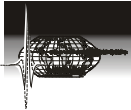
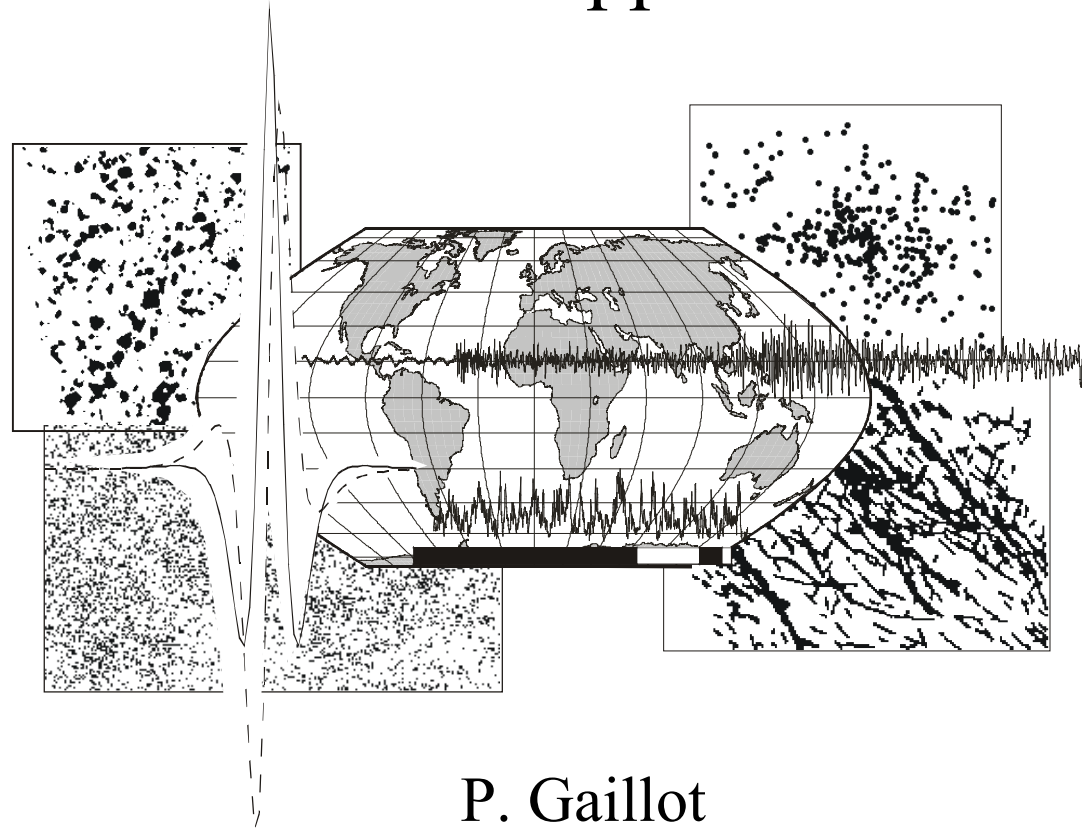


Continuous Wavelets in Earth Sciences - Methods and Applications -





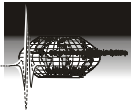
Outline

- Issue
- Formalism

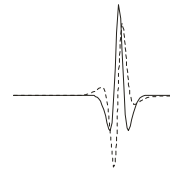
- ① - Relative geomagnetic paleointensity
- Automatic picking of seismic phases

- ② - Spatial distribution of seismic events
- Analysis of mineral organizations of magmatic rocks

- Conclusion



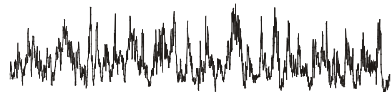
Issue



Geophysical - Geological signals

Non stationary behavior
Multi-scale components
singularities

- Indirect measurements (proxy)



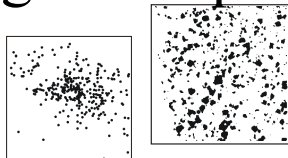
Geomagnetic paleointensity

- Noise - Spike

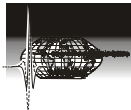


Seismograms

- Integrative quantification



Spatial distributions

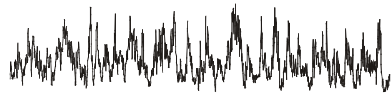


Issue

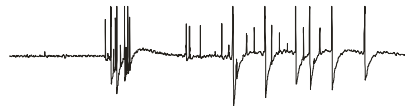
Geophysical - Geological signals

Non stationary behavior
Multi-scale components
singularities

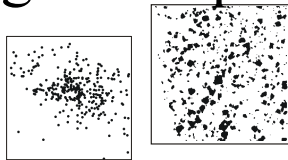
- Indirect measurements (proxy)



- Noise - Spike



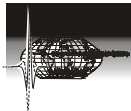
- Integrative quantification



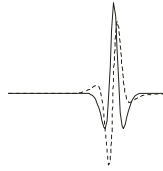
Main objectives:

