour de  $30\%$ . Si on considère le rapport entre réduction de bruit et efficacité quantique totale de détection, ce qui donne une indication sur la qualité de performances du laser (cf. 3.4.1), on trouve pour la mesure précédente une valeur de  $0,82$ , légèrement meilleure qu'à température ambiante (typiquement de  $0,75$ ).

A partir des données de la figure 6, on peut calculer le coefficient de corrélation  $C$  entre les modes de polarisation orthogonale. La définition de  $C$  est la suivante:

$$C = \frac{\langle \Delta n_{tot}^2 \rangle - (\langle \Delta n_{\parallel}^2 \rangle + \langle \Delta n_{\perp}^2 \rangle)}{2\sqrt{\langle \Delta n_{\parallel}^2 \rangle \langle \Delta n_{\perp}^2 \rangle}} \quad -1 \leq C \leq 1 \quad (3.2)$$

où  $\langle \Delta n_{tot}^2 \rangle$ ,  $\langle \Delta n_{\parallel}^2 \rangle$ ,  $\langle \Delta n_{\perp}^2 \rangle$ , représentent respectivement les variances de l'intensité totale, de la polarisation principale (parallèle à la jonction laser, d'où la notation  $\parallel$ ) et de la polarisation secondaire ( $\perp$ , orthogonale à la jonction laser). La normalisation adoptée assure que la valeur de  $C$  est comprise entre  $-1$  et  $1$ .  $C = -1$  correspond au cas où les fluctuations des deux modes de polarisation sont parfaitement anticorrélées, alors que  $C = 1$  correspond à une corrélation parfaite. A partir des mesures présentées sur la figure 6, nous avons calculé un degré de corrélation  $C = -0,4$  constant sur la bande de fréquence de 2 à 20 *MHz*.

### 3.6 Conclusion

Nous avons effectué une étude approfondie des propriétés de bruit des diodes laser commerciales SDL, avec des longueurs d'onde de fonctionnement de 810 et 850 *nm*, à température ambiante et en régime cryogénique (azote liquide).

A température ambiante, l'application du principe de la pompe régulière accompagné de l'utilisation des techniques classiques d'affinement spectral (diode sur réseau et diode injectée) a permis d'observer une réduction du bruit d'intensité sous le bruit quantique standard de 1,6 *dB* ( $\sim 31\%$ ) pour la diode sur réseau et de 2,3 *dB* ( $\sim 40\%$ ) pour la diode injectée. Le bruit de phase des diodes laser s'est révélé très au-dessus du bruit quantique standard (80 *dB* d'excès de bruit pour la diode libre) ; une sensible réduction est observée en utilisant les mêmes techniques (on obtient par exemple 46 *dB* d'excès de bruit pour une diode injectée).

L'analyse spectrale du rayonnement émis par la diode laser a permis d'expliquer le rôle des techniques d'affinement spectral et de démontrer expérimentalement que le bruit d'intensité des ces diodes est le résultat de la forte anticorrélation entre les fluctuations du mode principal et celles des faibles et nombreux modes longitudinaux. En particulier nous avons montré que la diode injectée constitue un exemple de laser multimode avec bruit d'intensité sous le bruit quantique standard, et que ce n'est que



pour la diode sur réseau que l'on peut parler de laser monomode à bruit d'intensité comprimé sous le bruit quantique standard.

La comparaison entre les résultats expérimentaux et les prévisions fournies par différents modèles quantiques du laser monomode est seulement partiellement satisfaisante et suggère la nécessité de développer un modèle plus complet qui puisse rendre compte correctement des propriétés de bruit des laser à semiconducteur.

Enfin, nous avons étudié les performances des diodes laser en régime cryogénique : parmi les nombreux lasers testés, seulement deux ont donnés de bons résultats. Les meilleures performances obtenues avec ces diodes fonctionnant dans les configurations expérimentales habituelles sont les suivantes : nous avons observé 9% de compression de bruit pour la diode libre, 25% pour la diode sur réseau et 31% pour la diode injectée. La mise en évidence des anticorrélations entre les fluctuations des modes de polarisation orthogonale dans le montage à injection a permis d'améliorer ce dernier résultat et d'atteindre ainsi 37% de compression de bruit. Cependant, les résultats que nous avons obtenus avec les diodes lasers refroidies bien que se situant parmi les meilleurs au niveau mondial, restent moins bons que ceux obtenus par le groupe de Steel sur le même type de composants [8, 21]. La raison principale de cette différence peut être attribuée au comportement fortement multimode que le régime cryogénique favorise sur nos diodes, en particulier pour un courant d'alimentation élevé. Cela entraîne l'impossibilité de faire fonctionner le laser très au-dessus du seuil, et de profiter pleinement de la sensible diminution du seuil d'oscillation à basses températures. Le montage cryogénique perd ainsi sa caractéristique principale, susceptible de constituer un avantage par rapport au fonctionnement à température ambiante. Nous n'avons pu obtenir des réductions de bruit que pour des taux de pompage inférieurs à 20, à comparer avec les meilleurs résultats du groupe de Steel, correspondant à des taux de pompage de l'ordre de 70. Les diodes dont dispose ce groupe conservent un comportement modal convenable aussi bien en régime cryogénique qu'à température ambiante (monomode avec taux de réjection des modes longitudinaux de 30  $dB$  pour la diode libre). Cela provient vraisemblablement du fait que ce groupe a pu avoir accès à des lasers sélectionnés par le fabricant.

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## 4 Bruit quantique dans les VCSELs

### 4.1 Introduction

La première démonstration de la possibilité de générer des états comprimés du rayonnement à l'aide de lasers à semiconducteur, comme nous l'avons vu dans le chapitre précédent, est due à Yamamoto et ses collaborateurs au milieu des années 1980. Pendant ces dix dernières années, les recherches menées par plusieurs groupes ont conduit à des progrès remarquables dans la compréhension des processus physiques qui sont à la base des caractéristiques de bruit que l'on observe dans les lasers à semiconducteur classiques. Grâce à ces connaissances, les meilleurs résultats de compression de bruit d'intensité obtenus avec des diodes laser ont pu atteindre la limite imposée par l'efficacité quantique du laser, en bon accord avec les prévisions théoriques.

Malgré le progrès enregistré dans le domaine des diodes conventionnelles, les performances fournies par les lasers semiconducteur à microcavité pompés par pompage électrique sont restées jusqu'à très récemment bien plus médiocres : le bruit d'intensité était typiquement un ordre de grandeur au-dessus du shot noise [28]. Cela est d'autant plus surprenant que les lasers semiconducteur à microcavité avaient été considérés comme les sources idéales pour la production d'états comprimés en intensité car ils présentent de nombreux et importants avantages par rapport aux diodes lasers conventionnelles. Dans les lasers à microcavité la longueur de la cavité est de l'ordre de la longueur d'onde de la lumière émise. Cette caractéristique implique que ces lasers fonctionnent dans un régime où des effets d'électrodynamique quantique deviennent non négligeables [29]. En particulier, l'extrême réduction du volume de la cavité a comme résultat de modifier l'émission spontanée du milieu actif et d'augmenter sensiblement la fraction qui est canalisée dans le mode lasant. L'inhibition complète de l'émission spontanée dans les modes non lasant pourrait conduire à une émission laser à bruit d'intensité comprimé indépendamment du taux de pompage : on aurait en effet une très grande efficacité quantique [30]. Si cet effet n'est pas encore atteint dans les

lasers dont on peut disposer à présent, néanmoins le volume réduit contribue à une sensible diminution du courant de seuil nécessaire à l'oscillation : celui-ci est inférieur au milliampère pour les meilleurs dispositifs. Cette valeur très basse du seuil permet d'atteindre des régimes de forts taux de pompage, nécessaires à la réduction du bruit quantique à température ambiante. De plus, les lasers semiconducteur à microcavité présentent des miroirs avec des valeurs très élevées du coefficient de réflexion ( $> 99\%$ ), nécessaires pour compenser le fait que le gain par passage est faible à cause des dimensions réduites de la cavité. Cela implique que la finesse de ces dispositifs est supérieure d'environ deux ordres de grandeur à celle de diodes laser traditionnelles.

Parmi les lasers semiconducteur à microcavité, un type particulier de dispositif a connu un développement important : les VCSELs, dont l'acronyme anglais signifie *vertical cavity surface emitting lasers*. Les avantages que ce nouveau type de laser présente par rapport aux diodes lasers habituelles sont liés au fait que la lumière est émise dans une direction verticale, c'est à dire orthogonale à la couche semiconductrice et parallèle au sens de propagation du courant d'alimentation. Cette caractéristique, qui s'accompagne de la miniaturisation du dispositif, ouvre la voie à des nombreuses applications dans le domaine opto-électronique : par exemple, il est possible de fabriquer des réseaux bi-dimensionnels de lasers pour la réalisation de liaisons tout optiques parallèles pour la transmission des données. En outre, la symétrie cylindrique très élevée autour de la direction de propagation de la lumière, réalisée dans ces lasers, est très avantageuse pour le couplage du faisceau émis dans des fibres optiques. Les propriétés de symétrie ont des conséquences sur les propriétés de polarisation de la lumière émise par les VCSELs : contrairement au cas des diodes lasers traditionnelles où la polarisation principale est fixée par la forte anisotropie de la jonction laser, dans les VCSELs, en principe, on n'a pas de contraintes sur la polarisation. L'état de polarisation du VCSEL est très sensible à des effets tels que des faibles anisotropies dans la structure cristalline ou dans les miroirs, et des phénomènes de compétition et de bistabilité entre les différents modes de polarisation, observés dans ces lasers, ont fait l'objet de nombreuses études théoriques et expérimentales [31–37].

En ce qui concerne plus précisément les caractéristiques de bruit auxquelles nous nous intéressons, les VCSELs offrent, de même que les lasers à semiconducteur classiques, la possibilité d'appliquer directement le principe de la pompe régulière par suppression du bruit du courant d'alimentation. Un autre atout des VCSELs est constitué par leur fonctionnement monomode longitudinal, imposé par la longueur de la cavité qui fixe un intervalle spectral libre supérieur à la largeur de la courbe de gain. Donc, dans ces dispositifs, le bruit associé à la compétition entre modes longitudinaux,

qui joue un rôle déterminant dans les diodes lasers conventionnelles (cf. 3.3.2) est automatiquement éliminé. Cependant, le fonctionnement monomode transverse n'est en général pas atteint et on peut observer une émission avec plusieurs modes transverses d'ordre supérieur [38]. L'oscillation simultanée de deux modes de polarisation orthogonale observée dans ces lasers a confirmé l'existence de phénomènes d'anticorrélation analogues à ceux que nous avons mis en évidence dans les diodes lasers conventionnelles en régime cryogénique (cf. 3.5.2). Malgré ce fonctionnement multimode, les VCSELs sont, parmi les lasers semiconducteur à microcavités, les dispositifs mieux adaptés à la production d'états comprimés en intensité. En effet, la première observation expérimentale de réduction de bruit d'intensité dans un laser à microcavité a été effectuée très récemment avec un VCSEL [39]. Jusqu'à présent deux autres groupes, dont le nôtre, ont pu produire des états comprimés en intensité avec des VCSELs [40, 41].

## 4.2 Observation d'états comprimés en intensité

Les VCSELs que nous avons utilisés sont fabriqués en Allemagne, au Département d'Opto-électronique de l'Université de Ulm, par le groupe du Professeur Ebeling [42]. Nous disposons de deux échantillons comprenant au total presque 200 lasers avec différents diamètres du milieu actif (entre 3 et 20  $\mu m$ ). Sur la base de l'expérience acquise avec les diodes lasers, pour observer des états comprimés nous avons sélectionnés parmi ces lasers ceux qui possèdent la meilleure efficacité quantique, le courant de seuil le plus bas et le comportement monomode le plus satisfaisant. Notons ici que ce sont les lasers de taille la plus petite (3  $\mu m$ ) qui présentent les meilleures caractéristiques ; malheureusement, dans notre montage, pour des raisons inexplicées, ils ont montré une grande fragilité et leur courte durée de vie a empêché une étude approfondie de leur caractéristiques de bruit. Les lasers de diamètre supérieur à 10  $\mu m$  présentent un bruit d'intensité assez élevé ( $> 5$  dB au-dessus du shot noise). Nous avons donc examiné en détail le bruit des lasers de diamètre 5, 7 et 10  $\mu m$ . Dans l'article qui suit nous présentons les résultats expérimentaux concernant la réduction de bruit dans les VCSELs. La meilleure compression mesurée est  $-0,75$  dB sous le bruit quantique standard et a été obtenue avec un laser de diamètre 5  $\mu m$  en fonctionnement multimode transverse (plusieurs modes transverses correspondant aux deux polarisations orthogonales oscillent simultanément). Notre analyse met en évidence l'influence du diamètre du laser sur le bruit d'intensité et montre que la compression de bruit obtenue

est due aux phénomènes d'anticorrélations entre modes transverses appartenant aux polarisations orthogonales.

#### 4.2.1 Reproduction de l'article : "Squeezed light generated by multimode VCSELs" (preprint)

## Squeezed light generated by multimode VCSELs

A. Bramati, J.-P. Hermier, A.Z. Khoury and E. Giacobino

*Laboratoire Kastler Brossel, Université Pierre et Marie Curie,*

*Ecole Normale Supérieure, CNRS*

*4, place Jussieu, F-75252 Paris Cedex 05, France*

P. Schnitzer, R. Michalzik and K.J. Ebeling

*University of Ulm Optoelectronics Dept. Albert-Einstein-Allee 45*

*D-89069 Ulm, Germany*

We demonstrate the possibility of generating intensity squeezed light with multimode VCSELs. Sub shot noise operation results from very strong anti correlations between the transverse modes. The influence of the active media diameter on the amount of squeezing is analysed.

### A. Introduction

VCSELs have been studied extensively in the past few years because of several useful characteristics and because they appear very promising both for industrial applications and for basic research. Indeed they show many advantages with respect to the previous standard semiconductor lasers architectures. They present a very low threshold, a high quantum efficiency and they can exhibit single longitudinal and transverse mode operation [1]. However, the maximum single mode power is limited by the onset of higher order transverse modes [2]. Moreover, many changes have been observed in the polarisation states of the emitted field as the driving current is increased [3,4].

In this letter, we investigate the intensity noise of high quantum efficiency oxide confined

VCSELs. Their features suggest that they are promising candidates for the generation of amplitude squeezed light. In general, single mode operation is most suitable and squeezing in such a situation has already experimentally been observed [5]. However, as theoretically predicted [6], in the case of two modes operation, the strong anticorrelations between the two transverse modes allow to achieve squeezed light [7]. In this letter, we generalise this result and report the experimental generation of amplitude squeezing with VCSELs operating with a large number of transverse modes.

The material of the paper is organised as follows : after this introduction, we present the experimental setup in section B. In section C, we analyse the experimental results. Finally, in section D, we summarize the results and present some possibilities for future work.

### B. Experimental setup

We use oxide confined VCSELs (made at the Department of Optoelectronics of the University of Ulm) with different active media diameters: 5, 7 and 10  $\mu m$ . The devices are schematically shown in figure 1. They consist of carbon doped p-type AlGaAs/AlGaAs and silicon doped n-type AlAs/AlGaAs Bragg reflectors with pairs of quarter wavelength thick layers. The top (respectively bottom) mirror has a reflectivity of 99,8 % (respectively 99%). They surround the three active 8 nm thick GaAs quantum wells, the cladding layers and the oxide aperture which provides both current and optical confinement. The devices are attached to a copper plate using silver paste and have an emission wavelength of about 840 nm.

Figure 2 shows the detail of the experimental setup. According to the principle of pump noise suppression [8], a low noise home made power supply with an appropriate LC filter

provides the regulated electrical current which drives the VCSELs. The VCSELs are also thermally stabilised with an active temperature stabilisation. With this stabilisation, we were able to operate at a fixed temperature with a drift less than  $0.01^{\circ}\text{C}$  per hour. The light beam is collimated by an antireflection coated microscope objective located at a distance of 2 mm from the laser output. This objective has a large numerical aperture ( $\text{N.A.} = 0.6$ ) to avoid optical losses which would deteriorate the squeezing. To measure the intensity noise and the corresponding shot noise, the standard scheme consists in a pair of two high quantum efficiency balanced photodiodes : this is the usual homodyne detection. The sum of the two photocurrents is proportional to the intensity noise while the difference is proportional to the corresponding shot noise [9]. However, in this case it was more appropriate to use only one photodiode (FND100, bandwidth 1-30 MHz, quantum efficiency of 90 %). Indeed, because of the multimode operation with two orthogonal linear polarisations, the shot noise obtained with a balanced detection would not be reliable and we preferred to use a separately calibrated shot noise. The shot noise is obtained by homodyne detection of a diode laser beam which has an intensity noise 0.5 dB below the shot noise in the range of frequency of 1-30 MHz. We checked carefully the linear dependance of the calibrated shot noise signal with the optical power incident on the photodiodes. The shot noise obtained with this method was in agreement within 0.1 dB with the noise obtained by a thermal light generating the same DC current on the photodiode. The photodiode is connected via a low noise home made amplifier (with a CLC425) and electronic amplifier (Nucleotide 4-40-1A) to a spectrum analyser (Tektronics 2753P). With this setup, the electronic noise was more than 6 dB below the signal we measured for a typical detected power of 1.5 mW. In our experiment, we could also perform a spectral analysis of the laser beam with a high resolution monochromator



(0.03 nm at 840 nm). At the output of the monochromator, a Glan polariser (extinction ratio  $10^{-4}$ ) allows us to measure the polarisation of the modes while a photodetector enables the measurement of the relative power of each transverse mode.

### C. Experimental results

First, we report the average threshold and the average differential quantum efficiency of the different kinds of VCSELs obtained after repeated measurements on several devices: the  $5 \mu\text{m}$  diameter VCSELs have an average differential quantum efficiency of 51.5 % and an average threshold of 0.67 mA, the  $7 \mu\text{m}$  diameter VCSEL an average differential quantum efficiency of 46.6 % and an average threshold of 1.15 mA while the  $10 \mu\text{m}$  diameter ones present an average differential quantum efficiency of 42 % and an average threshold of 1.73 mA. Since a low threshold and a high quantum efficiency are required to obtain squeezing, it appears that the lowest diameter VCSELs have the most interesting features.

In figure 3, we have plotted the measured normalised intensity noise versus the frequency for a VCSEL of  $5 \mu\text{m}$  driven with a current of  $I = 5.68 \text{ mA}$  and delivering an output power of 3.3 mW. The best squeezing obtained is about  $-0.75 \text{ dB}$  at the laser output (after correction for optical losses). The VCSELs with higher diameters present less favorable intensity noise characteristics. We obtained some intensity squeezing with the  $7 \mu\text{m}$  diameter VCSELs. The maximum amount (after correction for optical losses) was about  $-0.6 \text{ dB}$ . For the  $10 \mu\text{m}$  diameter VCSELs, the intensity noise we measured for various electrical driving currents was always above the shot noise level (at least 2 dB). These results confirm that, as expected, the lowest diameter VCSELs have the best characteristics for the generation of low intensity noise beams.

In figure 4, we have represented the results of the spectral analysis of the intensity squeezed light beam whose intensity noise is represented in figure 3. We have plotted the power (in dB) of the different modes versus their frequencies taking the lowest frequency TEM<sub>00</sub> mode as the reference for the two scales. We will see below that the noise of each mode is very large, so if the total intensity noise of the beam is under the shot noise, large anticorrelations between the fluctuations of the different modes are expected, as observed in other experiments with laser diodes and VCSELs [7,10,11].

To demonstrate the importance of these anticorrelations, we analysed the simple case of two transverse orthogonally linearly polarised modes oscillating together. Experimentally, we observed such a situation with the TEM<sub>00</sub> and TEM<sub>10</sub> modes. With a polariser placed just before the photodiode which measures the intensity noise, we can separate each mode and measure their respective intensity noise. In figure 5, we have plotted the results obtained with a 7 μm diameter VCSEL driven with a current of 2.58 mA. Even if the total intensity noise is above the shot noise, we see that it is much lower than the intensity noise of each mode. Using the data of figure 5, we can calculate the degree of correlation C between the two modes as follows :

$$C = \frac{\langle \Delta I_{tot}^2 \rangle - (\langle \Delta I_{00}^2 \rangle + \langle \Delta I_{10}^2 \rangle)}{2\sqrt{\langle \Delta I_{00}^2 \rangle \langle \Delta I_{10}^2 \rangle}} \quad -1 \leq C \leq 1$$

where  $\langle \Delta I_{tot}^2 \rangle$  is the intensity noise of the total beam, and  $\langle \Delta I_{00}^2 \rangle$  (respectively  $\langle \Delta I_{10}^2 \rangle$ ) is the intensity noise of TEM<sub>00</sub> (respectively TEM<sub>10</sub>) mode. In the case of perfect correlations, C is equal to 1 while, in the case of perfect anticorrelations, C is equal to -1. At 10 MHz, for example, we calculate that C is equal to -0.993. This confirms the strong anticorrelations between the transverse modes.

However the anticorrelations are not perfect resulting in an enhancement of the intensity noise with increasing number of lasing modes. Indeed in the case of perfect anticorrelations between the different lasing modes, the total intensity noise is expected to be independent of the number of the oscillating modes. It depends only on the quantum efficiency and the rate pumping as in the case of a perfect monomode laser. Experimentally, we have noticed that, for a given driving current, the number of oscillating transverse modes increases strongly with the diameter of the VCSEL. This characteristic is another explanation for the fact that the lowest diameter VCSELs present the best intensity noise features.

#### D. Conclusion

We have shown in this letter that high quality VCSELs, operating with a large number of transverse modes can exhibit a total intensity noise below the shot noise level. Our measurements also confirmed the influence of the active media diameter aperture on the intensity noise. The lowest diameter VCSELs which have the best quantum efficiency and the lowest threshold, have the best intensity noise characteristics.

Moreover, we have proved that the observed squeezing was due to very strong anticorrelations between the transverse modes. So it will be interesting to study these anticorrelations in more details. More accurate investigation of the anticorrelations will be presented in a forthcoming paper.

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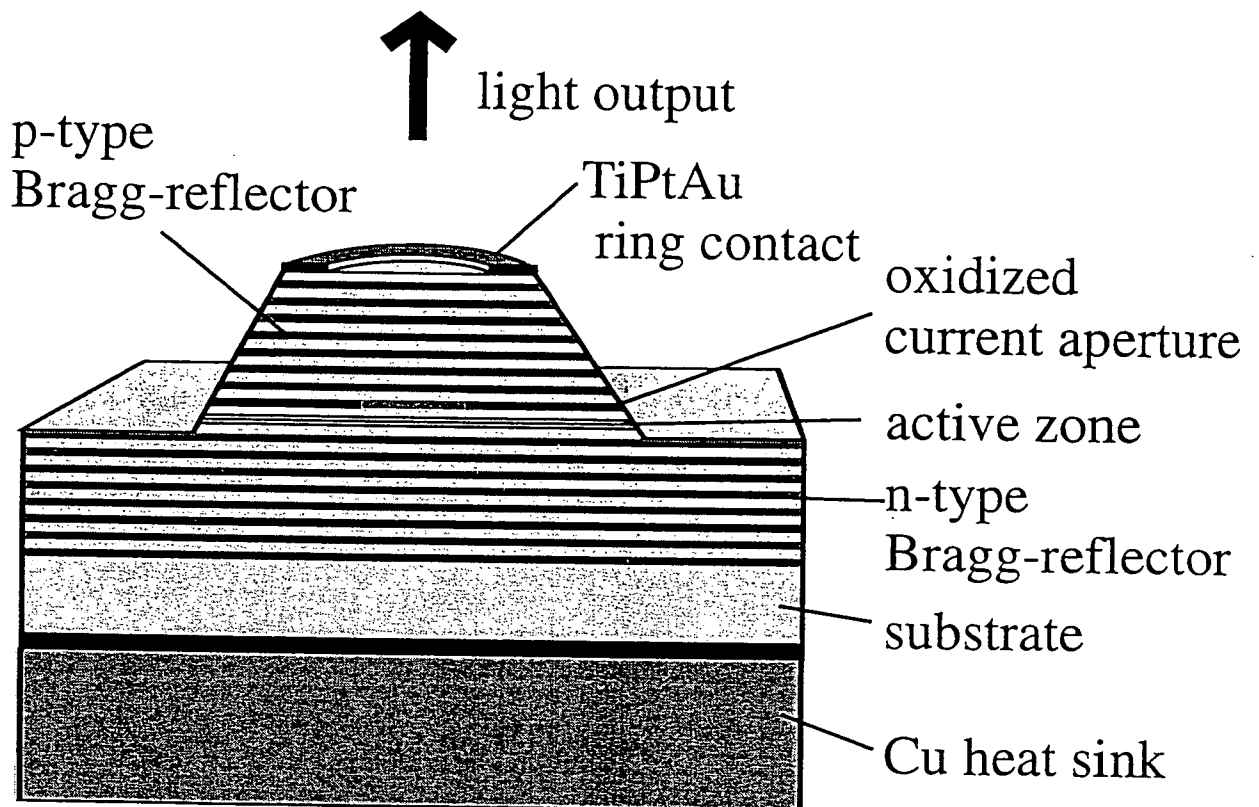


Fig.1. Schematic representation of the VCSEL.

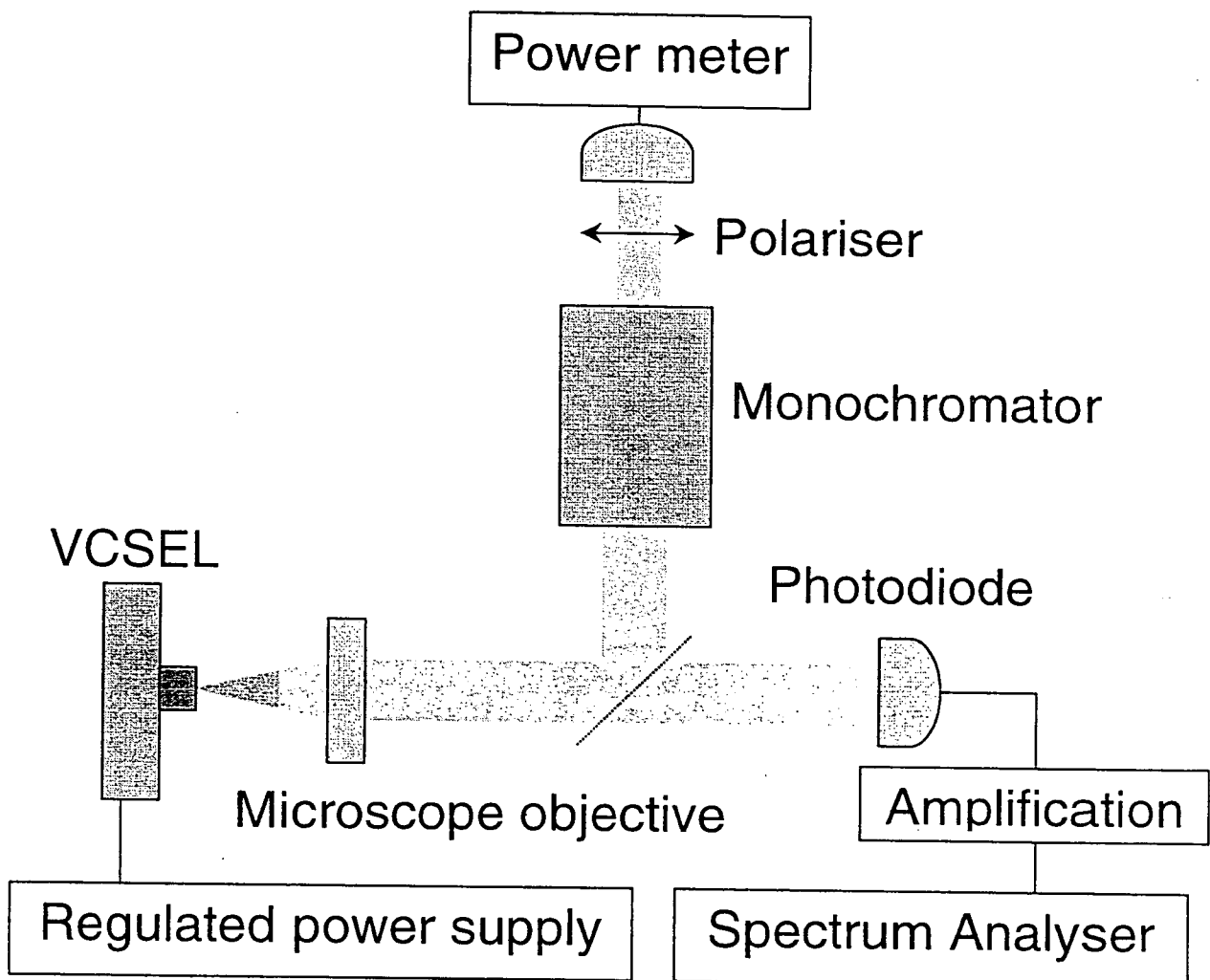


Fig.2. Experimental setup for the noise measurement on VCSELs.

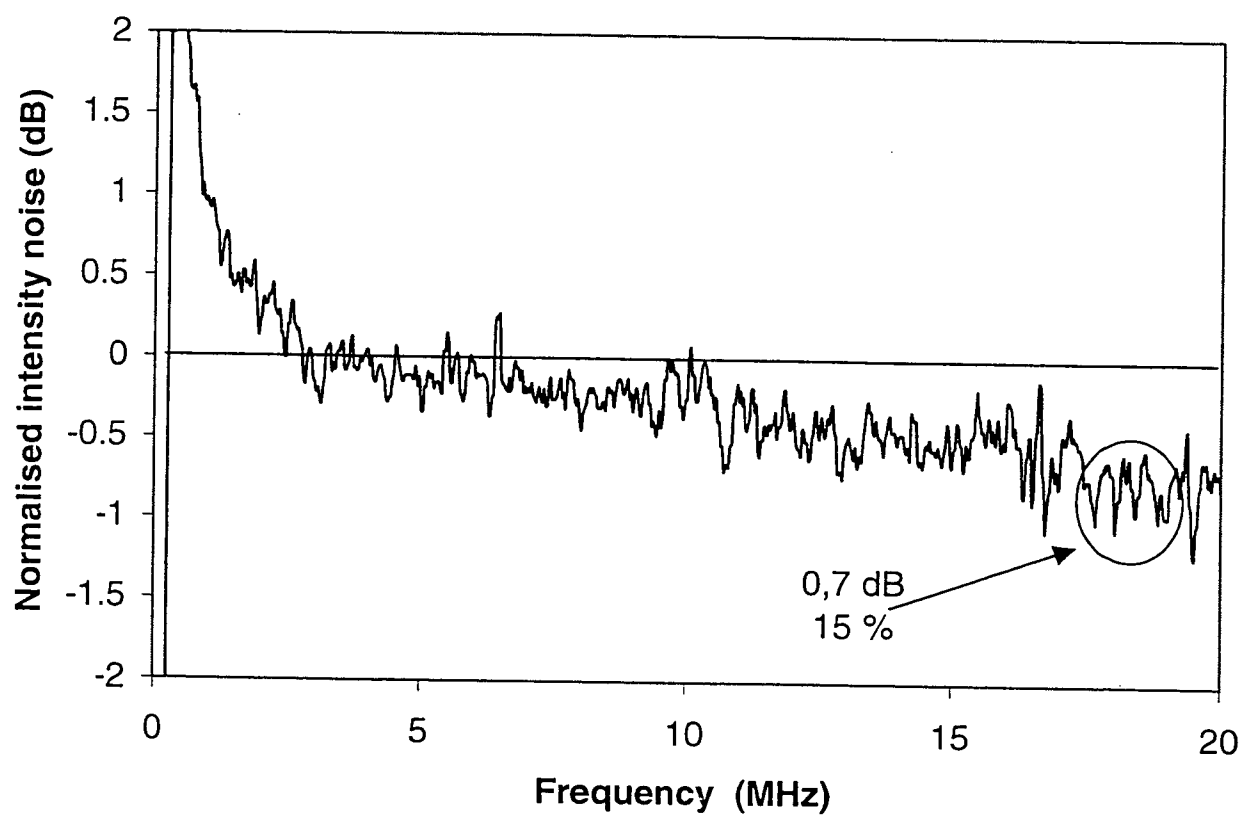


Fig.3. Normalised intensity noise spectrum (0 – 20 MHz) for a 5  $\mu m$  diameter VCSEL.

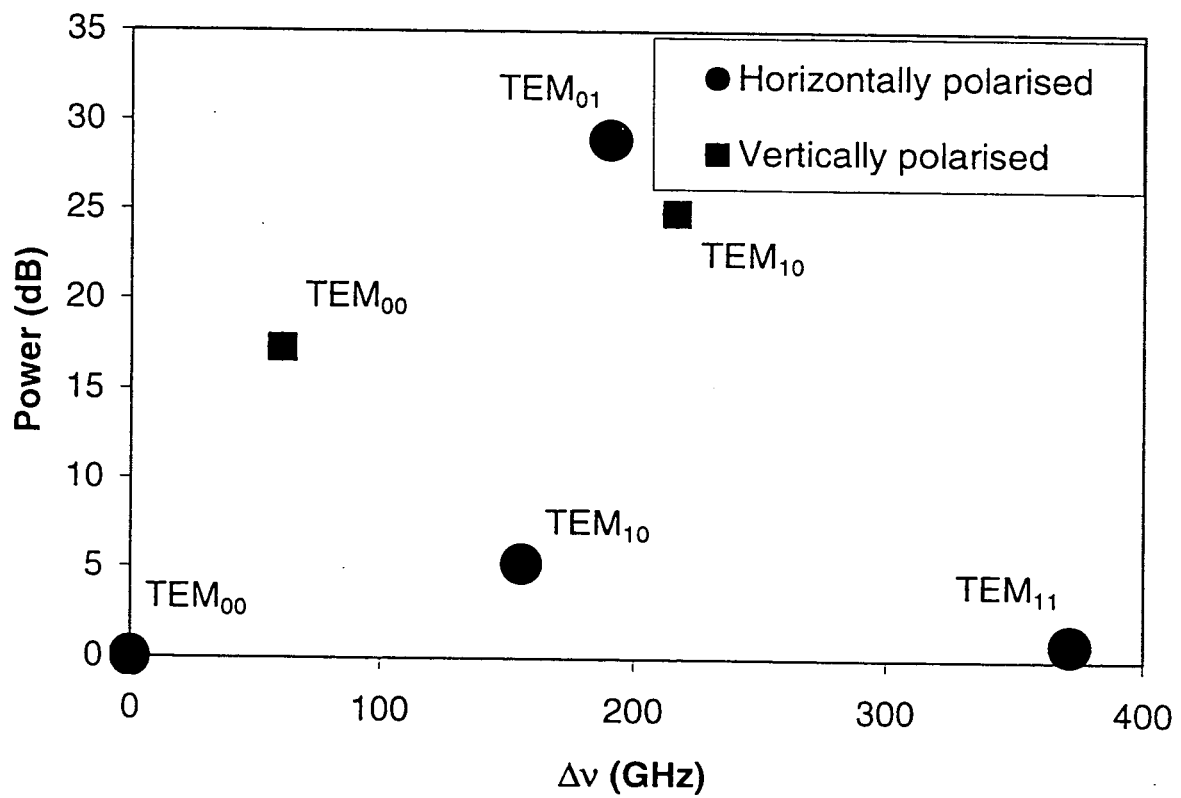


Fig.4. Spectral analysis of the intensity squeezed light beam of which intensity noise spectrum is represented in fig.3.



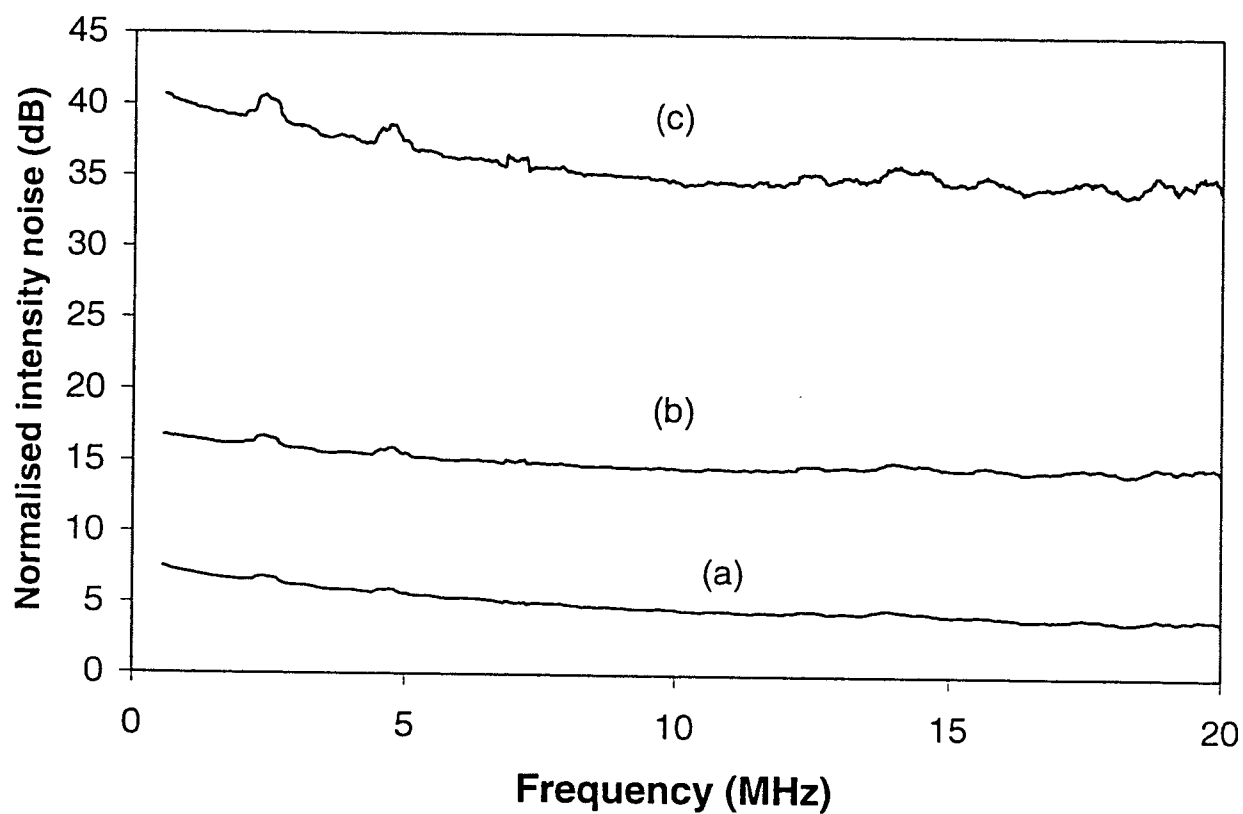


Fig.5. Normalised intensity noise of the total beam (a) and normalised intensity noise of each transverse mode composing the beam. The curve (b) corresponds to TEM<sub>00</sub> and the curve (c) to TEM<sub>10</sub>.

### 4.3 Distribution spatiale du bruit d'intensité

Dans le paragraphe précédent, nous avons montré l'importance fondamentale jouée par les corrélations entre les différents modes transverses dans la détermination des caractéristiques de bruit des VCSEls. Les corrélations ont été mises en évidence en comparant les mesures de bruit effectuées sur chaque mode de polarisation à la mesure du bruit de l'intensité totale. Une autre possibilité consiste à étudier la distribution spatiale du bruit d'intensité dans le faisceau et à en tirer des informations sur les corrélations entre les modes qui le composent [43]. C'est précisément ce que nous allons décrire dans ce paragraphe. Les mesures ont été faites en coupant progressivement le faisceau avec une lame de rasoir. Comme les différents modes transverses présentent différentes distributions d'intensité dans le plan transverse, la contribution de chaque mode au bruit d'intensité totale change en fonction de la position de la lame par rapport au faisceau. Nous nous sommes limités à l'étude des corrélations entre les modes  $TEM_{00}$  et  $TEM_{01}$  polarisés orthogonalement. Dans cette situation, il est possible, à l'aide d'un simple modèle, de prévoir les variations du bruit de l'intensité totale en fonction de la position de la lame et donc de les comparer aux résultats expérimentaux. L'article qui suit détaille les observations effectuées, la dérivation du modèle ainsi que la comparaison avec l'expérience. Il présente aussi une brève analyse théorique montrant que, en présence de corrélations et sous certaines conditions, la simple opération de couper un faisceau au-dessus du shot noise peut conduire à l'observation d'une réduction de bruit sur le faisceau coupé.