- extract relevant features
- reliable

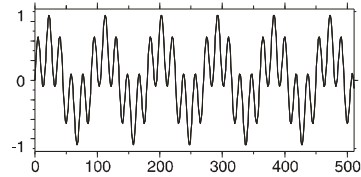
description
quantification



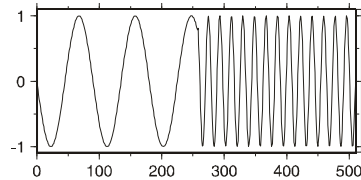
Limits of classical representations



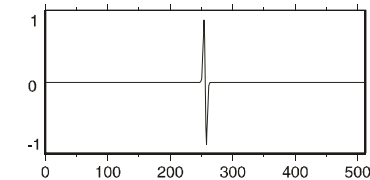
Stationary



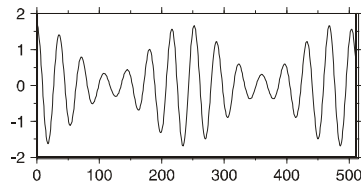
Frequency change



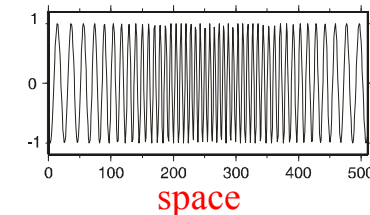
Abrupt time change



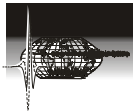
Amplitude modulation



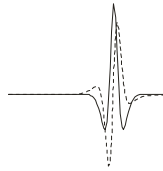
Frequency modulation



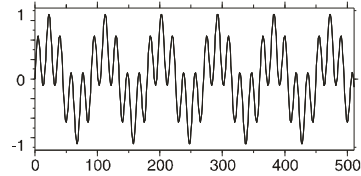
Statistical properties evolve in space



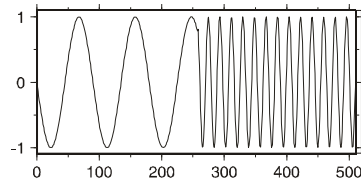
Limits of classical representations



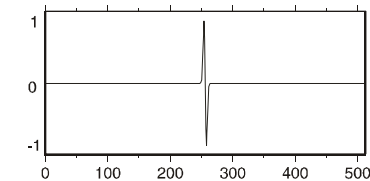
Stationary



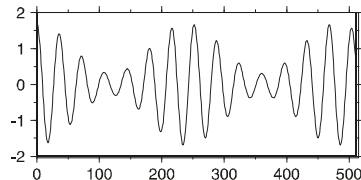
Frequency change



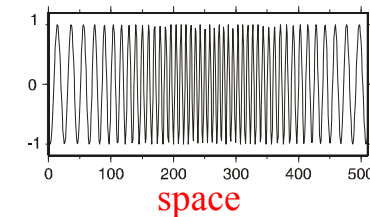
Abrupt time change



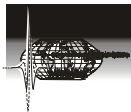
Amplitude modulation



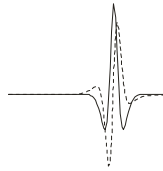
Frequency modulation



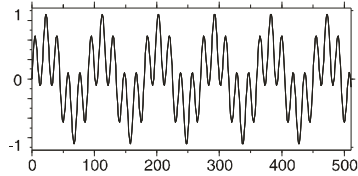
Catastrophic event having a long term impact



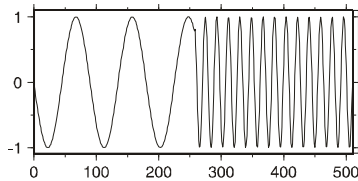
Limits of classical representations



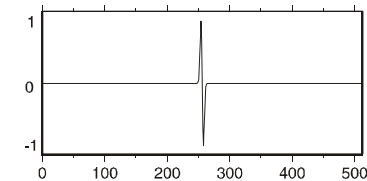
Stationary



Frequency change

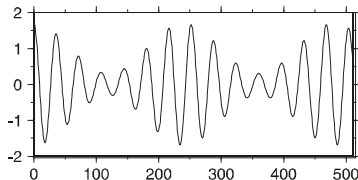


Abrupt time change

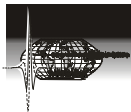
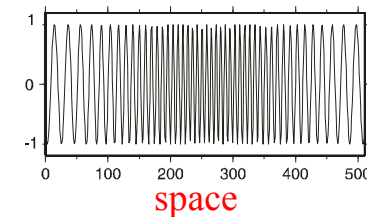


Catastrophic event having a short term effect

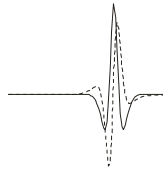
Amplitude modulation



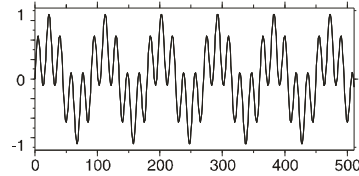
Frequency modulation



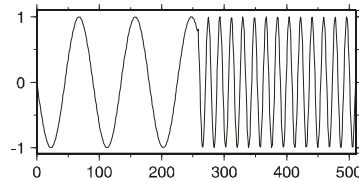
Limits of classical representations



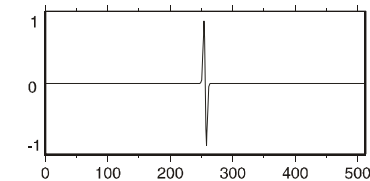
Stationary



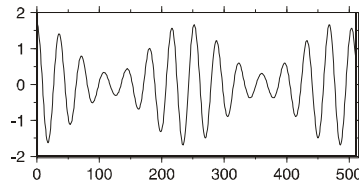
Frequency change



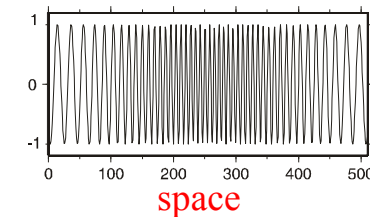
Abrupt time change



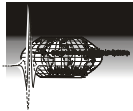
Amplitude modulation



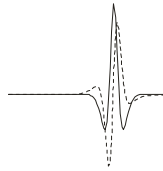
Frequency modulation



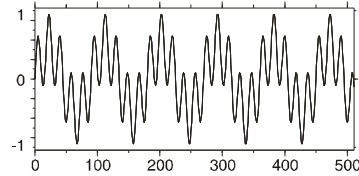
Non linear interactions between different scales
Interferences of frequency component from its sidebands