#### 4.3.1 Reproduction de l'article : "Spatial distribution of the intensity noise of a VCSEL"(preprint)

## Spatial distribution of the intensity noise of a VCSEL

A. Bramati, J.-P. Hermier, A.Z. Khoury and E. Giacobino

*Laboratoire Kastler Brossel, Université Pierre et Marie Curie,*

*Ecole Normale Supérieure, CNRS*

*4, place Jussieu, F-75252 Paris Cedex 05, France*

P. Schnitzer, R. Michalzik and K.J. Ebeling

*University of Ulm Optoelectronics Dept. Albert-Einstein-Allee 45*

*D-89069 Ulm, Germany*

J.Ph Poizat and Ph.Grangier

*Institut d'Optique, B.P. 147, F91403 Orsay Cedex, France*

We present a detailed study of the anticorrelations between the  $TEM_{00}$  and  $TEM_{01}$  transverse modes of a VCSEL through the transverse spatial distribution of the intensity noise. Our experimental results are found to be in good agreement with the predictions of a phenomenological model, that accounts for quantum correlations between modes.

### A. Introduction

During the last decade, a major effort has been put into the development of VCSELs. This new type of semiconductor lasers has some distinct advantages compared to conventional edge emitting semiconductor lasers. They present a lower threshold, a high quantum efficiency and can exhibit a single longitudinal and transverse mode operation [1].

These features make VCSELs good candidates for the generation of amplitude squeezed light. In general, single mode operation is most suitable. Unfortunately, as the driving current

is increased, high order transverse modes appear [2,3]. However, even in this case, strong anticorrelations between the transverse modes may allow to realise amplitude squeezed light [4,5].

In this paper, we investigate these anticorrelations through the spatial distribution of the intensity noise. We compare experimental results obtained with oxide confined VCSELs to the predictions of a phenomenological quantum model in the simple case of two orthogonally linearly polarised modes ( $TEM_{00}$  and  $TEM_{01}$ ). The measurement of the spatial distribution of the intensity noise is made by partially cutting the beam with a razor blade. The outline of this paper is the following : in section B, we present our model. In section C, we describe in detail the experimental setup. In section D, we present the experimental results and compare them to the theoretical predictions. Finally, we summarize the main results.

### B. Model

In this model, we consider a beam composed of two orthogonally linearly polarised transverse modes  $TEM_{00}$  and  $TEM_{01}$ . This situation is encountered rather often experimentally. We will see in section D that we can independently measure all the parameters introduced in the model. The beam emitted by the VCSEL is transversally cut with a razor blade (see fig.1). Since the two modes exhibit different transverse intensity distributions (see fig.2), their respective contribution to the total intensity noise will depend on the position of the blade.

In order to calculate the intensity noise of the detected part of the beam, the razor blade may be modeled as a beam splitter with position dependent transmissivity  $t(y)$  and reflectivity  $r(y)$  (see fig 2). Since the transverse intensity distribution is different for the two

modes, we will have to consider different transmissivities and reflectivities for each mode.

Let us now define some parameters (see fig.3):

- $y$  is the position of the blade,
- $t_i(y)$  is the amplitude transmissivity for the mode  $i$  ( $i = 00$  for the  $TEM_{00}$  and  $i = 01$  for the  $TEM_{01}$ ) for a given  $y$  position of the blade,
- $T_i(y)$  is the intensity transmissivity for the mode  $i$  for a given  $y$  position of the blade,
- $r_i(y)$  is the amplitude reflectivity for the mode  $i$  for a given  $y$  position of the blade,
- $a_i, a_i^\dagger$  are respectively the annihilation and creation operators of the mode  $i$  coming through the first port of the beam splitter,
- $b_i, b_i^\dagger$  are respectively the annihilation and creation operators of the mode  $i$  coming through the second port of the beam splitter. The input field in this port is the vacuum field for the two modes,
- $c_i, c_i^\dagger$  are respectively the annihilation and creation operators of the mode  $i$  transmitted by the beam splitter.

First, we can easily calculate  $t_{00}(y)$  and  $t_{01}(y)$  since we know the normalised amplitude profiles of the  $TEM_{00}$  and  $TEM_{01}$  modes :

$$t_{00}(y) = \left(\frac{2}{\pi}\right)^{1/4} \sqrt{1/w} \int_y^{+\infty} \exp(-y'^2/w^2) dy' \quad (1)$$

$$t_{01}(y) = \left(\frac{2}{\pi}\right)^{1/4} \sqrt{1/w} \int_y^{+\infty} 2(y'/w) \exp(-y'^2/w^2) dy' \quad (2)$$

We then deduce  $r_{00}(y)$  and  $r_{01}(y)$  from the relation  $t_i^2(y) + r_i^2(y) = 1$ :

$$r_{00}(y) = \sqrt{1 - \left(\frac{2}{\pi}\right)^{1/2} 1/w \left[ \int_y^{+\infty} \exp(-y'^2/w^2) dy' \right]^2} \quad (3)$$

$$r_{01}(y) = \sqrt{1 - \left(\frac{2}{\pi}\right)^{1/2} 1/w \left[ \int_y^{+\infty} 2(y'/w) \exp(-y'^2/w^2) dy' \right]^2} \quad (4)$$

With these definitions, we have the following relations between  $a_i$ ,  $b_i$  and  $c_i$  [6]:

$$c_i(y) = t_i(y)a_i + ir_i(y)b_i \quad (5)$$

We have also the following relations for the Hermitian conjugate operators,

$$c_i^\dagger(y) = t_i(y)a_i^\dagger - ir_i(y)b_i^\dagger \quad (6)$$

Since the two modes are orthogonally polarised, the photon number operator after the beam splitter is determined by:

$$n(y) = c_{00}^\dagger(y)c_{00}(y) + c_{01}^\dagger(y)c_{01}(y) \quad (7)$$

From the usual commutation relations between the different operators ( $[a_i, a_i^\dagger] = [b_i, b_i^\dagger] = [c_i(y), c_i^\dagger(y)] = 1$ ) and since the vacuum enters through the second port i.e.  $\langle b_i^\dagger b_i \rangle = 0$ , the mean photon number of the outgoing beam is obtained by a straightforward calculation :

$$\langle n(y) \rangle = \langle c_{00}^\dagger(y)c_{00}(y) \rangle + \langle c_{01}^\dagger(y)c_{01}(y) \rangle \quad (8)$$

$$= t_{00}^2(y) \langle a_{00}^\dagger a_{00} \rangle + t_{01}^2(y) \langle a_{01}^\dagger a_{01} \rangle \quad (9)$$

$$= T_{00}(y) \langle n_{00} \rangle + T_{01}(y) \langle n_{01} \rangle \quad (10)$$

where  $\langle n_{00} \rangle$  (respectively  $\langle n_{01} \rangle$ ) represents the mean photon number of the TEM<sub>00</sub> (respectively TEM<sub>01</sub>) mode.

We can also calculate  $\langle n^2(y) \rangle$ :

$$\begin{aligned} \langle n^2(y) \rangle &= T_{00}^2(y) \langle n_{00}^2 \rangle + T_{00}(y)(1 - T_{00}(y)) \langle n_{00} \rangle \\ &\quad + T_{01}^2(y) \langle n_{01}^2 \rangle + T_{01}(y)(1 - T_{01}(y)) \langle n_{01} \rangle + 2T_{00}(y)T_{01}(y) \langle n_{00}n_{01} \rangle \end{aligned} \quad (11)$$

Finally, we find that the photon number variance is given by  $\langle \Delta n^2(y) \rangle$ :

$$\langle \Delta n^2(y) \rangle = \langle n^2(y) \rangle - \langle n(y) \rangle^2 \quad (12)$$

$$\begin{aligned} &= T_{00}^2(y) \langle n_{00}^2 \rangle + T_{00}(y)(1 - T_{00}(y)) \langle n_{00} \rangle + T_{01}(y) \langle n_{01}^2 \rangle \\ &\quad + T_{01}(y)(1 - T_{01}(y)) \langle n_{01} \rangle + 2T_{00}(y)T_{01}(y) \langle n_{00}n_{01} \rangle \\ &\quad - T_{00}^2(y) \langle n_{00} \rangle^2 - T_{01}^2(y) \langle n_{01} \rangle^2 - 2T_{00}(y)T_{01}(y) \langle n_{00} \rangle \langle n_{01} \rangle \end{aligned} \quad (13)$$

Using the well known formula [7]:

$$: \Delta n_i^2 := \Delta n_i^2 - \langle n_i \rangle \quad (14)$$

(where  $: :$  means that the operators are normally ordered) we find that:

$$\begin{aligned} \langle \Delta n^2(y) \rangle &= T_{00}(y) \langle n_{00} \rangle + T_{00}^2(y) \langle : \Delta n_{00}^2 : \rangle + T_{01}(y) \langle n_{01} \rangle + T_{01}^2(y) \langle : \Delta n_{01}^2 : \rangle \\ &\quad + 2T_{00}(y)T_{01}(y) (\langle n_{00}n_{01} \rangle - \langle n_{00} \rangle \langle n_{01} \rangle) \end{aligned} \quad (15)$$

Let us define  $C$ , the degree of correlation between the two modes :

$$C = \frac{\langle \Delta n^2 \rangle - (\langle \Delta n_{00}^2 \rangle + \langle \Delta n_{01}^2 \rangle)}{2\sqrt{\langle \Delta n_{00}^2 \rangle \langle \Delta n_{01}^2 \rangle}} \quad -1 \leq C \leq 1 \quad (16)$$

A straightforward calculation shows that :

$$C = \frac{\langle n_{00}n_{01} \rangle - \langle n_{00} \rangle \langle n_{01} \rangle}{\sqrt{\langle \Delta n_{00}^2 \rangle \langle \Delta n_{01}^2 \rangle}} \quad (17)$$

Finally we find that :

$$\begin{aligned} \langle \Delta n^2(y) \rangle &= T_{00}(y) \langle n_{00} \rangle + T_{00}^2(y) \langle : \Delta n_{00}^2 : \rangle + T_{01}(y) \langle n_{01} \rangle + T_{01}^2(y) \langle : \Delta n_{01}^2 : \rangle \\ &\quad + 2T_{00}(y)T_{01}(y)C\sqrt{\langle \Delta n_{00}^2 \rangle \langle \Delta n_{01}^2 \rangle} \end{aligned} \quad (18)$$

In the case of perfect correlations,  $C$  is equal to 1 while, in the case of perfect anticorrelations,  $C$  is equal to -1. From eq (18), if  $C$  is equal to 0, we find :

$$\langle \Delta n^2(y) \rangle = T_{00}(y) \langle n_{00} \rangle + T_{00}^2(y) \langle : \Delta n_{00}^2 : \rangle + T_{01}(y) \langle n_{01} \rangle + T_{01}^2(y) \langle : \Delta n_{01}^2 : \rangle \quad (19)$$

If the transverse modes have an intensity noise above the shot noise level, we know [8] that  $\langle : \Delta n_{00}^2 : \rangle$  and  $\langle : \Delta n_{01}^2 : \rangle$  are positive which implies that  $T_{00}^2(y) \langle : \Delta n_{00}^2 : \rangle + T_{01}^2(y) \langle : \Delta n_{01}^2 : \rangle$  is also positive. Since  $T_{00}(y) \langle n_{00} \rangle + T_{01}(y) \langle n_{01} \rangle$  corresponds to the shot noise level when the razor blade is at the  $y$  position, it follows that the total intensity noise is above the shot noise level for every position of the razor blade.

Let us consider now a more interesting case :  $C = -1$ . In fig.4, we have plotted the normalised intensity noise versus the position of the blade normalised to the waist of the beam. As expected, the variations of the normalised intensity noise are very important. But the most interesting feature is that amplitude noise squeezing is even possible for some positions of the blade. Thus we can obtain amplitude squeezing from a beam having an intensity noise above the shot noise by partially cutting it, even if the intensity noise of each transverse mode is above the shot noise level.

### C. Experimental setup

For the measurements, we use high quantum efficiency oxide confined GaAs/AlGaAs VCSELs (made at the Department of Optoelectronics of the University of Ulm). The devices are schematically shown in fig.5. They consist of carbon doped p-type AlGaAs/AlGaAs and silicon doped n-type AlAs/AlGaAs Bragg reflectors with pairs of quarter wavelength thick layers. The top (respectively bottom) mirror has a reflectivity of 99,8 % (respectively 99%). They surround the three active 8 nm thick GaAs quantum wells, the cladding layers and the oxide aperture which provides both current and optical confinement. The devices are



attached to a copper plate using silver paste and have an emission wavelength of about 840 nm.

The detailed experimental setup is shown in fig.6. According to the principle of pump noise suppression [9], a low noise home made power supply with an appropriate LC filter provides the regulated electrical current which drives the VCSELs. The VCSELs are also thermally stabilised with an active temperature stabilisation. Thanks to this stabilisation, we were able to operate at a fixed temperature with a drift less than 0.01°C per hour. The light beam is collimated by an antireflection coated microscope objective located at a distance of 2 mm from the laser output. The objective has a large numerical aperture (N.A. = 0.6) to avoid optical losses which would deteriorate the squeezing. The measurements of the spatial distribution of the intensity noise were performed by partially cutting the beam with a razor blade. The blade is treated to avoid optical feedback into the laser. It is also mounted on a micrometric xy-translation stage controlled by a motor. The motor enables to cut gradually the beam over 50 seconds. To measure the intensity noise and the corresponding shot noise, the standard scheme consists in a pair of two high quantum efficiency balanced photodiodes, this is the usual homodyne detection. The sum of the two photocurrents is proportional to the intensity noise while the difference is proportional to the corresponding shot noise [10]. However, in this case it is more appropriate to use only one photodiode (FND100, bandwidth 10 kHz-30 MHz, quantum efficiency of 90 %). Indeed, because two modes with orthogonal linear polarisations oscillate simultaneously, the shot noise obtained with a balanced detection would not be reliable and we preferred to use a separately calibrated shot noise. The shot noise reference is obtained by homodyne detection of a diode laser beam which has an intensity noise 0.5 dB below the shot noise in the range

of frequency of 1-30 MHz. We checked carefully the linear dependance of the calibrated shot noise signal with the optical power incident on the photodiodes. The shot noise obtained with this method was in agreement within 0.1 dB with the noise obtained by a thermal light generating the same DC current on the photodiode. The photodiode is connected via a low noise home made amplifier (with a CLC425) and electronic amplifier (Nucletude 4-40-1A) to a spectrum analyser (Tektronics 2753P). With this setup, the electronic noise was more than 6 dB below the signal we measured for a typical detected power of 1.5 mW. In our experiment, we could also perform a spectral analysis of the laser beam with a high resolution monochromator (0.03 nm at 840 nm). At the output of the monochromator, a Glan polariser (extinction ratio  $10^{-4}$ ) allows us to measure the polarisation of the modes.

#### D. Experimental results

The experimental procedure is the following. To compare our experimental results to the theoretical predictions of the model presented in section B, we need first to measure the waist of the beam (see eq 18). To measure this waist, we choose an electrical driving current in order to have a single transverse mode operation. We find that this mode is the  $TEM_{00}$  mode. Under this conditions we measure the intensity of the transmitted beam versus the position of the blade normalised to the total intensity noise.

Since we know the spatial intensity distribution of the  $TEM_{00}$  mode, we know that the intensity when the razor blade reaches the  $y$  position is equal to:

$$I_{tot}(y) = \left(\frac{2}{\pi}\right)^{1/4} \sqrt{1/w} \int_y^{+\infty} \exp(-y'^2/w^2) dy' \quad (20)$$

The only free parameter in this equation is the beam waist  $w$ , which is obtained by fitting

the experimental results with the curve given by eq (20). This enables to deduce the value of the beam waist at the position of our photodiode. We can therefore calculate the value of  $t_{00}(y)$  and  $t_{01}(y)$ .

We then adjust the electrical driving current to have the VCSEL operating with only two transverse modes. This is possible for most of the VCSELs and the two transverse modes are  $TEM_{00}$  and  $TEM_{01}$  (or  $TEM_{10}$ ). Using a monochromator and a polariser we can check that the  $TEM_{00}$  and the  $TEM_{01}$  modes have linear and mutually orthogonal polarisations. We also measure the intensity of each mode (separating them with a Glan polariser), which gives the values of the shot noise  $\langle n_{00} \rangle$  and  $\langle n_{01} \rangle$ . From the measured intensity noise of each mode, we determine the values of the excess noise  $\langle : \Delta n_{00}^2 : \rangle$  and  $\langle : \Delta n_{01}^2 : \rangle$ . The measured total intensity noise enables to calculate the value of  $\langle \Delta n_{tot}^2 \rangle$ . Then we calculate  $C$  using eq. (16). Inserting these values in eq. (18), our model provides a value of  $\langle \Delta n^2(y) \rangle$  for every position of the blade.

Finally, we measure the total intensity noise and the total intensity of the beam versus the position of the blade. We normalise the total intensity noise to the shot noise corresponding to the intensity for every position of the blade.

In fig. 7, we have plotted the experimental results and the predictions of our model for a first VCSEL. The anticorrelations are equal to  $-0.7$  for these curves. As expected, we observe important variations of the intensity noise with the position of the blade. The agreement between theory and experiment is also very good even though there is a significant uncertainty for the experimental points at the right of fig. 7. This uncertainty is due to the fact that these points correspond to very low intensities. In fig. 8, we have plotted the experimental results and the predictions of our model for a second VCSEL presenting a

higher anticorrelation:  $-0.98$ . Hence, the variations of the intensity noise are even larger. If we compare the results of fig. 7 and fig. 8, we also notice that even the shape of the curves depends on the value of the correlations. The agreement between experimental results and theoretical predictions is again very good. This justifies that the hypothesis of our model were correct, in particular the fact that we have only taken two modes into account and that we have neglected the contribution of the non lasing modes.

#### E. Conclusion

We have studied in detail in this letter the anticorrelations between the transverse modes in high quantum efficiency VCSELs through the spatial distribution of the intensity noise. We have compared our experimental results to the predictions given by a phenomenological quantum model in the simple case of two transverse modes oscillating simultaneously. The agreement between theory and experiment is found to be very good.

Our model predicts that in some cases, we can get an amplitude squeezed light by partially cutting a beam which has an intensity noise above the shot noise level and which is composed of two transverse modes having also an intensity noise above the shot noise level. It also shows that it is possible to take only into account the lasing modes to study the intensity noise properties of the VCSELs.

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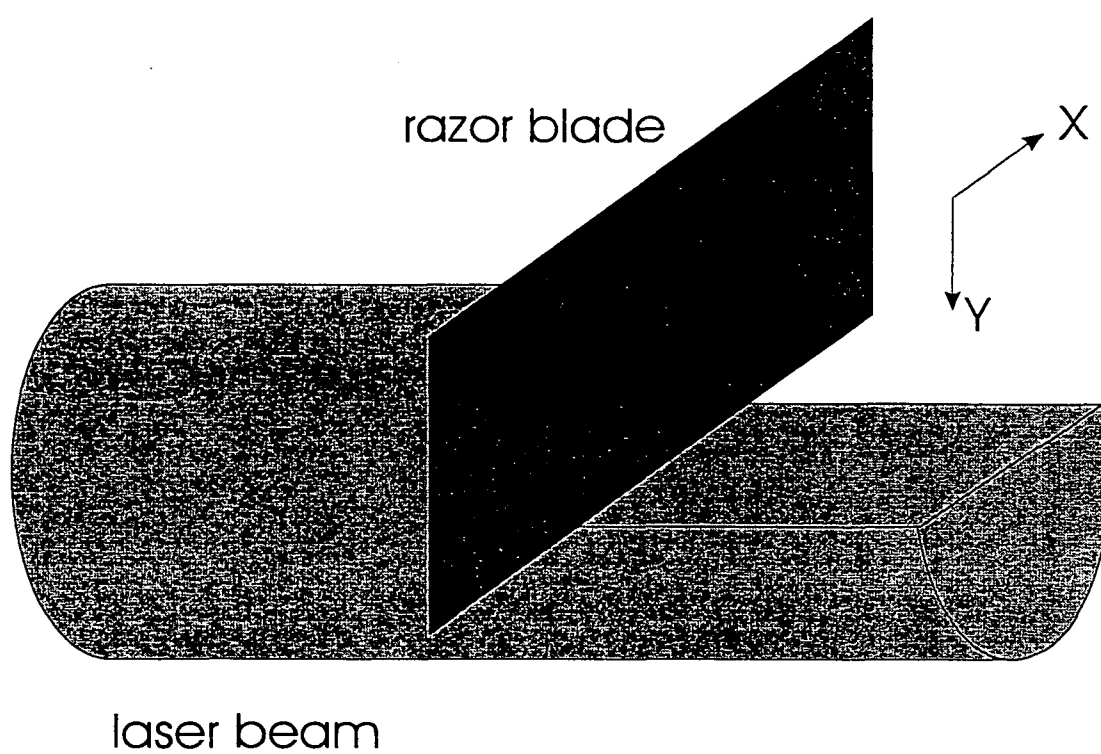


Fig.1. Schematic representation of the razor blade cutting the beam.

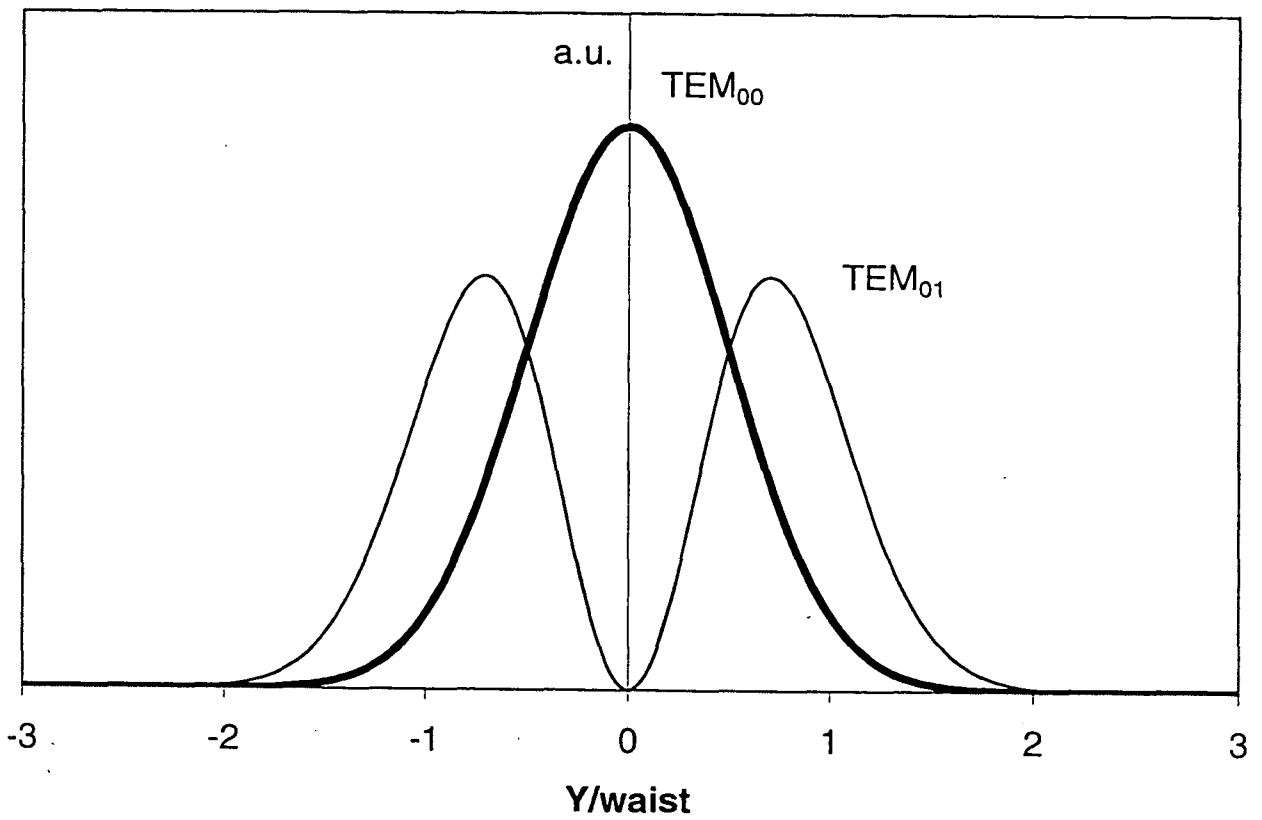


Fig.2. Schematic representation of the transverse spatial intensity distribution of the  $\text{TEM}_{00}$  and  $\text{TEM}_{01}$  modes.

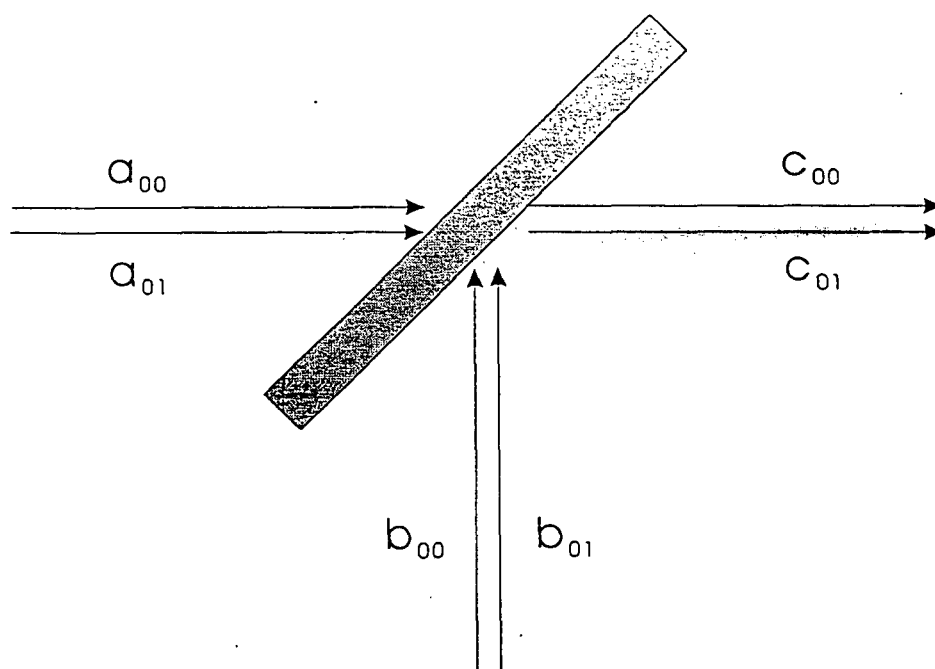


Fig.3. Schematic representation of the beam splitter used in our model to represent the losses of our razor blade.



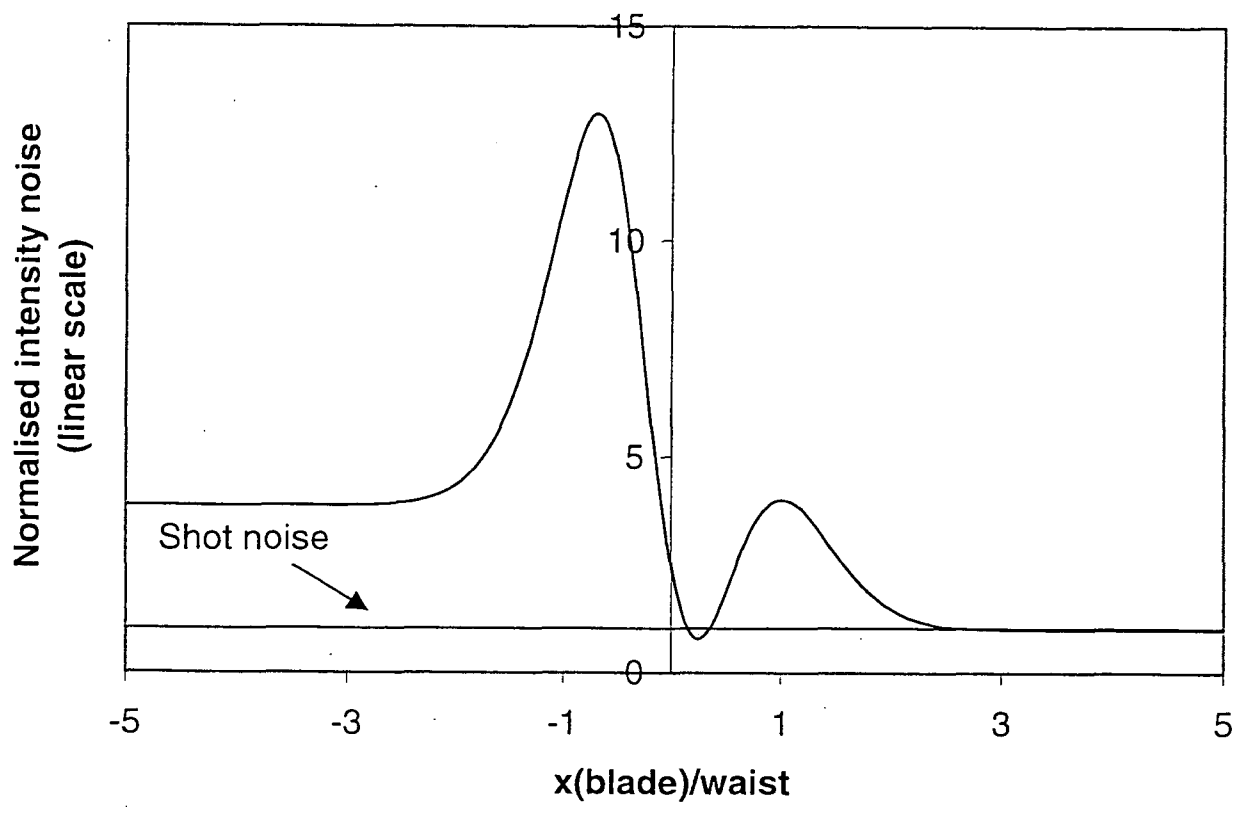


Fig.4. Normalised intensity noise spectrum versus the position of the razor blade normalised to the waist of the beam.

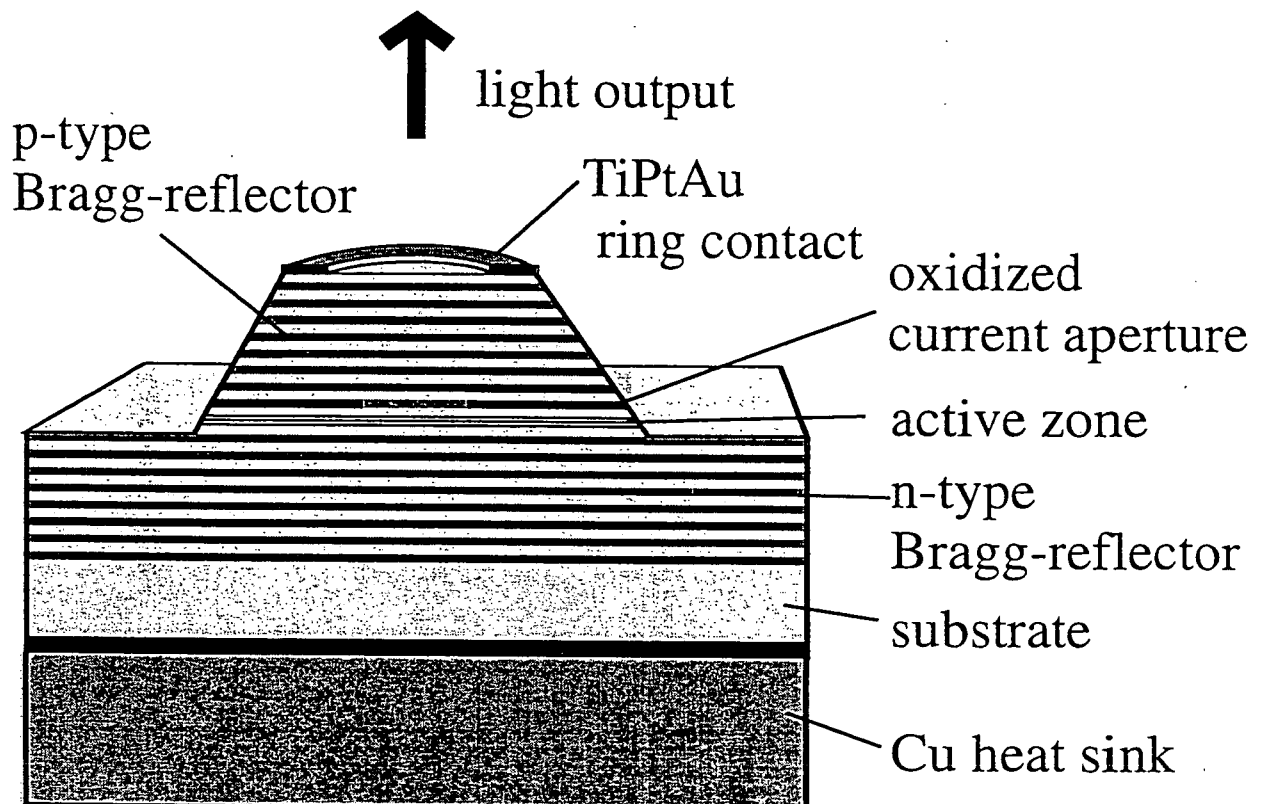


Fig.5. Schematic representation of the VCSEL.

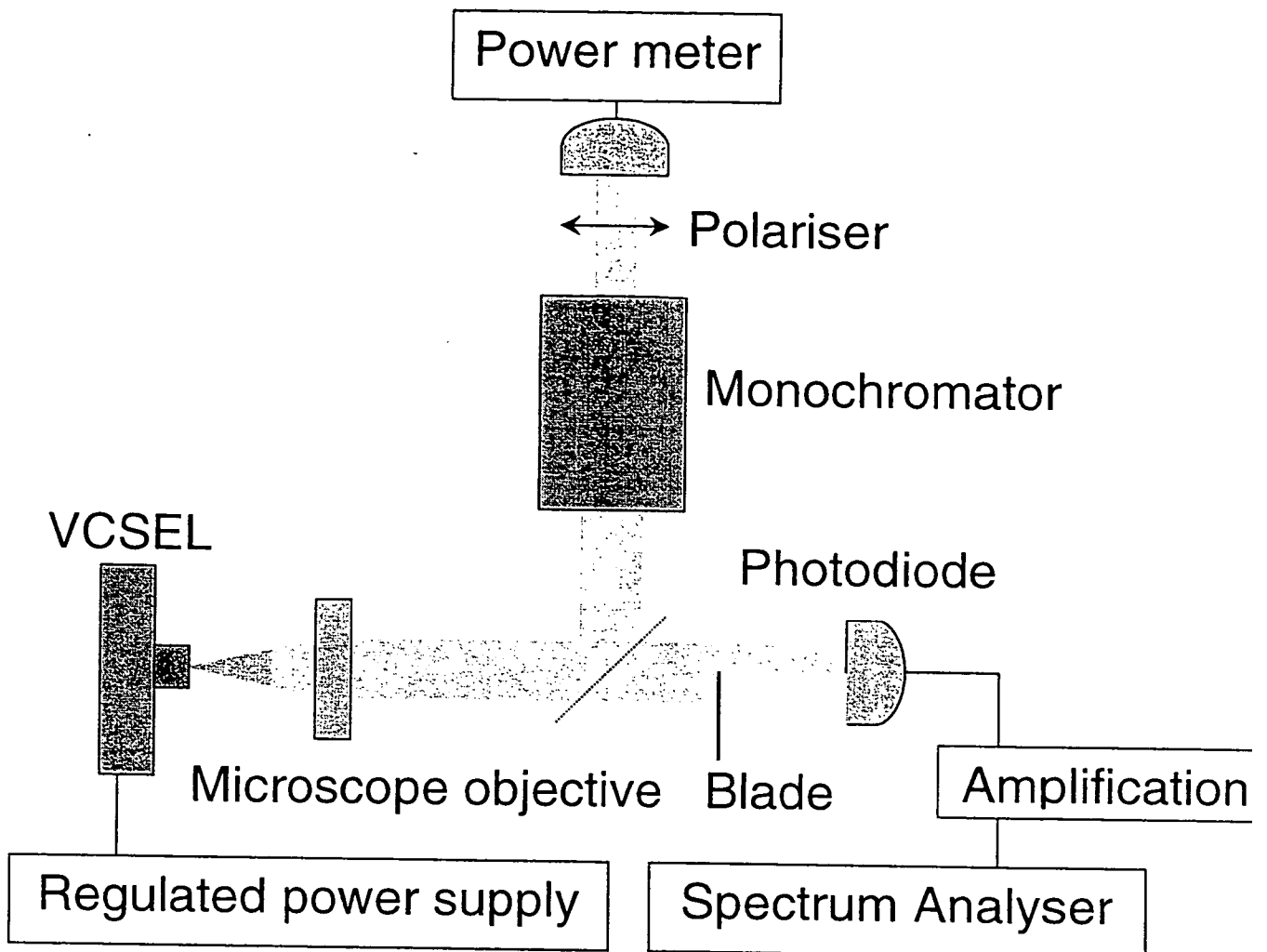


Fig.6. Experimental setup for the measurement of the spatial intensity noise distribution.

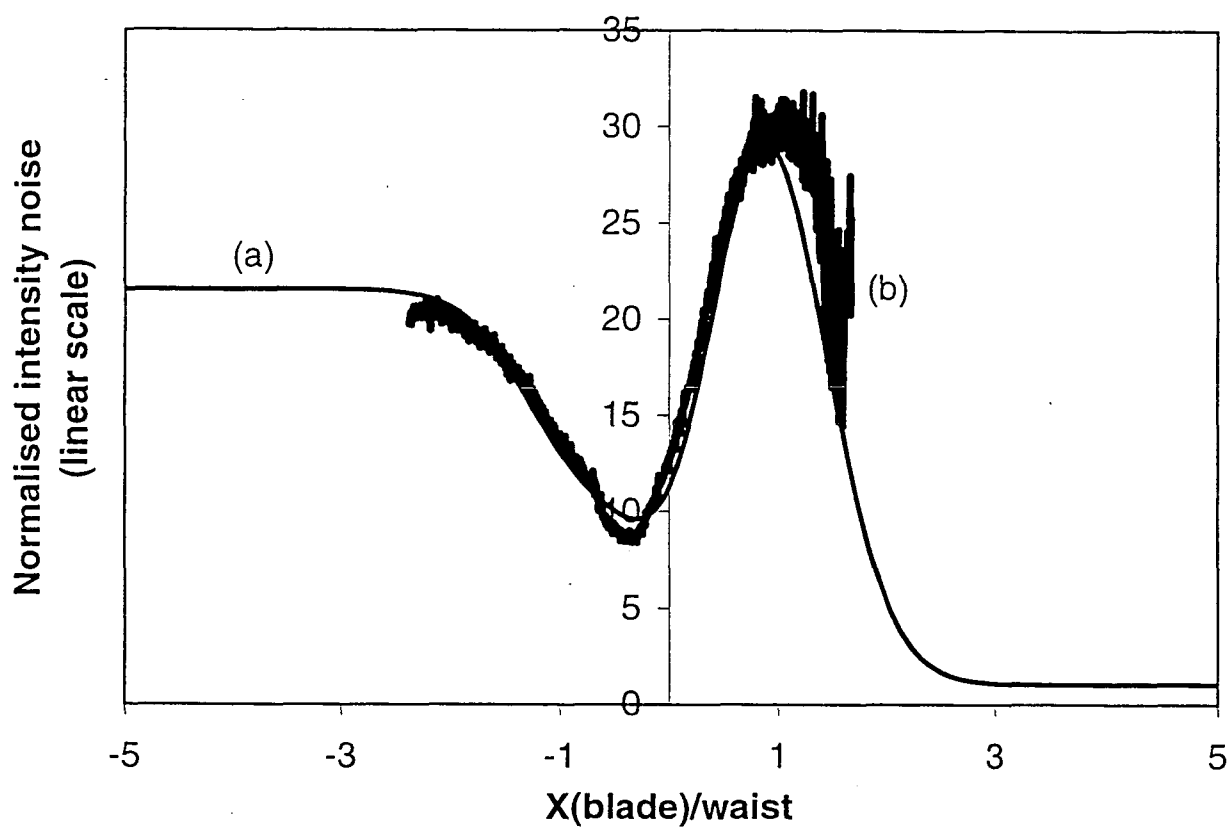


Fig.7. Normalised intensity noise versus the position of the razor blade normalised to the waist of the beam. Curve (a) is the theoretical prediction ; curve (b) correspond to experimental results.  $C$  is equal to  $-0.7$ .