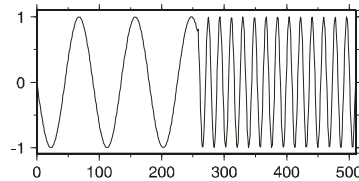
Limits of classical representations



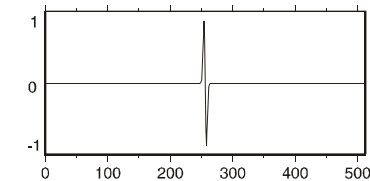
Stationary



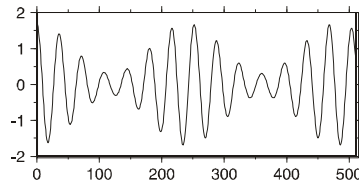
Frequency change



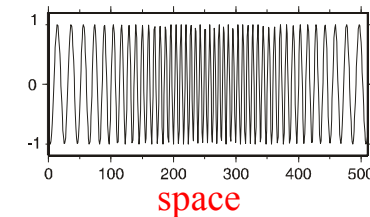
Abrupt time change



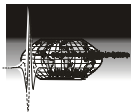
Amplitude modulation



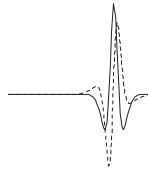
Frequency modulation



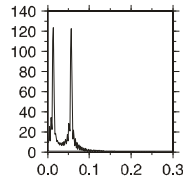
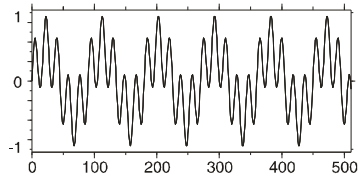
Instable systems with evolving fundamental frequencies



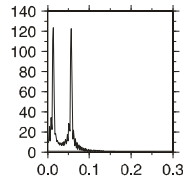
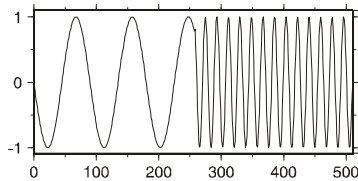
Limits of classical representations



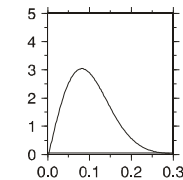
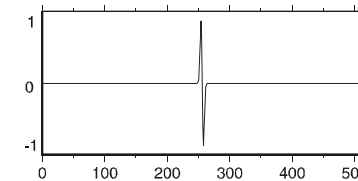
Stationary



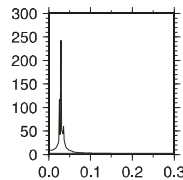
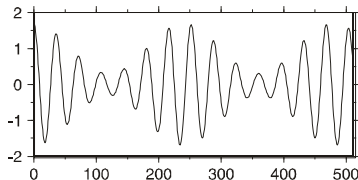
Frequency change



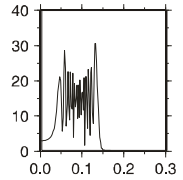
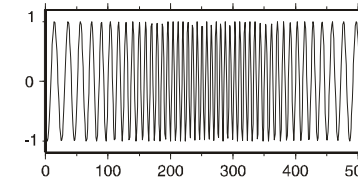
Abrupt time change



Amplitude modulation



Frequency modulation



space

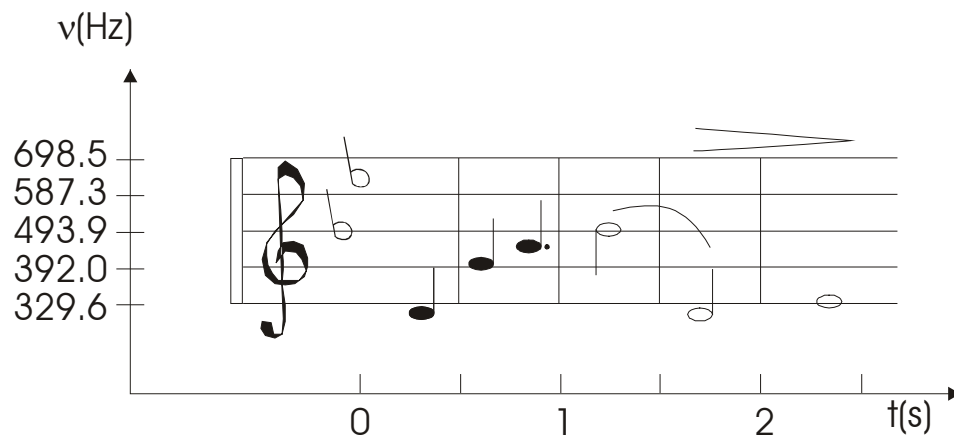
frequency

Space
Frequency | representations

NEW REPRESENTATION ...