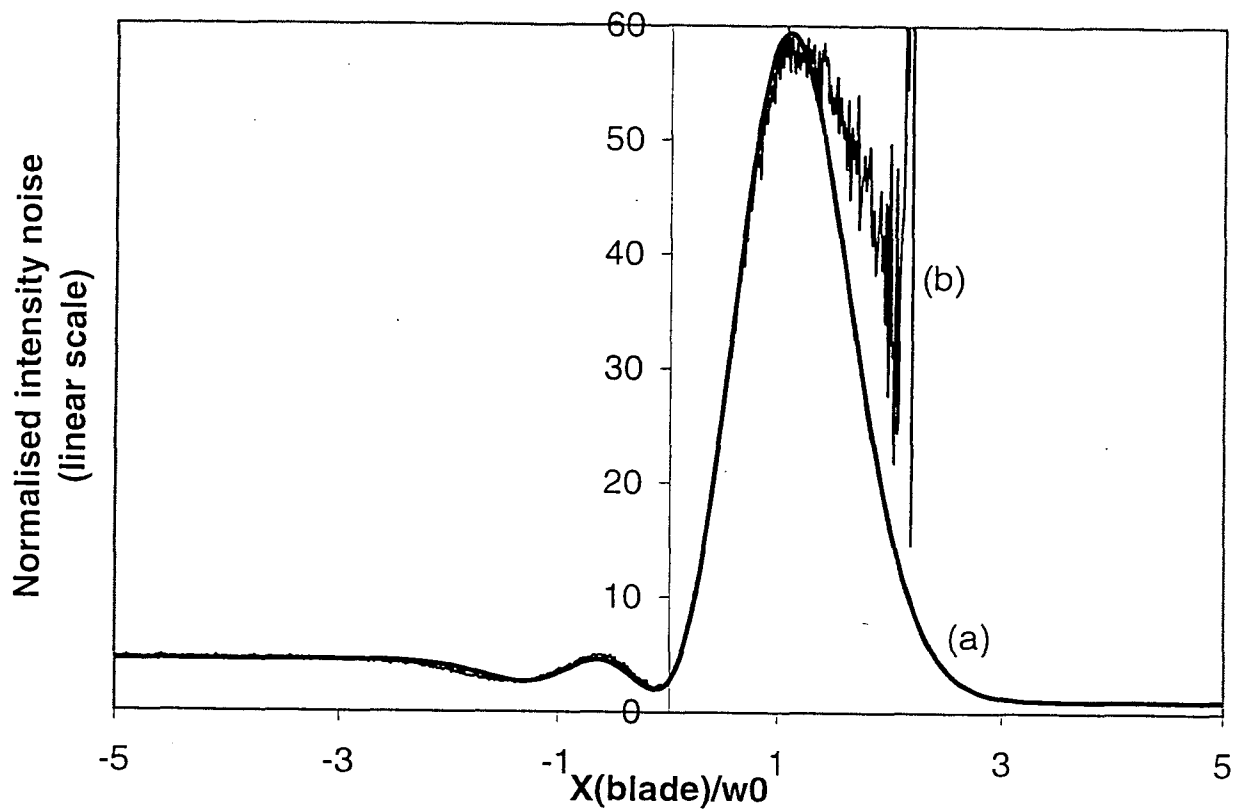


Fig.8. Normalised intensity noise versus the position of the razor blade normalised to the waist of the beam. Curve (a) is the theoretical prediction ; curve (b) correspond to experimental results.  $C$  is equal to  $-0.98$ .

## 5 Application du principe de la pompe régulière aux microlasers solides

### 5.1 Introduction

Les microlasers que nous avons étudiés sont de très petits cristaux d' $\text{YVO}_4$  dopés au Néodyme, de  $300 \mu\text{m}$  d'épaisseur, fabriqués au LETI à Grenoble. Ces lasers émettent de la lumière laser infrarouge à  $1064 \text{ nm}$ , lorsqu'ils sont pompés par un rayonnement vers  $810 \text{ nm}$ . Ces lasers sont déjà commercialisés dans certains dispositifs (pour la télémétrie, par exemple), en raison de leur compacité et leur qualités optiques. Dans ce travail, nous avons étudié la possibilité de leur appliquer le principe de la pompe régulière. Ces cristaux présentent en effet un certain nombre de traits communs avec les diodes laser qui indiquent l'intérêt de l'étude de l'influence du bruit de pompe et de la recherche des états comprimés du rayonnement qu'ils sont susceptibles de produire. Ces traits communs concernent notamment le bruit de pompe, qui peut être modifié dans de grandes proportions, et notamment réduit au-dessous du bruit quantique standard en utilisant des diodes comprimées en intensité, et l'efficacité de conversion de la pompe, qui se traduit, dans le cas des microlasers, par un rendement de conversion important des photons de longueur d'onde  $810 \text{ nm}$  en photons de longueur d'onde  $1064 \text{ nm}$  [44–47].

Dans un premier temps nous nous sommes attachés à comprendre et mettre en évidence les effets du bruit de pompe sur le bruit d'intensité du microlaser : le modèle théorique utilisé pour décrire la dynamique des microlasers et leurs propriétés de bruit est basé sur l'approche de Langevin quantique et prend en compte le bruit de pompe [14]. Les prévisions montrent que, si les caractéristiques des microlasers paraissent moins favorables à la production d'états comprimés que celles des diodes laser, il devrait être néanmoins possible d'obtenir des états dont le spectre de bruit d'intensité montre à basse fréquence une réduction du niveau du bruit au-dessous du bruit quantique

standard. Un tel résultat ne peut certes être obtenu que sous des conditions de pompage difficiles à réaliser, mais les récentes performances obtenues au moyen de diodes laser [8], avec 65 % de compression mesurée sous le bruit quantique standard, sont très encourageantes.

La réalisation expérimentale utilise comme source pour le pompage optique une diode laser SDL sur réseau ou injectée : le pompage optique peut être effectué dans des conditions de faible bruit. Nous avons comparé le bruit du laser YVO dans ces conditions au bruit observé avec une pompe bruyante (obtenue en dérégulant volontairement la diode). En cas de pompage à faible bruit, le niveau de bruit, dans la partie à basse fréquence du spectre, est considérablement plus faible (jusqu'à 30 *dB* de différence) qu'en cas de pompe bruyante. Cela montre bien les possibilités ouvertes à l'application du principe de la pompe régulière [48, 49].

Cependant, un inconvénient des microlasers réside dans la dynamique de ce type de lasers, marquée par une oscillation de relaxation qui affecte profondément l'ensemble du domaine spectral observable (à cause des dimensions réduites de la cavité laser, elle se situe entre 5 et 10 *MHz*), et qui tend à masquer les effets favorables d'un faible bruit de pompe. Ce n'est que pour des fréquences faibles devant celle du pic de relaxation que le modèle théorique prévoit, pour une pompe régulière, la réduction du bruit d'intensité au-dessous du bruit quantique standard.

Mais les niveaux de bruit d'intensité des faisceaux émis par les minicristaux restent trop élevés par rapport aux attentes du modèle théorique. Nous avons attribué cet écart à des phénomènes non linéaires dus à la présence de l'oscillation de relaxation. Ces phénomènes sont très visibles à haute fréquence et pourraient également être à l'origine des excès de bruit observés à basse fréquence.

Nous avons donc tenté de parvenir à des niveaux de plus faible bruit à basse fréquence en mettant au point un dispositif de rétroaction électro-optique qui réduise l'amplitude de l'oscillation de relaxation. L'implantation de dispositifs de rétroaction électro-optique est une pratique courante, par exemple sur les lasers à YAG, qui présentent un niveau de bruit très élevé, à cause de l'oscillation de relaxation mais aussi du bruit de pompe, particulièrement important si on utilise un réseau de diodes comme source de pompe. De nombreuses équipes se sont efforcées de mettre au point des dispositifs de rétroaction performants, ramenant le bruit d'intensité sur un large domaine spectral à des niveaux proches du bruit quantique standard [50–52]. Cependant, ce procédé se heurte aux limites introduites par la nature quantique du champ, qui ne permet pas d'obtenir par cette technique un champ lumineux à la fois disponible pour des mesures et de bruit égal au bruit quantique standard, puisqu'un tel dispositif

utilise nécessairement un séparateur optique et conduit à introduire dans la boucle de rétroaction le bruit du vide, non corrélé avec celui du laser. Par exemple, prélever un quart du faisceau laser pour la rétroaction conduit au moins à 6  $dB$  d'excès de bruit sur le faisceau sortant de la boucle. Cette problématique se retrouve de façon identique pour le cas du cristal d' $YVO_4$ .

C'est pourquoi notre démarche se distingue des techniques de rétroaction classiques, puisque nous évitons précisément de rétroagir sur le domaine de fréquence où nous comptons observer les effets quantiques attendus du principe de la pompe régulière. Sans cette précaution, ces effets se trouveraient détruits par la rétroaction. Cette contrainte supplémentaire rend la mise au point de la rétroaction très délicate. Nous avons cependant réussi à faire décroître le maximum de l'oscillation de relaxation d'une dizaine de  $dB$ , tout en nous assurant que l'effet du circuit de rétroaction à basse fréquence était complètement négligeable. Parallèlement à la mise en oeuvre expérimentale de la boucle de rétroaction, nous avons développé un traitement semi-classique de la rétroaction électro-optique [53, 54]. Son intégration au modèle pour le laser libre permet d'obtenir un modèle quantique décrivant les propriétés de bruit du laser en présence de la rétroaction. Les résultats que nous avons obtenus mettent clairement en évidence l'existence d'effets non linéaires dus à l'oscillation de relaxation responsables de l'excès de bruit constaté dans la partie à basse fréquence du spectre, par rapport aux prévisions théoriques.

Une autre méthode très utilisée pour éliminer le pic de l'oscillation de relaxation recourt à l'injection optique. Les analyses théoriques montrent en effet que, pour le laser injecté, l'amortissement de l'oscillation de relaxation est généralement beaucoup plus grand (deux ordres de grandeur dans notre cas) que pour le laser libre. L'injection, non seulement "écrase" l'oscillation de relaxation, mais, de plus, elle n'affecte pas le bruit à basse fréquence, qui reste étroitement lié aux fluctuations de la pompe [55–57]. Cette technique est donc bien adaptée à nos objectifs : elle pourrait nous permettre de profiter de la réduction du bruit par élimination de l'oscillation de relaxation, tout en gardant les effets bénéfiques provenant du pompage à faible bruit. Nous avons donc décidé de réaliser expérimentalement l'injection optique du microlaser. Pour l'interprétation théorique des résultats, nous avons utilisé un modèle quantique basé sur l'approche de Langevin [57] décrivant le laser injecté.

La mise en oeuvre expérimentale de l'injection s'est heurtée à quelques difficultés techniques : nous ne disposions que d'un laser à YAG Lightwave pour injecter le microlaser Nd:YVO<sub>4</sub>. Or, l'écart en fréquence entre les deux lasers est de 120  $GHz$ , c'est à dire beaucoup plus grande que la bande d'injection. Il a donc fallu stabiliser le



cristal d' $\text{YVO}_4$  à une température d'environ  $100^\circ\text{C}$  pour rendre possible l'injection. Le chauffage du cristal s'accompagne d'une nette dégradation des performances du microlaser, notamment en ce qui concerne le seuil d'oscillation et l'efficacité quantique. Cela constitue un obstacle majeur à l'observation d'une quelconque amélioration des performances du microlaser à basse fréquence par rapport à celles obtenus à température ambiante. Nous nous sommes donc contentés de vérifier que l'oscillation de relaxation est effectivement écrasée en régime d'injection et que les prévisions théoriques sont en bon accord avec l'expérience. Des expériences ultérieures devraient être conduites avec un laser d'injection adapté.

Le meilleur résultat a été obtenu dans le cas du microlaser avec rétroaction électronique : le bruit d'intensité est de  $7\text{ dB}$  au-dessus du bruit quantique standard à  $40\text{ kHz}$ . Nous sommes encore loin de la génération d'états comprimés : les deux obstacles principaux sont représentés par l'oscillation de relaxation et par le bruit de pompe. Le premier peut être surmonté à l'aide des techniques que nous venons de décrire (rétroaction ou injection). Quant au deuxième, il est vrai que nous disposons des diodes laser à bruit d'intensité comprimé, mais la compression est de l'ordre de  $40\%$  et, compte tenu des pertes optiques inévitables et de l'absorption imparfaite du cristal, la compression effective vue par le microlaser est considérablement plus faible (autour de  $15\%$ ). Les puissances délivrées par les diodes lasers sont, elles aussi, insuffisantes : typiquement nous disposons au maximum de  $70\text{ mW}$  correspondant à un fonctionnement du microlaser 10 fois au-dessus du seuil (les effets non classiques sont observables pour de taux de pompage nettement plus élevés, supérieurs à 100). Une autre difficulté, détaillée dans la suite de ce chapitre, vient du fait que le spectre de bruit des diodes lasers n'est pas plat mais présente un léger excès de bruit ( $\sim 3\text{ dB}$ ), notamment à basses fréquences ( $\leq 300\text{ kHz}$ ), où les effets non classiques dus à la réduction du bruit de pompe sont susceptibles d'être observés. L'amélioration des performances des diodes laser constitue donc un préalable à l'observation d'états comprimés du rayonnement dans les microlasers  $\text{Nd:YVO}_4$ . Notons cependant que le résultat obtenu,  $7\text{ dB}$  seulement d'excès de bruit au-dessus du bruit quantique standard, correspond à une des meilleures performances publiées à ce jour dans la littérature pour ce type de laser.

Dans ce chapitre nous allons présenter en détail les mesures de bruit effectuées sur un microlaser  $\text{Nd:YVO}_4$  pompé par diode à bruit d'intensité comprimé sous le bruit quantique standard et fonctionnant dans trois différentes configurations expérimentales : le laser libre, le laser avec rétroaction électro-optique et le laser injecté.

## 5.2 Bruit d'intensité du microlaser Nd:YVO<sub>4</sub>

Dans cette section nous présentons les propriétés de bruit du microlaser Nd:YVO<sub>4</sub>. L'article que nous reproduisons présente le modèle théorique utilisé et la comparaison avec les résultats expérimentaux.

### 5.2.1 Reproduction de l'article : "Effects of pump fluctuations on intensity noise of Nd:YVO<sub>4</sub> microchip lasers" (soumis à Eur. Phys. J. D)

Effects of pump fluctuations on intensity noise of Nd:YVO<sub>4</sub>  
microchip lasers

A. Bramati, J.P. Hermier, V. Jost and E. Giacobino

*Laboratoire Kastler Brossel, Université Pierre et Marie Curie,*

*Ecole Normale Supérieure, CNRS*

*4, Place Jussieu, F-75252, Paris Cedex 05, France*

J. J. Aubert, E. Molva and L. Fulbert

*LETI/CEA, Grenoble, France*

The principle of pump noise suppression is applied to a Nd:YVO<sub>4</sub> microchip laser, optically pumped by laser diodes. The noise of the microchip laser at low frequency (below the relaxation oscillation frequency) is compared for noisy and amplitude squeezed laser diodes. The minimum intensity noise of the microchip laser is 7 dB above SNL at a frequency of 40 kHz. Very good agreement between experimental results and theoretical predictions of a model based on quantum Langevin equations is found.

## I. INTRODUCTION

In recent years, diode pumped solid state lasers such as Nd:YAG or Nd:YVO<sub>4</sub> lasers have been overcoming gas lasers for many applications, because of their robustness and reliability [1,2]. However, their noise properties are quite different from those of the gas lasers [3-5]. Their intensity noise is somewhat similar to the one of the semiconductor lasers. Like the semiconductor lasers, they exhibit a large noise peak at high frequency due to the so-called

relaxation oscillation. The noise peak is in the gigahertz rang for the semiconductor lasers whereas it is in the kilohertz to megahertz range for the solid state lasers, depending on the parameters of the laser cavity. Well above the relaxation oscillation frequency, the intensity noise goes to the standard quantum limit, since it is mainly due to the vacuum fluctuations outside the laser cavity that are reflected off the coupling mirror. For frequencies well below the relaxation oscillation frequency, the intensity noise of the laser is related to the noise of the pump laser [6,7].

For semiconductor lasers, it was demonstrated that intensity squeezed light can be generated when high impedance pump noise suppression is used together with appropriate line narrowing techniques [8–10]. The case of solid state lasers differs from that of the semiconductor lasers by the pump mechanism, which is optical pumping for solid state lasers as opposed to electrical pumping for semiconductor lasers. In the present work, our aim is to precisely study the effect of the reduction of the optical pump noise on the intensity noise of solid state laser and to investigate the possibility to achieve sub-shot noise operation under pump noise suppression. We present an experimental investigation of the intensity noise properties of a Nd:YVO<sub>4</sub> microchip laser pumped by laser diodes having various noise characteristics. More specifically we have compared the effect of pumping with laser diodes exhibiting excess intensity noise, standard intensity quantum noise and sub-shot intensity noise. The sub-shot noise operation of the pump laser is achieved with two different configurations: grating extended cavity and injection locking respectively. Accurate comparison between theoretical predictions of a full quantum model based on quantum Langevin approach is presented. Our model is based on a microscopic theory of the laser and does not assume any approximation on the evolution time scales of the various quantities involved in

the problem. We show that low frequency noise decreases with pump noise, but we observe a discrepancy at low frequency that cannot be interpreted as due to excess pump noise (contrary to the case of [11]). Agreement between theory and experiment is retrieved when the amplitude of the relaxation oscillation is decreased.

## II. THEORETICAL MODEL

For a theoretical description of the solid state laser noise properties we have used a full quantum model based on the Langevin equations approach [12]. The model deals with a system of homogeneously broadened two-level atoms with transition frequency  $\omega_{ab}$ , assuming that the lower level is not the ground state. The atoms fill a resonant cavity of length  $L$  and volume  $V$  with intensity transmission coefficient of the coupling mirrors  $T$ . The atoms interact with the radiation field of a single excited mode of the cavity, which is considered as a plane wave with frequency  $\omega_c$ . The model is well adapted to describe the behavior of Nd:YVO<sub>4</sub> laser. The laser dynamics is described by the following stochastic  $c$ -number Langevin equations:

$$\dot{\mathcal{A}}(t) = -\kappa/2 \mathcal{A}(t) + g\mathcal{M}(t) + \mathcal{F}_\gamma(t) \quad (1)$$

$$\dot{\mathcal{M}}(t) = -\gamma_{ab}\mathcal{M}(t) + g[\mathcal{N}_a(t) - \mathcal{N}_b(t)]\mathcal{A}(t) + \mathcal{F}_\mathcal{M}(t) \quad (2)$$

$$\dot{\mathcal{N}}_a(t) = R - (\gamma_a + \gamma'_a)\mathcal{N}_a(t) - g[\mathcal{A}^*(t)\mathcal{M}(t) + \mathcal{M}^*(t)\mathcal{A}(t)] + \mathcal{F}_a(t) \quad (3)$$

$$\dot{\mathcal{N}}_b(t) = -\gamma_b\mathcal{N}_b(t) + \gamma'_a\mathcal{N}_a(t) + g[\mathcal{A}^*(t)\mathcal{M}(t) + \mathcal{M}^*(t)\mathcal{A}(t)] + \mathcal{F}_b(t) \quad (4)$$

The stochastic  $c$ -number variable  $\mathcal{A}(t)$  represents the electromagnetic field.  $\mathcal{N}_a$  and  $\mathcal{N}_b$  represent the macroscopic atomic population of the upper and lower level respectively.  $\mathcal{M}(t)$  represents the macroscopic atomic polarisation.  $\kappa$  is the total cavity damping constant. In

the original model, the cavity damping constant was assumed to be equal to the output coupling rate  $\kappa_{out}$ . Here we have included internal optical losses represented by the rate  $\kappa_{losses}$ . Subsequently, we have  $\kappa = \kappa_{out} + \kappa_{losses}$ .  $\gamma_a$  and  $\gamma_b$  are the decay rates of the populations of the upper and lower levels to the other atomic levels;  $\gamma'_a$  is the spontaneous decay rate between the lasing levels and  $\gamma_{ab}$  is the decay rate of the atomic polarisation.  $R$  is the mean pumping rate. The coupling constant  $g$  is given by

$$g = \sqrt{\frac{1}{2\varepsilon_0\omega_c V}} \omega_{ab}\mu \quad (5)$$

where  $\mu$  is the magnitude of the atomic dipole moment. The functions  $\mathcal{F}_k(t)$  with  $k = \gamma, \mathcal{M}, a, b$  are the stochastic  $c$ -number Langevin forces with the properties:

$$\langle \mathcal{F}_k(t) \rangle = 0 \quad (6)$$

$$\langle \mathcal{F}_k(t) \mathcal{F}_l(t') \rangle = 2\mathcal{D}_{kl}\delta(t-t') \quad (7)$$

where  $\mathcal{D}_{kl}$  represents the diffusion coefficient for the  $c$ -number Langevin force. The nonvanishing diffusion coefficients are given in appendix.

Solving the  $c$ -number Langevin equations allows us to calculate the steady state solutions for the field and atomic variables and to derive the spectra of the fluctuations of the field quadrature components. The steady state solutions for laser operation above threshold are obtained by dropping the noise terms in eqs. (1)-(4) and setting the time derivatives equal to zero. For the mean intensity of the laser field inside the cavity  $I_0 = \mathcal{A}_0^2$  we obtain:

$$I_0 = I_s (R/R_{th} - 1) \quad (8)$$

where the saturation intensity  $I_s$  is given by

$$I_s = \frac{\gamma_{ab}\gamma_b}{2g^2} \frac{\gamma_a + \gamma'_a}{\gamma_a + \gamma_b} \quad (9)$$

$R_{th}$  is the threshold pumping rate

$$R_{th} = \frac{\kappa\gamma_{ab}\gamma_b}{2g^2} \frac{\gamma_a + \gamma'_a}{\gamma_b - \gamma'_a} \quad (10)$$

The steady state populations of the upper and lower levels are given by

$$\mathcal{N}_{a0} = \frac{R - \kappa I_0}{\gamma_a + \gamma'_a} \quad (11)$$

$$\mathcal{N}_{b0} = \frac{\gamma'_a R - \kappa\gamma_a I_0}{\gamma_b(\gamma_a + \gamma'_a)} \quad (12)$$

The steady state value of the atomic polarisation can be expressed in terms of the mean value of the field

$$\mathcal{M}_0 = \frac{\kappa}{2g} \mathcal{A}_0 \quad (13)$$

The evolution of the quantum fluctuations are obtained by linearizing eqs. (1)-(4) around the steady state solution. A Fourier transform of the equations for fluctuations converts the differential equations into algebraic ones. The solutions of the linear system allow to analytically calculate the intensity noise spectrum at the output of the laser as a function of the relaxation constants of the upper and lower levels of the laser transition and of the pump noise.

In the original formulation of [12] the pump noise is described by the parameter  $p$  ranging from 0 to 1 ( $p = 0$  for poissonian pump;  $p = 1$  for regular pump). In order to reproduce the experimental conditions more precisely, we generalize the model by introducing the spectral density of the pump noise  $s(\tilde{\Omega})$  normalised to the shot noise.

According to [12] and setting  $p(\tilde{\Omega}) = 1 - s(\tilde{\Omega})$ , the normalised intensity noise spectrum at the laser output is given by:

$$V_{out}(\tilde{\Omega}) = 1 + \eta(V_A(\tilde{\Omega}) - 1) \quad (14)$$

$$\begin{aligned}
V_A(\tilde{\Omega}) = & 1 + \frac{2bc(a+a')}{b-a'} \frac{1}{D(\tilde{\Omega})} ((b^2 + \tilde{\Omega}^2) [(a+a')^2 + \tilde{\Omega}^2] \left[ r + n \left( \frac{c}{a+a'} - 1 \right) \right] \\
& + 2w^2 \{ [(b-a')^2 + \tilde{\Omega}^2] [n + (s(\tilde{\Omega}) - 1) r/2] \\
& - [(b-a')(a+a') + \tilde{\Omega}^2] \left( r - \frac{a+2a'}{a+a'} n \right) \\
& + [(a+a')^2 + \tilde{\Omega}^2] \left( \frac{a'}{a+a'} - \frac{bb-a'}{ca+a'} \right) \} \quad (15)
\end{aligned}$$

where  $\eta = \kappa_{out}/\kappa$  represents the correction for internal optical losses. The dimensionless parameters  $a$ ,  $b$ ,  $c$  and the dimensionless noise frequency  $\tilde{\Omega}$  are defined as follows:

$a \equiv \gamma_a/\kappa$  (respectively  $b \equiv \gamma_b/\kappa$ ) is the normalised decay rate of the upper level (respectively lower level);

$a' \equiv \gamma'_a/\kappa$  is the normalised spontaneous decay rate between the lasing levels;

$c \equiv \gamma_{ab}/\kappa$  is the normalised decay rate of the polarisation;

$\tilde{\Omega} \equiv \Omega/\kappa$ .

Also the following shorthands have been introduced:

$$D(\tilde{\Omega}) = \left| -i\tilde{\Omega} \left( \frac{1}{2} + c - i\tilde{\Omega} \right) (b - i\tilde{\Omega}) (a + a' - i\tilde{\Omega}) + 2w^2 (a + b - i2\tilde{\Omega}) (1 - i\tilde{\Omega}) \right|^2 \quad (16)$$

$$n = \frac{ra + b + a'(r-1)}{a+b} \quad w^2 = \frac{(a+a')bc}{2(a+b)} (r-1) \quad (17)$$

The normalised pump parameter  $r$  is defined as the ratio between the pump power  $p_{pump}$  and the threshold pump power  $p_{th}$ :  $r = p_{pump}/p_{th}$ .

Equation (14) is quite general and does not rely on any adiabatic elimination of variables. It may be used to deal with all types of lasers. In order to gain some physical insight



from this formula, we can however make some approximations which lead to a simplified expression. The laser under study is a Nd: YVO<sub>4</sub> laser, belonging to a third class lasers [13], characterized by a decay rate of the atomic polarisation much faster than the other relaxation rates. With respect to the dimensionless variables previously introduced, this condition is written  $c \gg 1 \gg a, b, a'$ . Applying this approximation to eq. (14) is equivalent to perform the adiabatic elimination of the atomic polarization. Moreover, the noise spectrum of these lasers is dominated by the relaxation oscillation (fig. 1): for low frequencies the noise increases from its minimum value corresponding to zero frequency; for high frequencies, the intensity noise spectrum decreases toward the quantum standard limit.

The minimum intensity noise for the third class lasers is hence obtained from eq. (14) setting  $\tilde{\Omega} = 0$  and  $c \rightarrow \infty$ . For the specific case of the Nd:YVO<sub>4</sub> laser, the condition  $b \gg a, a'$  is also verified. This leads to the very simple following formula:

$$V_m(0) \cong 1 - \eta + \eta \left[ s + \frac{s+1}{r-1} + \frac{2}{(r-1)^2} \right] \quad (18)$$

From eq. (18) we derive the condition for observing intensity noise below the shot noise level in the laser output:

$$r > \frac{3-s}{1-s} \equiv r_{th} \quad (19)$$

Equation (19) implies that intensity squeezing can only be observed with a squeezed pump ( $s < 1$ ) and that the laser must operate very far above threshold.

To get the detailed noise properties of the Nd:YVO<sub>4</sub> laser at all frequencies, we use eqs. (14) and (15). Figure 1 shows the normalised theoretical noise spectra for different values of the normalized pump parameter  $r$ , and for a perfectly noiseless pump. The other parameters correspond to the values of our microchip laser and are given in the experimental results section. As can be seen in fig. 1, suppression of the pump noise allows to generate squeezed states.

According to the eq. (18), the amount of squeezing increases with the pump rate  $r$  and it is limited by the quantum efficiency of the laser. The frequency range suitable to observe squeezed light is limited to low frequency region well below the relaxation oscillation peak, and it also increases with the pump parameter  $r$ , because the relaxation oscillation frequency ( $\tilde{\Omega}_{RO}$ ) shifts to higher frequencies, according to the equation:

$$\tilde{\Omega}_{RO} \cong \sqrt{(a + a')(r - 1)}. \quad (20)$$

The theoretical analysis indicates very clearly the features that the laser has to present in order to generate intensity squeezed light via pump noise suppression. First of all, the laser has to display a very low threshold to achieve far above threshold operation. The laser should also have a high quantum efficiency. Quantum efficiency of the laser may be limited by internal optical losses  $\eta$  and the conversion ratio  $\eta_p$  of the photons of the pump beam into lasing photons. These two mechanisms, however, have different effects on the noise properties of the laser. The first one reduces the maximum achievable squeezing, but it does not change the value of  $r_{th}$  given by eq. (19). In contrast, the second one influences directly the noise of the pump beam (which approaches the shot noise level:  $s \rightarrow 1$  in eq. (19)),

increasing the value of  $r_{th}$  necessary to generate squeezed light ( $r_{th} \rightarrow \infty$  for  $\eta_p \rightarrow 0$ ).

Finally, to avoid the competition between oscillating modes, the laser should have single mode operation. The requirements concerning low threshold and high quantum efficiency imply the use of a laser material with high stimulated emission gain and high pump absorption; the single mode operation can be achieved by a laser with a short Fabry-Perot cavity, and a narrow gain curve. The Nd:YVO<sub>4</sub> microchip laser used in our experiment is a good candidate: the high absorption coefficient of 70 cm<sup>-1</sup> at 808 nm (2% Nd doping) ensures the low threshold (4.2 mW) and good quantum efficiency ( $\sim 40\%$ ); the cavity length of 300  $\mu$ m together with a narrow gain bandwidth (0.95 nm [14]) ensure the single frequency oscillation.

### III. EXPERIMENTAL SET-UP

The experimental set-up for noise measurements in Nd:YVO<sub>4</sub> microchip laser is shown in figures 2a and 2b. The amplitude squeezed laser diode used for optical pumping of solid state laser is an index guided quantum well GaAlAs laser diode (SDL 5422H1) operating at 810 nm. Noise reduction in the pump beam is achieved by driving the laser diode with a high impedance constant current source and suppressing the side modes using two different configurations as described in [15], namely feedback from an external grating in an extended cavity laser (fig. 2a) and injection locking with a master laser (fig. 2b).

In the external grating configuration a reflection holographic grating is set at a distance of 10 cm from the laser, in Littrow configuration. By tilting the grating, the laser wavelength can be tuned to match the maximum of the Nd:YVO<sub>4</sub> line absorption at 808.5 nm.

In the injection locking set-up, the laser is injected by a master laser which is either a

grating extended cavity semiconductor laser. The injection locking set-up, with an extended cavity master laser is a device completely tunable over several nanometers, and allows us to match the frequency of the pump beam with the maximum of the absorption curve of the Nd:YVO<sub>4</sub>. In both configurations, astigmatism in the beam is corrected by means of anamorphic prisms. Two optical isolators (for a total isolation of 70 dB) are employed to prevent back reflection into the pump laser. The optical power available for pumping process is 70 mW for the injection locking set-up and 45 mW (due to the losses of the grating) in the grating extended cavity configuration. The intensity noise of the pump laser diode is measured by a standard balanced detection (two high efficiency EG&G FND100 PIN photodiodes), which allows to measure, under the same conditions, the shot noise and the intensity noise of the laser beam. We performed several tests in order to check the reliability of the shot noise measured in this way, as described in [15]. The common mode rejection of the balanced detection is better than 30 dB in the range of 0 - 30 MHz; electronic and dark noise are typically more than 10 dB below the shot noise level. The pump beam is sent to the microchip laser by mean of two mirrors and focused into the laser with a  $f = 8$  mm objective. The polarisation of the pump beam is fixed by a half-wave plate in order to achieve maximum absorption in the Nd:YVO<sub>4</sub> crystal. The microchip laser is mounted on a xyz-translation stage which allows an optimum alignment.

The Nd:YVO<sub>4</sub> microchip laser is 300  $\mu\text{m}$  long, with a plane-plane monolithic cavity (the stability is ensured by thermal lens effects) in which the mirrors were deposited directly onto the crystal. The output mirror and back reflector have reflectivities of 97% and 99.5% respectively at 1.064  $\mu\text{m}$ . The mirrors do not have special coatings for wavelength of pump radiation at 810 nm. Accurate measurements show a reflectivity of 24% and a transmissivity

of 7% for pump radiation.

For the intensity noise measurements on the beam emitted by the microchip we used a two-fold experimental set-up. For measurements in low frequency region (0 - 300 kHz) we use a balanced detection designed for 1.064  $\mu\text{m}$ . The 1.064  $\mu\text{m}$  light is detected by two high quantum efficiency (90%) InGaAs photodiodes. The electronic devices that complete the balanced detection are analogous to those previously described in [15]. The common mode rejection is better than 30 dB. For measurements at higher frequencies (up to 30 MHz), balanced detection is no longer reliable due to the very high excess noise of the relaxation peak (more than 80 dB) which exceeds the common mode rejection ratio. Therefore in this range of frequencies we choose to calibrate the shot noise level with an independent source. For the calibration we use the noise obtained by direct detection on one photodiode of attenuated radiation emitted by a shot noise limited laser diode. It is worth saying that no correction has to be calculated, due to the difference in the wavelength of the two beam. In fact we detect the noise of the photocurrent which is independent on the wavelength. We check carefully linear dependance of the calibrated shot noise signal with the optical power incident on the photodiode. The shot noise obtained in this way was in agreement within 0.1 dB with the noise obtained by a thermal light generating the same DC current on the photodiode.

#### IV. EXPERIMENTAL RESULTS

In this section we present the experimental results obtained with the microchip Nd:YVO<sub>4</sub> laser previously described. The laser threshold is 4.2 mW, the maximum single TEM<sub>00</sub> mode power output was 10 mW obtained by pumping with an injection locked laser diode (pump

power about 70 mW); in this conditions the normalised pump parameter assumes the value  $r \cong 9.5$ . If the pump laser is an extended cavity laser the maximum value achievable for  $r$  is about 6. In order to make a comparison between the experimental noise spectra and the theoretical predictions, the values of the parameters of the model are set as follows: the decay rate of the upper level  $\gamma_a$  and the spontaneous decay rate between the lasing levels  $\gamma'_a$  for a Nd:YVO<sub>4</sub> crystal doped at 2% are taken from [16,17]; we set  $\gamma_a = 3.3 \times 10^4 \text{ s}^{-1}$  and  $\gamma'_a = 3.3 \times 10^3 \text{ s}^{-1}$ . The values of  $\gamma_b$ , decay rate of the lower level, found in the literature [18,16] for Nd:YVO<sub>4</sub> crystal are affected by a huge uncertainty (more than two order of magnitude); within this range, we choose a value of  $\gamma_b = 3 \times 10^9 \text{ s}^{-1}$  that ensures an optimal fit of the experimental curve. The decay rate of the polarisation  $\gamma_{ab}$  is set equal to  $7 \times 10^{11} \text{ s}^{-1}$  according to [16,17]. The experimental determination of the optical pump power threshold and the measurement of the optical pump power allow us to easily calculate the normalised pump parameter  $r$ . The total cavity damping constant  $\kappa$  can be determined by the dependence of the relaxation oscillation frequency on the pump parameter  $r$ , according to the equation

$$\Omega_{RO} = \sqrt{\kappa (\gamma_a + \gamma'_a) (r - 1)} \quad (21)$$

we find  $\kappa = 1.56 \times 10^{10} \text{ s}^{-1}$ . The output coupling  $\kappa_{out}$ , calculated from the reflectivities of the cavity mirrors, is found to be  $8.36 \times 10^9 \text{ s}^{-1}$ .

We first consider the intensity noise characteristics of the two different pump laser diodes, the grating extended cavity laser diode and the injection locked laser diode. As described in [15] the feedback from grating and injection locking are useful techniques to generate amplitude squeezed light from laser diodes. The figures 3a and 3b show the normalized

intensity noise spectra of the grating extended cavity laser diode for low frequency region (0 - 300 kHz) and for higher frequencies (0 - 30 MHz) respectively. After correction for detection efficiency the amount of amplitude squeezing at the laser output is 0.7 dB, flat over the whole frequency range between 1 to 30 MHz. For frequencies lower than 300 kHz the pump laser diode exhibit slight excess noise ( $<3$  dB). For frequencies below 50 kHz technical  $1/f$  noise increases the excess noise up to 10 dB. Amplitude squeezing of 1.1 dB at the laser output in the range 0 - 30 MHz is achieved for the injection locked laser; this represents a slight improvement with respect to grating extended cavity laser. The intensity noise spectrum for the low frequencies region exhibits a behavior similar to that of the grating extended cavity laser.

We will now consider the noise properties of the Nd:YVO<sub>4</sub> microchip laser. All the noise spectra presented in the following are corrected by taking into account the detection efficiency, therefore they display the noise at the output of the laser. Figure 4 shows the normalized intensity noise spectrum of the Nd:YVO<sub>4</sub> microchip laser pumped with the extended cavity laser, with pump noise corresponding to fig. 3. For this measurement the power was about 6 mW and  $r = 6$ . The theoretical curve (thick line) is obtained from eq. (14) with the parameters given in the text;  $s(\tilde{\Omega})$  is given by empirical expression fitting the measured pump noise (thick line in fig. 3) and it is corrected for losses experienced by the pump beam (about 40%). The theoretical curve is in good agreement with the experimental one for a very large range of frequencies, particularly around the relaxation oscillation peak, well reproduced by theory (fig. 4a). However, a significant discrepancy is evident at very low frequencies (below 100 kHz) (fig. 4b). A minimum noise of 11 dB above shot noise level (SNL) is achieved at frequencies of 30 kHz, whereas the model predicts 5 dB above SNL. In

a recent experiment [11] performed on Nd:YVO<sub>4</sub> microchip laser pumped by an amplitude squeezed injection locked laser diode, an analogous discrepancy between theoretical predictions and experimental results is observed in low frequency range. The authors assumed that the low frequency extra noise could originate from the spectral distribution of the intensity squeezed light of the injection locked pump laser diode that they use in experimental set-up. In fact, in the case of the injection locked laser, the main mode can exhibit excess noise even if the total intensity noise is squeezed [19]. This excess noise typically ranges from 1 to 10 dB, depending on the optimisation of the injection parameters such as alignment, injected power, master frequency. As the microchip laser, due to its absorption linewidth of 1 nm, is pumped only by the main mode and few longitudinal side modes, it could experience excess pump noise.

However, this explanation does not hold in our case. We have checked that in the grating extended cavity configuration, as explained in [19], no difference is observed between the total intensity noise and the noise of the main mode alone: side modes are actually negligible and single mode squeezing is generated. Thus, when pumped with a grating extended cavity laser, Nd:YVO<sub>4</sub> microchip laser does not experience any excess pump noise; nevertheless, extra noise in the low frequency region is observed.

To further check the origin of the excess noise, we performed a noise measurement on the Nd:YVO<sub>4</sub> microchip laser in the same conditions of fig. 4, except for the pump noise, that exhibit a large excess noise peak up to 20 dB centered at 40 kHz, as shown in fig. 5b. The corresponding theoretical and experimental intensity noise spectrum of the microchip laser can be seen in fig. 5a. The peak of the pump noise is very well reproduced in the noise of the microchip laser. In this case the agreement between theory and experiment is good also



at low frequencies. Other measurements performed with a very noisy pump (more than 40 dB) clearly confirms the significant influence of the pump fluctuations on the low frequency noise of the Nd:YVO<sub>4</sub> microchip laser, as shown in fig. 6. This case is very illustrative of the high noise at low frequency that is obtained when microchip lasers are pumped with laser diodes arrays.

To improve these results and to obtain a reduction of the intensity noise of the Nd:YVO<sub>4</sub> with respect to the minimum noise of 11 dB above SNL achieved in the experimental conditions of fig. 4, we substitute the grating extended cavity pump laser with the injection locked one. In fact, as previously explained, with this new configuration we are able to operate the laser 10 times above threshold ( $r = 10$ ) and according with theory, we expect an improvement on the noise performances of the laser. The result is shown in fig. 7 for a quiet pump like that reported in fig. 3a. The minimum noise at 40 kHz is 7 dB above the SNL which is a significant improvement as compared to the previous case. However, the theoretical fit does not agree with experimental curve below 100 kHz.