New representation

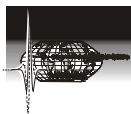


2 parameters : time - frequency

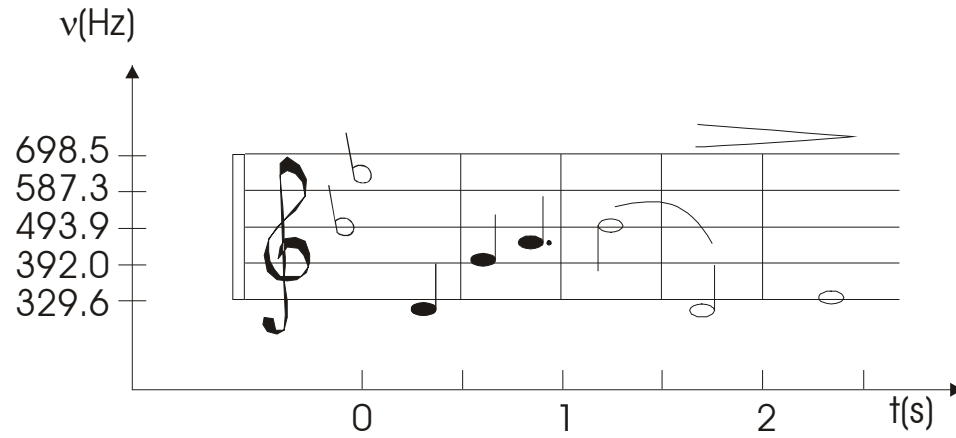
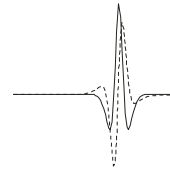
Beginning - End - Duration → Localization

Frequency → Multi-scale

Dynamic

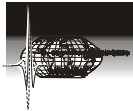


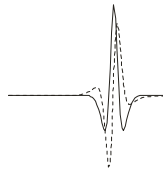
Formalism



- Local multi-scale analysis
- without any hypothesis

Wavelet Transform



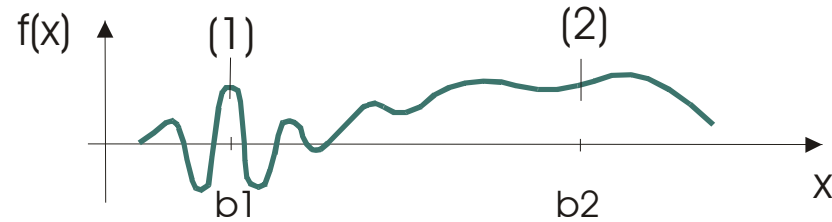


Continuous Wavelet Transform

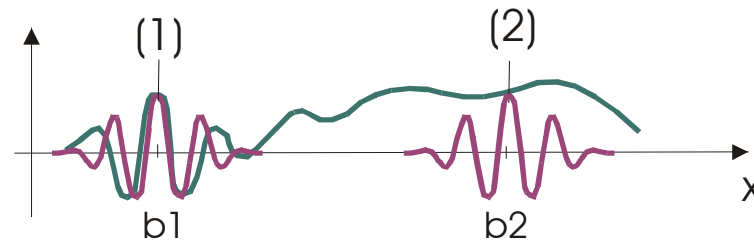
$$C(a, b) = \Psi(a, b) \otimes f(x)$$

a: scale parameter

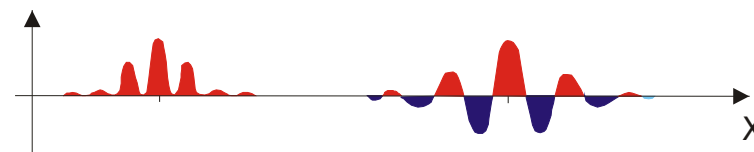
b: translation parameter



① Multiplication by the wavelet located in b

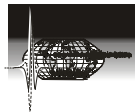
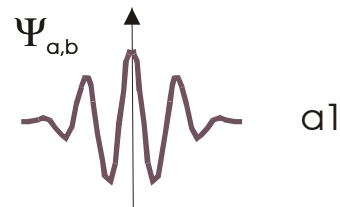