The fact that the theoretical noise spectrum is in good agreement with the measured noise spectrum over a very large range of frequencies seems to ensure that the Langevin approach is well adapted for description of the noise features of our Nd:YVO<sub>4</sub> microchip laser. The problem of modeling the noise behavior of this laser arises at low frequencies as can be seen in fig. 4b and 7. A noticeable discrepancy between theory and experiment occurs for frequencies below 100 kHz. One possible source of this low frequency extra noise could be the presence of non-linear effects generated by the very large noise peak of the relaxation oscillation. In fact, the intensity noise fluctuations at the level of the peak are typically 80 dB above SNL, corresponding to 1-3% of the steady state intensity. Moreover,

the occurrence of non-linear effects is clearly demonstrated by the presence of the harmonic of the relaxation oscillation peak (fig. 4b).

The theoretical noise spectrum obtained by linearisation of the stochastic  $c$ -number Langevin equation around the steady state solutions does not take into account the non-linear phenomena and its derivation is valid in the case of very small fluctuations with respect to the steady state values.

In order to investigate the possible non-linear effects at low frequency we have reduced the noise around the relaxation oscillations by an appropriate feedback loop reacting on the laser diode driving current. The feedback loop has a tailored frequency response in order to be active only in a narrow frequency region around the peak, while keeping unchanged the noise features of the laser at low frequencies. Nevertheless, we observed a sharp reduction (typically 5-7 dB) of the intensity noise at low frequencies. Although the feedback loop does not affect the low frequency part of the spectrum, the low frequency noise is now in agreement with the theoretical predictions. This behaviour that gives a strong indication for non-linear effects will be studied in detail in a forthcoming paper.

## V. CONCLUSION

In this work, we have presented a detailed investigation of the intensity noise of a Nd:YVO<sub>4</sub> microchip laser pumped by an amplitude squeezed laser diode. The squeezed light pump is obtained with two different configurations: grating extended cavity laser and injection locked laser. We have clearly demonstrated the effect of the reduced pump noise on the low frequency region of the intensity noise spectrum of the microchip laser. The sub-shot noise operation of the pump laser diode allows to achieve an intensity noise of 7

dB above SNL at 40 kHz, considerably reduced with respect to the performances achievable with pumping by standard laser diodes which present excess pump noise of typically 20 dB at low frequencies. The theoretical intensity noise spectra calculated from a full quantum model based on quantum Langevin approach are in very good agreement with experimental observations, except for very low frequencies ( $<100$  kHz). This extra low frequency noise may be a consequence of a non-linear effect due to the strong relaxation oscillation peak.

In conclusion, our analysis shows that the Nd:YVO<sub>4</sub> microchip laser could potentially exhibit sub-shot noise operation in the low frequencies region. At present, two dominant factors prevent to achieve noise reduction below SNL in this laser. On one hand, the pump laser diode exhibits a slight excess noise in low frequency range ( $<300$  kHz), where the microchip laser should take more relevant advantages from pump noise reduction, as shown by theoretical analysis. On the other hand, the previously described non-linear effects increase the noise in the same low frequency range. Therefore improvement on low frequency noise of the pump laser diode, together with specially designed feedback loop to eliminate extra low frequency noise, are necessary to a further reduction of the intensity noise of the Nd:YVO<sub>4</sub> microchip laser.

#### ACKNOWLEDGMENTS

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#### APPENDIX

We report the non-vanishing diffusions coefficients for the  $c$ -number Langevin forces:

$$2\mathcal{D}_{aa} = (\gamma_a + \gamma'_a) \langle \mathcal{N}_a(t) \rangle + R(1-p) - g [\langle \mathcal{M}^*(t) \mathcal{A}(t) + \mathcal{A}^*(t) \mathcal{M}(t) \rangle] \quad (\text{A1})$$

$$2\mathcal{D}_{bb} = \gamma_b \langle \mathcal{N}_b(t) \rangle + \gamma'_a \langle \mathcal{N}_a(t) \rangle - g [\langle \mathcal{M}^*(t) \mathcal{A}(t) + \mathcal{A}^*(t) \mathcal{M}(t) \rangle] \quad (\text{A2})$$

$$2\mathcal{D}_{ab} = -\gamma'_a \langle \mathcal{N}_a(t) \rangle + g [\langle \mathcal{M}^*(t) \mathcal{A}(t) + \mathcal{A}^*(t) \mathcal{M}(t) \rangle] \quad (\text{A3})$$

$$2\mathcal{D}_{\mathcal{M}\mathcal{M}} = 2g \langle \mathcal{M}(t) \mathcal{A}(t) \rangle \quad (\text{A4})$$

$$2\mathcal{D}_{\mathcal{M}^*\mathcal{M}} = (2\gamma_{ab} - \gamma_a - \gamma'_a) \langle \mathcal{N}_a(t) \rangle + R \quad (\text{A5})$$

$$2\mathcal{D}_{b\mathcal{M}} = \gamma_b \langle \mathcal{M}(t) \rangle \quad (\text{A6})$$

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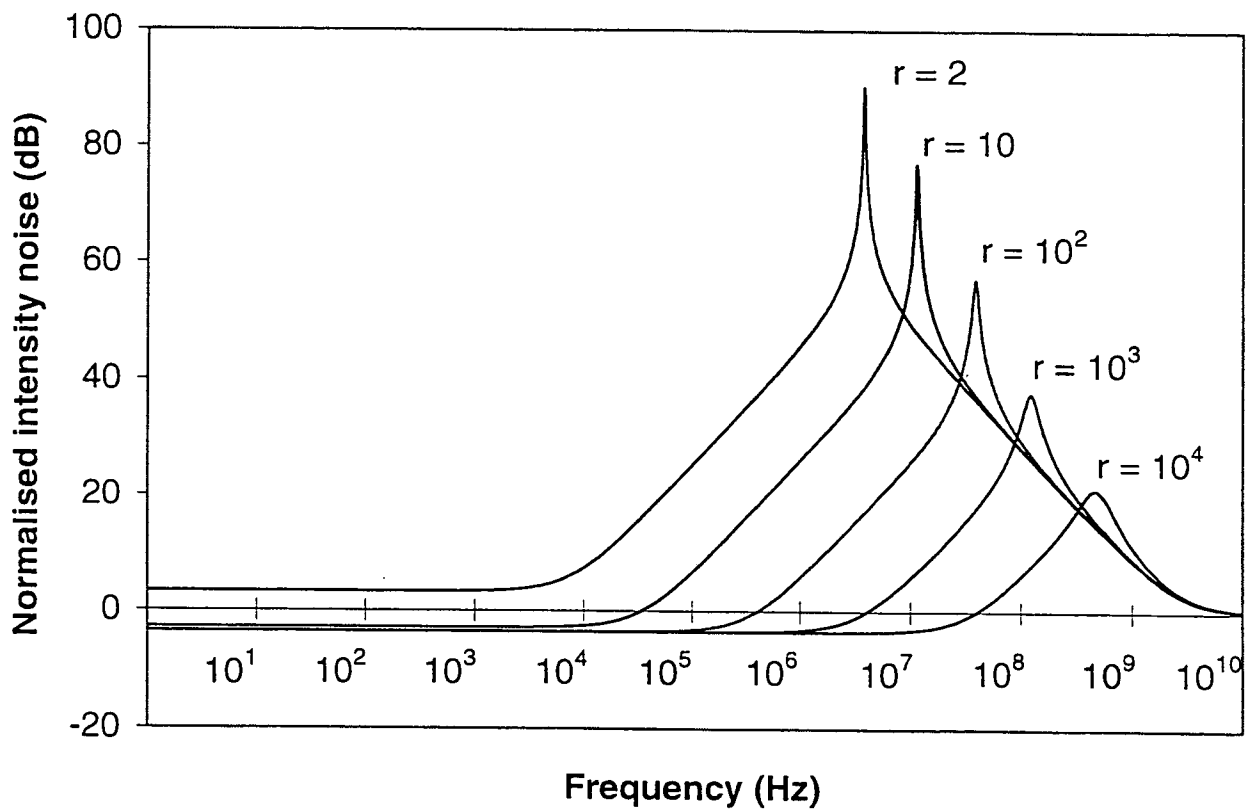


Fig.1. Calculated normalised intensity noise spectra of the Nd:YVO<sub>4</sub> microchip laser pumped with a noiseless pump ( $s(\tilde{\Omega}) = 0$ ) with the parameters given in the text for  $r = 2, 10, 100, 1000, 10000$ .

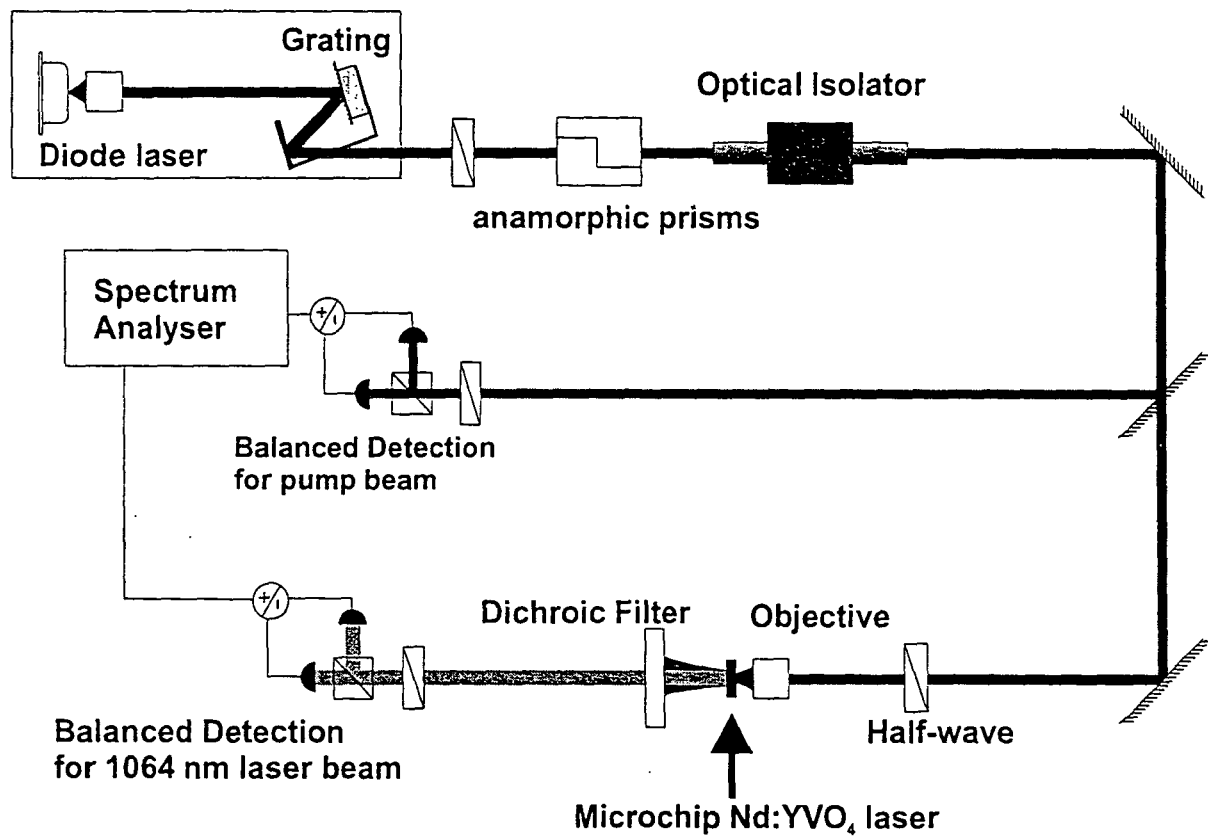


Fig.2a. Experimental setup for noise measurements in a Nd:YVO<sub>4</sub> microchip laser pumped by an extended grating cavity laser diode.

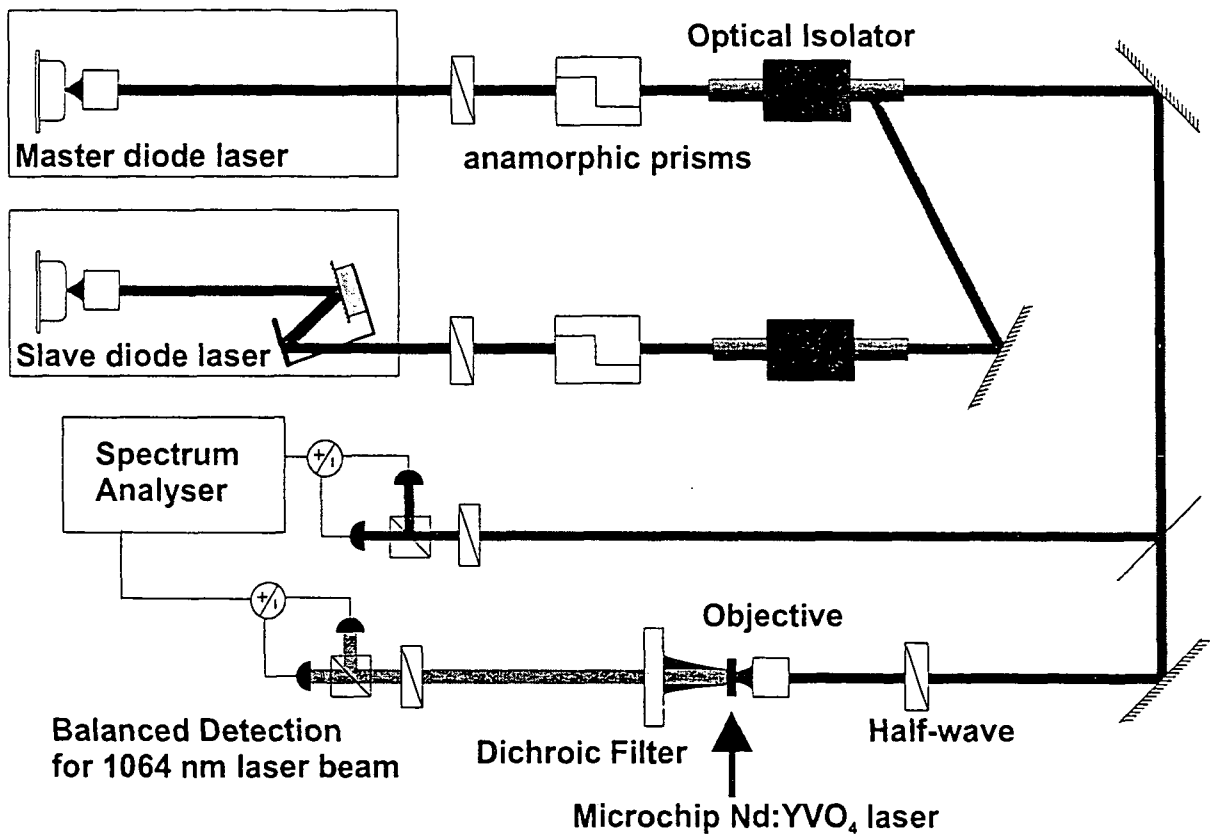


Fig.2b. Experimental setup for noise measurements in a Nd:YVO<sub>4</sub> microchip laser pumped by an injection locked laser diode.



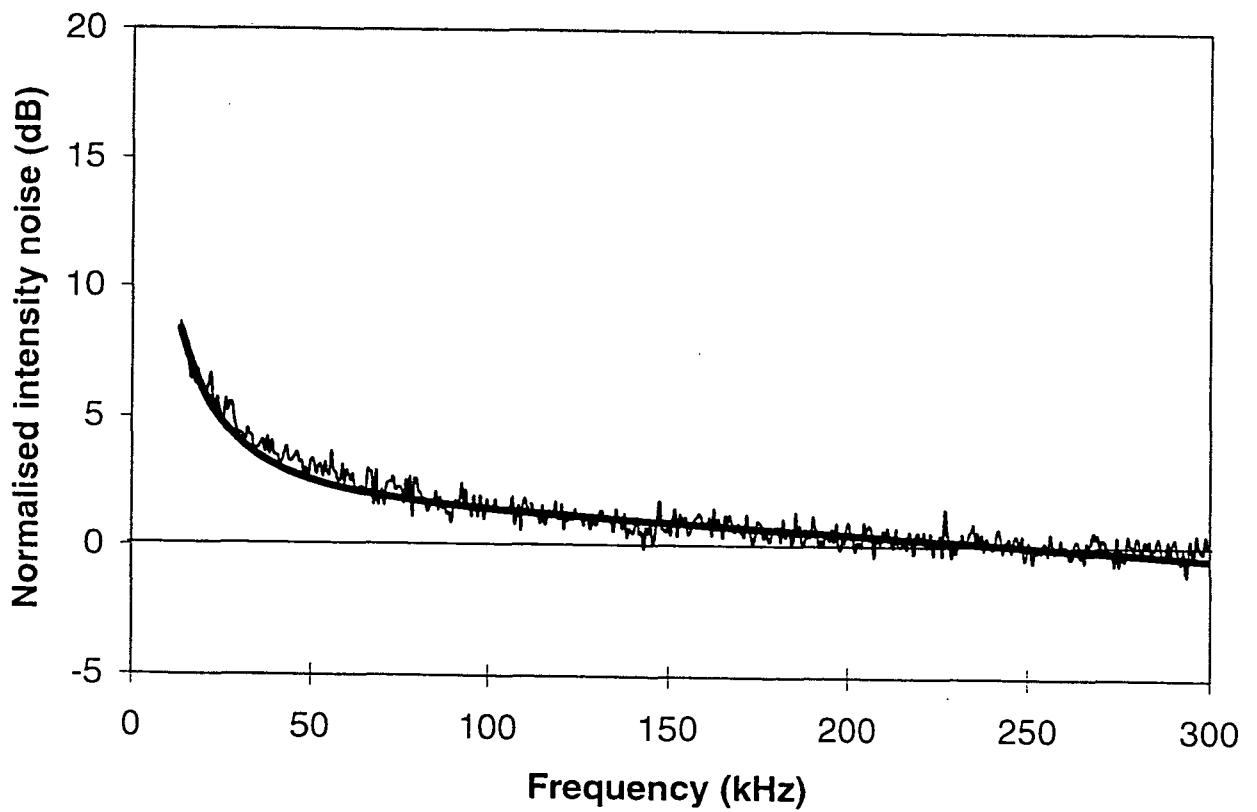


Fig.3a. Normalised intensity noise spectrum of the grating extended cavity laser diode in the low frequency region (0 – 300 kHz). The fit (thick line) is obtained with empirical expression.

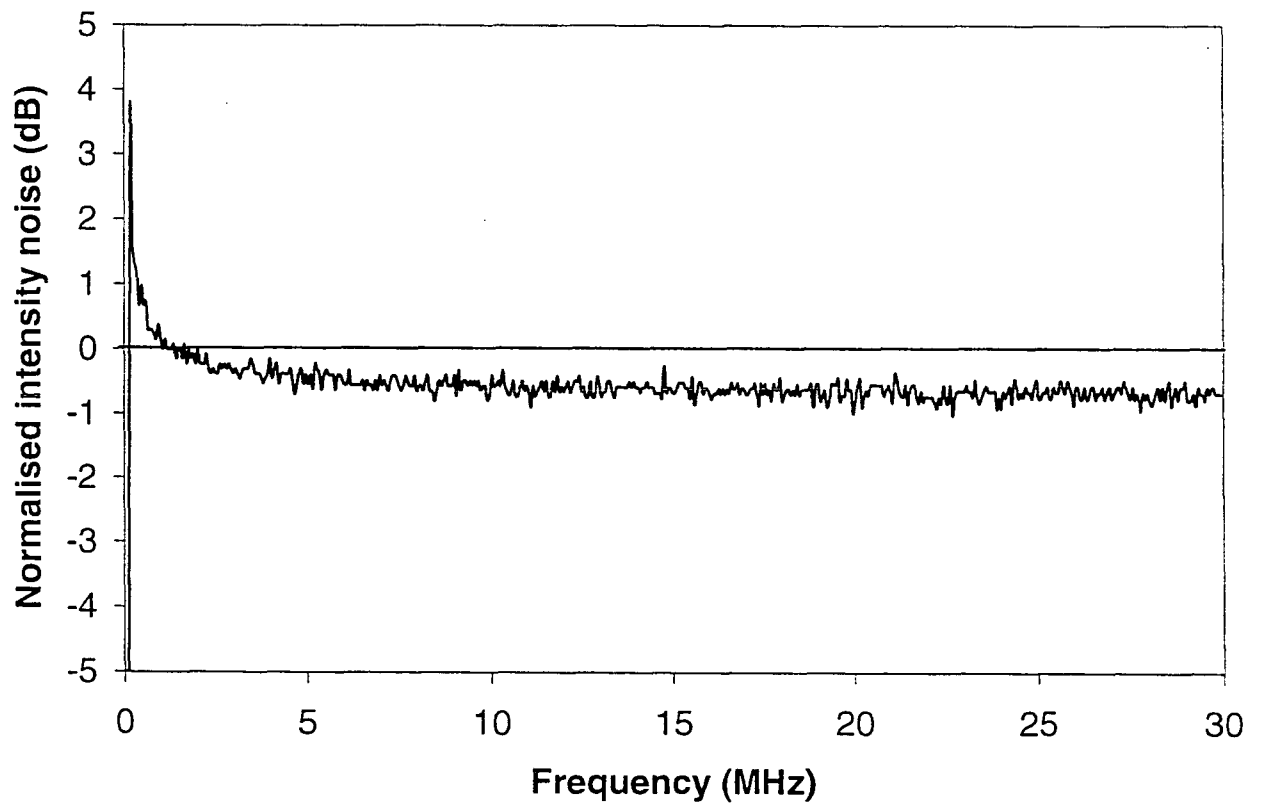


Fig.3b. Normalised intensity noise spectrum of the grating extended cavity laser diode in the high frequency region (0 – 30 MHz).

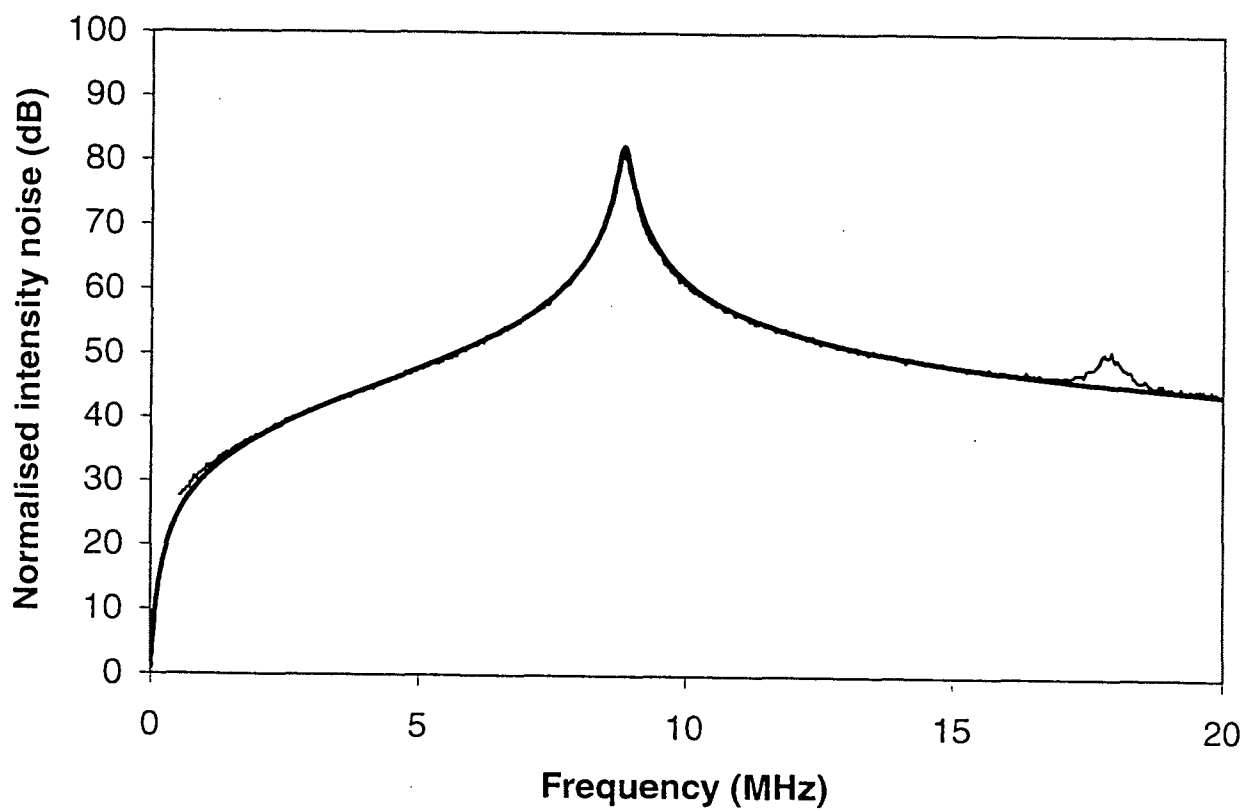


Fig.4a. Normalised experimental and theoretical (thick line) intensity noise spectra of the Nd:YVO<sub>4</sub> microchip laser pumped by a grating extended cavity laser diode in the 0 – 20 MHz frequency range. The corresponding pump noise is shown in fig. 3b.

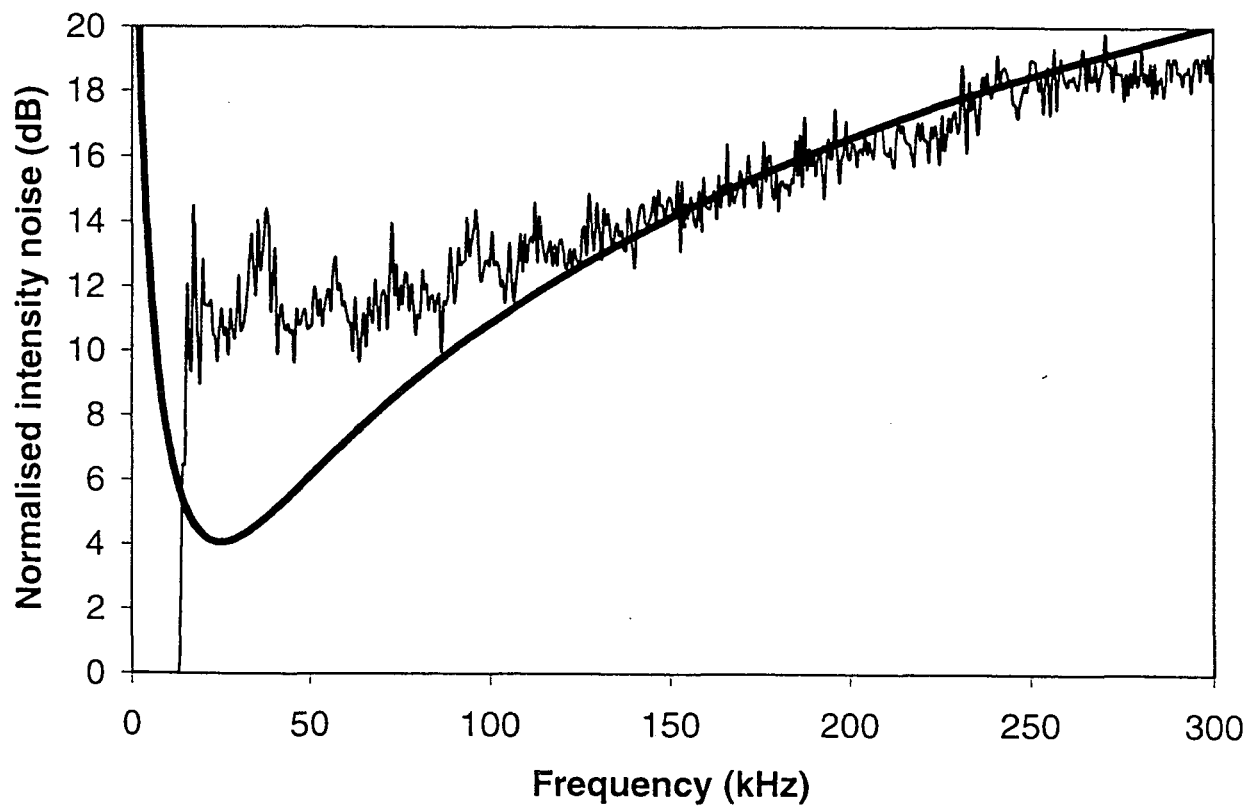


Fig.4b. Normalised experimental and theoretical (thick line) intensity noise spectra of the Nd:YVO<sub>4</sub> microchip laser pumped by a grating extended cavity laser diode in the 0 – 300 kHz frequency range. The corresponding pump noise is shown in fig. 3a.

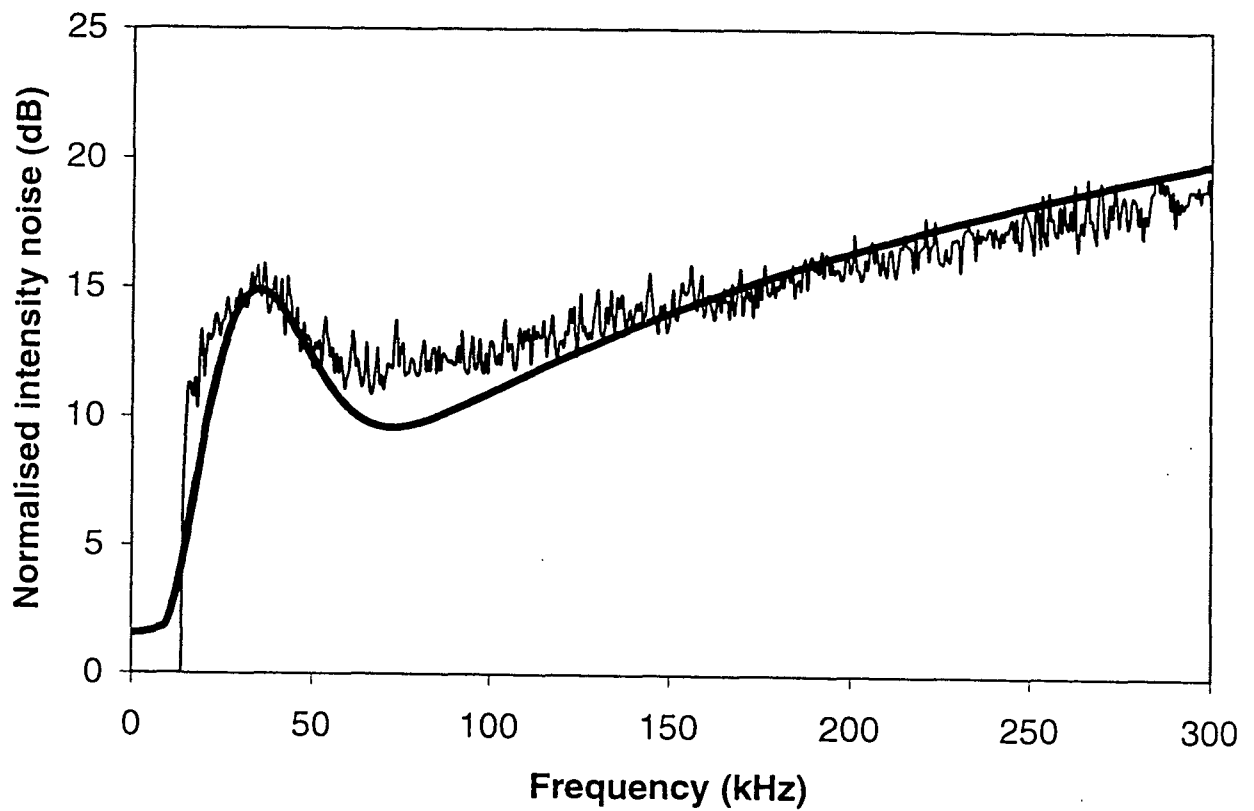


Fig.5a. Normalised experimental and theoretical (thick line) intensity noise spectra of the Nd:YVO<sub>4</sub> microchip laser pumped by a noisy grating extended cavity laser diode in the 0 – 300 kHz frequency range. The corresponding pump noise is shown in fig. 5b.

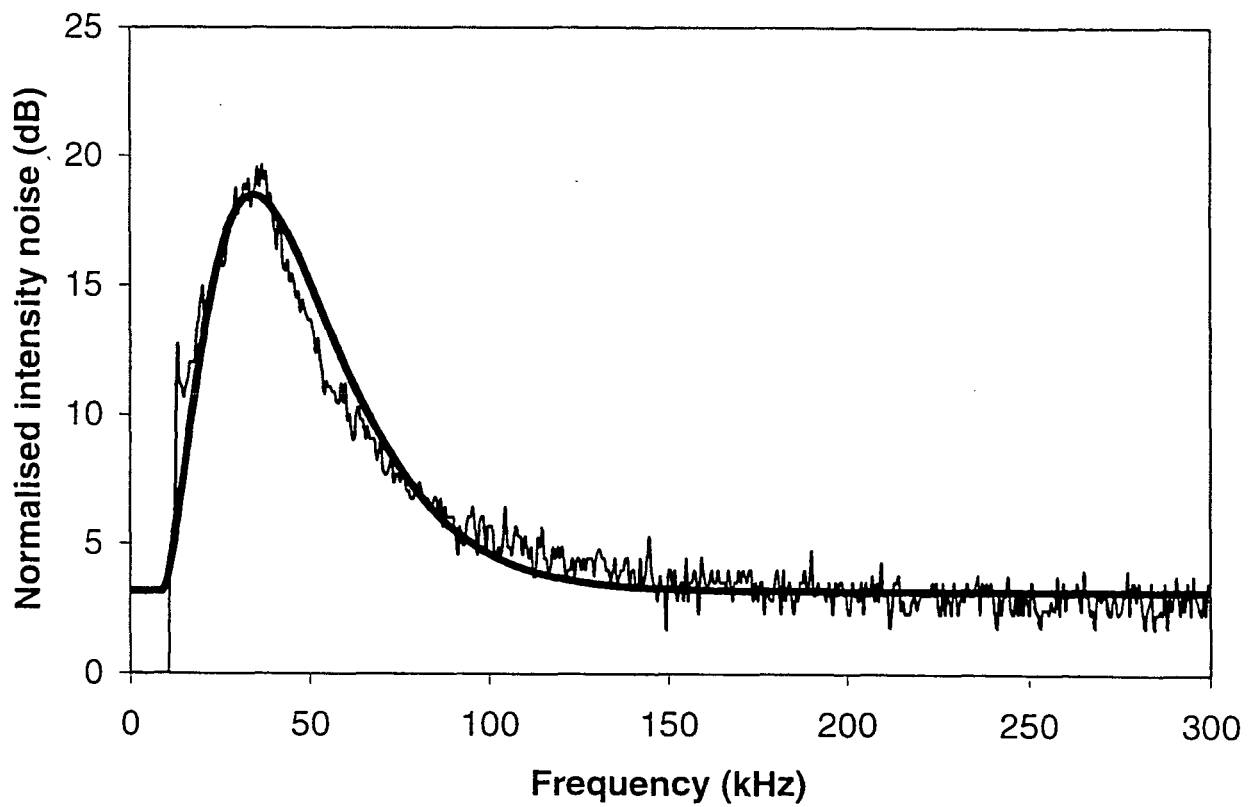


Fig.5b. Normalised intensity noise spectrum of the noisy grating extended cavity laser diode in the low frequency region (0 – 300 kHz). The fit (thick line) is obtained with empirical expression.

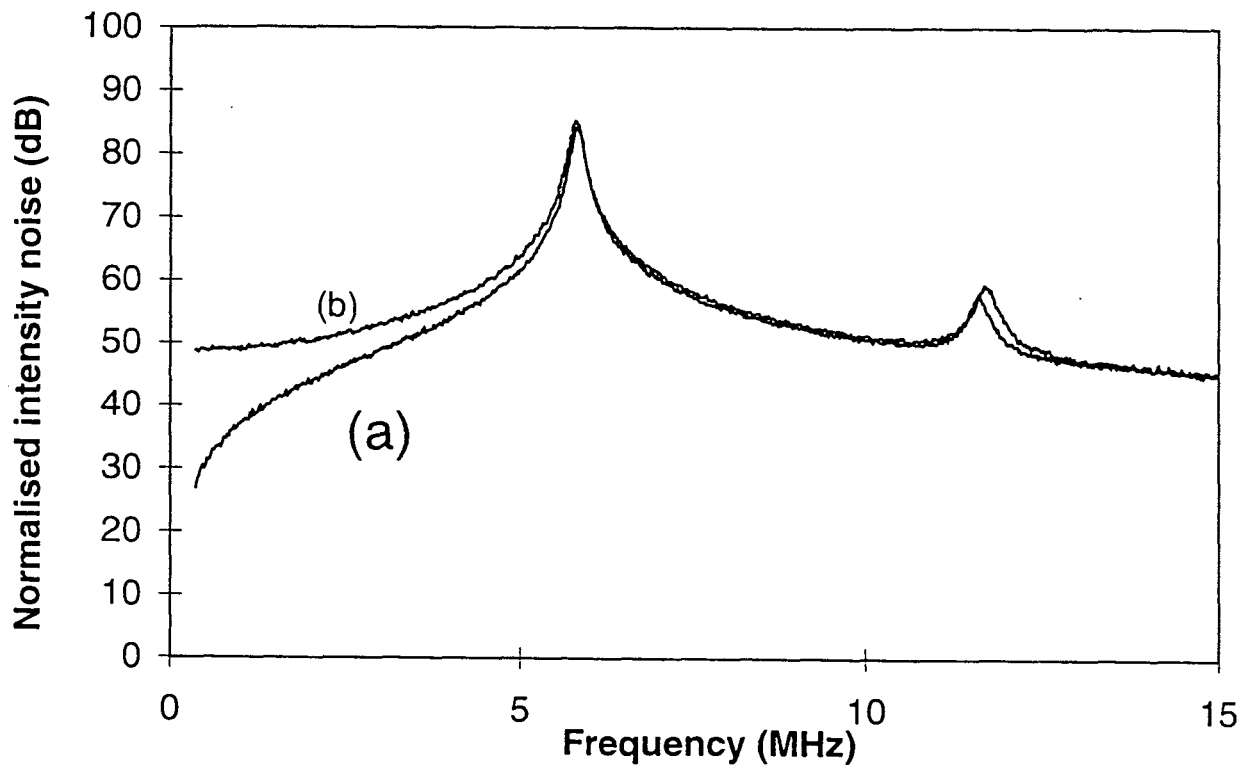


Fig.6. Normalised intensity noise spectra of the Nd:YVO<sub>4</sub> microchip laser for a squeezed pump (curve a) and for a noisy pump with an excess noise of more than 40 dB (curve b)

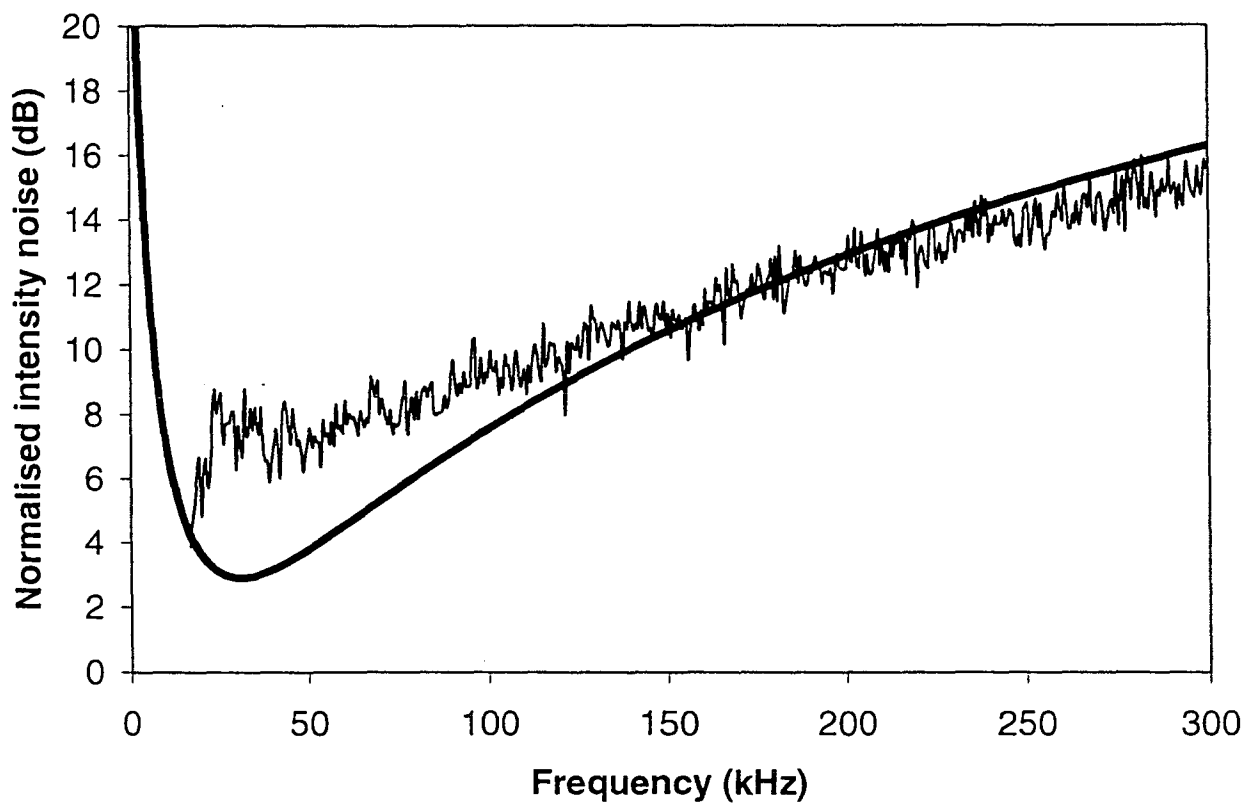


Fig.7. Normalised experimental and theoretical (thick line) intensity noise spectra of the Nd:YVO<sub>4</sub> microchip laser pumped by an injection locked laser diode in the 0 – 300 kHz frequency range.



### 5.3 Rétroaction électro-optique et bruit d'intensité du microlaser Nd:YVO<sub>4</sub>

Cette section est consacrée au microlaser avec rétroaction électro-optique. L'article qui suit détaille la dérivation du modèle théorique décrivant le laser avec rétroaction et les résultats expérimentaux.

#### 5.3.1 Reproduction de l'article : "Feedback control and intensity noise of Nd:YVO<sub>4</sub> microchip lasers" (preprint)

## Feedback control and intensity noise of Nd:YVO<sub>4</sub> microchip lasers

A. Bramati, J.-P. Hermier, V. Jost and E. Giacobino

*Laboratoire Kastler Brossel, Université Pierre et Marie Curie,*

*Ecole Normale Supérieure, CNRS*

*4, place Jussieu, F-75252 Paris Cedex 05, France*

A fully quantum model for a laser operating with an electronic feedback loop is derived. The implementation of a non-standard feedback loop on a Nd:YVO<sub>4</sub> microchip laser pumped with an amplitude squeezed diode laser permits to decrease the intensity of the relaxation oscillation noise peak while leaving the low frequency part of the spectrum unaffected. This procedure also results in a decrease of the noise in the low frequency region, showing evidence for non linear effects in the noise spectrum. The cancellation of these effects together with the pump noise suppression leads to a minimum intensity noise of 7 dB above the SNL at 40 kHz. Good agreement between the experimental results and theoretical predictions is found over the complete frequency range.

### I. INTRODUCTION

The necessity to use laser sources with high frequency stability and low intensity noise in numerous applications, such as gravitational wave detection and high sensitivity spectroscopy, points the need of a thorough investigation of the noise characteristics of various types of lasers [1,2]. Solid state lasers are good candidates to meet these requirements.

Single-mode output can be achieved using different cavity design (non- planar ring oscillator or monolithic planar microchip laser). The intensity noise spectrum of these lasers is determined by two dominant factors: the noise of the pump mechanism (for frequencies below the relaxation oscillation) and the resonant relaxation oscillation. The first factor can be partially eliminated using solid state lasers pumped with amplitude squeezed diode lasers [3,4]. The second factor is commonly controlled through the use of an electronic feedback loop [5–7]. This type of stabilisation scheme is well-known and efficient and it is present in many commercial diode pumped solid state lasers. However the combination of these techniques requires some care: in fact the use of a standard feedback loop (with a flat frequency response) should destroy completely the quantum intensity noise features of the laser and cancel the advantages coming from the squeezed pump.

In a previous work [4] we have investigated the effects of pump noise reduction in a free running Nd:YVO<sub>4</sub> microchip laser. The theoretical intensity noise spectra calculated from a full quantum model based on quantum Langevin approach were in very good agreement with experimental observations, except for very low frequencies (<100 kHz). This extra low frequency noise may be a consequence of indirect or non linear effects coming from the strong relaxation oscillation peak. In this work our aim is to investigate in detail the possibility to combine the pump noise reduction with an appropriate stabilisation scheme in order to minimize the intensity noise in the low frequency part of the spectrum of the diode pumped Nd:YVO<sub>4</sub> microchip laser. In order to investigate the possible non linear effects at low frequency we have reduced the noise around the relaxation oscillations by an appropriate non standard feedback loop reacting on the diode laser driving current. The feedback loop has a tailored frequency response in order to be active only in a narrow

frequency region around the peak, while keeping unchanged the noise features of the laser at low frequencies. We have observed a sharp reduction (typically 5-7 dB) of the intensity noise at low frequencies. Although the feedback loop does not affect the low frequency part of the spectrum, the low frequency noise is now in agreement with the theoretical predictions. This behaviour gives an experimental evidence for the occurrence of non-linear effects.

The plan of the paper is as follows: in Sec.II we present a fully quantum model for a laser operating in presence of feedback loop. An analytical expression is obtained for the intensity noise spectrum by combining the results of a quantum Langevin approach for the free running laser with a semiclassical model of the feedback loop. In Sec. III the experimental set-up and the designed non-standard feedback loop are described. Finally, in Sec. IV we present the experimental results.

## II. THEORETICAL MODEL

### A. The free running laser

The theoretical description of the free running solid state laser is accomplished by using a full quantum model based on Langevin equations approach. The details of the derivation of the intensity noise spectrum for the free running laser and the comparison with the experimental results can be found in [8] and [4] respectively. Here, we only give the final expression obtained for the normalized intensity noise at the laser output:

$$V_{out}(\tilde{\Omega}) = 1 + \eta(V_A(\tilde{\Omega}) - 1) \quad (1)$$

$$V_A(\tilde{\Omega}) = 1 + \frac{2bc(a+a')}{b-a'} \frac{1}{D(\tilde{\Omega})} ((b^2 + \tilde{\Omega}^2) [(a+a')^2 + \tilde{\Omega}^2] \left[ r + n \left( \frac{c}{a+a'} - 1 \right) \right])$$

$$\begin{aligned}
& +2w^2 \{ [(b-a')^2 + \tilde{\Omega}^2] [n + (s(\tilde{\Omega}) - 1) \tau / 2] \\
& - [(b-a')(a+a') + \tilde{\Omega}^2] \left( r - \frac{a+2a'}{a+a'} n \right) \\
& + [(a+a')^2 + \tilde{\Omega}^2] \left( \frac{a'}{a+a'} - \frac{b}{c} \frac{b-a'}{a+a'} \right) \} \quad (2)
\end{aligned}$$

where  $s(\tilde{\Omega})$  represents the normalized spectral density of the pump noise, and  $\eta = \kappa_{out}/\kappa$  represents the correction for internal optical losses.  $\kappa = \kappa_{out} + \kappa_{losses}$  is the total cavity damping constant,  $\kappa_{out}$  the output coupling, and  $\kappa_{losses}$  the internal optical losses. The dimensionless parameters  $a$ ,  $b$ ,  $c$  and the dimensionless noise frequency  $\tilde{\Omega}$  are defined as follows:

$a \equiv \gamma_a/\kappa$  (respectively  $b \equiv \gamma_b/\kappa$ ) is the normalised decay rate of the upper level (respectively lower level);

$a' \equiv \gamma_{a'}/\kappa$  is the normalised spontaneous decay rate between the lasing levels;

$c \equiv \gamma_{ab}/\kappa$  is the normalised decay rate of the polarisation;

$\tilde{\Omega} \equiv \Omega/\kappa$

Also the following shorthands have been introduced:

$$D(\tilde{\Omega}) = \left| -i\tilde{\Omega} \left( \frac{1}{2} + c - i\tilde{\Omega} \right) (b - i\tilde{\Omega}) (a + a' - i\tilde{\Omega}) + 2w^2 (a + b - i2\tilde{\Omega}) (1 - i\tilde{\Omega}) \right|^2 \quad (3)$$

$$n = \frac{ra + b + a'(r-1)}{a+b}, w^2 = \frac{(a+a')bc}{2(a+b)} (r-1) \quad (4)$$

The normalised pump parameter  $r$  is defined as the ratio between the pump power  $p_{pump}$  and the threshold pump power  $p_{th}$ :  $r = p_{pump}/p_{th}$ .

Good agreement was observed between experimental results and theoretical predictions for the free running Nd:YVO<sub>4</sub> laser.

### B. The semiclassical model for the feedback loop

We use a semiclassical approach in order to describe the properties of the feedback loop [9,10]. The experimental set up modeled by the theory is schematically shown in fig.1. A part  $I_2$  of the beam  $I_0$  emitted by the laser is split off by a beam splitter and is detected with a photodiode. The AC part of the signal  $\delta I_2$  is filtered, amplified and mixed with the driving current of the diode pump laser.  $I_2$  is the so-called in-loop beam. The other part of the beam  $I_1$  is the new outgoing beam (the out-of-loop beam) on which noise measurements are performed. The electric fields corresponding to  $I_0, I_1, I_2, I_p$  (intensity of the pump beam) are denoted by  $a_0, a_1, a_2, a_p$  (assumed to be real) and the real part of their fluctuations (amplitude fluctuations) by  $\delta p_0, \delta p_1, \delta p_2, \delta p_p$ . Assuming that the fields have a large mean value, their fluctuations can be related to the intensity fluctuations as:

$$\delta I = 2a\delta p \quad (5)$$

After the beam splitter the fluctuations of beams 1 and 2 are given by:

$$\delta p_1 = t\delta p_0 + r\delta p_v \quad (6)$$

$$\delta p_2 = -r\delta p_0 + t\delta p_v \quad (7)$$

where  $r$  and  $t$  are the amplitude reflection and transmission coefficients of the beam splitter and  $\delta p_v$  is the vacuum field that enters the second input (unused) port of the beam splitter.

The amplitude noise of the Nd:YVO<sub>4</sub> microchip laser can be separated in two parts, one,  $\delta p_0^{(p)}$ , coming from the pump fluctuations and the other one,  $\delta p_0^{(i)}$ , coming from other sources such as vacuum fluctuations or fluctuations of the atomic polarisation:

$$\delta p_0 = \delta p_0^{(p)} + \delta p_0^{(i)} \quad (8)$$

It is easy to express  $\delta p_0^{(p)}$  as a function of  $\delta p_p$  via the transfer function  $\mathcal{G}(\Omega)$  of the intensity modulation of the pump beam.  $\mathcal{G}(\Omega)$  is defined as follows:

$$\delta I_0 = \mathcal{G}(\Omega) \delta I_p \quad (9)$$

With this definition and using eq. (5) the amplitude fluctuations of the Nd:YVO<sub>4</sub> laser due to the pump fluctuations are written:

$$\delta p_0^{(p)} = (a_p/a_0) \mathcal{G}(\Omega) \delta p_p \quad (10)$$

In presence of the feedback loop, the pump noise contains two terms:

$$\delta p_p = \delta p_p^{(r)} + \delta p_p^{(i)} \quad (11)$$

$\delta p_p^{(r)}$  is a noise term due to the feedback, and  $\delta p_p^{(i)}$  represents the original noise of the pump beam. The quantity  $\delta p_p^{(r)}$ , proportional to  $\delta p_2$ , is superimposed to the pump amplitude in order to decrease the fluctuations on the outgoing field. It can be expressed as follows:

$$\delta p_p^{(r)} = - (a_2/a_p) g(\Omega) \delta p_2 \quad (12)$$

where  $g(\Omega)$  is a complex transfer function that relates a modulation of the pump beam of intensity  $I_p$  to the modulation of the beam 2. The definition of  $g(\Omega)$  is :

$$\delta I_p^{(r)} = -g(\Omega) \delta I_2 \quad (13)$$

$g(\Omega)$  represents the total transfer function of the electronic devices which constitute the feedback loop. The sign  $-$  is taken to be in agreement with the usual convention for negative feedback.

By combining the previous equations it is straightforward to obtain the fluctuations in the beams 1 and 2 in the presence of feedback:

$$\delta p_1 = \delta p_1^0 + \frac{tr\mathcal{G}(\Omega)g(\Omega)}{1+r^2\mathcal{G}(\Omega)g(\Omega)}\delta p_2^0 \quad (14)$$

$$\delta p_2 = \frac{\delta p_2^0}{1+r^2\mathcal{G}(\Omega)g(\Omega)} \quad (15)$$

where  $\delta p_1^0$  and  $\delta p_2^0$  are the fluctuations in the absence of feedback.

Finally, replacing  $\delta p_1^0$  and  $\delta p_2^0$  by their expressions as functions of  $\delta p_0^0$  and  $\delta p_v$  we obtain:

$$\delta p_1(\Omega) = \frac{t}{1+r^2\mathcal{G}(\Omega)g(\Omega)}\delta p_0^0(\Omega) + \frac{r(1+\mathcal{G}(\Omega)g(\Omega))}{1+r^2\mathcal{G}(\Omega)g(\Omega)}\delta p_v(\Omega) \quad (16)$$

$$\delta p_2(\Omega) = \frac{-r}{1+r^2\mathcal{G}(\Omega)g(\Omega)}\delta p_0^0(\Omega) + \frac{t}{1+r^2\mathcal{G}(\Omega)g(\Omega)}\delta p_v(\Omega) \quad (17)$$

The intensity noise spectra  $S_1(\Omega)$ ,  $S_2(\Omega)$  for both the beams 1 and 2 in presence of feedback can be easily calculated using the usual definition of the noise spectral density function:

$$2\pi S(\Omega)\delta(\Omega+\Omega') = \langle \delta p(\Omega)\delta p(-\Omega') \rangle \quad (18)$$

Straightforward calculations give:

$$S_1(\Omega) = \frac{t^2}{|1+r^2\mathcal{G}(\Omega)g(\Omega)|^2}S_0^0(\Omega) + \frac{r^2|1+\mathcal{G}(\Omega)g(\Omega)|^2}{|1+r^2\mathcal{G}(\Omega)g(\Omega)|^2}C_{\omega_0} \quad (19)$$

$$S_2(\Omega) = \frac{r^2}{|1+r^2\mathcal{G}(\Omega)g(\Omega)|^2}S_0^0(\Omega) + \frac{t^2}{|1+r^2\mathcal{G}(\Omega)g(\Omega)|^2}C_{\omega_0} \quad (20)$$

where  $S_0^0(\Omega)$  is the intensity noise spectrum of the free running laser and  $C_{\omega_0}$  is the intensity noise spectrum associated with the vacuum fluctuations. In deriving eq. (20) we have used the following relations, directly derived from eq. (18):



$$2\pi S_1(\Omega) = \langle \delta p_1(\Omega) \delta p_1(-\Omega) \rangle \quad (21)$$

$$2\pi S_2(\Omega) = \langle \delta p_2(\Omega) \delta p_2(-\Omega) \rangle \quad (22)$$

$$2\pi S_0^0(\Omega) = \langle \delta p_0^0(\Omega) \delta p_0^0(-\Omega) \rangle \quad (23)$$

$$2\pi C_{\omega_0} = \langle \delta p_v(\Omega) \delta p_v(-\Omega) \rangle \quad (24)$$

$$0 = \langle \delta p_v(\Omega) \delta p_0^0(-\Omega) \rangle \quad (25)$$

Eq. (25) expresses the fact that the fluctuations of the laser beam are not correlated with the vacuum fluctuations.

The term  $r^2 \mathcal{G}(\Omega) g(\Omega) \equiv G_{\text{olg}}(\Omega)$  which appears in the denominator of both eqs. (19) and (20) represents the open loop gain of the feedback loop. It can be seen from eqs. (19) and (20) that, for very high values of  $G_{\text{olg}}(\Omega)$ , the noise of the in-loop beam is decreased and goes to zero, as expected, but this noise reduction is confined inside the loop. On the other hand, the noise spectrum of the outgoing beam  $S_1(\Omega)$  can be decreased if  $S_0^0(\Omega)$  is large (classical noise) but has a lower limit equal to  $\frac{1}{r^2} C_{\omega_0}$ , given by the second term in eq. (20). This means that the intensity noise of the out-of-loop beam, as expected, cannot be reduced below the shot noise level, due to the contribution of the vacuum fluctuations which deteriorate the degree of correlations between the in-loop and the out-of-loop beams. Typically the best noise reduction achievable with a 50/50 beam splitter is of 3 dB above the shot noise level. By tailoring the open loop gain one can see that it is possible to reduce the large excess noise at the relaxation oscillation frequency while having basically no feedback at low frequency. The low frequency noise is however affected by the presence of the beamsplitter, but this effect can be made small if  $t^2 \gg r^2$ .

### C. Laser noise in the presence of feedback loop

By combining the intensity noise spectrum for the free running laser obtained with the quantum Langevin approach (eq. (1)) and the normalized eq. (19) derived from the semiclassical model, we are able to calculate the intensity noise spectrum for the laser operating in presence of feedback. In order to do this, we have to evaluate the open loop gain  $G_{\text{olg}}(\Omega)$  which appears in the denominator of the eq. (19). As previously explained,  $G_{\text{olg}}(\Omega)$  is the product of two different transfer functions,  $\mathcal{G}(\Omega)$ , transfer function of an intensity modulation of the pump beam as defined in eq. (9) and  $g(\Omega)$ , transfer function of the electronic part of the feedback loop;  $g(\Omega)$  takes into account also all the optical losses which occur in the loop via a simple coefficient.  $g(\Omega)$  is easily modeled by calculating the transfer function of each electronic device in the loop and making the product of all them.

For the determination of  $\mathcal{G}(\Omega)$  we use the results of the model for the free running laser. We are looking for a transfer function of an intensity modulation of the pump beam: in the original equations of the model [8] the intensity of the pump beam is represented by the mean pumping rate  $R$ . A modulation of the intensity pump beam is proportional to a modulation of the pumping rate  $R$ :

$$\delta R = \alpha \delta I_p \quad (26)$$

We replace  $R$  with  $R + \delta R$  in the equation (3.3) of [8] which gives the population  $\mathcal{N}_a(t)$  of the upper level of the laser transition

$$\dot{\mathcal{N}}_a(t) = R + \delta R(t) - (\gamma_a + \gamma'_a)\mathcal{N}_a(t) + ig[\mathcal{A}^*(t)\mathcal{M}(t) + \mathcal{M}^*(t)\mathcal{A}(t)] + \mathcal{F}_a(t) \quad (27)$$

After linearization around the steady state solution this equation reads:

$$\begin{aligned} \delta \dot{\mathcal{N}}_a(t) = & -(\gamma_a + \gamma'_a)\delta \mathcal{N}_a(t) - g\mathcal{A}_0[\delta \mathcal{M}(t) + \delta \mathcal{M}^*(t)] - g\mathcal{M}_0[\delta \mathcal{A}(t) + \delta \mathcal{A}^*(t)] \\ & + \mathcal{F}_a(t) + \delta R(t) \end{aligned} \quad (28)$$

We see that we have two source terms for the fluctuations: the Langevin force  $\mathcal{F}_a(t)$  et the pump modulation  $\delta R(t)$ . This source term can be written as:

$$\mathcal{F}'_a(t) = \mathcal{F}_a(t) + \delta R(t) \quad (29)$$

The amplitude fluctuations of the laser field inside the cavity are obtained directly by eq. (3.39) of [8], replacing  $\mathcal{F}_a(t)$  with  $\mathcal{F}'_a(t)$ . We have:

$$\begin{aligned} \delta X(\Omega) = & \frac{\gamma_a + \gamma'_a - i\Omega}{g(\gamma_a + \gamma_b - 2i\Omega)(\kappa - i\Omega)[C(\Omega) + 2\mathcal{A}_0^2]} \quad (30) \\ & \times \left\{ \frac{1}{2} [\mathcal{F}_\mathcal{M}(\Omega) + \mathcal{F}_\mathcal{M}^*(-\Omega)] (\gamma_b - i\Omega) \right. \\ & \left. + g\mathcal{A}_0 \frac{\gamma_b - \gamma'_a - i\Omega}{\gamma_a + \gamma'_a - i\Omega} [\mathcal{F}_a(\Omega) + \delta R(\Omega)] - g\mathcal{A}_0 \mathcal{F}_b(\Omega) \right\} \end{aligned}$$

with  $C(\Omega)$  given by

$$C(\Omega) = \frac{-i\Omega(\gamma_a + \gamma'_a - i\Omega)(\gamma_b - i\Omega)(\kappa/2 + \gamma_{ab} - i\Omega)}{g^2(\gamma_a + \gamma_b - 2i\Omega)(\kappa - i\Omega)} \quad (31)$$

Extracting the term proportional to  $\delta R(\Omega)$  we obtain the amplitude fluctuations originating from the pump modulation:

$$\delta X^R(\Omega) = \frac{\gamma_b - \gamma'_a - i\Omega}{(\gamma_a + \gamma_b - 2i\Omega)(\kappa - i\Omega)[C(\Omega) + 2\mathcal{A}_0^2]} \times \mathcal{A}_0 \times \delta R(\Omega) \quad (32)$$

The intensity fluctuations of the intracavity field are obtained from:

$$\delta I_i(\Omega) = 2 \mathcal{A}_0 \delta X^R(\Omega) \quad (33)$$

With the chosen normalization  $I_i = \mathcal{A}_0^2$  represents the number of photons inside the cavity. Hence, the intensity modulation  $\delta I_0$  of the field outgoing from the cavity is given by:

$$\delta I_0 = \kappa \delta I_i \hbar \omega_c \quad (34)$$

where  $\omega_c$  is the frequency of the Nd:YVO<sub>4</sub> laser beam. From eqs. (26), (32), (33) and (34) we obtain:

$$\mathcal{G}(\Omega) = \frac{\delta I_0(\Omega)}{\delta I_p(\Omega)} = 2 \frac{\gamma_b - \gamma'_a - i\Omega}{(\gamma_a + \gamma_b - 2i\Omega)(\kappa - i\Omega)[C(\Omega) + 2\mathcal{A}_0^2]} \kappa I_i \alpha \hbar \omega_c \quad (35)$$

At zero frequency the ratio  $\frac{\delta I_0(0)}{\delta I_p(0)}$  gives the conversion efficiency of the pump mechanism:

$$\frac{\delta I_0(\Omega)}{\delta I_p(\Omega)} = \eta \frac{\omega_c}{\omega_p} \quad (36)$$

where  $\eta$  is the quantum differential efficiency of the laser and  $\omega_p$  the frequency of the pump beam. Eq. (36) allows to calculate the value of the proportionality constant  $\alpha$  and then to obtain the normalised transfer function  $\mathcal{G}(\Omega)$ . The final expression of  $\mathcal{G}(\Omega)$  in terms of the dimensionless parameters previously introduced is written:

$$\mathcal{G}(\tilde{\Omega}) = \eta \frac{\omega_c}{\omega_p} \frac{2(b - a' - i\tilde{\Omega})\omega^2}{2\omega^2(a + b - 2i\tilde{\Omega})(1 - i\tilde{\Omega}) - i\tilde{\Omega}(\frac{1}{2} + c - i\tilde{\Omega})(b - i\tilde{\Omega})(a + a' - i\tilde{\Omega})} \frac{a + b}{b - a'} \quad (37)$$

Finally, we are able to write the expression for the intensity noise of the out-of-loop beam in presence of feedback:

$$V_{fb}(\tilde{\Omega}) = V_{out}(\tilde{\Omega}) \frac{t^2}{|1 + G_{o1g}(\tilde{\Omega})|^2} + \frac{r^2 \left| 1 + \frac{G_{o1g}(\tilde{\Omega})}{r^2} \right|^2}{|1 + G_{o1g}(\tilde{\Omega})|^2} \quad (38)$$

Let us make some comments on this result: our purpose is to reduce the huge relaxation oscillation peak which dominates the intensity noise spectrum of the free running Nd:YVO<sub>4</sub> microchip laser. This strong excess noise (more than 80 dB above the shot noise level) could generate via non linear frequency mixing an excess noise in the low frequency range and hence explain the discrepancy observed in different experiments [3,4] between theoretical predictions and experimental results at low frequencies. It is clear from eq. (38) that for

very high values of  $G_{oig}(\Omega)$  at all frequencies the intensity noise of the out-of-loop beam can be efficiently decreased and tends towards a constant value which depends on the reflectivity of the beam splitter. However, in this case, the intensity noise spectrum is flat and the noise features of the laser contained in the term  $V_{out}(\tilde{\Omega})$  are completely lost (in particular the effects of pump noise reduction and the sought non-linear effects). On the other hand is also evident that, for  $G_{oig}(\Omega) = 0$ , the intensity noise spectrum of the out of loop beam is substantially the same of that one of the free running laser except for some contaminations due to the beam splitter. In order to confirm the hypothesis of the occurrence of non linear effect at low frequency, we need a non standard feedback loop with a tailored frequency response reacting in the region of the relaxation oscillation to decrease its excess noise, and having no action at low frequency: in this conditions the observation of noise reduction at low frequency should constitute an experimental demonstration of the non linear phenomena. To be able to observe such kind of effects it is essential to eliminate the other sources of noise in the low frequency noise that could screen them: namely the noise coming from the pump mechanism, strongly reduced in our set-up by using an amplitude squeezed diode laser as a pump laser.

### III. EXPERIMENTAL SET-UP

The experimental set-up for the noise measurements on the Nd:YVO<sub>4</sub> microchip laser operating with feedback loop is shown in fig. 2. The amplitude squeezed diode laser used for optical pumping of solid state laser is an index guided quantum well GaAlAs laser diode (SDL 5422H1) operating at 810 nm. Noise reduction in the pump beam is achieved by driving the diode laser with a high impedance constant current source and suppressing the side modes

using feedback from an external grating in an extended cavity laser [11–17]: by tilting the grating, the laser wavelength can be tuned to match the maximum of the Nd:YVO<sub>4</sub> line absorption at 808.5. Astigmatism in the beam is corrected by means of anamorphic prisms. Two optical isolators (for a total isolation of 70 dB) are employed to prevent back reflection in the pump laser. The optical power available for pumping process is 45 mW (due to the losses of the grating). The intensity noise of the pump diode laser is measured by a standard balanced detection (two high efficiency EG&G FND100 PIN photodiodes), which allows to measure, under the same conditions, the shot noise and the intensity noise of the laser beam. We performed several tests in order to check the reliability of the shot noise measured in this way, as described in [16]. The common mode rejection of the balanced detection is better than 30 dB in the range of 0 - 30 MHz; electronic and dark noise are typically more than 10 dB below the shot noise level. The pump beam is sent to the microchip laser by means of two mirrors and focused into the laser with a  $f = 8$  mm objective. The polarisation of the pump beam is fixed by a half-wave plate in order to achieve the maximum absorption in the Nd:YVO<sub>4</sub> crystal. The microchip laser is mounted on a xyz-translation stage which allows an optimum alignment.

The Nd:YVO<sub>4</sub> microchip laser is 300  $\mu\text{m}$  long, with a plane-plane monolithic cavity (the stability is ensured by thermal lens effects) in which the mirrors were deposited directly onto the crystal. The output mirror and back reflector have reflectivities of 97% and 99.5% respectively at 1.064  $\mu\text{m}$ . The mirrors do not have special coatings for wavelength of pump radiation at 810 nm. Accurate measurements show a reflectivity of 24% and a transmissivity of 7% for pump radiation. The beam emitted by the microchip laser is sent to a beam splitter formed by a half wave plate followed by a polariser cube: this system is

equivalent to a beam splitter with variable transmissivity and reflectivity coefficients and allows to change continuously the fraction of emitted optical power which constitute the in loop beam. This beam is detected by a high quantum efficiency (90%) InGaAs photodiode which generates the error signal for the feedback control. The AC part of the error signal is sent to different electronics components specially designed in order to have an appropriated frequency response of our feedback loop, as explained in the previous section. The error signal is finally mixed with the driving current of the pump diode laser via AC coupling to avoid any perturbation to the DC working point of the diode laser. The characteristics of the total transfer function for our feedback loop are shown in the experimental Bode diagrams (gain and phase versus frequency) in fig. 3a and 3b. We note that the open loop gain is lower than  $-40$  dB below 1 MHz: this ensures that the noise features of the laser are unaffected in the low frequency range. The sharp slope (80 dB/dec) of the gain curve below the cut-off frequency of 6 MHz is obtained by inserting a double Butterworth filter in the feedback loop. Of course, this implies a very large phase shift; as well-known in the standard theory of electronic control, in order to have a stable feedback loop, the maximum phase lag at the two unity gain points has to be less than  $360^\circ$ . The task, naturally difficult due to the presence of the relaxation oscillation that introduces a  $180^\circ$  phase shift, is further complicated in our case due to the relatively high frequency at which the relaxation peak is located. In fact, the short cavity of our microchip laser ( $300 \mu\text{m}$ ) shifts it in the megahertz range (contrary to the usual kilohertz range in the standard cavity). In this frequency range the phase lags due to the delay time of the active electronics devices become relevant: for example the delay time of the CLC-425 op-amp is of the order of 10 ns corresponding to  $36^\circ$  at a frequency of 10 MHz. For this reason, the stability of the loop is rather critical and

this prevents from achieving larger open loop gain.

For the intensity noise measurements on the out of loop beam we used a two-fold experimental set-up. For measurements in low frequency region (0 - 300 kHz) we use a balanced detection designed for 1.064  $\mu\text{m}$ . The 1.064  $\mu\text{m}$  light is detected by two high quantum efficiency (90%) InGaAs photodiodes. The electronic devices that complete the balanced detection are analogous to those previously described in [16]. The common mode rejection is better than 30 dB. For measurements at higher frequencies (up to 30 MHz), balanced detection is no longer reliable due to the very high excess noise of the relaxation peak (more than 80 dB) which exceeds the common mode rejection ratio. Therefore in this range of frequencies we choose to calibrate the shot noise level with an independent source. For the calibration we use the noise obtained by direct detection on one photodiode of attenuated radiation emitted by a shot noise limited diode laser. It is worth saying that no correction has to be calculated, due to the difference in the wavelength of the two beam. In fact we detect the noise of the photocurrent which is independent on the wavelength. We check carefully linear dependance of the calibrated shot noise signal with the optical power incident on the photodiode. The shot noise obtained in this way was in agreement within 0.1 dB with the noise obtained by a thermal light generating the same DC current on the photodiode.

#### IV. EXPERIMENTAL RESULTS

In this section we present the experimental results obtained with the Nd:YVO<sub>4</sub> microchip laser operating with a feedback loop previously described. In order to make a comparison between theory and experiment we have to determine all the parameters of the model we have developed. A set of values for the parameters describing the microchip laser (relaxations



rates and cavity damping) has been determined in a previous work [4] in which an accurate investigation of the noise properties of the free-running laser is carried out. In the same work the experimental procedure allowing to measure the normalized pumping rate is described. We have reported the values of these parameters in Table 1.

With respect to the model for the free running laser, in this case we have to check the validity of the theoretical expression derived for the transfer function of the feedback loop  $G_{oig}(\Omega)$ . As previously explained, this function is the product of the transfer function of the electronic part of the feedback  $r^2g(\Omega)$  with the transfer function of a modulation of the pump intensity  $\mathcal{G}(\Omega)$ . The first one is easily obtained measuring the transfer function of each device in the loop; good agreement is found between the experimental transfer function and the theoretical one. The second transfer function  $\mathcal{G}(\Omega)$  is experimentally determined as follows: we modulate the driving current of the pump diode laser; the induced modulation on the pump beam is detected by a photodiode: gain and phase are observed on a spectrum analyser and on a digital oscilloscope respectively. Subsequently the pump beam with the same modulation as previously is sent to the microchip laser and the induced modulation on the emitted beam is detected by the same detection apparatus. With this experimental procedure the gain of the transfer function  $\mathcal{G}(\Omega)$  is simply given by the ratio of the gains of the two measured modulation with an appropriate correction coefficient, in order to take into account the different losses experienced by the two beams. The phase of  $\mathcal{G}(\Omega)$  is given by the difference of the phases of the two modulations also with an appropriate correction for the different phase shift introduced by different free space propagation of the two beams. We found a good agreement between the experimental and theoretical transfer function  $\mathcal{G}(\Omega)$  as shown in fig. 4a and 4b.

In the theoretical model the intensity noise of the Nd:YVO<sub>4</sub> microchip laser depends on the intensity noise of the pump beam. Of course, if the feedback loop is operating, we are not able to measure the pump noise; however the characteristics of the frequency response of the open loop gain (gain lower than  $-40$  dB below 1 MHz) ensure that in the low frequency range the intensity noise spectrum of the pump diode laser is not affected by the feedback. Hence, in the model we use the value of pump noise of the grating extended cavity diode laser in absence of feedback loop, measured with high accuracy in [4]: after correction for detection efficiency the amount of amplitude squeezing at the laser output is 0.7 dB, flat over the whole frequency range between 1 to 20 MHz. For frequencies lower than 300 kHz the pump laser diode exhibit slight excess noise ( $<3$  dB). For frequencies below 50 kHz technical  $1/f$  noise increases the excess noise up to 10 dB.

We now consider the experimental results obtained for the Nd:YVO<sub>4</sub> laser with feedback control. The pumping rate is adjusted in order to place the phase rotation due to the relaxation oscillation at a suitable frequency, ensuring the best performances of the feedback loop. A comparison between the intensity noise spectra of the free-running laser and the laser with feedback control is shown in fig. 5. We obtain a reduction of 8 dB of the relaxation oscillation peak due to the action of the feedback. As previously pointed out, the rather critical stability of the feedback loop prevents from achieving larger reduction. It is important to note that the reduction of the relaxation oscillation peak implies the disappearance of its harmonic. In the low frequency part of the spectrum the overlap of the experimental and theoretical curves is quite perfect, confirming that the feedback loop meets the requested features in order to put in evidence the occurrence of non-linear effects. The good agreement between experiment and theory over a large range of frequencies ensures

that the model developed is suitable and reliable to describe the noise features of the our microchip laser in presence of feedback. More accurate measurements at low frequency are shown in fig. 6. We observe a clear reduction of the noise (about 5 dB) in the case of the laser with feedback control, with respect to the free running laser, in spite of the fact that the feedback has no action at low frequency. In this case we have found a good agreement between the experimental results and the theoretical predictions of our model, which is derived from the linearization of the dynamical equations and then cannot take into account any non linear effect. This result constitutes an experimental evidence of the occurrence of non linear effects at low frequency. In this configuration a minimum noise of 7 dB above the shot noise level at 40 kHz is achieved.

## V. CONCLUSION

The intensity noise properties of a Nd:YVO<sub>4</sub> microchip laser with electronic feedback to the pump source have been investigated theoretically and experimentally. A fully quantum model is derived by combining the results of the quantum Langevin approach for the free-running laser with the semiclassical treatment of the feedback loop. The theoretical analysis shows that a feedback loop with a tailored frequency response is suitable to investigate the influence of non linear effects due to the large excess noise of the relaxation oscillation peak on the intensity noise of the microchip laser in the low frequency region.

We have implemented a non-standard feedback loop reacting only in a narrow frequency region around the relaxation oscillation peak, while keeping unchanged the noise features of the laser at low frequencies. A sharp noise reduction is observed at low frequency although the feedback loop does not affect this part of the spectrum. Moreover, in this case, good

agreement between the measured intensity noise and the predictions of the linearized model is found over the complete frequency range. This result constitutes an experimental evidence for the occurrence of non linear effects in the low frequency region. The cancellation of these effects together with the pump noise suppression leads to a minimum intensity noise of 7 dB above the SNL at 40 kHz.

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Parameter	Value
$\gamma_a$	$3.3 \times 10^4 \cdot s^{-1}$
$\gamma'_a$	$3.3 \times 10^3 \cdot s^{-1}$
$\gamma_b$	$3 \times 10^9 \cdot s^{-1}$
$\gamma_{ab}$	$7 \times 10^{11} \cdot s^{-1}$
$\kappa$	$1.56 \times 10^{10} \cdot s^{-1}$
$\kappa_{out}$	$8.36 \times 10^9 \cdot s^{-1}$

Table.1. Values of the parameters used for the theoretical calculations.

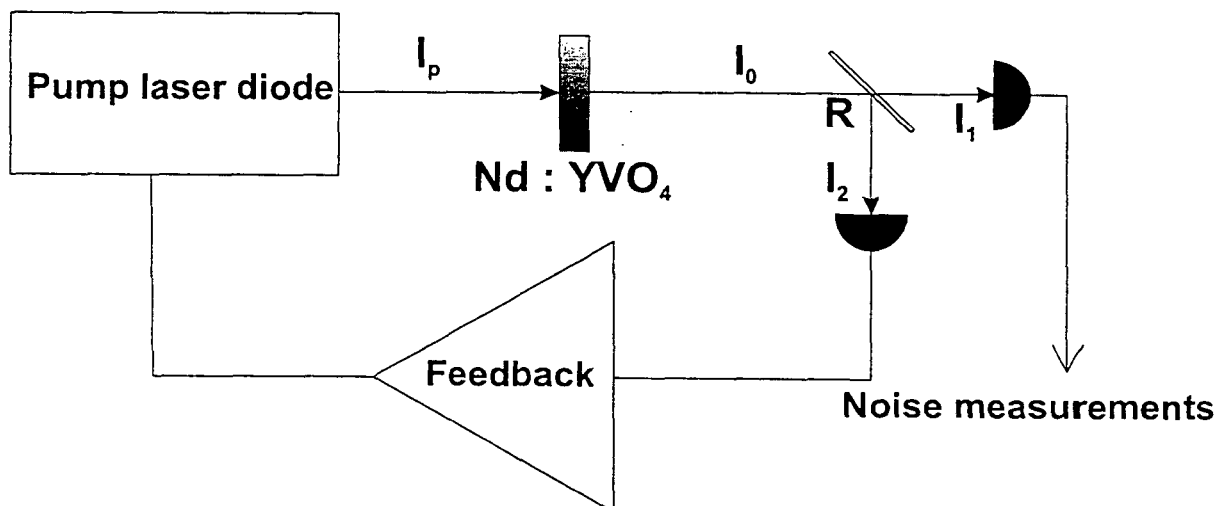


Fig.1. Schematic experimental setup modeled by theory.

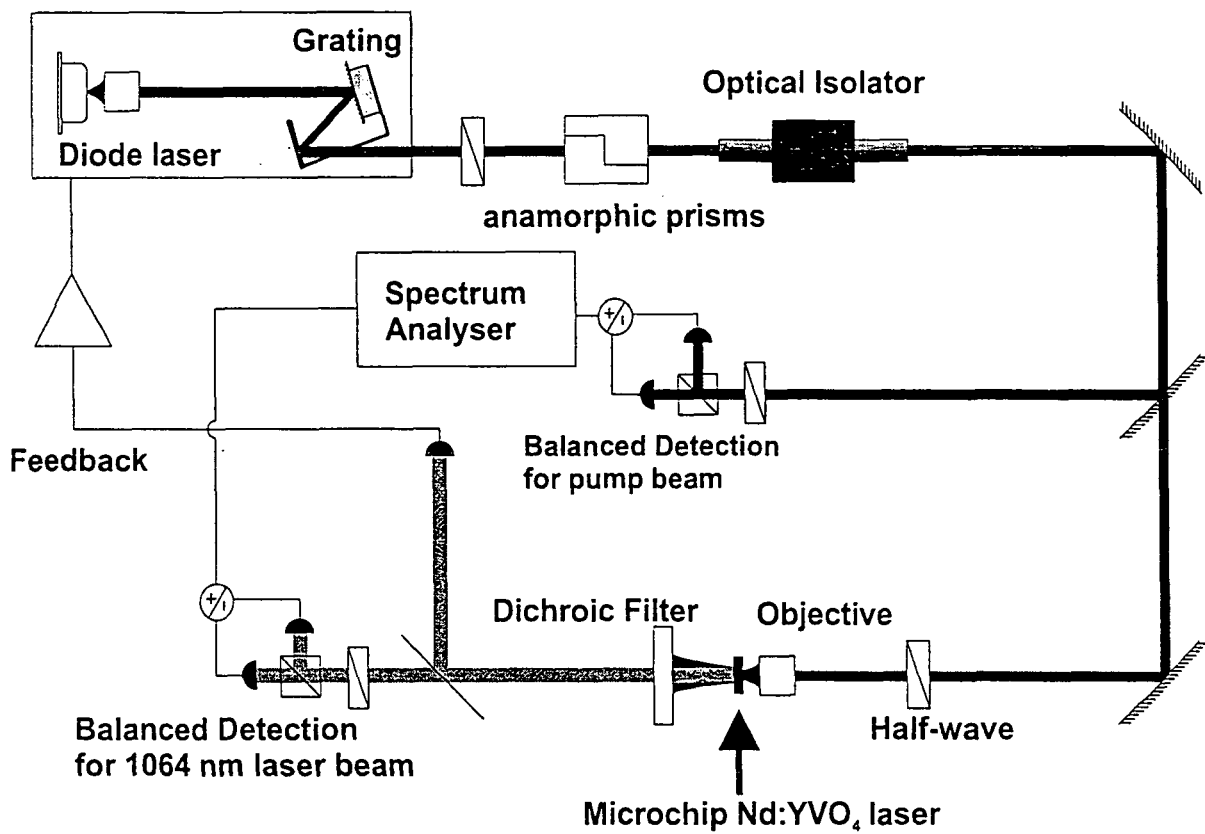


Fig.2. Experimental setup for feedback control of the diode pumped Nd: YVO<sub>4</sub> microchip laser.