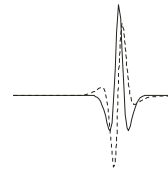
② Computation of the Product signal



High $C(a, b)$

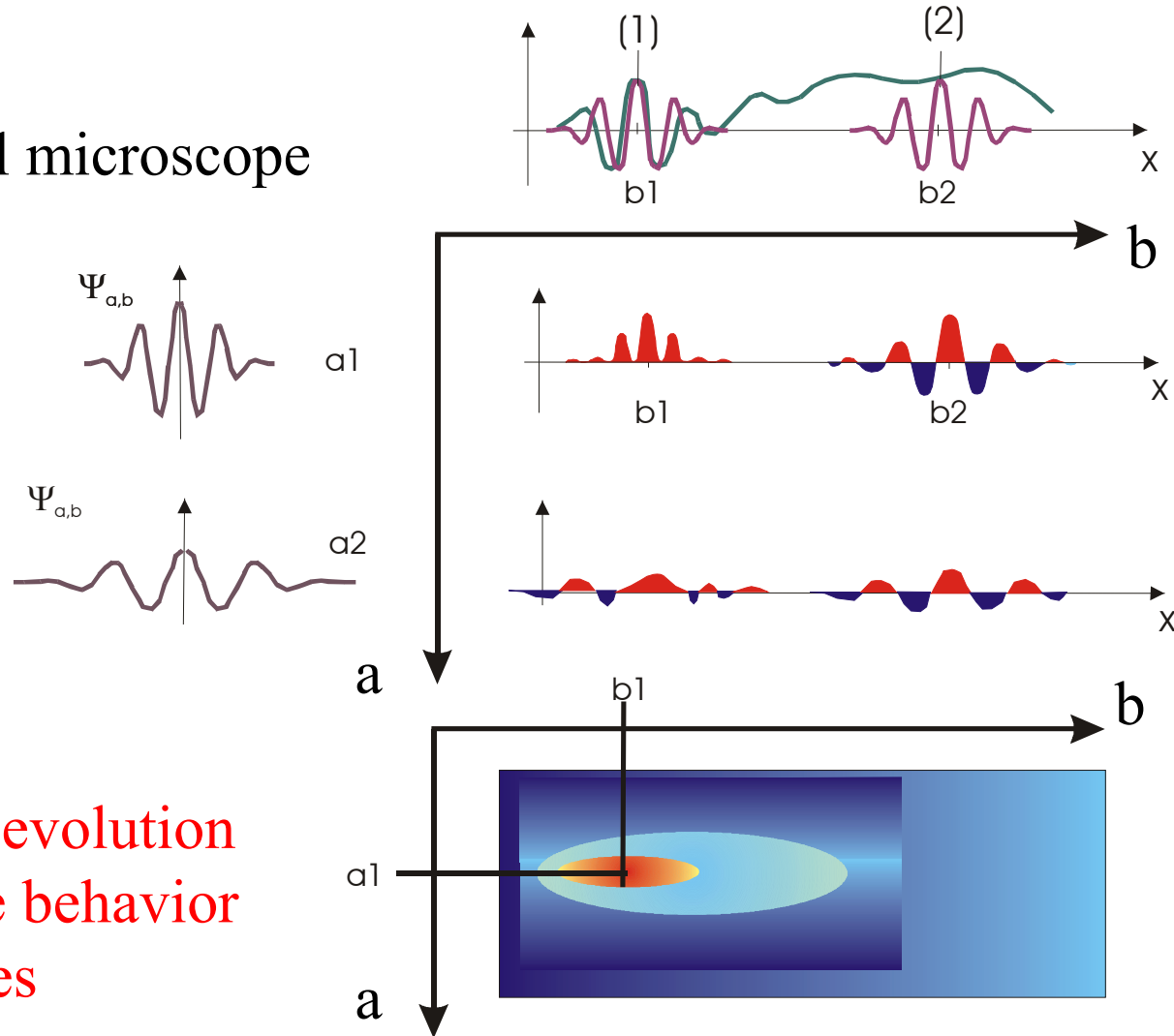
Low $C(a, b)$



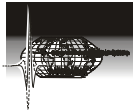


Continuous Wavelet Transform

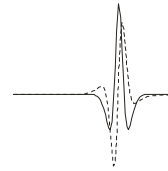
Mathematical microscope



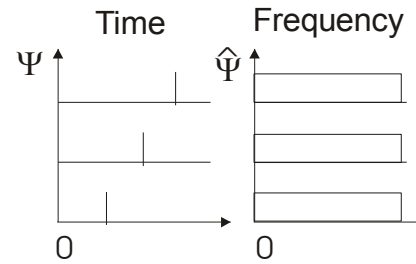
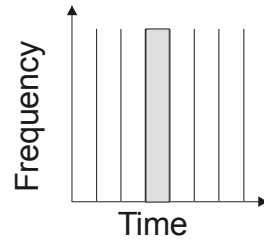
- Frequency evolution
- Multi-scale behavior
- Singularities



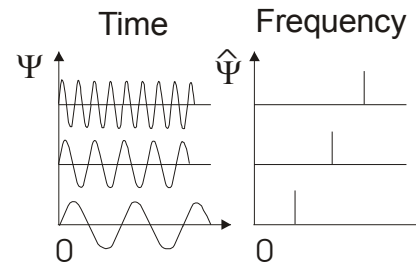
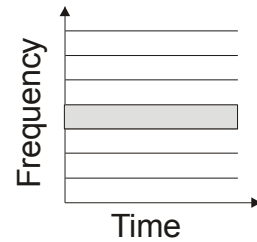
Time -frequency windows



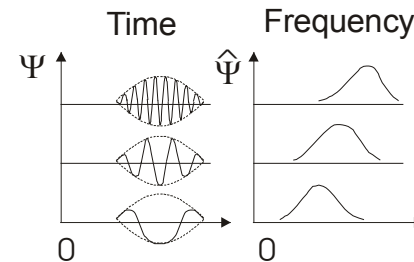
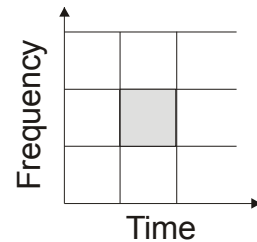
Temporal



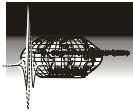
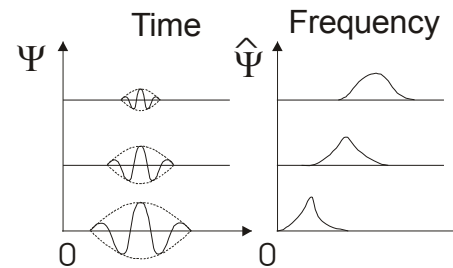
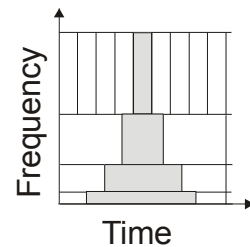
Frequency (FT)

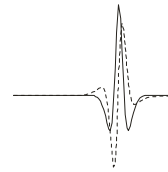


WFT



WT





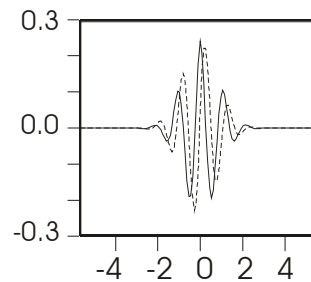
Mother functions

Compact support
Null mean

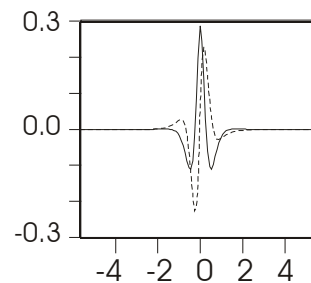
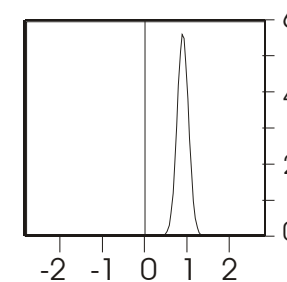
Domain

Space

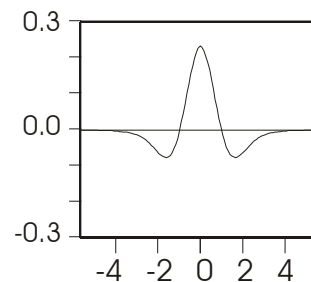
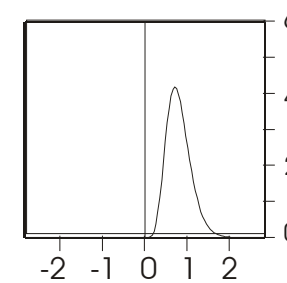
frequency



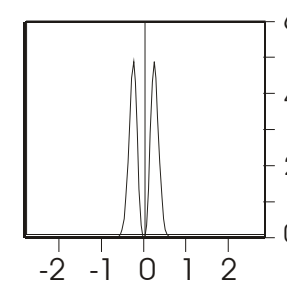
Morlet
 $\omega_0 = 5$



Paul
 $m = 4$



DOG
 $m = 2$



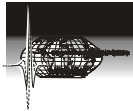
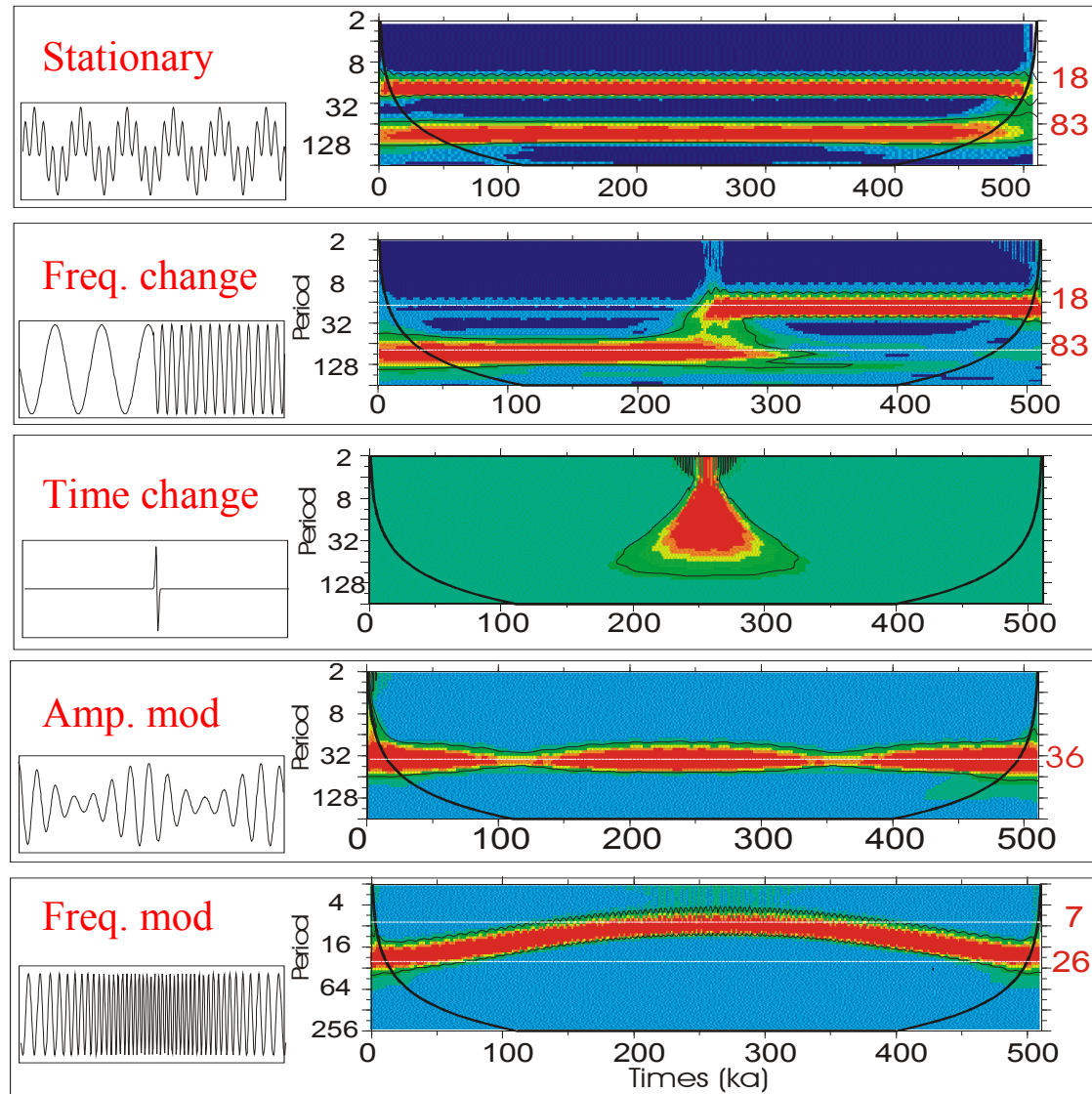
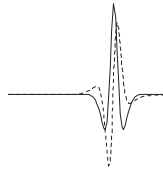
Environmental
applications