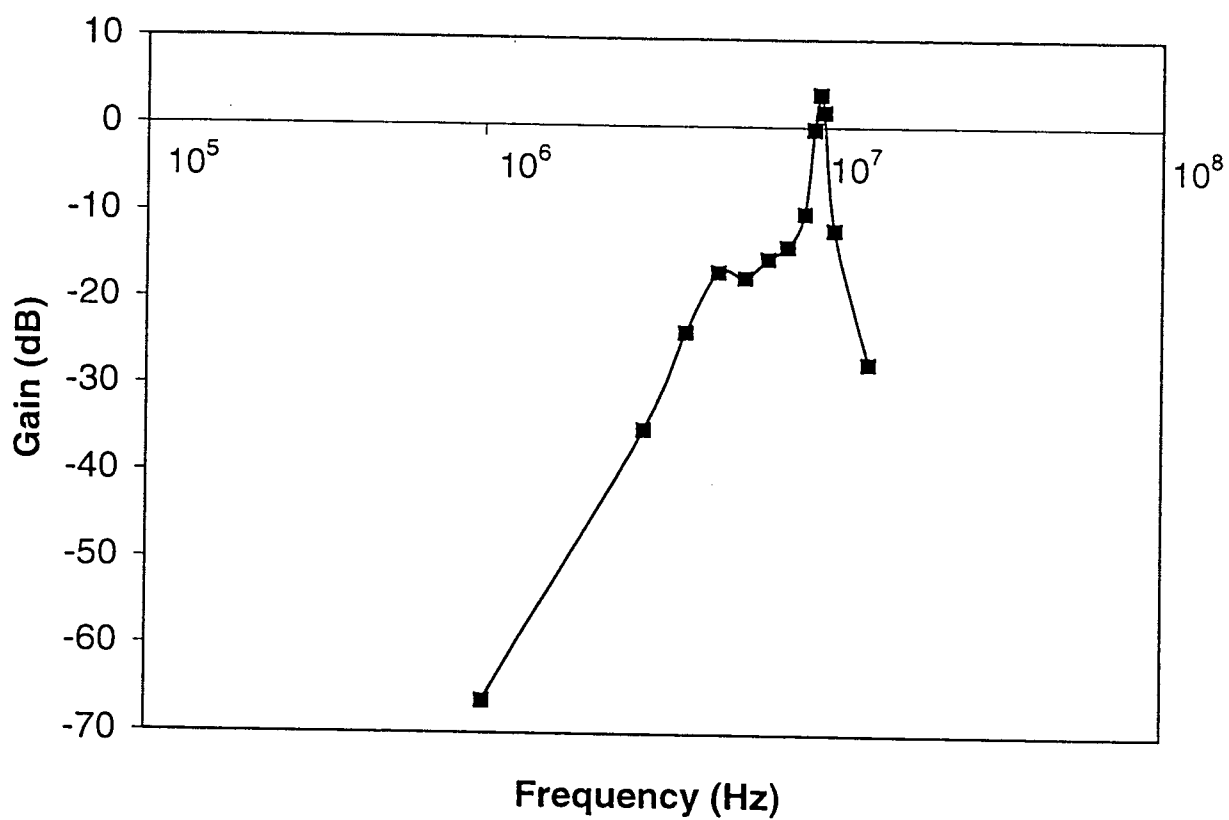


Fig.3a. Bode diagram for the gain of the total transfer function of the loop.

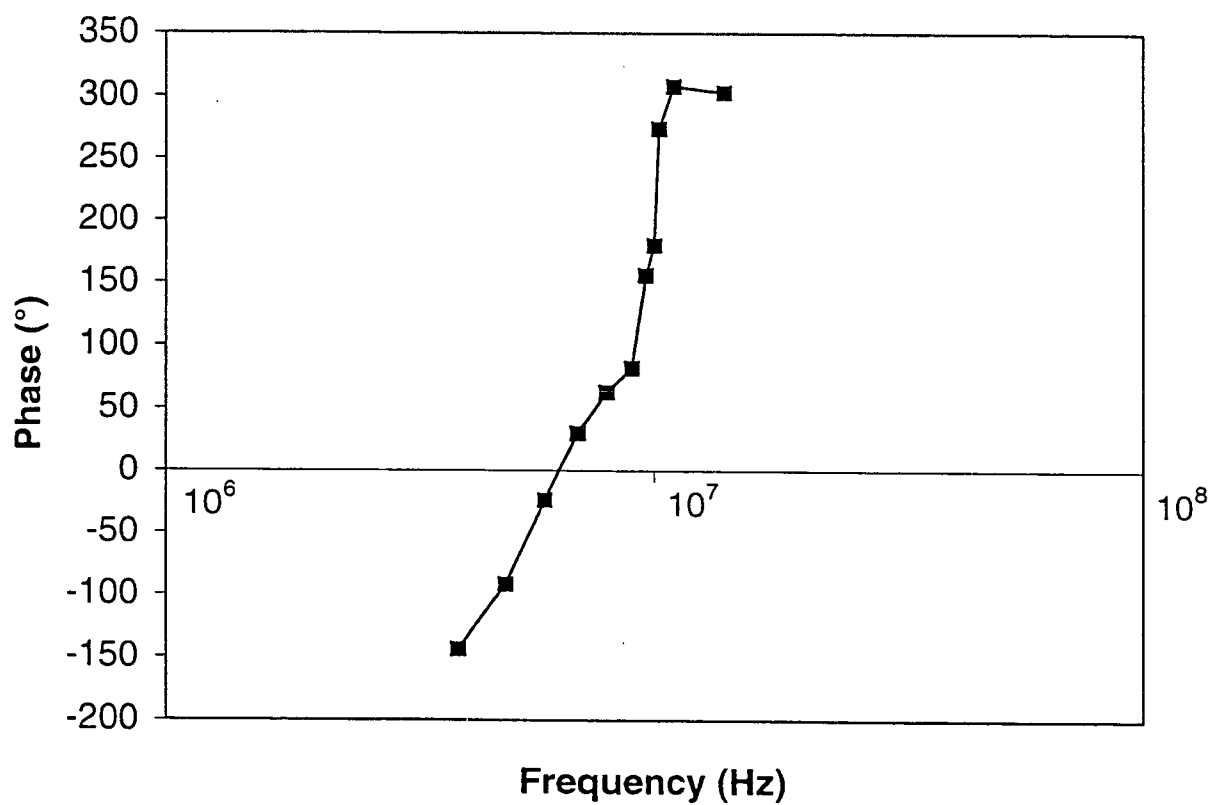


Fig.3b. Bode diagram for the phase of the total transfer function of the loop.

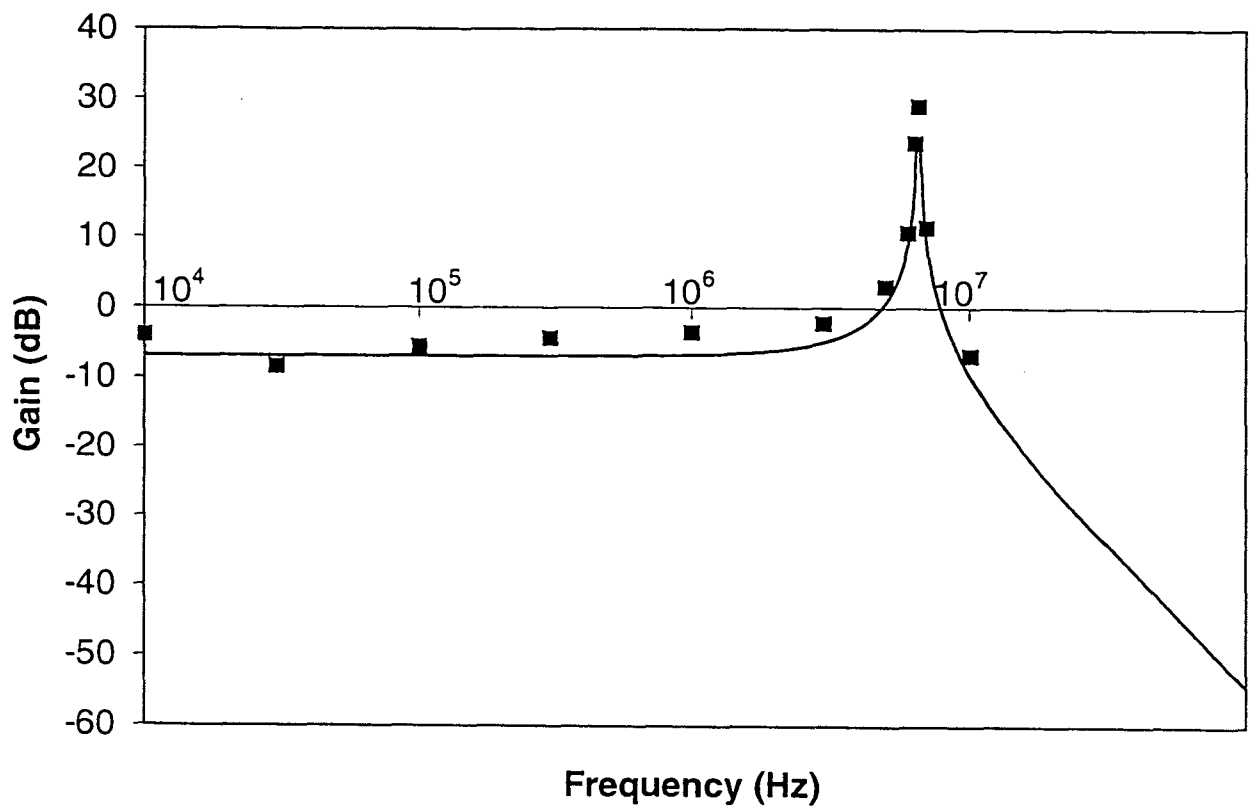


Fig.4a. Calculated gain of the transfer function  $\mathcal{G}(\Omega)$  of the Nd:YVO<sub>4</sub> microchip laser.  
Dots represent experimental data.

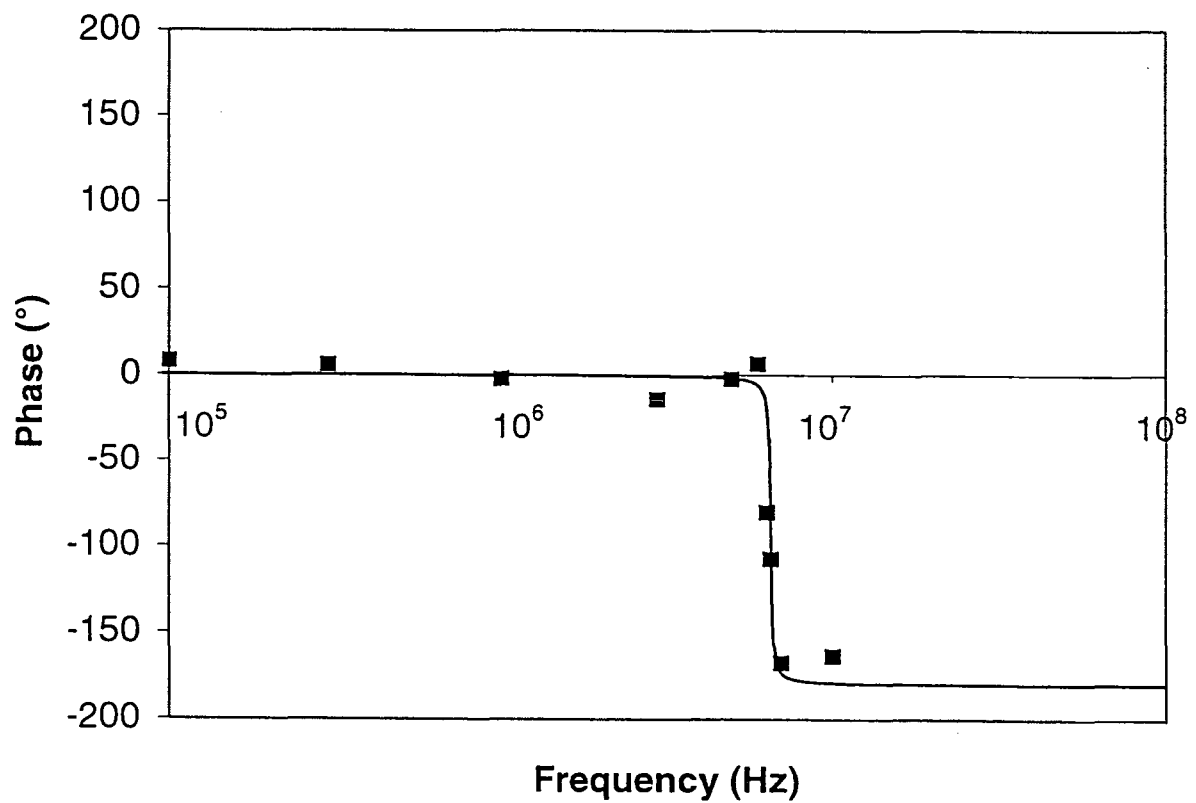


Fig.4b. Calculated phase of the transfer function  $\mathcal{G}(\Omega)$  of the Nd:YVO<sub>4</sub> microchip laser.  
Dots represent experimental data.

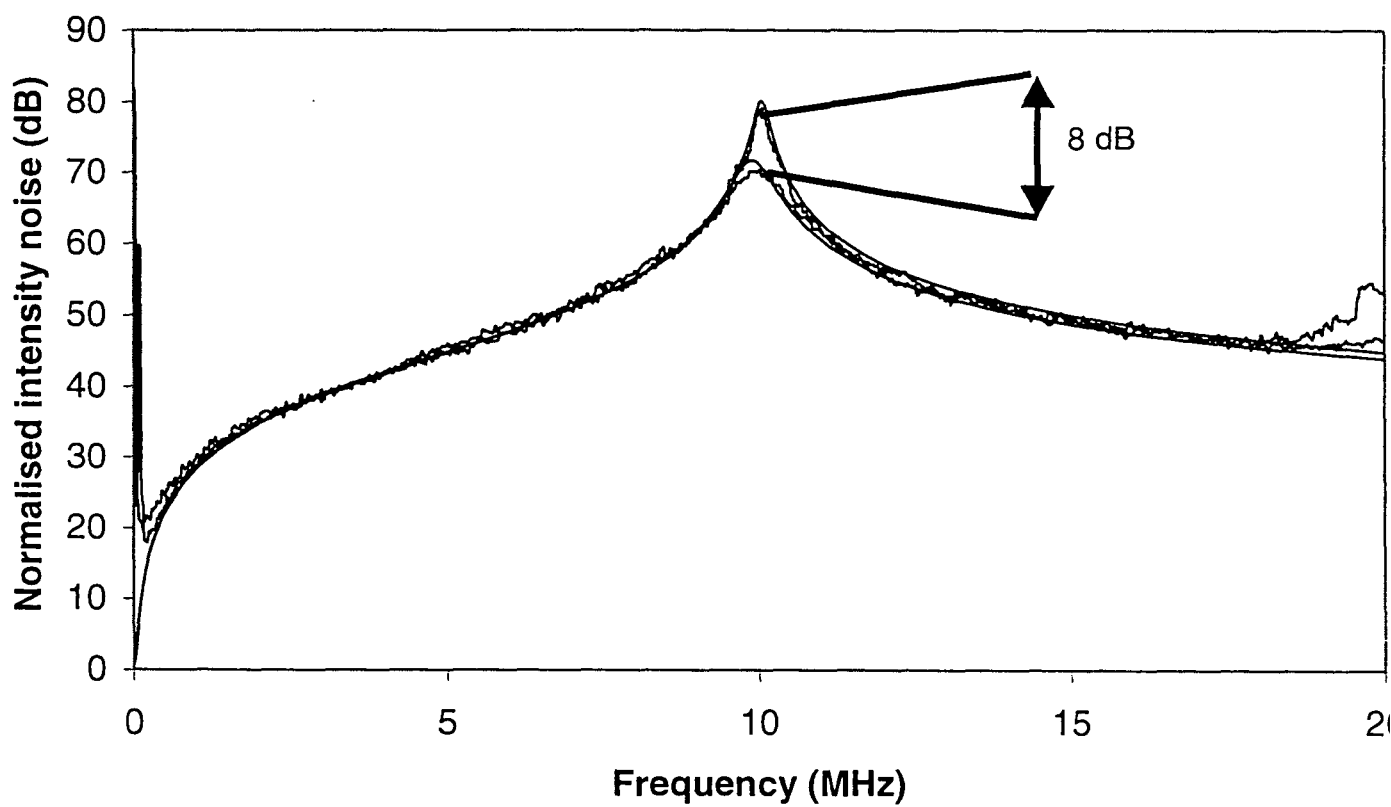


Fig.5. Normalised theoretical and experimental intensity noise spectra for the free running laser and for the laser with feedback control.

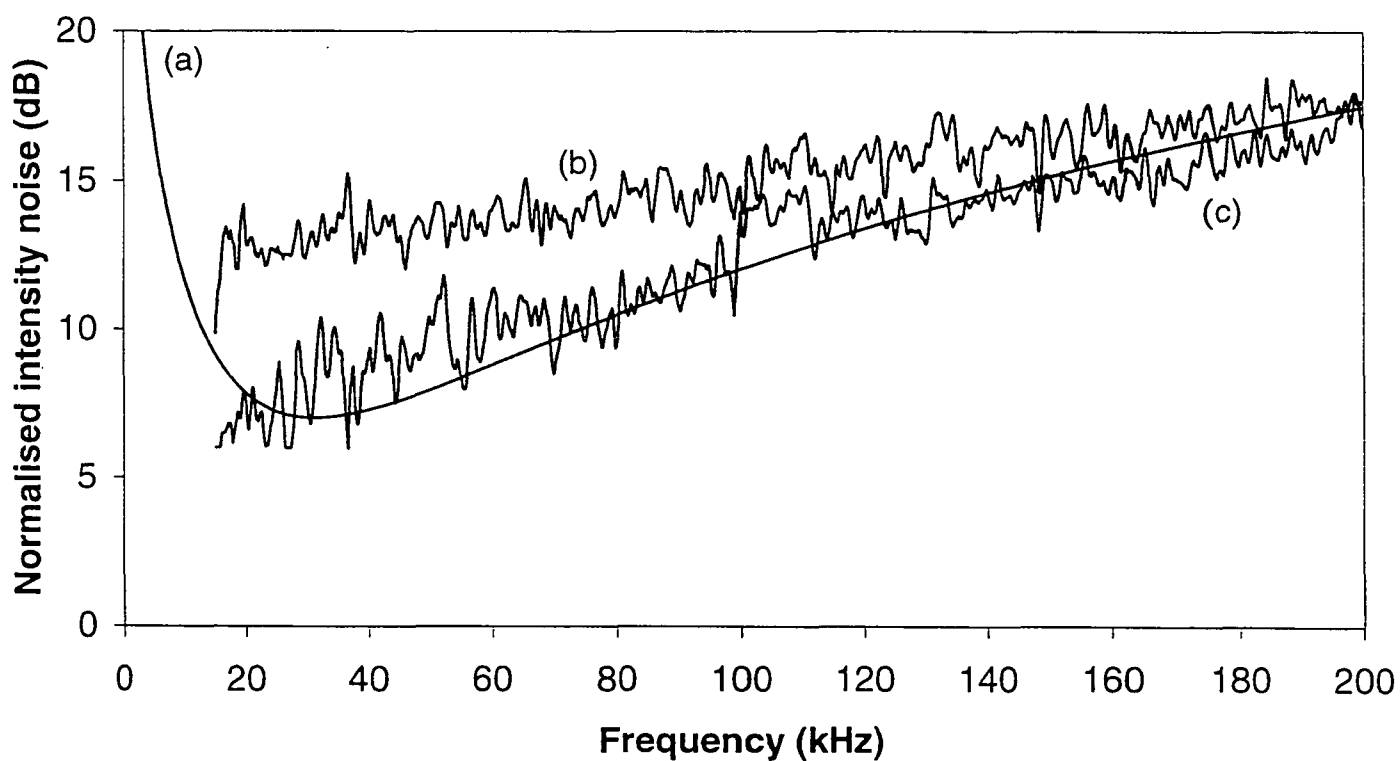


Fig.6. Normalised intensity noise spectra for the free running laser and for the laser with feedback control in the low frequency region (0 – 200 kHz). Curves (a) is a theoretical prediction; curves (b) and (c) are experimental results for the free running laser and for the laser with feedback loop respectively.

## 5.4 Injection optique et bruit d'intensité du microlaser Nd:YVO<sub>4</sub>

Les résultats expérimentaux et le modèle théorique utilisé pour décrire les propriétés de bruit du microlaser Nd:YVO<sub>4</sub> injecté sont détaillés dans l'article qui suit.

### 5.4.1 Reproduction de l'article : "Intensity noise of injected Nd:YVO<sub>4</sub> microchip lasers" (preprint)

## Intensity noise of injected Nd:YVO<sub>4</sub> microchip lasers

A. Bramati, J.-P. Hermier, V. Jost and E. Giacobino

*Laboratoire Kastler Brossel, Université Pierre et Marie Curie,*

*Ecole Normale Supérieure, CNRS*

*4, place Jussieu, F-75252 Paris Cedex 05, France*

Injection locking technique is applied to a diode pumped Nd:YVO<sub>4</sub> microchip laser, in order to suppress the relaxation oscillation peak. We found very good agreement between experimental results and theoretical predictions of a quantum model describing lasers with injected signal.

### I. INTRODUCTION

In recent years, intensity noise properties of different kind of injected lasers have been thoroughly investigated to the search of quiet light sources for applications in communications and high precision interferometry [1–3]. Great interest has been focused on injection locked diode pumped solid state lasers such as Nd:YAG or Nd:YVO<sub>4</sub> lasers [4–7]. The intensity noise spectrum of these lasers is determined by two dominant factors: the noise of the pump mechanism (for frequencies below the relaxation oscillation) and the resonant relaxation oscillation. The first factor can be partially eliminated using solid state lasers pumped with amplitude squeezed laser diodes [7,8]. The relaxation oscillation is overdamped when injection locking technique is applied [6].

In this work we present experimental and theoretical investigation of the intensity noise properties of an injected Nd:YVO<sub>4</sub> microchip laser pumped by an amplitude squeezed laser



diode.

Complete cancellation of the resonant relaxation oscillation is experimentally achieved and good agreement is found between experimental results and theoretical predictions of a quantum model describing lasers with injected signal.

## II. THEORY

For a theoretical description of the intensity noise properties of the injected solid state lasers, we have used a full quantum model based on the Langevin equations approach developed in [9]. The model considers a system of homogeneously broadened two-level atoms (assuming that the lower level is not the ground state) in resonant interaction with a mode of the electromagnetic field in a cavity of length  $L$  and volume  $V$ . The field in the cavity is driven by an external coherent optical signal, also resonant with the cavity mode. The laser dynamics is described by the following stochastic  $c$ -number Langevin equations:

$$\dot{\mathcal{A}}(t) = -\kappa/2 \mathcal{A}(t) + g\mathcal{M}(t) + \kappa/2 \lambda(t) + \mathcal{F}_\gamma(t) \quad (1)$$

$$\dot{\mathcal{M}}(t) = -\gamma_{ab}\mathcal{M}(t) + g[\mathcal{N}_a(t) - \mathcal{N}_b(t)]\mathcal{A}(t) + \mathcal{F}_\mathcal{M}(t) \quad (2)$$

$$\dot{\mathcal{N}}_a(t) = R - (\gamma_a + \gamma'_a)\mathcal{N}_a(t) - g[\mathcal{A}^*(t)\mathcal{M}(t) + \mathcal{M}^*(t)\mathcal{A}(t)] + \mathcal{F}_a(t) \quad (3)$$

$$\dot{\mathcal{N}}_b(t) = -\gamma_b\mathcal{N}_b(t) + \gamma'_a\mathcal{N}_a(t) + g[\mathcal{A}^*(t)\mathcal{M}(t) + \mathcal{M}^*(t)\mathcal{A}(t)] + \mathcal{F}_b(t) \quad (4)$$

The stochastic  $c$ -number variable  $\mathcal{A}(t)$  represents the electromagnetic field.  $\lambda(t)$  represents the injected coherent optical signal. The injected field is assumed to be in a coherent state, which means that its fluctuations are equivalent to the vacuum fluctuations. In some case, this could limit the possibility to apply the model to describe a realistic experiment. In fact, usually, the master laser used for the injection locking exhibits some excess noise (more

general models dealing with arbitrary noise of the injected field have been developed [5]). However, we will show in next section that our experimental conditions meet very well the assumptions of the model.  $\mathcal{N}_a$  and  $\mathcal{N}_b$  represent the macroscopic atomic population of the upper and lower level respectively.  $\mathcal{M}(t)$  represents the macroscopic atomic polarisation.  $\kappa$  is the total cavity damping constant: we assume here a more general definition of  $\kappa$  in order to take into account internal optical losses, neglected in the original formulation of the model:  $\kappa = \kappa_{out} + \kappa_{losses}$ , where  $\kappa_{out}$  represents the output coupling, and  $\kappa_{losses}$  the internal optical losses.  $\gamma_a$  and  $\gamma_b$  are the decay rates of the populations of the upper and lower levels to the other atomic levels;  $\gamma'_a$  is the spontaneous decay rate between the lasing levels and  $\gamma_{ab}$  is the decay rate of the atomic polarisation.  $R$  is the mean pumping rate. The constant  $g$  corresponds to the electric dipole coupling between the two-level atoms and the field. The functions  $\mathcal{F}_k(t)$  with  $k = \gamma, \mathcal{M}, a, b$  are the stochastic  $c$ -number Langevin forces with the properties:

$$\langle \mathcal{F}_k(t) \rangle = 0 \quad (5)$$

$$\langle \mathcal{F}_k(t) \mathcal{F}_l(t') \rangle = 2\mathcal{D}_{kl} \delta(t - t') \quad (6)$$

where  $\mathcal{D}_{kl}$  represents the diffusion coefficient for the  $c$ -number Langevin force. The nonvanishing diffusion coefficients are given in appendix.

The steady state is obtained from Eqs.(1)-(4) by neglecting the fluctuations and setting the time derivatives equal to zero. Expressing the atomic variables in terms of the steady state field  $\mathcal{A}_s$ , one gets:

$$\mathcal{N}_{as} = \frac{R}{\gamma_a + \gamma'_a} + \frac{\kappa \gamma_{ab} \gamma_b}{2g^2 (\gamma_a + \gamma'_a)} \left( 1 - \frac{\lambda}{\mathcal{A}_s} \right) \quad (7)$$

$$\mathcal{N}_{as} - \mathcal{N}_{bs} = \frac{\kappa \gamma_{ab}}{2g^2} \left( 1 - \frac{\lambda}{\mathcal{A}_s} \right) \quad (8)$$

$$\mathcal{M}_s = \frac{\kappa}{2g} \left( 1 - \frac{\lambda}{\mathcal{A}_s} \right) \mathcal{A}_s \quad (9)$$

where the steady state field  $\mathcal{A}_s$  is a solution of the equation

$$\lambda = \frac{I - I_0}{I + 1} \mathcal{A}_s \quad (10)$$

$I$  and  $I_0$  represent the normalised intensity for the laser with and without an injected signal respectively. Their definitions are as follows:

$$I = \frac{|\mathcal{A}_s|^2}{|\mathcal{A}_{sat}|^2}, \quad I_0 = \frac{|\mathcal{A}_0|^2}{|\mathcal{A}_{sat}|^2} \quad (11)$$

where

$$|\mathcal{A}_{sat}|^2 = \frac{\gamma_{ab}\gamma_b \gamma_a + \gamma_a'}{2g^2 \gamma_a + \gamma_b} \quad (12)$$

is the saturation intensity for the free-running laser ( $\lambda = 0$ ) and

$$|\mathcal{A}_0|^2 = |\mathcal{A}_{sat}|^2 (R/R_{th} - 1) \quad (13)$$

is the steady state intensity for the free-running laser.

By linearizing the Eqs. (1)-(4) around the steady state solutions and by applying the Fourier transform to the field and atomic variables, one gets a linear system of algebraic equations. Its solution allows to analytically calculate the intensity noise spectrum at the laser output as a function of the relaxation rates and the pump noise. The derived expression is quite general and does not rely on any adiabatic elimination of variables, hence it is suitable to describe any type of lasers. However, the laser under investigation in our experiment is a Nd:YVO<sub>4</sub> laser belonging to the third class lasers: for these lasers the decay rate of the atomic polarisation  $\gamma_{ab}$  is much faster than the other relaxation rates. In this condition, it

is possible to derive an approximate expression for the intensity noise spectrum at the laser output.

On the other and, in the original formulation of the model the pump noise is described by the parameter  $p$  ranging from 0 to 1 ( $p = 0$  for poissonian pump;  $p = 1$  for regular pump). In order to reproduce the experimental conditions more precisely, we generalize the model by introducing the spectral density of the pump noise  $s(\tilde{\Omega})$  normalised to the shot noise.

According with [9] the normalised intensity noise spectrum at the laser output for a third class laser is given by:

$$\begin{aligned}
 V_{out}(\tilde{\Omega}) = & 1 + \eta \frac{2b(a+a')}{b-a'} \frac{1}{D(\tilde{\Omega})} \left( (b^2 + \tilde{\Omega}^2) \left[ (a+a')^2 + \tilde{\Omega}^2 \right] \frac{n}{a+a'} \right. \\
 & + 2w^2 \left\{ \left[ (b-a')^2 + \tilde{\Omega}^2 \right] \left[ n + (s(\tilde{\Omega}) - 1) r/2 \right] \right. \\
 & - \left. \left[ (b-a')(a+a') + \tilde{\Omega}^2 \right] \left( r - \frac{a+2a'}{a+a'} n \right) \right. \\
 & \left. \left. + \left[ (a+a')^2 + \tilde{\Omega}^2 \right] \frac{a'}{a+a'} n \right\} \right) \quad (14)
 \end{aligned}$$

where we set  $p(\tilde{\Omega}) = 1 - s(\tilde{\Omega})$ , and  $\eta = \kappa_{out}/\kappa$  represents the correction for internal optical losses. The dimensionless parameters  $a$ ,  $b$  and the dimensionless noise frequency  $\tilde{\Omega}$  are defined as follows:

$a \equiv \gamma_a/\kappa$  (respectively  $b \equiv \gamma_b/\kappa$ ) is the normalised decay rate of the upper level (respectively lower level);

$a' \equiv \gamma'_a/\kappa$  is the normalised spontaneous decay rate between the lasing levels;

$x \equiv \lambda/\mathcal{A}_s$ ; then  $x^2$  corresponds to the ratio between the injected power and the power emitted by the laser;

$\tilde{\Omega} \equiv \Omega/\kappa$

We also used the following definitions:

$$D(\bar{\Omega}) = \left| \left( \frac{1}{2}x - i\bar{\Omega} \right) (b - i\bar{\Omega})(a + a' - i\bar{\Omega}) + 2w^2(a + b - 2i\bar{\Omega})(1 - x/2 - i\bar{\Omega}) \right|^2 \quad (15)$$

$$n = \frac{r(a + a') + (b - a')(1 - x)}{a + b} \quad \text{and} \quad w^2 = \frac{(a + a')b(r - 1 + x)}{2(a + b)(1 - x)} \quad (16)$$

In fig. 1 we have reported a comparison between the intensity noise spectra of the free-running laser and the injected laser for different values of the injected power. The spectra are calculated from the eq. (15) with the parameters corresponding to our experimental conditions. The effect of the injection results in a very large reduction of the excess noise (depending on the injected power) in the frequency region around the relaxation oscillation peak, while the low frequency part of the spectrum is unaffected, as predicted also in a more general model [5]. This is due to the fact that in this frequency range (well below the relaxation oscillation), the intensity noise of the laser is strongly coupled to the noise of the pump mechanism. Hence, the injection locking is found to be a very useful technique to obtain low intensity noise spectrum, quite flat versus the noise frequency, from the slave laser.

### III. EXPERIMENTAL SET-UP

The experimental set-up for the noise measurements on the injected Nd:YVO<sub>4</sub> microchip laser is shown in fig. 2. The amplitude squeezed laser diode used for optical pumping of solid state laser is an index guided quantum well GaAlAs laser diode (SDL 5422H1) operating at 810 nm. Noise reduction in the pump beam is achieved by driving the laser diode with a high impedance constant current source and suppressing the side modes using feedback from an external grating in an extended cavity laser [1,3,10-14]: by tilting the grating, the laser wavelength can be tuned to match the maximum of the Nd:YVO<sub>4</sub> line absorption at

808.5 nm. Astigmatism in the beam is corrected by means of anamorphic prisms. Two optical isolators (for a total isolation of 70 dB) are employed to prevent back reflection in the pump laser. The optical power available for pumping process is 45 mW (due to the losses of the grating). The intensity noise of the pump diode laser is measured by a standard balanced detection (two high efficiency EG&G FND100 PIN photodiodes), which allows to measure, under the same conditions, the shot noise and the intensity noise of the laser beam. We performed several tests in order to check the reliability of the shot noise measured in this way, as described in [14]. The common mode rejection of the balanced detection is better than 30 dB in the range of 0 - 30 MHz; electronic and dark noise are typically more than 10 dB below the shot noise level. The pump beam is sent to the microchip laser by mean of two mirrors and focused into the laser with a  $f = 8$  mm objective. The polarisation of the pump beam is fixed by a half-wave plate in order to achieve the maximum absorption in the Nd:YVO<sub>4</sub> crystal. The microchip laser is mounted on a xyz-translation stage which allows an optimum alignment.

The Nd:YVO<sub>4</sub> microchip laser is 300  $\mu\text{m}$  long, with a planar monolithic cavity (the stability is ensured by thermal lens effects) in which the mirrors were deposited directly onto the crystal. The output mirror and back reflector have reflectivities of 97% and 99.5% respectively at 1.064  $\mu\text{m}$ . The mirrors do not have special coatings for wavelength of pump radiation at 810 nm. Accurate measurements show a reflectivity of 24% and a transmissivity of 7% for pump radiation.

The injection locking was implemented by using a commercial Nd:YAG laser as the master laser. The frequency of the master laser is about 120 GHz below the Nd:YVO<sub>4</sub> laser frequency at room temperature. The frequency of the slave laser is tuned inside the

injection locking bandwidth by heating the laser crystal. The temperature operation is around  $100^{\circ}\text{C}$ ; the temperature stabilisation by a PID control ensures variations less than  $0.01^{\circ}\text{C}$ . The frequency shift versus the temperature for Nd:YVO<sub>4</sub> lasers was found to be  $-1.6\text{ GHz}/^{\circ}\text{C}$  (see also [15]): this implies that the jitter of the laser due to temperature variations is of the order of ten megahertz, and then negligible with respect to the observed injection bandwidth of about 200 MHz (the bandwidth calculated from the usual formula involving the cavity parameters, injected and emitted powers is expected to be 1 GHz). The master laser beam enters the output coupler of the slave laser through the escape port of an optical isolator which automatically match the polarisation of the master beam to the slave one; the Gaussian parameters of the master beam are reshaped with a telescopic lens system in order to have an efficient mode matching with the slave laser. The power effectively coupled into the slave laser is evaluated to be 10% of the incident power, typically ranging between 100 to 500  $\mu\text{W}$ . The emission wavelength of both the master and slave laser are measured by a high resolution monochromator. The injection locking is monitored with a Fabry-Perot analyser. With this experimental configuration we achieved a very stable injection locking operation over several hours.

For the intensity noise measurements on the emitted beam we used the following experimental set-up. For noise measurements up to 30 MHz, balanced detection is no longer reliable due to the very high excess noise of the relaxation peak (more than 80 dB) which exceeds the common mode rejection ratio (typically 30 dB). Therefore in this range of frequencies we choose to calibrate the shot noise level with an independent source. For the calibration we use the noise obtained by direct detection on one photodiode of attenuated radiation emitted by a shot noise limited diode laser. It is worth saying that no correction

has to be calculated, due to the difference in the wavelength of the two beam. In fact we detect the noise of the photocurrent which is independent on the wavelength. We check carefully linear dependence of the calibrated shot noise signal with the optical power incident on the photodiode. The shot noise obtained in this way was in agreement within 0.1 dB with the noise obtained by a thermal light generating the same DC current on the photodiode.

#### IV. EXPERIMENTAL RESULTS

In this section we present the experimental results obtained with the injected Nd:YVO<sub>4</sub> microchip laser previously described. In order to make a comparison between theory and experiment we have to determine all the parameters of the model. A set of values for the parameters describing the microchip laser (relaxations rates and cavity damping) has been determined in a previous work [8] in which an accurate investigation of the noise properties of the free-running laser is carried out. In the same work the experimental procedure allowing to measure the normalized pumping rate is described. However, in this case, the laser is operating at a temperature of 100°C and we have observed relevant variations with respect to the laser at room temperature. Assuming that the relaxation rates for the atomic populations and the atomic polarisation are substantially insensitive to temperature variations, and applying the same procedure described in [8], we have found a the total cavity damping  $\kappa = 9 \times 10^9 \text{ s}^{-1}$ ; the pump power threshold for oscillation is about 13 mW (4.2 mW at room temperature), the quantum efficiency of the pump mechanism 12.5% (40% at room temperature) and the maximum emitted power about 3.5 mW (10 mW at room temperature). We report in Table 1 the values of the parameters used in the numerical simulations. The degradation of the performances of the Nd:YVO<sub>4</sub> microchip



laser is related to the heating of the crystal: in fact, as explained previously, temperature allows to tune the slave frequency at a rate of  $-1.6 \text{ GHz} / ^\circ\text{C}$ ; on the other and the shift in the gain center frequency induced by heating is  $-0.53 \text{ GHz} / ^\circ\text{C}$  [15]. The oscillating mode, at resonance with the center of the gain bandwidth at room temperature, is more and more detuned off resonance, as the temperature is increased. In our case, the total detuning from the gain center frequency (estimated to be 80 GHz) together with the FWHM gain bandwidth of the Nd:YVO<sub>4</sub> (257 GHz [15,16]) prevents from obtaining better performances from our laser.

We now consider the noise features of the master laser: our commercial laser is equipped with a standard feedback loop on the pump in order to eliminate the relaxation oscillation peak. Thus, the intensity noise spectrum at the output of the laser, for an emitted power of 700 mW, is rather flat over the whole range of observation (0-30 MHz) and exhibits an excess noise of 30 dB above the shot noise level (SNL). The theoretical model assumes that the injected field is in a coherent state i.e. with intensity noise limited to the SNL. However, due to optical losses, the intensity noise of the master beam scales according the usual formula:

$$V_{inj}(\bar{\Omega}) = 1 + \eta(V_{out}(\bar{\Omega}) - 1) \quad (17)$$

where  $V_{inj}$  represents the actual normalised intensity noise of the injected beam and  $V_{out}$  is the normalised intensity noise at the output of the master laser;  $\eta$  is the optical attenuation. We use a strong attenuation of the master laser:  $\eta = 0.1 \text{ mW} / 700 \text{ mW} = 1.4 \times 10^{-4}$  in this case. It is then obvious from eq. (17) that the intensity noise of the injected beam can be identified with the SNL with a very good approximation, according to the assumption of the model.