Seismological
applications

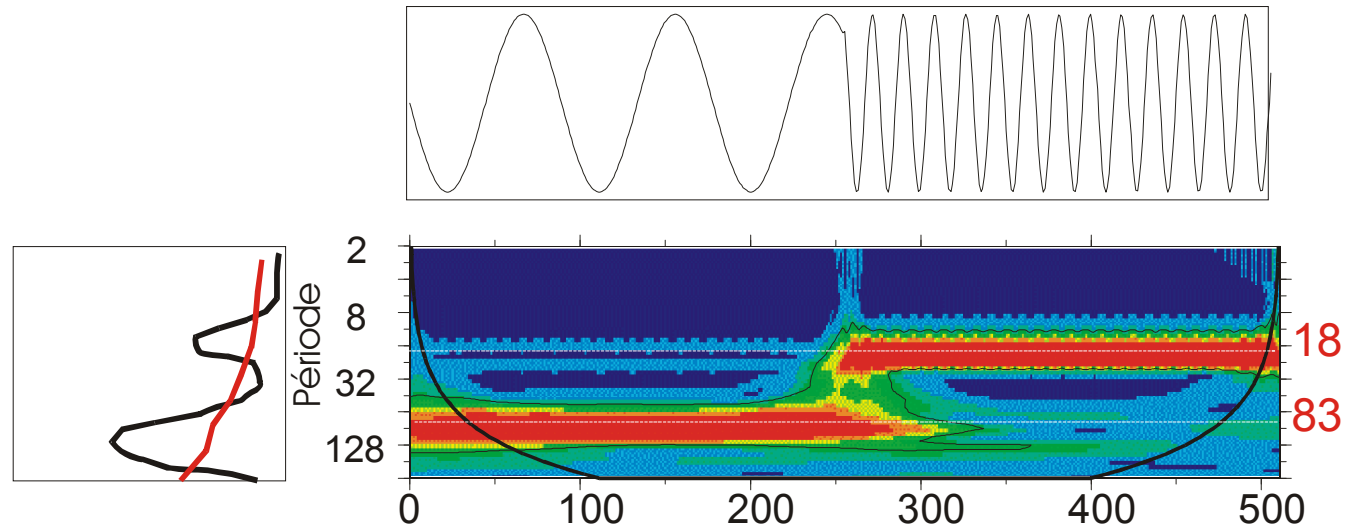
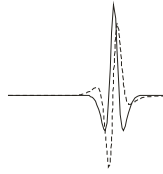
2D
applications



Illustration



Statistical significance test



Background power spectrum
 \times significance level χ^2

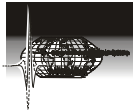
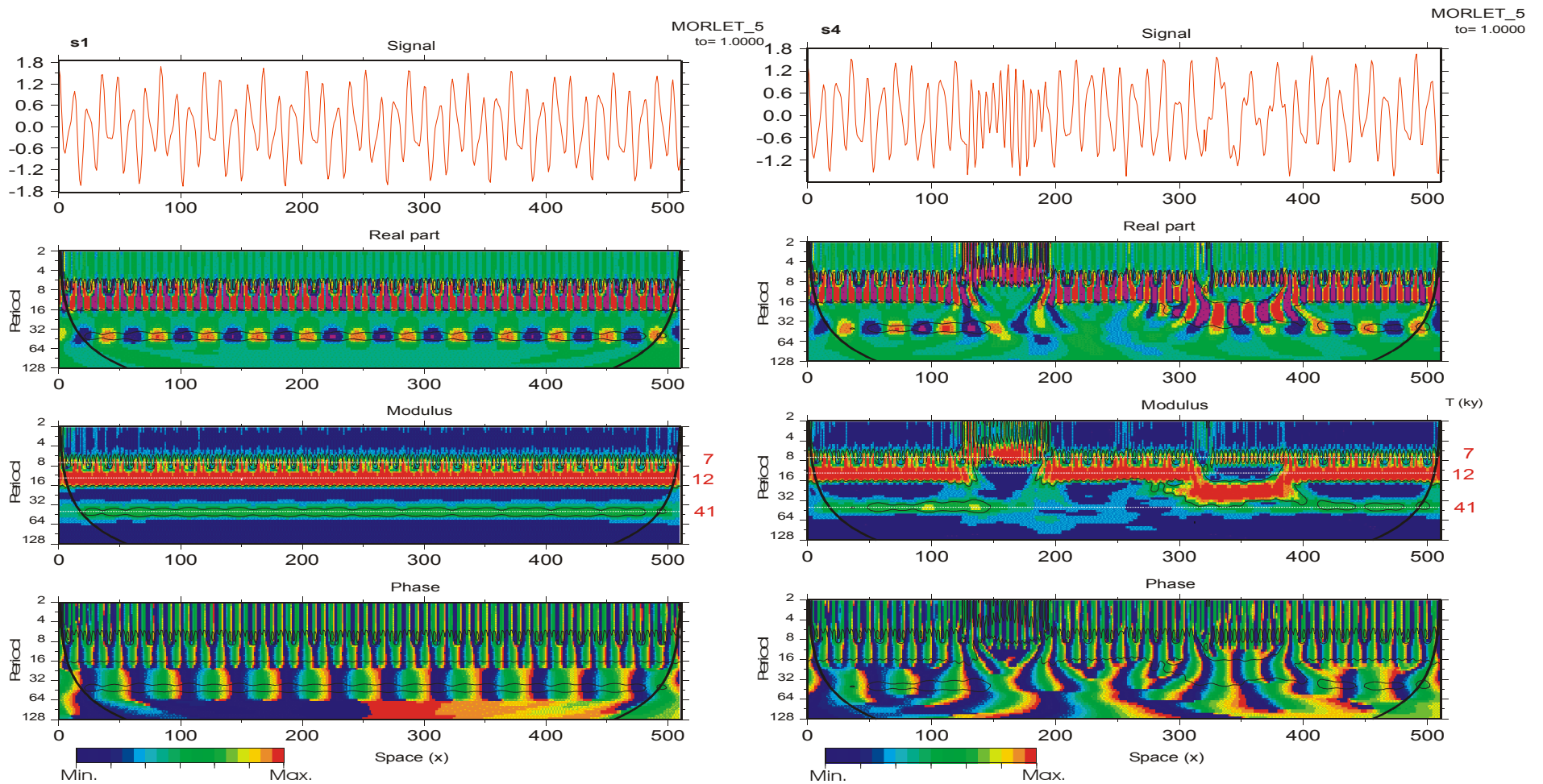
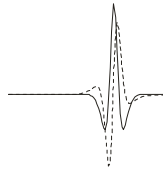