In the theoretical model the intensity noise of the Nd:YVO<sub>4</sub> microchip laser depends on the intensity noise of the pump beam. We use in the model the pump noise of the grating extended cavity laser diode, measured with high accuracy in [8]: after correction for detection efficiency the amount of amplitude squeezing at the laser output is 0.7 dB, flat over the whole frequency range between 1 to 20 MHz. For frequencies lower than 300 kHz the pump laser diode exhibit slight excess noise (<3 dB). For frequencies below 50 kHz technical 1/f noise increases the excess noise up to 10 dB.

The experimental results obtained with the injected Nd:YVO<sub>4</sub> microchip laser are shown in fig. 3. The graphic reports the normalised intensity noise spectra for different values of the injected master field amplitudes: good agreement is found between the predictions of the theoretical model (thick line) and experimental results over a large range of frequencies. The parameters used in the calculations are reported in Table 1. As expected, the relaxation oscillation peak is completely damped (more than 60 dB of reduction with respect to the free-running laser) and the microchip laser exhibits a flat intensity noise spectrum with a noise level varying between 25 to 17 dB above the shot noise level, decreasing with the injected power.

The quite high excess noise is due to the poor laser features (namely high oscillation threshold and low quantum efficiency observed at high temperature); significant improvement is expected for injection locked laser operating at room temperature.

## V. CONCLUSION

We have investigated the experimental behaviour of an injection locked Nd:YVO<sub>4</sub> microchip laser. The experimental intensity noise spectra show that the injection technique

allows to eliminate the relaxation oscillation peak, responsible for the huge excess noise that affect the spectrum of the free-running laser in the megahertz range. We found a good agreement between experimental results and theoretical predictions of a fully quantum model describing the noise features of an injected laser. The minimum intensity noise obtained with this experimental configurations is 17 dB above the SNL. Further improvements could be achieved by implementing the injection locking at room temperature and increasing the pump rate in order to operate the microchip laser far above threshold.

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#### APPENDIX

We report the non-vanishing diffusions coefficients for the  $c$ -number Langevin forces

$$2\mathcal{D}_{aa} = (\gamma_a + \gamma'_a) \langle \mathcal{N}_a(t) \rangle + R(1-p) - g[\langle \mathcal{M}^*(t)\mathcal{A}(t) + \mathcal{A}^*(t)\mathcal{M}(t) \rangle] \quad (\text{A1})$$

$$2\mathcal{D}_{bb} = \gamma_b \langle \mathcal{N}_b(t) \rangle + \gamma'_a \langle \mathcal{N}_a(t) \rangle - g[\langle \mathcal{M}^*(t)\mathcal{A}(t) + \mathcal{A}^*(t)\mathcal{M}(t) \rangle] \quad (\text{A2})$$

$$2\mathcal{D}_{ab} = -\gamma'_a \langle \mathcal{N}_a(t) \rangle + g[\langle \mathcal{M}^*(t)\mathcal{A}(t) + \mathcal{A}^*(t)\mathcal{M}(t) \rangle] \quad (\text{A3})$$

$$2\mathcal{D}_{\mathcal{M}\mathcal{M}} = 2g \langle \mathcal{M}(t)\mathcal{A}(t) \rangle \quad (\text{A4})$$

$$2\mathcal{D}_{\mathcal{M}\cdot\mathcal{M}} = (2\gamma_{ab} - \gamma_a - \gamma'_a) \langle \mathcal{N}_a(t) \rangle + R \quad (\text{A5})$$

$$2\mathcal{D}_{b\mathcal{M}} = \gamma_b \langle \mathcal{M}(t) \rangle \quad (\text{A6})$$

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Parameter	Value
$\gamma_a$	$3.3 \times 10^4 \text{ .s}^{-1}$
$\gamma'_a$	$3.3 \times 10^3 \text{ .s}^{-1}$
$\gamma_b$	$10^9 \text{ .s}^{-1}$
$\gamma_{ab}$	$7 \times 10^{11} \text{ .s}^{-1}$
$\kappa$	$9 \times 10^9 \text{ .s}^{-1}$
$\kappa_{\text{out}}$	$8.36 \times 10^9 \text{ .s}^{-1}$

Table.1. Values of the parameters used for the theoretical calculations.

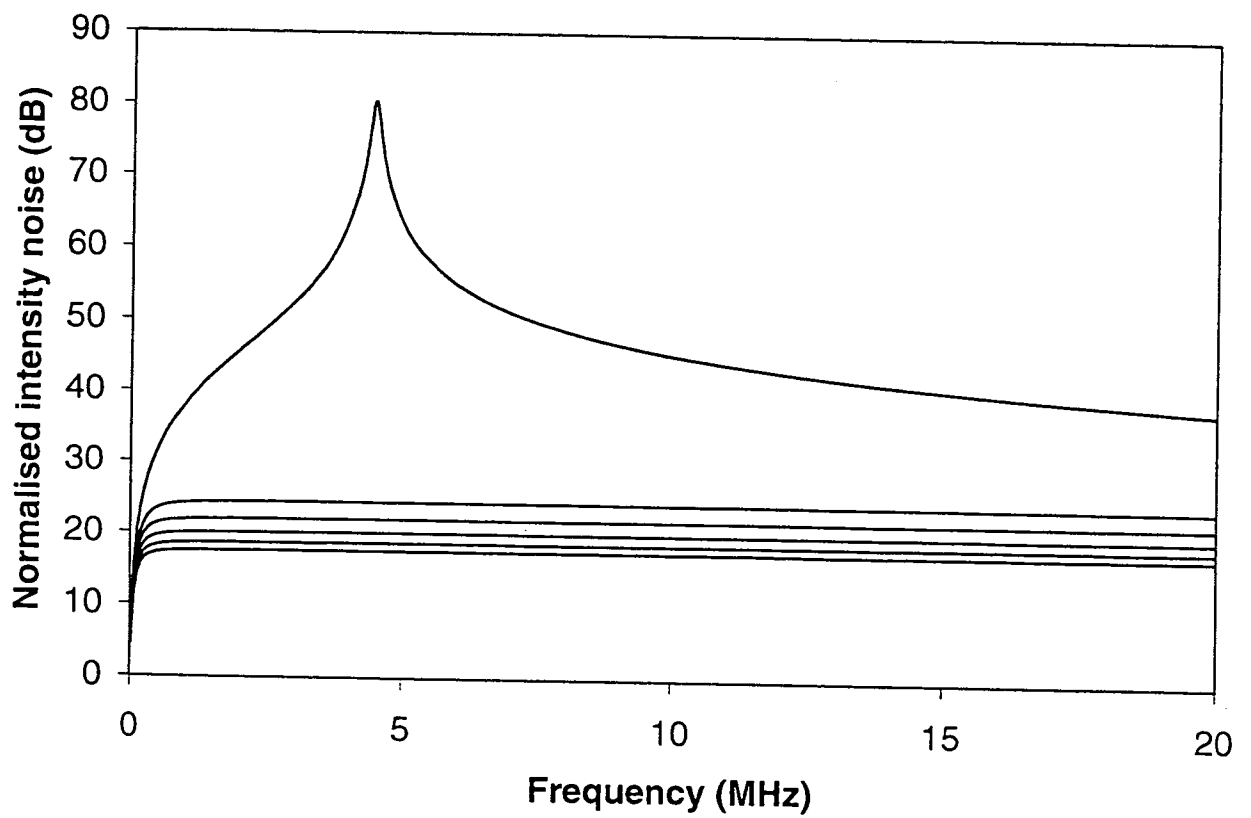


Fig.1. Calculated intensity noise spectra of the injected Nd:YVO<sub>4</sub> microchip laser for different injected power.

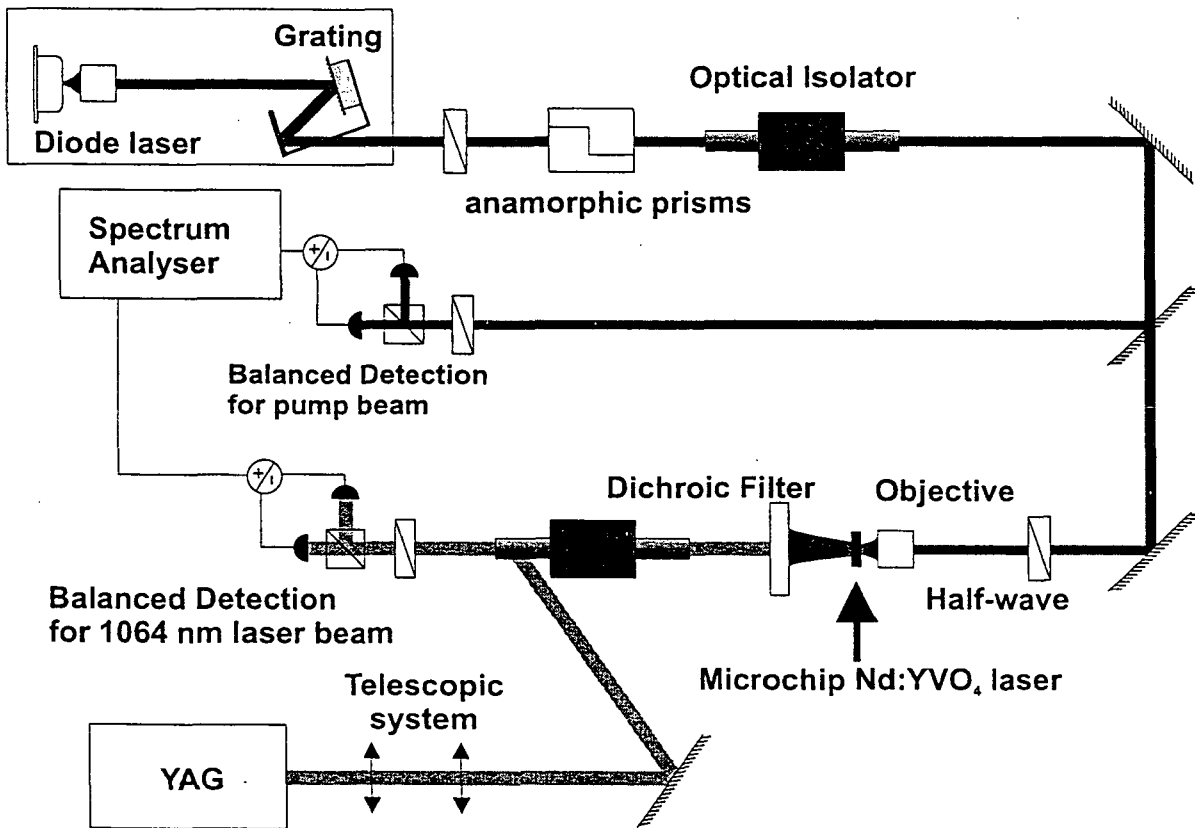


Fig.2. Experimental setup for noise measurements in the injected Nd:YVO<sub>4</sub> microchip laser.



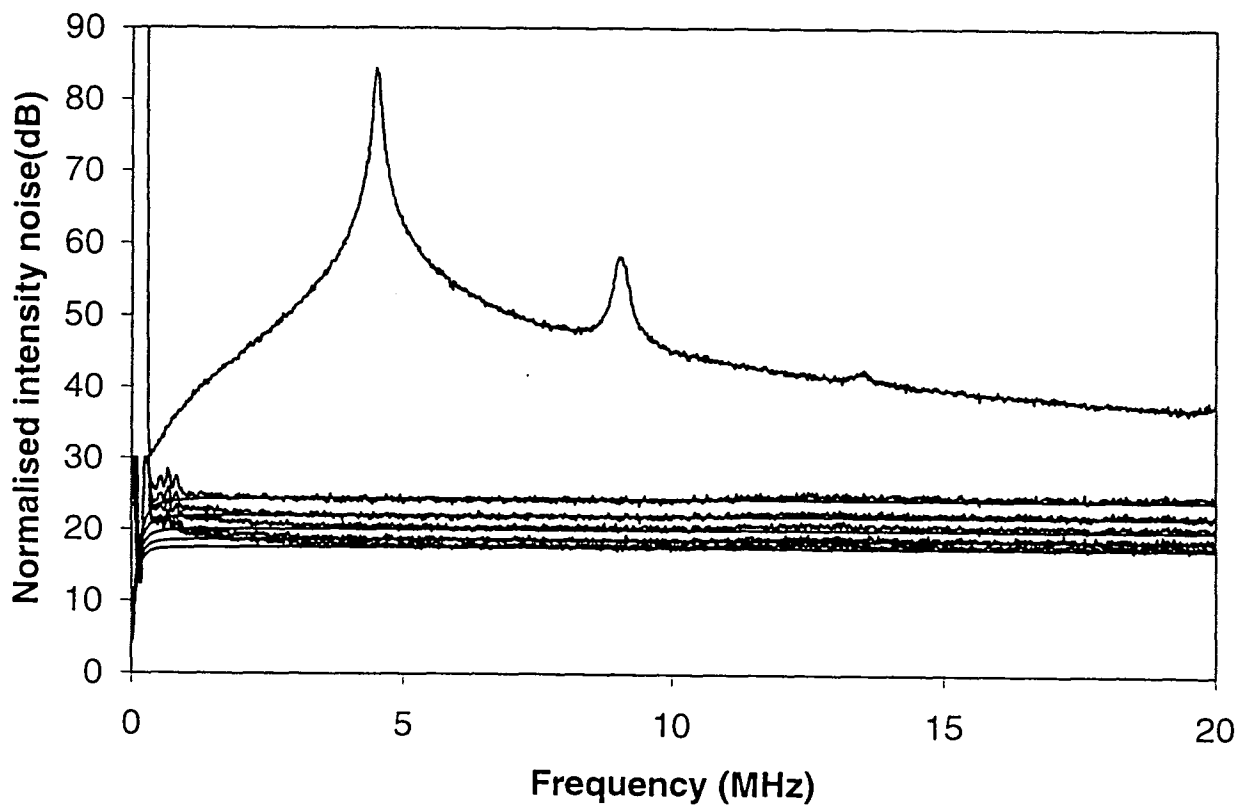


Fig.3. Normalised experimental and theoretical (thick line) intensity noise spectra of the injected Nd:YVO<sub>4</sub> microchip laser for different injected powers. The pump noise correspond to the experimental measured spectrum of the pump diode laser.

## 6 Spectroscopie de haute sensibilité sous le bruit quantique standard

### 6.1 Introduction

La sensibilité atteinte dans des mesures de spectroscopie laser traditionnelle est limitée par le bruit de photons des faisceaux lumineux qui sont utilisés dans l'expérience [58–61]. Les progrès réalisés dans la production d'états non classiques du rayonnement ont récemment ouvert des possibilités nouvelles pour améliorer la sensibilité dans diverses expériences d'optique : interférométrie, communications optiques et spectroscopie [62]. En particulier, en spectroscopie, des expériences utilisant de montages capables de réduire les fluctuations quantiques sous la limite quantique et donc d'accroître la sensibilité ultime de la mesure ont été réalisées. Nous pouvons rappeler ici l'expérience de Kimble [63] et celle conduite au laboratoire par l'équipe de Claude Fabre [64], utilisant des OPO, et les expériences de Steel [65] et de Yamamoto [66] qui utilisent de diodes lasers sous le bruit quantique standard.

Nous avons réalisé une expérience de spectroscopie FM de haute sensibilité [67] pour la détection des signaux d'absorption avec élargissement Doppler et sous-Doppler produits par la transition  $D_2$  du  $^{133}\text{Cs}$  à 852 nm, utilisant de diodes laser avec un bruit d'intensité d'environ 1 dB sous le shot noise. Cette propriété des sources laser permet en principe de mettre en évidence des signaux sous le bruit quantique standard qui ne seraient pas détectables en utilisant un laser avec un bruit d'intensité limité au bruit quantique standard. Nous avons essayé différentes configurations expérimentales pour les sources lasers et différentes techniques de modulation afin d'améliorer le rapport signal à bruit et tester la sensibilité ultime associée à cette technique :

- 1) Configuration en cavité étendue : la source est une diode laser sur réseau ; la modulation de fréquence est produite par un modulateur électro-optique.

2) Injection par une diode sur réseau : la source est une diode laser injectée ; le laser maître est modulé en phase par un modulateur électro-optique.

3) Injection par une diode laser DBR : la source est une diode laser injectée ; le laser maître est directement modulé en courant.

Le meilleur résultat, obtenu avec la troisième configuration, conduit à une sensibilité (pour un rapport signal à bruit égal à 1) de  $3.9 \times 10^{-8}$  avec une bande de détection de  $10 \text{ Hz}$  et un indice de modulation  $\beta = 0,25$ .

L'originalité de notre expérience par rapport aux expériences citées précédemment est que nous nous sommes placés dans les conditions exactes d'une expérience de spectroscopie de haute sensibilité, en évitant soigneusement toute saturation du signal par la sonde laser et que nous avons évalué en détail la sensibilité maximale accessible pour la comparer aux performances obtenues avec des lasers non comprimés.

Nous avons ainsi montré que les méthodes classiques de spectroscopie de haute sensibilité se transfèrent bien aux lasers à bruit comprimé. La sensibilité ultime que nous obtenons est supérieure aux valeurs publiées dans la littérature [61] et pourrait être encore améliorée avec des lasers plus comprimés.

## 6.2 Résultats expérimentaux

Le montage expérimental ainsi que les résultats obtenus dans les différentes configurations utilisées sont détaillés dans l'article que nous reproduisons dans le paragraphe suivant.

### 6.2.1 Reproduction de l'article : "Demonstration of high sensitivity spectroscopy with squeezed semiconductor lasers" (Optics Comm., 140, 146 (1997))



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## Demonstration of high sensitivity spectroscopy with squeezed semiconductor lasers

F. Marin, A. Bramati<sup>\*</sup>, V. Jost, E. Giacobino*Laboratoire Kastler Brossel, Université Pierre et Marie Curie, Case 74, 4 Place Jussieu, F-75252 Paris Cedex 05, France*

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### Abstract

We have employed amplitude squeezed semiconductor lasers in a frequency-modulation spectrometer for the detection of absorption signals of atomic cesium. The sub-quantum limit radiation allows to evidence signals that are below the shot noise limit of the laser source. We have used a set-up with extremely low effective absorptions in order to quantitatively assess the potential sensitivity enhancement in comparison with previous high sensitivity spectroscopy experiments. A narrow detection bandwidth and a high modulation index, together with the sub-shot noise source lead to a minimum detectable signal that is a few parts in  $10^8$ . © 1997 Elsevier Science B.V.

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### 1. Introduction

Using squeezed light has been proposed in various optical experiments, in order to reduce the quantum noise in spectroscopy, in interferometry and in optical communications. The possibility to improve the sensitivity in the detection of weak absorptions has been theoretically studied by Yurke and Wittaker [1] and demonstrated in experiments by Polzik, Kimble and co-workers [2], Steel and co-workers [3] and Yamamoto and co-workers [4]. The authors of Ref. [2] could obtain an improvement of 3.1 dB in the detection of a sub-Doppler atomic transition signal, using a quadrature squeezed field generated by an optical parametric oscillator. The authors of Refs. [3,4] used semiconductor lasers that are characterized by the possibility to produce light with photon number fluctuations below the standard quantum limit. This property allows to achieve a sub-shot noise sensitivity by direct detection, without homodyne or heterodyne techniques. Although the improvement in the signal to noise ratio obtained with laser diodes

is not yet as good as the one obtained with OPOs, it must be noticed that the squeezing bandwidth for diode lasers is much larger than the one available with an OPO. The above mentioned advantages add to various well known features of diode lasers, such as compactness, wide tunability, reliability, that have lead to a widespread application of these lasers in the last years (see Ref. [5] and references therein). In particular, semiconductor lasers are now the most common sources for the investigation of trace species, and the near-infrared AlGaAs diode lasers are widely used for the detection of simple molecule overtone transitions, due also to their capability to be modulated up to a few GHz.

In this work, we have concentrated on the demonstration of the ultimate possibilities of the method, in conditions where the absorption signal is extremely weak and with experimental parameters employed in actual high sensitivity experiments. More specifically, we have used high modulation indices of the diode lasers and very narrow detection bandwidths in order to detect ultra-low effective absorption signals. Our results are then directly comparable to the state of the art in high sensitivity spectroscopy.

<sup>\*</sup> Corresponding author. E-mail: bramati@spectro.jussieu.fr.

Quantum noise reduction in lasers based on pump noise suppression was first predicted by Golubev and Sokolov [6] and by Yamamoto for semiconductor lasers [7]. It was first demonstrated [8] in 1987 by Yamamoto and co-workers. The present state of the art for noise reduction in the intensity of a collimated beam of a single mode semiconductor laser at the front laser facet is 2.3 dB below shot noise at room temperature [9] and 5.9 dB at low temperature [10]. It was shown that, in addition to noise suppression in the driving current, suppression of the very weak side modes, using line narrowing techniques such as feedback from external grating and injection-locking, was very useful [11,12]. In previous work [13], we have also observed that the total intensity noise results from a cancellation between very large anticorrelated fluctuations of the main mode and the other longitudinal modes [14] and for this reason the observed amplitude squeezing is not always "single mode" squeezing. This feature could lead to increased noise in some measurements, since in general, only the laser main mode interacts with the sample. However, for weak absorptions, the degradation of the anticorrelation is negligible in the signal and the background noise, that involves all the modes, is not modified: the result is a sub-shot noise background.

The experiment described here relies on semiconductor lasers with an amplitude noise more than 1 dB below the standard quantum level. Frequency modulated absorption spectroscopy is performed on Doppler broadened and sub-Doppler signals of the  $D_2$  transition of  $^{133}\text{Cs}$  at 852 nm with these sources, improving the sensitivity of the detection beyond the shot noise level. We have tested different laser configurations and modulation techniques and we have optimized our apparatus to achieve an effective minimum detectable signal down to a few parts in  $10^8$ , with a detection bandwidth of 3 Hz and a high modulation depth.

## 2. The frequency-modulation technique

The frequency-modulation (FM) spectroscopy was introduced by Bjorklund in 1980 [15] and has proved to be a very efficient method for ultrasensitive detection of atoms and molecules [16,17]. The extension to semiconductor lasers [18] is of particular interest because they can be modulated at RF frequencies by acting directly on the injection current. But most commercially available free running semiconductor lasers exhibit an amplitude noise higher than the shot noise at RF frequencies and the best FM spectroscopy experiments were performed with dye lasers, that usually have a lower noise. Despite the very high efficiency of the FM techniques, only a few experimentalists could indeed demonstrate a shot noise limited sensitivity in a linear absorption spectroscopy experiment: first, the groups of Bjorklund [19] and Hall [20], employed dye lasers with external electro-optic modulator and a double-beam configuration in order to cancel the excess

noise; later, Cooper and co-workers, used a current-modulated lead-salt laser in a two-tone FM spectroscopy experiment [21], with appropriate filtering [22] or with the second beam subtraction [23]. It must be remarked that in a double-beam configuration [19,22,23] only one half of the detected laser radiation interacts with the sample, hence the detection limit is 3 dB higher than the shot noise of the beam carrying the signal. Carlisle et al. [22] could achieve a minimum detectable absorption of  $5 \times 10^{-8}$ , with a 1 Hz bandwidth, optimizing the modulation parameters. This is, to our knowledge, the highest sensitivity obtained in such a kind of experiments. We demonstrate hereafter that this sensitivity can be further improved with amplitude squeezed laser diodes. A few other experimentalists [24,25] could achieve the same level of sensitivity, exploiting a stronger laser power than the previous ones, but at the expense of a background noise higher than the shot noise.

The classical theory of the signal obtained in a FM spectroscopy experiment has already been thoroughly investigated (for a recent overview of the single-tone detection for a wide range of parameters, see Ref. [17]). Let us first recall its main features.

The modulated electric field can be written as

$$E(t) = E_0 e^{i\omega_0 t} [1 + M \sin(\Omega t + \varphi)] e^{i\beta \sin \Omega t}, \quad (1)$$

where  $\beta$  is the modulation index,  $\Omega$  is the modulation frequency and  $M$  is the residual amplitude modulation coefficient. In our experiment  $M$  is negligible, and we will neglect the amplitude modulation in the following calculations.  $E(t)$  can be expanded in a series of Bessel functions  $J_n(\beta)$  of order  $n$ :

$$E(t) = E_0 e^{i\omega_0 t} \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{in\Omega t}. \quad (2)$$

Assuming a weak linear interaction with the sample, the transmission of each frequency component is

$$T_n \approx 1 - (A/2)(\delta_n + i\phi_n) \quad (3)$$

and  $AE_0^2 \delta_0$  is the dc absorption signal. For a Lorentzian absorption, we have

$$\delta = \Gamma^2 / [\Gamma^2 + (\omega - \omega_0)^2], \quad \phi = -\delta(\omega - \omega_0) / \Gamma, \quad (4)$$

where  $\Gamma$  is the width HWHM of the signal.

The signal that can be detected at frequency  $\Omega$  by a photodiode originates from the beat note between sidebands separated by  $\Omega$ . It can be written as

$$I_\Omega(t) = AE_0^2 e^{-\delta_0} \sum_{n=0}^{+\infty} J_n J_{n+1} \times [(\delta_{-n-1} - \delta_{n+1} + \delta_{-n} - \delta_n) \cos \Omega t - (\phi_{-n-1} - \phi_{-n} + \phi_{n+1} - \phi_n) \sin \Omega t]. \quad (5)$$

In a phase-sensitive detection, the phase and quadrature signals (i.e. absorption and dispersion) can be separated. If

the detection is performed by means of a spectrum analyzer, this is no more possible, and it is the sum of the powers in the two quadratures that is displayed.

For  $\beta \ll 1$ ,  $J_n(\beta)$  is negligible for  $n > 1$  and the signal amplitude is a linear function of  $\beta$ . For  $\Omega \leq \Gamma$ , which is the case of our experiment, the lineshape is similar to the derivative of an absorption lineshape. In a spectrum analyzer detection (working as a peak detection, without phase sensitivity) it is transformed in a characteristic 'M' shape [2].

In our experiment, we have detected saturated absorption signals with both pump and probe beams phase-modulated. The signal can be obtained by solving the Bloch equations, expanding the Rabi frequencies in Fourier series of  $\Omega$ . The calculation is given in Appendix A. In the limit of weak saturation and weak modulation the signal comprises two contributions. The first one is due to the probe beam modulation; it is the same signal that would be obtained by sweeping the probe laser on a linear Lamb dip-like absorption, but with the sidebands spaced by  $\Omega/2$  instead of  $\Omega$  (because both the probe and the pump beam frequencies are swept at the same time). The second part corresponds to the modulation transfer from the pump to the probe beam. We show in Appendix A that the resulting signal has a linear dependence on the modulation index  $\beta$  on a rather broad range that extends up to values of  $\beta$  of the order of 0.4, with a shape similar to the one obtained in the linear case described above.

Frequency modulation of the diode lasers was achieved either by means of an external electro-optic modulator (EOM) or by injection locking with a frequency modulated laser. At this stage, the question of noise added by the modulation device should be addressed. Let us stress that, from this view point, the externally modulated laser and the injected laser are not equivalent. In the first case, the electro-optic modulator acts like a beam splitter that couples in vacuum noise through an input port with an amplitude coupling coefficient  $\beta$ . If the intensity noise at the input of the electro-optic modulator is below the shot noise, this effect adds a shot noise contribution proportional to  $\beta^2$ , resulting in a degradation of the intensity squeezing. In our experiment, the squeezing was 1.4 dB at best and the maximum modulation index achieved with external modulation was 0.06. Using the above argument, the degradation of squeezing is predicted to be below our experimental accuracy, in agreement with the measurements.

In the second case, injection locking by a modulated master laser causes the slave laser to oscillate on two side modes in addition to the main mode. In as much as the quantum correlation between the side modes and the main mode is preserved, the quantum noise on the total intensity of the outgoing beam should be the same as with unmodulated injection locking. Measurements show that, within our experimental precision, the intensity squeezing is not degraded by this procedure for values of  $\beta$  up to 0.25. A

more detailed experimental comparison of the two methods would require a larger amount of squeezing.

### 3. Experimental apparatus

The lasers we used are quantum-well AlGaAs semiconductor lasers (SDL 5422-H1) operating at 850 nm. The free-running lasers exhibit a low threshold current (18 mA) and a high differential quantum efficiency (66%). We used home built temperature stabilizers and low noise power supplies. Appropriate LC filters and inputs for fast current modulation are set just before the lasers.

As described in Ref. [9], two configurations have been used to improve the intensity noise of the diode laser, namely feedback from an external grating in an extended-cavity laser and injection-locking with a master laser. For the external-grating configuration (Fig. 1a), a reflection holographic grating (Jobin-Yvon, 1200 lines/mm) is set at a distance of 10 cm from the laser, in the Littrow configuration. The grating is glued on a piezoelectric transducer, which allows a continuous laser frequency scan over about 10 GHz. By tilting the grating, the laser wavelength can be tuned by  $\pm 10$  nm around the free-running wavelength. Semiconductor lasers are easily frequency-modulated by modulating the injection current. However, it is difficult to apply this method to the extended-cavity laser because the grating damps the frequency modulation. In this case, the frequency modulation of the laser beam is achieved by means of an external electro-optic modulator (EOM) (Gsänger PM25), with Brewster windows, located at the output of the laser.

In the injection locking set-up, the master laser is either a cavity extended semiconductor laser (Fig. 1b) or a DBR

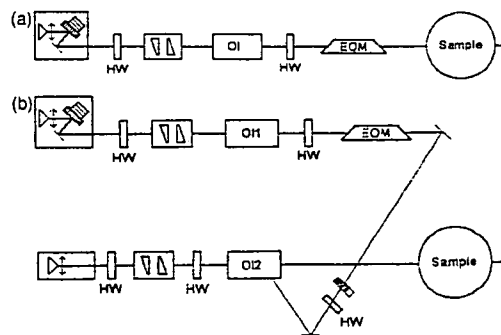


Fig. 1. (a) Scheme of the external grating laser modulated with an electro-optic modulator. HW – half-wave plate; OI – optical isolator; EOM – electro-optic modulator. A pair of anamorphosing prisms is placed after the first half-wave plate to correct the shape of the laser beam. (b) Scheme of an injection-locked semiconductor laser, the master laser being an extended-cavity diode laser. The symbols are the same as in (a).

semiconductor laser (SDL 5712), with a set-up similar to the one described in Fig. 1b. The master beam enters through the escape port of an optical isolator. The injected power (1.5 to 2 mW, measured at the entrance of the slave laser) is controlled either by a half-wave plate or by a variable grey filter. In order to achieve frequency locking, it is sufficient to have a slave laser longitudinal cavity mode close to the master laser frequency within a few GHz. In this way, a single-mode emission, phase-locked to the master laser, is obtained even starting from a multi-mode laser (a situation which often happens for this kind of lasers, particularly for high driving currents). In the laser that we have tested, the longitudinal modes are separated by 0.12 nm and we could observe the frequency locking together with a good squeezing up to a wavelength shift of about 3 nm between master laser and free-running slave laser. Therefore, the injection-locking setup, with an extended-cavity master laser, is a device completely tunable over several nanometers. Moreover, if the master laser is frequency-modulated, the slave laser follows the frequency modulation. The extended-cavity master laser was modulated by an external EOM. We also used a DBR laser as a master laser. It was frequency-modulated by simple modulation of the laser injection current.

In order to calibrate the shot noise level (SNL), we use a double balanced detection. The laser beam is precisely divided into two parts of equal intensity by means of a beam splitter formed by a half-wave plate and a polarizer. Each output of the beam splitter is sent to a high efficiency (90%) PIN-photodiode (Eg&g FND100). The ac parts of the signals from the two photodiodes are amplified and sent to a RF  $\pm$  power combiner. When set on the difference position, the circuit gives a signal proportional to the shot noise, while in the sum position, it gives the full amplitude modulation and noise of the beam impinging on the beam splitter. We performed various tests in order to check the reliability of the shot noise measured in this way, as described in Ref. [9]. The output of the  $\pm$  power combiner is sent to a spectrum analyzer and the spectra are recorded and stored in a computer for the sum and the difference signals. The shot noise level measured in this way has been checked to be the same in the presence and in the absence of modulation, as it should. The common mode rejection (with the power combiner in the difference position) is about 25 dB and the electronic and dark noise at 7.5 MHz (the frequency chosen for the modulation), measured by stopping the laser beam, is typically  $-15$  to  $-18$  dB below the laser SNL.

We could observe intensity squeezing in the laser emission with all the experimental configurations that we tested, in the frequency range from 0.5 MHz up to about 30 MHz (limited by the detection amplifiers bandwidth). The signal recordings were taken in "zero span" with the spectrum analyzer frequency fixed at the frequency of the modulation, that is 7.5 MHz, while the laser wavelength was swept slowly. The spectrum analyzer scan was triggered

by the ramp used to sweep the wavelength of the laser. Only single sweeps were taken, without trace averaging.

A 3.5 cm long vapor cell, with Brewster windows, contains the sample of  $^{133}\text{Cs}$ . In the first part of the experiment, the full laser beam goes through the cell, then is detected. The cold finger can be kept at liquid nitrogen temperature. However, in such a configuration, the modulated absorption at the  $\text{D}_2$  resonance is still significant, and in particular is too high to investigate the high sensitivity performance of the system. To circumvent this problem and demonstrate sub-shot noise sensitivity even in a medium that has a sizable absorption, several options are available. Some of them were explored by other authors, as having a low modulation index in the probe beam, or detecting the modulation transferred to the probe beam by a pump beam, the modulation index of which is adjusted to a low enough value. We have chosen a different solution, in order to work in a regime closer to the one of the spectroscopists and to use experimental parameters that are currently employed in high sensitivity and trace detection experiments. In particular, this implies significant modulation indices.

In addition, an ideal absorption measurement implies both weak photon and atom densities. This can be achieved in cesium vapor in various configurations. One of them consists in expanding the laser beam (to avoid saturation) and cooling the cesium cell further (to decrease the absorption) (Fig. 2a). Another one, that we have used, consists in having only a small part of the laser beam interact with the sample while detecting the full beam (Fig. 2b). In this case, the cesium cell itself does not have to be cooled down to very low temperatures, which would require an elaborate evacuated temperature controlled chamber. Let us emphasize that, for the purpose of a demonstration

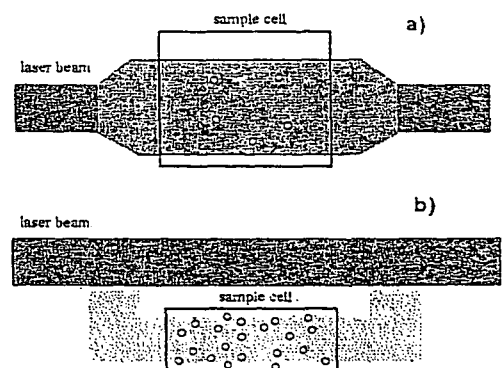


Fig. 2. Simplified experimental schemes for the demonstration of high sensitivity spectroscopy with low saturation and low absorption. In (a), the laser beam is expanded and goes through a cell with low density. In (b), the laser beam is split into two parts, one propagating freely, the other one going through a cell with larger atomic density.

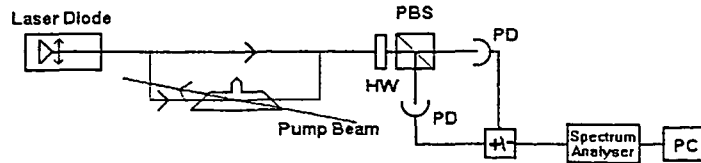


Fig. 3. Experimental apparatus for FM spectroscopy. PBS – polarizing beam splitter; HW – half-wave plate; PD – photodiode.

experiment, the second configuration is equivalent to the first one, as far as both the signal and the noise<sup>1</sup> are concerned. In this way we could simulate a very low absorption signal in cesium vapor.

The experimental scheme is sketched in Fig. 3. Part of the modulated and intensity squeezed beam going out of the laser system is split off by means of a  $R = 3\%$  beam splitter and passed through the cesium cell, while the main beam propagates freely to the detector where the two parts are incoherently recombined. To obtain a Doppler-free signal, a counterpropagating pump beam crosses the probe beam in the cell. The pump beam is itself rather weak (it is obtained from the reflection off the second side of the beam splitter used for the probe beam).

The diameter of the beam is about 2 mm, the pump power in the sample is of the order of 1 mW. The probe beam absorption (for the Doppler-broadened transition starting from the  $F = 4$  level) is about 80%. The sub-Doppler signal of the  $F = 4 \rightarrow 5$  transition exhibits a FWHM linewidth of 14 to 19 MHz (due to residual Doppler broadening) and its amplitude corresponds to a decrease in the absorption of the order of  $1/10$ .

#### 4. Measurements

##### 4.1. Demonstration of sub-shot noise sensitivity

As mentioned above, the first experiments we performed used a cell with a liquid nitrogen cooled finger and a very low modulation index. The absorption of the cooled sample cell placed directly on the beam is about 0.5% for the transition starting from the  $F = 3$  level (the transition is strongly saturated by the full laser power and the absorption is reduced by self-induced transparency). In this case there is no counterpropagating beam. In Fig. 4, we show an example of a Doppler-broadened signal recorded by the spectrum analyzer (Tektronix 2753P), with a resolution bandwidth (RBW) of 1 MHz. The modulation parameter is about  $\beta = 0.007$ . The asymmetry in the lineshape is

<sup>1</sup> Because it is a very much attenuated part of the full squeezed beam, the beam going through the cell is no more squeezed. However, this has no incidence on the total measured noise, which is the one of the full beam.

due to the overlap of several hyperfine structure transitions.

Due to the large phase noise of the semiconductor laser, a similar signal can be obtained also without modulating the laser [26]. An example is shown in Fig. 5, for the cell at room temperature and an absorption of about 30%. In this case, we have verified that the signal-to-noise ratio is independent of the RBW and of the spectrum analyzer frequency, since both amplitude and phase noise are white noises. The amplitude of the signal in Fig. 5 corresponds to the calculations, assuming a phase noise of 60 dB above the SNL [9]. These two experiments show the potentialities of the method to detect sub-shot noise signals.

To investigate the sensitivity of the method, we reduced the absorption signal further, by splitting off part of the laser beam, as explained above. We could subsequently increase the modulation index. In Fig. 6 we show a spectrum analyzer recording of the sub-Doppler signals of the  $F = 4 \rightarrow 5$  transition and of the  $F = 4 \rightarrow 4, 5$  cross-over transition, with a RBW of 1 MHz. The modulation index is

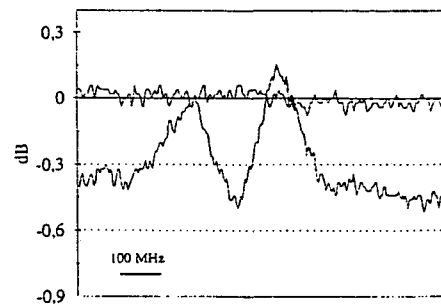


Fig. 4. Recording of the Doppler-broadened signal of the  $D_2$  line of Cs at 852 nm, for the transition starting from the  $F = 3$  hyperfine level as the laser wavelength is scanned. The signal is obtained by means of FM spectroscopy with an extended cavity laser, modulated by an EOM with modulation index  $\beta = 0.007$ . The cooled sample cell is placed before the photodetectors; the absorption is about 0.5%. The trace reported represents the spectral density of the photodiodes current, detected by means of a spectrum analyzer whose frequency is fixed at the modulation frequency, i.e. 7.5 MHz. The device is triggered by the laser wavelength scan. The trace at 0 dB is the corresponding laser shot noise; the 0 dB level is the average of the shot noise signal. The resolution bandwidth is 1 MHz, the video bandwidth is 3 kHz.



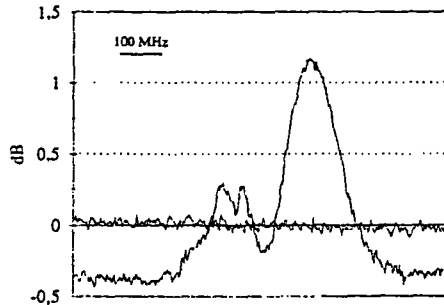


Fig. 5. The same as in Fig. 4, but with the frequency modulation switched off. The absorption from the room-temperature cell is  $\sim 30\%$ . The signal is due to the phase noise of the semiconductor laser, transformed into amplitude noise by the interaction with the sample.