local spectrum

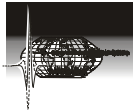
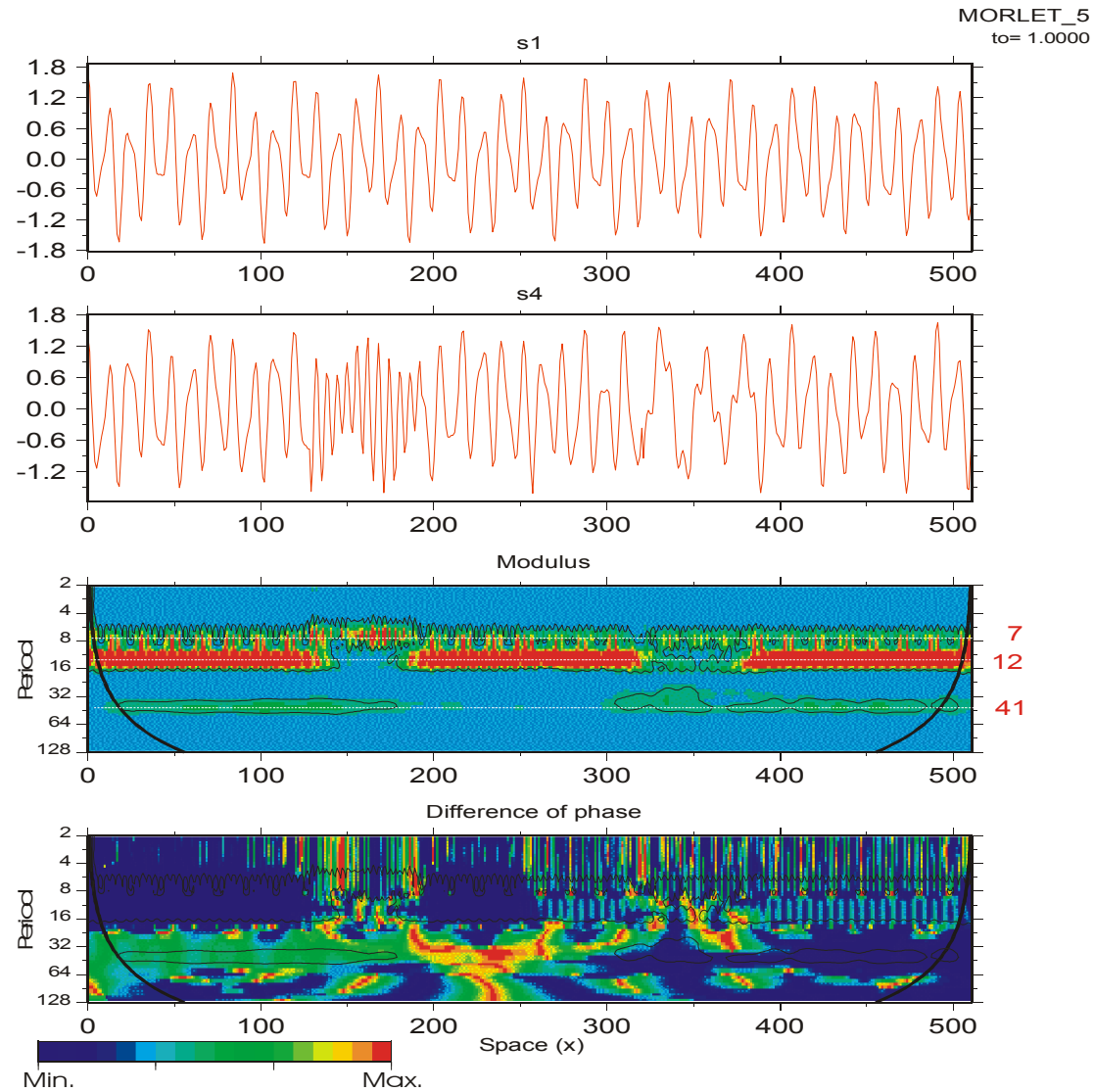
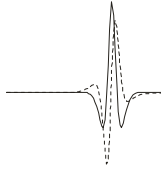
Torrence and Compo, 1998

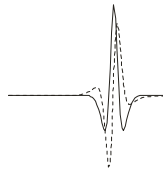


Cross Wavelet Spectrum



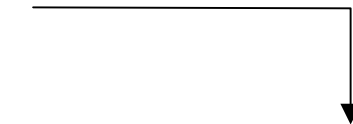
Cross Wavelet spectrum





Summary

Wavelets



local and multi-scale analysis

no hypothesis on the nature of the analyzed signal

space-scale representation

dynamics

multi-scale behavior



Processes