$\beta = 0.04$ . The laser noise is at  $-0.6$  dB below the SNL. The EOM slightly increases the laser noise, which would be otherwise at about  $-0.8$  dB. This is probably due a slight coupling with the large laser phase noise, originating from the birefringence of the EOM. We can notice that the signal is almost completely below the SNL. The total laser power at the detectors is 29 mW, while the probe beam power is about  $8 \mu\text{W}$ . The  $F = 4 \rightarrow 5$  sub-Doppler transition signal corresponds to decrease in absorption of 13% of the probe beam (measured with a photodiode detecting only the probe beam). Hence, the signal corresponds to  $3.5 \times 10^{-5}$  of the laser power.

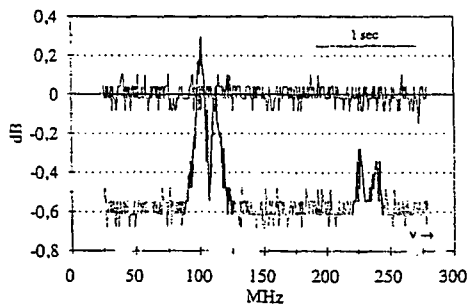


Fig. 6. Spectrum analyzer recording of the sub-Doppler signals of the  $F = 4 \rightarrow 5$  (at higher frequency) and  $F = 4 \rightarrow 4, 5$  cross-over transitions, obtained by FM spectroscopy with an extended cavity laser modulated with an EOM. The experimental configuration is shown in Fig. 3. The laser injection current is 87 mA; the total photodiodes current is 17.6 mA, for 29 mW of laser radiation at the detectors, while the probe beam power is about  $6 \mu\text{W}$ . The modulation index is  $\beta = 0.04$ . The parts of the traces displayed with a thinner line have been used to calculate the variance. The one-second marker indicates the sweep time. The spectrum analyzer parameters are the following: RBW = 1 MHz; VBW = 3 kHz; sample-rate = 100 sample/s. The 0 dB level (shot noise) corresponds to  $-62$  dBm.

The residual amplitude modulation is negligible. It is reduced to less than  $-20$  dB below the SNL (in a bandwidth of 1 MHz) by optimizing the laser polarization before the EOM and the EOM alignment. In anticipation of larger modulation indices, we have further improved it by superimposing an attenuated amplitude- and phase-controlled part of the modulation to the laser driving current. This allows to minimize the amplitude modulation down to  $-30$  dB below the SNL of the laser.

#### 4.2. Improvement of the "classical" sensitivity

The fact that the background noise is below shot noise is a non classical feature which is independent of the resolution bandwidth and of the detection bandwidth. The signal shown in Fig. 6 is a clear demonstration of how an amplitude squeezed emission can increase the sensitivity in FM spectroscopy with a large detection bandwidth. This is mainly useful for fast continuous monitoring of the sample. However, the signal to noise ratio can be improved further by reducing the bandwidth.

This could be achieved by reducing the resolution bandwidth of the spectrum analyzer, but it requires very stable lasers and modulators. In standard measurements, that do not use a spectrum analyzer, this is usually done by means of a filter placed after the phase-sensitive detection and an average over several scans. In our experiment, this is equivalent to decreasing the bandwidth of the video filter (VBW) and increasing the trace averaging of the spectrum analyzer. A careful analysis of the signal processing done by the spectrum analyzer shows that the variance (mean square deviation of the fluctuations measured at the output of the spectrum analyzer) of the signal and of the background are proportional (with a factor of  $\sim 0.5$ ) to the one of the same signal recorded with a usual lock-in technique (Appendix B). We have used this variance to evaluate the ultimate sensitivity of the set-up.

In the situation of the signal in Fig. 6, the VBW is 3 kHz, the spectrum analyzer works in the "average mode" acquisition, the sample-rate is 100 sample/s and the effective final detection bandwidth is 65 Hz. We have measured the variance of the signal (with the power combiner in the sum position), in the wavelength regions where the signal lineshape is flat, i.e. out of resonance, (the part of the signal used for this measurement has been drawn with a thinner line in Fig. 6 and in the following figures). The ratio between the variance of the signal and the one of the shot noise is  $0.91 \pm 0.15$ . We can conclude that the residual amplitude noise is still negligible, at this level of modulation index, even within a 65 Hz bandwidth. The sensitivity deduced for this experimental configuration (i.e. the minimum detectable signal, assuming a unity signal-to-noise ratio) is  $0.84 \times 10^{-6}$ .

For higher modulation indices, the residual amplitude noise at the frequency of the modulation becomes signifi-

cant, reducing the sensitivity. This additional noise is not the laser low-frequency amplitude noise, shifted towards the detection frequency by the residual amplitude modulation, as in Refs. [24,27,28] (in fact, the amplitude modulation index is of the order of  $10^{-7}$ ). Its origin is related to the EOM. A small angle between the polarization of the incoming beam and the axis of the EOM gives rise to a modulation of the polarization, and thus, due to the polarization-dependent transmission of the EOM (Brewster windows and crystal), it leads to an amplitude modulation. This amplitude modulation fluctuates with the alignment and the refraction index of the crystal (for example, for thermal noise and vibrations), resulting in a residual amplitude noise [20]. We have then decided to turn to an injection locked laser.

In the injection locking conditions, the amplitude fluctuations of the master laser are significantly attenuated in the slave laser emission, while keeping the full phase modulation. We have verified that a master amplitude modulation (obtained by modulating the laser current) is attenuated by about 40 dB (see also Ref. [29]). However, it must be considered that the master beam is reflected by the output polarizer of an optical isolator (OI 2 in Fig. 3), thus a polarization modulation produced by the EOM is completely converted into amplitude modulation. In order to minimize this effect, we have used a half-wave plate to maximize the master beam transmission through the optical isolator, and we have adjusted the injected power by means of a variable grey filter.

An example of the recorded signal is shown in Fig. 7. The laser noise is 1 dB below the SNL, and the sub-Doppler transition signal is again below the SNL. The modulation index is  $\beta = 0.06$  and the signal corresponds to  $2.8 \times 10^{-5}$  of the total laser power. The recording has been taken with the same spectrum analyzer parameters as

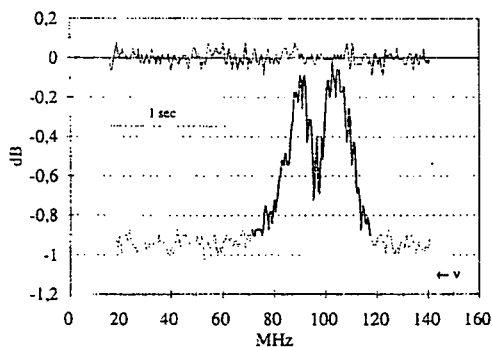


Fig. 7. Recording of the signal corresponding to the  $F = 4 \rightarrow 4, 5$  cross-over transition, with a two-point linear smoothing, obtained with the injection locked laser. The total detectors current is 33.3 mA, corresponding to 56 mW; the slave laser current is 100 mA. The modulation index is  $\beta = 0.06$ ; the spectrum analyzer parameters are the same as in Fig. 6.

in Fig. 6b (i.e. RBW, 1 MHz; VBW, 3 kHz; sample-rate 100 sample/s). The signal in Fig. 7 corresponds to an average over a couple of subsequent samples of the same laser frequency scan. We have verified that this smoothing procedure allows to reduce the effective bandwidth down to 45 Hz, without decreasing the signal amplitude, even if the central dip is smoothed. As for the previous configuration, the variance of the signal out of resonance is slightly lower than the one of the shot noise: their ratio is  $0.83 \pm 0.16$ , indicating again the absence of residual modulation. The corresponding maximum sensitivity is  $2.1 \times 10^{-7}$ .

With the same experimental configuration, we have further increased the modulation index up to  $\beta = 0.1$ , at the limit of the possibilities of our EOM. In order to reduce the detection bandwidth, we have used a different spectrum analyzer (Hewlett-Packard HP 8560 E), which allows a better VBW choice and can display data in the "sample mode". Besides, the higher vertical resolution of the HP spectrum analyzer (1/600 of the full scale) allows to appreciate the variance of the signal even with a narrower VBW. We have reduced the VBW down to 3 Hz, with a RBW of 100 kHz and a sample-rate of 20 sample/s (sweep time: 30 s). With these values, we have reached an effective maximum sensitivity of  $5.6 \times 10^{-8}$ , much improved with respect to the results obtained with the simple external grating laser.

However, the modulation index is at the limit of our EOM and the adjustment is rather critical, if we want to avoid fluctuations of the residual amplitude modulation, originating from fluctuations of the polarization. In order to achieve better results, we have replaced the external-grating laser with a free-running DBR laser as a master laser. With this setup the EOM is no more necessary, since the frequency modulation is obtained by modulating the laser current. Of course, this also produces a strong master laser amplitude modulation, which is however rather stable. The frequency modulation efficiency of the DBR laser around 7 MHz is about 100 MHz/mA. We can deduce that, for our experimental conditions, the ratio between amplitude and phase modulation index is  $M/\beta \cong 2 \times 10^{-3}$ . In the slave laser, this ratio is reduced by about two orders of magnitude. For a detection current of 30 mA, the residual amplitude modulation is still at the level of the shot noise in a bandwidth of 1 MHz, for  $\beta = 0.2$ . We have further reduced it by slightly modulating the slave laser current, as already described.

We have tested modulation indices up to  $\beta = 0.2$ , keeping the residual amplitude modulation at  $-25$  dB below the SNL (with a RBW of 1 MHz). An example of the recorded signal is displayed in Fig. 8 with a laser power of 50 mW. With this configuration, we have 0.9 dB of squeezing. The signal reported in Fig. 8, for the  $F = 4 \rightarrow 5$  transition, corresponds to an equivalent fractional saturated absorption of  $1.6 \times 10^{-3}$ . The dip in the center of the lineshape does not reach the level of the background noise, because of the large linewidth of the master laser (a

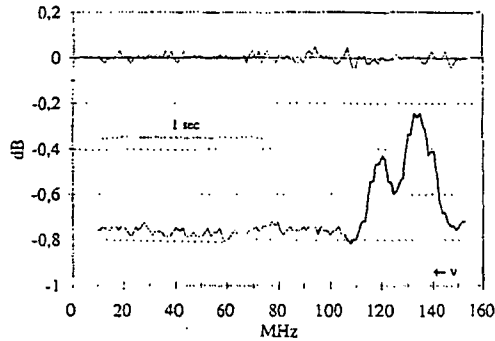


Fig. 8. The signal displayed corresponds to the  $F = 4 \rightarrow 4, 5$  cross-over transition, obtained with the injection locked laser. The total detectors current is 30.2 mA, for a laser power of 50 mW; the laser current is 94 mA. The modulation index is  $\beta = 0.2$ , the spectrum analyzer parameters are: RBW = 1 MHz; VBW = 10 Hz; sample rate = 60 sample/s.

few MHz), compared to the signal linewidth (here the FWHM is 16 MHz for the  $F = 4 \rightarrow 5$  transition, and 23 MHz for the  $F = 4 \rightarrow 4, 5$  cross-over signal). In the external grating configuration, the laser linewidth was about a few hundred kHz. The recording in Fig. 8 was taken by means of the HP spectrum analyzer, with a VBW of 10 Hz and a sample rate of 60 sample/s. As in the previous measurements, the variance of the signal is slightly lower than the one of the shot noise (this ratio is  $0.88 \pm 0.27$ ). The sensitivity that can be deduced from the spectrum in Fig. 9 is  $5.2 \times 10^{-8}$ .

Increasing the modulation depth of the master laser further causes an instability of the slave laser emission. Nevertheless, we were able to use a modulation index up to  $\beta = 0.25$ ; an example is shown in Fig. 9. Here the recording was taken immediately after adjusting the can-

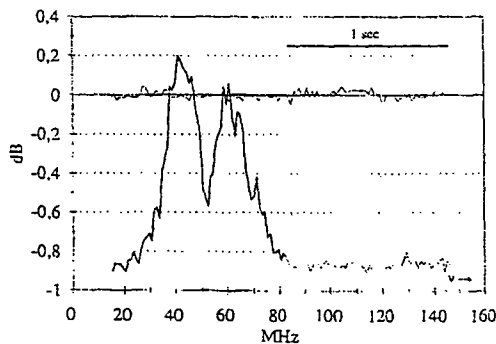


Fig. 9. The  $F = 4 \rightarrow 5$  transition signal has been recorded with the same experimental conditions used for the acquisition in Fig. 8, but with the modulation index increased up to  $\beta = 0.25$ .

cellation of the residual amplitude modulation: the signal corresponds to  $7 \times 10^{-5}$  of the total laser power, the VBW is still 10 Hz and the variance is slightly lower, by a factor  $0.91 \pm 0.25$ , for the signal than for the shot noise. However, the slave laser amplitude stability rapidly deteriorates: the residual amplitude modulation increases and fluctuates, up to about  $-10$  dB below the SNL in a 1 MHz bandwidth. The inferred sensitivity is now  $3.9 \times 10^{-8}$ , a figure which is at the level of the best ones recorded to date.

### 5. Conclusion and perspectives

In this work, intended to investigate the possibilities of spectrometers based on squeezed laser diodes, we have demonstrated a potential sensitivity comparable to the one of the most refined FM experiments and shown that the improvement of the signal to noise ratio due to sub-shot noise operation of the laser diodes can be transferred to high resolution spectroscopy. We have shown that the squeezing is conserved and can really improve the final sensitivity. This will be even more relevant with the use of the strongly squeezed semiconductor lasers recently demonstrated at low temperature [10], if they become widely available.

Our set-up was primarily designed for the study of intensity squeezing in the laser emission; in particular, the use of the spectrum analyzer and of the balanced detection allows a precise measurement of the noise and an accurate calibration of the shot noise. Nevertheless, with a detailed study of the variance of the signal given by the spectrum analyzer, we were able to determine the ultimate sensitivity achievable in frequency-modulation spectroscopy using a range of parameters typical of high sensitivity measurements, i.e. high modulation indices and narrow detection bandwidths. We have achieved a minimum detectable signal of a few parts in  $10^{-8}$  in a 10 Hz bandwidth.

As far as the modulation depth is concerned, we have reported in Fig. 10, together with the plot of the signal amplitude as a function of the modulation index  $\beta$ , the maximum values of  $\beta$  that we have tested for the three different laser configurations. The highest modulation index, obtained with the injection by the DBR laser, leads to a signal amplitude which is only 3.5 times smaller than the maximum theoretically obtainable with the FM technique. Indeed, as can be seen in Fig. 10, the amplitude of the signal saturates for  $\beta \approx 1.9$ .

The detection bandwidth was reduced down to 3 Hz for the injection with the external grating laser, and 10 Hz for the injection using the DBR laser. In both configurations, we were limited by the master laser wavelength stability (we have in fact detected narrow signals) and by slow amplitude fluctuations. Of course, the rather short frequency scans that we have used to detect narrow sub-Doppler transitions have helped to reduce the problems con-

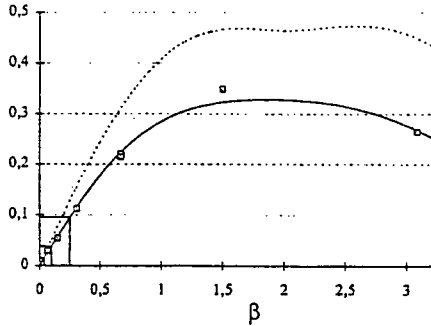


Fig. 10. Theoretically calculated signal amplitude for FM spectroscopy and detection by means of a spectrum analyzer, for a Lorentzian lineshape with  $\Omega = \Gamma$ , as a function of the modulation index  $\beta$ . The solid line corresponds to a saturated absorption with both probe and pump beam phase modulated; the dashed line is for a linear absorption of a laser sweeping on a signal with the same lineshape. The squares are experimental values. We have reported in the plot the limit values of  $\beta$  tested with the different experimental configurations, keeping an amplitude squeezed laser emission (see text).

cerning possible etalon fringes, that have often limited the sensitivity of previous FM spectroscopy works (see, e.g., Refs. [19–23] and references therein).

In order to detect signals characterized by a broader linewidth, like molecular absorptions, a two-tone FM technique [21,27,28] would be more suitable than the single-tone, exploited in this work. In this way, the detection frequency can be kept in a region (a few MHz) where the availability of high efficiency detectors allows to maintain and take advantage of the amplitude squeezing. In any case, the use of a slave laser injected by a master free-running semiconductor laser with current modulation seems to be the best choice.

Both modulation index and detection bandwidth can be easily further optimized by exploiting detection techniques more suitable to the high sensitivity spectroscopy, for example an RF phase-sensitive detection, the use of a mechanical chopper together with a second, low frequency, phase-sensitive detection and an average over several fast laser wavelength scans.

#### Acknowledgements

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#### Appendix A. Signal in FM spectroscopy

In FM saturated absorption spectroscopy, when both pump and probe beams are phase-modulated, the signal can be obtained by solving the Bloch equations, expanding the Rabi frequencies in Fourier series of  $\Omega$ . In the limit of weak saturation, the result is

$$I_{\Omega}(t) = AE_0^2 \sum_{k,m,n} J_{m+n-k}(\beta_2) J_{n-k}(\beta_2) J_{m+k}(\beta_1) \times \frac{1}{1 + (m\Omega T_1)^2} \times [(J_{k+1}(\beta_1) + J_{k-1}(\beta_1))(\delta_n - m\Omega T_1 \phi_n) \cos \Omega t + (J_{k+1}(\beta_1) - J_{k-1}(\beta_1))(m\Omega T_1 \delta_n + \phi_n) \sin \Omega t], \quad (A1)$$

where  $T_1$  is the longitudinal relaxation time (i.e. the lifetime of the upper level),  $\beta_1$  and  $\beta_2$  are respectively the modulation indices of the probe and pump beams, and the sum must be extended from  $-\infty$  to  $+\infty$ . Here we have

$$\delta_m = \delta(\omega - \Omega m/2), \quad \phi_m = \phi(\omega - \Omega m/2), \quad (A2)$$

and  $AE_0^2 \delta_0$  is the Lamb dip, as detected with dc coupling, without laser modulation. For  $\beta_1, \beta_2 \ll 1$  the signal reduces to

$$I_{\Omega}(t) \approx AE_0^2 \frac{\beta_1}{2} [(\delta_1 - \delta_{-1}) \cos \Omega t + (2\phi_0 - \phi_{-1} - \phi_1) \sin \Omega t] + AE_0^2 \frac{\beta_2}{2} (1 + (\Omega T_1)^2)^{-1/2} \times [(-\delta_{-2} + \delta_{-1} - \delta_1 + \delta_2) \cos(\Omega t - \alpha) + (-\phi_{-2} + \phi_{-1} + \phi_1 - \phi_2) \sin(\Omega t - \alpha)], \quad (A3)$$

with  $\alpha = \tan^{-1}(\Omega T_1)$ .

The first part is due to the probe beam modulation; it is the same signal that would be obtained by sweeping the probe laser on a linear "Lamb dip-like" absorption, but with the sidebands spaced by  $\Omega/2$  instead of  $\Omega$  (because the pump and the probe are swept in frequency at the same time). The second part corresponds to the modulation transfer signal, with four resonances at  $\pm \Omega/2$  and  $\pm \Omega$ . The observed signals are the superposition of all these resonances which have a large overlap since  $\Omega < \Gamma$ .

It is interesting to remark that in the case where  $\beta_1 = \beta_2 = \beta$ , and for  $\Omega T_1 \rightarrow 0$ , the resonances at  $\pm \Omega/2$  of the two parts cancel, leading to a signal which is the same as in linear FM spectroscopy (Eq. (5)), with the sidebands at  $\pm \Omega$ .

Our experimental parameters are the following:  $\Omega/2\pi$  is fixed at 7.5 MHz, while  $\Gamma/2\pi$  can vary between 7 and 9.5 MHz depending on pressure broadening and residual Doppler broadening (pump and probe beam are not perfectly aligned); the upper level decay rate gives  $1/2\pi T_1 = 2.6$  MHz. In Fig. 10 we have reported the signal amplitude calculated from Eq. (A1), for the detection with a spectrum analyzer, with  $\Gamma/2\pi = 7.5$  MHz and  $AE_0^2 = 1$ . The signal is given mostly by the terms proportional to  $(J_{k+1} + J_{k-1})\delta_n$  in Eq. (A1), while the other terms are smaller by at least a factor of 5. We have also drawn in Fig. 10 the signal amplitude that would be obtained in a linear FM experiment (i.e. calculated with Eq. (5) of the text), with an absorption similar to our Lamb dip and the same laser modulation. The squares in Fig. 10 are experimental data for the signal amplitude. Let us note that the values of the modulation index can be determined directly from the signal fed into the modulation system. However, this determination is not very accurate. In our experiments, we have used the theoretical calculation to infer the values of the modulation index  $\beta$  associated to the various recordings, starting from the observed signals (for this we have used the  $F = 4 \rightarrow 5$  sub-Doppler transition signal, which is less distorted by the Doppler background and the nearby transitions).

#### Appendix B. Noise measurement with a spectrum analyzer

In this appendix, we examine the way the spectrum analyzer processes the noise and we derive the formulae relating the average signal given by a spectrum analyzer and its variance to the output noise when the input is noise. The spectrum analyzer (SA) performs a super-heterodyne detection of the input signal using several mixer stages. The final mixer is followed by a band-pass filter (BPF) centered at the mixer output frequency or "intermediate frequency" (IF), then by a rectifier and an integrator ("video filter").

The cut-off frequencies of a filter are usually defined as the  $-3$  dB or the  $-6$  dB frequencies. For the noise measurements, it is more useful to use an equivalent width, defined as the bandwidth of an ideal filter (with unity transmission within the bandwidth and zero transmission outside) transmitting the same noise power, with a white noise input. This bandwidth corresponds to the integral of the spectral transmission curve. The SA band-pass filter (BPF) is usually rather flat, with steep upward and downward edges, and has an equivalent bandwidth that can slightly differ from the nominal resolution bandwidth (RBW). In the following, we will not distinguish between them, but a measured equivalent bandwidth should be considered for accurate results.

If the input noise is approximately (within the RBW) white and gaussian, it has a gaussian distribution with zero

average at the output of the BPF. Its variance  $\sigma_G^2$  (defined as usual as  $\sigma^2 = \langle (x - \langle x \rangle)^2 \rangle$  where  $x$  is the signal and  $\langle \rangle$  can be interpreted as temporal average or sample average) is equal to the total noise power, integrated over the spectrum. In practice, the integration is performed over the RBW and  $\sigma_G^2$  is proportional to  $\nu_{RBW}$ , the resolution bandwidth.

The rectifier can be sketched as a diode followed by a parallel RC. If its bandwidth  $1/RC$  (which is a fraction of the IF) is larger than the RBW, it can be shown that the rectifier transforms the gaussian distribution of the noise into a Rayleigh distribution:

$$R(x) = (1/\sigma_G^2) \exp(-x^2/2\sigma_G^2) \text{ for } x > 0;$$

$$R(x) = 0 \text{ for } x < 0.$$

This distribution has an average

$$\langle x \rangle = \sqrt{\pi/2} \sigma_G \quad (B1)$$

and a variance

$$\sigma_R^2 = \langle x \rangle^2 (4/\pi - 1) = (2 - \pi/2) \sigma_G^2. \quad (B2)$$

The output of the rectifier is integrated by the video filter. If the video filter bandwidth (VBW) is much narrower than the RBW, the output has a gaussian distribution. The average is conserved and the variance is reduced according to the ratio between the RBW and the equivalent bandwidth of the video filter. This final filter is usually a single-pole low-pass filter, and the nominal video bandwidth  $\nu_{VBW}$  is defined at  $-3$  dB. Thus the noise power transmission factor is  $(1 + (\nu/\nu_{VBW})^2)^{-1}$  and the equivalent video bandwidth is  $(\pi/2)\nu_{VBW}$ . The variance of the output signal is then

$$\begin{aligned} \sigma_{OUT}^2 &= \langle x \rangle^2 (2 - \pi/2) \nu_{VBW} / \nu_{RBW} \\ &= (2 - \pi/2) (\pi/2) \sigma_G^2 \nu_{VBW} / \nu_{RBW}. \end{aligned} \quad (B3)$$

The signal can be further integrated, depending on the acquisition mode of the SA. While the "sample mode" is characterized by a sequence of instantaneous measurements, and does not change the statistics of the signal, the "average mode" performs an integration of the waveform between two consecutive data points. Thus, the effective final bandwidth can be much reduced, depending on the sweep time. In particular, the effective bandwidth is proportional to the sample rate if the latter is lower than the VBW. The Tektronix 2753 P spectrum analyzer is characterized by the "average mode" acquisition. We have verified, with an electronic white noise as input, that the variance of the output waveform is proportional to the sample rate when it is lower than the VBW, and converges to the calculated value of  $\sigma_{OUT}^2$  for high sample rates. We have used this variance to measure the effective video bandwidth, normalized to the nominal VBW for high sample rate. For the Hewlett-Packard HP 8560 E spectrum analyzer, working in "sample mode", we have verified

that the variance of the output waveform is in agreement with the calculations. Of course, this variance can be measured only with a sufficient vertical resolution (it must be at least  $\sqrt{\nu_{\text{VBW}}/\nu_{\text{RBW}}}$ ).

A final remark concerning noise measurements with a SA is that the SA is actually a peak voltmeter, but it is corrected to give output rms values. Thus, the displayed signal is decreased by a factor of  $\sqrt{2}$ . In order to obtain the real output, the data must be multiplied by  $\sqrt{2}$ .

Finally, taking the  $\sqrt{2}$  factor into account, Eq. (B1) shows that if a voltage  $V$  is read at the output of the SA, the noise power at the output of the band-pass filter of bandwidth  $\nu_{\text{RBW}}$  is given by

$$\sigma_G^2 = (\sqrt{2} \langle V \rangle)^2 / \pi. \quad (\text{B4})$$

Eqs. (B3) and (B4) permit to relate  $\sigma_V^2$ , the variance of  $V$  to the average value  $\langle V \rangle$  and to the noise power  $\sigma_G^2$

$$2\sigma_V^2 = \sigma_{\text{OUT}}^2 = (\sqrt{2} \langle V \rangle)^2 (2 - \pi/2) \nu_{\text{VBW}} / \nu_{\text{RBW}} \\ = [\sigma_G^2 \nu_{\text{VBW}} / \nu_{\text{RBW}}] (\pi/2) (2 - \pi/2), \quad (\text{B5})$$

where an effective detection bandwidth  $\nu_{\text{VBW}}$  has been introduced to account for the case of "average mode" acquisition. It can be seen that  $\sigma_{\text{OUT}}^2$  is proportional to  $\sigma_G^2$ , the noise power at the output in a bandwidth equal to  $\nu_{\text{VBW}}$ .

$$\sigma_G'^2 = \sigma_G^2 \nu_{\text{VBW}} / \nu_{\text{RBW}}.$$

Let us point out that the variance at the output of a phase sensitive lock-in detection having a detection bandwidth  $\nu_{\text{VBW}}$  would be  $\sigma_G'^2 (\pi/2)$ .

The measurement of the mean level of the output waveform allows a precise determination of the input noise power integrated over the BPF. If there is also a modulated signal at the input, the ratio between the output signal amplitude and the output background level corresponds to the signal-to-noise ratio (S/N) obtained with a detection bandwidth equal to RBW.

In real high sensitivity experiments, the detection bandwidth is usually reduced, in order to increase the signal-to-noise ratio. The same result is obtained at the output of the SA by means of the video filter. However, this procedure does not change the level of the background and the latter cannot be used any more to evaluate the signal to noise ratio. As a measure of the output noise, we have taken the square root of the variance  $\sigma_{\text{OUT}}^2$  of the output waveform, according to Eq. (B5), which corresponds to our specific detection apparatus. This is the noise that is used to calculate the signal to noise ratio and the maximum sensitivity (in terms of fractional absorption) from the recordings shown in Figs. 6–9. The maximum sensitivity is evaluated from the smallest signal that can be detected assuming a signal to noise ratio of 1.

The output noise is sensitive to fluctuations of the residual amplitude modulation, which introduce additional noise around the modulation frequency. This additional

noise often prevents from achieving a sensitivity limited only by the unmodulated laser noise. In particular, for a frequency modulated and amplitude squeezed laser, it is of critical importance to check that the modulation does not degrade the noise characteristics around the modulation frequency.

In the case of the detection by SA, this additional amplitude noise can be hidden by the original laser intensity noise in the BPF, but it becomes significant in the output noise (i.e. after the video filter). In order to achieve a large sensitivity, the additional noise must be negligible with respect to the intensity noise not only within a large detection bandwidth, but also in a narrow bandwidth around the modulation frequency. This condition is the most stringent for narrow detection bandwidths. If it is not fulfilled, the intensity squeezing of the laser is destroyed in a narrow bandwidth around the modulation frequency, resulting in a degradation of the sensitivity.

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## 7 Conclusion

Dans ce travail de thèse nous avons effectué une étude approfondie du bruit quantique dans les lasers à semiconducteur et à solide et des techniques expérimentales qui permettent de le réduire. L'idée physique commune à toutes nos expérimentations est l'application aux différents types de lasers du principe de la pompe régulière.

Dans les cas des diodes laser à température ambiante, l'utilisation des techniques traditionnelles d'affinement spectral nous a permis d'obtenir une réduction du bruit d'intensité de 1,6 *dB* ( $\sim 31\%$ ) et de 2,3 *dB* ( $\sim 40\%$ ) sous le bruit quantique standard pour la diode sur réseau et pour la diode injectée respectivement. L'analyse approfondie du contenu spectral du rayonnement laser nous a conduit à la compréhension du lien étroit qui existe entre comportement modal et bruit d'intensité dans une diode laser. Le résultat marquant de cette recherche est la démonstration expérimentale que le bruit d'intensité d'une diode laser est souvent le résultat de la forte anticorrelation entre les fluctuations du mode principal et celles des faibles et nombreux modes longitudinaux. Cela conduit naturellement à la distinction entre la notion de réduction de bruit en régime monomode (*single-mode squeezing*), valable pour la diode sur réseau, et celle en régime multimode, (*multimode squeezing*), applicable à la diode injectée. L'importance de cette distinction pour des applications de spectroscopie où de pompage optique est évidente.

Après avoir obtenu un comportement monomode du laser, nous avons effectué une comparaison détaillée des résultats expérimentaux avec les prévisions théoriques de différents modèles quantiques du laser : l'écart constaté pourrait être expliqué en supposant la présence d'un excès de bruit sur le courant de pompe. Cependant, cela est en contradiction avec le principe de la pompe régulière mis en oeuvre dans les expériences et met en évidence la nécessité d'une théorie plus élaborée pour rendre compte des propriétés de bruit dans les lasers à semiconducteur.

Pour améliorer les performances obtenues à température ambiante, l'approche typiquement utilisée consiste à opérer en régime cryogénique, pour profiter de la diminution



du seuil d'oscillation du laser et de l'augmentation de son efficacité quantique. Les expérimentations que nous avons menées en régime cryogénique se sont heurtées à une difficulté inattendue sur la base des résultats obtenus par d'autres groupes : le comportement fortement multimode des diodes lasers lorsqu'elles sont refroidies. La meilleure réduction de bruit, observée pour la diode injectée, est de 37% et résulte de l'anticorrélation entre modes de polarisation orthogonale.

Dans le but d'atteindre une meilleure compréhension des caractéristiques de bruit des diodes lasers que nous avons utilisées, deux points restent à éclaircir : d'une part l'excès de bruit constaté à basse fréquence ( $< 300 \text{ kHz}$ ) et d'autre part les moindres performances fournies par les lasers fonctionnant à  $810 \text{ nm}$  bien qu'ils présentent les mêmes caractéristiques essentielles à la réduction de bruit (faible seuil d'oscillation, efficacité quantique élevée, comportement monomode). Le premier a été attribué très récemment à des phénomènes d'asymétrie spectrale dans la corrélation entre le mode principal et les modes longitudinaux [68] ; une autre interprétation possible met en cause la dépendance en fréquence de l'excès de bruit dû à la présence de modes de cavité non orthogonaux [69]. Le deuxième, à présent inexplicable, pourrait être compris dans le cadre d'une théorie plus complète dont on a mentionné plus haut le besoin.

L'expérience acquise dans l'étude du bruit quantique des diodes lasers à ruban (*edge emitting semiconductor lasers*) nous a été très utile lorsque nous avons décidé d'étudier les propriétés des lasers semiconducteur à microcavité (*vertical cavity surface emitting lasers*). Les résultats expérimentaux témoignent d'une réduction de bruit de 15% sous le bruit quantique standard obtenue en régime multimode grâce à la forte anticorrélation entre modes transverses de polarisation orthogonale. Nous avons étudié en détail les corrélations entre les modes transverses à travers l'analyse de la distribution spatiale transverse du bruit d'intensité : les résultats expérimentaux sont en excellent accord avec les prévisions d'un simple modèle phénoménologique. Les techniques d'affinement spectral précédemment utilisées avec succès dans les diodes à ruban devraient conduire, aussi dans les cas des VCSELs, à une amélioration des performances des ces lasers. L'injection optique et le réseau devraient en effet assurer la stabilisation de la polarisation du rayonnement émis, permettre d'atteindre un fonctionnement monomode et en conséquence une meilleure réduction de bruit.

La continuation naturelle de ce travail d'analyse des caractéristiques du bruit quantique dans les lasers à semiconducteur dans un régime de pompage à faible bruit, nous a amenés à nous intéresser aux microlasers à solide pompés par diodes. La motivation qui a déterminé cette étude est double : d'une part le grand intérêt de la communauté scientifique envers ces lasers rendait nécessaire une caractérisation précise de

leur propriétés de bruit et, d'autre part, le fait qu'ils possèdent une dynamique similaire à celle des lasers à semiconducteur laissait présager la possibilité de leur appliquer avec succès le principe de la suppression du bruit de pompe. De plus, la tâche était facilitée du fait que nous disposions déjà des faisceaux comprimés pour le pompage optique du microlaser, produits par les diodes lasers. La difficulté principale est liée à la présence de l'oscillation de relaxation qui affecte le spectre de bruit à basse fréquence (quelques megahertz). L'étude effectuée s'est déroulée en trois étapes distinctes. Nous avons d'abord analysé les effets du bruit de pompe sur le laser libre ; les résultats expérimentaux sont comparés avec les prévisions d'un modèle quantique du laser dans lequel les paramètres décrivant notre laser sont inclus. L'accord entre théorie et expérience est très satisfaisant sauf à très basse fréquence où le microlaser présente un bruit en excès par rapport aux prévisions.

La réalisation d'une boucle de rétroaction électro-optique expressement étudiée pour réduire le bruit autour de l'oscillation de relaxation sans affecter les parties à basse fréquence du spectre, a néanmoins conduit à la diminution du bruit dans la région des basses fréquences, témoignant de l'existence d'effets non linéaires dus à l'excès de bruit très élevé de l'oscillation de relaxation. L'élimination de ces effets a conduit à un bon accord sur tout le spectre entre résultats expérimentaux et prévisions du modèle théorique décrivant le laser en présence de rétroaction que nous avons développé.

La technique de l'injection optique a aussi été utilisée avec succès pour supprimer l'oscillation de relaxation : un bon accord entre théorie et expérience est observée dans ce cas.

Le meilleur résultat obtenu avec le microlaser  $\text{Nd:YVO}_4$  est de 7 dB d'excès de bruit à 40 kHz, en présence de rétroaction électro-optique. Les perspectives d'obtenir des réductions ultérieures passent par l'amélioration des performances des diodes de pompe. Notamment l'analyse théorique montre clairement la nécessité d'atteindre des taux de pompage bien plus élevés (un ordre de grandeur au moins) que ceux disponibles à présent. Une possibilité à explorer est constituée par l'emploi d'une diode de puissance (diode à ruban large) comme laser de pompe. La difficulté principale est ici représentée par le bruit de la diode de puissance : en général elle présente un fonctionnement multimode très bruyant. Nous avons envisagé de l'injecter avec une diode sur réseau pour réduire son bruit : les premiers essais sont encourageants. On obtient un fonctionnement monomode longitudinal, mais la présence de modes transverses limite les performances ; un meilleur recouvrement spatial entre faisceau maître et esclave devrait améliorer l'efficacité de l'injection et permettre d'atteindre un comportement monomode avec des effets bénéfiques sur le bruit d'intensité. Cependant,

il s'agit d'une tâche difficile à réaliser à cause de la qualité optique assez médiocre des faisceaux des diodes laser, et en particulier de la diode de puissance.

Enfin, nous avons présenté une application des diodes laser comprimés : la technique de la modulation de fréquence est utilisée sur une diode laser à bruit d'intensité réduit dans une mesure d'absorption en spectroscopie de haute sensibilité. Les résultats obtenus montrent que les techniques classiques de spectroscopie de haute sensibilité peuvent être transférées aux lasers à bruit comprimé, conduisant à une amélioration de la sensibilité maximale accessible à l'aide de lasers limités au bruit quantique standard.

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## Résumé

La thèse est consacrée à l'étude du bruit quantique dans les lasers à semiconducteur et à solide. Le principe de la suppression du bruit de pompe est utilisé avec différentes méthodes pour réduire le bruit d'intensité.

Les lasers semiconducteur à ruban ont été étudiés en utilisant différents types de techniques d'affinement spectral à température ambiante : diode sur réseau et injection optique. La compression de bruit mesurée est de 1.6 dB et 2.3 dB respectivement. La principale conclusion de cette étude est que le bruit d'intensité des diodes laser résulte de l'annulation entre les fluctuations fortement anticorrélées du mode principal et des faibles et nombreux modes longitudinaux. Les mesures effectuées à basse température montrent l'importance des corrélations quantiques entre modes de polarisation orthogonale.

L'étape suivante est l'étude des lasers semiconducteur à microcavité (VCSELS). Une compression de bruit de 0.7 dB sous le bruit quantique standard, résultant de fortes anticorrélations entre les modes transverses, a été observée dans un VCSEL multimode. Une étude approfondie de ces anticorrélations est effectuée analysant la distribution spatiale transverse du bruit d'intensité.

Un microlaser Nd:YVO<sub>4</sub> pompé par diode à bruit comprimé a aussi été étudié. Les effets du bruit de pompe sur le bruit d'intensité du microlaser sont clairement mis en évidence. La réalisation d'une boucle de rétroaction non standard sur la diode laser de pompe révèle l'existence d'effets non linéaires dans le spectre de bruit. La technique de l'injection optique a aussi été utilisée avec succès pour supprimer l'oscillation de relaxation.

Enfin, une application des diodes laser à bruit comprimé est présentée. La technique de la modulation de fréquence est employée avec des laser à bruit d'intensité sous le bruit quantique standard pour la détection de signaux d'absorption dans une expérience de spectroscopie de haute sensibilité.

**Mots-clés :** Réduction du bruit, Corrélations quantiques, Lasers à semiconducteur, Lasers à solide, Spectroscopie de haute sensibilité

## Abstract

The thesis focuses on the study of quantum noise in semiconductor and solid state lasers. The principle of pump noise suppression is employed together with different methods to reduce the intensity noise.

First, edge emitting laser diodes were investigated using different types of line narrowing techniques at room temperature: feedback from an external grating and injection locking. The measured intensity squeezing is 1.6 dB and 2.3 dB respectively. The intensity noise of the laser diodes is shown to result from a cancellation between very large anticorrelated fluctuations of the main mode and of many weak longitudinal side modes. Measurements performed at low temperature point to the relevance of quantum correlations between orthogonal polarised modes.

The next step was the investigation of vertical cavity semiconductor lasers (VCSELS). Sub shot noise operation (0.7 dB) is observed in a multimode VCSEL, resulting from very strong anticorrelations between transverse modes. A detailed study of these anticorrelations is performed through the analysis of the transverse spatial distribution of the intensity noise.

A Nd:YVO<sub>4</sub> microchip laser pumped by a squeezed laser diode was also studied. The effects of the pump noise on the intensity noise of the microchip laser are clearly shown. The implementation of a non-standard feedback loop on the pump laser diode shows evidence for non linear effects in the noise spectrum. Injection locking technique is also successfully employed to suppress the relaxation oscillation noise peak.

Finally, an application of intensity squeezed laser diodes is presented. Frequency modulation technique together with sub shot noise laser diodes is employed for detection of absorption signals in a high sensitivity spectroscopy experiment.

**Key words:** Squeezing, Quantum correlations, Semiconductor lasers, Solid state lasers, High sensitivity spectroscopy